Exact solution of a stochastic model of rain-induced shallow landslides in hollows

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ABSTRACT: D’Odorico & Fagherazzi (2003) have proposed a probabilistic model of rainfall-triggered shallow landslides in hollows. Their model describes the long-term evolution of colluvial deposits through a probabilistic soil mass balance at a point. The model accounts for hollow infilling, expressed as a deterministic function of the deposit thickness, and soil erosion by shallow landslides, modeled as a time-dependent stochastic point process related to the occurrence of triggering precipitation events. Further building blocks of the model are: an infinite-slope stability analysis; a steady-state kinematic wave model of hollow groundwater hydrology; and a statistical model relating intensity, duration, and frequency of extreme precipitation. The authors provide an analytical solution to their model under the simplifying assumption that the occurrence rate of landslide-triggering rain events is independent of the colluvial deposit thickness. We present an exact solution to the stochastic landslide model for the general case where the triggering rain event occurrence rate depends on the soil thickness and hence on time.

1 INTRODUCTION

The stochastic properties of rain-induced shallow landslides in topographic hollows are controlled by the probability of occurrence of extreme precipitation events and the hollow hydrological response to such events. The objective of this paper is to establish explicit relations for the coupling between the intensity-duration-frequency (IDF) curves characterizing the regime of extreme (i.e. landslide-triggering) precipitation and the probability distributions of scar depth (i.e. colluvium thickness when landslides occur), landslide return period (recurrence interval), and colluvium thickness in topographic hollows through a simple model of hollow hydrological reponse to landslide-triggering precipitation.

The starting point of our analysis is the probabilistic model of rainfall-triggered shallow landslides in hollows proposed by D’Odorico & Fagherazzi (2003) and D’Odorico et al. (2005), which is related to the stochastic landslide model of Iida (1999, 2004). Their model describes the long-term evolution of colluvial deposits through a probabilistic soil mass balance at a point (D’Odorico 2000; D’Odorico et al. 2001). The model accounts for hollow infilling, expressed as a deterministic function of the deposit thickness, and soil erosion by shallow landslides, modeled as a time-dependent stochastic point process related to the occurrence of triggering precipitation events. Further building blocks of the model are: an infinite-slope stability analysis; a steady-state kinematic wave model of hollow groundwater hydrology; and a statistical model relating intensity, duration, and frequency of extreme precipitation. These are common assumptions in landslide modeling (e.g. Dietrich et al. 1986; Montgomery & Dietrich 1994; Wu & Sidle 1995; Montgomery et al. 1997; Hennrich & Crozier 2004; Sidle & Onda 2004; Rosso et al. 2006; Talebi et al. 2007, 2008a,b,c).

D’Odorico & Fagherazzi (2003) provide an analytical solution to their model under the simplifying assumption that the occurrence rate of landslide-triggering rain events is independent of the colluvial deposit thickness. We present exact solutions to the stochastic landslide model for the general case where the triggering rain event occurrence rate depends on the soil thickness and hence on time.

2 METHODOLOGY

Following D’Odorico & Fagherazzi (2003), the modeling approach adopted here is to establish a stochastic soil mass balance at a point, describing the long-term evolution of colluvial deposits in topographic hollows. The modeled soil mass balance is affected by two counteracting processes: (1) a *deterministic* model of hollow infilling (accretion) by transport of soil and/or debris from surrounding hillslopes; (2) a *probabilistic* model of soil erosion (denudation) through shallow landslides triggered by extreme precipitation. The main elements of the stochastic model can be summarized as follows: (1) an infinite planar slope stability analysis (Fig. 1); (2) a steady-state kinematic wave model of hillslope hydrology; (3) a statistical model relating intensity, duration and frequency of extreme precipitation based on the Gumbel extreme value distribution; (4) the growth of colluvial deposits in hollows is assumed to occur only due to transport of soil from uphill, not from physical weathering of the underlying bedrock; (5) landslides are assumed to scour hollows to the bedrock.

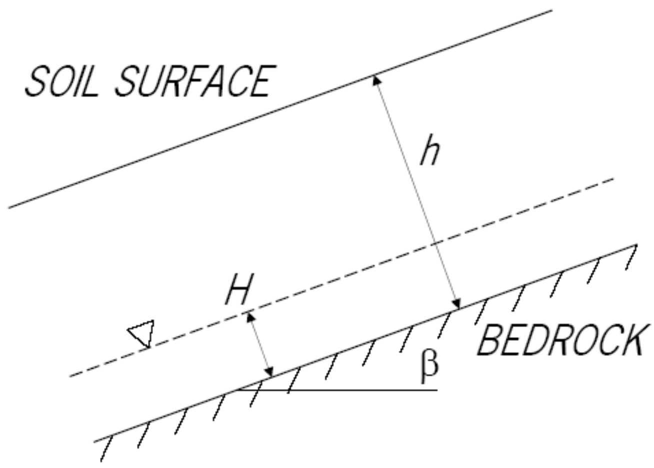

Figure 1. Schematic representation of soil-mantled slope; h = soil thickness, H = saturated water depth, β = hollow slope angle (after D'Odorico & Fagherazzi, 2003).

Following D'Odorico & Fagherazzi (2003), the geometry of the hollow model is defined by a tipped triangular trough and planar side slopes (Fig. 2).

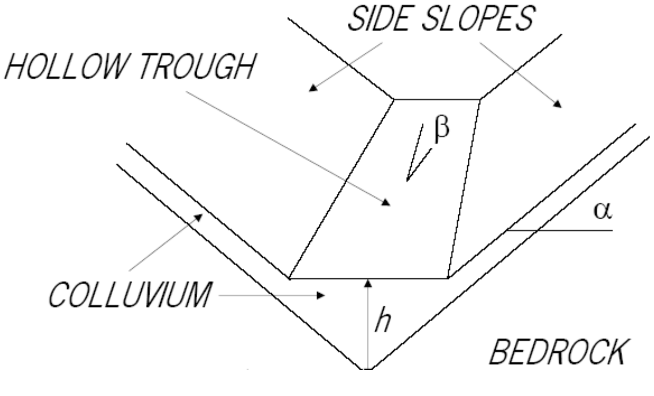

Figure 2. Schematic representation of hollow geometry; h = soil thickness, α = side slope angle, β = hollow slope angle (after D'Odorico & Fagherazzi, 2003).

Simulations of the temporal evolution of soil thickness in hollows using the stochastic landslide model are carried out in a Monte Carlo framework. First, the topographical, hydraulic and geotechnical properties of the hollow are defined. Based on these, the concentration time (T_c) of the hollow is determined. Subsequently, the parameters of the Gumbel distribution for extreme rainfall of duration T_c are determined. Assuming soil thickness to be zero at the start of the simulation, for each year: (1) the saturated water depth (H) able to trigger landslides is computed on the basis of an infinite slope stability analysis; (2) the rain rate (R) required to produce H is computed, assuming a steady-state kinematic wave hillslope hydrology; (3) the probability (P) that R is exceeded in a given year is computed from the Gumbel extreme value distribution parameters; (4) a random number is drawn and compared to P to determine if a landslide occurs; (5) if a landslide-triggering storm occurs, it scours the hollow entirely; (6) if such a storm does not occur, the soil thickness (h) is increased by transport from uphill.

On the basis of this algorithm we have been able to derive the exact probability density functions of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness. The mathematical expressions for these functions are quite tedious and consequently their derivation will be reported elsewhere. However, in the following section we will graphically compare our exact analytical solution of the presented stochastic landslide model both with the results of Monte Carlo simulations and with an approximate analytical solution of the model.

3 RESULTS

On the basis of the Gumbel parameters provided by D'Odorico & Fagherazzi (2003) for four hollows with different concentration times in the Oregon coastal range, we have derived an explicit (power-law) relation between the expected annual maximum rain rate and its duration (Fig. 3).

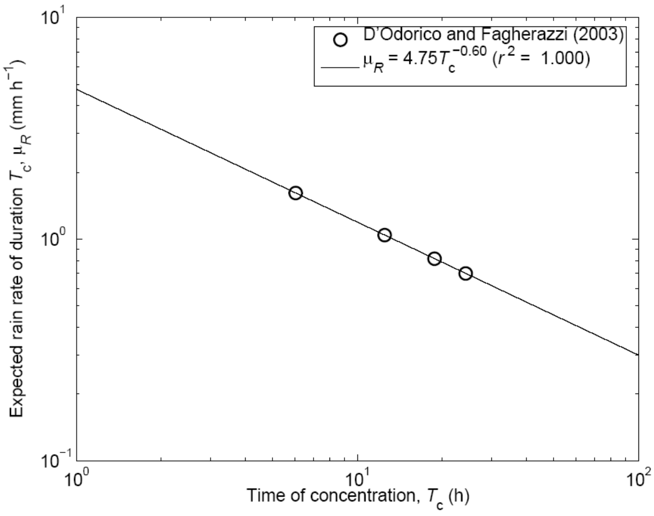

Figure 3. Power-law relation (solid line) between expected annual maximum rain rate (μ_R) and its duration (T_c), fitted to Gumbel extreme value distribution parameters for four hollows with different concentration times (circles), derived from rainfall data from Alleghany, Oregon (1951–2000).

The topographical, hydraulic and geotechnical properties of the four hollows considered have been taken from Dietrich et al. (1986), Montgomery & Dietrich (1994), and Montgomery et al. (1997). Results of our Monte Carlo simulations of landslide occurrence in the hollows and comparisons of the empirical (based on 1000 simulated landslides) and exact probability density functions of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness are provided in Figures 4-7.

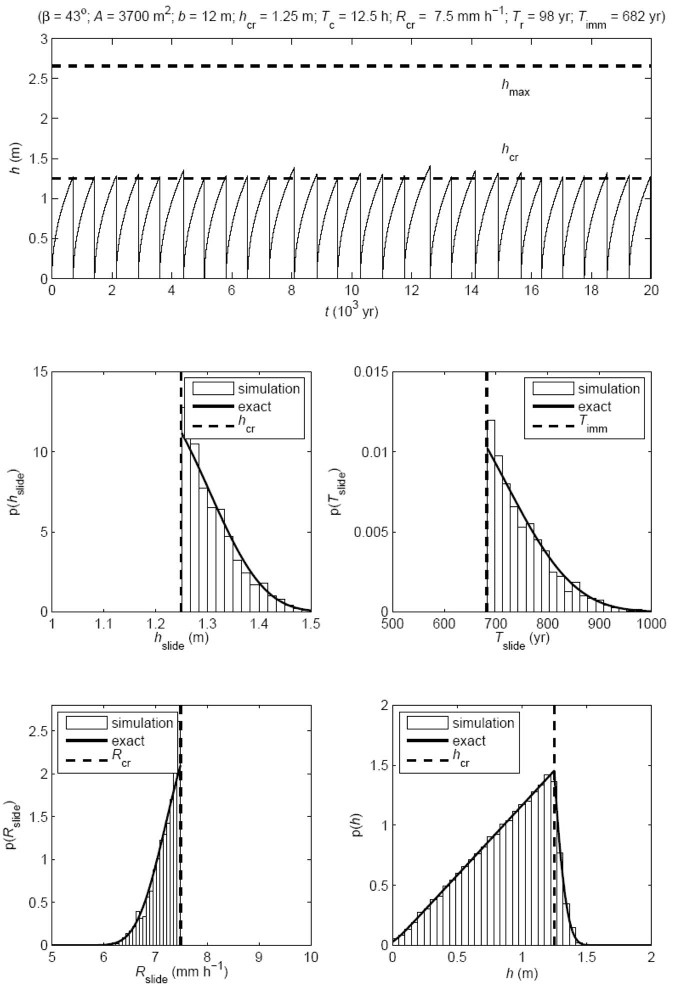

Figure 4. Results for steep hollow, supply-limited regime. (top panel) Monte Carlo simulations of landslide occurrence in hollow (β = hollow slope angle; A = drainage area; b = outlet width; h_{cr} = immunity depth, i.e. soil depth below which hollow is unconditionally stable; T_c = concentration time; R_{cr} = rain rate saturating h_{cr}; T_r = return period of R_{cr}; T_{imm} = immunity period, i.e. time needed to accumulate h_{cr}; $h_{\max}$ = colluvium thickness above which hollow is always unstable). (lower panels) Comparisons of empirical (bars) and exact probability density functions (solid lines) of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness.

In terms of hollow slope, we can distinguish between *steep* hollows (Figs 4, 5), where the hollow slope (β) exceeds the angle of internal friction (φ, soil repose angle), and *gentle* hollows (Figs 6, 7), where β < φ. In terms of landslide occurrence regime, we can distinguish between the *supply-limited* regime (Figs 4, 6), where the immunity period (T_{imm}) greatly exceeds the return period of landslide-triggering rain rates (T_r), and the *event-limited* regime (Figs 5, 7), where $T_r \gg T_{imm}$. In the former case, the occurrence of landslides is controled by the supply rate of soil from uphill, whereas in the latter case the occurrence of landslides is controled by the occurrence probability of extreme (landslide-triggering) rain events.

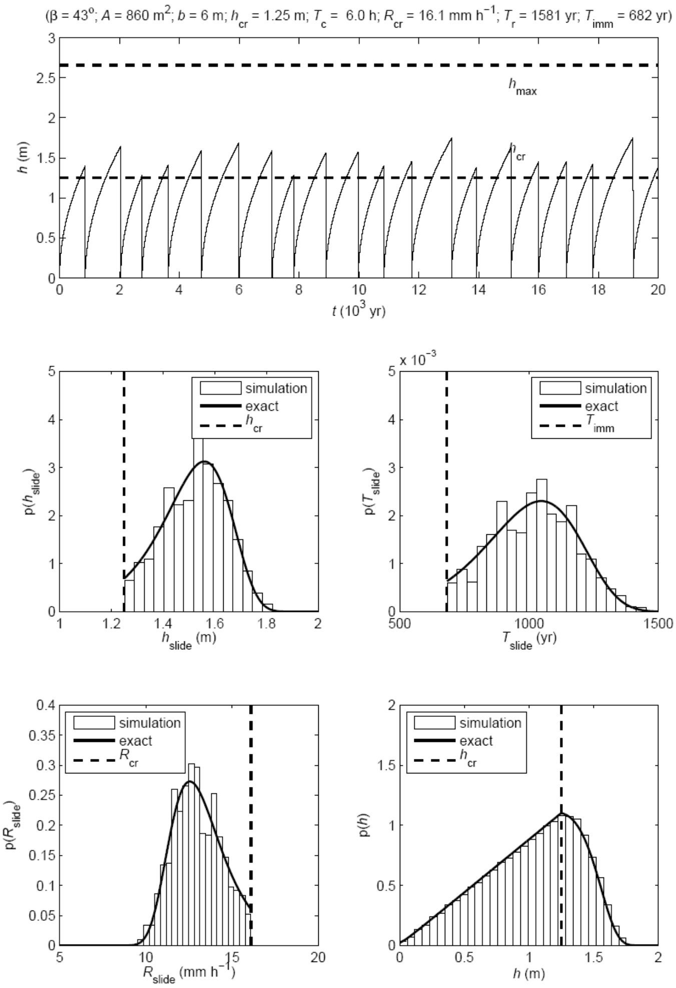

Figure 5. Results for steep hollow, event-limited regime.

There are pronounced differences between the probability density functions of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness for the four different cases considered. The immunity depth (h_{cr}) is the colluvium thickness below which a hollow is always (unconditionally) stable. The maximum depth ($h_{\max}$), on the other hand, is the colluvium thickness above which a hollow is always (unconditionally) unstable. For steep hollows in the supply-limited regime, landslides always occur shortly after the immunity depth has been exceeded (Fig. 4). For steep hollows in the event-limited regime, this may take a while (Fig. 5). For such hollows, the larger the colluvium thickness, the smaller the rain rate required to trigger a landslide. For gentle hollows, on the other hand, the larger the accumulated soil depth, the larger the rain rate needed to produce a landslide. In the supply-limited regime, landslides still occur relatively shortly after the immunity depth has been exceeded (Fig. 6). However, if $T_r \approx T_{imm}$, there is a transition from the event-limited regime to an unconditionally stable state (Fig. 7). In this case, the probability density functions are undefined (D'Odorico & Fagherazzi 2003). For all cases considered, our exact probability density functions of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness closely follow the empirical histograms based on 1000 simulated landslides.

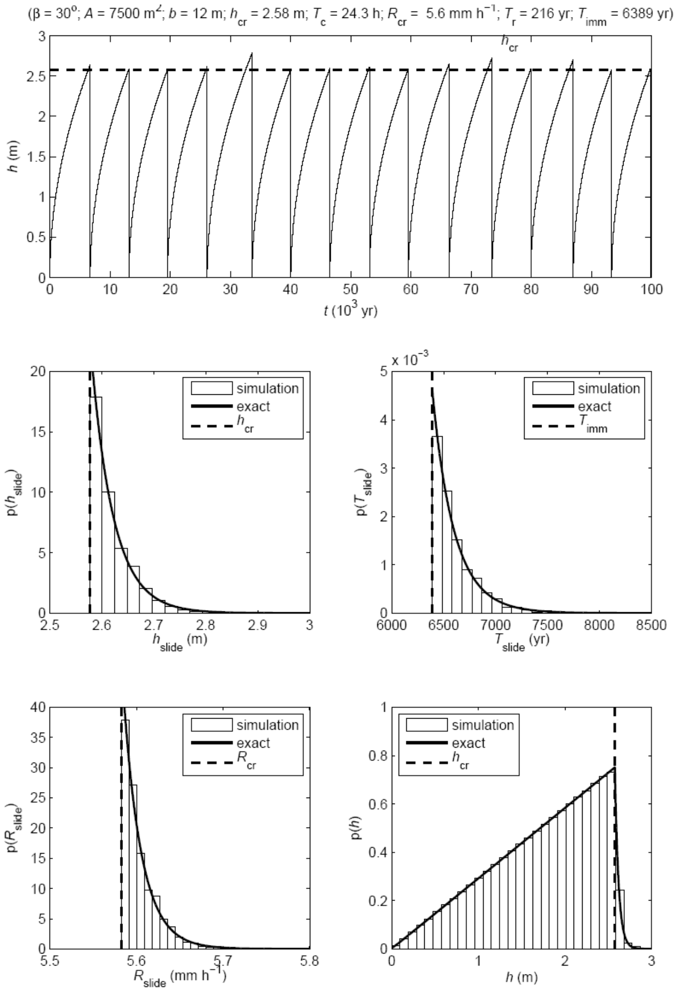

Figure 6. Results for gentle hollow, supply-limited regime.

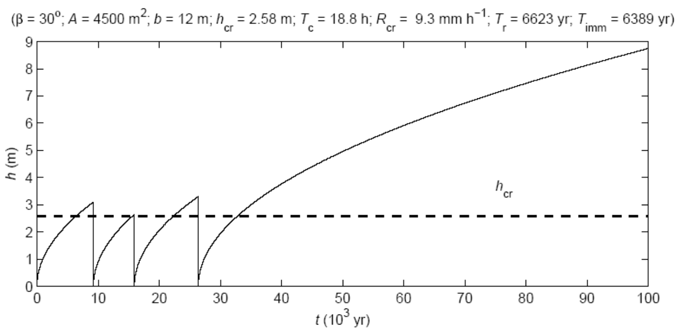

Figure 7. Results for gentle hollow, event-limited regime.

D'Odorico & Fagherazzi (2003) provide an analytical solution for the probability density function of soil thickness in their stochastic landslide model under the simplifying assumption that the occurrence rate of landslide-triggering rain events is independent of the colluvial deposit thickness. This condition is met in the supply-limited regime, where landslides occur shortly after the immunity depth has been exceeded. D'Odorico & Fagherazzi (2003) state that 'in the event-limited condition, the analytical solution cannot be applied since [the occurrence rate of landslide-triggering rain events] strongly depends on the soil thickness'. We have been able to derive an exact solution to the presented stochastic landslide model for the general case where the triggering rain event occurrence rate does depend on the soil thickness and hence on time. The analytical solution of D'Odorico & Fagherazzi (2003) for the supply-limited regime can be shown to be a limiting case of our general solution.

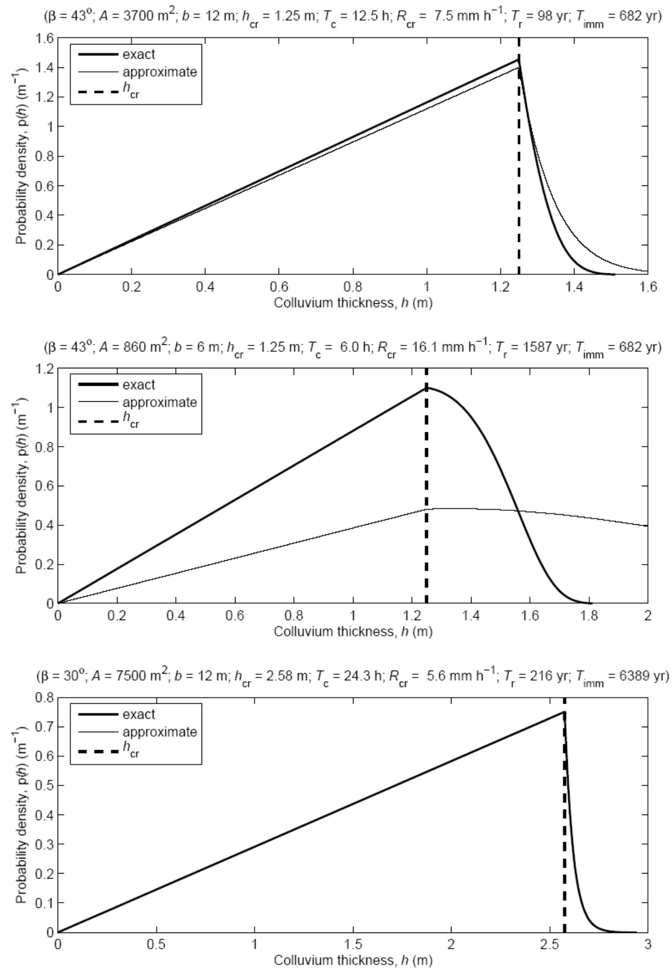

Figure 8. Comparison of exact general solution for probability density function of soil thickness (bold curves) with approximate analytical solution of D'Odorico & Fagherazzi (2003) for supply-limited regime (thin curves) for hollow geometries of Figs. 4-6, respectively (dashed vertical lines indicate immunity depths).

Figure 8 shows, for the three hollow geometries of Figures 4-6, comparisons of our exact general solution for the probability density function of soil thickness with the approximate analytical solution of D'Odorico & Fagherazzi (2003) for the supply-limited regime. For the steep hollow in the supply-limited regime (top panel, corresponding to Fig. 4), the approximate analytical solution follows the exact general solution reasonably well. For the steep hollow in the event-limited regime (middle panel, corresponding to Fig. 5), however, the analytical solution of D'Odorico & Fagherazzi (2003) is clearly not a good approximation to our exact general solution. Finally, for the gentle hollow in the supply-limited regime (bottom panel, corresponding to Fig. 6), the approximate solution does a very good job and cannot be distinguished from the exact solution. This is not surprising, because in this limiting case the assumption on which the analytical solution of D'Odorico & Fagherazzi (2003) has been based (i.e. the independence of the occurrence rate of landslide-triggering rain events from the soil thickness) is met almost perfectly.

4 CONCLUSIONS

We have presented an exact solution to the probability density function of soil thickness related to the stochastic rain-induced shallow landslide model of D'Odorico & Fagherazzi (2003) for general case where the landslide-triggering rain event occurrence rate is *dependent* on soil thickness and hence on time. This generalizes the analytical solution provided by authors for the restrictive case where the triggering rainfall is *independent* of colluvial deposit thickness.

We have also performed Monte Carlo simulations using the stochastic landslide model for four hollows in the Oregon coastal range with different concentration times. For all cases considered, our exact probability density functions of landslide depth, landslide return period, landslide-triggering rain rate, and colluvium thickness closely follow the empirical histograms based on 1000 simulated landslides.

Acknowledgments This research is partly funded by the European Commission through projects HYDRATE (FP6) and IMPRINTS (FP7).

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