n of field water use
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R.A.Feddes, P.J.Kowalik and H.Zaradny


## Simulation of field water use and crop yield

## Simulation Monographs

Simulation monographs is a series on computer simulation in agriculture and its supporting sciences

# Simulation of field water use and crop yield R.A.Feddes, P.J.J.Kowalik and H.Zaradny 



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## 1 Introduction

Crop production is generally determined by the prevailing environmental conditions, i.e. by the existing complex of physical, chemical and biological factors. This study concentrates on the effects of the physical environment on plant development, with special regard to aspects of soil physics and meteorology. The main aim is to estimate field water use and crop yield under limiting water conditions.

The amount of water available for transpiration strongly influences dry matter production. Actual transpiration depends not only on the weather but also on physical properties of the soil and on factors related to the type of crop. Therefore attention is paid to the effects of soil physical properties and, in connection with capillary rise, to the influence of the groundwater table on the amount of water available for transpiration.

A calculation of actual transpiration from meteorological, crop and soil properties is of value when designing water management projects under a given climate. The relationship between crop production and transpiration indicates whether supplemental irrigation is justified or not.

As the book probably will be read by scientists, field workers and students of various disciplines, the material is presented step by step so that it is easier to understand.

Two models are presented which can be used either separately or conjointly. The first model, program SWATR, calculates the actual transpiration of a crop. The second model, program CROPR, calculates the actual yield of a crop. Some main differences with other models published are:

- more emphasis on soil physical aspects;
- application to heterogeneous soil systems;
- use of a new function to account for water uptake by roots;
- consideration of the depth of the groundwater table fluctuating with time;
- micrometeorological data on a daily basis taken from field experiments having a practical background;
- a different approach to the boundary condition at the soil surface;
- main interest in total dry matter production rather than in plant phenological stages;
- the crop to be supposed optimally supplied with nutrients, with soil moisture as the main factor limiting growth.

An illustration of the approach is given in Fig. 1. It shows the flow patterns and the action of various factors in the soil-plantatmosphere system.
In this monograph, the underlying theory is given in the first two parts: 'Theory of field water use' and 'Theory of crop production'. The third part: 'The programs', gives the main features of SWATR and CROPR as well as experimental verifications and complete descriptions of the two programs.
In Chapter 2 the principles of the energy status of soil water and


Fig. 1. Flow chart of the integrated model approach for computing the influence of water use on crop yield.
the flow of water in the unsaturated zone are treated.
Chapter 3 deals with the water uptake by roots described by a sink term depending on soil moisture pressure head, rooting depth and potential transpiration. The latter sets an external limit for the root water uptake. Both potential transpiration and soil evaporation are estimated from a combined energy balance-vapour transport approach, based on rather easily measurable meteorological and crop quantities.

The initial and boundary conditions to be applied to the flow equations in the soil-root system are presented in Chapter 4. A one-dimensional numerical (finite difference) scheme approximating the flow in this system is derived and evaluated.

Chapter 5 is devoted to a mathematical description of plant growth resulting in a growth equation that accounts for the growth factor water and the potential growth rate. An expression for the relation between transpiration and production is presented in Chapter 6. In Chapter 7 a method is given for the calculation of potential production.

In Chapter 8 the set-up of the program SWATR is described in general terms. The model is verified with data from field experiments with red cabbage and potatoes. Also some numerical experiments are given to investigate the influence of changes in soil physical properties, rooting depth, meteorological conditions, etc. on field water use.

In Chapter 9 the main points of program CROPR are presented. Also with verifications (with data of field experiments with red cabbage, grass and potatoes) and numerical experiments with regard to crop production.

Finally Chapters 10 and 11 give complete listings of the programs, instructions for input and examples of factual input.

The models and programs are meant to be applied in evaluating existing as well-as planned soil, water and crop management practices in humid, semi-humid and arid areas.

## I Theory of field water use

## 2 Basics of water flow in unsaturated soils

### 2.1 Soil water potential

Water in soil moves from points where it has a high energy status to points where it has a lower one. The energy status of water is called the water potential $\Psi$ and is composed of several components

$$
\begin{equation*}
\psi=\psi+\psi_{\mathrm{g}}+\left(\psi_{o s m}+\psi_{\mathrm{gas}}\right) \tag{2.1}
\end{equation*}
$$

where
$\psi=$ matric potential, arising from local interacting forces between soil and water
$\psi_{\mathrm{g}}=$ gravitational potential, arising from the gravitational force
$\psi_{\text {osm }}=$ osmotic potential, arising from osmotic forces
$\psi_{\text {gas }}=$ pneumatic potential, arising from changes in external gas pressure

The potentials are defined relative to the reference status of water (of composition identical to the soil solution) at atmospheric pressure, $293 \mathrm{~K}\left(20^{\circ} \mathrm{C}\right)$ and datum elevation zero. The potential is often expressed as energy per unit weight of soil water. Then energy has the dimension of length, i.e. cm . This is equivalent to about $10^{-1} \mathrm{~J} \cdot \mathrm{~kg}^{-1}\left(10^{3}\right.$ erg. $\mathrm{g}^{-1}$ ) and to about $10^{2} \mathrm{~Pa}\left(10^{-3}\right.$ bar, 1 mbar or $10^{-3} \mathrm{~atm}$.).

In studies on soil moisture flow, one may usually neglect the potentials put between the brackets in Eqn 2.1. The influence of $\psi_{\text {osm }}$ is low because the osmotic potential is measured of water that is assumed to have the same chemical properties all over the profile. As gas pressures in natural soil generally do not differ from the atmospheric pressure, $\psi_{\mathrm{gas}}=0$.
The matric potential ( $\psi$ ) in unsaturated soil is negative, because work is needed to withdraw water against the soil matric forces. It is not essential to specify these forces in detail: it suffices that $\psi$ can be measured by tensiometry or other techniques. At the phreatic surface, $\psi=0 \mathrm{~cm}$.

The gravitational potential $\left(\psi_{\mathrm{g}}\right)$ at each point is determined by the
height of that point relative to some (arbitrary) reference level. If we consider the origin of $z$ at the soil surface and positive in downward direction $\psi_{\mathrm{g}}=-z \mathrm{~cm}$.

When dealing only with the sum of matric and gravitational potential, one usually speaks of the hydraulic head, $H$. Thus

$$
\begin{equation*}
H=\psi-z \quad \text { cm }) \tag{2.2}
\end{equation*}
$$

with $\psi$ now called the soil moisture pressure head and $z$ the gravitational head.

### 2.2 Soil moisture characteristic and hydraulic conductivity curve

Water in the unsaturated zone is retained in the soil mainly by the matric forces. In wet, coarse-textured media capillary forces are dominant, while in dry soils adsorption is most important. In finetextured media exhibiting colloidal properties double-layer effects may become significant. At zero pressure head all the pores are supposed to be filled with water. This situation occurs at the phreatic surface. Under equilibrium conditions with increasing height above this surface, the pressure head decreases and progressively smaller pores will empty. So one may expect a certain relation between the pressure head and the moisture content of a soil, $\theta=\mathrm{f}(\psi)$. Since soils differ in physical properties, such a relationship is different for each soil.

After exerting a certain pressure head upon a soil sample, the equilibrium soil moisture content can be determined. Applying different pressure heads step by step one can obtain a graph of pressure head $(\psi)$ versus moisture content $(\theta)$. Such a graph is called the soil moisture retention curve or the soil moisturecharacteristic.

It is convenient to refer to a negative pressure head $(\psi)$ as a positive suction or tension (h). Thus

$$
\begin{equation*}
h=-\psi \tag{2.3}
\end{equation*}
$$

The value of $h$ ranges from 0 to $10^{7} \mathrm{~cm}$. To present this range easily in a graph, Schofield (1935) introduced the quantity pF, defined as

$$
\begin{equation*}
\mathrm{pF}=\log _{10} h \tag{2.4}
\end{equation*}
$$

The tension curves are usually determined by removing water from an initially wet soil sample (desorption). If one adds water to an initially dry sample (adsorption), the moisture content will be different at corresponding tensions. This phenomenon is referred to


Fig. 2. Examples of soil moisture retention curves for clay, sandy loam and sand.
as hysteresis, which occurs because it takes more energy to get water out of the soil than in. In this monograph we only consider desorption curves. In general the variability of the soil is unknown and its influence often exceeds that of hysteresis. In Fig. 2 examples of soil moisture retention curves are shown for clay, sandy loam and sand.

For saturated (groundwater) flow the total soil pore space is available for water flow. With unsaturated flow, however, part of the pores are filled with air. Therefore, the hydraulic conductivity ( $K$ ) must be smaller than for saturated flow. So for unsaturated soils $K$ is not a constant but depends on the soil moisture content $\theta$ [because


Fig. 3. Examples of hydraulic conductivity curves for clay, sandy loam and sand.
$\theta=\mathrm{f}(\psi)]$ on the pressure head

$$
\begin{equation*}
K=\mathrm{f}(\theta) \quad \text { or } \quad K=\mathrm{f}(\psi) \tag{2.5}
\end{equation*}
$$

As an example of the influence of $\psi$ on $K$, see Fig. 3 which pertains to the three soil types of Fig. 2. For more information about measurement techniques of soil physical properties, see Bouma (1977).

The soil moisture retention curve $h(\theta)$ and the hydraulic conductivity curve $K(\psi)$ or $K(\theta)$ can be described in three alternative ways:
(a) $h$ as a table of $\theta$
$K$ as function of $h$. According to Rijtema (1965)

$$
\begin{array}{lll}
K=K_{\mathrm{s}} & \text { for } & h \leqslant h_{a} \\
K=K_{\mathrm{s}} \mathrm{e}^{-\eta\left(h-h_{a}\right)} & \text { for } & h_{a}<h<h_{\text {lim }} \\
K=a h^{-1.4} & \text { for } & h \geqslant h_{\text {lim }} \tag{2.8}
\end{array}
$$

where $K_{\mathrm{s}}$ is saturated hydraulic conductivity; $h_{a}$ is suction at airentry point, i.e. the suction at which a water-saturated porous medium starts to let air pass through it; $h_{\text {lim }}$ is some arbitrary suction above which Eqn 2.7 is no longer valid; $a$ and $\eta$ are constants.
(b) $h$ as a function of $\theta$ (see Fig. 4A)
$K$ as a function of $h$ (see Fig. 4B)

$$
\begin{array}{ll}
h=\mathrm{e}^{a_{1}\left(b_{1}-\theta\right)} & \text { for } \theta_{1} \leqslant \theta \leqslant \theta_{\mathrm{s}} \\
h=\mathrm{e}^{a_{2}\left(b_{2}-\theta\right)} & \text { for } \theta_{2} \leqslant \theta<\theta_{1} \\
h=\mathrm{e}^{a_{3}\left(b_{3}-\theta\right)} & \text { for } \theta<\theta_{2}
\end{array}
$$



Fig. 4: A, Description of the soil moisture retention curve by three line segments (see Eqns 2.9-2.11); B, description of the hydraulic conductivity curve by three line segments (see Eqns 2.12-2.14).

$$
\begin{array}{ll}
K=K_{\mathrm{s}} \mathrm{e}^{-\alpha_{1}\left(h-\beta_{1}\right)} & \text { for } h \leqslant h_{1} \\
K=K_{\mathrm{s}} \mathrm{e}^{-\alpha_{2}\left(h-\beta_{2}\right)} & \text { for } h_{1}<h<h_{2} \\
K=\left(\alpha_{3}+\beta_{3} \log _{10} h\right) h^{-1.4} & \text { for } h \geqslant h_{2} \tag{2.14}
\end{array}
$$

With Eqns 2.9-2.11 one can easily find the value of $\theta$ and the differential moisture capacity $C_{h}$ (see also Eqn 2.22) according to

$$
\begin{align*}
& \theta_{i}=b_{i}-\frac{\ln h}{a_{i}} \quad i=1,2,3  \tag{2.15}\\
& C_{h_{i}}=\frac{\mathrm{d} \theta}{\mathrm{~d} h}=-\frac{1}{a_{i} h}
\end{align*}
$$

(c) $h$ as a table of $\theta$
$K$ as a table of $\theta$

### 2.3 Soil water flow

To describe the flow of water in soil systems, it is customary to use Darcy's law. For one dimensional vertical flow, the volumetric flux $q\left(\mathrm{~cm}^{3} . \mathrm{cm}^{-2} . \mathrm{day}^{-1}\right)$ can be written as

$$
\begin{equation*}
q=-K \frac{\partial H}{\partial z} \quad\left(\mathrm{~cm}^{2} \cdot \mathrm{day}^{-1}\right) \tag{2.17}
\end{equation*}
$$

Substitution of Eqn 2.2 into Eqn 2.17 yields

$$
\begin{equation*}
q=-K\left(\frac{\partial \psi}{\partial z}-1\right) \tag{2.18}
\end{equation*}
$$

In order to get a complete mathematical description for unsaturated flow, we apply the continuity principle (Law of Conservation of Matter)

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=-\frac{\partial q}{\partial z} \quad\left(\mathrm{day}^{-1}\right) \tag{2.19}
\end{equation*}
$$

where $\theta$ is expressed in $\mathrm{cm}^{3} . \mathrm{cm}^{-3}$ and $t$ is time in days.
Substitution of Eqn 2.17 into Eqn. 2.19 yields the partial differential equation, in terms of hydraulic head

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial z}\left(K \frac{\partial H}{\partial z}\right) \tag{2.20}
\end{equation*}
$$

Substitution of Eqn 2.18 into Eqn 2.19 yields the pressure head form of the flow equation

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial z}\left[K\left(\frac{\partial \psi}{\partial z}-1\right)\right] \tag{2.21}
\end{equation*}
$$

Eqn 2.21 is a second-order, parabolic type of partial differential equation which is non-linear because of the dependency of $K$ and $\psi$ on $\theta$ (linearity means that the coefficients in a differential equation are only functions of the independent variables $z$ and $t$ ). To avoid the problem of the two dependent variables $\theta$ and $\psi$, the derivative of $\theta$ with respect to $\psi$ can be introduced, which is known as the differential moisture capacity $C$

$$
\begin{equation*}
C=\frac{\mathrm{d} \theta}{\mathrm{~d} \psi} \quad\left(\mathrm{~cm}^{-1}\right) \tag{2.22}
\end{equation*}
$$

In Eqn 2.22 a normal instead of a partial derivative notation is used, because $\psi$ is considered here as a single-value function of $\theta$ (no hysteresis!).

Writing

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{\mathrm{d} \theta}{\mathrm{~d} \psi} \cdot \frac{\partial \psi}{\partial t} \tag{2.23}
\end{equation*}
$$

and substituting Eqn 2.22 into Eqn 2.21 yields

$$
\begin{equation*}
C(\psi) \frac{\partial \psi}{\partial t}=\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}-1\right)\right] \tag{2.24}
\end{equation*}
$$

In Eqn 2.24 the coefficients $C$ and $K$ are functions of the dependent variable $\psi$, but not functions of the derivatives $\partial \psi / \partial t$ and $\partial \psi / \partial z$. Written in this form, Eqn 2.24 provides the basis for predicting soil water movement in layered soils of which each layer may have different physical properties. Dividing both sides of Eqn 2.24 by $C(\psi)$ gives the flow equation

$$
\begin{equation*}
\frac{\partial \psi}{\partial t}=\frac{1}{C(\psi)} \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}-1\right)\right] \tag{2.25}
\end{equation*}
$$

## 3 Water uptake by plant roots

### 3.1 Mathematical description

When dealing with water uptake by roots two approaches are customary. The first approach relies on the properties of a single root, the second one on the integrated properties of the entire root system. Here the second one will be followed. In this macroscopic approach, water uptake by the roots is represented by a volumetric sink term, which simply is added to the continuity equation (see Eq. 2.19 and Fig. 5). So

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=-\frac{\partial q}{\partial z}-S \tag{3.1}
\end{equation*}
$$

where $S$ represents the volume of water taken up by the roots per unit bulk volume of the soil in unit time $\left(\mathrm{cm}^{3} . \mathrm{cm}^{-3} . \mathrm{day}^{-1}\right)$. Usually flow of water from roots into the soil is not taken into account. This behaviour only occurs under very special conditions and is neglectably small (Molz \& Peterson, 1976). The integral of the sink term


Fig. 5. Schematic representation of water uptake by plant roots by a sink term $S$ which is added to the continuity equation for one-dimensional vertical soil water flow; $q$ defined according to Darcy.


Fig. 6. Actual transpiration $E_{p t}$ defined as the integral of water uptake over rooting depth $L_{\text {r }}$.
over the rooting depth $L_{r}$ then gives the actual transpiration, $E_{p l}$

$$
\begin{equation*}
E_{p l}=\int_{z=0}^{z=L_{r}} S \mathrm{~d} z \tag{3.2}
\end{equation*}
$$

which is schematically depicted in Fig. 6.
A major difficulty in solving Eqn 3.1 stems from the function of $S$ being unknown. For field measurements of root water uptake patterns on sugar-beet and winter wheat, see Strebel et al. (1975) and Ehlers (1975); on grasses Rijtema (1965), Rice (1975) and Flühler et al. (1975); on cotton Rose \& Stern (1967); on alfalfa Nimah \& Hanks (1973); on soybeans Reicosky et al. (1972), Arya (1973), Stone et al. (1976); on sorghum Stone et al. (1973), Reicosky \& Ritchie (1976); on a Douglas-Fir forest Nnyamah (1977); on red cabbage Feddes (1971).

Several authors tried to describe the flow of liquid water through the rooted soil zone (and the root-stem-leaf-stomata-air path) in terms of Ohm's law. The rate of water uptake is then assumed to be directly proportional to the difference in pressure head between the soil and the root interior, to the hydraulic conductivity of the soil and to some empirical 'root effectiveness' or 'root density' function. This 'root density' function is interpreted and evaluated differently by various investigators. For a literature survey see e.g. Feddes et al. (1974), van Bavel \& Ahmed (1976).

One of the major difficulties in such a description is the determination of this root effectiveness function or some equivalent thereof. Therefore a different description has been developed here in which the water uptake by roots is considered to be a function of the


Fig. 7. General shape of the sink term $S$ as a function of the absolute value of the soil moisture pressure head $|\psi|$.
pressure head $\psi$. The sink term used here is shown in Fig. 7. We assume that under conditions wetter than a certain 'anaerobiosis point' $\left(\psi_{1}\right)$, water uptake by roots is zero (Assumption 1) or quickly reaches zero (Assumption 2). Under conditions drier than 'wilting point' $\left(\psi_{3}\right)$, water uptake by roots is also zero.

A fixed 'anaerobiosis point' at which deficient aeration conditions exist and root growth is seriously hampered, is hard to define. To characterize soil aeration, one can use the so-called oxygen diffusion rate (ODR), which is defined as the flux of oxygen towards a platinum wire inserted in the soil. Stolzy \& Letey (cf. Wesseling, 1974) found that many plants do not grow in soils with oxygen diffusion rates (ODR) below $20 \times 10^{-8} \mathrm{~g} . \mathrm{cm}^{-2} \cdot \mathrm{~min}^{-1}$. This value corresponds to critical gas porosities of about 0.04 to $0.25 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}$ for different soils (see Bakker, 1970; Kowalik, 1972; Gawlik, 1975).

Soil aeration can also be characterized by the possible gas exchange through the gas filled soil pores, i.e. by the gas diffusion coefficient. Calculations on the necessary transport of oxygen towards the roots of normal growing plants show that below an oxygen diffusion coefficient of $1.5 \times 10^{-4} \mathrm{~cm}^{2} . \mathrm{s}^{-1}$, the oxygen demand of the plants can never be met (Bakker et al., 1978). This coefficient corresponds to gas filied porosities of less than $0.05 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}$ for good structured soils and to about $0.10 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}$ for single-grained structures.

Most of the experiments on the effect of soil aeration on plant growth have been conducted under conditions where the entire root system was subjected to constant air regimes. Letey et al. (1961)
reported from such an experiment that in the beginning there is hardly any effect of low oxygen content on transpiration. Only after the amount of root surface and the permeability of the roots are reduced by the low oxygen content, is water uptake also decreased. The outcome of their experiment indicates that the effect of low oxygen content on water use is a resultant of root behaviour.

An experiment where only part of the root system is subjected to a given aeration condition was described by Letey et al. (1965). They found that root development in a given soil compartment was independent of the condition in other compartments. The rate of water removal from a compartment was dependent on the amount of roots. Water removal per net weight of roots was not influenced by the aeration treatments. Purvis \& Williamson (1972) showed that a (corn) crop can withstand a soil $\mathrm{O}_{2}$-concentration as low as $1 \%$ for two days without detectable injury to the plant. If such a low $\mathrm{O}_{2}$-concentration continues for more than two days, injury may occur. In general, injuries at low oxygen concentrations largely depend on soil temperature. Sojka et al. (1972) emphasized the possible antagonistic effect of high soil temperatures and poor aeration.
In field experiments it was observed (Feddes, 1971) that roots of red cabbage on clay soil, close to the rather shallow groundwater table, grew well at gas-filled porosities of $0.01 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}$. There also is some experimental evidence that with enough air being present in the upper part of the root zone, water can be extracted by roots in the lower part of the root zone under nearly water saturated conditions (e.g. Gosiewski \& Skapski, 1976).

From the literature cited, it is difficult to decide whether Assumption 1 or 2 in Fig. 7 is true. Until more evidence becomes available, the two options are kept in this study.
The water uptake by the roots is assumed to be maximal when the pressure head in the soil is between $\psi_{1}$ and $\psi_{2}$ (Fig. 7). It is known that at increasing desiccation the availability of water for the plant decreases progressively (e.g. van Keulen, 1975). The pressure head at which soil water begins to limit plant growth seems to range between pF -value of 2.6 and 3 . This range corresponds to values of $-1000<\psi_{2}<-500 \mathrm{~cm}$. The value of $\psi_{2}$ is in fact not a constant, as it varies with the evaporative demand of the atmosphere. Under conditions of high evaporative demand, a drop in root water uptake generally occurs at higher $\psi$-values than under conditions of low demand (see e.g. Yang \& de Jong, 1972). In the model, however, we take $\psi_{2}$ to be a constant.

When $\psi$ is below $\psi_{2}$ but larger than $\psi_{3}$, we assume that the water uptake decreases linearly with $\psi$ to zero (Fig. 7, Assumption 3). Hence

$$
\begin{equation*}
S(\psi)=S_{\max } \frac{\psi-\psi_{3}}{\psi_{2}-\psi_{3}} \tag{3.4}
\end{equation*}
$$

Or, the decrease may be linear to a certain small value of residual water uptake (Assumption 4). In practice it can be taken that $-20,000<\psi_{3}<-15,000$.
Let us now define the value of $S_{\text {max }}$. For convenience sake a dimensionless variable is introduced

$$
\begin{equation*}
\alpha(\psi)=\frac{S(\psi)}{S_{\max }} \tag{3.5}
\end{equation*}
$$

Substituting Eqn 3.5 into Eqn 3.2, one gets

$$
\begin{equation*}
E_{\mathrm{pl}}=\int_{0}^{L_{\mathrm{L}}} \alpha(\psi) S_{\max } \mathrm{d} z \tag{3.6}
\end{equation*}
$$

For unit time, e.g. one day, $S_{\text {max }}$ may be taken to be constant, so

$$
\begin{equation*}
E_{\mathrm{pl}}=S_{\max } \int_{0}^{L_{\mathrm{L}}} \alpha(\psi) \mathrm{d} z \tag{3.7}
\end{equation*}
$$

When the transpiration is maximal ( $=$ potential $=E_{p p}^{*}$ ) then $E_{p l}=$ $E_{p l}^{*}$ and $\alpha(\psi)$ must be equal to 1 . So

$$
E_{p l}^{*}=S_{\max } \int_{0}^{L_{0}} d z \quad\left(\mathrm{~cm}_{0} \mathrm{day}^{-1}\right)
$$

or

$$
\begin{equation*}
S_{\max }=\frac{E_{\mathrm{pl}}^{*}}{L_{r}} \quad\left(\text { day }^{-1}\right) \tag{3.8}
\end{equation*}
$$

Some plant species, whether or not under particular conditions (e.g. drought), have a root system that does not reach from the rooting depth $L_{r}$ to the surface. For that reason a reduction $L_{r}^{\text {na }}$ (rooting depth non-active) is introduced to correct the rooting depth $L_{r}$. The effective rooting depth is then found as

$$
\begin{equation*}
L_{r}^{e f f}=L_{r}-L_{r}^{\text {na }} \tag{3.9}
\end{equation*}
$$

The expression of $S_{\text {max }}$ therefore reads

$$
\begin{equation*}
S_{\max }=\frac{E_{p l}^{*}}{L_{r}^{e f f}} \tag{3.10}
\end{equation*}
$$

Adding the sink term to the continuity equation (see Eqns 2.25 and 3.1) yields the equation describing flow of water in the soil-root system

$$
\begin{equation*}
\frac{\partial \psi}{\partial t}=\frac{1}{C(\psi)} \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}-1\right)\right]-\frac{S(\psi)}{C(\psi)} \tag{3.11}
\end{equation*}
$$

which is applied in the calculations in this book.

### 3.2 Limitation of uptake by potential transpiration

The maximum possible water uptake by roots per unit area of soil (potential transpiration) is dependent on the conditions of the atmosphere and kind, stage and condition of the crop. This quantity can be calculated according to

$$
\begin{equation*}
E_{\mathrm{pl}}^{*}=E^{*}-E_{\mathrm{s}}^{*} \quad\left(\mathrm{~mm}^{*} \mathrm{day}^{-1}\right) \tag{3.12}
\end{equation*}
$$

where $E^{*}$ is potential evapotranspiration from both crop and soil, and $E_{s}^{*}$ is potential evaporation from the soil only.

Potential evapotranspiration $E^{*}$ can according to Penman (1948) be derived from a combination of the energy balance and the transport of water vapour. He applied this combination method to a water surface. Independently of each other, Monteith (1965) and Rijtema (1965) extended this method to crops. For a recent reevaluation of Penman's equation, see Thom \& Oliver (1977).
The method applied to a water saturated surface briefly can be described as follows. The energy balance equates all incoming and outgoing energy per unit area at the surface

$$
\begin{equation*}
R_{n}=H+L E+G \quad\left(\mathrm{~W} \cdot \mathrm{~m}^{-2}\right) \tag{3.13}
\end{equation*}
$$

where $R_{n}$ represents the energy flux of net incoming radiation, $H$ the flux of sensible heat into the air, $L E$ the flux of latent heat into the air and $G$ the flux of heat into the soil. In the expression $L E$ is $L$ the latent heat of vaporization of water per unit mass in $\mathrm{J} . \mathrm{kg}^{-1}$, and $E$ the vapour flux in $\mathrm{kg} \cdot \mathrm{m}^{-2} \cdot \mathrm{~s}^{-1}$. In this way an energy flux can be converted into evaporation equivalents. In Eqn $3.13, G$ is small over a day period and assumed to be zero. Hence

$$
\begin{equation*}
R_{n} \approx H+L E \tag{3.14}
\end{equation*}
$$

Now one can calculate $E$ when $H / L E$ (Bowen ratio) is known. This ratio can be derived from the transport equations of heat and water vapour in air.
The situation depicted in Fig. 8 shows that radiation energy is


Fig. 8. Illustration of the variables involved in the energy balance at the soil surface.
transformed into heat and water vapour which are transported to the air according to

$$
\begin{align*}
& H=c_{1}\left(T_{s}-T_{a}\right) / r_{a}  \tag{3.15}\\
& L E=c_{2}\left(e_{s}-e_{d}\right) / r_{a} \tag{3.16}
\end{align*}
$$

where

$$
c_{1}, c_{2}=\text { constants }
$$

$T_{s}, T_{a}=$ temperature ( $K$ ) at soil surface and of the air at a certain height
$e_{s}, e_{d}=$ saturated and prevailing vapour pressure (mbar) at the surface and of air at the same height at temperature $T_{s}$ and $T_{a}$, respectively
$r_{a}=$ aerodynamic diffusion resistance, assumed to be the same both for heat and water vapour (s.m ${ }^{-1}$ )

When the concept of similarity of transport of heat and water vapour is applied, the Bowen ratio yields

$$
\begin{equation*}
\frac{H}{L E}=\frac{c_{1}}{c_{2}} \frac{T_{s}-T_{a}}{e_{s}-e_{d}} \tag{3.17}
\end{equation*}
$$

where $c_{1} / c_{2}=\gamma=$ psychrometer constant (mbar. $\mathrm{K}^{-1}$ )
The problem is that generally the surface temperature $T_{s}$ is unknown. Penman therefore introduced an additional equation

$$
\begin{equation*}
e_{s}-e_{a}=\delta\left(T_{s}-T_{a}\right) \tag{3.18}
\end{equation*}
$$

where the proportionality constant $\delta\left(\right.$ mbar. $\left.^{-1}\right)$ is the first derivative of the function $e_{s}(T)$ known as the saturated vapour pressure curve (Fig. 9). Note that in Eqn $3.18 e_{a}$ is the saturated vapour pressure at temperature $T_{a}$. Rearranging gives

$$
\begin{equation*}
\delta=\frac{e_{s}-e_{a}}{T_{s}-T_{a}} \approx \frac{\Delta e_{a}}{\Delta T_{a}} \approx \frac{d e_{a}}{d T_{a}} \tag{3.1}
\end{equation*}
$$

In Fig. 9 the slope $\delta$ can be determined at temperature $T_{a}$, provided that ( $T_{s}-T_{a}$ ) is small.

From Eqn 3.19 it follows that $T_{s}-T_{a}=\left(e_{s}-e_{a}\right) / \delta$. Substitution into Eqn 3.17 yields

$$
\begin{equation*}
\frac{H}{L E}=\frac{\gamma}{\delta} \frac{e_{s}-e_{a}}{e_{s}-e_{d}} \tag{3.20}
\end{equation*}
$$

Replacing ( $e_{s}-e_{a}$ ) by ( $e_{s}-e_{d}-e_{a}+e_{d}$ ), then Eqn 3.20 can be written as

$$
\begin{equation*}
\frac{H}{L E}=\frac{\gamma}{\delta}\left(1-\frac{e_{a}-e_{d}}{e_{s}-e_{d}}\right) \tag{3.21}
\end{equation*}
$$

Under isothermal conditions one may assume that $T_{s} \approx T_{a}$. This implies that $e_{s} \approx e_{a}$. Then we may introduce this assumption in Eqn 3.16 and the isothermal evaporation $L E_{a}$ reads as

$$
\begin{equation*}
L E_{a}=c_{2}\left(e_{a}-e_{d}\right) / r_{a} \tag{3.22}
\end{equation*}
$$

Dividing Eqn 3.22 by Eqn 3.16 yields

$$
\begin{equation*}
\frac{E_{a}}{E}=\frac{e_{a}-e_{d}}{e_{s}-e_{d}} \tag{3.23}
\end{equation*}
$$



Fig. 9. Saturated water vapour pressure $e_{a}$ as a function of air temperature $T_{a}$.

This ratio was already mentioned in Eqn 3.21, which can now be written as

$$
\begin{equation*}
\frac{H}{L E}=\frac{\gamma}{\delta}\left(1-\frac{E_{a}}{E}\right) \tag{3.24}
\end{equation*}
$$

From Eqn 3.14 it follows that $H=R_{n}-L E$. After some rearrangement, substitution into Eqn 3.24 gives

$$
\begin{equation*}
L E=\frac{\delta R_{n}+\gamma L E_{a}}{\delta+\gamma} \quad\left(\mathrm{W} \cdot \mathrm{~m}^{-2}\right) \tag{3.25}
\end{equation*}
$$

Dividing both sides of Eqn 3.25 by $L$ yields the formula of Penman (1948)

$$
\begin{equation*}
E=\frac{\delta R_{n} / L+\gamma E_{a}}{\delta+\gamma} \quad\left(\mathrm{kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right) \tag{3.26}
\end{equation*}
$$

In Eqn 3.26, $R_{n} / L$ is the evaporation equivalent of the net flux of radiant energy to the surface and $E_{a}$ the corresponding aerodynamic term.
Potential evapotranspiration from a crop can be described by an equation very similar to Eqn 3.26. But then one has to take into account the differences between a crop surface and a water surface:

- the reflection coefficient for solar radiation is different for a crop surface ( 0.23 say) and a water surface ( $0.05-0.07$ );
--when water shortage occurs plants have a biological control (by closing their stomata) to restrict evaporation while a water surface
- has not;
- a crop surface has a roughness length (dependent on crop height and wind speed) and therefore a diffusion resistance, $r_{a}$, that can differ considerably from that of a water surface.

In a way similar to that applied to Eqn 3.22 and replacing the coefficient $c_{2}$ by its proper expression, one can write $E_{a}$ for a crop as

$$
\begin{equation*}
E_{a}=\frac{\varepsilon \rho_{a}}{p_{a}}\left(e_{a}-e_{d}\right) / r_{a} \tag{3.27}
\end{equation*}
$$

where
$\varepsilon=$ ratio of molecular weight of water vapour to dry air
$p_{a}=$ atmospheric pressure (mbar)
$\rho_{a}=$ density of the air $\left(\mathrm{kg} \cdot \mathrm{m}^{-3}\right)$

For a wet crop surface with ample water supply the equation of Penman 3.26 can then be modified (Monteith, 1965; Rijtema, 1965) into

$$
\begin{equation*}
E^{*}=\frac{\delta R_{n} / L+\gamma \frac{\varepsilon \rho_{a}}{p_{a}}\left(e_{a}-e_{d}\right) / r_{a}}{\delta+\gamma} \tag{3.28}
\end{equation*}
$$

Because the psychrometric constant $\gamma=c_{p} p_{a} / L \varepsilon$, Eqn 3.28 reduces to

$$
\begin{equation*}
E^{*}=\frac{\delta R_{n}+c_{p} \rho_{a}\left(e_{a}-e_{d}\right) / r_{a}}{(\delta+\gamma) L} \quad\left(\mathrm{~kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right) \tag{3.29}
\end{equation*}
$$

where $c_{\mathrm{p}}$ is the specific heat of air at constant pressure ( $\mathrm{J} . \mathrm{kg}^{-1} \cdot \mathrm{~K}^{-1}$ ). The resistance $r_{a}$ can under conditions of neutral stability ( $T_{\text {crop }} \approx$ $T_{\text {air }}$ ) be expressed (Feddes, 1971) as

$$
\begin{equation*}
r_{a}=\frac{\varepsilon \rho_{a}}{p_{a}}\left[f(l) 1.15 u^{0.75}\right]^{-1} \quad\left(\mathrm{~s} . \mathrm{m}^{-1}\right) \tag{3.30}
\end{equation*}
$$

where $u$ is the wind velocity ( $\mathrm{m} . \mathrm{s}^{-1}$ ) measured at 2 m height and $\mathrm{f}(l)$ is a function ( $\mathrm{m}^{-2} . \mathrm{s}^{2}$ ) dependent on crop height, taken from Rijtema (1965) and as confirmed by Slabbers (1977) for semi-arid and arid conditions. Values of $r_{a}$ for various crop heights and wind velocities are presented in Table 1.

The potential evaporation of a soil under a crop cover can be computed from a simplified form of Eqn 3.29 by neglecting the aerodynamic term and taking into account only that fraction of $R_{n}$ which reaches the soil surface (Ritchie, 1972)

$$
\begin{equation*}
E_{\mathrm{s}}^{*}=\frac{\delta}{(\delta+\gamma) L} R_{\mathrm{n}} \mathrm{e}^{-0.39 \mathrm{I}} \quad\left(\mathrm{~kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right) \tag{3.31}
\end{equation*}
$$

where $I$ is the leaf area index. This index generally can be related to soil cover, as will be shown later in Figs. 25 and 31.

In Eqns 3.29 to 3.31, one can use the following values which apply to conditions of $293 \mathrm{~K}\left(=20^{\circ} \mathrm{C}\right)$ or $1013 \mathrm{mbar}: \varepsilon=0.622$; $\rho_{a}=1.2047 \mathrm{~kg} . \mathrm{m}^{-3} ; \quad c_{\mathrm{p}}=1004 \mathrm{~J} . \mathrm{kg}^{-1} \cdot \mathrm{~K}^{-1} ; \quad p_{a}=1013 \mathrm{mbar} ; \quad \gamma=$ $0.67 \mathrm{mbar} . \mathrm{K}^{-1} ; L=2.451 \times 10^{6} \mathrm{~J} . \mathrm{kg}^{-1}$. To convert from $L E$ in W. $\mathrm{m}^{-2}$ to $E$ in $\mathrm{kg} \cdot \mathrm{m}^{-2}$. $\mathrm{day}^{-1}$, one must multiply by the factor $86400 /\left(2.451 \times 10^{6}\right)=0.352 \times 10^{-1}$. And as a vapour flux of $1 \mathrm{~kg} \cdot \mathrm{~m}^{-2}$. day $^{-1}$ is equivalent with an evaporation of $1 \mathrm{~mm} . \mathrm{day}^{-1}$ one

| $l(\mathrm{~m})$ | $u\left(\mathrm{~m} . \mathrm{s}^{-1}\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.3 | 0.5 | 0.7 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 | 6.0 | 7.0 |
| 0.00 | 1020 | 693 | 539 | 412 | 304 | 245 | 207 | 181 | 161 | 146 | 123 | 108 | 95.8 |
| 0.01 | 962 | 656 | 509 | 390 | 288 | 232 | 196 | 171 | 152 | 138 | 117 | 102 | 90.6 |
| 0.02 | 793 | 541 | 420 | 322 | 237 | 191 | 162 | 141 | 126 | 114 | 96.2 | 83.9 | 74.7 |
| 0.03 | 590 | 402 | 312 | 239 | 176 | 142 | 120 | 105 | 93.4 | 84.5 | 71.5 | 62.4 | 55.6 |
| 0.04 | 468 | 319 | 248 | 190 | 140 | 113 | 95.4 | 83.2 | 74.1 | 67.1 | 56.7 | 49.5 | 44.1 |
| 0.05 | 389 | 265 | 206 | 158 | 116 | 93.7 | 79.3 | 69.2 | 61.6 | 55.7 | 47.1 | 41.1 | 36.6 |
| 0.06 | 338 | 231 | 179 | 137 | 101 | 81.5 | 69.0 | 60.2 | 53.6 | 48.5 | 41.0 | 35.8 | 31.9 |
| 0.07 | 305 | 208 | 161 | 123 | 91.1 | 73.4 | 62.1 | 54.2 | 48.2 | 43.6 | 36.9 | 32.2 | 28.7 |
| 0.08 | 281 | 192 | 149 | 114 | 84.1 | 67.8 | 57.4 | 50.0 | 44.6 | 40.3 | 34.1 | 29.7 | 26.5 |
| 0.09 | 261 | 178 | 138 | 106 | 78.1 | 62.9 | 53.2 | 46.4 | 41.3 | 37.4 | 31.6 | 27.6 | 24.6 |
| 0.10 | 247 | 168 | 131 | 100 | 73.8 | 59.5 | 50.3 | 43.9 | 39.1 | 35.4 | 29.9 | 26.1 | 23.2 |
| 0.12 | 226 | 154 | 120 | 91.5 | 67.5 | 54.4 | 46.0 | 40.1 | 35.8 | 32.3 | 27.4 | 23.9 | 21.3 |
| 0.14 | 210 | 143 | 111 | 85.2 | 62.9 | 50.7 | 42.9 | 37.4 | 33.3 | 30.1 | 25.5 | 22.2 | 19.8 |
| 0.16 | 199 | 135 | 105 | 80.5 | 59.4 | 47.9 | 40.5 | 35.3 | 31.5 | 28.5 | 24.1 | 21.0 | 18.7 |
| 0.18 | 190 | 130 | 101 | 77.1 | 56.9 | 45.9 | 38.8 | 33.8 | 30.1 | 27.3 | 23.1 | 20.1 | 17.9 |
| 0.20 | 183 | 125 | 96.8 | 74.1 | 54.7 | 44.1 | 37.3 | 32.5 | 29.0 | 26.2 | 22.2 | 19.3 | 17.2 |
| 0.25 | 171 | 116 | 90.5 | 69.2 | 51.1 | 41.2 | 34.8 | 30.4 | 27.1 | 24.5 | 20.7 | 18.1 | 16.1 |
| 0.30 | 163 | 111 | 86.5 | 66.2 | 48.8 | 39.3 | 33.3 | 29.0 | 25.9 | 23.4 | 19.8 | 17.3 | 15.4 |
| 0.35 | 156 | 106 | 82.7 | 63.3 | 46.7 | 37.6 | 31.8 | 27.8 | 24.7 | 22.4 | 18.9 | 16.5 | 14.7 |
| 0.40 | 150 | 102 | 79.4 | 60.7 | 44.8 | 36.1 | 30.5 | 26.6 | 23.7 | 21.5 | 18.2 | 15.8 | 14.1 |
| 0.45 | 144 | 98.1 | 76.2 | 58.3 | 43.0 | 34.7 | 29.3 | 25.6 | 22.8 | 20.6 | 17.4 | 15.2 | 13.6 |
| 0.50 | 138 | 94.4 | 73.3 | 56.1 | 41.4 | 33.4 | 28.2 | 24.6 | 21.9 | 19.8 | 16.8 | 14.6 | 13.0 |
| 0.55 | 135 | 92.3 | 71.7 | 54.9 | 40.5 | 32.6 | 27.6 | 24.1 | 21.4 | 19.4 | 16.4 | 14.3 | 12.8 |
| 0.60 | 133 | 90.9 | 70.6 | 54.1 | 39.9 | 32.1 | 27.2 | 23.7 | 21.1 | 19.1 | 16.2 | 14.1 | 12.6 |
| 0.65 | 130 | 89.0 | 69.1 | 52.9 | 39.0 | 31.5 | 26.6 | 23.2 | 20.7 | 18.7 | 15.8 | 13.8 | 12.3 |
| 0.70 | 129 | 87.7 | 68.2 | 52.2 | 38.5 | 31.0 | 26.2 | 22.9 | 20.4 | 18.4 | 15.6 | 13.6 | 12.1 |
| 0.80 | 124 | 84.8 | 65.9 | 50.4 | 37.2 | 30.0 | 25.4 | 22.1 | 19.7 | 17.8 | 15.1 | 13.1 | 11.7 |
| 0.90 | 122 | 83.1 | 64.6 | 49.4 | 36.4 | 29.4 | 24.8 | 21.7 | 19.3 | 17.5 | 14.8 | 12.9 | 11.5 |

can write: $L E\left(\mathrm{~W}^{-2}\right) \times 0.352 \times 10^{-1}=E\left(\mathrm{~mm} . \mathrm{day}^{-1}\right)$.
Subtracting the values obtained from Eqn 3.31 from those of Eqn 3.29 gives the potential transpiration, as mentioned in Eqn 3.12.

For crops with incomplete cover in the first growth stages, some alternative relationships to calculate $E^{*}, E_{\mathrm{pl}}^{*}$ and $E_{\mathrm{s}}^{*}$ have been proposed. Rijtema (1965) introduced therefore an internal canopy resistance $r_{s}$ built up of a stomatal resistance depending on light intensity ( $r_{1}$ ) and a resistance dependent on the fraction of soil covered ( $r_{c}$ )

$$
\begin{equation*}
r_{s}=r_{l}+r_{c} \quad\left(\mathrm{~s} . \mathrm{m}^{-1}\right) \tag{3.32}
\end{equation*}
$$

Taking into account that during periods with precipitation, evaporation increases due to evaporation of intercepted water, he arrived at an expression which can be described as

$$
\begin{equation*}
E^{* *}=\frac{\delta+\gamma}{\delta+\gamma\left(1+r_{s} / r_{a}\right)}\left(E^{*}-E_{i}\right)+E_{i} \quad\left(\mathrm{~kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right) \tag{3.33}
\end{equation*}
$$

where $E_{i}$ is the evaporation flux of the intercepted water. Because $E^{*}$ is the maximum possible evaporation of a cropped surface, $E^{*} \geqslant E_{i}$. Due to the interception total evapotranspiration increases but transpiration reduces, because part of the incoming energy is used for evaporation of intercepted water $\left(E_{i}\right)$. The amount of water intercepted by a crop can be measured by covering the ground around a number of individual plants with plastic sheets. The amounts of water reaching these covers i.e. the throughfall, can be compared with rainfall measurements. In general large errors in estimating $E_{i}$ result in relatively small errors in $E^{* *}$ (see Feddes, 1971). Values of $r_{1}, r_{c}$ and $E_{i}$ are given in Tables 2, 3 and 4, respectively.
Ritchie \& Burnett (1971) proposed to estimate $E_{\mathrm{pl}}^{*}$ from $E^{*}$ for cotton and sorghum, with the expression

$$
\begin{array}{llrl}
E_{\mathrm{p}}^{*}=E^{*}(-0.21+0.70 \sqrt{I}) & \text { for } & 0.1 \leqslant I \leqslant 2.7 \\
E_{\mathrm{pl}}^{*}=0.01 E^{*} & \text { for } & 0 \leqslant I<0.1 \tag{3.35}
\end{array}
$$

Table 2. Diffusion resistance $r_{1}$ depending on mean short-wave radiation flux $\bar{R}_{3}\left[=(1 / N) \int_{0}^{N} R_{3} \mathrm{~d} N\right]$. Adapted from data of Rijtema, 1965.

| $\bar{R}_{s}\left(\mathrm{~W} \cdot \mathrm{~m}^{-2}\right)$ | 100 | 150 | 200 | 250 | $\geqslant 275$ |
| :--- | :--- | :--- | ---: | ---: | ---: |
| $r_{\mathrm{l}}\left(\mathrm{s} \cdot \mathrm{m}^{-1}\right)$ | 237 | 141 | 69 | 10 | 0 |

Table 3. Diffusion resistance $r_{c}$ depending on fraction of soil covered $S_{c}$. Adapted from data of Rijtema \& Ryhiner 1966 and Feddes, 1971.

| $S_{c}$ (fraction) | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r_{c}\left(s . \mathrm{m}^{-1}\right)$ | 336 | 250 | 200 | 168 | 144 | 120 | 82 | 48 | 25 | 14 | 10 | 0 | 0 |


| Table 4. Evaporation flux of intercepted precipitation $E_{i}$ depending on precipitation flux $\boldsymbol{X}$. Adapted from data of |
| :--- |
| Rijtema, 1965 and Feddes, 1971 . |
| $\boldsymbol{x}$ (mm.day ${ }^{-1}$ ) |
| $E_{i}\left(\right.$ mm.day $\left.^{-1}\right)$ |

Tanner \& Jury (1976) found that Eqn 3.34 gave reasonable results also for a potato crop. They showed for a variety of crops and climates that a good alternative for estimating maximum possible evapotranspiration under non-advective conditions is the Priestley \& Taylor (1972) equation

$$
\begin{equation*}
E^{*}=\alpha \frac{\delta}{\delta+\gamma} R_{n} \tag{3.36}
\end{equation*}
$$

which is valid for all leaf area indexes with $\alpha=1.35 \pm 0.10$. For similar type of models for soybean and sorghum, see Kanemasu et al. (1976), for corn see Rosenthal et al. (1977) and for wheat Denmead (1973).
Idso et al. (1977) recently proposed the following expression for the estimation of potential evapotranspiration of different kinds of surfaces based on net solar radiation absorbed by the surface, $R_{\mathrm{s}}$, incoming thermal radiation from the atmosphere, $R_{t a}$, and outgoing thermal radiation from the surface, $R_{\mathrm{tg}}$ :

$$
\begin{equation*}
E^{*}=1.72 \times 10^{-2}\left[R_{\mathrm{sn}}+1.56\left(R_{\mathrm{ta}}-R_{\mathrm{tg}}\right)+156\right] \quad\left(\mathrm{mm} . \mathrm{day}^{-1}\right) \tag{3.3}
\end{equation*}
$$

where $R_{\mathrm{ta}}$ and $R_{\mathrm{tg}}$ are obtained from the daily averages of screen air temperatures and radiometrically measured surface temperatures, respectively.

In most studies direct measurements of $R_{n}$ are not available. Then $R_{n}$ has to be derived with empirical formulae. Net radiation flux can be written as

$$
\begin{equation*}
R_{n}=(1-\nu) R_{s}-R_{t} \quad\left(\mathrm{~W} \cdot \mathrm{~m}^{-2}\right) \tag{3.38}
\end{equation*}
$$

where
$R_{s}=$ flux of incident short-wave radiation
$R_{t}=$ flux of net outgoing thermal radiation
$\nu=$ surface reflection coefficient of short-wave radiation
The coefficient $\nu$ depends on various kinds of conditions as for example the reflectivity of the soil, which varies with its moisture content, soil cover, structure of the surface, latitude, etc. Some values are: 0.23 for a green crop; 0.08 and 0.14 for wet and dry clay, respectively; 0.10 and 0.17 for wet and dry sandy loam, respectively; 0.07 for water (Feddes, 1971).
An empirical expression frequently used for the calculation of $R_{s}$ is the one proposed by Kimball (1927)

$$
\begin{equation*}
R_{\mathrm{s}}=\left(p+q \frac{n}{N}\right) R_{\mathrm{s}}^{\mathrm{top}} \quad\left(\mathrm{~W} \cdot \mathrm{~m}^{-2}\right) \tag{3.39}
\end{equation*}
$$

where $R_{s}^{\text {top }}$ is the extra terrestrial radiation flux at the top of the atmosphere, $n$ is the duration in minutes of bright sunshine in a day length, $N$ is the maximum $n$ can reach on clear days. The values of $N$ and $R_{s}^{\text {top }}$ depend on latitude and time of the year (see Smithson. Meteor. Tables, 1951; Tables 171 and 132). Linacre (1967) presented values for $p$ and $q$ from 39 stations, that are mostly near 0.25 and 0.50 , respectively.

The most applied equation for thermal radiation $R_{t}$ is a Brunttype formula as used by Penman (1948)

$$
\begin{equation*}
R_{t}=5.67 \times 10^{-8} T_{a}^{4}\left(0.56-0.80 \sqrt{e_{d}}\right)\left(0.10+0.90 \frac{n}{N}\right) \quad\left(\mathrm{W} \cdot \mathrm{~m}^{-2}\right) \tag{3.40}
\end{equation*}
$$

Instead of using empirical formulae such as Eqn 3.39 and Eqn $3.40, R_{n}$ is often derived from $R_{s}$ data only. A relation found by Feddes (1971) from experiments in the Netherlands for different crops and soils which agreed strikingly well with Australian data is

$$
\begin{equation*}
R_{n}=0.649 R_{s}-23 \quad\left(\mathrm{~W} \cdot \mathrm{~m}^{-2}\right) \tag{3.41}
\end{equation*}
$$

The formula to be chosen will in practice be determined by the kind of data available.

### 4.1 Initial and boundary conditions

To obtain a solution for the one-dimensional vertical flow equation in the soil-root system, Eqn 3.11 must be supplemented by appropriate initial and boundary conditions.
As initial condition (at $t=0$ ) the pressure head is specified as a function of the depth $z$

$$
\begin{equation*}
\psi(z, t=0)=\psi_{0} \tag{4.1}
\end{equation*}
$$

As hysteresis is not considered in this study, this condition is equivalent to

$$
\begin{equation*}
\theta(z, t=0)=\theta_{0} \tag{4.2}
\end{equation*}
$$

One can then easily obtain the value of $\psi$ (and vice versa) from the expression: $\psi=\mathrm{f}(\theta)$.

To describe the boundary conditions of the depth-time Region $R$ (see Fig. 10) as a function of position on $R_{1}$ and $R_{2}$ (boundaries at $z=0$ and $z=L$ of the Region $R$ satisfying Eqn 3.11) one can distinguish between three types:
(a) Dirichlet condition: specification of the dependent variable, the pressure head


Fig. 10. Depth-time Region under consideration, with upper boundary $\boldsymbol{R}_{1}$ at the soil surface $(z=0)$ and lower boundary $R_{2}$ at the bottom $(z=L)$.
$\psi(z=0, t)=\psi^{U}$
$\psi(z=L, t)=\psi^{L}$
These conditions are equivalent to
$\theta(z=0, t)=\theta^{U}$
$\theta(z=L, t)=\theta^{L}$
(b) Neumann condition: specification of the derivative of the pressure head. For the soil water problem this condition means a specification of the flow through the boundaries (see Eqn 2.18)

$$
\begin{equation*}
q(t)=-K(\psi)\left(\frac{\partial \psi}{\partial z}-1\right) \tag{4.7}
\end{equation*}
$$

(c) 'mixed' condition, a combination of the first two types. In particular this can specify
$\psi$ at the lower boundary
$q$ at the upper boundary
In agrohydrological studies the use of the mixed condition has some advantages:

- the value of $\psi$ at the lower boundary can easily be measured in the field by a piezometer ( $\psi \geqslant 0$ ) or a tensiometer ( $\psi \leqslant 0$ );
- the flux $q$ at the upper boundary is governed by the meteorological conditions. The soil can lose water to the atmosphere by evaporation or gain water by infiltration. While the maximum possible (potential) rate of evaporation from a given soil depends only on atmospheric conditions, the actual flux across the soil surface is limited by the ability of the porous medium to transmit water from below. Similarly if the potential rate of infiltration (e.g. the rain or irrigation intensity) exceeds the absorption capacity of the soil, part of the water will be lost by surface run-off. Here, again, the potential rate of infiltration is controlled by atmospheric (or other) external conditions, whereas the actual flux depends on antecedent moisture conditions in the soil. Thus, the exact boundary condition to be assigned at the soil surface is not known a priori, but a solution must be sought by maximizing the absolute value of the flux (Hanks et al., 1969).

If one takes $q^{*}(z=0, t)$ as the maximum possible flux, the following expression must always be satisfied

$$
\begin{equation*}
\left|q^{*}(z=0, t)\right| \geqslant|q(z=0, t)|=\left|-K(\psi)\left(\frac{\partial \psi}{\partial z}-1\right)\right| \tag{4.8}
\end{equation*}
$$

Also during rainfall, the condition

$$
\begin{equation*}
\psi(z=0, t) \leqslant 0 \quad\left[\text { or } \theta(z=0, t) \leqslant \theta_{s}\right] \tag{4.9}
\end{equation*}
$$

must hold where $\theta_{s}$ is the moisture content at saturation. During evaporation the requirement

$$
\begin{equation*}
\psi(z=0, t) \geqslant \psi_{t} \tag{4.10}
\end{equation*}
$$

holds where $\psi_{i}$ is the minimum pressure head to be allowed under air-dry conditions. Assuming that the pressure head at the soil surface is at equilibrium with the atmosphere, then $\psi_{l}$ can be derived from the well-known relationship

$$
\begin{equation*}
\psi_{l}=\frac{R T}{M g} \ln (F) \tag{4.1}
\end{equation*}
$$

where $R$ is the universal gas constant (J.mole ${ }^{-1} \cdot \mathrm{~K}^{-1}$ ), $T$ is the absolute temperature ( K ), $g$ is acceleration due to gravity ( $\mathrm{m} . \mathrm{s}^{-2}$ ), $M$ is the molecular weight of water ( $\mathrm{kg} \cdot \mathrm{mole}^{-1}$ ) and $F$ is the relative humidity of the air (fraction). From Eqns 4.9 and 4.10 it follows that under all circumstances

$$
\begin{equation*}
\psi_{l} \leqslant \psi \leqslant 0 \tag{4.12}
\end{equation*}
$$

The way the actual flux $q$ can be determined will be explained later in Section 4.2.
In the field one may encounter the following situations (see Fig. 11):

Case A: a semi-infinite soil profile. Here one needs to prescribe the boundary condition at $t=0$ and $z=0$.
Case B: a finite soil profile with a constant depth of the groundwater table or with known values of $\psi\left(z=L^{*}, t\right)$. In addition to the initial condition described above, one must specify the conditions on the upper and the lower boundary as Dirichlet conditions.
Case C: a soil profile without any flow through the lower boundary. Water can only enter or disappear through the upper boundary. This case pertains to a lysimeter (or pot) closed at the bottom. By solving Eqn 3.11 for this problem, the actual depth of the watertable can be estimated.
Case D: a soil profile with a shallow watertable fluctuating with time. This case is equivalent to a lysimeter in which the same


Fig. 11. One-dimensional flow situations that may occur in the field, plotted against time: A, semi-infinite soil profile; B, finite soil profile with constant depth of the groundwater table; C, finite soil profile with no flow through the bottom boundary; D , finite soil profile with a fluctuating groundwater table at the bottom (Case D is discussed in detail in the text).
(fluctuating) watertable is present as in the field, or to an area (e.g. a polder) with ditches or canals. The processes in the unsaturated zone are governed both by the meteorological conditions at the soil surface and the conditions in the saturated zone of the soil. Here one has a mixed type of boundary conditions: a Dirichlet condition at the bottom and a Neumann condition at the soil surface. At the top the additional requirement of Eqn 4.12 must be satisfied. This case will be elaborated in the text.

### 4.2 Finite difference approximation

Eqn 3.11 is a non-linear partial differential equation (PDE) because the parameters $K(\psi), C(\psi)$ and $S(\psi)$ depend on the actual solution of $\psi(z, t)$. The relations between these parameters and the dependent variable are schematically shown in Fig. 12. Also depicted in Fig. 12 are the initial and boundary conditions to be provided. The non-linearity of Eqn 3.11 causes problems in its solution. Analytical solutions are known for special cases only (e.g. Raats, 1974; Lomen \& Warrick, 1978). The majority of practical field problems can only be solved by numerical methods. In this respect one can use either explicit or implicit methods. Although an implicit approach is more complicated, it is preferable because of its better stability and convergence. Moreover, it permits relatively large time steps thus keeping computer costs low.

Let us now solve Eqn 3.11 by a finite difference technique and use the initial and boundary conditions specified in Figs 11D and 12.


Fig. 12. Illustration of the relation between the independent variables, the initial and other boundary conditions, the functional parameters and the dependent variable $\psi$ as used in the partial differential equation PDE (Eqn 3.11).


Fig. 13. Finite difference mesh superimposed on the depth-time Region $\boldsymbol{R}$.

First a grid is laid over Region $R$ occupied by the independent variables $z$ and $t$ (Fig. 13).

With grid spacings $\Delta z$ and $\Delta t$, respectively, one can write for every grid or nodal point ( $j, i$ ):
distance coordinate

$$
\begin{equation*}
z_{i}=\left(j-\frac{1}{2}\right) \Delta z \quad j=1,2, \ldots, N \tag{4.13}
\end{equation*}
$$

time coordinate

$$
\begin{equation*}
t^{i}=\sum_{n=0}^{i} \Delta t^{n} \quad i=0,1,2, \ldots, M \tag{4.14}
\end{equation*}
$$

The partial differential equation (PDE) can be approximated by a finite difference equation (FDE) replacing $\partial t$ and $\partial z$ by $\Delta t$ and $\Delta z$, respectively, in the following way

$$
\begin{equation*}
\mathrm{PDE} \approx \mathrm{FDE} \tag{4.15}
\end{equation*}
$$

which, when expanded, becomes

$$
\begin{align*}
& \frac{\psi_{j}^{i+1}-\psi_{j}^{i}}{\Delta t^{i+1}} \\
\approx & \frac{1}{C_{i}^{i+\frac{1}{2}} \Delta z}\left[\left[K\left(\psi_{j+\frac{1}{2}}^{i+\frac{1}{2}}\right)\left(\frac{\partial \psi}{\partial z}\right)_{j+\frac{1}{2}}^{i+\frac{1}{2}}-K\left(\psi_{j+\frac{1}{2}}^{i+\frac{1}{2}}\right)-0.5 \Delta z S\left(\psi_{j+\frac{1}{2}}^{i+\frac{1}{2}}\right)\right]+\right. \\
& \quad-\left[K\left(\psi_{j-\frac{1}{2}}^{i+\frac{1}{2}}\left(\frac{\partial \psi}{\partial z}\right)_{j-\frac{1}{2}}^{i+\frac{1}{2}}-K\left(\psi_{j-\frac{1}{2}}^{i+\frac{1}{2}}\right)+0.5 \Delta z S\left(\psi_{j-\frac{1}{2}}^{i+\frac{1}{2}}\right)\right]\right] \tag{4.16}
\end{align*}
$$

The derivatives of $\psi$ with respect to $z$ can be written as (see Fig. 14A)

$$
\begin{align*}
& \left(\frac{\partial \psi}{\partial z}\right)_{j+\frac{1}{2}}^{i+\frac{1}{2}} \approx \frac{1}{2 \Delta z}\left[\left(\psi_{j+1}^{i+1}+\psi_{j+1}^{i}\right)-\left(\psi_{j}^{i+1}+\psi_{j}^{i}\right)\right]  \tag{4.17}\\
& \left(\frac{\partial \psi}{\partial z}\right)_{j-\frac{1}{2}}^{i+\frac{1}{2}} \approx \frac{1}{2 \Delta z}\left[\left(\psi_{j}^{i+1}+\psi_{j}^{i}\right)-\left(\psi_{j-1}^{i+1}+\psi_{j-1}^{i}\right)\right] \tag{4.18}
\end{align*}
$$



Fig. 14. A, location of the derivatives of pressure head $\psi$ with respect to depth $z$ in the depth-time diagram (Eqns 4.17 and 4.18); B, location of pressure heads $\psi$ in the depth-time diagram (Eqns 4.19 and 4.20).

The values of $\psi_{i+\frac{1}{2}}^{i+\frac{1}{2}}$ and $\psi_{i-\frac{1}{2}}^{i+\frac{1}{2}}$ can be approximated by (see Fig. 14B)

$$
\begin{align*}
& \psi_{i+\frac{1}{2}}^{i+\frac{1}{2}} \approx \frac{1}{2}\left(1+\frac{\Delta t^{i+1}}{2 \Delta t^{i}}\right)\left(\psi_{j+1}^{i}+\psi_{j}^{i}\right)-\frac{1}{4} \frac{\Delta t^{i+1}}{\Delta t^{i}}\left(\psi_{j+1}^{i-1}+\psi_{j}^{i-1}\right)  \tag{4.19}\\
& \psi_{i-\frac{1}{2}}^{i+\frac{1}{2}} \approx \frac{1}{2}\left(1+\frac{\Delta t^{i+1}}{2 \Delta t^{i}}\right)\left(\psi_{j}^{i}+\psi_{j-1}^{i}\right)-\frac{1}{4} \frac{\Delta t^{i+1}}{\Delta t^{i}}\left(\psi_{j}^{i-1}+\psi_{j-1}^{i-1}\right) \tag{4.20}
\end{align*}
$$

Substitution of Eqns 4.17 to 4.20 in Eqn 4.16 yields the following linear algebraic equation valid for each nodal point

$$
\begin{equation*}
-A_{j} \psi_{j+1}^{i+1}+B_{j} \psi_{j}^{i+1}-D_{j} \psi_{j-1}^{i+1}=F_{j} \tag{4.21}
\end{equation*}
$$

where

$$
\begin{align*}
& A_{i}=\frac{\Delta t^{i+1}}{(\Delta z)^{2} C_{i}^{i+2}} K\left(\psi_{i+\frac{2}{2}}^{i+\frac{1}{2}}\right)  \tag{4.22}\\
& D_{j}=\frac{\Delta t^{i+1}}{(\Delta z)^{2} C_{j}^{i+\frac{1}{2}}} K\left(\psi_{i-\frac{1}{2}}^{i+\frac{1}{2}}\right)  \tag{4.23}\\
& B_{j}=2+A_{i}+D_{i}  \tag{4.24}\\
& F_{j}=A_{i} \psi_{j+1}^{i}+\left(2-A_{j}-D_{j}\right) \psi_{j}^{i}+D_{i} \psi_{j-1}^{i}-2 \Delta z\left(A_{j}-D_{j}\right)+ \\
& -\frac{\Delta t^{i+1}}{C_{j}^{i+\frac{1}{2}}}\left[S\left(\psi_{j+\frac{1}{2}}^{i+\frac{1}{2}}\right)+S\left(\psi_{i-\frac{2}{2}}^{i+\frac{1}{2}}\right)\right]  \tag{4.25}\\
& C_{i}^{i+\frac{1}{2}}=C\left(\psi_{i}^{i+\frac{1}{2}}\right)  \tag{4.2}\\
& \psi_{j}^{i+\frac{1}{2}}=0.5\left(\psi_{j+\frac{1}{2}}^{i+\frac{1}{2}}+\psi_{j-\frac{1}{2}}^{i+\frac{1}{2}}\right) \tag{4.27}
\end{align*}
$$

The time step $\Delta t$ can be estimated according to an expression given by Zaradny (1978)

$$
\begin{equation*}
\Delta t^{i+1}<\frac{\zeta \cdot \Delta z}{|q|^{i}} \tag{4.28}
\end{equation*}
$$

where $q$ is the actual flux at the boundary $R_{1}$ or $R_{2}$ for the previous stage of computation and $\zeta$ is a factor where $0.015<\zeta<0.035$. For problems with rapid variations in boundary conditions (e.g. infiltration), the lower value of $\zeta$ might be taken. Higher values of $\zeta$ can be used if there is only a slow change in boundary conditions (continuous upward flow of water).

When Eqn 4.24 is applied at all nodes of the depth-time diagram, the result is a system of simultaneous linear algebraic equations with a tridiagonal coefficient matrix and unknown values of $\psi$ at time
$i+1$. At that time level, for node $j=1$ and node $J=N$ the boundary conditions to be applied reduce Eqn 4.21 to

$$
\begin{equation*}
-A_{1} \psi_{2}^{i+1}+B_{1} \psi_{1}^{i+1}=F_{1} \quad \text { for } \quad j=1 \tag{4.29}
\end{equation*}
$$

and

$$
\begin{equation*}
B_{N} \psi_{N}^{i+1}-D_{N} \psi_{N-1}^{i+1}=F_{N} \quad \text { for } \quad j=N \tag{4.30}
\end{equation*}
$$

where the values of $F$ depend on the boundary conditions at time $i+\frac{1}{2}$.

For all nodal points $1<j<N$, the hydraulic conductivity $K\left(\psi_{j+}^{i+}\right.$ is taken as the arithmetic mean. At nodal point $j=1$, large differences in boundary conditions can occur and in two vertically adjacent top nodal points large differences in $\psi$ and $K$ can be expected. Following suggestions of Haverkamp \& Vauclin (1978) for the top nodal point, we can take the geometric mean of $K$

$$
\begin{equation*}
K_{\frac{1}{4}}^{i+\frac{1}{2}}=\sqrt{K\left(\psi_{\frac{1}{2}}^{i+\frac{1}{2}}\right) \cdot K\left(\psi_{1}^{i+\frac{1}{2}}\right)} \quad j=1 \tag{4.31}
\end{equation*}
$$

The procedure of maximizing the possible flux through the soil surface as mentioned under Section 4.1 (see Eqns 4.8 to 4.12), leads to the following numerical expression

$$
\begin{align*}
& q_{\frac{1}{2}}^{i+\frac{1}{2}}=K_{\frac{1}{4}}^{i+\frac{1}{2}}\left(\frac{\psi_{1}^{i+\frac{1}{2}}-\psi_{1}^{i+\frac{1}{2}}}{\Delta z / 2}+1\right)  \tag{4.32}\\
& q_{\frac{1}{2}}^{i+\frac{1}{2}}=q_{s}^{*} \text { for }\left|\hat{q}_{\frac{2}{i}}^{i+\frac{1}{2}}\right|>\left|q_{s}^{*}\right| \tag{4.33}
\end{align*}
$$

and

$$
\begin{equation*}
q_{\frac{1}{2}}^{i+\frac{1}{2}}=\hat{q}_{\frac{1}{2}}^{i+\frac{1}{2}} \text { for }\left|\hat{q}_{\frac{1}{2}}^{i+\frac{1}{2}}\right| \leqslant\left|q_{s}^{*}\right| \tag{4.34}
\end{equation*}
$$

where at $z=0$ and $t=t^{i+\frac{1}{2}}$
$q_{s}^{*}=$ potential evaporation or infiltration flux at time $t^{i+\frac{1}{2}}$
$\hat{q}_{\hat{2}}^{i+\frac{1}{2}}=$ maximum flux corresponding to the actual conditions
$q_{\frac{1}{2}}^{i^{+\frac{1}{2}}}=$ actual flux
It is to be noticed that the following expression is always valid

$$
\begin{equation*}
\left|q_{\frac{1}{2}}^{i+\frac{1}{2}}\right| \leq\left|q_{s}^{*}\right| \tag{4.35}
\end{equation*}
$$

In Eqn 4.32 the following values for $\psi_{\frac{1}{2}}^{i+\frac{1}{2}}$ are assumed
$\psi_{\frac{1}{2}}^{i+\frac{1}{2}}=0$ for precipitation
$\psi_{\frac{1}{2}}^{i+\frac{1}{2}}=\psi_{l} \quad$ for evaporation

The value of $\psi_{l}$ can be found from Eqn 4.11 with known values of air temperature and relative humidity. As far as precipitation (irrigation) is concerned, differences between potential and actual flux determine the so-called run-off $\left(R_{\text {off }}\right)$, which is calculated according to

$$
\begin{equation*}
R_{o f f}=\sum_{n=0}^{i}\left(q_{s}^{*}-q_{\frac{1}{2}}^{n+\frac{1}{2}}\right) \Delta t^{n} \tag{4.38}
\end{equation*}
$$

where $q_{s}^{*}$ is the flux at time $t^{n+\frac{1}{2}}$. In matrix notation Eqn 4.21 can be written as

$$
\begin{equation*}
A \stackrel{\rightharpoonup}{\psi}=\stackrel{\rightharpoonup}{F} \tag{4.21a}
\end{equation*}
$$

or as shown in Fig. 15.

$$
\left[\begin{array}{ccccc}
B_{1} & -A_{1} & 0 & \cdot & 0 \\
-D_{2} & B_{2} & -A_{2} & \cdot & \cdot \\
0 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & -D_{N-1} & B_{N-1}-A_{N-1} \\
0 & \cdot & 0 & -D_{N} & B_{N}
\end{array}\right] \times\left[\begin{array}{c}
\vec{\psi} \\
\Psi_{1} \\
\Psi_{2} \\
\cdot \\
\cdot \\
\cdot \\
\Psi_{N-1} \\
\Psi_{N}
\end{array}\right]=\left[\begin{array}{c}
\vec{F} \\
F_{1} \\
F_{2} \\
\cdot \\
\cdot \\
\cdot \\
F_{N-1} \\
F_{N}
\end{array}\right]
$$

Fig. 15. System of simultaneous linear algebraic equations (Eqn 4.21) written in matrix notation as $A \vec{\psi}=\vec{F}$.
$A$ is a tridiagonal coefficient matrix with zero elements outside the diagonals. Values of the elements $B_{j}(1,2, \ldots, N)$ along the principal diagonal are larger than $A_{j}$ and $D_{i}\left(B_{i}=2+A_{j}+D_{i}\right)$. Because the elements $A_{j}, B_{j}$ and $D_{j}$ differ from unity, $A$ is not a unit matrix (sometimes called an identity matrix) and thus a nontrivial solution $\psi_{1}, \psi_{2}, \ldots, \psi_{N}$ exists. In solving this system of equations, a so-called direct method was used by applying a tridiagonal algorithm of the kind discussed by Remson et al. (1971).

II Theory of crop production

The course of dry matter production of a crop ( $Q$ ) with time ( $t$ ) can be presented as an $S$-shape curve (Fig. 16). The curve starts at time zero ( $t_{0}$ ) with an amount of dry matter equal to the quantity of dry matter in seeds or roots ( $Q_{0}$ ). In spring and summer the dry matter yield increases rather quickly, but this levels off towards the end of the growing season.
Since yield is a function of time, one can write the first order differential equation as

$$
\begin{equation*}
\frac{\mathrm{d} Q}{\mathrm{~d} t}=\mathrm{f}(Q, t) \tag{5.1}
\end{equation*}
$$

where $\mathrm{d} Q / \mathrm{dt}$ is the growth rate which is a function of cumulative yield $Q$ and time $t$. If instead of $\mathrm{d} t$ we take a time difference $\Delta t=1$ day, then

$$
\begin{equation*}
\dot{q}=\frac{\Delta Q}{\Delta t} \approx \frac{\mathrm{~d} Q}{\mathrm{~d} t} \tag{5.2}
\end{equation*}
$$

where $\dot{q}$ is the growth rate in kg.ha ${ }^{-1}$. day ${ }^{-1}, Q$ is the yield in $\mathrm{kg} . \mathrm{ha}^{-1}$ and $t$ is the time in days. The growth rate gradually


Fig. 16. Illustration of the course of dry matter production $Q$ of a crop with time $t$.


Fig. 17. Illustration of the variation of growth rate $\dot{q}$ of a crop with time $t$.
increases to a maximum in the summer, then slowly decreases and finally becomes zero at the end of the growing season ( $t_{e}$ ) (Fig. 17).

As $\dot{q}$ is influenced by such growth factors as solar radiation, temperature, water, nutrients, oxygen and carbon dioxide, only when all these factors are adequately available, will potential growth be reached. Both growth rate and yield will be potential ( $\dot{q}_{\mathrm{pot}}$ and $Q_{\text {pot }}$ ). Then this potential growth depends only on the biological growth capacity of the plant. When one of the growth factors is limiting, growth rate and yield are limited ( $\dot{q}_{\text {act }}$ and $Q_{\text {act }}$ ). Although other growth factors may still be optimal, potential growth cannot be reached. The main idea can be illustrated by Fig. 18 (after Gaastra, 1963) where the measured growth rate of a single cucumber leaf is shown in relation to the solar radiation flux involved in photosynthesis and the temperature at a limiting ( $0.03 \%$ ) and a non-limiting ( $0.13 \%$ ) or 'saturated' $\mathrm{CO}_{2}$ concentration. Under conditions of Curve $\mathrm{A}, \mathrm{CO}_{2}$-diffusion is limiting photosynthesis. Increasing the $\mathrm{CO}_{2}$-concentration up to 'saturation' yields Curve B. Here temperature is the limiting factor, since a temperature increase of $10^{\circ} \mathrm{C}$ causes a strong increase in photosynthesis (Curve C).

Growth is proportional to the flux $v_{x}$ of the factor $x$ from the surrounding media into the plant system. Thus under the condition that $\dot{q}_{\text {act }} \leqslant \dot{q}_{\text {pot }}$, the mathematical description must be

$$
\begin{equation*}
\dot{q}_{x}=c_{x} \cdot v_{x} \tag{5.3}
\end{equation*}
$$

where $c_{x}$ is a coefficient of proportionality. This relationship was discovered by Liebig in 1840 and explained in detail by Blackman in 1905. The influence of each growth factor is supposed to proceed


Fig. 18. Growth rate of a cucumber leaf in relation to photosynthetically active radiation flux for a limiting ( $0.03 \%$ ) and a 'saturated' ( $0.13 \%$ ) $\mathrm{CO}_{2}$-concentration at two different temperatures. For Curve A, the limiting growth factor is $\mathrm{CO}_{2}$, for Curve B temperature and for Curve C radiation. After Gaastra, 1963.
according to a simple linear transport function, such as is valid for mass flux and diffusion (for example the Law of Darcy or Fick). The general form is

$$
\begin{equation*}
v_{x}=K_{x} F_{x} \tag{5.4}
\end{equation*}
$$

where $K_{x}$ is a conductivity coefficient and $F_{x}$ is the driving force. The latter can be defined as a gradient of potential (concentration) of a factor $x$ over a distance $\Delta s$, hence

$$
\begin{equation*}
F_{x}=\frac{\Delta x}{\Delta s} \tag{5.5}
\end{equation*}
$$

Substitution of Eqn 5.5 and Eqn 5.4 in Eqn 5.3 yields

$$
\begin{equation*}
\dot{q}_{x}=\frac{c_{x} K_{x}}{\Delta s} \Delta x \tag{5.6}
\end{equation*}
$$

Writing for $c_{x} K_{x} / \Delta s=A_{x}$ one obtains

$$
\begin{equation*}
\dot{q}_{x}=A_{x} \Delta x \tag{5.7}
\end{equation*}
$$

where $A_{x}$ is the slope of the line $\dot{q}_{x}$ versus $\Delta x$ (see Line $l$ in Fig. 19) and an indicator of the productivity of the crop in a certain environment for the growth factor $x$. Line $p$ in Fig. 19 represents the production level under conditions of adequate supply of growth factor $x$ and limited supply of another growth factor $y$. Thus on the linear function of growth factor $x$, a limitation is imposed by growth factor $y$, constituting a ceiling to plant growth. The minimum amount of growth factor $x$ necessary to obtain maximum growth rate is given by the intersection of the lines $l$ and $p$. Here $\dot{q}_{x}=\dot{q}_{x p o r}$. The actual rate of growth $\dot{q}_{\text {act }}$ is usually smaller than $\dot{q}_{\mathrm{x}}$. Thus the slope of the actual growth curve ( $a_{\mathrm{x}}$ ) is smaller than the boundary value $A_{x}$, the latter seeming to depend on the plant species. It can be considered as the maximum efficiency, i.e. the initial slope of the $\dot{q}_{x} / \Delta x$ curve.
From Fig. 19 it follows that

$$
\left(A_{x} \Delta x-a_{x} \Delta x\right)>0
$$

or

$$
\begin{equation*}
A_{x}-a_{x}>0 \tag{5.8}
\end{equation*}
$$

The existence of differences between actual efficiency $a_{\mathrm{x}}$ and maximal efficiency $A_{x}$ can be interpreted in terms of a stress $S_{x}$ in a plant growing under sub-optimal conditions (Visser, 1969). So

$$
\begin{equation*}
S_{x}=A_{x}-a_{x} \tag{5.9}
\end{equation*}
$$

Writing for $a_{\mathrm{x}}=\dot{q}_{\text {acc }} / \Delta x$ Eqn 5.9 yields

$$
\begin{equation*}
S_{x}=A_{x}-\frac{\dot{q}_{a c t}}{\Delta x} \tag{5.10}
\end{equation*}
$$

Similar derivations can be set up for other growth factors $y, z$, etc. Now for convenience, the units of the factors $x, y$ and $z$ on the horizontal axis of the growth rate-growth factor graphs, are chosen in such a way that $\dot{q}_{a c t}=a_{x} \Delta x=a_{y} \Delta y=a_{z} \Delta z$, etc. Thus in ordinary terms it is taken that for plants under an equilibrium of stresses, the growth rate is equal when for example a certain number of kilograms $N$ or a certain number of millimeters $\mathrm{H}_{2} \mathrm{O}$ are added or when the temperature is increased by a certain number of degrees centigrade. So one may write

$$
\begin{equation*}
S_{x}=A_{x}-\frac{\dot{q}_{\text {act }}}{\Delta x} ; \quad S_{y}=A_{y}-\frac{\dot{\dot{q}}_{\text {act }}}{\Delta y} ; \quad S_{z}=A_{z}-\frac{\dot{q}_{\text {act }}}{\Delta z} \tag{5.11}
\end{equation*}
$$

All the stresses $S_{x}, S_{y}, S_{z}, \ldots$, are present in the plants in such a way that a dynamic equilibrium exists, enabling the plant to cope with changing environmental circumstances. Thus the resulting total stress $S$ is as small as possible. According to assumptions of Visser (1969), the equilibrium for different growth factors $j$ depends on the ratio $\mathrm{d} S_{i} / S_{i}$ (relative stress) and the sum of these ratios tends to be equal to zero. This can be expressed as

$$
\begin{equation*}
\sum_{j=1}^{n} \frac{\mathrm{~d} S_{\mathrm{j}}}{S_{j}}=0 \tag{5.12}
\end{equation*}
$$

where $j$ represents the growth factors $x, y, z, \ldots$ Using the previous notations, this is equivalent to


Fig. 19. Actual growth rate $\dot{q}$ of a crop as a function of a certain growth factor $x$. Line $l$ indicates the productivity of the crop for growth factor $x$. Line $p$ represents the production level under conditions of adequate supply of growth factor $x$ and limited supply of some other growth factor $y$.

$$
\begin{equation*}
\frac{\mathrm{d} S_{x}}{S_{x}}+\frac{\mathrm{d} S_{y}}{S_{y}}+\frac{\mathrm{d} S_{z}}{S_{z}}+\cdots=0 \tag{5.13}
\end{equation*}
$$

Integration of Eqn 5.13 yields

$$
\begin{equation*}
\ln S_{x}+\ln S_{y}+\ln S_{z}+\cdots=\ln C \tag{5.14}
\end{equation*}
$$

where $\ln C$ is the integration constant. A similar notation for Eqn 5.14 is

$$
\begin{equation*}
S_{x} \cdot S_{y} \cdot S_{z} \cdots=C \tag{5.15}
\end{equation*}
$$

Substitution of Eqn 5.11 into Eqn 5.15 gives a formulation of crop production

$$
\begin{equation*}
\left(A_{x}-\frac{\dot{q}_{a c t}}{\Delta x}\right)\left(A_{y}-\frac{\dot{q}_{a c t}}{\Delta y}\right)\left(A_{z}-\frac{\dot{q}_{a c t}}{\Delta z}\right) \cdots=C \tag{5.16}
\end{equation*}
$$

Eqn 5.16 gives $\dot{q}_{\text {act }}$ as a function of the growth factors $x, y, z$. It is written in a rather complicated implicit form, but in principle it is an $n$-order polynomial, i.e. a non-rectangular hyperboloid in a $n$ dimensional space.
In this study the growth factors are split up in two groups, one consisting of the growth factor water only, the other of all other growth factors together. This division reduces Eqn 5.16 to

$$
\begin{equation*}
\left(A_{x}-\frac{\dot{q}_{a c t}}{\Delta x}\right)\left(A_{y}-\frac{\dot{q}_{\text {act }}}{\Delta y}\right)=C^{\prime} \tag{5.19}
\end{equation*}
$$

Dividing the left and right hand terms of Eqn 5.17 by $A_{x} A_{y}$ gives

$$
\begin{equation*}
\left(1-\frac{\dot{q}_{\text {act }}}{A_{x} \Delta x}\right)\left(1-\frac{\dot{q}_{\text {act }}}{A_{y} \Delta y}\right)=\xi \tag{5.18}
\end{equation*}
$$

where $\xi$ is a mathematical flexibility constant which should be close to zero. Eqn 5.18 is a second-order polynomial, i.e. a nonrectangular hyperbola in the two-dimensional space.
Assuming that Fig. 19 depicts the situation that all growth factors except water are at their optimum, the given curve represents the top level of a vertical sequence of curves as given in Fig. 18. Then from Fig. 19 (where $\xi=0$ ), it is clear that the actual growth rate is delimited by two asymptotes: one that shows a proportional increase of the growth rate with increasing supply of the growth factor water $w\left(A_{x} \Delta x=A w\right)$ and the second one, imposed by all growth factors together, that limits the growth rate to a certain maximum or ceiling level ( $\dot{q}_{\text {xpot }}=\dot{q}_{\text {pot }}$ ). Then Eqn 5.18 can be written as

$$
\begin{equation*}
\left(1-\frac{\dot{q}_{a c t}}{A w}\right)\left(1-\frac{\dot{q}_{a c t}}{\dot{q}_{p o t}}\right)=\xi \tag{5.19}
\end{equation*}
$$

where $0<\xi \lll 1$.
After multiplication and rearrangement Eqn 5.19 becomes

$$
\begin{equation*}
\dot{q}_{\mathrm{act}}^{2}-\dot{q}_{\mathrm{act}}\left(\dot{q}_{\mathrm{pot}}+A w\right)+A w \cdot \dot{q}_{\mathrm{pot}}(1-\xi)=0 \tag{5.20}
\end{equation*}
$$

which gives for the two asymptotes of the hyperbola

$$
\begin{align*}
& \dot{q}_{a c t}=\dot{q}_{p o t}(1-\xi)  \tag{5.21}\\
& \dot{q}_{a c t}=A w+\dot{q}_{p o t} \xi \tag{5.22}
\end{align*}
$$

The graphical interpretation of Eqn 5.20 is shown in Fig. 20. If $\xi=0$, asymptotes are as shown in Fig. 19, which implies the so-called Blackman's response to growth factors.

For the growth factors light and carbon dioxide, Rabinowitch (1951) found relationships similar to Eqn 5.20. Recently Thornley (1976), working on Rabinowitch's theory, derived an equation for plant response to growth factors also similar to Eqn 5.20.


Fig. 20. Actual growth rate $\dot{q}$ versus the growth factor water $w$ described as a non-rectangular hyperbola (Eqn 5.20) bounded by the asymptotes $l$ and $p$.

If one is working with consecutive separate time intervals denoted by $i$, Eqn 5.20 must be evaluated for separate days

$$
\begin{equation*}
\left(\dot{q}_{\text {act }}^{i}\right)^{2}-\dot{q}_{a c t}^{i}\left(\dot{q}_{\mathrm{pot}}^{i}+A w^{i}\right)+A w^{i} \cdot \dot{q}_{\mathrm{pot}}^{i}(1-\xi)=0 \tag{5.23}
\end{equation*}
$$

where $i$ is an arbitrary day of the growing period ( $i=1,2,3, \ldots, n$ day). The graphical interpretation of Eqn 5.23 is shown in Fig. 21. The values of $A$ and $\xi$ are taken to be independent of time.

From the above it would seem that in the approach presented only the effect of water and potential growth rate is taken into account. But in the elaboration given in Chapter 7, it will be shown that the effects of actual radiation, air temperature, respiration losses and soil cover on potential growth are also included. The values of $\dot{q}_{\text {pot }}^{i}$ are thus taken to vary with time: $\left(\dot{q}_{\text {pot }}^{1}, \dot{q}_{\text {pot }}^{2}, \ldots, \dot{q}_{\text {pot }}^{n}\right)$.
Eqn 5.23 is of the type

$$
a x^{2}+b x+c=0
$$



Fig. 21. Actual growth rate $\dot{q}$ versus the growth factor water $w$ for arbitrary days of the growing period showing different potential production levels.

Because the condition holds that $0<x\left(=\dot{q}_{\text {act }}^{i}\right) \leqslant \dot{q}_{\text {pot }}^{i}$, only one root of the quadratic equation is valid

$$
x_{1}=\frac{-b-\sqrt{b^{2}-4 a c}}{2 a}
$$

The solution of Eqn 5.23 then becomes

$$
\begin{equation*}
\left.\dot{q}_{\mathrm{act}}^{i}=\frac{A}{2} w^{i}+\frac{\dot{q}_{\mathrm{pot}}^{i}}{2}-\frac{1}{2}\left[\dot{q}_{\mathrm{pot}}^{i}+A w^{i}\right)^{2}-4 \dot{q}_{\mathrm{pot}}^{i} A w^{i}(1-\xi)\right]^{\frac{1}{2}} \tag{5.24}
\end{equation*}
$$

In principle the parameter $\xi$ is to be determined from field experiments. However, in our study $\xi$ is taken to be a constant during the growing period, e.g. $\boldsymbol{\xi}=0.01$.

The final yield $Q_{\text {act }}$ then can be calculated as the sum of the daily growths over the growing period

$$
\begin{equation*}
Q_{a c t}=\sum_{i=1}^{n} \dot{q}_{a c t}^{i} \cdot \Delta t \tag{5.25}
\end{equation*}
$$

In a similar way one can calculate the potential yield

$$
\begin{equation*}
Q_{\mathrm{pot}}=\sum_{i=1}^{n} \dot{q}_{\mathrm{pot}}^{i} \cdot \Delta t \tag{5.26}
\end{equation*}
$$

where $\Delta t$ in both equations represents a period of 1 day.

The relationship between water use by plants and field water requirements has been intensively investigated over the years. According to Stanhill (1973), meteorological literature on water loss by evapotranspiration now totals about 18000 items, increasing with papers of in total more than 3000 pages every year. For a recent review on crop water requirements and their application in irrigation schemes, the reader is referred to Doorenbos \& Pruitt (1977).

The terminological suggestion of Stanhill (1973) is accepted throughout this study, that 'evapotranspiration' should be reserved for the total water loss to the atmosphere per unit ground surface, 'evaporation' for water loss to the atmosphere from bare soil or free water, and 'transpiration' to be retained to describe the loss of water vapour to the atmosphere through plant surfaces. In many publications this distinction is not made and water use by plants is considered as total evapotranspiration also called 'water consumptive use'. This approach is too simple, because one might then get the idea that water use is not related to crop yield at all.

In Chapter 5, we have seen that growth and yield are proportional to the amount of the various growth factors taken up by the plant. Thus for water crop yield is simply directly proportional to the amount of water used by the crop, i.e. to the total transpiration

$$
\begin{equation*}
Q=A_{1} W \tag{6.1}
\end{equation*}
$$

where $Q$ is in $\mathrm{kg} . \mathrm{ha}^{-1}$ and cumulative plant transpiration $W$ is in mm . Therefore $A_{1}$ must be in units of $\mathrm{kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1}$. Stanhill (1960) did show for pastures that the slope $A_{1}$ depends on climate and that it changes with latitude. De Wit (1958) found that Eqn 6.1 is valid for temperate climates and from experiments in the Netherlands reported that $A_{1}=26$ for oats, $A_{1}=61$ for sugar-beet and $A_{1}=34 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1}$ for peas. He pointed out that 'in this approach no attention has been paid to the distribution of water during growth, because the value of $A_{1}$ depends not or to a small extent only on the age of the plant'. Thus one can derive from Eqn 6.1 that

$$
\begin{equation*}
\mathrm{d} Q / \mathrm{d} t=A_{1}(\mathrm{~d} W / \mathrm{d} t) \tag{6.2}
\end{equation*}
$$

which more generally can be written as

$$
\begin{equation*}
\dot{q}=A_{1} E_{p l} \tag{6.3}
\end{equation*}
$$

where $\dot{q}$ is the growth rate ( $\mathrm{kg.ha}^{-1} \mathrm{dav}^{-1}$ ) and $E_{\mathrm{pl}}$ is the transpiration rate of the crop ( $\mathrm{mm} \cdot \mathrm{day}^{-1}$ ). So there exists more or less the same relationship between $\dot{q}$ and $E_{p l}$ as between $Q$ and $W$.

For arid areas, de Wit (1958) found that yield and transpiration could be related as

$$
\begin{equation*}
Q=A_{2} \frac{W}{E_{o}} \tag{6.4}
\end{equation*}
$$

where $E_{o}$ is the evaporation rate from a free water surface $\left(\mathrm{mm} . \mathrm{day}^{-1}\right.$ ), being a measure of the solar radiation intensity (i.e. the transpiration demand of the atmosphere). Here $A_{2}$ has the dimension of kg.ha ${ }^{-1} \cdot \mathrm{day}^{-1}$. He reported for wheat $A_{2}=115$, for sorghum $A_{2}=207$ and for alfalfa $A_{2}=55 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{day}^{-1}$. Furthermore he showed that the above mentioned relationships are hardly affected by small variations in water potential and nutrient level, because the resulting changes in increase of leaf area affect both evaporation and photosynthesis in the same way.

As the relationships between yield and transpiration discussed so far seem to be different for humid and arid locations at similar latitudes, Bierhuizen \& Slatyer (1965) proposed to use another relationship depending on the vapour pressure deficit of the air. Transpiration ( $E_{\mathrm{pl}}$ ) and photosynthesis ( $\dot{q}$ ) can be described in terms of molecular diffusion equations depending on a gradient and a resistance according to

$$
\begin{align*}
& E_{p l}=\frac{\varepsilon \rho_{a}}{p_{a}} \frac{\Delta e}{r_{a}+r_{s}}  \tag{6.5}\\
& \dot{q}=\frac{\Delta C O_{2}}{r_{a}^{\prime}+r_{s}^{\prime}+r_{m}^{\prime}}  \tag{6.6}\\
&\left(\mathrm{kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right) \\
&\left(\mathrm{kg} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~s}^{-1}\right)
\end{align*}
$$

where $\Delta e$ is the vapour pressure gradient between leaves and air; $\Delta \mathrm{CO}_{2}$ the difference in carbon dioxide concentration; $r_{a}, r_{s}$ and $r_{m}$ are the resistances of the laminar boundary layer, the stomata and the mesophyll, respectively; the diffusion resistances for $\mathrm{CO}_{2}$ transport $\left(r^{\prime}\right)$ are related to the one for water vapour ( $r$ ) by the ratio of their diffusion coefficients ( $r^{\prime} / r=D_{\mathrm{CO}_{2}} / D_{\mathrm{H}_{2} \mathrm{O}}$ ). Now one can write

$$
\begin{equation*}
\frac{E_{p l}}{\dot{q}}=\left(\frac{\varepsilon \rho_{a}}{p_{a}} \frac{r_{a}^{\prime}+r_{s}^{\prime}+r_{m}^{\prime}}{r_{a}+r_{s}} \frac{1}{\Delta C O_{2}}\right) \Delta e \tag{6.7}
\end{equation*}
$$

In the field $\mathrm{CO}_{2}$ conditions are nearly constant. If we take the ratio of the resistances to be approximately constant for a certain crop, Eqn 6.7 reduces to

$$
\begin{equation*}
\dot{q}=A \frac{E_{p l}}{\Delta e} \tag{6.8}
\end{equation*}
$$

For $\Delta e$, one can take the difference between the saturated and actual water vapour pressure in the air. (In fact one can only do this when the temperature of the leaves equals that of the air and when the substomatal cavities inside the leaves are saturated with water vapour.)
Now $\dot{q}$ is in units of $\mathrm{kg} . \mathrm{ha}^{-1}{\text {. } \mathrm{day}^{-1} \text {, } E_{p l} \text { in mm.day }}^{-1}$ and $\Delta e$ in mbar, $A$ must be in units of $\mathrm{kg} \cdot \mathrm{ha}{ }^{-1} \cdot \mathrm{~mm}^{-1}$.mbar. Climatic zones showing different $\Delta e$ will result in different values of $\dot{q}$ for the same. $E_{p l}$. From 'water consumptive use' data, the following values for $A$ have been found (Table 5).

Table 5. Some examples of maximum water use efficiencies A (kg.ha ${ }^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar}$ ) for a number of crops as found in the literature

| Crop | A | Author |
| :--- | :--- | :--- |
| Dwarf French beans | 155 | Feddes (1971) |
| Red cabbage | 100 | Feddes (1971) |
| Celery | 107 | Feddes (1971) |
| Grass | 68.5 | Rijtema (1969) |
| Lilies* | 156 | van der Valk (1978) |
| Potatoes** | 154 | Rijtema \& Endrödi (1970) |
| Tulips | 245 | van der Valk (1978) |

## * With exclusion of roots

** For an 'average' year

Recently there has been some indication that $A$ differs for different climatic circumstances (Slabbers et al., 1978). This idea would imply that $A$ would be higher for a 'dry' year than for an 'average' or a 'wet' year. For data on alfalfa, on sorghum and on maize, see Slabbers et al. (1978).
From Eqn 6.8 one can write for conditions of potential transpiration $E_{p l}^{*}$

$$
\begin{equation*}
\dot{q}_{\mathrm{pot}}=A^{*} \frac{E_{\mathrm{pl}}^{*}}{\Delta e} \tag{6.9}
\end{equation*}
$$

and thus derive that relative yield is related to relative transpiration according to

$$
\begin{equation*}
\frac{\dot{q}}{\dot{q}_{p o t}}=B \frac{E_{p l}}{E_{p l}^{*}} \tag{6.10}
\end{equation*}
$$

where $B=1$ if $A=A^{*}$ and $B \neq 1$ if $A \neq A^{*}$.
For a crop without water sensitive growth stages (i.e. effects of water stress on yield during all growth stages are similar), $B$ can be considered as a constant. For crops showing different effects of water stress during various physiological stages of growth, $B$ may vary. For the implications on the calculation of the final yield, see Jensen (1968). Stewart et al. (1973) report for maize $B=1.26$ and for sorghum $B=0.98$.

In this study we consider the influence of the growth factor water on growth according to Eqn 6.8. The value of $A$ in this equation generally has to be determined from field experiments. When in Eqn 5.24 the growth factor $w^{i}$ is replaced by $E_{p i}^{i} / \Delta e^{i}$, the actual growth rate for an arbitrary day $i$ of the growing period is

$$
\begin{equation*}
\dot{q}_{\mathrm{act}}^{i}=\frac{A}{2} \frac{E_{\mathrm{pl}}^{i}}{\Delta e^{i}}+\frac{\dot{q}_{\mathrm{pot}}^{i}}{2}-\frac{1}{2}\left[\left(\dot{q}_{\mathrm{pot}}^{i}+A \frac{E_{\mathrm{pl}}^{i}}{\Delta e^{i}}\right)^{2}-4 \dot{q}_{\mathrm{pot}}^{i} A \frac{E_{\mathrm{pl}}^{i}}{\Delta e^{i}}(1-\xi)\right]^{\frac{1}{2}} \tag{6.11}
\end{equation*}
$$

where $A$ has the dimension of $\mathrm{kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar}, E_{\mathrm{pl}}$ of $\mathrm{mm} \cdot \mathrm{day}^{-1}$, $\Delta e$ of mbar, $\dot{q}_{\mathrm{act}}$ and $\dot{q}_{\mathrm{pot}}$ of $\mathrm{kg} \cdot \mathrm{ha}^{-1} . \mathrm{day}^{-1}$.

The potential growth rate $\dot{q}_{\text {pot }}$ appears in Eqn 6.11 as an input value. In the following a procedure will be outlined to calculate this potential production rate.

Photosynthesis is the process by which radiant energy is converted into chemical energy by the reduction of $\mathrm{CO}_{2}$ in the presence of $\mathrm{H}_{2} \mathrm{O}$ to carbohydrates, $\mathrm{CH}_{2} \mathrm{O}$

$$
\underset{\text { from air) }}{\mathrm{CO}_{2}}+\underset{\text { (from soil) }}{\mathrm{H}_{2} \mathrm{O}} \xrightarrow[\text { radiation }]{\text { solar }} \underset{\text { (plant biomass) (into air) }}{\mathrm{CH}_{2} \mathrm{O}}+\underset{\mathrm{O}_{2}}{\mathrm{O}^{2}}
$$

This process occurs in the chloroplasts of green plants. Because energy is needed for the maintenance of the structure (constant turnover of plant constituents) and growth (synthesis of new material) (Hansen \& Jensen, 1977), some of the stored carbohydrates are oxidized to deliver the required energy by the process of respiration: $\mathrm{CH}_{2} \mathrm{O}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{O}$. For a model approach to describe respiration, see e.g. McCree (1970). The difference between gross photosynthesis and respiration is called net photosynthesis.

Gross potential photosynthesis of a crop canopy can be calculated according to a model of de Wit (1965) taking into account the height of the sun, the condition of the sky, the canopy architecture and the photosynthesis function of the individual leaves. Considering light energy as the main factor in production, de Wit (1965) developed a model to calculate gross potential photosynthesis rates of a 'standard canopy' from production rates on both clear and overcast days for any day and any place. A 'standard canopy' is defined as a canopy with a leaf area index $I=5$ ( 5 ha of leaves over 1 ha of soil surface) that is fully supplied with nutrients and water. The results of de Wit can be illustrated by Fig. 22 pertaining to the Netherlands $\left(52^{\circ} \mathrm{N}\right.$ latitude), which shows the variation of light energy on clear days ( $R_{c}$ ) over the year. The light energy on overcast days is assumed to be $0.2 R_{c}$. De Wit, used this assumption and certain energy and leaf distributions he developed, to calculate gross growth rates for clear days $\left(P_{c}\right)$ as well as overcast days ( $P_{0}$ ) (Fig. 22). From this graph it can be seen that $P_{0} \approx 0.5 P_{c}$.


Fig. 22. Annual variation of gross photosynthesis rate at $52^{\circ}$ North latitude (Netherlands) of a 'standard canopy' on clear days ( $P_{c}$ ), and on overcast days ( $P_{o}$ ). Also shown is the variation of the solar radiation flux $\left(R_{c}\right)$ involved in photosynthesis on clear days. After de Wit, 1965.

The gross potential growth rate of the standard canopy $\left(P_{\mathrm{st}}\right)$ on an arbitrary day is found from the expression

$$
\begin{equation*}
P_{\mathrm{st}}=\Lambda \cdot P_{0}+(1-\Lambda) P_{c} \quad\left(\mathrm{~kg}_{\mathrm{c}} \cdot \mathrm{ha}^{-1} \cdot \mathrm{day}^{-1}\right) \tag{7.2}
\end{equation*}
$$

where $\Lambda$ is the fraction of time the sky under the actual conditions is overcast. The value of $\Lambda$ can be taken either from cloud cover measurements or from the expression

$$
\begin{equation*}
\Lambda=\frac{R_{c}-R}{0.8 R_{c}} \quad(-) \tag{7.3}
\end{equation*}
$$

The light energy flux on an arbitrary day ( $R$ ) is taken to be half of the solar radiation flux $R_{s}$, so $R=0.5 R_{s}$. The values of $P_{c}$ and $P_{0}$ for a certain latitude and day can be derived from figures such as Fig. 23 by reading the graphs at the $R_{c}$-value of that specific day.

In Table 6, $R_{c}, P_{0}$ and $P_{c}$ values are given at various times of the year for a number of different latitudes. Under actual field condi-


Fig. 23. Gross photosynthesis rate at $52^{\circ}$ North latitude (Netherlands) of a 'standard canopy' on clear days ( $P_{c}$ ) and on overcast days ( $P_{o}$ ) in relation to solar radiation flux $R_{c}$ involved in photosynthesis on clear days. After de Wit, 1965.
tions, the photosynthesis rates mentioned in Table 6 are not reached. They must be corrected for various reasons:

Respiration Respiration is only slightly dependent on the amount of biomass. It is about 20 to $50 \%$ of the gross potential production. From field trials in the Netherlands, Sibma (1968) found that a green crop surface from the time the soil is completely covered until maturity, produces on the average $225 \mathrm{~kg} \cdot \mathrm{ha}^{-1}$.day ${ }^{-1}$. As the gross potential rate is about $320 \mathrm{~kg} \cdot \mathrm{ha}^{-1}$. $\mathrm{day}^{-1}$, the respiration loss is about $30 \%$. The lowest respiration losses reported in literature are about $20 \%$. For some examples of respiration coefficients of various types of crops, see Table 7. Thus to account for respiration losses, one must multiply gross photosynthesis rates by a so-called photorespiration factor, $\phi_{r}$, where $0.4<\phi_{r}<0.8$.

Temperature Photosynthesis varies with temperature. Fig. 24 shows the influence of air temperature on the photosynthesis of the

Table 7. Some examples of respiration losses (\%) for a number of crops as found in the literature

| Crop | Respiration losses | Author |
| :--- | :---: | :--- |
| Alfalfa (lucerne) | $35-49$ | Gaastra (1963) |
| Dwarf French beans | 33 | Feddes (1971) |
| Sugar-beet | $29-33$ | Gaastra (1963) |
| Sugar-beet | 23 | Penning de Vries (1972) |
| Red cabbage | $49-60$ | Feddes (1971) |
| Celery | $49-58$ | Feddes (1971) |
| Grasses | $30-50$ | de Wit (1969) |
| Italian ryegrass | $20-30$ | Hansen and Jensen (1977) |
| Maize | 28 | Penning de Vries (1972) |
| Potatoes | $20-25$ | Burton (1964) |
| Potatoes | 32 | Rijtema \& Endrödi (1970) |
| Tulips | 20 | van der Valk (1978) |
| Winterwheat | $25-35$ | see Sibma (1977) |
| Winterwheat | 47 | Hodges \& Kanemasu (1977) |

[^0]

Fig. 24. Influence of temperature, as indicated by the temperature parameter $\alpha_{\mathrm{T}}$, on photosynthesis of red cabbage (after Wiebe, 1975), of potatoes (after Winkler, 1961) and of grass (after Goudriaan, 1973).
three crops treated in this study. The parameter $\alpha_{T}$ indicates the limiting effect of temperature on growth. If the temperature is optimal for growth, $\alpha_{T}=1$. For the temperature function of alfalfa (lucerne), sorghum and maize see e.g. Slabbers et al. (1978), for lettuce, tomato, cucumber and melon Klapwijk (1969), for grasses Saugier (1974).

Soil cover During the beginning and first part of the growing season, photosynthesis is only performed by the fractional area of the soil covered by plants ( $S_{c}$ ). Multiplying gross photosynthesis rate by $S_{c}$, gives the correction for soil cover. A qualitative example of the variation in $S_{c}$ with time is presented in Fig. 25.


Fig. 25. Example of the variation in soil cover $S_{c}$ of a crop with time $t$; $I=$ leaf area index.

Harvested part Gross potential production rates are calculated for shoots plus roots. If one wants to know only the dry matter production of for example the above ground dry matter, one has to apply a correction for the amounts of roots. Therefore we introduced the shoot/(shoot + root) parameter $\beta_{h}$, with which the gross potential growth rate can be multiplied. This ratio can also be interpreted as the harvested part/total plant ratio. It is to be noted that $\beta_{h}$ may vary considerably during the growing season (Fig. 26). In spring, roots generally constitute the main part of the plant, while later on the ratio gradually decreases in favour of the shoots. Some data mentioned in literature are presented in Table 8.


Fig. 26. Example of the variation in the shoot/(shoot + root ) parameter $\beta_{h}$ of a crop with time $t$.

Table 8. Some examples of the harvested part/total plant ratio, $\boldsymbol{\beta}_{h}$, for a number of crops as found in the literature

| Crop | $\beta_{h}$ | Author |
| :--- | :---: | :--- |
| Alfalfa, first year of |  |  |
| $\quad$ growth | $0.25-0.50$ | see Slabbers et al. (1978) |
| Alfalfa, well established | $0.25-0.95$ | see Slabbers et al. (1978) |
| Dwarf French beans | 0.92 | Feddes (1971) |
| Sugar-beet | 0.67 | see Sibma (1977) |
| Red cabbage | 0.90 | Feddes (1971) |
| Grass swards | 0.60 | Alberda \& de Wit (1961), |
|  |  | Kowalik (1973) |
| Maize | $0.50-0.90$ | see Slabbers et al. (1978) |
| Potatoes | 0.85 | see Sibma (1977) |
| Sorghum | $0.50-0.87$ | see Slabbers et al. (1978) |
| Winterwheat | 0.40 | see Sibma (1977) |

Taking into account all the factors mentioned above, one can calculate the potential dry matter production, $\dot{q}_{p o t}$, according to

$$
\begin{equation*}
\dot{q}_{\mathrm{pot}}=P_{\mathrm{st}} \cdot \phi_{\mathrm{r}} \cdot \alpha_{\mathrm{T}} \cdot S_{c} \cdot \beta_{\mathrm{h}} \tag{7.4}
\end{equation*}
$$

Substitution of Eqn (7.2) in Eqn (7.4) yields

$$
\begin{equation*}
\dot{q}_{\mathrm{pot}}=\left[\Lambda P_{0}+(1-\Lambda) P_{c}\right] \cdot \phi_{r} \cdot \alpha_{T} \cdot S_{c} \cdot \beta_{h} \tag{7.5}
\end{equation*}
$$

In Fig. 27 the entire procedure is summarized by a Forrester notation. The rate of transformation is indicated by valve symbols, the flow of material or energy by solid lines and the cumulative values by rectangles.


Fig. 27. Flow chart (in Forrester notation) of the transformation of solar radiation into actual crop yield.

The values of $\phi_{r}, \alpha_{T}, S_{c}$ and $\beta_{h}$ are a priori information and inputs into the model. So one must realize that in this model there is no feedback with calculated actual production rates.

## III The programs

## 8 Program for field water use, SWATR

### 8.1 General description

The program SWATR is written in FORTRAN IV and was run on a CYBER 72 computer. It can be applied to problems dealing with the transient water flow in a non-homogeneous soil-root system which is under groundwater influence (see the flow chart, Fig. 28). It goes without saying that simpler flow cases (no roots present, without groundwater table, etc.) can also be handled by the program.

For convenience, we use in the program 'suctions' or 'tensions' $h$, which are always positive, instead of pressure heads $\psi$.

### 8.1.1 Coding of the program

The program has a special array, entitled KOD, which gives the user a selection of inputs to choose from. KOD sets up 6 elements, having the following characteristics:

KOD (1) $\begin{array}{r}-1-h \text { as a function of } \theta \\ K \text { as a function of } h\end{array}$
$2-h$ as a table of $\theta$ $K$ as a table of $\theta$
$\operatorname{KOD}$ (2) $\left[\begin{array}{l}0-\text { calculations start from time } t=0 \\ 1-\text { calculations start from } t>0\end{array}\right.$
KOD (3) $\left[\begin{array}{l}0-\text { depth root zone is varying with time } \\ 1-\text { depth root zone is constant with time }\end{array}\right.$
$[0-z=0$ at the soil surface so vertical flow with coordinate $z$ is positive downwards
KOD (4) 1-horizontal flow (not yet included in present program)
$2-z=0$ at the bottom of the system so vertical flow with $z$ positive upwards (not yet included in present program)


KOD (5) $\left[\begin{array}{l}0 \text {-initial value is given as a value of } \theta \\ 1 \text {-initial condition is given as a value of } h\end{array}\right.$
[ 0 - prescribed value of potential transpiration rate and water content at the surface at various moments of time

- 1 -prescribed soil surface flux, maximum suction value at the surface and potential transpiration rate all at various moments of time
KOD (6) $\quad 2$ - prescribed values of soil surface flux, maximum suction value at the surface and potential transpiration rate are calculated by SWATR from meteorological and external conditons as functions of time.


### 8.1.2 Soil physical properties

In the program SWATR Eqns 2.6 to 2.16 are given as follows:
Eqns 2.6-2.8

$$
\begin{align*}
& \text { upper layer (index U) } \\
& \mathrm{CU}=\mathrm{FAC} * \mathrm{CSAT} 1 \text { for } \mathrm{SS} \leqslant \mathrm{CUA} 1 \\
& \mathrm{CU}=\mathrm{FAC} * \mathrm{CSAT} 1 * \operatorname{EXP}(-\mathrm{CUA} 2 *(S S-C U A 1)) \text { for CUA1 } \\
& \text { <SS<CUB1 (2.7) } \\
& \mathrm{CU}=\mathrm{FAC} * \mathrm{CUB} 2 *(\mathrm{SS} * *(-1.4)) \text { for } \mathrm{SS} \geqslant \mathrm{CUB} 1 \\
& \text { lower layer (index L) } \\
& \mathrm{CL}=\mathrm{FAC} * \mathrm{CSAT} 2 \text { for } \mathrm{SS} \leqslant \mathrm{CLA} 1  \tag{2.6}\\
& \mathrm{CL}=\mathrm{FAC} * \operatorname{CSAT} 2 * \operatorname{EXP}(-\mathrm{CLA} 2 *(S S-C L A 1)) \text { for CLA1 } \\
& <\text { SS < CLB1 (2.7) } \\
& \mathrm{CL}=\mathrm{FAC} * \mathrm{CLB} 2 *(\mathrm{SS} * *(-1.4)) \text { for } \mathrm{SS} \geqslant \mathrm{CLB} 1 \tag{2.8}
\end{align*}
$$

Eqns 2.9-2.14

$$
\begin{align*}
& \text { upper layer (index U) } \\
& \text { SU }=\operatorname{EXP}(\text { SUA1 } *(\text { SUB1 }-W) \text { ) for } \text { SUC } \leqslant W \leqslant \text { SUB1 }  \tag{2.9}\\
& \text { SU }=\text { EXP(SUA2 } *(S U B 2-W) \text { ) for } \operatorname{SUD} \leqslant W<\text { SUC }  \tag{2.10}\\
& \text { W < SUD }  \tag{2.11}\\
& \text { CU }=\text { FAC } * \operatorname{CSAT} 1 * \operatorname{EXP}(-\mathrm{CUA} 1 *(S S-C U B 1)) \text { for SS } \\
& \leqslant \text { CUC }  \tag{2.12}\\
& \mathrm{CU}=\mathrm{FAC} * \operatorname{CSAT} 1 * \operatorname{EXP}(-\mathrm{CUA} 2 *(\mathrm{SS}-\mathrm{CUB} 2) \text { ) for CUC } \\
& \text { <SS<CUD }  \tag{2.13}\\
& \mathrm{CU}=\mathrm{FAC} *(\mathrm{CUA} 3+\mathrm{CUB} 3 * \text { ALOG10(SS)) } \\
& \text { *(SS**(-1.4)) for } \mathrm{SS} \geqslant \text { CUD } \tag{2.14}
\end{align*}
$$


where $\mathrm{FAC}=$ factor depending on the units used in the problem
Eqn 2.15

```
upper layer (index U)
    W(I) = SUB(I) - ALOG(SS)/SUA(I) where I=1,2,3
    and e.g. SUB(1)=SUB1, etc. and e.g. \(\operatorname{SUB}(1)=\) SUB1, etc.
```

```
lower layer (index L)
```

lower layer (index L)
W(I) = SLB(I) - ALOG(SS)/SLA(I) where I = 1, 2, 3
W(I) = SLB(I) - ALOG(SS)/SLA(I) where I = 1, 2, 3
and e.g. SLB(1) = SLB1, etc.
and e.g. SLB(1) = SLB1, etc. and e.g. $\operatorname{SLB}(1)=$ SLB1, etc.

```
Eqn 2.16

Eqn 2.16 expresses the differential moisture capacity \(C_{h}\), if suction is given in the form of Eqns 2.9-2.11.
```

upper layer (index U)
$\mathrm{CHU}=-1.0 /(\mathrm{SUA}(\mathrm{I}) * \mathrm{SS}) \quad$ where $\quad \mathrm{I}=1,2,3$
and SUA(1) = SUA1, etc.

```
lower layer (index L)
\(\mathrm{CHL}=-1.0 /(\mathrm{SLA}(\mathrm{I}) * \mathrm{SS}) \quad\) where \(\quad \mathrm{I}=1,2,3\) and SLA(1) \(=\) SLA1, etc.

\subsection*{8.1.3 Discretization of the soil profile}

The program SWATR has been designed for a two-layered soil profile (Fig. 29) because such situations often occur, for example:
- the upper layer is different in texture and structure from the lower layer,


Fig. 29. Schematic representation [for \(K O D(4)=0\), depth \(x\) and time \(t\) ] of the layered soil profile under consideration, with the notations used in SWATR.
- the upper layer is of the same texture as the lower layer but its density is different because of tillage operations, etc. Consequently the physical properties of the two layers are different.

The program is able to handle maximally 25 nodal points, with constant depth increments, \(\Delta z=\mathrm{DX}\). The relation between the depth of nodal point \(J\) and \(D X\) is
\[
\begin{equation*}
\mathrm{X}(\mathrm{~J})=\mathrm{DX} *(\mathrm{~J}-0.5) \tag{4.13}
\end{equation*}
\]

This means that \(\mathrm{X}(\mathrm{J})=0\) is equivalent to \(\mathrm{J}=0.5\). As during the growing season the watertable is usually fluctuating, the unsaturated part of the soil profile is varying according to the actual position of the watertable. The actual number of nodal points under consideration N is estimated according to the expression
\[
\begin{equation*}
\mathrm{N}=\mathrm{DWT} / \mathrm{DX}+0.49 \tag{8.1}
\end{equation*}
\]
where DWT is the depth of the watertable (Fig. 29).

\subsection*{8.1.4 Initial and boundary conditions}

As initial condition one has to prescribe for each nodal point either the water content or the suction. As daily average values of the boundary conditions one can use
at the bottom: depth of the watertable (prescribed suction) at the surface: soil water content or the maximum possible surface flux as governed by atmospheric or other external conditions.
The conditions at the surface can thus be described as a Dirichlet condition (water content or suction) or as a Neumann condition (value of the flux). In the latter case the maximum soil surface flux, \(E_{3}^{*}\), must be prescribed (see Eqn 3.31). From the discussion in Chapter 4, we have seen that for the actual flux a solution must be sought by maximizing the absolute value of the flux subject to some requirements (see Eqns 4.8 and 4.11). In the program this procedure is formulated as
FLUXM < |FLUXA|
and
\[
\begin{equation*}
0 \leqslant S S \leqslant S G L \tag{4.12}
\end{equation*}
\]
where
FLUXA \(=\) prescribed potential surface flux in mm.day \({ }^{-1}\) (positive, when directed downwards as in the case of rain or infiltration, and negative when directed upwards as in the case of evaporation)
SGL = maximum allowed suction \(h_{1}\) at the surface in cm , which can be found from Eqn \(4.11\left(h_{1}=-\psi_{1}\right)\)
The potential surface flux can be found from
FLUX \((=\) FLUXA \()=\) PREC - ES - FIN
where
PREC \(=\) precipitation rate \(\left(m m\right.\). day \(\left.^{-1}\right)\)
ES \(=\) potential soil evaporation rate \(\left(m m . d a y ~{ }^{-1}\right)\)
FIN \(=\) interception rate \(\left(\mathrm{mm}\right.\). day \(\left.^{-1}\right)\)
At the surface also the maximum possible transpiration rate EP must be defined. From Eqn 3.12 we have seen that
\(\mathrm{EP}=\mathrm{EWET}-\mathrm{ES}\)

The values of EWET and ES are calculated with Eqns 3.29 and 3.31 , respectively. When in the input \(\mathrm{KOD}(6)\) is set equal to 2 , SWATR calculates ES and EP. In that case, daily values of TEM, RH, U, HNT, CH, SC and FLUX must be prescribed, where
TEM \(=\) air temperature at 2 m height \(\left({ }^{\circ} \mathrm{C}\right)\)
\(\mathrm{RH}=\) relative humidity of the air at 2 m height (fraction)
\(\mathrm{U}=\) wind velocity at 2 m height (m.s. \({ }^{-1}\) )
HNT \(=\) net radiation flux [W. \(\mathrm{m}^{-2}\) if \(\mathrm{L}(7)=0\), or cal. \(\mathrm{cm}^{-2}\). day \(^{-1}\) if \(\mathrm{L}(7) \neq 0]\)
\(\mathrm{CH}=\) crop height \((\mathrm{cm})\)
SC = soil cover (fraction)
FLUX \(=\) precipitation rate in mm. day \(^{-1}\), i.e. PREC in Eqn 8.2
In addition the following functions must be described: the crop height function \(f(l)\) (see Eqn 3.30), the leaf area index relationship with soil cover \(I\left(S_{c}\right)\) and the interception function depending on precipitation, in the program denoted as \(G(C H)\) ( \(\mathrm{m}^{-2} . \mathrm{s}^{2}\) ), LAI and \(\operatorname{FIN}(\) PREC \()\left(\mathrm{mm}\right.\). day \(^{-1}\) ), respectively.
If the values of the parameters \(L(8), L(9)\) and \(L(10)\) in the program are set \(\neq 0\), then calculations are performed for standard functions of G(CH), LAI and FIN(PREC) just as they are presented in Figs. 30, 31 and 32, respectively. If one takes \(L(8), L(9)\) and \(\mathrm{L}(10)\) equal to 0 , then different coefficients can be used, for which one has to prescribe:
-6 values of \(\mathrm{G}(\mathrm{CH}):\) FGA, FGB, FGC, FGD, FGM and FMCH, where according to Fig. 30
\[
\begin{align*}
& \mathrm{G}(\mathrm{CH})=\mathrm{FGA} * \mathrm{CH} * * \mathrm{FGB} \text { for } \mathrm{CH} \geqslant \mathrm{FMCH}, \\
& \text { with } \mathrm{FGA}=a \text { and } \mathrm{FGB}=b \tag{8.3}
\end{align*}
\]
and
\[
\begin{align*}
& \mathrm{G}(\mathrm{CH})=\mathrm{FGC} * \mathrm{CH} * * \mathrm{FGD} \text { for } \mathrm{CH}<\mathrm{FMCH}, \\
& \text { with } \mathrm{FGC}=a \text { and } \mathrm{FGD}=b \tag{8.4}
\end{align*}
\]
and
\[
\begin{equation*}
\mathrm{G}(\mathrm{CH})=\mathrm{FGM} \text { is the maximum value of } \mathrm{G}(\mathrm{CH}) \tag{8.5}
\end{equation*}
\]
-3 values of LAI: FLA, FLB and FLC,(where according to Fig. 31 \(\mathrm{FLA}=a, \mathrm{FLB}=b\) and \(\mathrm{FLC}=c\) )
\[
\begin{equation*}
\mathrm{LAI}=\mathrm{FLA} * \mathrm{SC}+\mathrm{FLB} * \mathrm{SC} * * 2+\mathrm{FLC} * \mathrm{SC} * * 3 \tag{8.6}
\end{equation*}
\]
-6 values of FIN: FIA, FIB, FIC, FID, FMP and FMI (where according to Fig. 32, \(\mathrm{FIA}=a, \mathrm{FIB}=b, \mathrm{FIC}=c\) and \(\mathrm{FID}=d\) )


Fig. 30. Function dependent on crop height CH used in Eqn 3.30 to calculate the aerodynamic resistance of a crop. After Rijtema, 1965.
\[
\begin{array}{r}
\mathrm{FIN}(\mathrm{PREC})=\mathrm{FIA} * \mathrm{PREC} * *(\mathrm{FIB}-\mathrm{FIC}(\mathrm{PREC}-\mathrm{FID})) \\
\text { for PREC }<\mathrm{FMP} \tag{8.7}
\end{array}
\] and

FIN \((\) PREC \()=\mathrm{FMI}\) for PREC \(\geqslant \mathrm{FMP}\)
FMI describes the maximum value of precipitation PREC that can be intercepted. For precipitation rates exceeding a certain value FMP, the interception is equal to FMI. Interception must depend on soil cover. We assume that it is proportional to soil cover according to
\[
\begin{equation*}
\text { INTERC }=\mathrm{SC} * \mathrm{FIN}(\text { PREC }) \quad\left(\mathrm{mm} \cdot \mathrm{day}^{-1}\right) \tag{8.9}
\end{equation*}
\]

The saturated water vapour pressure EV (mbar) at temperature TEM (K), and the shape of the saturation vapour pressure curve DEL (mbar. \(\mathrm{K}^{-1}\) ) are described by the empirical expressions
\[
\begin{equation*}
\mathrm{EV}=1.3332 * \operatorname{EXP}((1.088719061 * \mathrm{TEM}-276.4883955) / \mathrm{WED}) \tag{8.10}
\end{equation*}
\]
\(\mathrm{DEL}=13.73150407 * \mathrm{EV} / \mathrm{WED} * * 2\)


Fig. 31. Relationship between leaf area index LAI and fraction of soil covered SC for red cabbage on sticky clay. After Feddes, 1971.


Fig. 32. Relationship between flux of intercepted precipitation FIN and precipitation rate PREC. Adapted from grass data of Rijtema, 1965 and red cabbage data of Feddes, 1971.


Fig. 33. Saturated water vapour pressure EV and slope of the saturation vapour pressure curve DEL at temperature TEM.
where
\[
\begin{equation*}
\text { WED }=0.058302635 * \text { TEM }-2.19386068 \tag{8.12}
\end{equation*}
\]

These functions are graphically presented in Fig. 33.

\subsection*{8.1.5 Sink term}

The main idea of the description of the sink term \(S\) was explained in Section 3.1. In Fig. 7 the basic function \(S(\psi)\) was given. The program allows for the use of alternative shapes of the \(S(\psi)\) function, according to the four assumptions discussed in Section 3.1. These are depicted in Fig. 34.

For two different soil layers, as treated in SWATR, one has to provide 6 critical values for the input:

SMB - starting point: the suction at which the roots start to extract water from the soil
SMU1, SML1 - the suction value for the upper and lower soil,
respectively, when optimal conditions for water extraction occur
SM2-limiting point: the suction at which extraction by roots starts to decrease
SM3 - wilting point: the suction at which water extraction is no longer possible anymore
BQ-factor describing the character of extraction in the suction range SM2 \(<h \leqslant\) SM3

In Fig. 34 one will notice that \(\mathrm{AQ}=1.0-\mathrm{BQ}\). If \(\mathrm{SMB}=\mathrm{SMU1}\) (or SML1) and \(\mathrm{BQ}=1.0\), the calculations are performed for the sink term function described in Section 3.1.




Fig. 34. Graphical representation of the various alternative shapes of the sink term S in relation to suction SS, which can be used in SWATR.

\subsection*{8.1.6 Compilation of subroutines}

The program SWATR consists of a main program and 8 subroutines. The dimensions of the arrays are fixed, covering a year of input data. They are:

365 values of the upper and lower boundary condition
80 values of suction and hydraulic conductivity for each layer [if \(\mathrm{KOD}(1)=2]\)
25 nodal points of the soil profile
52 outputs (if \(\mathrm{TM}=7\) days; \(52 \times \mathrm{TM}=364\) days)
Using the statement EQUIVALENCE, there are 7 main arrays:
CH - elements of crop height CH and potential transpiration rate EP
RH - elements of relative humidity RH , maximum allowed suction SGL and moisture content as a boundary condition at the surface WCS
SC- elements of soil cover SC, and depth of watertable DWT
HNT - elements of net radiation HNT, depth of the root zone DRZ, and transpiration rate actual TRA
FLUX - elements of precipitation rate FLUX (=PREC) and calculated values of FLUXA = FLUX-ES-FIN
TEM- elements of temperature TEM, of differential moisture capacity of the upper and the lower layer CHU and CHL, of suction of upper layer as a function of water content SU, of coefficients used in solving procedure R1 and R2, of water extraction rate for each nodal point QR, of water content for each nodal point W and of hydraulic conductivity for each nodal point used to estimate the flux at the nodes W2.
U - elements of wind velocity U , of hydraulic conductivity as a table of water content for upper and lower layer CU and CL, of suction of the lower layer as a table of water content SL, of suction at each nodal point for ( \(i-1\) ) stage of computation S 1 , of suction at each nodal point for ( \(i-2\) ) stage of computation S2, of calculated suction for \(i\) stage of computation S, of suction for each nodal point \(j+\frac{1}{2}\) and \(j-\frac{1}{2}\) i.e. SN1 and SN2 respectively

The remaining arrays are:
IA - 99 elements for a graphical output of the program; this array is also used as a table IB ( 69 elements in subroutine PARAM for a graphical output) and table K (12 elements: number of days in each month of the year)
\(\mathrm{L}-\mathrm{L}(1)\) : first day of calculation (reckoned from the beginning of the year)
\(\mathrm{L}(2): \quad\) last day of calculation (the same)
L(3): number of days in February (28 or 29)
\(\mathrm{L}(4): \quad\) first day in the first month of calculation (reckoned from the beginning of the month)
\(\mathrm{L}(5): \quad\) first month of calculation (reckoned from the beginning of the year)
L(6): last month of calculation (the same)
\(\mathrm{L}(7)=0\) : if HNT in W. \(\mathrm{m}^{-2}\)
\(=1\) : if HNT in cal. \(\mathrm{cm}^{-2}\). day \(^{-1}\)
\(\mathrm{L}(8)=0: 6\) coefficients of GCH must be described
\(\mathrm{L}(9)=0: 3\) coefficients of LAI must be described
\(\mathrm{L}(10)=0: 6\) coefficients of FIN must be described
If \(\mathrm{L}(8), \mathrm{L}(9)\) and \(\mathrm{L}(10) \neq 0\), then calculations are performed for the GCH, LAI and FIN functions of Figs. 30, 31 and 32, respectively.

In order to print results in the form of tables one extra array is used, denoted IX with dimension of \(52 \times 25 \times 5\) integer elements. The IX array collects the following values:
\[
\begin{aligned}
\text { IX-IX(L,M,1) } & =1000 \times \mathrm{W}(\mathrm{~L}, \mathrm{M}): \text { water content } \\
\text { IX }(\mathrm{L}, \mathrm{M}, 2) & =10 \times \mathrm{VOL}(\mathrm{~L}, \mathrm{M}): \text { integr. water content over depth } \\
\text { IX(L,M,3) } & =100 \times \mathrm{PF}(\mathrm{~L}, \mathrm{M}): \text { logarithm of suction } \\
\text { IX } \mathrm{IL}, \mathrm{M}, 4) & =1000 \times \mathrm{Q}, \mathrm{M}): \text { fux at nodal points } \\
\text { IX }(\mathrm{L}, \mathrm{M}, 5) & =10,000 \times \mathrm{QR}(\mathrm{~L}, \mathrm{M}): \text { root extraction rate }
\end{aligned}
\]
where \(L\) denotes the time axis and \(M\) denotes the depth axis.
The aim of the main program and the subroutines in sequence of appearance can briefly be summarized as follows:

SWATR - main program for the solution of the flow equations and printing of results
PARAM - subroutine to read and print input data. If \(\mathrm{KOD}(6)=\) 2 , this segment estimates the boundary condition at the soil surface from meteorological and external data.

If \(\mathrm{KOD}(1)=0\) or 2 , this segment calculates the differential moisture capacity for the upper (CHU) and for the lower (CHL) layer in the form of a table
WACO-subroutine to calculate water contents at the nodal points from suction data
BOCO-subroutine to calculate intermediate values of the boundary conditions at any stage of computation
HEPR - subroutine to calculate suctions for each nodal point when the initial condition is given as a value of water content \(-\mathrm{KOD}(5)=0\)
HEPAS-subroutine to calculate suctions at the soil surface when the boundary condition is given as a value of water content \(-\mathrm{KOD}(6)=0\)
DMC-subroutine to calculate differential moisture capacities at the suctions prevailing in the nodal points
CON -subroutine to calculate hydraulic conductivities from suction values
RER - subroutine to calculate root extraction rates at each nodal point

\subsection*{8.2 Field experiments}

\subsection*{8.2.1 Red cabbage on sticky clay}

A field experiment was performed by the first author in 1967 (Feddes, 1971) at the 3 ha experimental field Geestmerambacht in the Netherlands where groundwater tables were maintained at different depths. The main aim of this experiment was to investigate thoroughly the influence of some environmental conditions on plant growth, based on data from the Geestmerambacht area. Red cabbage (Brassica oleracea L. cv. Rode Herfst) was grown under optimum conditions of nutrient supply on a heavy clay ( \(48 \%<\) \(2 \mu \mathrm{~m}\) ) in rows 75 cm wide and spaced 65 cm apart.

Water balance studies were carried out with a specially designed non-weighable lysimeter in which the groundwater table could be continuously maintained at the same depth (approximately 1 cm accuracy) as it was in the surrounding field. Upward flow from and downward flow towards the watertable were measured in the lysimeter every day. Moisture content was measured weekly with a \(20 \mathrm{mc}{ }^{137} \mathrm{Cs}\) sealed gamma radiation source, which had a peak gamma energy of 0.662 MeV . Measurements were in duplicate at 10 cm depth intervals, with the deepest measurement under the groundwater table. Precipitation was measured at a neighbouring
weather station with a recording rain gauge having its rim at ground level. Interception was indirectly determined by measuring the through-fall collected from a number of plants and averaged per unit of soil area. Actual evapotranspiration was the only unknown in the water balance equation and could therefore be easily calculated for each week.

Soil water retention curves were determined (by desorption) in the laboratory. Hydraulic conductivity data were obtained in the Laboratory by an infiltration method (Wesseling \& Wit, 1966) and in the field from flow measurements during dry periods.
Meteorological data for the calculation of maximum possible transpiration and soil evaporation were obtained from the weather station. In a Stevenson screen air temperature ( \(T_{a}\) ) and relative humidity ( \(100 e_{d} / e_{a}\) ) were recorded continuously with two-bimetallic thermographs and two hair-hygrographs, respectively. Daily ( 24 hours) mean values were estimated with an Ott-integrimeter. Wind velocities ( \(u\) ) were measured 2 m above soil surface with a totalizing cup anemometer, with a mechanical counter system. The duration of bright sunshine ( \(n\) ) was measured with a Campbell-Stokes sunshine recorder. Short-wave radiation ( \(R_{s}\) ) was recorded with a MollGorczynski solarimeter. Net radiation \(\left(R_{n}\right)\) was measured at a height of about 1 m above the red cabbage crop with a miniaturized unaspirated net radiometer developed by Funk. The reflection coefficient ( \(\nu\) ) was derived by measuring the reflected solar radiation ( \(R_{s}^{\text {re }}\) ) with a second solarimeter mounted in an inverted position at a height of about 2.10 m above the crop surface on a movable 'sulky' type installation ( \(\nu=R_{s}^{r} / R_{s}\) ).

Growth and development of the crop were determined at regular intervals by measuring crop height, soil cover, leaf area and rooting depth. The fraction of soil covered was estimated with aid of a frame of \(1 \mathrm{~m}^{2}\). The leaf area was determined by measuring length and width of all leaves of some individual plants.
The case investigated is depicted in Fig. 35.

\subsection*{8.2.2 Potatoes on loamy sand}

Reactions of potato plants to soil compaction were studied in the field by van Loon \& Bouma (1978) \({ }^{1}\), as potato growth generally is strongly influenced by soil structure. One of their studies was carried out on a loose sandy loam soil. Data from this study were taken to test SWATR and CROPR.
\({ }^{1}\) The authors are most grateful to Ir C. D. van.Loon and Dr J. Bouma for making their experimental data available.

In one of the new polders in the Netherlands (Oost Flevoland) on 14 April 1976, mini-sprouted seeds of the variety Bintje, size 40 to 45 mm , were planted 33 cm apart with a 4-row automatic planter on rows 75 cm apart. A loose ridge of 20 cm height was overlying the layer of 20 to 40 cm (both having the texture of: \(10 \%\) clay, \(35 \%\) silt, \(2.3 \%\) organic matter, \(7 \%\) calcium carbonate). From 40 to 70 cm a loamy fine sand was present ( \(5 \%\) clay, \(15 \%\) silt, \(0.9 \%\) organic matter, \(3 \% \mathrm{CaCO}_{3}\) ). From 70 to 90 cm a moderately stratified half ripened soil composed of thin layers of sandy loam, loam and sand was present. Below 90 cm the soil was unripened.

The part of the growing period under consideration was from 10 May to 16 August. The year 1976 was an unusually dry year, marked by high rainfall deficits during summer. The groundwater table in May and June was situated at about 100 to 150 cm below soil surface, which gradually decreased to about 230 cm on 16 August. Soil moisture contents were determined gravimetrically at depth intervals 0 to 10,10 to \(20, \ldots, 70\) to 80 cm and time intervals ranging from 6 to 13 days. The rooting depth ranged from about 40 cm depth at the beginning to about 92 cm at the end of the period considered. Meteorological data such as air temperature, relative humidity, short-wave radiation, wind speed, and precipitation were taken from the site Lelystad-Haven, about 10 km from the experimental field. Net radiation was calculated from short-wave radiation, using Eqn 3.41.

During the growing season, a growth analysis was carried out. Length of foliage was determined on 10 and 25 June, 12 July and 16 August. The percentage of ground covered by green foliage was estimated on 4, 22 and 29 June, 9 July and on 20 September. The leaf area index was determined for a number of different fractions of soil coverage.

\subsection*{8.3 Experimental verification}

\subsection*{8.3.1 Red cabbage on sticky clay}

From Fig. 35 it is seen that the soil profile consists of two layers with different hydrological properties. The roots are initially growing in the upper layer but with time they extend to the lower layer.


Fig. 35. Schematic representation of the situation and circumstances of red cabbage on sticky clay as used for experimental verification of SWATR and CROPR.

At the soil surface the meteorological and the crop conditions vary with time, while at the bottom there is a fluctuating watertable. This case is similar to the one shown in Fig. 11 as Case D, but it incorporates a root system.

In the input the following data were used.
Physical properties of the soil layers Soil sampling showed that the bulk density of the soil varied with depth. It increased from values of about \(0.90{\mathrm{~g} . \mathrm{cm}^{-3}}^{\text {at }}\) the surface to about \(1.35 \mathrm{~g} . \mathrm{cm}^{-3}\) at a depth of 25 to 35 cm and remained nearly constant below this depth. Therefore for the upper 32.5 cm soil layer a soil moisture retention and hydraulic conductivity pertaining to an average bulk density of \((0.90+1.35) / 2=1.125 \mathrm{~g} . \mathrm{cm}^{-3}\) was used. For the lower layer, data pertaining to a density of 1.350 were applied (Figs. 36 and 37). The \(\psi(\theta)\) as well as the \(K(\theta)\) relationship (both for the upper and lower soil layer) are presented in the form of a table (see Groups \(P\) and \(R\), and Groups \(S\) and \(T\), respectively of Section 10.2).


Fig. 36. Soil moisture retention curves for the two soil layers for the case in Fig. 35.

Depth of the root zone DRZ (Group M of Section 10.2) The rooting depth of the red cabbage crop varied with time as shown in Fig. 38. After planting ( 21 June 1967, \(L(1)=172\) days), it remained constant for about a week, then increased gradually to about 83 cm at Day 214, and remained at this depth for the rest of the growing season.


Fig. 37. Hydraulic conductivity \(K\) versus water content \(\theta\) of the two soil layers for the case in Fig. 35.


Fig. 38. Variation in rooting depth DRZ and maximum possible transpiration rate \(\mathrm{EP}\left(=E_{p \mathrm{p}}^{*}\right)\) with time \(t\) for the case in Fig. 35.

Reduction factor RNA During the field experiment, it was noted that in the top 5 to 10 cm layer the roots slowly died off. Therefore it was decided to use the reduction factor RNA (Roots Non Active) to correct for the depth of the root zone DRZ as follows
\[
\begin{array}{llr}
\text { RNA }=\text { RNAM } *(T 1-T B) /(T E-T B) & \text { for } & \mathrm{TB}<\mathrm{T} 1<\mathrm{TE} \\
\text { RNA }=\text { RNAM } & \text { for } & \mathrm{T} 1 \geqslant \mathrm{TE} \tag{8.13}
\end{array}
\]

The symbols are explained under Group \(C\) of Section 10.2. In our study we took \(\mathrm{RNAM}=10 \mathrm{~cm}, \mathrm{~TB}=172\) days, \(\mathrm{TE}=221\) days. These values imply a monotonically increasing RNA-value from 0 at \(t=172\) to 10 cm at \(t=221\) days.

Critical suction values of the sink term (see Fig. 7) We took from the pF -curve for the lower layer a corresponding 'anaerobiosis' pressure head value \(\psi_{1}\) of -4.7 cm . For the upper layer a gas-filled porosity of \(0.05 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}\) was taken as the upper limit for water uptake by roots (corresponding to a \(\psi_{1}\) of -32.5 cm ). As 'limiting point' we set \(\psi_{2}=-500 \mathrm{~cm}\) and 'wilting point' we set \(\psi_{3}=\) \(-20,000 \mathrm{~cm}\). The computation was not started at \(\psi=0\), but at \(\psi=-0.1\) for computational reasons. This all leads to the following suction values (see Fig. 33 and Group D of Section 10.2): SMB = \(0.1 ;\) SMU1 \(=32.5 ;\) SML1 \(=4.7\); SM2 \(=500 ;\) SM3 \(=20,000 \mathrm{~cm}\); factor \(\mathrm{BQ}=1.0\).

Initial condition As the initial water content of the soil profile at \(t=t_{0}=172\) days, we used the data measured by the gamma transmission method, as shown in Fig. 39.

Boundary conditions at the soil surface \((z=0)\) (Group G of Section 10.2) These were collected from the meteorological station and the field. They are TEM, RH, U, HNT, CH, SC, FLUX. In addition standard functions G(CH), LAI and FIN were used, as presented in the Figs 30, 31 and 32, respectively. With these data maximum possible values of evapotranspiration rate (EWET; Eqn 3.29), of soil evaporation rate (ES; Eqn 3.31), of transpiration rate (EP; Eqn 3.12), of cumulative transpiration (SEP), and of surface flux (FLUXA; Eqn 8.2) are calculated. The course of EP with time is shown in Fig. 38. For infiltration, the difference in potential rate (FLUXA) and actual rate (FLUXM) yields the amount of water lost by surface run-off (RUNOFF):


Fig. 39. Initial soil water content at \(t=t_{0}=172\) days for the case in Fig. 35 .
\[
\begin{equation*}
\text { RUNOFF }=\sum_{t_{0}}^{i}(\text { FLUXA-FLUXM }) * D T \text { for } \text { FLUXA } \geqslant 0.0 \tag{8.14}
\end{equation*}
\]

The variation of FLUXA with time is shown in Fig. 40. Positive values here represent maximum possible infiltration rates, negative values maximum possible evaporation rates.

Boundary conditions at the bottom (Group L of Section 10.2) The depth of the watertable DWT as measured in the field lysimeter is given as the bottom boundary condition (see also Fig. 40). The soil profile was divided into \(j_{\max }=20\) nodes, according to
\[
\begin{equation*}
z_{\mathrm{i}}=(j-0.5) \Delta z \quad(\mathrm{~cm}) \tag{4.13}
\end{equation*}
\]

With \(L_{\text {max }}=z_{\text {max }}=100 \mathrm{~cm}, \Delta z\) becomes 5 cm . With \(z_{\mathrm{j}}=32.5 \mathrm{~cm}\), the transition between the upper and lower soil layer is at \(j=7\). When the depth of the watertable was above 97.5 cm , a smaller number of nodes was taken accordingly.

Upward flow from the groundwater table ('capillary rise') is denoted by DELTA in the program. It is calculated according to
\[
\begin{equation*}
\text { DELTA }=\text { VOL2 }- \text { VOL1 }+ \text { GG }- \text { GG1 -FLUXM } * \text { DT } \tag{8.15}
\end{equation*}
\]


Fig. 40. Variation in depth of groundwater table DWT and maximum possible infiltration (evaporation) flux FLUXA with time \(t\) for the case in Fig. 35.
where (VOL2-VOL1) is the change in water storage of the profile over the time period DT and (GG-GG1) the difference in cumulative transpiration over that same period. The complete list of input data is given in Section 10.3 so that others can perform the calculations mentioned above.

The main results of the computations are presented in Figs. 41-46. In Fig. 41 curves of cumulative flows are given: first the measured cumulative evapotranspiration ( \(E_{\text {water balance }}\) ) as obtained from the lysimeter; second the cumulative transpiration \(E_{\text {plant }}^{\text {comp }}\) as computed with the model by integration of the sink term over depth; third the cumulative soil evaporation \(E_{\text {soil }}^{\text {comp }}\) which is not yet printed as an output. It can, however, easily be calculated from the following cumulative quantities: CUM.WATER ( \(t\) ), CUM.WATER \(\left(t_{0}\right)\), CUM.TRANS., RUNOFF, SDELTA (cumulative upward flow), CUM. INFILT. (cumulative potential infiltration) by writing


Fig. 41. Computed cumulative evapotranspiration \(E_{w b}^{\text {comp }}\) and measured (via lysimeter) cumulative evapotranspiration \(E_{\text {water balance }}\) for the case in Fig. 35.
\[
\begin{align*}
E_{\text {soil }}^{\text {comp }}=\text { CUM.WATER }\left.\right|_{t} ^{t_{0}=\text { TINIT }} & \text {-CUM.TRANS + CUM.INFILT } \\
& - \text { RUNOFF + SDELTA } \tag{8.16}
\end{align*}
\]

Fig. 41 shows that there is a rather good agreement between computed and measured evapotranspiration, especially at the beginning and end of the period considered.

Cumulative evapotranspiration is one of the means to verify the results of the numerical model. Another possibility is to check for various days the computed soil moisture profiles comparing them with the measured ones. In Figs \(42 A, B, C, D\), the computed soil moisture profiles are compared with measured data for \(t=199,206\), 214 and 221 days, respectively. The agreement between computation and actual data is rather good. From this it might be concluded that the numerical model provides satisfying results. For the period 199 to 221 days, the maximum difference between cumulative \(E_{\text {plant }}^{\text {comp }}\) and \(E_{\text {plant }}^{\text {meas }}\) is about \(10 \%\) at \(t=221\) days (Fig. 41). At the same time the discrepancy between computed and measured soil moisture
profile (Fig. 42) is most pronounced when compared with the results shown in Fig. 42A,B,C, but still acceptable.

\section*{DAYE199.80}


DAYE2060015


Fig. 42. Computed and measured soil water content profiles for the case in Fig. 35: A, \(t=199\) days; B, \(t=206\) days; C, \(t=214\) days and \(D, t=221\) days.


DAYE221.0月


Fig. 42 continued

Another way of comparing measured and computed moisture contents is by plotting for each day the data found at the various depths as depicted in Fig. 43. For the three-week period between Days 199 and 221, the maximum deviation between \(\theta_{\text {comp }}\) and \(\theta_{\text {meas }}\)


Fig. 43. Comparison between measured and computed soil moisture contents \(\theta\) for the case in Fig. 35 at depths of \(10,20, \ldots, 90 \mathrm{~cm}\) for 27, 34, 42 and 49 days after the beginning of the experiment. As boundary condition at the soil surface a Neuman condition (flux, see Eqns 4.8 and 4.11) as well as a Dirichlet condition (pressure head or water content, Eqn 4.11) were taken.
is less than \(0.025 \mathrm{~cm}^{3} . \mathrm{cm}^{-3}\). Fig. 43 also shows the results when the Dirichlet condition of a prescribed pressure head (i.e. a prescribed moisture content) is kept as boundary condition at the top instead of the flux. Assuming that the pressure head at the soil surface is at equilibrium with the atmosphere, \(\psi\) was found from Eqn 4.11. Via the \(\psi-\theta\) curve of the upper layer shown in Fig. 36, the water content at the soil surface could be prescribed. It is clear from Fig. 43 that for the situation and conditions met there is not much difference in water content with either the flux or the pressure head as the boundary condition at the soil surface.

Fig. 44 shows how the computed sink term (root extraction rate) changes as a function of time and depth. Since the measured sink term could be derived only from lysimeter and soil moisture data as an average over periods of one week, the calculated values cannot be compared directly with field data. The shape and order of magnitude, however, are similar to the time averaged root extrac-


Fig. 44. Computed root extraction rates \(S\) for the case in Fig. 35.
tion rates derived from field data of the same profile (Feddes, 1971). From Fig. 44 it is seen that the magnitude of the root extraction rate is generally small at the top of the profile. It increases to a certain maximum zone and decreases to zero at the bottom of the root zone. The zone of maximum activity moves downwards with time making water uptake from the upper layers less important. The height of maximum uptake depends on the demands the atmosphere makes on the plant system, on the depth to which the roots penetrate, and on the soil water suction. From Day 243 until Day 304, there was considerable rain (Fig. 40) and water extraction from the upper layer again became important. In this period the groundwater table did rise (Fig. 40) and anaerobic conditions occurred in the lower part of the profile. Therefore the activity of the roots diminished there, and water uptake became less important. Generally it can be concluded that the shape of the sink term versus depth changed from a triangular shape at the beginning to a more trapezoidal shape with the root system extending.

The computed upward flow from the groundwater table summed


Fig. 45. Computed cumulative run-off for the case in Fig. 35.


Fig. 46. Actual transpiration rate \(E_{p l}\) as a function of time computed by SWATR for the case in Fig. 35.
over the growing season amounted to 224 mm , showing that high groundwater tables may considerably contribute to the evapotranspiration of a crop canopy.

In Fig. 45, the computed runoff during the growing season is presented. For Days 172 to 242 no runoff occurred. From Day 242 onwards runoff occurred, as was also observed in the field. Runoff increased to relatively large values during the last week, when rainfall reached a maximum of 28 mm. day \(^{-1}\).

In Fig. 46 the actual transpiration as computed with the model as integrated water uptake by roots, is plotted versus time. These data given in the form of a table in the output of SWATR will later function as an input for the production model CROPR.
A comparison between cumulative actual and potential transpiration is presented in Fig. 47. One can see that the differences between the two are considerable. At the beginning of the growing


Fig. 47. Computed cumulative potential transpiration \(E_{p l}^{*}\) and cumulative actual transpiration \(E_{p l}\) for the case in Fig. 35.
period, potential transpiration is reduced more than during the later stages of growth, when the roots extend to the lower layer. During the last stage of growth considerable rain occurred, which wet the upper soil layer and permitted a more favourable root extraction. Potential transpiration was not yet reached, however, because root activity was reduced by rising watertables.

\subsection*{8.3.2 Potatoes on loamy sand}

The case investigated is schematically depicted in Fig. 48. The soil profile consists of a number of layers with different hydrological properties. The rooting depth is increasing with time and there is a fluctuating watertable at the bottom of the system.

Because of the shape and surface of the potato ridge, one should in fact treat the problem two-dimensionally as for example with the unsaturated-saturated finite element program UNSAT 2 shown by Neuman et al. (1975) and by Feddes et al. (1975). However,


Fig. 48. Schematic representation of the situation at time \(t\) of potatoes on loamy sand as used for experimental verification of SWATR and CROPR.

SWATR was applied taking the vertical axis in the centre of the ridge with the top of the ridge as zero level.

The one-dimensional model is rather far from the actual situation and can be considered as a rough approximation. Nevertheless our final goal is not to predict very accurately the phenomena in the unsaturated zone, but to predict a realistic magnitude of production with CROPR from the transpiration data obtained with SWATR.

Physical properties of the soil layers For the various soil layers separate \(\psi(\theta)\) and \(K(\psi)\) relationships were determined. As the retention curves of the layers from 20 to 90 cm of Fig. 48 were quite similar, it was decided to neglect geometrical dimensions of the ridge and to treat the entire soil profile as homogeneous. The soil moisture retention curve and hydraulic conductivity curve taken are shown in Fig. 49.


Fig. 49. Soil moisture suction \(h\) and hydraulic conductivity \(K\) versus water content \(\theta\) for the case in Fig. 48.

Depth of the root zone The rooting depth was 40 cm deep on 22 May, 70 cm on 14 June, 90 cm on 17 July and 92 cm on 15 August.

Reduction factor RNA. This factor was taken to be zero throughout the growing season.

Critical suction values of the sink term It is known that for optimum top quality yields of potatoes, soil moisture contents should be maintained at high levels, especially from the time of flowering and tuber initiation almost until the tubers are mature. Therefore we set as 'limiting point' \(\psi_{2}=-400 \mathrm{~cm}\). Wilting point was set at \(\psi_{3}=-16,000 \mathrm{~cm}\). Under the dry conditions of 1976 , it was relatively unimportant what value to assign to the 'anaerobiosis point' \(\psi_{1}\), therefore we took -2.5 cm . For technical reasons the computations were started at \(\psi=-0.1\). This all leads to the following values: \(\mathrm{SMB}=0.1, \mathrm{SMU} 1=2.5, \mathrm{SML} 1=2.5, \mathrm{SM} 2=400, \mathrm{SM} 3=16000\), factor \(\mathrm{BQ}=1.0\).

Initial condition The initial soil moisture profile was based on the gravimetrically measured profile on 13 May.

Boundary conditions at the soil surface As 1976 was a very dry year, we expected some difficulties in the determination of the boundary conditions at the soil surface, i.e. potential soil evaporation and transpiration rate. Moreover potato is a special kind of crop, being a row crop because for a relatively large part of the growing season the soil is covered incompletely. As a first method (I) to estimate \(E^{*}\) and \(E_{s}^{*}\) we used the same approach as described for red cabbage i.e. applying the Eqns 3.29 and 3.31 respectively. As a second method (II) we estimated potential evapotranspiration \(E^{* *}\) according to Eqn 3.33 which includes diffusion resistances depending on stomatal resistance ( \(r_{1}\) ) and a resistance dependent on soil covered. Potential transpiration \(E_{\text {pl }}^{*}\) was estimated according to the Eqns 3.34 and 3.35. For the first approach, \(\operatorname{KOD}(6)=2\) was used and the program automatically computed \(E^{*}, E_{s}^{*}, E_{p l}^{*}\) and FLUXA (potential surface flux). The input data included TEM, RH, U, HNT, CH, SC, FLUX (=PREC). The relation between leaf area index and soil cover was described as
\[
\begin{equation*}
\mathrm{LAI}=3.625 * \mathrm{SC}-1.605 * \mathrm{SC} * * 2+2.105 * \mathrm{SC} * * 3 \tag{8.17}
\end{equation*}
\]

For \(\mathrm{G}(\mathrm{CH})\) and \(\mathrm{FIN}(\mathrm{PREC})\) the standard functions were taken (Fig. 30 and Fig. 32). In the second approach \(\mathrm{KOD}(6)\) was set equal to 1
and the potential fluxes were calculated with a pocket calculator.
Boundary condition at the bottom In the beginning of the period considered the groundwater table was at a depth of 100 cm . From half May to half July it stayed between 120 and 150 cm and then gradually dropped to 230 cm at the end of August. The soil profile was divided into \(j_{\max }=23\) nodes, with \(L_{\text {max }}=z_{\text {max }}=230 \mathrm{~cm}\).

The main results of the computations are presented in Fig. 50, where potential and computed cumulative transpiration, as obtained with the two different top boundary conditions, are shown. During July and August, both methods I and II gave rather high values of potential transpiration, sometimes a dozen mm per day. From Fig. 50 one can see that the sink term used in our model gives a strong reduction in transpiration. It is generally known that under conditions of high transpirative demand of the atmosphere, even when potatoes are fully supplied with water, transpiration is reduced. The


Fig. 50. Computed potential \(E_{p l}^{*}\) and actual cumulative transpiration \(E_{p l}\) using two alternative methods to estimate the boundary condition at the top for the case in Fig. 48. Method I uses Eqns 3.29 and 3.31; Method II uses Eqns 3.33, 3.34 and 3.35.
differences in potential transpiration computed with both methods mainly occur in the first 30 days of the growing season, when the soil is sparsely covered. With Method II practically no potential transpiration is computed: until Day 30 about 77 mm less than with Method I. During the remainder of the growing season, the general trend and shape of both potential transpiration curves are the same. The same behaviour can be found in the actual transpiration curves. Here the final difference amounts to about 100 mm at the end of the season, which is mainly due to effects occurring in the beginning.
The actual transpiration was not measured in the field, and therefore a comparison of computed with measured data was not possible. However, the values computed are within the limits for potatoes as given by Doorenbos \& Pruitt (1978). Pseudo-steady state calculations performed by van Loon \& Bouma (1978) on water uptake by roots resulted in \(E_{p l} \approx 425 \mathrm{~mm}\), which is close to curve \(E_{p l}\) II in Fig. 50.

Comparison of computed and measured soil moisture contents showed that Method I gave better results than Method II. Later it will be shown that prediction of actual yield with the transpiration data of Method I gives a closer approximation of measured production than the data of Method II.

Finally it can be concluded that the model computations compare favourably well with field data. The actual transpiration rates computed with the SWATR model can, if so desired, be introduced now in CROPR to calculate actual production.

\subsection*{8.4 Numerical experiments}

It was shown that experimental verification of the SWATR model gave reasonable agreement between measured and calculated values of soil water content, transpiration and evapotranspiration. The model was applied to certain local conditions. Can the model be extrapolated for different crops, soils, meteorological and agricultural conditions? In general, it is necessary to know about how a system behaves to be able to guide actual soil water management and future field experiments with the aid of simulation. Thus numerical experiments are of great importance.

Sensitivity analysis is generally applied to evaluate the effect of structural changes in a model and to determine the relative importance of parameters and boundary conditions. For example, the effect of a structural change in SWATR was evaluated by comparing two different concepts of the upper boundary condition: the flux
boundary condition and the moisture content boundary condition. Sensitivity analysis connected with the variation of input values include all the initial and boundary conditions and all parameters of the model. Changes in output values are a measure of the change in overall system behaviour as compared with the reference case. It is useful to evaluate how an error in each parameter affects the overall system performance. Examples of sensitivity analysis for ecological system models are given by Miller (1974).

In our numerical experiments with the SWATR model, input data are:
- initial condition of soil water content
- upper boundary condition
- lower boundary condition
- potential transpiration
- depth of root penetration
- differences in physical properties of soil layers
- parameters of sink term, as anaerobiosis point, limiting point and wilting point

Output data are:
- soil water content
- water uptake by roots
- actual transpiration (cumulative over time)

For a sensitivity analysis of the SWATR model we changed the following inputs with respect to the red cabbage reference case: initial water content in the soil profile, upper boundary condition (as a water content at soil surface), lower boundary condition (as a differently fluctuating groundwater table), soil profiles (two soil layers with different hydraulic conductivities and water retention curves), sink terms (described by different anaerobiosis, limiting and wilting points). For all cases the values for both the potential transpiration rate and the advance of the rooting depth were taken to be the same as in the reference case of the red cabbage experiment.

The following situations were compared:
A. reference conditions for initial water content, upper and lower boundaries as registered in the field during the experiment, with
the same soil profile of loose clay over dense clay (actual field data), sink term with pressure head values \(\psi_{1}=-40, \psi_{2}=-630\) and \(\psi_{3}=-15,000 \mathrm{~cm}\).
B. the initial and boundary conditions, soil profile and rooting depth as given under A, but the sink term with pressure head values \(\psi_{1}=-32.5, \psi_{2}=-500\) and \(\psi_{3}=-20,000 \mathrm{~cm}\).

Differences in shape of the sink term (see Fig. 7) will affect actual transpiration as an output of the model. For Case B a \(10 \%\) higher transpiration was found than for Case A. Even relatively small changes in sink function will affect the system.

The next numerical experiment was connected with the evaluation of the influence of initial and upper boundary conditions upon actual transpiration. For the situation of Case A, two changes were introduced:
C. instead of the initial soil water distribution as measured on 21 June for Case A, a dry soil moisture profile as actually measured in the field at the end of August was assumed and lower values of \(\theta\left(z, t=t_{0}\right)\) were taken;
D. in Case A the soil water content at the surface was changed. Precipitation was assumed to be zero and water content at the soil surface was assumed to be similar to that on dry days in the middle of summer.

The results of the simulation were somewhat surprising, because the actual transpiration, obtained for Cases C and D was slightly higher than for Case A, 6 and \(8 \%\), respectively. The drier soil appeared to be a better environment for the plant roots.

Another sensitivity analysis was made by inducing changes in the soil profile. The situation described in Case B has a dense clay as subsoil. Now the subsoil was assumed to be a fine sand and parameters of clay over sand were introduced into the model. The cases can be described as:
E. conditions as in Case B, but instead of the dense clay, a sandy subsoil.
F. conditions as E , but the fluctuating groundwater table changed to a constant depth of 100 cm and a dry soil surface with no precipitation.

The response of the model to these changes was negligible for Case E but higher for Case F. Actual transpiration for Case F was \(13.5 \%\) higher than for Case B. The explanation for the similar results of Case E and Case B is that in the wet range (subsoil close to the watertable) there is not much difference in upward flow from the groundwater table taking either clay or sand. The reason for the higher transpiration obtained in Case F is that less anaerobiosis is encountered with a constant watertable at 100 cm depth, than with the fluctuating one, yielding a higher root water uptake.

In the following four computer drawn figures, one can get an impression of the main features of water content and root water uptake versus depth, in time.


Fig. 51. Computer drawn variation of water content \(\theta\) versus depth and time for Case A.

In Fig. 51 (Case A) the upper boundary condition was given as a soil water content. At the bottom a fluctuating groundwater table was present. One can see that higher soil moisture contents at the soil surface (after rains) induce high water contents deeper in the profile. The lowest moisture content in the topsoil occurred during August because of intensive water uptake by roots and almost no rainfall. Because the topsoil is more porous than the subsoil ( 0.60 against 0.50 ), the water content in October, after a rise of the groundwater level, is much higher in the upper part of the soil (above 40 cm depth) than in the lower part (below 40 cm depth). In Fig. 52 (Case D), data are presented for zero rainfall. Influence of zero rainfall on the redistribution of water is not large in this soil, because the capillary supply from the fluctuating groundwater table


Fig. 52. As Fig. 51, but for Case D.


Fig. 53. Computer drawn variation of root water uptake rate \(S\) versus depth and time for Case B.
was already dominant, as one can see from comparing Figs. 51 and 52. This behaviour indicates the importance of groundwater in water supply to crops.

In Fig. 53 (Case B) water uptake by roots is shown. Development of the roots can be seen from the left side of Figs. 53 and 54 by an increase of the sink term at larger depths. Later root development was limited to a depth of approximately 80 cm . From Figs. 53 and 54 , it is evident that the mean rate of water uptake during September and October was much lower than during summer. The highest uptake (over a small depth) was found for the shallowest rooting depths, just after planting of red cabbage after 21 June. Random daily distribution of uptakes is connected with random values of evaporative demand of the atmosphere. A fluctuating and -alatively shallow groundwater level causes anaerobic conditions in


Fig. 54. As Fig. 53, but for Case F.
the subsoil resulting in zero water uptake. This feature is clearly visible in Fig. 53 during the end of September and the whole period of October. This behaviour is not present at all in Fig. 54, where the groundwater level was kept constant at a depth of 100 cm . Uptake of water was limited here only by a certain maximum penetration of the root zone. Different patterns of water uptake can be observed close to the soil surface as well. Figs. 53 and 54 indicate that it would be interesting to study the limits for water uptake in relation to overmoistening of soil.

A more detailed sensitivity analysis was made to check the influence of the sink term function on actual transpiration. The purpose of this exercise was to investigate the effect of \(\psi_{1}\) and \(\psi_{2}\) on the cumulative water uptake by the roots. The parameters used in the investigations are specified in Table 9.

In Fig. 55, computed values of cumulative transpiration are

Table 9. Values of \(\psi_{1}\) (corresponding with gas filled porosities \(\theta_{\text {air }}\) \(\left.\mathrm{cm}^{3} . \mathrm{cm}^{-3}\right), \psi_{2}, \psi_{3}(\mathrm{~cm})\) and \(L_{r}^{\text {na }}(\mathrm{cm})\) used in the computations, the results of which are presented in Fig. 55
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Case} & \multicolumn{4}{|l|}{Upper layer} & \multicolumn{4}{|l|}{Lower layer} & \multirow[t]{2}{*}{\(L_{r}^{\text {na }}\)} \\
\hline & \(\psi_{1}\) & & \(\psi_{2}\) & \(\psi_{3}\) & \(\psi_{1}\) & \(\theta_{\text {air }}\) & \(\psi_{2}\) & \(\psi_{3}\) & \\
\hline 1 & -32.5 & 0.05 & -1000 & -15000 & -23.1 & 0.02 & -1000 & -15000 & 5 \\
\hline 2 & -32.5 & 0.05 & -1000 & -15000 & -183.2 & 0.05 & -1000 & -15000 & 5 \\
\hline 3 & -32.5 & 0.05 & -500 & -15000 & -183.2 & 0.05 & -500 & -15000 & 5 \\
\hline 4 & -40 & 0.06 & -630 & -15000 & -40 & 0.025 & -630 & -15000 & 5 \\
\hline
\end{tabular}


Fig. 55. Cumulative transpiration for Cases \(1-4\) as defined in Table 9.
presented for different values of \(\psi_{1}\) and \(\psi_{2}\). Computations were performed for a period of 130 days, taking \(\psi_{3}=-15,000 \mathrm{~cm}\) and \(L_{r}^{\text {na }}=5 \mathrm{~cm}\). Too low (dry) values of \(\psi_{2}\) result in too low values of cumulative transpiration. This result is not surprising as it is generally known that for various vegetable crops the admissable pressure head at which soil moisture begins to limit plant growth is about -400 to -500 cm (Feddes, 1971). One can observe that differences in \(\psi_{1}\) of the lower layer have the largest influence upon cumulative transpiration. The highest cumulative transpiration is obtained at
the lowest value of \(\psi_{1}\) (Case 1, \(\theta_{\text {air }}=0.02\) ). Increasing \(\theta_{\text {air }}\) from 0.02 to 0.05 yields a strong reduction in cumulative flow (Case 2). The effect of changes in \(\psi_{2}\) seems, at least for the situation investigated, to be of less importance (compare Case 2 with Case 3). Keeping this small effect of changes in \(\psi_{2}\) in mind one will notice the sharp reduction in flow due to a rather small change in \(\psi_{1}\) when comparing Case \(1\left(\theta_{\text {air }}=0.02\right)\) and Case \(4\left(\theta_{\text {air }}=0.025\right)\). There is some experimental evidence that with enough air present in the upper part of the root zone, water can be extracted by roots in the lower part of the root zone under nearly water-saturated conditions. It should be emphasized that investigations dealing with effects of anaerobiosis upon growth should provide the model user with clear insight about aeration limits of the crop occurring under different external conditions of the soil-plant-atmosphere system.

\subsection*{8.5 Conclusions}

The model SWATR was verified with two field experiments. The first one concerned a water balance study on red cabbage grown on clay, the second one a soil tillage study on potatoes grown on a loam sand. Both crops were grown under the influence of a watertable.

For the first case good numerical results were obtained in simulating cumulative soil evaporation, transpiration as well as soil moisture content. In the second case, two different methods were applied in describing the bounfary conditions at the soil surface. Although the model did not predict distribution of soil-water content with depth in accurate detail, cumulative effects over the entire depth yielded values that are encountered in practice.

From the sensitivity analysis it can be concluded that relatively small changes in the sink function may affect the system. For roots growing in very wet soil, water uptake may become limited because of anaerobic conditions, as is especially true for roots growing near a fluctuating groundwater table: small variations in the anaerobiosis point may then considerably affect transpiration. This behaviour émphasizes the need for a closer study of the effects of anaerobiosis upon water use by plants.

From the material presented it may be concluded that the rather simple model SWATR can provide a useful tool in solving actual flow problems in the field.
If desired the actual transpiration rate obtained as an output from SWATR can be used as an input to the model CROPR.

\section*{9 Program for crop production, CROPR}

\subsection*{9.1 General description}

Program CROPR (see the flow chart, Fig. 56) consists of a main program and two short subroutines:

SQUE - solves quadratic algebraic equations,
TEINF - calculates the influence of temperature on production if ALFA is given as table of temperature TEM: \(\mathrm{L}(8)=2\).

The program is built up in two parts:
Part 1: calculates the potential dry matter yield of the crop, Part 2: calculates the actual dry matter yield of the crop.

If only Part 1 is used, \(L(7)\) must be set equal to 1 . To calculate crop production, the following values must be prescribed beforehand for Part 1 and Part 2 respectively:

\section*{Part 1 of CROPR}

BETA - ratio \(\beta_{h}\) of harvested part over total plant production
PHF - photorespiration factor \(\phi_{r}\) to account for respiration losses
WG - latitude of the area concerned
ALFA - parameter \(\alpha_{T}\) to account for the influence of temperature on production.

In CROPR three possibilities are provided:
Case \(1-\mathrm{L}(8)=0: \mathrm{ALFA}=\operatorname{SIN}(\mathrm{II}(\mathrm{TEM}+\mathrm{AL}) / \mathrm{BL})\)
for TEM \(<0.5 \mathrm{BL}-\mathrm{AL}\)
\(\mathrm{ALFA}=1.0\) for \(\mathrm{TEM} \geqslant 0.5 \mathrm{BL}-\mathrm{AL}\)
Case \(2-\mathrm{L}(8)=1: \mathrm{ALFA}=1-(\mathrm{TEM}-\mathrm{AL})^{2} / \mathrm{BL}^{2}\)
Case \(3-L(8)=2\) : as a table of TEM
\(\mathrm{L}-12\) values describing the dimension of the array and governing the computation process


Fig. 56. Flow chart of the main operations of program CROPR.


Fig. 56 continued
TAB \((10,12,3)\) - which consists of 3 times 120 values: of the solar radiation flux (RC) involved in phosynthesis ( 0.4 to \(0.7 \mu \mathrm{~m}\) ) on clear days ( \(\mathrm{W} . \mathrm{m}^{-2}\) ), of potential photosynthetic rates on clear days (PC) and of potential photosynthetic rates on overcast days (PO) in kg.ha \({ }^{-1} \cdot\) day \(^{-1}\). These values are presented in Table 6 for 10 different northern latitudes ( \(0^{\circ}, 10^{\circ}, \ldots, 90^{\circ}\) ) and 12 different points in time, which have to be prescribed in array \(\mathrm{D}(12)\).

In addition to the above mentioned data, the following daily values must be given:

TEM-temperature of the air at 2 m height
SC-soil cover (fraction)
SRF-daily values of actual solar radiation flux; if \(L(9)=0\) in W. \(\mathrm{m}^{-2}\), otherwise in cal. \(\mathrm{cm}^{-2}\).day \({ }^{-1}\). One must notice that if SRF is prescribed, the value of \(L(10)\) must be set equal to 0 ; if \(L(10) \neq 0\), the degree of cloud cover CLO must be used as input.

Part 2 of CROPR
A - initial slope of the ratio \(\dot{q}\) over \(E_{p} / \Delta e\), i.e. maximum efficiency of water use (Eqn 6.8)
FKSI - mathematical flexibility constant in growth equation 6.11; it is recommended to set FKSI equal to 0.01
EP - daily values of actual transpiration rate ( mm .day \(^{-1}\) ) obtained as an output from program SWATR, or from other sources
RH-if \(\mathrm{L}(12) \neq 0\), daily values of relative humidity of the air (fraction), otherwise daily values of vapour pressure deficit VPD in mbar. VPD can, for example, be obtained from program SWATR, if the boundary condition at the soil surface is prescribed by meteorological and other external conditions

Other remarks
In CROPR the statement 'EQUIVALENCE' is used. As a maximum input one can use 365 daily values ( 1 year); for ALFA as a table. of TEM 10 values.
RH-collects values of relative humidity RH and vapour pressure deficit VPD
SRF - collects values of solar radiation flux SRF, of cloudiness CLO and of potential rate of growth PRG
SC-collects values of soil cover SC and of potential cumulative yield PCY
Daily totals of light on clear days RC, of photosynthetic rates on clear days PC and on overcast days PO for different latitudes, can be obtained from the data given in Table 6 using linear interpolation. From the graphical presentation of RC, PC and PO for 15 June depicted in Fig. 57, one can state that for latitudes between \(0^{\circ}\) and \(60^{\circ}\) a linear variation of RC, PC and PO can be assumed. This approach seems unsatisfactory for latitudes between \(60^{\circ}\) and \(80^{\circ}\), particularly for the estimation of PC. Using linear interpolation also for this range, the error in estimating RC, PC and PO is less than 2 to \(3 \%\), however. On the other hand, the variation of RC, PC and PO with time is rather far from linear (see Fig. 22). Description of these variations by sine curves give rather good approximations. Assuming that the maximum values occur on 22 June ( 174 days after the beginning of the year) the periodicity of the sinus function can be expressed as


Fig. 57. Daily totals of gross photosynthesis rate on 15 June for various northern latitudes of a 'standard canopy' on clear days ( \(P_{c}\) ) and on overcast days ( \(P_{\mathrm{o}}\) ), as well as solar radiation flux \(R_{c}\) involved in photosynthesis (data taken from Table 6).
\[
\begin{equation*}
\omega=\pi / 348 \quad \text { (radians) } \tag{9.3}
\end{equation*}
\]

For a certain point in time \(T\) between for example \(R C(I)\) and \(\mathrm{RC}(\mathrm{I}+1)\) one can write
\[
\mathrm{T}(\mathrm{I}) \leqslant \mathrm{T} \leqslant \mathrm{~T}(\mathrm{I}+1)
\]
\[
\mathrm{RCA}=\mathrm{RC}(\mathrm{I})+\frac{\mathrm{RC}(\mathrm{I})-\mathrm{RC}(\mathrm{I}+1)}{\operatorname{SIN}(\omega \cdot \mathrm{T}(\mathrm{I}))-\operatorname{SIN}(\omega \cdot \mathrm{T}(\mathrm{I}+1))}[\mathrm{SIN}(\omega \cdot \mathrm{~T})+
\]

Similar expressions can be written for PCA and POA, i.e. for values of potential photosynthetic flux for clear days (PC) and for overcast days (PO) at actual time T .

If \(L(10)=0\), the solar radiation flux on actual days SRF is used. For \(L(10) \neq 0\), cloudiness CLO is used. The mean fraction of time
the sky under the actual condition is overcast, DELTA, is found from
\[
\begin{equation*}
\text { DELTA }=(\operatorname{RCA}-0.5 * \operatorname{SRF}(\mathrm{~J})) /(0.8 * \mathrm{RCA}) \tag{7.3}
\end{equation*}
\]

The mean gross potential photosynthetic flux of the standard canopy is then obtained from the expression
\[
\begin{equation*}
\mathrm{P}_{\mathrm{st}}=\text { DELTA } * \mathrm{POA}+(1.0-\mathrm{DELTA}) * \text { PCA } \tag{7.2}
\end{equation*}
\]

The output of CROPR is simultaneously given in a numerical and in a graphical form. The maximum yield to be plotted is 400 units of 100 kg.ha \({ }^{-1}\).

\subsection*{9.2 Field experiments}

\subsection*{9.2.1 Red cabbage on sticky clay}

In addition to the details given in Section 8.2.1, the following production experiments were carried out. Fresh and dry weight production of leaves and heads were measured weekly. At the same time the shoot/root ratio was estimated on a neighbouring sandy loam where the total root weight could be easily obtained. The ratio for the clay profile was approximated from the sandy loam data, measured differences in rooting depth being taken into account.

\subsection*{9.2.2 Potatoes on loamy sand}

In addition to the details given in Section 8.2.2, the following production experiments were carried out. The fresh and dry weight of the foliage was measured for all treatments, on 14 June and 16 August. Tuber weight was determined on 12 July, 16 August and at maturity on 20 September. At every harvest; three plots of \(6 \mathrm{~m}^{2}\) each per treatment were lifted.

\subsection*{9.2.3 Grass on silty clay}

A field experiment of grass production on a deep profile of silty clay, and a silty clay ( 30 to 40 cm ) over medium and fine sand in the polders of the River Vistula area (Poland) was performed by Brandyk \& Trzeciecki (1976). Data from this experiment were used to compare theoretically computed potential yields with actual data obtained under optimum conditions. During the experimental period the groundwater level was relatively shallow with a depth of
approximately 80 cm . The fluctuations in the groundwater table were relatively small as a result of subsurface irrigation from ditches. Only data from the relatively wet year 1972 have been considered, since over the 10 years of the experiment only in this year there was no shortage of water in the soil observed during the whole growing season. Fertilizers were applied three times: in spring and after the first and second cut every time \(60 \mathrm{~kg} \mathrm{~N}, 18 \mathrm{~kg} \mathrm{P}\) and 30 kg K per ha. Grass was cut at the end of May, July and September. Every 10 days, growth was measured 6 times on \(1 \mathrm{~m}^{2}\) plots. The calculations of potential production were checked against the highest values of measured yields.

\subsection*{9.3 Experimental verification}

\subsection*{9.3.1 Red cabbage on sticky clay}

For the simulation of the dry matter production of red cabbage (from 21 June to 31 October, 1967) the field data as reported by Feddes (1971) were used. The following input data were applied:

Ratio of harvested part of the plant over total production BETA For red cabbage on sandy loam a constant shoot/(shoot plus root) ratio of 0.90 was found during the growing season. The same ratio held for cabbage grown on sandy loam covered with clay. This crop had an effective rooting depth of 40 cm , similar to that in the sandy loam. In accordance with the larger rooting depth (up to 83 cm ) in the sticky clay profile a ratio of 0.885 was adopted. Thus BETA \(=\) 0.885 .

Photorespiration factor PHF The first author compared computed maximum production rates with dry matter production rates obtained from periodical harvests. From linear regression of measured on computed production, reduction factors were derived for red cabbage grown on clay, sandy loam and clay on sandy loam. For the pooled plots a value \(\mathrm{PHF}=0.51\) was found. This value is quite low when compared with data found by other investagators for different crops. One has to realize, however, that on some of the plots investigated lack of water, nitrogen deficiency or air deficiency was observed. In general the reduction factor should be determined under optimum conditions of water supply, nitrogen supply, etc. Therefore in the calculations a PHF value of 0.56 was adopted, being the highest value obtained on the clay on sandy loam plot,
where both nitrogen and water availability were the most favourable.

Latitude of the area concerned WG The latitude of the Geestmerambacht experimental area in the Netherlands is \(52^{\circ}=\) WG. Values of solar radiation on clear days (RC), potential photosynthesis rates on clear days (PC) and on overcast days (PO) were taken by interpolation from Table 6 for this latitude [in the program \(\operatorname{TAB}(10,12,3)\) ].

Influence of temperature on production ALFA According to interpretation of the research work of Wiebe (1975) on the influence of temperature on the production of cabbage, we took for red cabbage the curve
\[
\begin{array}{lll}
\text { ALFA }=\sin [\Pi(\text { TEM }+2.0) / 44.0] & \text { for } & \text { TEM }<20^{\circ} \mathrm{C}  \tag{9.1}\\
\text { ALFA }=1.0 & \text { for } & 20^{\circ} \mathrm{C} \leqslant T E M \leqslant 25^{\circ} \mathrm{C}
\end{array}
\]

This pertains to Case 1 in the program and to the red cabbage line in Fig. 24.

Daily values of TEM, SC and SRF These were taken from the field or the meteorological station. For SRF we used units of cal. \(\mathrm{cm}^{-2} \cdot \mathrm{day}^{-1}(\mathrm{~L}(9)=1)\).

The above mentioned data were used for the calculation of potential crop production. In order to calculate actual production the following additional input data were used:

Water use efficiency A From data of Feddes (1971) of measured production versus measured \(E / \Delta e\) obtained for clay, sandy loam and clay on sandy loam an overall maximum slope \(A=\) \(100 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1}\). mbar was found.

Mathematical constant FKSI This constant appears in Eqn (6.11). A value \(\mathrm{FKSI}=0.01\) was chosen.

Daily values of actual transpiration EP These values were obtained as an output in the form of a table from program SWATR. For a plot of actual transpiration as a function of time, see Fig. 46. One should notice that the symbol EP in CROPR means actual transpiration rate, in SWATR potential transpiration rate.

Daily values of vapour pressure deficit VPD These values were obtained as an output from SWATR, as performed by subroutine PARAM.

All the data used as an input in CROPR are listed in Section 11.3.

Only one cabbage plant per treatment was harvested at one time to avoid destroying too many plants by weekly harvests. With a heterogeneous crop like cabbage, a relatively large variation in dry matter production is then to be expected. This variation is reflected in the scatter of the yield data. At the end of the growing season, the cabbage crop of the entire field was harvested, so a rather representative final dry matter production could be determined.

In Fig. 58 a graph is given of cumulative dry matter yields versus time, i.e. computed potential, computed actual and measured actual yield, respectively. It is seen that calculated actual yield compares quite well with the measured actual yield. The measured 'single cabbage points' show a random scatter around the calculated actual curve but final yield was predicted quite well. The potential production amounts to 9.32 ton. \(\mathrm{ha}^{-1}\) ( \(\mathrm{ton}=10^{3} \mathrm{~kg}\) ). The actual production was 8.19 ton.ha \({ }^{-1}\). Thus the difference between potential and actual production was \(12 \%\).


Fig. 58. Computed potential \(Q_{\text {pot }}\) and computed actual \(Q_{\text {act }}\) cumulative crop yield as compared with measured data for the case in Fig. 35.


Fig. 59. Variation in computed potential growth rate \(\dot{q}_{\text {pot }}\), computed actual growth rate \(\dot{q}_{a c t}\), as well as the derivative \(\mathrm{d} Q_{a c t} / \mathrm{dt}\) of computed actual yield curve of Fig. 58 for the case in Fig. 35.

In Fig. 59 the variation of both the potential and actual rate of growth with time is shown. The growth rate was small in the beginning (when plants were still small), then rapidly increased to a maximum in August and the beginning of September, and then slowly decreased in September and October. At the end of October the growth rate was similar to that occurring around the middle of July. Also drawn in Fig. 59 is the derivative of the computed actual yield curve of Fig. 58. This curve is an indication for the average variation in growth rate during the growing season.

\subsection*{9.3.2 Potatoes on loamy sand}

For the simulation of the dry matter production of potatoes (from 10 May to 16 August, 1976) the following particular input data were applied (for non-specified input data see Section 9.3.1):

Ratio of harvested part of the plant over total production BETA This factor was experimentally determined and changes strongly with time as is shown in Fig. 60. During the first 30 days this factor is zero. From then on it is increasing heavily, reaching a value of 0.6 within three weeks. After this period it increases relatively slowly towards a value of 0.79 at the end of the period considered. Increase in production then is mainly due to an increase


Fig. 60. Distribution of foliage and root production as against tuber production during the year 1976 for the case in Fig. 48.
in tuber weight. For this crop BETA then can hardly be a constant with time. Now one can follow two ways in the program either taking BETA varying with time and adapt the program (by inserting BETA as a table or a function of time) or set BETA equal to 1.0 , calculate total production and correct total production afterwards with Fig. 60 to find the dry matter production of the tubers. We chose the second approach.

Photorespiration factor PHF From Burton's data (1964) (see also Beukema, 1972) who estimated a \(\mathrm{CO}_{2}\) balance of growing potato plants in the field, it can be derived that at \(20^{\circ} \mathrm{C}, \mathrm{PHF}=0.81\). From this value one might conclude that a potato crop is highly efficient in its production.

Influence of temperature on production ALFA According to data of Winkler (1961), ALFA was described by the curve given for potatoes in Fig. 24. The input was in the form of a table [Case 3 in the program, \(\mathrm{L}(8)=2]\).

Water use efficiency A For the 'average' years of 1961 to 1966, Rijtema \& Endrödi (1970) report an A value of \(154 \mathrm{~kg} . \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar}\). For the very dry year of 1959 , they found \(A \approx 260\), but remarked that, according to the procedures they applied, actual transpiration might have been underestimated. As


Fig. 61. Comparison of computed potential \(Q_{\text {pot }}\) and computed actual \(Q_{\text {act }}\) dry matter yield with measured data, taking into account different approaches for estimation of potential surface flux boundary conditions for the case in Fig. 48 (see also Fig. 50).
the tendency is towards higher A-values for dry years, we took, for the dry year 1976 an average of the two mentioned values, i.e. \(\mathrm{A}=207 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar}\).

Daily values of actual transpiration EP The output of the applied Methods I and II (see Section 8.3.2) of SWATR were used as input data.

The main results of the computations are presented in Fig. 61, where potential yields ( \(Q_{\text {pot }}\) ), actual yields ( \(Q_{\text {act }}\) ) according to Methods I and II and measured yields of tubers are shown. During the first 30 days of the growing season, computed potential tuber yield was below the measured yield. This is probably due to an underestimation of the fraction of soil covered \(\left(S_{c}\right)\) in the beginning of the growing period. The maximum final production to be reached for the conditions and period considered is 18.6 ton.ha \({ }^{-1}\). Looking
at the computed actual yield curves, we see that with the boundary conditions taken according to Method I, the best prediction of final actual yield is obtained. With Method II a too low production is predicted during the final part of the growing season. It seems that there the measured growth rate is about \(300 \mathrm{~kg} \cdot \mathrm{ha}^{-1} . \mathrm{day}^{-1}\). This gives a steeper slope than for \(Q_{\text {pot }}\) : theoretically a potential growth rate of about \(230 \mathrm{~kg} . \mathrm{ha}^{-1}\). \(\mathrm{day}^{-1}\) is possible. So one might conclude that actual measured plot yields were somewhat too favourable.

\subsection*{9.3.3 Grass on silty clay}

For the simulation of potential dry matter production of grass (from 1 April to 27 September, 1972) the following particular input data were applied (for non-specified input data see Section 9.3.1):

Ratio of harvested part of the crop over total production BETA For grass, root production usually amounts to some 50 to \(60 \%\) of the production of tops, which is larger than that of other crops. In spring, the production rate of new roots is about two to three times that in summer (Williams, 1969; Rose et al., 1972). Thus the value of BETA may range from 0.3 to 0.8 during the growing season. According to Kowalik (1973) an average value of BETA \(=0.6\) was adopted.

Photorespiration factor PHF According to de Wit (1969), this factor varies for grasses from about 0.5 to 0.7 . Therefore an average PHF \(=0.6\) was taken.

Latitude WG The northern latitude of the area under consideration was \(54^{\circ}=\) WG.

Influence of temperature on production ALFA For this factor the curve for grass of Fig. 24 was taken.

Water use efficiency A According to Kowalik (1973) a value \(\mathrm{A}=68.5 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} . \mathrm{mbar}\) was taken.

Daily values of actual transpiration EP As the year 1972 was wet, actual transpiration approached potential transpiration, which was calculated with Eqn 3.33.

Daily values of vapour pressure deficit VPD These values were derived from data of air temperature and air humidity.

Cloudiness factor CLO The fraction of cloud cover was taken from estimates made three times a day; \(\mathrm{L}(10) \neq 0\).

The main results of the computations on growth rates and cumulative yield are presented in Fig. 62. As the measured yields in 1972 were the highest over the 10 -year period of investigation, it was assumed that yields were close to potential yields. After mowing usually a regeneration period occurs, in which the leaf area index


Fig. 62. Comparison of computed potential growth rate \(\dot{q}_{\mathrm{pot}}\) and potential yield \(Q_{\text {pot }}\) with measured maximum yield data of a grass crop in a wet year.
must be corrected. It was assumed that within 20 days the full leaf area index again was reached. The values of \(\dot{q}_{\text {pot }}\) were multiplied by a coefficient of 0.05 for the first day, of 0.10 for the second day and so forth, taking steps of 0.05 per day until the 20th day after mowing, for which the coefficient was set equal to 1 .

The variation in growth rate during the year is clearly shown in Fig. 62. Maximum growth rates are reached in June and July. Computed potential cumulative yield agrees fairly well with the data measured on both soil types.

From the relatively good agreement between measured and computed yields, it can be concluded that the CROPR-model is also useful for calculating crop yields of grasslands.

\subsection*{9.4 Numerical experiments}

The general discussion presented in Section 8.4 about the necessity of a sensitivity analysis for the SWATR model also applies to the CROPR model.
The parameters of CROPR subjected to a sensitivity analysis are:
- coefficient of photo respiration \(\phi_{r}\)
- coefficient of the temperature influence on the rate of growth \(\alpha_{T}\)
- coefficient of the harvested part of plant \(\beta_{h}\)
-flexibility constant \(\xi\) as influenced by all growth factors
- water use efficiency \(A\)
- rooting depth \(L_{r}\)
-soil profile.
Two series of analyses were made. In the first series the response of potential yields of red cabbage for changes in \(\phi_{r}, \beta_{h}\) and \(\alpha_{T}\) was evaluated. The second one evaluated the response of actual yield to changes of the coefficients \(A, \xi\), of the value \(L_{r}\), and of the soil profile.

\subsection*{9.4.1. Influence on potential yield \(Q_{\text {pot }}\)}
A. changes in photo respiration coefficient \(\phi_{r}\); with \(\beta_{h}=0.90\) and \(\alpha_{T}=1\) for temperature \(\geqslant 17^{\circ} \mathrm{C}\) :
\[
\phi_{r}=0.51
\]
\[
\rightarrow Q_{\text {pot }}=9,108 \text { ton. } \mathrm{ha}^{-1}
\]
\(\phi_{r}=0.60 \quad \rightarrow Q_{\text {pot }}=10,708\) ton. ha \(^{-1}\)
B. changes in coefficient of harvested part of plant \(\beta_{h}\); with \(\phi_{r}=\) 0.56 and \(\alpha_{T}=1\) for temperature \(\geqslant 20^{\circ} \mathrm{C}\) :
\[
\begin{array}{ll}
\beta_{h}=0.90 & \rightarrow Q_{\text {pot }}=9,475 \text { ton. } \mathrm{ha}^{-1} \\
\beta_{h}=0.885 & \rightarrow Q_{\text {pot }}=9,315 \text { ton.ha }
\end{array}
\]
C. changes in temperature coefficient \(\alpha_{T}\); with \(\phi_{r}=0.56\) and \(\beta_{h}=\) 0.885
\[
\begin{aligned}
& \alpha_{T}=1 \text { for temperature } \geqslant 17^{\circ} \mathrm{C} \rightarrow Q_{\mathrm{pot}}=9,834 \text { ton. } \mathrm{ha}^{-1} \\
& \alpha_{T}=1 \text { for temperature } \geqslant 20^{\circ} \mathrm{C} \rightarrow Q_{\mathrm{pot}}=9,315 \text { ton. } \mathrm{ha}^{-1}
\end{aligned}
\]

Of course the results depend on the meteorological conditions in the year of study (1967). They provide, however, at least an impression of the order of magnitude of the differences in potential yield that can be expected when changing the values of the various coefficients.

\subsection*{9.4.2. Influence on actual yield \(Q_{\text {act }}\)}
D. changes in water use efficiency \(A\); with \(\phi_{r}=0.60, \beta_{h}=0.90\), \(\alpha_{T}=1\) for temperature \(\geqslant 17^{\circ} \mathrm{C}\) and \(\xi=0.01\) :
\[
\begin{array}{ll}
A=80 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar} & \rightarrow Q_{\text {act }}=8.22 \text { ton. } \mathrm{ha}^{-1} \\
A=100 \mathrm{~kg} \cdot \mathrm{ha}^{-1} \cdot \mathrm{~mm}^{-1} \cdot \mathrm{mbar} & \rightarrow Q_{\text {act }}=9.04 \text { ton. } \mathrm{ha}^{-1}
\end{array}
\]
E. changes in flexibility constant \(\xi\); with \(\phi_{r}=0.60, \beta_{h}=0.90, \alpha_{T}=1\) for temperature \(>17^{\circ} \mathrm{C}\) and \(A=100\) :
\[
\begin{array}{ll}
\xi=0.01 & \rightarrow Q_{\text {act }}=9.04 \text { ton. } \mathrm{ha}^{-1} \\
\xi=0.04 & \\
\hline Q_{\text {act }}=8.26 \text { ton. } \mathrm{ha}^{-1}
\end{array}
\]
F. changes in rooting depth, \(L_{r}\); with \(\phi_{r}=0.56, \beta_{h}=0.885\) (0.90), \(\alpha_{T}=1\) for temperature \(\geqslant 20^{\circ} \mathrm{C}, A=100\) and \(\xi=0.01\) :
\[
\begin{array}{ll}
L_{r}=82.5 \mathrm{~cm}(\text { as in A, B, C, D and E }) & \rightarrow Q_{a c t}=8.19{\text { ton. } . \mathrm{ha}^{-1}}_{L_{r}=42.5 \mathrm{~cm}} \\
& \rightarrow Q_{a c t}=7.00 \text { ton. } \mathrm{ha}^{-1}
\end{array}
\]
G. changes in soil profile; with \(\phi_{r}=0.56, \beta_{h}=0.90, \alpha_{T}=1\) for temperature \(\geqslant 20^{\circ} \mathrm{C}, A=100, \xi=0.01\) and \(L_{r}=42.5 \mathrm{~cm}\) :
\(\begin{array}{ll}\text { clay over sandy loam } & \rightarrow Q_{\text {act }}=8.24 \text { ton. } \mathrm{ha}^{-1} \\ \text { clay over clay } & \rightarrow Q_{a c t}=7.00 \text { ton. } \mathrm{ha}^{-1}\end{array}\)
From the results presented and for the situations investigated it seems worthwhile to give attention to a proper estimation of \(A\) and also of \(\xi\). From Case \(F\) it is clear that when the rooting depth on clay is reduced, a relatively strong reduction in yield may be expected. This is caused by the limitation set by this soil to transport water from the groundwater table to the relatively shallow root zone. A subsoil of sandy loam (Case G) improves the water transmitting properties considerably. Case \(G\) is illustrated in more detail in Fig. 63. Comparing actual yield of the clay covered sandy loam with 42.5 cm rooting depth (Fig. 63) with the yield curve obtained on the


Fig. 63. Computed potential \(Q_{\text {pot }}\) and computed actual \(Q_{a c t}\) yield of a red cabbage crop on two different soil profiles with a restricted rooting depth of 42.5 cm .
clay soil with 82.5 cm rooting depth (Fig. 58), i.e. Case \(F\) for \(L_{r}=82.5\), we see that there is practically no difference.

\subsection*{9.5 Conclusions}

With CROPR the potential and actual yields of red cabbage on sticky clay, potatoes on sandy loam and the potential yield of grass on silty clay and silty clay over sand was simulated. For the cases investigated it was shown that actual yields for red cabbage and potatoes and the potential yield for grass could be predicted fairly well.

From the sensitivity analysis it can be concluded that a proper estimation of the water use efficiency of a crop and the mathematical flexibility constant is relatively important.

The need of a soil profile that offers no restriction to root growth was quantified, as well as the importance of a subsoil which can transmit water easily from the groundwater table to the root zone.

The material presented did show that with model CROP it is possible to predict with fair accuracy the dry matter production of a crop.

\subsection*{10.1 Listing: of program}
```

    i= PROGRAM SWATR[INPUT,OUTPUT]
    2=C*****SIMULATION MODEL OF SOIL WATER DYNAMICS FOR LAYERED SOIL PROFILE
    3=C*****WITH FLUCTUATING WATER TABLE ANO WATER UPTAKE BY ROOTS
    A=C*****THIS PROGRAM IS DEUELOPED BY R.A.FEDDES, INSTITUTE FOR LAND AND
    5=C####*WATER MANAGEMENT RESEARCH,P.O.BOX 35,6700 AA UAGENINGEN,
    6=C*****THE NETHERLANDS; P.J.KOWALIK, INSTITUTE OF HYDROTECNICS,TECHNICAL
    7=C*****UNIUERSITY,P.O.BOX 642,80-952 GDANSK,POLAND: H.ZARADNY,INSTITUTE
    8=C*****OF HYDRO- ENGINEERING,POLISH ACADEMY OF SCIENCES,UL.CYSTERSOW 44.
    9=C*****80-953 GDANSK,POLAND.
    10=C
11=C"""n"THE NAME OF THIS PROGRAM CONSISTS OF THE FIRST LETTERS OF 5 WORDS:
42=C" """"*SOIL*,*UATER*,*ACTUAL*,*TRANSPIRATION*,*RATE*-I.E.-S-W-A-T-R-
43=C
44=C======THE FOLLOWING UALUES MUST BE PRESCRIBED:
15=C-----THE INITIAL CONDITION-VALUE OF THETA[THEN KOD[5]=0]
46=C -KOD[S]=4 MUST BE SET
47=C -SUCTION(NEGATIUE VALUE OF PRESSURE HEAD),
48=C-----THE BOUNDARY CONDITIONS [DAILY UALUES):
19=C.....AT THE BOTTOM-DEPTH OF WATER TABLE
20=C.....AT THE SURFACE-AJ TEM-TEMPERATURE OF AIR[DEGREES CELCIUSJ
24=C RH-RELATIUE HUMIDITY OF AIR[FRACTIONJ
22=C U-WIND VELOCITY AT 2 M HEIGHTCM/SJ
23=C HNT-NET RADIATION FLUX(W/M**2 IF L[7]=0,OTHER-
WISE IN CAL/CM**2/DAY IF L(7)>QJ
CH-CROP HEIGHT(CM)
SC-SOIL CQUERCFRACTIONJ
FLUX-PRECIPITATION(MM/DAY)
FOR CASE A KOD(6)=2
OR BJ-EP-POTENTIAL PLANT TRANSPIRATION[MM/DAYJ
FLUX-SURFACE FLUX-[UP:SIGN-,DOWN:SIGN+][MM/DAY)
SGL-CRITICAL UALUE OF SUCTION AT THE SURFACE[CM)

```
    FOR CASE B KOD(6]=4
    OR CJ-THETA(CM**3/CM**3) IF KOD(6)=0
\(33=\mathrm{C}\)
\(34=\mathrm{C}---\) THE DEPTH OF ROOT THETACCM
\(35=\mathrm{C}\)
36=C-----HYORAULIC PARAMETERS OF SOIL MUST BE PRESCRIBED AS FOLLOWS:
37=C IF KODC \(1=0\)-CONDUCTIUITY AS A FUNCTION OF SUCTION,SUCTION AS
\(38=C\)
\(39=\mathrm{C}\)\(\quad\) A TABLE OF WATER CONTENT \(\quad\) KOD \(43=4-\) SUCTION AS A FUNCTION OF WATER CONTENT,
40=C CONDUCTIUITY AS A FUNCTION OF SUCTION
4化 IF KOD(1)=2-SUCTION AND CONDUCTIUITY MUST BE GIUEN AS
42=C A TABLE OF WATER CONTENT
\(43=C\)
\(44=\mathrm{C}===x=\) MAXIMALLY CAN BE USED:
45=C 365-VALUES OF THE BOUNDARY CONDITION\{ 4 YEARJ
46=C 80-VALUES OF PRESSURE HEAD AND CONDUCTIUITY[FOR EUERY LAYERJ
\(47=\mathrm{C} \quad 25\)-NODAL POINTS OF THE SOIL PROFILE
48=C 52-OUTPUTS


\(52=\) 1SC(365), FLUX(365), DWT(365), DRZ(365), CU(80), CL(80), SU(80), WCS(365),
53= 2SL(80), CHU(80),CHL(80),R1(25),R2(25),OK(25), W(25),W2(25), S(25),
54= \(354(25), S 2(25), S N 4(25), S N 2(25), X(25), E P(365], S G L(365), I B[69), K A(5)\),
\(55=4 \operatorname{HED}(20], \operatorname{TR}(52), L(40], I X[52,25,5), \operatorname{TRA}(364), K M(12)\)
56: EQUIVALENCE (CH,EP),(RH,SGL,WCS),(SC,DWT),[HNT,DRZ,TRA],[TEMT4),
\begin{tabular}{|c|c|}
\hline 57 &  \\
\hline 58= &  \\
\hline 59= &  \\
\hline \(60=\) &  \\
\hline \(61=\) & 5),KM ( 1 ) \({ }^{\text {d }}\) \\
\hline 62= & COMMON/CONDU/ CSAT4, CSAT2, SUA1, SUA2, SUA3, SUB4, SUB2, SUB3, SUC, SUD, \\
\hline \(63=\) & 1SLA4, SLA2, SLA3, SLB , SLB2, SLB3, SLC, SLD, CUA , CUA2, CUA3, CUB , CUB2, \\
\hline \(64=\) & 2CUB3, CUC, CUD, CLA4, CLA2, CLA3, CLB , CLB2, CLB3, CLC, CLD, KODC6], NNL, IW4, \\
\hline \(65=\) & 3IW2,L6,SWCU, SWCL, LU, LL, MU, ML, FAC \\
\hline \(66=\) & COMMON/BONC/ DWT(365).SGL(365), EP(365), DRZ(365], FLUXC365) \\
\hline \(67=\) & COMMON/SINK/ SMB, SMU4,SML4,SM2,SM3, QM, SMM, PRZ, AQ, BO \\
\hline \(68=\) & COMMON/FACT/ TEM, U \\
\hline \(69=\) & COMMON/DECL/ HED(20],L(40) \\
\hline 70= & DATA KA/40H4000*W[J], 40H 40*U , 40H 400*PF, 40H 4000*O, 10 \\
\hline \(74=\) & 4H10000*OR /,L3,L4,LS,L6,L7,L8, ITER,ITERM, ITIME/9*0/,GG, GG4,Z,ZZ, \\
\hline 72= & 2TINIT,RUNOFF,VOL4,SDELTA/8*日.8/,END/4HEND/,RESTAR/4HREST/.L4/4*4/ \\
\hline \(73=\) & NER \(=0\) \\
\hline \(74=10\) & READ 20, HED \\
\hline \(75=20\) & FORMAT (20A4) \\
\hline 76= & IFCHED[ 1 ].EQ.END J STOP \\
\hline \(77=\) C= & GENERAL INFORMATION: \\
\hline \(78=\mathrm{C}\) & LU,MU,LL,ML-NUMBERS DESCRIBING LIMIT OF ARRAYCPRESSURE HEAD, \\
\hline \(79=\) C & CONDUCTIUITYJ \\
\hline \(80=C\) & NM-MAXIMUM NUMBER OF NOOAL POINTS \\
\hline 84=C & NNL-NODAL POINT WHERE THE SOIL PROFILE IS LAYERED \\
\hline \(82=C\) & L2-MAXIMUM NUMBER OF OUTPUT \\
\hline \(83=C\) & IMAX-MAXIMUM NUMBER OF ITERATIONS \\
\hline \(84=C\) & FAC, FAC4-TIME CONSTANTS DEPENDING ON UNITS USED IN PROBLEM \\
\hline \(85=C\) & SWCU, SWCL-SATURATED WATER CONTENT OF UPPER AND LOWER LAYER \\
\hline 86=C & AA-FACTOR(0.7 (AA) 1.0 ) \\
\hline \(87=\) C & RNAM, TB, TE-MAX. VALUE OF ROOTING DEPTH NON-ACTIUE AND: \\
\hline \(88=C\) & BEGINNING AND END RNA OCCURS \\
\hline 89 = C & SMB, SML \(4, S M U 4, S M 2, S M 3, A Q, B Q\) VALUES DESCRIBING SINK TERM \\
\hline \(90=C\) & DT,STM, TM, WSP, DS-STARTING TIME STEP AND UALUES DESCRIBING \\
\hline 94=C & UARIATION OF TIME STEP FOR NEXT STAGES OF COMPUTATION \\
\hline \(92=\) C & STM-IT IS RECOMMENDED TO SET STM EQUAL TO 40*DT \\
\hline \(93=C\) & TM-OUTPUTS TIME STEP \\
\hline \(94=C\) & USP-IT IS RECOMMENDED TO SET WSP BETWEEN 0.045 AND 0.035 \\
\hline \(95=C\) & DS-ESTIMATED MAX. TIME STEP OF COMPUTATION[DTMAX=TM*DSJ \\
\hline \(96=C\) & DSP-DEPTH OF SOIL PROFILE[DSP = DX*NM] \\
\hline \(97=\) C & EPS-MAXIMUM RELATIUE CHANGE IN THE VALUES OF SUCTION BETUEEN \\
\hline \(98=C\) & ANY TWO SUCCESSIVE ITERATIONS[FRACTION] \\
\hline \(99=C\) & L-10 UALUES: L[9]-FIRST DAY OF CALCULATION(FROM BEGINNING OF YEAR] \\
\hline 400=C & L(2)-LAST DAY OF CALCULATION \\
\hline 104=C & L(3)-NUMBER OF DAYS IN FEBRUARYC28 OR 293 \\
\hline 102=C & L(4)-DATE OF THE BEGINNING OF CALCULATION \\
\hline 103x & L(5)-FIRST MONTH OF CALCULATION \\
\hline 104=C & L(6)-LAST MONTH OF CALCULATION \\
\hline 105=C & L[7]-IF EQUALS 0-NET RADIATION IN W/M**2; \\
\hline 106=C & IF EQUALS 4 -NET RADIATION IN CAL/CM**2/DAY \\
\hline \(407=\) C & L(8).L(9), L( 40]-UALUES OF 0 OR 4 MUST BE SET \\
\hline 108= & READ 30, KOD,LU,MU,LL,ML,NM, NNL, IMAX,L2,L \\
\hline \(109=30\) & FORMAT (46I5) \\
\hline 110= & READ 4B, AA, SWCU, SWCL, RNAM, TB, TE \\
\hline 119= & READ 40, SMB, SMU4,SML \(4,5 \mathrm{SM} 2,5 \mathrm{SH}, 80\) \\
\hline 142= & READ 40, DT, STM, TM, WSP, DS, DSP, EPS,FAC \\
\hline 113=40 & FORMAT( 8 F 40.3 ) \\
\hline 144= & IW \(=\) MU-LU 4 \\
\hline 145= & IW2=ML-LL+ 4 \\
\hline 146= & \(10=L(2)-L(1)+1\) \\
\hline 147= & \(A O=4.8-B C\) \\
\hline 148= & CALL PARAM (ID, NMJ \\
\hline \(119=\) & CHL [ 1 )=TEM ( 1 ) \\
\hline 120= & CHU[ 1 )=TEM 8 8 ] \(^{\text {- }}\) \\
\hline 124= & SUC 4 ]=TEM ( 464 ) \\
\hline 122= & W[ 4 )=TEM 3 (36) \\
\hline 123= & CL[ 1 ]=U( 4 ) \\
\hline 124= & SLC 4 )=U(84) \\
\hline 125= & \(\operatorname{CUS}\) (1)=U(161) \\
\hline 126= & S4(1)=U(244) \\
\hline 127 = & IF(KOD(5).EQ.0] CALL HEPR[W,S4,SU,SL,NM] \\
\hline
\end{tabular}
484: IFCTi.GE.TEJ RNA=RNAM
482= CALL BOCO[EPA,SGLA,FLUXA,DRZA,SN4N,CFWT,DX,NA,ID,L,KOD,T4]
483=
484=
185=
486표
187=70
488=
489=
490=
491=80
492=
193=90
494프․
495=
496=
497=
\(498=440\)
IF[KOD(5].EQ.4) CALL WACO[SU,SL.W,WCL.NM,S4)
\(A M=4.8\)
\(B M=4.5\)
STN=DT
DT \(4=D T\)
DX=DSP / NM
DO 50 J=4.NM

S2[J]=Sイ〔〕)
\(\mathrm{x}(\mathrm{J})=0 \mathrm{X} *(\mathrm{~J}-0.5)\)
CONTINUE
\(N=N M\)
N4 \(=N M\)
H4=DT/DX
\(H 2=H 4 / D X\)
IF (KOO (4).E日. 0 ) \(Z=4.0\)
IF (KOOC4].EO.2) Z=-4.0
IF(KOD(2).EQ.1) READ 40. TINIT,GG, (TR(J),J=4.L2]
IF(KOO(2).NE.1) TR[1]=TM
T=TINIT
TMA=T+TR[1]
IF (KOD(6).E日.0) CALL HEPAS(UCS,SU,ID)
DXH=日. 5 *DX
SSS \(=T+S T M\)
TER \(4=\mathbf{G G}\)
KN=4
YY\{ \(=\mathbf{G G}\)
TEE2=T
LPA=L4
FWT =FAC*CSAT2
\(T=T+D T\)
GG4=GG
RUNOFF \(4=\) RUNOFF
\(K Y=L[1]+K N\)
IF[T.LE.KYJ GO TO 64日
IFCTM.EO.1.0.AND.KN.EO.1] ABC=AUTR
IF[TM.EO.1.B.ANO.KN.GT.4] TRA[KN-4]=AUTR
IFCTM.E日.1.0] 60 TO 650
TE2ㅍT-DT
IF[TE2.LT.KY] YY4=GG
IF[TEZ.LT.KYJ TEEZ=TEZ
IF[TE2.LT.KYJ GO TO 648
AT \(=\) YY \(1+[G G-Y Y 1\) ]* \((K Y-T E E 2] /[T E 2-T E E 2]\)
IF[KN.EQ.1) ABC=10.0*[AT-TER4]

TEE2=KY
YY\{ \(=A T\)
TER\{=AT
\(K M=K N+1\)
\(L 6=L 6+1\)
\(T 4=T-9.5 * D T\)
IF (T1.LE.TB) RNA=B.E
IF[T4.GT.TB.AND.T4.LT.TE] RNA=RNAM*(T4-TB)/(TE-TB)
IF[DRZA-RNA.LE.B.D] OM=0.0
IF[DRZA-RNA.GT.O.O] OMz. 1 \%EPA/(DRZA-RNA]
SMM=日0*OM/[SM3-SM2]
PRZ=DRZA/DX+.504
IF[SN4N.GE.DXH] GO TO 80
DX4=SN4N
SNイN=0.0
GO TO 90
SN4N=SN4N-DXH
\(D \times 1=D \times H\)
RR=DX/DX1
IF(N.GE.N4) GO TO 108
AGPF=[S[N]-SN4N]/(DX*(N4-N)+DX4)

\(J=N 1+1\)
J=Jーイ

499＝
200＝
\(201=\)
202＝
\(203=180\)
204＝
205＝
IF［LB．EO．1］GO TO 440
207＝
\(208=148\)
\(209=450\)
\(248=430\)
241＝
242＝
213＝
214＝
\(215=720\)
246＝
247＝
218＝
\(249=\)
\(220=\)
\(221=\)
222＝
\(223=\)
\(224=\)
\(225=\)
\(226=\)
227＝
228＝
\(229=\)
\(230=\)
\(231=460\)
232＝
\(233=\)
234＝
\(235=\)
236＝
237＝
238＝
\(239=480\)
240＝
244＝490
242＝749
\(243=200\)
244＝
245＝
\(246=170\)
247＝
\(248=240\)
249＝
250＝
254＝
252＝
253＝
254＝
255\％
256＝
257＝
258＝
259＝
\(268=\)
\(261=\)
262＝
263＝
264플
265＝
266＝
267＝
\(268=230\)
\(269=\)
IF（J．LE．N＋4）GO TO 100
SN2［J］＝SN4（J－4）
GOTO 140
\(N 2=N-1\)
DO \(430 \mathrm{~J}=4, \mathrm{~N} 2\)

GO TO 450
SN2 \((J+4)=S N\{(J)\)
CONTINUE
\(\mathrm{N}=\mathrm{N} 1\)
\(J=4\)
IF［L8．EO．4）SG4＝0．S＊（S（4）＋54（4））
SG＝．S＊（SG4＋SGLA）
C2U＝SORTCC4＊C2 J
C2＝C2U
CALL DMCCJ，CH4，SG4，CHU，CHL，SU，SL．
\(C=0.0\)
IFCKOO［6］．EO．0］ \(\mathrm{C}=-\mathrm{H} 2 * \mathrm{C} 2 / \mathrm{CH} 4\)
A＝－H2＊C4／CH4
\(8=2.0+A+2 . \theta * C\)
OK（ 7\()=0.0\)
IF［J．GT．PRZ］GO TO 460
CALL RER［J，02，01．SG，SN4（J），NNL）

IF［L8．EO．8）GG＝GG＋OK［J］＊DX＊DT
IF（KOD（6）．EO．Q）GO TO 470
FLUXM＝C2＊（SG4－SGL．A＋Z＊DXH）／DXH

IF（FLUXA．GT．B．日）GO TO 740
IF（FLUXA．LE．B．8）GO TO 200
GO TO 200

4OK［ 4］／CH4
GO TO 240
43／CH4
R4（1）＝A／B
\(\operatorname{R2}[1]=-E / A\)
\(I I I=N-4\)
\(00220 \mathrm{~J}=2, I I I\)
SN12＝0．5＊［SN4［J］＋SN2［J］）
\(\mathrm{C}=-\mathrm{H} 2 * \mathrm{C} 2 / \mathrm{CH} 4\)
\(A=-H 2 * C 4 / C H 4\)
\(B=2 \cdot \theta+A+C\)
OK（J）\(=0.0\)
DXL＝J※DX
DXU＝［J－1）※DX
DXM＝［J－0．5］＊DX

IF（RNA．LE．DXU）OK［J］\(=0.5 *(01+02)\) 10K［J］／CH4


SN4［J）\(=.25 *(54(J+4)+54(J)+S(J)+5(J+4))\)

IF（L8．NE． 13 SG4天BMnS4（4）－．5＊AMnS2（4）
CALL CONCJ，C4，C2，SG4，SGLA，CU，CL，SU，SLJ
CALL CON：J，C4，C2，SN4［J］，SG，CU，CL，SU，SLJ

IF［RNA．LE．DXH）OK［J］＝0．5＊［O4＋02＊\((D X H-R N A) / D X H)\)
IF［RNA．GT．DXH．AND．RNA．LE．DX］OK［J］＝04＊［DX－RNA］／DX

IF［FLUXM．LE．日．B．AND．FLUXA．LE．©．日］GO TO 480
IF［FLUXH．GT．E．Q．AND．FLUXA．GT．B．日J GO TO 490
IF FFLUXM．GT．日．日．AND．FLUXA．LE．日． 0 ］FLUXM＝0． 8
IFCFLUXM．LE．B．E．AND．FLUXA．GT．B．8）FLUXM＝B．8

IF（FLUXM．LT．8．4＊FLUXA）FLUXM＝0．1＊FLUXA
IF［FLUXM．GT．O．1＊FLUXAJ FLUXM＝日．1＊FLUXA
RUNOFF＝RUNOFF＋CFLUXA－10． \(0 * F L U X M\) ）＊DT


EmAnS4［2］＋［4．0－8］＊S4［4］＋4．0＊C＊SGLA－2．0＊Z＊H4＊［C4－C2］／CH4－2．\＃DT＊OKC4

CALL CON［J，C4，C2，SN4［J］，SN2［J］，CU，CL，SU，SL］
CALL DHCCJ，CH4，SN42，CHU，CHL，SU，SLJ

IF［J．GE．PRZ＋1．OR．RNA．GT．DXLJ GO TO 230

CALL RER［J，Q4，02，SN4［J］，SN2（J），NNL］
IF［J．E日．PRZ）Of＝04＊（DRZA－DX＊（J－0．5））／DXH
IF（RNA．LE．DXM．AND．RNA．GT．DXU］OK［J］＝0．5＊［O4＋02＊（DXM－RNA）／DXH）
IF（RNA．GT．DXM．AND．RNA．LE．DXLJ OK［J］＝01＊［DXL－RNA］／DX


270＝R4［J］＝A／［B－C＊R4（J－1］）
\(271=\)
272＝
\(273=220\)
\(274=\)
R2 \((J)=(C * R 4(J-4) * R 2(J-4)-E) / A\)
IF（L8．EO．8）GG＝GG＋QK（J）＊DT＊DX
CONTINUE
J＝N
275＝CALL CON［J，Cイ，C2，SNイN，SN2［N］，CU，CL，SU，SL］
276＝IFCDX4．EQ．DXH］SN42＝0．5＊（SN2（N）＋SN1N）
277＝IF（DX1．NE．DXH）SN12＝SN2（N）＊（4．0－DX／（DX＋2．0＊DX4））
278＝CALL DMC（J，CH4．SN42．CHU，CHL，SU，SL3
279＝
280＝
\(281=\)
\(A=-\mathrm{H} 2 * \mathrm{C} / \mathrm{ClH}_{4}\)
\(\mathrm{C}=-\mathrm{H} 2 * \mathrm{C} 2 / \mathrm{CH} 4\)
\(8=2 . \theta+A * R R+C\)
OK（J）\(=0.0\)
\(D \times M=(N-0.5) * D X\)
283＝
284＝
285＝
\(D \times L=D \times M+D \times 4+S N 4 N\)
IF（RNA．GT．DXL．OR．J．GE．PRZ＋4］GD TO 240
CALL RER［J，Q1，Q2，SN4N，SN2\｛J］，NNL）
DXU＝（N－1）＊DX
IF（J．EQ．PRZ）Q4＝Q4＊（DRZA－DX＊（PRZ－0．5）\() / D X H\)
IF（RNA．LE．DXU］OK（J）＝0．5＊［01＋02）
IF（RNA．LE．DXM．AND．RNA．GT．DXU）OK［J］＝0．5＊［O\｛＋02＊［DXM－RNA］／DXH）
IF（RNA．GT．DXM．AND．RNA．LE．DXL 3 OK（J）\(=01 *(D X L-R N A) / D X\)
IF（L8．EQ．Q）GG＝GG＋QK（J）＊DT＊DX
\(E=[4.0-8] * S\{[N]+C * S\{(N-4]+2.0 * A * R R * S N\) 价－2．0＊2＊H\｛＊［C\｛－C2］／CH\｛－2．日＊
TDT＊OK［N］／CH4
\(S(N)=(E-C * R 1(N-1) * R 2(N-1)) /(B-C * R 1(N-1))\)
\(J=N+4\)
\(J=コ ー 4\)
IF［J．LT．2）GO TO 250
S（J－1）＝Rイ（J－1）＊（S［J］－R2（J－4）］
IF（S［J－4）．LT．0．004）S（J－4）＝0．004
GO TO 710
\(\mathrm{N} 2=\mathrm{N}-1\)
DO 260 J＝4，N2
\(D E V=A B S[S(J)+S[J+4)+S\{(J)+S 4[]+4)-4.0 * S N 4[J]) * 0.25\)
IF（DEU．GT．4．0．AND．DEV．GT．EPS＊SN\｛［J］）GO TO 270
CONTINUE
ITER＝0
GO TO 286
\(I T E R=I T E R+1\)
IF（ITER．LT．IMAX）GO TO 298
ITERM＝ITERM＋1
ITIME＝ITIME 4
IF（SN4\｛J］．NE．0．8）EPSM＝DEV／SNイ［J］
IF（SN4［J］．EQ．B．日］EPSM＝EPS
GO TO 340
DO 320 J＝4，N2
SN4 \((J)=0.25 *(S(J)+S[J+4]+S 1[J)+S\{[J+4)\}\)
IF（J．GT． 1\()\) SN2［J］＝SN4（J－4）
CONTINUE
J＝4
GG＝GG 1
RUNOFF＝RUNOFF 4
SG4＝0．5＊（S4［4］＋S（4）］
GO TO 728
IF［LPA．EQ．L43 PRINT 330，EPS，ITIME，ITERM，J，EPSM，DEU，T
IF［LPA．NE．L4］PRINT 730，EPS，ITIME，ITERM，J，EPSM，DEU，T
FORMATC \(4 \mathrm{H} 4,49 \mathrm{H}\) NUMBER OF IMAX NOT ENOUGH TO REACH ACCURACY EPS＝FS．
\(44 /\)
247H VALUE OF ITIME＝15，8H ITERM＝15，15H NODE POINT J＝12，7H EPSM＝ 3F6．4，18H UALUE OF DEU［J］＝E9．3．7H TIME＝F7．3／3
FORMATE49H NUMBER OF IMAX NOT ENOUGH TO REACH ACCURACY EPS＝F5．4／ 447H UALUE OF ITIME＝IS，8H ITERM＝IS，45H NODE POINT J＝I2，7H EPSM＝ 2F6．4，48H UALUE DF DEU［J］＝E9．3．7H TIME＝F7．3／J
\(L P A=L P A+4\)
\(N S=N+4\)
IF［NS．GT．NMJ GO TO 460
DO 620 I＝NS，NM
IF（KOD（4）．EQ．4）S［I）＝4．8
IF［KOD 4 1）．NE．1）S［1］＝0．001
OK（I）\(=0.0\)
\(344=620\)
\(342=400\)
343＝
\(344=\)
345＝
\(346=630\)
347＝
\(348=\)
349＝
\(350=\)
\(351=\)
352＝
\(353=\)
\(354=\)
\(355=\)
\(356=\)
\(357=\)
\(358=\)
\(359=\)
368＝
36体
\(362=\)
\(363=\)
\(364=\)
\(365=\)
\(366=360\)
\(367=\)
368＝
\(369=340\)
370＝
\(374=\)
372＝
\(373=\)
\(374=\)
\(375=350\)
376＝
377＝
\(378=\)
379＝
\(380=\)
\(384=390\)
382＝
383＝
384＝
\(385=440\)
386＝
387＝
388＝
\(389=\)
\(390=\)
\(391=420\)
392＝
393＝
394＝
395＝
396＝
\(397=\)
398＝
\(399=\)
\(400=\)
401玉
402＝
403＝
\(404=\)
\(405=\)
406＝
407＝
\(408=\)

\section*{\(489=\)}
\(410=\)
414＝430

CONTINUE
CALL WACOCSU，SL，W，WCL，NM，S \()\)
VOL2 \(=0.0\)
DO 630 I \(=4\) ，NM
VOL2＝VOL2＋W［I］＊DX
CONTINUE
DELTA＝VOL2－VOL4＋GG－GG4－FLUXM＊DT
VOL \(4=\) VOL 2
SDELTA＝SOELTA＋DELTA
IF［T．LE．SSS GO TO 340
IF［LB．NE．日］GO TO 350
FLOW＝ABS（0．5＊C4／DX4＊（2．0＊SN4N－S（N）－S4（N）＋2．0＊2＊DX4］）
IF（KOD（6）．EQ．0）FLUXM＝C2U／DXH＊［SG4－SGLA＋Z＊DXH］
IF（FLOW．LE．ABS（FLUXM））FLOW＝ABS（FLUXM）
IF［LS．NE．0］GO TO 340
ST＝WSP＊DX／FLOW
\(L 3=L 3+4\)
CFWT＝CFWT＊FWT
IF［CFWT．GT．4．0）CFWT＝4．0
IF（ST．GT．（C）FWT＊DS＊TM）ST＝CFWT＊DS＊TM
IF（L3．EQ．4）SS＝ST
IF（ \((L 3-L 4) . N E .4)\) GO TO 368
DT4＝DT4＋STN
STN＝DT4
\(L 4=L 4+1\)
IFCL4．EQ． \(8 . A N D . D T 4 . L T . S S J\) GO TO 340
DT4＝SS
L4＝0
IF（ABS［T－TMA）．GT．．日04＊DT）GO TO 370
IFCLB．EQ．1］GO TO 358
T \(4=1\)
L8＝4
RUNOFF＝RUNOFF 4
GO TO 300
DO \(390 \quad I=1, N\)
\(K=1 / 2\)
IF（I．GT．2．AND．I．NE．2＊K）CALL CON（I，W2（I），W2（I－4），SCI］，SCI－4），CU，CL
1，SU，SL）
IF（I．EQ．1）CALL CON（I，W2（I），C2，S［I］，SG，CU，CL，SU，SL）
IF［I．EQ．NJ CALL CON（I，C4，W2（I），SN4N，S（I），CU，CL，SU，SL）
CONTINUE
C4＝SORT（C4＊W2CN3）
AUTR＝10．8＊［GG－ZZ］／TR（Li）
PRINT 440，T，GG，AUTR，L6，ITERM，RUNOFF，DELTA，SOELTA，WCL
FORMATC 1H4．5H DAY＝F6．2．24H CUMULATIUE TRANS．\(=\mathrm{F} 6.3 .24 \mathrm{H} \mathrm{CM}\) AUERA 4GE TRANS．\(=\) FS． \(2,29 \mathrm{H}\) MM／DAY NUMBER OF TIME STEP＝I4．49H NUMBER OF 2ITER．\(=14 / /\)
38 H RUNOFF＝F6．2，12H MM DELTA＝F6．2，8H SDELTA＝F6．2．53H CM THETA 0 4F LOWER LAYER AT THE CONTACT WITH UPPER＝FS．4，42H CM＊＊3／CM＊＊3J
PRINT 420
FORMATC／／84H \(Z\) THETA CUM．WATER SUCTION

30AY 4／DAY／J
\(V=0.8\)
DO \(430 I=4\) ．NM
\(V=U+W[I)\) \＃DX
IF（I．EQ．1） \(00=F L U X M\)
IF（I．GE．N）OO＝C4／DX4＊（SN4N－S（N）＋Z＊DX4）
IF［I．NE．4．AND．I．LT．N］\(O Q=0.5 * W 2(I) / D X *[S(I+\{ )-S(I-4)+2 . \theta * Z * D X]\)
IF（S（I）．LT．4．0）S2（I）\(=4.0\)

PRINT 448，X［I］，W［I］，V，S［I］，00，OK［I］
IT＝L． 1
RETC＝ALOG48［S2（I）
IX［IT，I，i］＝W［I］＊4000．＋0．5
IX \(\{1 T, 1,2\}=U * 10 .+0.5\)
IX \(I T, I, 3]=R E T C * 400 .+0.5\)
IX \([\) IT，I， 4\(\}=00 * 4000 .+0.5\)
IX［IT，I，5 \(\}=0 \mathrm{~K}(I) * 40000 .+8.5\)
CONTINUE


483= PRINT 590, ( \(X(J), J=4, N M)\)
484=
T=TINIT
\(485=\)
00600 J=4.L2
\(486=\quad T=T+T R(J)\)
487 = LJ=T
488= PRINT 640, LJ, (IX[J,I,KKK),I=4,NM)
\(489=600\)
\(490=570\)
\(491=580\)
\(492=590\)
\(493=618\)
\(494=668\)
\(495=\)
\(496=\)
CONTINUE
CONTINUE

FORMATE7X,25F5.8]
FDRMAT \{ \(4 \mathrm{X}, 14,2 \mathrm{X}, 2515\) \}
FDRMAT( \(4 \mathrm{H} 4,39 \mathrm{H}\) ACTUAL TRANSPIRATION [MM/DAY)///3
PRINT 660
PRINT 670, \(0,[I, I=4,40)\)
\(498=\)
\(499=\)
FORMAT(44140)
\(K X=I D / 40+9\)
\(L Y=0\)
\(500=\)
\(501=\)
\(502=680\)
503=
\(504=\)
\(585=\)
506=
IF[LY.EQ.1) PRINT 700, LC,ABC,[TRA(KN),KN=LA,LB]
IF(LY.GT.1] PRINT 700, LC, (TRACKN),KN=LA,LB)
\(509=\quad \quad L C=L B+1\)
\(540=\quad\) GO TO 680
S41=780 FORMATEI40.10F40.21
\(542=690\) STOP
513=
END
\begin{tabular}{|c|c|}
\hline \[
\begin{aligned}
& 544= \\
& 545= \\
& 546=
\end{aligned}
\] &  \\
\hline 547= & 45), DWT (365), DRZ (365), CU( 80), SU( 80), CHU(80), CL(80), SL(80), CHL (80), \\
\hline 548= & 2S4(25), W( 25 ), SGL (365), IB(69), KM ( 42\()\), THETA[2), WCS(365), EP (365), HED \\
\hline \(549=\) & 320), L( 40), KOD(6), LC4(5) \\
\hline \(520=\) & EQUIVALENCE (CH,EP), (RH,SGL,WCS), (SC, DWT), (HNT, DRZ), (TEM (4), CHL ( 1 ) \\
\hline 524= &  \\
\hline 522= &  \\
\hline 523= & COMMON/CONDU/ CSAT4, CSAT2,SUA4,SUA2, SUA3, SUB4, SUB2, SUB3, SUC, SUD, \\
\hline 524= & 4SLA4, SLA2, SLA3, SLB 4, SLB2, SLB3, SLC, SLD, CUA , CUA2, CUA3, CUB 4, CUB2, \\
\hline 525= & 2CUB3, CUC, CUD, CLA4, CLA2, CLA3, CLB 4, CLB2, CLB3, CLC, CLD, KODC63, NNL, IW4, \\
\hline 526= & 31W2,L6, SWCU, SWCL,LU,LL, MU, ML, FAC \\
\hline 527 \(=\) & COMMON/BONC/ DWT(365),SGL(365), EP (365), DRZ(365), FLUX(365) \\
\hline 528= & COMMON/FACT/ TEM, U \\
\hline 529= & COMMON/DECL/ HED (20), L (40) \\
\hline \(530=\) &  \\
\hline 53 4x & 4,KM(9),KM(44)/4*3日/, GAMMA/4*8.66743/, SEP/4*0.0/ \\
\hline 532= & KM( 2 ) \(=\) L ( 3 ) \\
\hline 533=C= & \(=\) BROUNDARY CONDITIONS \\
\hline 534 \(=\) C & IF KOD(6)=6-PRESCRIBED THETA AT THE SURFACE \\
\hline 535=C & If KOOC 6 ]=1-PRESCRIBED FLUX, SGL AND EP AT THE SURFACE \\
\hline 536 \(=\) C & IF KOD 6 ] 2 2-BOUNDARY CONDITION AT THE SURFACE IS ESTIMATED FROM \\
\hline 537ec & METEOROLOGICAL DATA: TEM, RH, U, HNT,CH AND FLUX \\
\hline \(538=\) C & IF L(7)=8-HNT IS GIUEN IN W/M**2, OTHERWISE IN CAL/CM**2/DAY \\
\hline 540= & PRINT 10, HED \\
\hline \(544=40\) & FORMAT ( H4, 20A4///J \(^{\text {/ }}\) \\
\hline 542= & PRINT 38 \\
\hline 543=30 & FORMATC \(4 \mathrm{H}, 34 \mathrm{H}\) BOUNDARY CONDITIONS AT THE TOP/J \\
\hline 544= & IF (KOD (6).EQ. 1) READ 640, [FLUX[I],EP[I], SGL [I], I= 1, ID \\
\hline \(545=\) & IF[KOD (6).EQ.2] READ 658, [TEMCI],RH[I],U[I], HNT[I], CH[I], SC[I], \\
\hline 546= &  \\
\hline 547= & IF [KOD (6).EQ.0] READ 20, (EP(I), WCS [I],I=4,ID) \\
\hline \(548=20\) & FORMAT(BF40.4) \\
\hline \(549=640\) & FORMAT 2 [F40.3,F40.3.E40.4]) \\
\hline \(558=650\) & FORMAT 7 76.3) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline \[
\begin{aligned}
& 551= \\
& 552=
\end{aligned}
\] & IF(KOD(6).NE.2) GO TO 300 PRINT 40 \\
\hline 553=40 & FORMATT//7X,3HDAY,5X,5HTEMP.,4X, BHREL HUM.,4X,9HWIND VEL.. 4 X , 9HNET \\
\hline 554= & 4 RAD.,5X,44HCROP HEIGHT,4X,40HSOIL COVER,4X,9HPRECIPIT./J \\
\hline 555= & DO \(501=9.10\) \\
\hline 556= & \(L C=L[1]+I-4\) \\
\hline 557 \(=\) & IF (L[7].NE.0) HNT \((1)=0.48426 * H N T(I)\) \\
\hline 558= & TEM [ I \()=\) TEM ( I \()+273.45\) \\
\hline 559 = &  \\
\hline \(560=58\) & CONTINUE \\
\hline 561=60 & FORMATEI \(40.3 \mathrm{X}, \mathrm{F6.2,7X,F5.3,6X,F6.2,6X,F7.2,7X,F7.2,9X,F5.2,9X,FS.2}\) \\
\hline 562= & 4) \\
\hline 563=C & \\
\hline 564=C & FGA,FGB,FGC,FGO,FGM, FMCH-COEFFICIENTS OF G(CH)-FUNCTION \\
\hline 565=C & \\
\hline 566= & FGA \(=.370 E-07\) \\
\hline \(567=\) & \(F G B=.283\) \\
\hline 568= & FGC=.164E-67 \\
\hline 569= & FGO \(=.59\) \\
\hline 570= & \(F G M=1.3 E-07\) \\
\hline \(574=\) & FMCH=20.6 \\
\hline 572= & IFCLC8].EQ.0) READ 430, FGA,FGB,FGC,FGD,FGM,FMCH \\
\hline 573=C & \\
\hline 574=C & FLA,FLB,FLC-COEFFICIENTS OF LAI-FUNCTION \\
\hline 575=C & \\
\hline \(576=\) & FLA \(=1.479\) \\
\hline 577= & \(F L B=.25\) \\
\hline 578= & FLC=1.174 \\
\hline 579= & IFCL[9].EQ.0] READ 28, FLA,FLB,FLC \\
\hline \(580=C\) & \\
\hline 584=C & FIA,FIB,FIC,FID,FMP,FMI-COEFFICIENTS OF INTERCEPTION [FIN[PRECJ]- \\
\hline 582=C & \\
\hline 583=C & \\
\hline 584= & FIA \(=.55\) \\
\hline 585= & \(F I B=.53\) \\
\hline 586= & FIC=.0085 \\
\hline S87 = & FID \(=5.0\) \\
\hline 588= & \(F M P=28.0\) \\
\hline 589 = & \(F M I=4.85\) \\
\hline \(590=\) & IF[L(10).EQ.0) READ 20, FIA,FIB,FIC,FID,FMP,FMI \\
\hline \(591=\) & PRINT 70 \\
\hline \(592=70\) & FORMAT \(/ / 50 \mathrm{H}\) THE FUNCTIONS OF G(CH),LAI AND FIN[PRECJ/J \\
\hline 593= - & -PRINTING OF THE G[CH]-FUNCTION \\
\hline 594= & PRINT 80, FGA, FGB, FMCH, FGC, FGD, FMCH, FGM \\
\hline \(595=80\) & FORMATC \(46 \mathrm{H} \quad \mathrm{G}(\mathrm{CH})=\mathrm{E}\) 10.3,5H*CH**F6.3,29H \\
\hline 596= & 4FOR CH.GE.F7.2,3H CM/ \\
\hline 597 \(=\) & 246 H G CH\(]=\) [40.3.5H*CH**F6.3.29H FOR CH. \\
\hline \(598=\) & 3LT.F7.2.3H CM/ \\
\hline 599= & 433H MAXIMUM UALUE OF G(CH)=E \(10.3 / 3\) \\
\hline 680=C-- & -PRINTING OF THE LAI-FUNCTION \\
\hline \(604=\) & PRINT \(100, F L A, F L B, F L C\) \\
\hline \(602=100\) & FORMATC 44H LAI=F6.3.4H*SC+F6.3.7H*SC**2+F6.3.6H*SC**3/3 \\
\hline 603=C- & -PRINTING OF THE FINCPRECJ-FUNCTION \\
\hline 604= & PRINT 140, FIA,FIB,FIC,FID,FMP,FMI,FMP \\
\hline \(605=410\) &  \\
\hline 606= & (REC-FS.2.24H) FOR.PREC.LT.FS.2,7H MM/DAY/ \\
\hline 687= & 223H FINCPRECJ=SC*FS.2,57 H \\
\hline 698= & 3 FOR.PREC.GE.FS.2,7H MM/DAY/J \\
\hline \(609=C\) & \\
\hline \(640 \times C=\) & \(=\) CALCULATION ANO PRINTING OF THE VALUES-EWET-,-ES-, -EP-,-SEPLANT-, \\
\hline 6419 \({ }^{\text {c }}\) & FLUX-,-SGL \\
\hline 612=C & ESOIL=ES,-EPLANT=EP,-SEPLANT IS THE SUM OF THE EP-VALUES \\
\hline \(643=C\) & FLUX=PREC-ES-FIN \\
\hline \(614=C\) & FIN IS INTERCEPTION, SGL IS THE MINIMUM ALLOWED SUCTION AT \\
\hline 645=C & THE SOIL SURFACE,EV IS THE SATURATED WATER VAPOUR PRESSURE,DL IS \\
\hline \(646=C\) & THE SLOPE OF SATURATION UAPOUR PRESSURE CURVE \\
\hline 647=C & UPD IS THE VAPOUR PRESSURE DEFICIT OF AIR \\
\hline 648=C & \\
\hline 619= & PRINT 420 \\
\hline \(620=426\) & FORMATC 4H4//30X, SOHCALCULATION OF MAXIMUM POSSIBLE EVAPOTRANSPIRAT \\
\hline 624= & 410N///J \\
\hline
\end{tabular}

622＝
\(623=430\)
624＝
\(625=440\)
626＝
627＝
\(628=\)
\(629=450\)
\(630=\)
\(634=\)
\(632=\)
\(633=\)
\(634=\)
\(635=\) \(636=\)
\(637=\)
\(638=\)
\(639=\)

\section*{\(640=\)}
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\(663=\) \(664=180\) \(665=\) \(666=190\) \(667=\)
\(668=\)
\(669=170\)
\(670=\) \(674=46\) \(672=200\) \(673=\) \(674=\) \(675=300\) \(676=\) \(677=230\) \(678=\) \(679=\) \(680=\) \(681=\) \(682=680\) \(683=\) \(684=690\) \(685=240\) \(686=250\) \(687=\) \(688=220\) \(689=270\) \(690=\) \(691=\) \(692=\)

PRINT 130
FORMATC／／55X，38HPOTENTIAL TRANSPIRATION RATE［MM／DAY］／／］
PRINT 140
FORMATE 4H，32HDATE DAY EWET ESOIL EPLANT 0．0．7X，3H2．0，7X，3H4．0．7X
4，3H6．\(, 7 \mathrm{FX}, 3 \mathrm{HB}, 0,6 \mathrm{X}, 4 \mathrm{H} 4 \mathrm{Q}, \mathrm{B}, 6 \mathrm{X}, 4 \mathrm{H} 42.0,6 \mathrm{X}, 4 \mathrm{H} 44,0,4 \mathrm{X}, 7 \mathrm{HSEPLANT}, 4 \mathrm{X}, 5 \mathrm{HFL}\)
2UX，5X，3HSGL，4X，3HUPD 3
PRINT 450


\(N E=L(4)-4\)
\(L F=L(S)\)
\(L E=L(6)\)
Lf＝L（1）
L4＝L（4）
\(00468 \mathrm{M}=\mathrm{LF}, \mathrm{LE}\)
［2 \(=\mathrm{KM} \mathrm{M}(\mathrm{M}]\)
00170 J＝L4，I2
\(N E=N E+4\)
\(I=N E-L 4+4\)
WED \(=.858302635 *\) TEMC I J－2． 19386068
\(E V=1.3332 * E X P(\{1.088749864 * T E M[\) I \(\}-276.4883955) / W E D)\)
\(D E L=43.73450407 * E V /(\) UED＊＊2 \()\)
IF（CH［I）．GE．FMCH GCH＝FGA＊CHCI \(3 * * F G B\)
IF（CH（I）．LT．FMCH）GCH＝FGC＊CH（I）＊＊FGD
IF（GCH．GT．FGM）GCH＝FGM
LAI＝FLA＊SC（I）＋FLB＊SC［I \() * * 2+F L C * S C[I] * * 3\)
UPD \(=\)（4．B－RH（I）\(] * E V\)
EWET＝．0352＊（DEL＊HNTCI \()+4.8804 E+08 * G C H *(U[I) * * .75) * U P D) /[D E L+G A M M A)\)
ES＝0．0352＊DEL＊HNT（I）＊EXP \([-0.39 * L A I] /(D E L+G A M M A]\)
IF（ES．GT．EWET）ES＝EWET
CHE I J＝EWET－ES
IF（FLUX［I］．LE．FMP］FIN＝SC［I］＊FIA＊FLUX［I］＊＊［FIB－FIC＊［FLUX［I］－FID］］
IF（FLUX（I）．GT．FMP）FIN＝SC（I）＊FMI
FLUX［I］＝FLUX［I］－FIN－ES
\(S E P=S E P+C H(I)\)
IF［FLUX［I］．GT．0．0］SGL［I］＝0．084
IF［FLUX［I］．LE．日．0］SGL［I］＝－4708．0＊TEM［I］＊ALOG［RH［I］）
\(I I=\{C H(I) * 5.0+0.5\}\)
DO 180 13 \(=4.69\)
IF（II．GT．I3）IB（I3）＝4H－
IF（II．EQ．13）IB（I3）\(=4 \mathrm{H}+\)
IF（II．LT．I3）IB（I3）＝4
CONTINUE
PRINT 199，J，M，NE，EWET，ES，CH［I］，IB，SEP，FLUX［I），SGL（I J，UPD

4X，F6．2，4X，F6．2，4X，E9．3，4X，F5．4\}
IF（NE．GE．L［2）］GO TO 200
CONTINUE
L4 \(=1\)
CONTINUE
PRINT 150
PRINT 148
GO TO 240
IF［KOD（6）．EQ．8］GO 10220
PRINT 230
FORMATC／／2［7X，3HDAY，4X，6HEPLANT，6X，4HFLUX， \(7 \mathrm{X}, 3 \mathrm{HSGL}\) J／J
DO \(248 \mathrm{I}=1,1 \mathrm{D}, 2\)
\(00680 \mathrm{~J}=1,2\)
LC 1 （ \()\) ）\(=\) L \((1)+1-2+〕\)
IF（LCiCJ）．EQ．L（2）J GO TO 690
CONTINUE
コロコー 4

CONTINUE
FORMATE2（I40，3X，F7．3，3X，F7．2，4X，E9．3］）
GO TO 216
PRINT 270
FORMAT \(/ / / 4[7 X\), 3HDAY， \(4 X\), 6HEPLANT， \(5 X, 5 H T H E T A 3]\)
DO \(280 \mathrm{I}=1, I \mathrm{D}, 4\)
DO \(290 \mathrm{~J}=1.4\)
LC\｛（J）\(=\)（ \((4)+I-2+〕\)

\(764=\)
\(765=\)
766＝
767＝
768x
\(769=510\)
\(770=\)
\(774=\)
772＝
\(773=\)
\(774=\)
\(775=\)
\(776=520\)
\(777=\)
778＝
\(779=\)
\(780=\)
\(784=\)
782＝
783＝
\(784=\)
\(785=\)
786＝
\(787=540\)
\(788=\)
789＝
\(790=550\)
\(791=\)
792＝
793＝
\(794=530\)
\(795=\)
796＝
797＝
798＝
\(799=570\)
800＝
\(804=\)
892＝
803＝
\(884=\)
805＝
\(806=\)
\(807=500\)
808：
\(809=\)
\(810=\)
814＝
842＝
\(843=\)
\(814=580\)
\(845=\)
\(816=\)
8イ7＝
\(818=\)
849＝
\(820=\)
821＝
\(822=700\)
823＝
824＝
825＝
\(826=\)
827＝
828＝
\(829=560\)
\(830=\)
831＝
\(832=\)
833＝
\(834=\)

\begin{tabular}{|c|c|}
\hline \(835=\) & D0 \(600 \mathrm{I}=4, \mathrm{IW} 2,2\) \\
\hline \(836=\) & D0 610 J＝1，2 \\
\hline \(837=\) & THETA \(]\) ］\(=\) THETA \((J)+0.82\) \\
\hline \(838=\) & IF［THETA［J］．GE．（ML＊0．04］）GO TO 620 \\
\hline \(839=640\) & CONTINUE \\
\hline \(840=\) & \(J=J-4\) \\
\hline \(844=620\) &  \\
\hline 842＝ & 1IL－ 1 ），IL＝ \(1, \mathrm{~J}\) ） \\
\hline \(843=\) &  \\
\hline \(844=\) & 4J］ \\
\hline \(845=600\) & CONTINUE \\
\hline 846＝ & IFCKOD（ 1）．EQ．2）GO TO 630 \\
\hline 847 \(=\) & PRINT 570，2，CSAT2，CLA1，CSAT2，CLA2，CLA1，CLA1，CLB4，CLB2，CLB4 \\
\hline 848＝ & GO TO 638 \\
\hline \(849=590\) & PRINT 580，2，SLA1，SLB4，SLB 1 ，SLC，SLA2，SLB2，SLC，SLD，SLA3，SLB3，SLD \\
\hline \(850=\) & PRINT 700，CSAT2，CLAi，CLB1，CLC，CSAT2，CLA2，CL82，CLC，CLD，CLA3，CLB3， \\
\hline \(851=\) & 1CLD \\
\hline 852＝630 & RETURN \\
\hline 853＝ & END \\
\hline 854 \(=\) & SUBROUTINE WACO［SU，SL，W，WCL，NM，S \\
\hline 855＝ & INTEGER P \\
\hline 856＝ & DIMENSION KOD（6），SU［80），SL（80），W（25），S（25） \\
\hline \(857=\) & COMMON／CONDU／CSAT4，CSAT2，SUA4，SUA2，SUA3，SUB4，SUB2，SUB3，SUC，SUD， \\
\hline 858＝ & 1SLA1，SLA2，SLA3，SLB ，SLB2，SLB3，SLC，SLD，CUA \(4, C U A 2, C U A 3, C U B 4, C U B 2, ~\) \\
\hline 859＝ & 2CUB3，CUC，CUD，CLA ，CLA2，CLA3，CLB, CLE2，CLB3，CLC，CLD，KOD 6 ，NNL，IW4， \\
\hline \(860=\) & 3IW2，L6，SWCU，SWCL，LU，LL，MU，ML，FAC \\
\hline \(864=\) & IF［KOD \(43 . E 0.13\) GO TO 40 \\
\hline 862 \(=\) & \(\mathrm{I}=0\) \\
\hline \(863=20\) & \(\mathrm{I}=\mathrm{I}+1\) \\
\hline \(864=\) & IF［I．GT．NNLJ GO TO 30 \\
\hline 865 \(=\) & DO \(40 \mathrm{~J}=2\) ，IW 4 \\
\hline 866＝ & \(X=S(1)\) \\
\hline 867＝ & \(Y=4.0\) \\
\hline \(868=\) & IF［X．LT．SU［4］．AND．X．GE．SU［J）］\(Y=(J+L U-4+[S U C J]-X] /[S U C J-4)-S U(J)])\) \\
\hline 869＝ & 4／400．B \\
\hline \(870=\) & IF［X．GE．SU［ 1 ） \(\mathrm{Y}=0.04 * \mathrm{LU}\) \\
\hline \(874=\) & IF［X．LT．SU［IW4］．AND．SU［IWイ）．GT．0．00イ］\(Y=S W C U-[S W C U-B .04 *[1 W 4+L U-4)\) \\
\hline 872＝ & 1）／SU［IW1］＊X \\
\hline 873＝ & IF（X．LT．SU（IW4）．AND．SU［IW4）．LE．0．004］\(Y=S W C U\) \\
\hline \(874=\) & IF（Y．NE．4．0）W（I）\(=\) Y \\
\hline 875＝ & IF（Y．NE．4．0）GO TO 20 \\
\hline \(876=40\) & CONTINUE \\
\hline \(877=30\) & \(P=\) NNL－ 4 \\
\hline \(878=50\) & \(P=P+1\) \\
\hline 879＝ & IF（P．GT．NM）GO TO 60 \\
\hline 880＝ & \(X=S(P)\) \\
\hline 881＝ & \(Y=4.0\) \\
\hline 882＝ & DO 70 J＝2，IW2 \\
\hline 883 \(=\) & IF（X．LT．SL（1）．AND．X．GE．SL（J））\(Y=(J+L L-1+\{S L[J)-X] /(S L(J-4)-S L(J)])\) \\
\hline 884＝ & 1／100．0 \\
\hline 885＝ &  \\
\hline 886＝ & IF X ．LT．SL（IW2）．AND．SL［IW2）．GT．0．001）\(Y=S W C L-[S W C L-0.01 *(1 W 2+L L-1)\) \\
\hline \(887=\) & 4）／SL（IW2）＊X \\
\hline \(888=\) & IF［X．LT．SL［IW23．AND．SL［IW2］．LE．0．004］\(Y=\) SWCL \\
\hline 889＝ & IFCY．NE．4．0．AND．P．EQ．NNL）WCL＝Y \\
\hline 890＝ &  \\
\hline \(891=\) &  \\
\hline 892＝70 & CONTINUE \\
\hline \(893=10\) & IFCL6．GT． 43 GO TO 80 \\
\hline 894픅 & SUM \(=\) EXP［SUA \(4 *\)（SUB 1－SUC）\({ }^{\text {a }}\) \\
\hline 895＝ & SUM2＝EXP［SUA2＊（SUB2－SUD） \\
\hline 896 \(=\) &  \\
\hline 897＝ & SLM2 \(=\) EXP（SLA2＊\({ }^{\text {（SLB2－SLD）}}\) \\
\hline \(898=80\) & \(J=0\) \\
\hline \(899=92\) & J＝J＋1 \\
\hline \(980=\) & IF［J．GT．NNL GO TO 100 \\
\hline \(901=\) & \(\mathrm{X}=\mathrm{S}(\mathrm{J})\) \\
\hline 902＝ & IF（X．LT． 1.8\() \mathrm{X}=4.8\) \\
\hline 983＝ & IF［X．LE．SUMイ］W［J］＝SUB 4－ALOG［X］／SUAイ＊ \\
\hline
\end{tabular}
\(904=\)
\(905=\) \(906=\) \(907=100\) \(908=410\) \(989=\) \(910=\) 914= \(912=\) \(943=\) \(914=\)
\(945=\)
\(916=\)
\(947=\)
\(918=60\) \(919=\)

IF(X.GT.SUM4.AND.X.LE.SUM2) W[J]=SUB2-ALOG(X)/SUAZ
IF[X.GT.SUM2] \(W[J]=S U B 3-A L O G[X] / S U A 3\)
GO TO 90
\(P=\) NNL -1
\(P=P+1\)
IF(P.GT.NMJ GO TO 60
\(X=S(P)\)
IF(X.LT. 1.0 ) \(X=1.0\)
IF(X.LE.SLMi) \(Y=\) SLB 1 -ALOG(X)/SLA1
IF[X.GT.SLM4.AND.X.LE.SLM2] \(Y=S L B 2-A L O G(X) / S L A 2\)
IF (X.GT.SLM2) \(Y=\) SLB3-ALOG[X]/SLA3
IF(P.EO.NNL) WCL=Y
IF (P.GT.NNL) \(W(P)=Y\)
GO TO 140
RETURN
END
\(920=\)
SUBRDUTINE BOCDCEPA,SGLA,FLUXA,DRZA,SN4N,CFWT,DX,N4,ID,L,KOD,T1]
\(921=\)
922 \(=\)
923=
924=
\(925=\)
926=
\(927=\)
928=
\(929=\)
\(930=\)
\(931=\)
932=
\(933=\)
\(934=\)
\(935=\)
936=
\(937=\)
\(938=\)
\(939=\)
\(940=40\)
944=
942=
\(943=\)
\(944=\)
\(945=\)
\(946=\)
\(947=\)
\(948=20\)
949=

\section*{INTEGER P}

DIMENSION EP(365), DWT(365), DRZ(365), SGL (365), FLUX(365), L( 10 ), KOD(6 4)

COMMON/BONC/ DWT(365),SGL(365),EP(365),DRZ(365),FLUX(365)
\(P=T 4-L\{1]+4\)
IF(P.GE.ID) GO TO 40
TA=T4-L(4)-P+1
\(E P A=E P(P)+[E P(P+1)-E P(P)] * T A\)
\(\operatorname{IF}(K O D(3) . N E .4) \quad D R Z A=D R Z(P)+C D R Z(P+4)-D R Z(P)] * T A\)
IF (KOD(3).EQ.1) DRZA=DRZ(P)
SGLA=SGL(P)
CFWT \(=\operatorname{DWT}(P)+[\operatorname{DWT}(P+4)-\operatorname{DWT}(P)) * T A\)
N4=CFWT/DX+0. 49
SNイN=CFWT-DX*[Nイ-0.5]+0.001
\(\operatorname{IF}(D W T(P+1) . E Q\). DUT \((P)] \quad C F W T=D X / 0.1\)
IF[DWT(P+1).NE.DWT(P)) CFWT=DX/ABS(DWT(P+4]-DWT(P)\}
IF[KOD[6].EQ.0] GO TO 20
FLUXA=FLUX(P)
GO 1020
\(E P A=E P(1 D)\)
DRZA \(=\) DRZ(ID)
SGLA \(=\) SGL(ID)
N4=DWT(ID]/0X+0.49
SN \(\left\{\begin{array}{c}\text { N }=D W T(I D)-D X *(N 4-0.5)+0.004 ~\end{array}\right.\)
\(C F W T=4.8\)
IF \((K O D(6) . E Q .0] 60\) TO 20
FLUXA=FLUX(ID)
RETURN
END

950=
954=
952=
\(953=\)
954=
\(955=\)
956=
\(957=\)
\(958=\)
\(959=20\)
960=
\(961=\)
962 \(=\)
\(963=\)
964=
965=
\(966=\)
967=
```

SUBROUTINE HEPR[W,S,SU,SL,NM]
INTEGER P
DIMENSION L(25),S(25),SU(80),SL(80),KOD(6)
COMMON/CONDU/ CSAT4,CSAT2,SUA4,SUA2, SUA3,SUB4, SUB2,SUB3,SUĆ, SUD,
4SLA1, SLA2, SLA3, SLB 1, SLB2, SLB3, SLC, SLD, CUA1, CUA2, CUA3, CUB 1, , CUB2,
2CUB3, CUC, CUD, CLA4, CLA2, CLA3, CLB 4, CLB2, CLB3,CLC, CLD, KOD(6), NNL, IW4, 3IW2,L6, SWCU, SWCL,LU,LL,MU,ML, FAC
$\mathrm{I}=0$
IF[KOD[i].EO.1] GO TO 40
$\mathrm{I}=\mathrm{I}+4$
IF[I.GT.NNL.) GO TO 30
$x=-4.6$
$0046 \mathrm{P}=2$.IW 4

```

``` 4LU+2J
IF(WCI].LE.(0.04*LU]) \(X=S U[4]\)
IF[W[IJ.GE.SWCU) \(X=0.001\)
IF (X.NE.-4.8) S[I]=X
```

968=
$969=40$
$970=30$
$974=58$
972=
973=
$974=$
975=
$976=$
977=
978=
979=
$980=$
$984=70$
$982=10$
983=
984=
$985=$
$986=80$
987 =
988=
989 =
990 =
994=
$992=90$
993=60
994=

IF (X.NE.-1.0) GO TO 20
CONTINUE
$I=$ NNL
$\mathrm{I}=\mathrm{I}+4$
IF(I.GT.NM) GO TO 60
$X=-4.8$
DO $70 \quad \mathrm{P}=2$. IW2
IF(W[I).LE.0.01*[LL+P-4)] $X=S L(P-4)+(S L(P)-S L(P-4)) *(100.8 * W(I)-P-$ 1LL+23
IF(W[I).LE.(0.04*LL)] $X=S L(4)$
IF(WCI).GE.SWCL $X=0.001$
IF (X.NE. - $1 . \theta$ ) S(I)=X
IF(X.NE.-4.0) GO TO 50
CONTINUE
DO $80 I=1$. NNL
IF[W(I).GE.SUC) S(I)=EXP [SUA\{*(SUB 1-WCI)])
IF(WCI).LT.SUC.AND.W[I].GE.SUD $\operatorname{S[I]=EXP(SUA2*(SUB2-W[I)])~}$
IF[W[I].LT.SUD] S[I]=EXP[SUA3*(SUB3-W[I])]
CONTINUE
$P=N N L+1$
DO $90 \mathrm{I}=\mathrm{P}$, NM
IF(LCI].GE.SLC] S[I]=EXP(SLA1*(SLB4-WCI)])
IF(WCI).LT.SLC.AND.W(I).GE.SLD) S(I)=EXP(SLA2*(SLB2-WCI)J)
IF(WCI].LT.SLD] S(I)=EXP(SLA3*\{SLB3-W(I])\}
CONTINUE
RETURN
END

## 995=

996=
997=
998=
999=
4000=
100 亿=
4002=
1003 =
4004=20
1005=
4006=
4007=
4008=
1009 =
1018=
$4014=$
但42=
40 13=
$1044=40$
$1015=10$
$1046=$
4847=
4848= $1049=$
$1820=50$
1021=30
1022=

## SUBROUTINE HEPAS(HCS,SU,ID)

INTEGER P
OIMENSION WCS(365), SU[80),KOD(6)
COMMON/CONDU/ CSAT4,CSAT2,SUA4,SUA2,SUA3,SUB4,SUB2,SU83,SUC,SUD,
4SLA4, SLA2, SLA3, SLB4, SLB2, SLB3, SLC, SLD, CUA 1, CUA2, CUA3, CUB 4, CUB2, 2CUB3, CUC, CUD, CLA4, CLA2, CLA3, CLB4, CLB2, CLE3, CLC, CLD, KOD 6 3, NNL, IW4, 3IW2,L6,SWCU,SWCL,LU,LL, MU, ML, FAC
IF[KOD[4].EQ. 1] GO TO 40
$I=0$
$I=I+4$
IF(I.GT.IDJ GO TO 30
$X=-4.0$
DO $40 \mathrm{P}=2.1 W 4$
IF(WCS(I).LE.0.04* (LU+P-4) X=SU(P-4)+(SUCP)-SU(P-4)]*(400.0*WCS(I
13-P-LU+2)
IF(WCS[I].LE.[0.04*LU]) X=SUC 1 )
IF(WCS[I).GE.SWCU) $X=8.081$
IF $(X . N E .-1.0)$ WCS $\{I)=X$
IF $(X . N E .-4.0)$ GO TO 20
CONTINUE
DO $50 \mathrm{I}=4,10$
IF (UCS[I].GE.SUC) X=EXP(SUA4*[SUB4-WCS[I])]
IF(WCS[I).LT.SUC.AND.WCS(I).GE.SUD) X=EXP(SUA2*[SUB2-WCS(I)]) IF(HCS[I).LT.SUD) X=EXP(SUA3*(SUB3-WCS[I])]
WCS (I) $=\mathrm{X}$
CONTINUE
RETURN
END

4023=
1024=
$1025=$
1026=
1027=
4028=
4029=
1030=
4031=

SUBROUTINE DMC(J,CH4,X,CHU,CHL,SU,SLJ
INTEGER $P$
DIMENSION CHU(80), CHL(B0), SU(80), SL(80), KOD(6)
COMMON/CONDU/ CSAT1,CSAT2,SUA 1, SUA2,SUA3, SUB 1, SUB2, SUB3, SUC, SUD, 4SLA1, SLA2, SLA3, SLB 1, SLB2,SLB3, SLC, SLD, CUA1, CUA2, CUA3, CUB 1, CUB2, 2CUB3, CUC, CUD, CLA 1, CLA2, CLA3, CLB 4, CLB2, CLB3, CLC, CLD, KOD[6], NNL, IW 4 , 3IW2,L6,SWCU,SWCL,LU,LL,MU,ML,FAC
IF[KOD[4].EG.1] GO TO 40
IF(J.GT.NNL) GO TO 20

1032＝
1033＝
1034＝
1035＝
1036＝
1037＝
$1038=$
$4039=30$
$1840=40$
1044＝68
1042＝
$1043=20$
1044＝70
1045＝
1046＝
1047＝
4048＝
1049＝
4050＝
405 亿＝80
4052＝98
4053＝
1054＝40
4055＝
1056＝
1057＝
4058＝
$1059=100$
4060＝
1064＝
1062＝
1063＝
1064＝
$1065=120$
$4066=$
1067＝410
4068＝
4069＝430
$4070=$
$1074=$
1072＝
$1073=50$
4074＝
$\mathrm{CH} 4=0.0$
DO $30 \mathrm{P}=4,1 \mathrm{IW}$
IF［X．GE．SU［4］］CH4＝CHU［ 1］
IF（X．LE．SU（IW4）$C H\{=C H U(I W 4)$

4X］／［SU［P］－SU（P－4］）
IF（CH\｛．NE．日．0）GO TO 40
CONTINUE
IF（J－NNL $30,60,20$
SI＝4．0
GO TO 70
$S I=0.0$
$\mathrm{CH} 2=8.8$
DO $80 \quad P=1$ ，IW2
IF（X．GE．SL（1））CH2＝CHL（1）
IF（X．LE．SL（IW2）］CH2＝CHL（IW2）
IF（X．LT．SL（4）．AND．X．GT．SL（P）$C H 2=C H L[P)-(C H L(P)-C H L(P-4)] *(S L(P)-$
（X）／［SL（P）－SL（P－4）］
IF（CH2．NE．日．0］GO TO 90
CONTINUE
CH4＝0．5＊（［2．0－SI ）＊CH2＋SI＊CH4）
GO TO 58
IFCL6．NE． 43 GO TO 400
SUM $1=$ EXP（SUA $1 *$（SUB 4－SUC））
SUM2＝EXP（SUA2＊（SUB2－SUD））
SLM $=\operatorname{EXP}(S L A 4 *(S L B 1-S L C))$
SLM2 $=E X P(S L A 2 *(S L B 2-S L D])$
IF［J．GT．NNL GO TO 140
IF（X．LT．4．0）$X=4.0$
IF（X．LE．SUM4）CHi＝－1．B／［SUA $1 * X]$
IF［X．GT．SUM4．AND．X．LE．SUM2］$C H\{=-4 . B /[S U A 2 * X]$
IF（X．GT．SUM2）CH $4=-1.0 /(S U A 3 * X)$
IF（J－NNL）S8，120．440
$S I=4.0$
GO TO 130
$S I=0.0$
IF（X．LT．1．0）$X=1.0$

IF（X．GT．SLM4．AND．X．LE．SLM2］CH2 $=-1.0 /[S L A 2 * X]$
IF（X．GT．SLM2）CH2＝－4．0／（SLA3＊X）
CH\｛ $=0.5 *(2.0-S I) * C H 2+S I * C H 4)$
RETURN
END

| 1075＝ | SUBROUTINE CON［J，A，B，SA，SB，CU，CL，SU，SL） |
| :---: | :---: |
| 1076＝ | INTEGER P |
| 1077＝ | DIMENSION CL（80），SL（80），CU（80），SU（80），KOD 6 （ ${ }^{\text {（ }}$ |
| 1078＝ | COMMON／CONDU／CSAT4，CSAT2，SUA4，SUA2，SUA3，SUB4，SUB2，SUB3，SUC，SUD， |
| 1879＝ | 4SLA4，SLA2，SLA3，SLB ，SLB2，SLB3，SLC，SLD，CUA ，CUA2，CUA3，CUB ，CUB2， |
| 1880＝ | 2CUB3，CUC，CUD，CLA ，CLA2，CLA3，CLB4，CLB2，CLB3，CLC，CLD，KOD 6 ），NNL，IH4， |
| 1084＝ | 3IW2，L6，SWCU，SWCL，LU，LL，MU，ML，FAC |
| 1082＝ | LK＝4 |
| 4083＝ | IF ${ }^{\text {J．GT．NNL }}$ GO TO 40 |
| 1084＝ | SS＝S8 |
| 1085＝ |  |
| 1086＝30 | IF［SS．LE．CUAイ）A4＝FAC＊CSAT4 |
| 1087＝ | IF（SS．GT．CUA4．AND．SS．LT．CU84）A4＝FAC\＃CSAT4＊EXP（－CUA2＊（SS－CUAイ） |
| 1088＝ |  |
| 1089＝ | IFCLK．E日．4］$B=A 4$ |
| 1090＝ | IFCLK．EQ．O］$A=A 4$ |
| 1094＝ | IFCLK．EO．0）GO TO 40 |
| 1092＝ | LK＝0 |
| 1093＝ | IF ${ }^{\text {d．GE．NNL }}$ GO TO 40 |
| 1894＝ | SS＝SA |
| 1895＝ | GO TO 30 |
| 1096 $=10$ | IFCKOD（ 1 ］．NE．0）GO TO 20 |
| 1097＝ | SS＝SA |
| $4098=56$ | IFCSS．LE．CLAイ3 B4＝FAC＊CSAT2 |

1099＝ $1400=$ $4101=$ 1402＝ $4183=$ 1184＝ $1105=$ 4406＝ 1107＝ $1908=28$
4409＝ 1440＝ 1114＝98 1412＝ 1143＝ 1414＝ $1415=$ $1146=$ 1147＝ 4148＝80 1449＝440 1420＝ 1121＝ 4122＝ 4123＝ $1124=$ 1125＝ $1126=70$ 1427＝ $1428=440$ 1429＝ 4130＝ 1431＝ 1432＝ 1433＝ 1434＝100 $1135=150$ 1136＝ 1437＝ 1438＝ $1139=$ 4140＝ $1444=$ 1442＝ $1143=60$ 4444＝ $1445=130$ 1446＝ 1447＝
1448＝
1449＝
$1450=$
4151＝
4452＝
$1153=$
$1454=$
$1455=$
$4456=120$
$4457=160$
1458＝
4459＝
$1460=$
1169＝
1462＝
4463＝
4464＝
4465＝
$1466=$
$1467=$
$4468=40$
4169＝

IF［SS．GE．CLB4］B\｛＝FAC＊CLB2＊（SS＊＊\｛－4．4］）
IF［LK．NE．2）$A=B 4$
IF［LK．EO．0）GO TO 40
IF（LK．EQ．4）SS＝SB
IFCLK．EO．2）B＝B 1
IF（LK．EQ．2）GO TO 40
$L K=2$
GO TO 50
IFCKOD（1］．NE．2］GO TO 68
IFCJ．GT．NNLJ GO TO 70
SS＝S8
$A$ $=0.0$
DO 80 P＝\｛，IWイ
IF（SS．GE．SUCP］．AND．SS．LT．SU（1））A1＝FAC＊（CU［P－4）＋CCU（P）－CU（P－1））＊
\｛（SU（P－1）－SS）／（SU（P－1）－SU（P）\}\}
IF［SUC 1］．LE．SS］A1＝FAC＊CU［1］
IF（SU［IWイ）．GE．SS）A\｛＝FAC＊CU（IWイ）
IF［A1．NE．0．0］GO TO 440
CONTINUE
IF（LK．EQ． 1 ）B＝A 1
IF（LK．EQ．0）$A=A 1$
IFCLK．EO．OJ GO TO 40
LK＝0
IF［J．GE．NNL）GO TO 70
SS＝SA
GO TO 90
SS＝SA
B $1=0.0$
DO $100 \mathrm{P}=$ 亿，IW2
IF［SS．GE．SL［P）．AND．SS．LT．SL（1）］B4＝FAC＊［CL（P－4）＋［CL（P）－CL（P－4）］＊
\｛（SL（P－4］－SS）／［SL［P－1］－SL［P）］］
IF［SS．GE．SL（4）$B$ i＝FAC＊CL［4）
IF（SS．LE．SL［IW2J）B4＝FAC＊CL（IW2］
IF（B4．NE．0．0）GO TO 450
CONTINUE
IF（LK．NE．2）$A=B 1$
IF［LK．EQ．0）GO TO $4 \theta$
IF［LK．EQ．1］SS＝SB
IF［LK．EQ．2］$B=B 4$
IF（LK．EQ．2）GO TO 40
LK＝2
B $1=0.0$
GO TO 4 18
IF（J．GT．NNL 3 GO TO 120
SS＝SB
IF［SS．LE．CUC］A伊FAC＊CSAT4＊EXP（－CUA1＊［SS－CUB4］）
IF〔SS．GT．CUC．AND．SS．LT．CUD）A4＝FAC＊CSAT4＊EXP［－CUA2＊［SS－CUB2J］
IFCSS．LT． 1.0 S SS＝4．0
IF（SS．GE．CUD）A4＝FAC＊（CUA3＋CUB3＊ALOG48（SS）（\＃（SS＊＊（－4．4））
IF（LK．EQ．1）B＝A4
IF［LK．EQ．D］$A=A 4$
IF LLK．EQ．0）GO TO 40
$L K=0$
IF（J．GE．NNL）GO TO 420
SS＝SA
GO TO 430
SS＝SA
IF（SS．LE．CLC］B4＝FAC＊CSAT2＊EXP［－CLA1＊［SS－CLB1］）
IF［SS．GT．CLC．AND．SS．LT．CLD］B4＝FAC＊CSAT2＊EXP［－CLA2＊（SS－CLB2）］
IF（SS．LT．1．B）SS＝4．0
IF［SS．GE．CLD］ 8 4＝FAC＊［CLA3＋CLB3＊ALOG40（SS］）＊［SS＊＊［－4．4］）
IFCLK．NE．2）$A=B$ i
IF（LK．EQ． 0 ）GO TO 48
IF（LK．EO．4）SS＝SB
IFCLK．EQ．2］ $8=84$
IF［LK．EQ．2）GO TO 40
LK＝2
GO TO 468
RETURN
END

1470＝ 147イ＝ $1472=$ 1473＝ $1474=$ $1475=$ $1176=$ $1477=36$ 1478＝ $1479=$ $1480=$ 148行 4482＝ 1483＝ $1484=$ $1485=$ 1486＝ $1187=$ $1488=20$ $1489=50$ $1490=$ 199行 1492＝ 1493＝ $1494=$ $1495=$ $1196=$ $1197=$ $1198=$ $1199=$ 1200＝10 $1204=$ 1202＝40 $1203=$

SUBROUTINE RER［J，A，B，SA，SB，NNLJ
INTEGER PRZ COMMON／SINK／SMB，SMU4，SML4，SM2，SM3，OM，SMM，PRZ，AQ，BO LK＝4
IF（J．GT．PRZ）GO TO 10
IF（J．GT．NNL）GO TO 28 SS＝SA
A $1=0.0$
IF［SS．GT．SMB．AND．SS．LT．SMU4）A $1=0 \mathrm{OM}$（SS－SMB）／（SHU4－SMB）
IF（SS．GE．SMU4．AND．SS．LE．SM2］A $1=0 \mathrm{M}$
IF［SS．GT．SM2．AND．SS．LE．SM3］A4＝SMM＊（SM3－SS $3+A 0 * O M$
IF［LK．EO．1］A＝A
IF［LK．EQ．8）B＝A
IFCLK．EQ．0J GO TO 40
LK＝0
IF（J．GE．NNL GO TO 28
SS＝SB
GO TO 30
SS＝SB

## 8 $=0.0$

IF（SS．GT．SMB．AND．SS．LT．SML1］B $1=0 \mathrm{M} *(S S-S M B) /(S M L 4-S M B)$
IF（SS．GE．SHL4．AND．SS．LE．SM2）B $4=0 \mathrm{M}$
IF（SS．GT．SM2．AND．SS．LE．SM3）B $1=5 M M *(S M 3-S S)+A Q * O M$
IFCLK．NE．2］ $\mathrm{B=B4}$
IF（LK．ER．8）GO TO 40
IFCLK．EQ．13 SS＝SA
IFCLK．EO．2）A＝B1
IFCLK．EQ．2J GO TO 46
LK＝2
GO TO 50
$A=0.0$
$8=0.8$
RETURN
END

## 10．2 Instructions for input

The data input has been arranged in 24 groups $\mathrm{A}, \mathrm{B}, \ldots, \mathrm{Y}, \mathrm{Z}$ and should be punched on cards in FORTRAN－code according to the instructions given on page $144-161$ ．

| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| A | 1-80 | 20A4 | HED | desired heading to be printed |
|  |  |  |  | Group A consists of 1 card |
| B | 1-5 | I5 | KOD(1) |  |
|  | 6-10 | I5 | KOD(2) |  |
|  | 11-15 | I5 | KOD(3) | the meaning of KOD is described in Section 8.1.1 |
|  | 16-20 | I5 | KOD(4) |  |
|  | 21-25 | I5 | KOD(5) |  |
|  | 26-30 | 15 | KOD(6) |  |
|  | 31-35 | I5 I5 | LU | minimum value $\left(\mathrm{LU}=100, \theta_{\text {min }}\right)$ of water content of upper soil layer |
|  | 36-40 | I5 | MU | maximum value ( $\mathrm{MU}=100 . \theta_{\text {sat }}$ ) of water content of upper soil layer |
|  | 41-45 | I5 | LL | as LU, but of lower soil layer |
|  | 46-50 | I5 | ML | as MU, but of lower soil layer |
|  | 51-55 | I5 | NM | maximum number of nodal points in the profile under consideration |
|  | 56-60 | I5 | NNL | number of the nodal point where the physical properties of the soil profile change (boundary between upper and lower layer) |
|  | 61-65 | 15 | IMAX | desired maximum number of iterations |
|  | 66-70 | I5 | L2 | maximum number of daily outputs to be printed (see also Group Z) |



| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
|  | 41-50 | F10.3 | TB | point of time at which roots in the top soil layer start |
|  |  |  |  | die because of drought conditions (for $t \leqslant \mathrm{~TB}, \mathrm{RNA}=0$ |
|  | 51-60 | F10.3 | TE | point of time at which roots in the top soil layer reach |
|  |  |  |  | $\begin{aligned} & \text { therr ma } \\ & \text { RNAM) } \end{aligned}$ |
|  |  |  |  | for $\mathrm{TB}<t<\mathrm{TE}$, the non-activity zone is equal to RNA $=$ |
|  |  |  |  | Group C consists of 1 card |
| D |  |  |  | Group D describes the sink term (see Fig. 34) minimum value of suction at which roots start to extract water from the soil (starting point) minimum suction at which roots start to extract water optimally from the upper soil layer as above but for the lower soil layer maximum suction at which roots still have optimal conditions to extract water from the soil (limiting point) suction above which no water uptake by roots is possible (wilting point) <br> factor describing shape of sink term. BQ must be set less or equal to 1.0 <br> Group D consists of 1 card |
|  | 1-10 | F10.3 | SMB |  |
|  | 11-20 | F10.3 | SMUI |  |
|  |  |  |  |  |
|  | 21-30 | F10.3 | SMLI |  |
|  | 31-40 | F10.3 | SM2 |  |
|  |  |  |  |  |
|  | 41-50 | F10.3 | SM3 |  |
|  |  |  |  |  |
|  | 51-60 | F10.3 | BQ |  |
|  |  |  |  |  |


Fig. 64. Explanation of the iteration procedure used in SWATR.

| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
|  | 71-80 | F10.3 | FAC | The iteration procedure continues until all changes are less than EPS or until the number of iterations IMAX is exceeded (see Group B). If the number of iterations is equal to IMAX and EPS is not reached, SWATR prints a statement and continues the calculation for $\mathrm{T}=\mathrm{T}^{i+1}+\mathrm{DT}^{i+2}$ constant depending on the unit of conductivity $K$ used in the problem; $\mathrm{FAC}=\frac{\text { unit of conductivity (as an input) }}{\mathrm{cm} \cdot \text { day }^{-1}(\text { working unit })}$ <br> i.e. $\mathrm{FAC}=1.0$ for K in cm .day ${ }^{-1}$ <br> Group E consists of 1 card |
| Groups F, G, H describe the boundary conditions at the top |  |  |  |  |
| F | 1-10 | F10.3 | FLUX(1) | Omit Group F if $\operatorname{KOD}(6) \neq 1$ <br> potential flux at the soil surface (mm.day ${ }^{-1}$ ) for 1st day of calculation; for infiltration the sign is + , for evaporation the sign is -. FLUX can be estimated with Eqn 8.2. In this equation PREC is measured, ES can be computed with e.g. Eqn 3.31 and FIN can be derived from the Eqn 8.7 or Eqn 8.8 or from Fig. 32 (see also Group K) |


| Group | Columns | Format | Symbol - | Description |
| :---: | :---: | :---: | :---: | :---: |
|  | 11-20 | F10.3 | EP(1) | potential transpiration rate (mm.day ${ }^{-1}$ ). This value can be determined from e.g. Eqn 3.12, or equations like 3.34 and 3.35 <br> maximum allowed suction (cm) at the soil surface to be calculated with Eqn 4.11 <br> as above, but for the 2nd day, etc. <br> Maximally 183 cards can be used in Group F ( 1 card for 2 days) |
|  | 21-30 | E10.4 | SGL(1) |  |
|  | 31-40 | F10.3 | FLUX(2) |  |
|  | 41-50 | F10.3 | EP(2) |  |
|  |  |  |  |  |
| G | 1-10 | F10.3 | TEM(1) | Omit Group G if $\operatorname{KOD}(6) \neq 2$ <br> air temperature $\left({ }^{\circ} \mathrm{C}\right)$ for first day of calculation <br> relative humidity of air (fraction) <br> wind velocity at 2 m height $\left(\mathrm{m} . \mathrm{s}^{-1}\right)$ <br> net radiation flux W. $\mathrm{m}^{-2}$ if $\mathrm{L}(7)=0$; otherwise cal. $\mathrm{cm}^{-2}$. $\mathrm{day}^{-1}$ |
|  | 11-20 | F10.3 | RH(1) |  |
|  | 21-30 | F10.3 | U(1) |  |
|  | 31-40 | F10.3 | HNT(1) |  |
|  | 41-50 | F10.3. | $\mathrm{CH}(1)$ | crop height (cm) |
|  | 51-60 | F10.3 | SC(1) | soil cover (fraction) |
|  | 61-70 | F10.3 | FLUX(1) | preciptiation ( $=$ PREC; $\mathrm{mm}_{\text {d }}$ day $^{-1}$ ) |
|  |  |  |  | Maximally 365 cards can be used in Group G (1 card for each day) |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| H | 1-10 | F10.4 | EP(1) | Omit Group H if $\operatorname{KOD}(6) \neq 0$ <br> potential transpiration rate (mm.day ${ }^{-1}$ ) for first day of calculation. This value can be determined from e.g. Eqn |
|  | 11-20 | F10.4 | WCS(1) | 3.12 or equations like 3.34 and 3.35 <br> prescribed water content at the surface for first day of calculation. From Eqn 4.11 the equilibrium pressure head can be determined and via the soil moisture retention curve the corresponding water content |
|  | 21-30 | F10.4 | EP(2) |  |
|  | 31-40 | F10.4 | WCS(2) |  |
|  | 41-50 | F10.4 | EP(3) | as above, but for 2nd, 3rd, 4th day of calculation, etc. |
|  | 51-60 | F10.4 | WCS(3) | as above, but for 2nd, 3rd, 4th day of calculation, etc. |
|  | 61-70 | F10.4 | EP(4) |  |
|  | 71-80 | F10.4 | WCS(4) |  |
|  |  |  |  | Maximally 92 cards can be used in Group H (1 card for 4 days) |

[^1]| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| I | 1-10 | E10.4 | FGA | Omit Group I if $L(8) \neq 0$ <br> coefficients of $\mathrm{G}(\mathrm{CH})$-function, which is used to estimate potential evapotranspiration flux. The $\mathrm{G}(\mathrm{CH})$-function is applied if $\operatorname{KOD}(6)=2$, i.e. if the boundary condition at the surface is estimated from meteorological and other external data (see Eqns 8.3, 8.4 and 8.5) <br> Group I consists of 1 card |
|  | 11-20 | E10.4 | FGB |  |
|  | 21-30 | E10.4 | FGC |  |
|  | 31-40 | E10.4 | FGD |  |
|  | 41-50 | E10.4 | FGM |  |
|  | 51-60 | E10.4 | FMCH |  |
| J |  |  |  | Omit Group J if $L(9) \neq 0$ coefficients of the LAI-function used to estimate potential soil evaporation flux (see Eqn 8.6) <br> Group J is used in the program if $\operatorname{KOD}(6)=2$ |
|  | $1-10$ $11-20$ | F10.4 F10.4 | FLA |  |
|  | 21-30 | F10.4 | FLC |  |
|  |  |  |  |  |



| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| M | 1-10 | F10.4 | DRZ(1) | Group M describes the depth of the root zone (cm) DRZ(1) is rooting depth on the first day of calculation |
|  | 11-20 | F10.4 | DRZ(2) |  |
|  | 21-30 | F10.4 | DRZ(3) |  |
|  | 31-40 | F10.4 | DRZ(4) |  |
|  | 41-50 | F10.4 | DRZ(5) | if $\operatorname{KOD}(3)=0$ : as above, but for $2 \mathrm{nd}, 3 \mathrm{rd}, \ldots, 8$ th day of calculation, etc. |
|  | 51-60 | F10.4 | DRZ(6) |  |
|  | $\begin{aligned} & 61-70 \\ & 71-80 \end{aligned}$ | F10.4 | DRZ(7) |  |
|  |  | F10.4 | DRZ(8) |  |
|  |  |  |  | If $\operatorname{KOD}(3)=1$ : the rooting depth is constant; then only one value of $\operatorname{DRZ}(1)$ is to be given. |
|  |  |  |  | Maximally 46 cards can be used in Group M |
| Groups N and O describe the initial condition in the soil profile |  |  |  |  |
| N | 1-10 | E10.4 | S1(1) | Omit Group N if $\mathrm{KOD}(5)=0$ <br> $\mathrm{S} 1(1)$ is suction at the 1st nodal point |
|  | 11-20 | E10.4 | S1(2) |  |
|  | 21-30 | E10.4 | S1(3) |  |
|  | 31-40 | E10.4 | S1(4) | as above, but at $2 \mathrm{nd}, 3 \mathrm{rd}, \ldots, 8$ th nodal point of the soil |
|  | 41-50 | E10.4 | S1(5) | profile, etc. |
|  | 51-60 | E10.4 | S1(6) |  |
|  | 61-70 | E10.4 | S1(7) |  |
|  | 71-80 | E10.4 | S1(8) |  |
|  |  |  |  | Maximally 4 cards can be used in Group N |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1-10 | F10.4 | W(1) | Omit Group O if $\mathrm{KOD}(5)=1$ |
|  | 11-20 | F10.4 | W(2) | $\mathrm{W}(1)$ is water content at the 1st nodal point |
|  | 21-30 | F10.4 | W(3) |  |
|  | 31-40 | F10.4 | W(4) |  |
|  | 41-50 | F10.4 | W(5) | as above, but at 2 nd, 3 rd, ..., 8 th nodal point of the soil |
|  | 51-60 | F10.4 | W(6) |  |
|  | 61-70 | F10.4 | W(7) |  |
|  | 71-80 | F10.4 | W(8) |  |
|  |  |  |  | Maximally 4 cards can be used in Group O |



| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| R | 1-10 | E10.4 | SL(1) | Omit Group R if $\operatorname{KOD}(1)=1$ <br> suction (cm) for lower soil layer at water content $\theta_{1}=$ $0.01 \cdot \mathrm{LL}\left(\mathrm{cm}^{3} . \mathrm{cm}^{-3}\right)$ |
|  | 11-20 | E10.4 | SL(2) | as above, but at water content |
|  | 21-30 | E10.4 | SL(3) | $\theta_{j}=0.01 \cdot(\mathrm{LL}+j-1) ; j=2,3, \ldots,(\mathrm{ML}-\mathrm{LL}+1)$ |
|  | 31-40 | E10.4 | SL(4) | $\mathrm{LL}=$ minimum value of water content for |
|  | 41-50 | E10.4 | SL(5) | lower soil layer ( $L L=100 \cdot \theta_{\text {min }}$ ) |
|  | 51-60 | E10.4 | SL(6) | $\mathrm{ML}=$ maximum value of water content for |
|  | 61-70 | E10.4 | SL(7) | upper soil layer (ML $=100 \cdot \mathrm{SWCL}$ ) |
|  |  |  |  | Maximally 10 cards can be used in Group R |
| S | 1-10 | E10.4 | CU(1) | Omit Group S if $\operatorname{KOD}(1) \neq 2$ <br> conductivity for upper soil layer at water content $\theta_{1}=$ $0.01 \cdot \mathrm{LU}$ |
|  | 11-20 | E10.4 | $\mathrm{CU}(2) 7$ |  |
|  | 21-30 | E10.4 | CU(3) |  |
|  | $31-40$ $41-50$ | E10.4 E10.4 | CU(4) $\mathrm{CU}(5)$ | $\theta_{i}=0.01 \cdot(\mathrm{LU}+j-1) ; j=2,3, \ldots,(\mathrm{MU}-\mathrm{LU}+1)$ |
|  | 51-60 | E10.4 | CU(6) |  |
|  | 61-70 | E10.4 | CU(7) |  |
|  | 71-80 | E10.4 | $\mathrm{CU}(8)$ |  |
|  |  |  |  | Maximally 10 cards can be used in Group S |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| T | 1-10 | E10.4 | CL(1) | Omit Group T if $\operatorname{KOD}(1) \neq 2$ conductivity for lower soil layer at water content $\theta_{1}=$ $0.01 \cdot \mathrm{LL}$ |
|  | 11-20 | E10.4 | CL(2) | as above, but at water content$\theta_{j}=0.01 \cdot(\mathrm{LL}+j-1) ; j=2,3, \ldots,(\mathrm{ML}-\mathrm{LL}+1)$ |
|  | 21-30 | E10.4 | CL(3) |  |
|  | 31-40 | E10.4 | CL(4) |  |
|  | 41-50 | E10.4 | CL(5) |  |
|  | 51-60 | E10.4 | CL(6) |  |
|  | 61-70 | E10.4 | CL(7) |  |
|  | 71-80 | E10.4 | CL(8) |  |
|  |  |  | Maximally 10 cards can be used in Group T |  |
| Groups U and W describe the conductivity of soils if $\operatorname{KOD}(1)=0$ |  |  |  |  |
| U | 1-10 | F10.4 | $\begin{aligned} & \text { CSAT1 } \\ & \text { CUA1 } \\ & \text { CUA2 } \\ & \text { CUB1 } \\ & \text { CUB2 } \end{aligned}$ | Omit Group U if $\mathrm{KOD}(1) \neq 0$ |
|  | 11-20 | F10.4 |  |  |
|  | 21-30 | F10.4 |  | coefficients to describe conductivity for upper soil layer (see Eqns 2.6, 2.7 and 2.8, Section 8.1.2) |
|  | 31-40 | F10.4 |  |  |
|  | 41-50 | F10.4 |  |  |
|  |  |  |  | Group U consists of 1 card |

\(\left.\begin{array}{lclll}\hline Group \& Columns \& Format \& Symbol \& Description <br>
\hline \& \& \& \& Omit Group W if KOD(1) \neq 0 <br>
\mathrm{~W} \& 1-10 \& F10.4 \& CSAT2 <br>
\& 11-20 \& F10.4 \& CLA1 <br>
\& 21-30 \& F1.4 \& CLA2 <br>
\& 31-40 \& F10.4 \& CLB1 <br>

\& 41-50 \& F10.4 \& CLB2\end{array}\right] \quad\)| (seefficients to describe conductivity for lower soil layer |
| :--- |
|  |

Groups X and Y describe the suction and conductivity of the soil layers if $\operatorname{KOD}(1)=1$

## Omit Group X if $\operatorname{KOD}(1) \neq 1$

coefficients to describe suction, conductivity and differen-
tial moisture capacity for upper soil layer (see Eqns 2.9 ,
$2.10,2.11 ; 2.12,2.13,2.14 ; 2.15$ and 2.16, Section 8.1.2)
Group X consists of 3 cards ( 17 coefficients)

F10.4
F10.4
F10.4
F10.4
F10.4
F10.4
F10.4
F10.4

F10.4
F10.4
F10.4
F10.4
F10.4
F10.4
F10.4
F10.4
F10.4

1-10
1-10

## Description



| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| Y |  |  |  | Omit Group Y if $\operatorname{KOD}(1) \neq 1$ |
|  | $1-10$ $11-20$ | F10.4 F10.4 | CSAT2 ${ }^{\text {SLA }}$ |  |
|  | 21-30 | F10.4 | SLA2 |  |
|  | 31-40 | F10.4 | SLA3 |  |
|  | 41-50 | F10.4 | SLB1 |  |
|  | 51-60 | F10.4 | SLB2 |  |
|  | 61-70 | F10.4 | SLB3 |  |
|  | 71-80 | F10.4 | SLC | coefficients to describe suction, conductivity and differen- |
|  | 1-10 | F10.4 | SLD | tial moisture capacity for lower soil layer (see Eqns 2.9, |
|  | 11-20 | F10.4 | CLA1 | 2.10, 2.11; 2.12, 2.13, 2.14; 2.15 and 2.16, Section |
|  | 21-30 | F10.4 | CLA2 | 8.1.2) |
|  | 31-40 | F10.4 | CLA3 |  |
|  | 41-50 | F10.4 | CLB1 |  |
|  | 51-60 | F10.4 | CLB2 |  |
|  | 61-70 | F10.4 | CLB3 |  |
|  | 71-80 | F10.4 | CLC |  |
|  | 1-10 | F10.4 | CLD $]$ |  |
|  |  |  |  | Group Y consists of 3 cards ( 17 coefficients) |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| Z | 1-10 | F103 |  | Omit Group Z if $\operatorname{KOD}(2) \neq 1$ <br> Use Group Z if TINIT $\neq 0$ if the required frequency of the various outputs is not constant |
|  | 11-20 |  |  | Starting time of calculation |
|  | $11-20$ $21-30$ | F10.3 | TR(1) | cumulative transpiration at time $\mathrm{T}=$ TINIT time increment for which first output must be printed |
|  | 31-40 | F10.3 | TR(2) | the second output will be at time T $=$ TINIT + TR ( 1 ) + TR( 2 ) |
|  | $\begin{aligned} & 41-50 \\ & \text { etc. } \end{aligned}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | $\begin{aligned} & \operatorname{TR}(3) \\ & \operatorname{TR}(\mathrm{L} 2) \end{aligned}$ | as above, but for 3rd output, etc. <br> as above, but for last daily output to be printed, L2 <br> which is the maximum value as defined under Group B |
|  |  |  |  | For Group $\mathbf{Z}$ maximally 7 cards can be used |

### 10.3 Example of input

|  | MULATION | MODEL OF S | SOIL HATER | AND ACTUAL | TRANSPIRATION | RATECDATA | R.A.F | FEDDES-67 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2= | 24 | B 0 | 02 | 060 | B 5020 | 910 | 49 | 472304 |
| 3= | $28 \quad 24$ | 640 | 14 | 14 |  |  |  |  |
| 4= | 4.0 | 0.68 | 0.50 | 18. | 472. | 224. |  |  |
| 5= | 0.1 | 32.5 | 4.7 | 500. | 28000. | 1. |  |  |
| 6= | . 05 | 0.5 | 7. | 0.845 | . 8358 | 400. | . 04 | 24. |
| 7= | 13.0 | 8.77 | 4.77 | 395. | 44. | .848 | . 0 |  |
| 8= | 14.8 | 0.93 | 5.58 | 307. | 42. | . 048 | . 0 |  |
| $9=$ | 43.6 | 0.89 | 4.89 | 240. | 12. | .640 | . 8 |  |
| 40= | 43.6 | 0.95 | 2.46 | 176. | 43. | . 010 | 2.0 |  |
| 412 | 46.7 | 0.86 | 5.49 | 384. | 43. | . 810 | 7.7 |  |
| 12= | 13.3 | 0.89 | 3.92 | 428. | 13. | . 028 | 8 |  |
| 43= | 43.0 | 0.90 | 1.15 | 164. | 13. | . 020 | . 0 |  |
| 44= | 14.2 | 0.90 | 5.40 | 484. | 14. | . 830 | . 8 |  |
| 45= | 13.9 | 0.93 | 5.33 | 436. | 14. | . 038 | . 8 |  |
| 16= | 14.8 | 0.78 | 1.95 | 394. | 44. | . 840 | . 8 |  |
| $17=$ | 14.8 | 0.84 | 4.80 | 270. | 45. | . 858 | . 0 |  |
| 18= | 15.8 | 0.88 | 2.56 | 302. | 45. | . 858 | . 0 |  |
| 19= | 14.8 | 0.80 | 4.20 | 365. | 46. | . 060 | . 0 |  |
| 20= | 13.6 | 0.83 | 2.76 | 253. | 46. | . 078 | . 0 |  |
| 24= | 43.3 | 0.80 | 1.96 | 334. | 47. | . 080 | 0 |  |
| 22= | 17.8 | 8.79 | 4.57 | 275. | 48. | . 890 | . 0 |  |
| 23= | 47.6 | 0.82 | 1.77 | 292. | 18. | . 408 | . 8 |  |
| 24= | 44.5 | 0.89 | 5.08 | 486. | 19. | . 448 | 2.5 |  |
| 25= | 42.7 | 0.83 | 2.46 | 303. | 20. | . 128 | . 8 |  |
| 26= | 16.1 | 0.80 | 2.77 | 329. | 21. | . 138 | . 8 |  |
| $27=$ | 47.0 | 0.82 | 2.55 | 363. | 22. | . 148 | . 8 |  |
| $28=$ | 47.6 | 0.84 | 4.23 | 343. | 23. | . 168 | . 8 |  |
| $29=$ | 20.6 | 0.86 | 3.87 | 326. | 25. | . 170 | . 8 |  |
| $30=$ | 47.3 | 0.95 | 2.72 | 488. | 26. | . 480 | . 8 |  |
| $34=$ | 45.1 | 0.96 | 4.59 | 142. | 27. | . 290 | . 0 |  |
| 32= | 15.8 | 0.89 | 1.44 | 312. | 28. | . 220 | . 0 |  |
| $33=$ | 28.3 | 0.80 | 3.44 | 345. | 29. | . 230 | . 0 |  |
| 34= | 24.8 | 8.84 | 2.48 | 329. | 30. | . 250 | . 4 |  |
| 35= | 17.6 | 0.94 | 4.08 | 346. | 34. | . 270 | . 7 |  |
| 36= | 16.4 | 0.87 | 2.62 | 356. | 32. | . 280 | 2.3 |  |
| $37=$ | 44.2 | 0.88 | 4.75 | 250. | 33. | . 380 | . 0 |  |
| 38= | 46.4 | 0.74 | 3.45 | 272. | 33. | . 320 | . 0 |  |
| 39= | 43.3 | 0.94 | 2.45 | 60. | 34. | . 340 | . 6 |  |
| $40=$ | 47.3 | 0.88 | 2.90 | 333. | 35. | . 36 | . 0 |  |
| 41= | 16.1 | 0.92 | 2.93 | 125. | 35. | . 38 | . 0 |  |
| 42= | 17.3 | 0.94 | 4.54 | 255. | 36. | . 40 | . 0 |  |
| 43= | 16.7 | 0.95 | 3.69 | 154. | 37. | . 42 | . 0 |  |
| 44= | 45.5 | 0.95 | 2.90 | 483. | 37. | .45 | . 0 |  |
| $45=$ | 47.0 | 0.83 | 4.40 | 244. | 38. | .47 | 3.9 |  |
| $46=$ | 18.2 | 0.97 | 5.73 | 180. | 39. | . 50 | . 0 |  |
| $47=$ | 19.1 | 0.87 | 2.65 | 324. | 39. | . 52 | . 0 |  |
| $48=$ | 24.5 | 0.83 | 2.84 | 252. | 48. | . 54 | . 4 |  |
| 49= | 48.5 | 8.92 | 3.84 | 485. | 40. | . 57 | 1.5 |  |
| $50=$ | 43.6 | 0.87 | 2.94 | 79. | 40. | . 59 | 3.4 |  |
| $51=$ | 43.0 | 0.83 | 3.08 | 179. | 44. | . 62 | 2.7 |  |
| 52= | 42.4 | 0.88 | 2.82 | 177. | 44. | . 64 | . 0 |  |
| $53=$ | 43.2 | 0.84 | 1.43 | 129. | 44. | . 67 | . 0 |  |
| 54* | 46.4 | 0.75 | 2.75 | 260. | 44. | . 69 | . 0 |  |
| 55= | 20.0 | 0.79 | 2.89 | 280. | 42. | . 72 | 3.2 |  |
| $56=$ | 47.3 | 0.84 | 2.84 | 471. | 42. | . 737 | . 8 |  |
| 57 = | 17.8 | 0.88 | 2.35 | 283. | 42. | . 755 | . 0 |  |
| 58= | 46.4 | 0.87 | 5.44 | 250. | 42. | . 772 | 40.6 |  |
| 59= | 45.1 | 0.80 | 5.15 | 223. | 42. | . 790 | . 2 |  |
| $68=$ | 45.4 | 0.94 | 3.94 | 138. | 42. | . 794 | 4.0 |  |
| $64=$ | 45.1 | 0.90 | 6.28 | 404. | 42. | . 798 | 4.2 |  |
| $62=$ | 44.2 | 0.98 | 4.44 | 168. | 42. | . 882 | 5.8 |  |
| $63=$ | 46.1 | 0.85 | 6.14 | 444. | 42. | . 806 | . 0 |  |
| $64=$ | 45.5 | 0.86 | 5.83 | 458. | 12. | . 810 | 1.5 |  |
| 65= | 45.8 | 0.93 | 5.54 | 412. | 42. | . 845 | 2.4 |  |
| $66=$ | 13.6 | 8.94 | 4.64 | 50. | 42. | . 828 | 2.4 |  |
| $67=$ | 42.7 | 0.84 | 4.35 | 265. | 42. | . 826 | . 8 |  |
| 68= | 47.0 | 0.89 | 4.77 | 189. | 42. | . 832 | . 0 |  |
| $69=$ | 14.5 | 0.93 | 3.25 | 189. | 42. | . 838 | . 8 |  |
| $70=$ | 45.8 | 0.94 | 1.85 | 245. | 42. | . 844 | . 0 |  |
| 74= | 45.5 | 0.89 | 4.69 | 248. | 42. | . 858 | . 0 |  |
| 72= | 44.2 | 0.96 | 1.86 | 412. | 42. | . 855 | - 0 |  |
| 73= | 14.2 | 0.94 | 4.44 | 179. | 42. | .868 | - 0 |  |


| $74=$ | 45.5 | 0.92 | 9.21 | 206. | 42. | . 865 | . 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $75=$ | 15.8 | 8.88 | 4.33 | 218. | 42. | . 870 | . 8 |  |
| 76= | 45.4 | 0.94 | 2.34 | 108. | 42. | . 875 | . 8 |  |
| 77= | 12.4 | 0.84 | 5.66 | 466. | 42. | . 888 | . 0 |  |
| $78=$ | 44.8 | 0.89 | 4.24 | 108. | 42. | . 885 | 8.4 |  |
| $79=$ | 15.8 | 0.89 | 3.74 | 197. | 42. | . 890 | 42.1 |  |
| $80=$ | 16.4 | 0.80 | 5.80 | 405. | 42.1 | . 896 | . 8 |  |
| $81=$ | 44.5 | 0.89 | 7.92 | -14. | 42.2 | . 902 | 3.6 |  |
| 82= | 44.8 | 0.87 | 5.74 | 425. | 42.3 | . 988 | 4.8 |  |
| 83= | 13.6 | 0.89 | 8.19 | 15. | 42.4 | . 914 | 11.8 |  |
| 84: | 13.8 | 0.85 | 3.74 | 459. | 42.6 | . 920 | 7.7 |  |
| $85=$ | 9.7 | 0.94 | 4.27 | 85. | 42.8 | . 925 | 4.9 |  |
| $86=$ | 18.3 | 8.89 | 1.45 | 434. | 43. | . 930 | . 2 |  |
| $87=$ | 44.5 | 0.90 | 4.53 | 434. | 43. | . 934 | . 8 |  |
| $88=$ | 43.0 | 0.89 | 1.23 | 425. | 43. | . 938 | . 9 |  |
| 89= | 12.7 | 8.95 | 1.23 | 37. | 43. | . 943 | . 8 |  |
| 90=. | 13.0 | 0.92 | 1.74 | 449. | 43. | . 948 | . 8 |  |
| 919 | 44.5 | 0.92 | 3.58 | 187. | 43. | . 952 | 2.8 |  |
| 92= | 45.1 | 0.97 | 3.43 | 15. | 43. | . 956 | 3.1 |  |
| 93= | 14.5 | 0.96 | 2.54 | 70. | 43. | . 960 | 4.9 |  |
| $94=$ | 43.9 | 0.92 | 4.88 | 48. | 43. | . 968 | . 3 |  |
| $95=$ | 13.3 | 0.95 | 0.99 | 69. | 43. | . 960 | . 8 |  |
| $96=$ | 43.4 | 0.93 | 2.80 | 80. | 43. | . 960 | 4.3 |  |
| $97=$ | 42.7 | 0.88 | 4.82 | 45. | 43. | . 960 | 5.6 |  |
| 98= | 12.4 | 0.85 | 4.04 | 424. | 43. | . 968 | 9.1 |  |
| 99: | 44.5 | 8.92 | 2.28 | 64. | 43. | . 968 | 2.8 |  |
| 100= | 13.8 | 0.93 | 3.36 | 22. | 43. | . 966 | 5.8 |  |
| $101=$ | 43.9 | 0.90 | 2.72 | 115. | 43. | . 950 | . 9 |  |
| 402= | 46.4 | 0.94 | 3.64 | 25. | 43. | . 940 | . 0 |  |
| 103= | 47.0 | 8.93 | 1.49 | 62. | 43. | . 930 | . 8 |  |
| 404= | 18.2 | 0.89 | 3.15 | 22. | 42.5 | . 920 | . 0 |  |
| 105= | 45.2 | 0.90 | 4.56 | 422. | 42.0 | . 945 | . 0 |  |
| 406= | 44.5 | 0.94 | 4.18 | 401. | 44.5 | . 913 | . 0 |  |
| 107= | 17.6 | 0.91 | 3.88 | 63. | 44.8 | . 918 | 2.8 |  |
| 108= | 45.5 | 0.78 | 6.46 | 69. | 48.8 | . 988 | . 0 |  |
| 189= | 44.5 | 0.82 | 6.63 | 53. | 39.7 | . 967 | . 0 |  |
| 410= | 43.5 | 0.73 | 5.82 | 49. | 39.4 | . 986 | 3.8 |  |
| 111= | 13.3 | 0.84 | 7.92 | 39. | 39.1 | . 904 | 42.8 |  |
| 142= | 12.4 | 0.79 | 6.76 | 66. | 38.7 | .983 | 6.8 |  |
| 413= | 41.8 | 0.78 | 4.86 | 38. | 38.3 | . 984 | . 8 |  |
| 144= | 42.7 | 0.94 | 5.80 | 27. | 38.8 | . 980 | . 2 |  |
| 145= | 45.8 | 0.93 | 6.16 | -44. | 37.5 | . 980 | . 4 |  |
| 116= | 44.5 | 0.96 | 3.90 | 43. | 37.0 | .900 | . 4 |  |
| 117= | 16.3 | 0.94 | 7.09 | 58. | 36.5 | . 900 | . 0 |  |
| 148= | 45.5 | 0.98 | 3.67 | 22. | 36.0 | . 900 | 4.4 |  |
| 119= | 44.8 | 0.94 | 4.80 | 48. | 35.5 | . 980 | . 1 |  |
| 128= | 44.5 | 0.97 | 2.90 | -44. | 35.0 | . 900 | 43.9 |  |
| 124= | 42.7 | 0.79 | 5.36 | -17. | 34.8 | . 908 | . 05 |  |
| 422= | 12.9 | 0.86 | 7.78 | 54. | 33.9 | . 900 | 5.8 |  |
| 123= | 13.3 | 0.79 | 6.20 | 66. | 33.8 | . 908 | 46.3 |  |
| 124= | 43.9 | 0.89 | 7.64 | 44. | 33.6 | . 988 | 44.2 |  |
| 425= | 10.6 | 0.76 | 48.29 | -22. | 33.4 | . 908 | 5.6 |  |
| 426= | 8.2 | 0.86 | 3.85 | 24. | 33.3 | .900 | 0.3 |  |
| 127= | 9.7 | 0.88 | 5.53 | 26. | 33.4 | . 900 | . 7 |  |
| 128= | 13.0 | 8.89 | 4.23 | 30. | 33.8 | . 900 | .8 |  |
| 429= | 13.9 | 0.96 | 3.45 | 18. | 32.8 | . 900 | 3.4 |  |
| 139= | 9.7 | 0.94 | 2.04 | -4. | 32.6 | . 900 | . 0 |  |
| 434= | 10.6 | 0.95 | 3.68 | 45. | 32.4 | . 900 | . 0 |  |
| 432= | 10.3 | 0.94 | 2.56 | -29. | 32.3 | . 989 | . 8 |  |
| 133= | 12.4 | 0.94 | 7.09 | 0. | 32.2 | . 980 | 4.0 |  |
| 434= | 13.3 | 0.79 | 7.18 | -3. | 32.1 | . 908 | . 0 |  |
| 435= | 40.9 | 0.90 | 9.47 | 6. | 32.0 | . 900 | 44.1 |  |
| 436\% | 9.7 | 0.98 | 6.46 | -5. | 32.8 | . 900 | 29.8 |  |
| 437= | 8.5 | 0.98 | 3.54 | -29. | 32.0 | . 900 | 3.1 |  |
| 438= | 8.2 | 0.94 | 4.24 | -23. | 32.8 | . 900 | 3.8 |  |
| 439= | 7.2 | 8.94 | 3.58 | -23. | 32.8 | . 980 | 8.8 |  |
| 440= | 90.0 | 95.8 | 93.0 | 99.8 | 98.0 | 108.8 | 99.0 | 92.0 |
| 944 | 93.8 | 93.8 | 94.0 | 86.0 | 84.0 | 85.8 | 88.0 | 88.9 |
| 442= | 90.0 | 98.0 | 92.8 | 85.0 | 87.0 | 92.0 | 92.0 | 91.0 |
| 143= | 92.0 | 93.8 | 93.0 | 79.0 | 82.0 | 85.0 | 88.0 | 89.0 |
| 444* | 94.0 | 95.8 | 94.8 | 95.6 | 87.8 | 90.0 | 95.8 | 98.0 |
| 445= | 94.0 | 90.0 | 96.8 | 86.0 | 92.0 | 90.0 | 96.8 | 95.0 |
| 446= | 98.8 | 409.8 | 96.8 | 95.0 | 90.8 | 98.8 | 94.8 | 88.0 |
| 147= | 90.0 | 96.8 | 93.8 | 92.8 | 93.8 | 98.0 | 97.8 | 95.8 |
| 148= | 98.0 | 97.0 | 95.0 | 99.0 | 183.0 | 94.0 | 408.8 | 108.8 |
| 149= | 400.0 | 98.8 | 93.0 | 95.8 | 96.8 | 79.8 | 79.0 | 99.8 |
| 450= | 96.0 | 96.0 | 94.0 | 94.8 | 94.8 | 92.0 | 94.0 | 93.8 |


| $454=$ | $95.0$ | $94.0$ | 92.0 | 80.0 | 79.0 | 86.0 | 80.0 | 90.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $152=$ | $92.0$ | $94.0$ | 94.0 | 96.0 | 95.0 | 94.0 | 84.8 | 79.8 |
| 453= | 85.0 | 73.0 | 78.0 | 76.0 | 86.0 | 88.0 | 95.0 | 94.8 |
| 454= | 89.0 | 94.0 | 70.8 | 72.0 | 76.0 | 68.0 | 55.8 | 60.8 |
| 455= | 67.0 | 76.0 | 84.0 | 87.0 | 87.8 | 89.0 | 90.0 | 86.0 |
| 456= | 94.0 | 67.8 | 36.0 | 47.0 | 54.0 |  |  |  |
| 457= | 25.5 | 25.5 | 25.6 | 25.6 | 25.7 | 25.8 | 25.9 | 26.8 |
| 458= | 26.6 | 27.2 | 27.8 | 28.4 | 29.0 | 29.6 | 38.2 | 34.4 |
| 159: | 32.7 | 34.0 | 35.2 | 36.5 | 37.7 | 40.2 | 42.7 | 45.2 |
| 168= | 47.7 | 50.2 | 52.8 | 55.4 | 56.6 | 57.8 | 59.8 | 60.4 |
| 464= | 64.2 | 62.3 | 63.4 | 65.8 | 68.2 | 70.6 | 73.1 | 75.6 |
| 462= | 78.1 | 80.6 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 | 83.0 |
| 463= | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 |
| 164= | 83.0 | 83.8 | 83.8 | 83.0 | 83.0 | 83.8 | 83.8 | 83.8 |
| 165= | 83.0 | 83.0 | 83.8 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 |
| 466= | 83.8 | 83.8 | 83.8 | 83.8 | 83.0 | 83.8 | 83.0 | 83.8 |
| 167= | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 |
| 168= | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 | 83.8 | 83.6 |
| 469= | 83.0 | 83.8 | 83.8 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 |
| $470=$ | 83.0 | B3. 0 | 83.8 | 83.8 | 83.0 | 83.0 | 83.0 | 83.0 |
| 474 = | 83.0 | 83.8 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 |
| 172= | 83.0 | 83.8 | 83.0 | 83.0 | 83.0 | 83.0 | 83.0 | 83.8 |
| 473= | 83.0 | 83.8 | 83.8 | 83.6 | 83.0 |  |  |  |
| 474= | 0.184 | 0.292 | 0.318 | 0.343 | 0.380 | 0.424 | 0.447 | 0.460 |
| 475= | 0.468 | 0.472 | 0.474 | 0.475 | 0.476 | 0.478 | 0.480 | 0.484 |
| 476= | 0.482 | 0.487 | 0.492 | 0.496 |  |  |  |  |
| 477= | . $1000 E+08$ | . 5248E+07 | . $3462 E+07$ | . 2042E+07 | . $1445 E+07$ | - $1072 \mathrm{E}+07$ | . $8544 \mathrm{E}+86$ | . $6607 E+86$ |
| 478= | $.5433 E+86$ | . $4365 E+86$ | . $3548 E+86$ | . $29515+86$ | . $2399 E+06$ | . $1985 E+06$ | . $1622 E+06$ | - 1288E+86 |
| 479= | . $1096 E+06$ | . $8740 \mathrm{E}+85$ | . $7079 E+05$ | . 5689E+05 | . $4677 E+05$ | . 37 15E+05 | . $3462 \mathrm{E}+05$ | - $2630 E+05$ |
| 480= | $.2463 E+85$ | . $4758 \mathrm{E}+85$ | . $1443 E+05$ | - $1475 \mathrm{E}+85$ | . $9333 E+04$ | .7586E+04 | . $6466 E+04$ | - $5129 E+04$ |
| $184=$ | . $4169 E+84$ | . $3344 \mathrm{E}+84$ | . $2692 E+04$ | - $2213 \mathrm{E}+04$ | - $4820 \mathrm{E}+04$ | . $4429 E+04$ | . $1184 \mathrm{E}+04$ | $.9550 E+83$ |
| 482= | . $7499 E+03$ | . $6346 \mathrm{E}+0 \dot{0}$ | . $5500 E+83$ | . $5850 \mathrm{E}+03$ | . $4609 E+03$ | .4450E+03 | - 3700E+03 | . $3250 \mathrm{E}+03$ |
| 483= | . $2800 \mathrm{E}+03$ | $.2350 E+03$ | -1980E+03 | . $1450 \mathrm{E}+03$ | . $4000 \mathrm{E}+83$ | $.7750 E+02$ | $.5500 E+02$ | $.3250 E+02$ |
| 484= | $.1000 E+02$ | . $7750 \mathrm{E}+04$ | . $5500 E+04$ | $.3750 E+04$ | $.0040 E+80$ |  |  |  |
| 485= | . $1000 \mathrm{E}+08$ | $.5248 \mathrm{E}+07$ | . $3462 E+07$ | . $2042 \mathrm{E}+07$ | - $1445 \mathrm{C}+07$ | . $1872 E+07$ | . 85 14E+06 | . $6607 E+86$ |
| 486 $=$ | . $5433 \mathrm{E}+86$ | . $4365 E+06$ | . $3548 E+06$ | . $2954 E+86$ | . $2399 E+06$ | . $1905 E+06$ | . $1622 E+06$ | - $1288 \mathrm{E}+06$ |
| 487= | . $1896 E+06$ | . $8710 \mathrm{E}+05$ | . $7079 E+05$ | . $5689 \mathrm{E}+05$ | -4677E+85 | . $3745 E+85$ | . $3162 \mathrm{E}+05$ | . $2630 E+85$ |
| 188= | $.2463 E+05$ | -1758E+85 | - $1443 E+05$ | - $1475 E+05$ | . $9333 \mathrm{E}+04$ | . $7586 \mathrm{E}+04$ | . $6466 E+04$ | . $5129 E+04$ |
| 189\% | $.4469 E+84$ | . $3314 \mathrm{E}+04$ | . $2692 E+84$ | -2243E+04 | . 1820E+04 | - $1429 E+04$ | . $1484 \mathrm{E}+04$ | $.9550 E+03$ |
| 498= | . $7499 \mathrm{E}+03$ | $.6346 E+03$ | . $5042 \mathrm{E}+93$ | . $3745 E+03$ | -2609E+03 | - 4832E+03 | . $4340 \mathrm{E}+03$ | . $8350 E+82$ |
| 191: | . $2310 \mathrm{E}+02$ | -4700E +84 | .004E+00 |  |  |  |  |  |
| 192= | 0.100E-09 | 0.200E-89 | 0.380E-09 | 0.500E-89 | 0.900E-09 | $0.430 E-08$ | 0. 180E-08 | $0.250 E-88$ |
| 193= | 0.330E-08 | 0.440E-08 | 0.589E-88 | 0.740E-88 | 0.989E-08 | 0.134E-07 | 0.466E-07 | 0.227E-07 |
| 194= | $0.282 E-87$ | 8.385E-07 | 8.599E-07 | 0.684E-07 | 0.891E-07 | 0.122E-06 | 0.451E-06 | B. $194 \mathrm{E}-06$ |
| 195= | $0.252 E-86$ | 8.334E-06 | 0.448E-06 | 0.575E-06 | 0.785E-06 | 0.104E-05 | 0.437E-05 | 0.476E-05 |
| 496= | $0.233 E-85$ | 0.348E-05 | 0.428E-05 | Q.548E-05 | 0.713E-65 | 0.988E-85 | 8. 433E-04 | 0.470E-84 |
| 197= | 0.236E-84 | 0.298E-64 | 0.487E-04 | 6.900E-04 | 0.100E-03 | 8.140E-03 | 0. 120E-03 | 0. $448 \mathrm{E}-83$ |
| 198= | B. $470 \mathrm{E}-03$ | 0.280E-63 | Q.230E-03 | 0.270E-03 | 0.380E-03 | 0.360E-03 | 0.440E-03 | 0.660E-03 |
| 499= | 0.120E-02 | 0.220E-02 | 0.508E-02 | 0.170E-81 | 0.230E-01 |  |  |  |
| 200= | 0.109E-09 | 0.200E-09 | 0.380E-89 | 0.S00E-69 | 0.900E-89 | 0.430E-08 | $0.480 E-88$ | 0.250E-88 |
| 204= | 0.339E-08 | 0.440E-08 | 0.580E-08 | 6.740E-08 | 0.980E-08 | 0.134E-07 | 0:166E-07 | 0.227E-07 |
| 202= | $0.282 E-07$ | 0.385E-87 | 0.509E-87 | 0.684E-07 | 0.89 1E-07 | B.122E-06 | 0.451E-06 | 0.194E-86 |
| 283= | $0.252 E-86$ | 0.334E-06 | B.448E-06 | 0.575E-86 | $0.785 E-86$ | 8. 404E-95 | 8. 137E-05 | 0.176E-85 |
| 204= | $0.233 \mathrm{E}-05$ | $0.348 \mathrm{E}-85$ | 0.420E-65 | $0.548 E-05$ | $0.713 E-85$ | 0.988E-85 | 0. $433 \mathrm{E}-04$ | 0.178E-04 |
| 205: | 0.233E-04 | 0.298E-04 | 0.407E-04 | 0.683E-04 | 0.982E-84 | 0.458E-03 | 0.249E-03 | 0.457E-63 |
| 286= | 0.500E-82 | 0.476E-01 | 0.230E-04 |  |  |  |  |  |
| 287= | 472. | 0. | 6. | 7. | 7. | 7. | 7. | 8. |
| 208= | 7. | 7. | 7. | 7. | 7. | 7. | 7 。 | 7. |
| 209: | 7. | 7. | 7. | 7. | 6 |  |  |  |

## 11 Execution of CROPR

### 11.1 Listing of program



$127=130$
428=440
$429=450$
430=
131=
132=
$133=170$
134=160
135=
136=
137=
138 =
439=
440=
$141=$
442=
$443=480$
144=
$145=190$
446=
$447=200$
$148=$
149=
450=210
151=
452=
$153=220$
154=
455=
156=
$157=$
158=
459=
160=
161=
162=
463=
164=
465=
166=
467=
468=
$169=$
478=
47 1=250
172=
473=260
$174=$
$475=$
476=
$477=280$
$178=290$
179=
$480=$
$184=$
$482=300$
483:
484=
185=
186=340
487=
488=
489=
198=
494=
492=
193=
194=
195=
196=278
$497=$


| $498=$ | $P C A=A 1+A 2 * V A L 3$ |
| :---: | :---: |
| $199=$ | $A 2=(P O(N M)-P O(N M+1) 3 /$ VAL2 |
| 200= | A1=PO[NM]-A2*UAL4 |
| 204= | $P O A=A 1+A 2 * \cup A L 3$ |
| 202= |  |
| 203=320 | DELTA $=(R C A-0.5 * S R F(J) 3 /(0.8 * R C A)$ |
| $204=$ | RNET $=[D E L T A * P O A+[4.0-D E L T A] * P C A] * P H F$ |
| 205= | PRG(J)=RNET*BETA*ALFA*SC[J] |
| $286=$ | GO TO 340 |
| $207=330$ | $D N=1.0-C L O[J] / 10$. |
| 208= | RNET $=[P O A+0.9 * D N *[P C A-P O A)\} * P H F$ |
| 209= | PRG(J) $=$ RNET*BETA*ALFA*SC[J) |
| 210=340 | IF (J.EQ.4) PCY[J] $=0.04 * P \mathrm{RG}(\mathrm{J})$ |
| 214= | IF(J.GT. 1) PCY(J) =PCY(J-4)+0.01*PRG[J] |
| 212= |  |
| 213= | I2=PRG[J]/4.0+0.5 |
| $214=$ | DO $350 \mathrm{~K}=1,99$ |
| $215=$ |  |
| $216=$ | $I A(K)=4 \mathrm{HO}$ |
| 247= | GO TO 350 |
| 218=360 | IF [12.LT.K) GO TO 370 |
| 249= | $I A(K)=4 H=$ |
| 220= | GO TO 350 |
| 221=370 | $I A[K]=4 \mathrm{H}$ |
| 222=350 | CONTINUE |
| $223=$ | PRINT 380, I,M,N,PRG[J],PCY[J],IA |
| 224= |  |
| $225=240$ | CONTINUE |
| 226 $=$ | LC= 4 |
| 227=230 | CONTINUE |
| 228=580 | PRINT 220 |
| 229= | PRINT 210 |
| $230=$ | PRINT 200 |
| $234=380$ |  |
| 232= | IFCLC7].EQ. 13 STOP |
| 233= | PRINT 390, A,FKSI |
| 234=390 | FORMATC $141,2 \mathrm{X}, 97 \mathrm{H}$ RATIO OF ACTUAL TRANSPIRATION/VAPOUR PRES |
| 235= | SSURE DEFICIT OF AIR UERSUS THE RATE OF GROWTH A=F7.2/ |
| 236= | 244X,21HTHE COEFFICIENT FKSI=F4.3) |
| 237 = | PRINT 420 |
| 238= | IFCL( 12].E0.0) PRINT 400 |
| 239= | IF[L( 12).NE.0] PRINT 440 |
| $240=406$ | FORMATC2[30H DAY EP UPD 〕] |
| 2412410 | FORMAT[2[40H DAY TEM EP RH ]) |
| 242= | DAY( 1 ) $=1(4)-2.0$ |
| $243=$ | DAY(2)=L( 1 )-1.0 |
| $244=$ | IFCL(42].NE.0) GO TO 420 |
| 245= | DO $430 \mathrm{I}=1,10.2$ |
| 246= | $\mathrm{J}=1$ |
| 247 $=$ | DAY $\{$ \{ $\}=\operatorname{DAY}\{1\}+2.0$ |
| 248= | $\operatorname{DAY}(2)=\operatorname{DAY}(2)+2.8$ |
| 249= | IF(DAY(2).GT.L(2)) J=0 |
| $250=$ | $J=J+1$ |
| $251=$ | PRINT 440, [DAY[K], EP [I+K-4],RH(I+K-4), $K=4, \mathrm{~J}$ ] |
| 252=430 | CONTINUE |
| 253= | GO TO 460 |
| $254=440$ | FORMAT 2 [F90.0.2F10.2] |
| 255=420 | DO $450 \mathrm{I}=1,10,2$ |
| 256= | $J=4$ |
| 257= | DAY 4 1)=DAY( $13+2.0$ |
| 258= | DAY(2)=DAY(2)+2.8 |
| 259= | IF(DAY(2).GT.L(2)] J=0 |
| 260= | $J=J+1$ |
| 261= | PRINT 150, [DAY(K), TEM $1+K-4), E P(I+K-4), R H(I+K-4), K=4, J]$ |
| $262=450$ | CONTINUE |
| 263= | DO 478 I= 4,10 |
| 264= |  |
| 265= |  |
| $266=470$ | CONTINUE |
| 267=460 | PRINT 480 |
| $268=480$ | FORMATC 4 H///50X, 27HSIMULATION OF ACTUAL YIELD J |


| $\begin{aligned} & 269= \\ & 278=490 \end{aligned}$ | PRINT 490 FORMAT $/ / / / 4 \mathrm{H}, 40 \mathrm{X}, 43 \mathrm{HACTUAL}$ RATE OF GROWTHCARGJ－KG／HA／DAY，SIGN $=/ /$ |
| :---: | :---: |
| 27 亿＝ | $141 \times, 45 \mathrm{HACTUAL}$ CUMULATIVE YIELD［ACY］－400＊KG／HA，5IGN 0／／ |
| 272＝ | $244 \mathrm{X}, 53 \mathrm{HDPAY}$ IS DIFFERENCE BETWEEN POTENTIAL AND ACTUAL YIELD／／3 |
| 273＝ | PRINT 500 |
| 274＝500 | FORMATE4H，34H DATE DAY ARG ACY DPAY 0．0，20X，5Hi00．0，20X，5H200 |
| 275＝ | 1．0，20X，5H30日．0，20X，5H400．0］ |
| 276＝ | PRINT 220 |
| $277=$ | $N=L[1]-1$ |
| 278＝ | LC＝L（4） |
| 279x | DO $520 \mathrm{M}=\mathrm{LA}, \mathrm{LB}$ |
| 280＝ | $13=K M(M)$ |
| 284＝ | DO $530 \mathrm{I}=\mathrm{LC,I3}$ |
| 282＝ | $N=N+1$ |
| 283＝ | $J=N-L(4)+1$ |
| 284＝ | C＝A＊EP［J］／UPD［J］ |
| $285=$ | $\theta=[4.0-F K S I) * P R G(J) * C$ |
| 286＝ | $P=P R G(J)+C$ |
| 287＝ | CALL SOUE $(P, Q, X, Y, R E)$ |
| 288＝ | ARG $=X$ |
| 289＝ | $A C Y=A C Y+0.0$ 4＊ARG |
| 290＝ | DPAY＝PCY［J］－ACY |
| 291＝ | $14=A C Y / 4.0+0.5$ |
| 292＝ | I2＝ARG／4．0＋日． 5 |
| 293 $=$ | DO $540 \mathrm{~K}=1.99$ |
| $294=$ |  |
| 295＝ | $I A(K)=4 \mathrm{HO}$ |
| 296＝ | GO TO 540 |
| 297＝550 | IF（I2．LT．K）GO TO 560 |
| 298＝ | $I A(K)=14=$ |
| 299＝ | GO TO 540 |
| $300=560$ | $I A(K)=4 \mathrm{H}$ |
| $301=540$ | CONTINUE |
| 382＝ | PRINT 570，I，M，N，ARG，ACY，DPAY，IA |
| $303=$ | IFCN．GE．L［2］）GO TO 590 |
| $304=530$ | CONTINUE |
| 305＝ | LC＝ 4 |
| $306=520$ | continue |
| $307=590$ | PRINT 220 |
| $308=$ | PRINT 588 |
| 309＝ | PRINT 490 |
| $340=570$ | FORMAT $4 \mathrm{H}, \mathrm{I} 2,4 \mathrm{X}, \mathrm{I} 2,4 \mathrm{X}, \mathrm{I} 3,4 \mathrm{X}, \mathrm{FS}, 1,4 \mathrm{X}, \mathrm{FS}, 4,4 \mathrm{X}, \mathrm{F6}, 2,4 \mathrm{X}, 4 \mathrm{H}+, 99 \mathrm{~A}, 4 \mathrm{H}+\mathrm{J}$ |
| $341=$ | STOP |
| $312=$ | END |


| 343＝ | SURROUTINE SQUE $(P, Q, X, Y, R E)$ |
| :---: | :---: |
| $314=$ | LOGICAL RE |
| $345=$ | $X=P / 2.0$ |
| 316＝ | $D P=X * X-0$ |
| $347=$ | RE＝DP．GE．8．6 |
| 318＝ | $D P=S Q R T(A B S(D P))$ |
| 319＝ | IF（RE．AND．．TRUE．）GO TO 40 |
| $328=$ | $Y=D P$ |
| $321=$ | GO TO 20 |
| $322=40$ | $Y=X+D P$ |
| 323＝ | $X=X-D P$ |
| $324=20$ | RETURN |
| 325＝ | END |

326＝SUBROUTINE TEINF［ALFA，X，NCA］
$327=$ DIMENSION ALTE［ 10，2］
328＝
329＝
$330=$
COMMON／ALF／ALTE
$K=N C A-4$
DO $40 I=4, K$
$331=\quad \operatorname{IF}(X . L E . A L T E(4,2))$ GO TO 20
$332=$

IF (X.GT.ALTE 1,2$\}$.AND.X.LE.ALTE $1+1,2])$ GO TO 40 CONTINUE
ALFA=ALTE[1,1]
GO TO 50
ALFA $=A L T E(N C A, 1)$
GO TO 50
ALFA $=A L T E[I, 1]-(A L T E[I, 1]-A L T E[I+4,1)] *(X-A L T E(I, 2)] /(A L T E[I+1,2]-$ \{ALTE (I,2]J
RETURN
END

### 11.2 Instructions for input

The data input has been arranged in 11 groups $\mathrm{A}, \mathrm{B}, \ldots, \mathrm{L}$ and should be punched on cards in FORTRAN code according to the instructions given on page 171-176.

| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| A | 1-80 | 20A4 | HED | desired heading |
|  |  |  |  | Group A consists of 1 card |
| B | 1-5 | I5 | L(1) | first day of calculation (reckoned from beginning of the year) <br> last day of calculation (the same) <br> number of days in February ( 28 or 29) <br> first day in the first month of calculation (reckoned from the beginning of the month) <br> first month of calculation (reckoned from the beginning of the year) <br> last month of calculation (the same) <br> if $L(7)=1$ : only Part 1 (calculates potential growth rate and potential dry matter yield of the crop) is performed if $L(7)=0$ : also Part 2 (calculates actual growth rate and actual dry matter yield of the crop) is executed if $L(8)=0$ : the influence of temperature on production is to be given as a function according to Eqn 9.1 if $L(8)=1$ : the influence of temperature on production is to be given as a function according to Eqn 9.2 if $L(8)=2$ : the influence of temperature on production is given as a table, entitled ALTE (see Group D) <br> if $\mathrm{L}(9)=0$ : actual solar radiation flux SRF is in W.m ${ }^{-2}$ |
|  | 6-10 | I5 | L(2) |  |
|  | 11-15 | I5 | L(3) |  |
|  | 16-20 | I5 | L(4) |  |
|  | 21-25 | 15 | L(5) |  |
|  | 26-30 | I5 | L(6) |  |
|  | 31-35 | I5 | L(7) |  |
|  | 36-40 | I5 | L(8) |  |
|  | 41-45 | I5 | L(9) |  |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | if $\mathrm{L}(9)=1: \mathrm{SRF}$ is in cal. $\mathrm{cm}^{-2} \cdot \mathrm{day}^{-1}$ |
|  | 46-50 | 15 | L(10) | if $\mathrm{L}(10)=0: S R F$ is to be given as an input |
|  |  |  |  | if $\mathrm{L}(10)=1$ : cloudiness CLO is to be given as an input |
|  | 51-55 | 15 | L(11) | number of lines in ALTE array. The value of L(11) can |
|  | 56-60 | 15 | L(12) | be set maximally equal to 10 <br> if $L(12)=0$ : vapour pressure deficit VPD is to be given |
|  |  |  |  | as an input <br> if $L(12)=1$ : relative humidity of the air RH is to be given as an input |
|  |  |  |  | Group B consists of 1 card |
| C |  |  |  | Omit Group C if $\mathrm{L}(8)=2$ |
|  | 1-10 | F10.3 | BETA | ratio of harvested part of plant to total production |
|  | 11-20 | F10.3 | PHF | photorespiration factor to account for respiration losses |
|  | 21-30 | F10.3 | WG | latitude of area concerned, e.g. for $52^{\circ} 15^{\prime}$ : $\mathrm{WG}=52.25$ |
|  | 31-40 | F10.3 | AL | parameter in ALFA function (see Eqns 9.1 and 9.2) |
|  | 41-50 | F10.3 | BL | parameter in ALFA function (see Eqns 9.1 and 9.2) |
|  |  |  |  | Group C consists of 1 card |
| D | 1-10 | F10.3 | BETA | Omit Group D if $L(8) \neq 2$ see Group C |
|  | 11-20 | F10.3 | PHF | see Group C |



| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| F | 1-10 | F10.3 | D(1) | time (days) from the beginning of the year at which in $\mathrm{TAB}(\mathrm{I}, \mathrm{J}, \mathrm{K}) \mathrm{J}=1$ (for time $\mathrm{D}(1)=15.0$, i.e. 15 January) as above but for $\mathrm{J}=2$ (for time $\mathrm{D}(2)=46.0$, i.e. 15 February) |
|  | 11-20 | F10.3 | D(2) |  |
|  | 21-30 | F10.3 | $\mathrm{D}(3)$ | $\mathrm{D}(3)=75.0$ |
|  | 31-40 | F10.3 | D(4) | $D(4)=106.0$ |
|  | 41-50 | F10.3 | D (5) | $\mathrm{D}(5)=136.0$ |
|  | 51-60 | F10.3 | D(6) | $\mathrm{D}(6)=167.0$ |
|  | 61-70 | F10.3 | D(7) | $\mathrm{D}(7)=197.0$ |
|  | 71-80 | F10.3 | D(8) | $\mathrm{D}(8)=228.0$ |
|  | 1-10 | F10.3 | D(9) | $\mathrm{D}(9)=259.0$ |
|  | 11-20 | F10.3 | $\mathrm{D}(10)$ | $\mathrm{D}(10)=289.0$ |
|  | 21-30 | F10.3 | D(11) | $\mathrm{D}(11)=320.0$ |
|  | 31-40 | F10.3 | D(12) | $\mathrm{D}(12)=350.0$ |
|  |  |  |  | Group F consists of 2 cards (12 values) |
| G | 1-10 | F10.3 | TEM(1) | average temperature $\left({ }^{\circ} \mathrm{C}\right)$ for first day of computation as above, but for second day, etc. <br> Group G consists maximally of 365 values ( 46 cards) |
|  | 11-20 | F10.3 | TEM(2) |  |
| H | $\begin{array}{r} 1-10 \\ 11-20 \end{array}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | $\begin{aligned} & \mathrm{SC}(1) \\ & \mathrm{SC}(2) \end{aligned}$ | soil cover (fraction) for first day of computation as above, but for the second day, etc. <br> Group H consists maximally of 365 values ( 46 cards) |
|  |  |  |  |  |
|  |  |  |  |  |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| I | $\begin{array}{r} 1-10 \\ 11-20 \end{array}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | $\begin{aligned} & \operatorname{SRF}(1) \\ & \operatorname{SRF}(2) \end{aligned}$ | If $L(10)=0$ : SRF is actual solar radiation flux <br> If $L(9)=0$ : in W. $\mathrm{m}^{-2}$; if $\mathrm{L}(9)=1:$ in cal. $\mathrm{cm}^{-2}$.day ${ }^{-1}$ <br> If $\mathrm{L}(10)=1:$ SRF is actual degree of cloud cover CLO (scale 1 to 10 ) <br> value for first day of computation as above, but for second day, etc. <br> Group I consists maximally of 365 values ( 46 cards) |
| J | $\begin{array}{r} 1-10 \\ 11-20 \end{array}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | A FKSI | Omit Group J if $L(7)=1$ <br>  initial slope of the ratio $\dot{q}$ to $E_{p l} / \Delta e$ (see Eqn 6.8) mathematical flexibility constant used in Eqn 6.11. Usually one can set $\mathrm{FKSI}=0.01$ <br> Group J consists of 1 card |
| K | $\begin{gathered} 1-10 \\ 11-20 \end{gathered}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | $\mathrm{EP}(1)$ $\mathrm{EP}(2)$ | Omit Group K if $L(7)=1$ <br> actual transpiration rate $\left(\mathrm{mm}^{2} \mathrm{day}^{-1}\right)$ as obtained from SWATR (or possibly otherwise) for the first day of computation <br> as above, but for second day, etc. <br> Group K consists maximally of 365 values ( 46 cards) |


| Group | Columns | Format | Symbol | Description |
| :---: | :---: | :---: | :---: | :---: |
| L | $\begin{array}{r} 1-10 \\ 11-20 \end{array}$ | $\begin{aligned} & \text { F10.3 } \\ & \text { F10.3 } \end{aligned}$ | $\begin{aligned} & \text { RH(1) } \\ & \text { RH(2) } \end{aligned}$ | Omit Group $L$ if $L(7)=1$ <br> If $L(12)=0: R H$ is vapour pressure deficit VPD of the air in mbar (from SWATR or otherwise) <br> If $L(12)=1$ : RH is relative humidity of the air (fraction) value for first day of computation as above, but for second day, etc. <br> Group L consists maximally of 365 values ( 46 cards) |



| $74=$ | . 72 | . 74 | .76 | . 77 | . 79 | . 79 | . 88 | . 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $75=$ | . 84 | . 81 | . 82 | . 82 | . 83 | . 83 | . 84 | . 84 |
| $76=$ | . 85 | . 86 | . 86 | . 87 | . 87 | . 88 | . 88 | . 89 |
| $77=$ | . 89 | . 90 | . 98 | . 94 | . 91 | . 92 | . 93 | . 93 |
| $78=$ | . 93 | . 94 | . 94 | . 95 | . 95 | . 96 | . 96 | . 96 |
| $79=$ | . 96 | . 96 | . 96 | . 96 | . 96 | . 96 | . 95 | . 94 |
| $80=$ | . 93 | .92 | . 92 | . 91 | . 91 | . 91 | . 91 | . 91 |
| $84=$ | . 90 | .98 | .90 | . 90 | . 98 | . 90 | . 98 | . 90 |
| 82x | . 90 | .90 | . 90 | . 98 | . 90 | . 98 | . 90 | . 98 |
| 83\% | . 90 | .90 | . 90 | . 90 | . 90 | .90 | . 98 | .90 |
| 84= | . 98 | . 90 | . 98 | . 90 | . 98 |  |  |  |
| 85= | 647. | -498. | 358. | 255. | 547. | 743. | 286. | 658. |
| $86=$ | 236. | 676. | 488. | 537. | 635. | 463. | 583. | 496. |
| 87\% | 523. | 359. | 548. | 579. | 632. | 682. | 491. | 284. |
| 88: | 285. | 495. | 495. | 509. | 578. | 570. | 459. | 540. |
| 89= | 176. | 689. | 271. | 438. | 283. | 465. | 484. | 236. |
| 98= | 554. | 440. | 318. | 232. | 398. | 357. | 305. | 503. |
| $94=$ | 460. | 299. | 507. | 464. | 427. | 284. | 207. | 262. |
| 92= | 257. | 329. | 283. | 174. | 481. | 343. | 258. | 462. |
| $93=$ | 389. | 223. | 358. | 373. | 485. | 272. | 324. | 484. |
| 94= | 342. | 157. | 58. | 308. | 102. | 383. | 230. | 293. |
| $95=$ | 303. | 344. | 448. | 301. | 202. | 75. | 154. | 285. |
| 96= | 459. | 285. | 436. | 323. | 479. | 92. | 258. | 452. |
| 97= | 436. | 475. | 268. | 266. | 183. | 248. | 197. | 163. |
| 98= | 446. | 169. | 129. | 89. | 94. | 408. | 434. | 47. |
| $99=$ | 140. | 29. | 447. | 489. | 220. | 143. | 83. | 486. |
| 100= | 189. | 204. | 76. | 440. | 189. | 106. | 450. | 194. |
| 184= | 96. | 66. | 84. | 79. | 79. |  |  |  |
| 182= | 400. | . 84 |  |  |  |  |  |  |
| 183= | 4.24 | . 74 | . 53 | . 85 | 4.14 | . 55 | . 65 | . 85 |
| 404= | . 85 | . 96 | . 87 | 4.36 | 1.53 | 4.12 | 4.44 | 1.43 |
| 185= | 4.25 | 4.27 | 1.42 | 4.64 | 1.80 | 2.00 | 4.27 | . 45 |
| 186= | . 62 | 1.85 | 2.87 | 2.69 | 1.74 | 4.77 | 2.72 | 2.33 |
| 407= | 1.92 | 2.03 | 4.57 | 1.51 | . 92 | 2.14 | 2.28 | 1.82 |
| 488= | 2.59 | 2.47 | 4.75 | 2.22 | 2.28 | 4.74 | 2.85 | 4.16 |
| 499= | 3.28 | 2.45 | 3.08 | 4.49 | 3.36 | 2.30 | 2.44 | 3.82 |
| 148= | 3.56 | 2.60 | 1.29 | 4.42 | 2.04 | 9.65 | 4.69 | 4.95 |
| 144= | 1.35 | 1.35 | 1.74 | 1.71 | 4.69 | 2.92 | 3.28 | 2.43 |
| 142= | 3.79 | 3.74 | 2.96 | 3.81 | 2.44 | 4.63 | 4.05 | 1.32 |
| 143= | 4.32 | . 91 | . 98 | 4.56 | 4.17 | . 70 | . 94 | . 84 |
| 144= | 1.83 | 4.97 | 2.84 | 2.82 | 4.44 | 4.54 | 1.58 | 1. 14 |
| 415 $=$ | 4.54 | 1.84 | 4.46 | 4.72 | 4.18 | 5.57 | 5.35 | 5.38 |
| $446=$ | 4.80 | 4.33 | 2.52 | 1.46 | 1.35 | 4.52 | 4.28 | 4.88 |
| $417=$ | . 95 | 2.03 | 3.37 | 4.83 | 3.59 | 3.58 | 2.93 | 4.58 |
| 148= | 4.85 | 1.27 | . 68 | . 62 | . 78 | 4.55 | 3.89 | 4.25 |
| $419=$ | 2.35 | .92 | . 36 | . 34 | .34 |  |  |  |
| 420= | 3.4 | 4.2 | 4.7 | . 8 | 2.7 | 4.7 | 4.5 | 4. |
| 124= | 1.1 | 3.0 | 3.2 | 2.2 | 3.4 | 2.6 | 3.1 | 4.4 |
| 422= | 3.6 | 4.8 | 2.5 | 3.7 | 3.5 | 3.2 | 3.4 | 1.8 |
| 123= | . 7 | 2.0 | 4.8 | 5.0 | 1.8 | 2.4 | 3.2 | 4.8 |
| 124= | 1.4 | 3.9 | 4.5 | 1.8 | 1.8 | . 9 | 3.3 | . 6 |
| 125= | 2.9 | 4.4 | 4.7 | 2.6 | 2.5 | 1.7 | 2.4 | 4.7 |
| 126= | 4.9 | 3.2 | 2.3 | 2.4 | 3.4 | 4.5 | 4.7 | 4.6 |
| 127= | 2.7 | 2.5 | 1.3 | . 9 | 2.3 | 2.1 | 1.2 | 1.6 |
| 128= | 4.9 | . 6 | 3.1 | 1.4 | 2.2 | 4.5 | 2.7 | 1.9 |
| 129= | 2.8 | 3.7 | 1.8 | 2.2 | 4.7 | 2.2 | 1.1 | 4.4 |
| 430= | 1.4 | 4.6 | . 7 | 4.2 | 4.3 | .5 | . 7 | 4.3 |
| 434= | . 8 | 1.1 | 1.8 | 2.2 | 4.1 | 1.0 | 1.6 | 4.1 |
| 432= | 4.4 | 2.3 | 4.7 | 4.5 | 4.8 | 3.9 | 3.0 | 4.2 |
| 433= | 2.4 | 3.0 | 3.8 | . 9 | 1.3 | . 7 | 1.1 | . 4 |
| 434= | 4.6 | . 5 | 3.4 | 2.1 | 3.2 | 1.7 | 3.1 | 1.5 |
| 135= | 4.4 | 1.6 | . 6 | . 7 | . 6 | 1.1 | 1.3 | 3.2 |
| 436= | 4.3 | 4.2 | 4.1 | .7 | . 6 |  |  |  |

## Appendix A List of used symbols

Some incidental symbols are defined in the text only. Some letters are also used for any given constant. For conversion of SI units into other units, see Table 10. Explanation of computer symbols is given in Part III.

| Symbol | Interpretation | Dimension |
| :---: | :---: | :---: |
| A | Maximum water use efficiency (see Eqn |  |
|  | 6.8 and Figs 19-21) | $\mathrm{M}^{2} \cdot \mathrm{~L}^{-4} \cdot \mathrm{~T}^{-2}$ |
| C | Differential moisture capacity | $\mathbf{L}^{-1}$ |
| $\mathrm{c}_{\mathrm{p}}$ | Specific heat per unit mass of air at constant pressure | $L^{2} \cdot .^{-2} \cdot \mathrm{~T}^{-1}$ |
| $E, E^{*}$ | Actual and potential evapotranspiration | M.L ${ }^{-2} . \mathrm{t}^{-1}$ or L. $\mathrm{t}^{-1}$ |
| $E_{\text {pl }}, E_{\text {pl }}^{*}$ | Actual and potential transpiration | M. L ${ }^{-2} . \mathrm{t}^{-1}$ or L. $\mathrm{t}^{-1}$ |
| $E_{s}, E_{s}^{*}$ | Actual and potential soil evaporation | M. L ${ }^{-2} . \mathrm{t}^{-1}$ or L. $\mathrm{t}^{-1}$ |
| $E_{i}$ | Actual evaporation flux of intercepted water | M. $L^{-2} . \mathrm{t}^{-1}$ or L. $\mathrm{t}^{-1}$ |
| $e_{a}, e_{d}, e_{s}$ | Saturated and prevailing vapour pressure at air temperature $\mathrm{T}_{\mathrm{a}}$, saturated vapour pressure at soil surface temperature $\mathrm{T}_{\mathrm{s}}$ | M.L ${ }^{-1} \cdot \mathrm{t}^{-2}$ |
| $F$ | Relative humidity of the air | - . |
| $f$ | Function | - |
| G | Heat flux into the soil | M. $\mathrm{t}^{-3}$ |
| g | Acceleration due to gravity | L. $\mathrm{t}^{-2}$ |
| H | Heat flux into the air | M. $\mathrm{t}^{-3}$ |
| $h$ | Suction | L |
| I | Leaf area index | - |
| $K, K_{s}$ | Unsaturated and saturated hydraulic conductivity | L. $\mathrm{t}^{-1}$ |
| $L$ | Latent heat of vaporization of water per unit mass (Chapter 3) | $L^{2} . \mathrm{t}^{-2}$ |
| L | Lower boundary of flow region (Chapter |  |
|  | 4) | L |


| Symbol | Interpretation | Dimension |
| :---: | :---: | :---: |
| $L_{r}, L_{r}^{\text {eff }}, L_{r}^{\text {na }}$ | Rooting depth, effective rooting depth and non-active rooting depth, respectively | L |
| $l$ | crop length or height | L |
| $N, n$ | Maximum possible and actual duration of sunshine per day | t |
| $P_{c}, P_{0}, P_{s t}$ | Gross photosynthesis rate of a 'standard canopy' on clear, overcast and arbitrary days, respectively. | M. $\mathrm{L}^{-2} . \mathrm{t}^{-1}$ |
| $p_{\text {a }}$ | Atmospheric pressure | M.L $L^{-1} . \mathrm{t}^{-2}$ |
| $Q, Q_{\text {act }}, Q_{\text {pot }}$ | Dry matter yield of a crop, actual and maximum dry matter yield | M.L ${ }^{-2}$ |
| $q, q^{*}$ | Actual and maximum possible volume flux of water passing through a unit horizontal area per unit time | L. $\mathrm{t}^{-1}$ |
| $\dot{q}, \dot{q}_{\text {act }}, \dot{q}_{\text {pot }}$ | Growth rate, actual and potential growth rate of a crop | M.L $\mathrm{L}^{-2} . \mathrm{t}^{-1}$ |
| $R_{n}, R_{s}, R_{1}$ | Net, short-wave and thermal radiation flux | M. $\mathrm{t}^{-3}$ |
| $R^{\text {Lop }}$ | Short-wave radiation flux at the top of the atmosphere | M. $\mathrm{t}^{-3}$ |
| $R, R_{\text {c }}$ | Solar radiation flux involved in photosynthesis ( 0.4 to $0.7 \mu \mathrm{~m}$ ) on actual resp. clear days | M. $\mathrm{t}^{-3}$ |
| $r_{a}$ | Diffusion resistance to water vapour of the air layer surrounding the leaves | t. $L^{-1}$ |
| $r_{c}, r_{1}$ | Diffusion resistance to water vapour dependent on fraction of soil covered and on solar radiation flux, respectively | t. $\mathrm{L}^{-1}$ |
| $r_{s}$ | Diffusion resistance to water vapour of both crop and soil surface | t. $\mathrm{L}^{-1}$ |
| $\boldsymbol{S}, S_{\text {max }}$ | Actual and maximal possible volume of water taken up by roots per unit volume of soil per unit time | $\mathrm{t}^{-1}$ |
| $S_{\text {c }}$ | Fraction of soil covered | - |
| $S_{\text {i }}$ | Stress of growth factor $\mathbf{j}$ | - |
| $T_{a}, T_{s}$ | Air temperature in a Stevenson screen and at the soil surface | T |
| $t$ | Time | t |
| $u$ | Horizontal wind velocity | L. $\mathrm{t}^{-1}$ |


| Symbol | Interpretation | Dimension |
| :---: | :---: | :---: |
| $w$ | Growth factor water | - |
| $z$ | Vertical distance from the soil surface taken positive in downward direction | L |
| $\alpha_{T}$ | Parameter accounting for effect of temperature on growth | - |
| $\boldsymbol{\alpha}(\psi)$ | Sink term variable ( $\alpha(\psi)=S / S_{\text {max }}$ ) | - |
| $\beta_{h}$ | Ratio of harvested part to total plant |  |
| $\boldsymbol{\gamma}$ | Psychrometric constant | M. $\mathrm{L}^{-1} \cdot \mathrm{t}^{-2} \cdot \mathrm{~T}^{-1}$ |
| $\boldsymbol{\delta}$ | Slope of the saturation vapour pressure curve | $\mathrm{M} . \mathrm{L}^{-1} \cdot \mathrm{t}^{-2} \cdot \mathrm{~T}^{-1}$ |
| $\boldsymbol{\varepsilon}$ | Ratio molecular weight of water vapour and dry air | M. $L^{-1} \cdot t^{-2} \cdot \mathrm{~T}^{-1}$ |
| $\boldsymbol{\theta}$ | Volume of water per unit volume of soil | - |
| $\theta_{s}$ | Moisture content at saturation | - |
| $\Lambda$ | Fraction of time the sky is overcast | - |
| $\nu$ | Reflection coefficient of short-wave radiation | - |
| $\boldsymbol{\xi}$ | Mathematical flexibility constant in growth equation [see Eqn (6.11)] |  |
| $\rho_{a}$ | Density of moist air | M.L ${ }^{-3}$ |
| $\rho$ | Bulk density of dry soil | M.L ${ }^{-3}$ |
| $\phi_{r}$ | Factor to account for the respiration of a crop |  |
| $\chi$ | Flux of precipitation | M.L ${ }^{-2} \cdot \mathrm{t}^{-1}$ or L.t ${ }^{-1}$ |
| $\Psi$ | Total water potential expressed as energy per unit weight | $L^{2} \cdot t^{-2}$ |
| $\psi$ | Soil moisture pressure head | L |
| $\psi_{2}$ | Minimum soil moisture pressure head to be allowed under air-dry conditions | L |

Table 10. Conversion of units. From Feddes 1971, Table 20.


[^2]
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[^0]:    * Young plants

[^1]:    Groups I, J and K describe the $\mathrm{G}(\mathrm{CH})$, LAI and FIN-functions

[^2]:    *Latent heat consumption in evaporating $1 \mathrm{~kg} \mathrm{H}_{2} \mathrm{O}$ at $293 \mathrm{~K}^{* *}$ Power cons. per $\mathrm{m}^{2}$ in evapor. $\mathrm{H}_{2} \mathrm{O}$ at a flux of 1 mm .day ${ }^{-1}$ at 293 K

