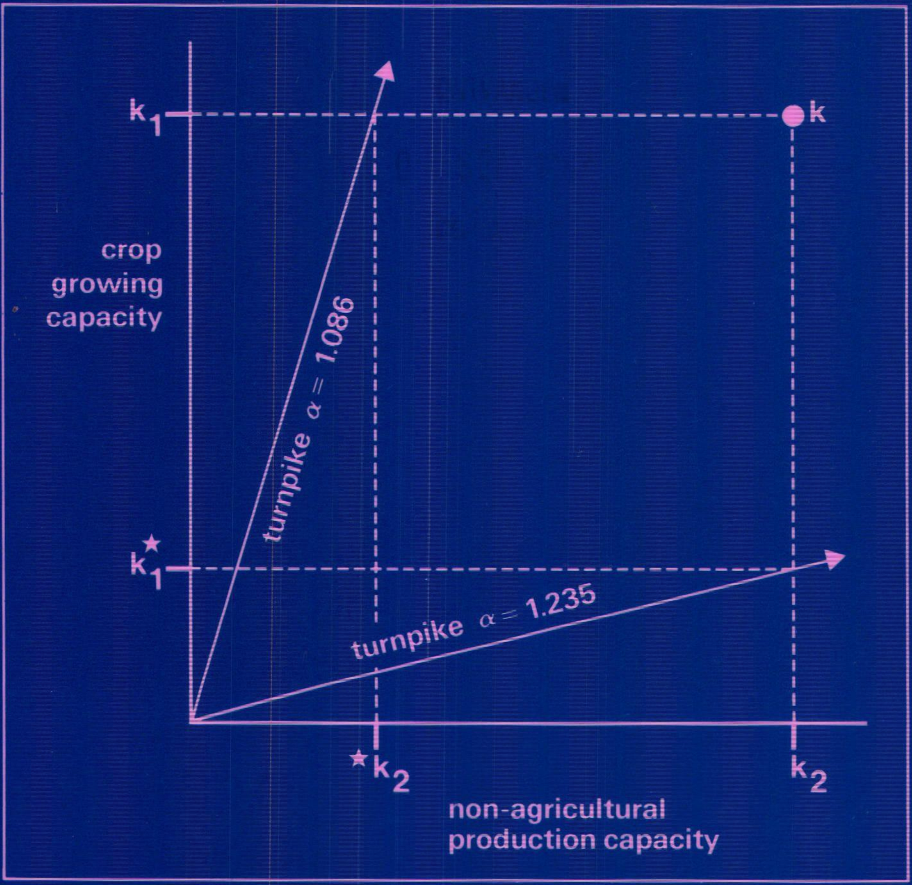
OPTIONS FOR ECONOMIC GROWTH IN BANGLADESH

AN APPLICATION OF THE VON NEUMANN MODEL



H.J.J. STOLWIJK

STELLINGEN

1. In tegenstelling tot het optimaliseringsmodel dat de WRR gebruikt om de middellange termijn groeimogelijkheden van de Nederlandse economie te onderzoeken, heeft het Von Neumann model de volgende eigenschappen:

   (i) De relaties binnen het model hebben een duidelijke economische, technologische en/of institutionele betekenis. Er verschijnen geen niet of moeilijk te interpreteren ad hoc restricties als 'dei ex machina' op het (model-)toneel.

   (ii) Het model staat de specificatie van meerdere produktietechnieken toe. Binnen de projectieperiode kunnen endogene wisselingen van produktietechniek optreden.

   (iii) Het niveau van een (produktie-)activiteit in de modeluitkomsten wordt bepaald door haar relatieve economische aantrekkelijkheid; en niet door de bijdrage die geleverd wordt aan een doelfunctie die een zeer ongeloofwaardig gedrag van de achterliggende economische agenten impliceert.

   (iv) Het handelsregime van een goed (export, import, autarkie) is endogeen en kan binnen de projectieperiode wisselen.

   Onder andere om deze redenen verdient het Von Neumann model meer aandacht als instrument waarmee de middellange termijn groeimogelijkheden van een economie kunnen worden geanalyseerd.

   WRR (1987): 'Ruimte voor groei'. Rapport 29, Staatsuitgeverij, 's-Gravenhage.
   Dit proefschrift.

2. De 'revolutie' die de hypothese van de rationele verwachtingen in de macro-economie heeft teweeg gebracht, doet nogal onevenwichtig aan als men beziet met welk een gemak er binnen diezelfde macro-economie geabstraheerd wordt van allerlei zeer relevante kenmerken van de economische werkelijkheid. Het feit dat er binnen de macro-economische modelbouw überhaupt geen sprake is van expliciet gedefinieerde economische agenten is in dit verband wel het meest in het oog springend.

   David K.H. Begg (1984): 'The rational expectations revolution in macro-economics'. Philip Allan Publishers, Oxford.

3. De door Veerman gebruikte argumenten in zijn kritiek op de 'gangbare (landbouw-)economische theorie' zijn deels onjuist; in zoverre ze juist zijn, zijn ze niet bruikbaar en bovendien inconsistent met zijn conclusies. De door hem bepleite 'grondige bezinning op de uitgangspunten van de landbouweconomie' kan daarom maar beter achterwege blijven.

   C.P. Veerman (1987): 'Over landbouweconomie en landbouwpolitiek'. Inaugurele rede, KUB, Tilburg.

4. De economische wetenschap wordt gekenmerkt door de (bijna) onmogelijkheid tot het doen van experimenten onder gecontroleerde externe omstandigheden en door een in het algemeen zeer matige kwaliteit van het beschikbare datamateriaal. Bij discriminatie en beoordeling van model en modeluitkomsten dienen daarom modelconsistentie en theoretische onderbouwing een belangrijker rol te spelen dan t-waarden, r-kwadraten en ex-post voorspelkracht.

5. Het ene algemene evenwichtsmodel is het andere niet.

6. Met behulp van een serieus empirisch onderbouwd algemeen evenwichtsmodel zou de discussie over het in Nederland te voeren sociaal-economische beleid op een hoger plan kunnen worden gebracht.

7. De bewering van Rutten dat de door hem geformuleerde 'vaste vuistregels' meer houvast bieden voor de algemeen-economische politiek dan 'modelresultaten', is vooral interessant als illustratie van het gemak waarmee men binnen de economische wetenschap politieke knollen voor economische citroenen kan verkopen.

   F.W. Rutten (1987): 'Economische wetenschap en economisch beleid'. Economenblad, jaargang 9, nr. 6.

8. Argumenten die pleiten voor een vrij internationaal verkeer van goederen en kapitaal zijn evenzeer van toepassing op een vrij internationaal verkeer van arbeid. Het is een bedenkelijke vorm van opportunisme dat met betrekking tot arbeid de vrijhandelsargumenten zo goed als volledig genegeerd worden.

9. Het EG-landbouwbeleid stimuleert de consumptie van plantaardige vetten ten koste van die van dierlijke. Het positieve effect hiervan op de volksgezondheid speelt in de huidige discussie over dit beleid ten onrechte geen rol van betekenis.

H.J.J. Stolwijk
Options for economic growth
in Bangladesh
Wageningen, 9 september 1987

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PROEFSCHRIFT

ter verkrijging van de graad van
doctor in de landbouwwetenschappen,
op gezag van de rector magnificus,
dr. C.C. Oosterlee,
in het openbaar te verdedigen
op woensdag 9 september 1987
des namiddags te vier uur in de aula
van de Landbouwuniversiteit te Wageningen

Promotor:	dr.ir. J. de Hoogh, *hoogleraar in de algemene agrarische economie, in het bijzonder de grondslagen, de doeleinden en instrumenten van het landbouwbeleid.*
Co-promotor:	dr. M.A. Keyzer, *adjunct-directeur Stichting Onderzoek Wereldvoedselvoorziening.*

H.J.J. STOLWIJK

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CONTENTS

PREFACE

This study has its origins in 1981, when my colleague at the time, Michiel Keyzer, drew my attention to the book of Morgenstern and Thompson (MT), 'Mathematical theory of expanding and contracting economies' [69]*. In this book MT present their research results on the Von Neumann model. In various places of the text they express the wish that their book will be the start of concrete applications of this model.

I worked at that time at the Centre for World Food Studies, on a large linear programming model of Bangladesh agriculture. Michiel Keyzer suggested to me that it might be interesting to analyze the linear programming matrix along the lines proposed by MT. This thesis can be considered the result of this suggestion.

I would never have been succesful in carrying the project to its conclusion if I had not had time, help and support from a number of persons. I am delighted to have the opportunity to thank them.

The first person I would like to mention is Michiel Keyzer. I was extremely fortunate to have him as one of my thesis advisors. His very detailed comments on draft versions of all chapters, always containing many useful suggestions and his readiness to discuss specific problems, however busy he was with his own work, did not only contribute much to the quality of the work but also made working on the thesis a pleasure. The existence of this book owes much to him.

Secondly, I would like to express my sincere gratitude to professor Jerrie de Hoogh, who also consented to be a thesis advisor. His valuable and thoughtful comments have especially helped to improve chapters 7 and 8.

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* The numbers in brackets refer to the literature at the end of the study.

Thirdly, I would like to thank professor Wouter Tims, the director of the Centre for World Food Studies. When, in 1985, I left the Centre, he kindly consented to the use of the Centre's computational and secretarial facilities. Only in retrospect I fully realize how decisive this 'farewell-present' was for the completion of my thesis. He also made a number of useful comments on chapter 5.

My present employer, the Central Planning Bureau of the Netherlands, has liberally allowed me to spend a part of my research time on finishing this thesis and to use its secretarial and other support facilities. For this I am very grateful.

Thanks also go to Geert Overbosch and Wim van Veen who have helped in the gathering and updating of statistical information on the non-agricultural sector and who have also assisted in implementing the software; and to Richard Rosenbrand who has made the graphic designs.

In particular, I like to take the opportunity to express my deep gratitude to mrs. Lioe Jacobs-Sie for the cheerful and tireless way in which she has typed, retyped and carefully edited the manuscript.

Notwithstanding all the above assistance and support I have received, I must claim all the errors and shortcomings of this work as mine and mine alone.

Herman Stolwijk
June 22, 1987

Chapter 1

INTRODUCTION

1.1 AIMS

In this study we are concerned with economic growth in Bangladesh. More precisely: we want to analyze and quantify the growth potential of the Bangladesh economy. In this respect, some pertinent questions arouse, e.g.: (i) what is the maximum (balanced) growth rate of the economy; (ii) what does the economy look like at such growth rate; (iii) is it possible to identify specific constraints causing the growth rate to slow down; (iv) how would different assumptions with regard to world-market prices, technological progress and the like affect the economic growth prospects; etc.

Our main analytical tool will be the Von Neumann model of an expanding economy. This highly celebrated model, which was presented for the first time as early as 1932[1], became, among others, the starting point for a great number of publications on economic growth. Although one cannot be but impressed by the number of publications and research findings based on the Von Neumann model, at least as striking is the nearly complete lack of any real world application. Hamburger, Thompson and Weil [30], Truchon [101], Tsukui and Murakami [102] and Weil [106] have in effect undertaken some calculations with empirical data. Using the data in the form of a Leontief input-output system, these studies nevertheless failed to show the great analytical power of the Von Neumann model to full advantage. Therefore, the testing and evaluation of the 'Von Neumann tool' on its empirical usefulness will be the second objective of our study. Before giving a brief outline of the study (section 1.3), we shall first explain in more detail the a priori considerations for choosing the Von Neumann model.

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[1] This was in Princeton. In 1937 the model was published for the first time (in German). In 1945 it was translated into English and published as 'A Model of Economic Equilibrium' in the Review of Economic Studies [70].

1.2 THE CHOICE FOR THE VON NEUMANN MODEL

The relevance of studying options for economic growth in Bangladesh will need no further explanation. The achievement of sustained growth can be considered as the paramount economic objective of most of the developing countries. On the other hand, the lack of real world applications of a model of such a venerable age as the Von Neumann model, may raise the question as to why we have chosen for this particular framework. In answering this question we shall, at first, even go further back and briefly go into the matter why a model is used at all to analyze growth options of the Bangladesh economy.

Thinking about growth is, by its very nature, thinking about change. As we are interested in options for growth, especially change that can take place in the future is at issue. In principle, such 'thinking' can be attempted in many ways. One could, for example, disregard laws of nature and logic and, instead dream up a future. As fruitful as this may be for writers of science-fiction, for every-day policy it is not likely to disclose relevant insights. Everything is thinkable and if the profession were not to impose restrictions, a policy-maker has no alternative but to rely on the nicest story. Economics would be reduced to rhetorics. Although to some economists rhetorics is a fair description of what economics actually amounts to (see e.g. Klamer [47] or McCloskey [56]), this would be a deplorable state of affairs. Instead of concentrating on the 'nicest story' it is better to use a theory, i.e. a consistent framework, a model, to discriminate between what is conceivable and what is, in addition, interesting in a policy context.

One could argue in this connection that, in a poor and strongly agricultural oriented country as Bangladesh where a great part of the population does not get an adequate daily diet, options for growth have mainly to do with the agricultural potentials of the country. Therefore a model that allows for studying and evaluating these potentials would be most suited. There is certainly an element of truth in this. It is safe to say that agricultural potentials are of an overwhelming importance for Bangladesh. And, if options for growth of the country are evaluated, one can certainly not do without them. However, a purely agronomic model has its limitations also. On the one hand because such a model is partial; if, for example, no chemical fertilizer at all is available, a discussion on modern farming methods in order to realize the computed yield potential, does not serve much purpose. On the other hand purely agronomic models abstract from the economic aspect, which is too important an aspect to lose sight of; if, for example, irrigation water is too expensive, yield potential will not be realized either.

The above considerations result almost automatically in the choice of a model of economic growth. What has economics to offer in this respect? Since the time-aspect is so inherent to the economic process, it is only natural that economists have been much concerned with economic growth. Roughly speaking one can distinguish two types of growth theories. First, there are the all-embracing theories. These are never purely economic in character: a large variety of political, sociological, cultural and psychological factors are intermixed to produce an all-encompassing 'vision' on the long term development of society. The great classical economists like Smith, Ricardo, Malthus, Mill and, in particular, Marx were all growth theorists in this sense. Rostow's 'Stages of Economic Growth' [82] can be considered as a relatively recent example of such a 'grand' theory (see Jones [37], p. 4).

The importance of these 'grand' theories is beyond doubt. However, they are too general and too imprecise to result in explicit quantitative statements on growth options of a specific economy; they (may) provide a philosophical background for interpreting 'reality'. It will be clear that they are not appropriate for our purpose.

The principal characteristic of the second type of growth theory is the explicit mathematical structure. The models of Harrod and Domar, the neo-classical growth model and the many variants on it are outstanding examples of this category.

Innate to several of these formal models is their high level of aggregation and abstraction. Often, only one or two goods are distinguished. Consequently many pertinent economic issues cannot be traced separately. The threads connecting model with reality become too thinly spaced.

Growth theorists generally recognize this problem. Many textbooks acknowledge the lack of practice-oriented shape of growth theory. Discussing macro-economic growth theory, Solow warns, for example, that

> "we are dealing with a drastically simplified story, a parable . . ."

([87], p. 1). Hahn tells the reader

> "not to expect that he will learn all about economic change and growth, but rather that he will see rather good minds struggling with the most elementary aspects of what may become such a theory"

([29], p. XI); Hacche ([28], p. 21) speaks about disillusionment with the results of the effort put into growth theory; etc. The remarks of Solow, Hahn and Hacche originate in the 1970s, but there is little reason to assume that the situation has fundamentally changed since.

"In part", as Sen ([84], p. 10) explains, "the difficulty arises from the innate complexity of the processes of economic growth". Especially with respect to intertemporal issues there are still many problems to be settled. In such a situation, we think, an applied modelbuilder should follow relevance and pragmatism as his guides.

There are a number of reasons why the Von Neumann model comes to the limelight then. In contrast with the models of Harrod and Domar or the neo-classical growth models, the Von Neumann model distinguishes many sectors and goods. Moreover the model can handle joint production while the input-output matrices by which the production relations are described, need not necessarily be square. The advantages of these characteristics are manyfold.

To start with, the technical relations of the production processes can be taken into account explicitly. Further, the non-squareness of the technology matrices implies that alternative technology levels can be introduced. The technology level, according to which production actually takes place, need not be specified a priori but is endogenous to the model. Similarly, it is not necessary for trade regimes to state in advance whether a good is imported or exported. The joint production property allows also for an explicit treatment of by-products. In the case of Bangladesh where by-products play such an important role, this model property cannot easily be overvalued.

Finally, because many sectors can be distinguished, the Von Neumann model permits a studying of the interdependencies that exist among them.

Given these advantages, it may be intuitively clear that the potential empirical content of the Von Neumann model is superior to that of the traditional macro-economic growth models.

With respect to intertemporal issues, the Von Neumann model takes a pragmatic position. The model describes states where balanced growth is at its maximum. A balanced growth path is certainly interesting on its own. However, because a balanced growth path consists of equilibrium states, i.e. states which are consistent with (i) the laws of nature (volume balances are satisfied) and (ii) optimizing behaviour of economic agents (the price system is such that all processes carried out yield the same profit rate, while processes not carried out cannot yield higher profits), such paths are, in our opinion, highly relevant as well,

Of course, the optimality property of the outcomes presupposes a specific institutional structure of the economy. Uncertainty, lack of information and the like do not explicitly enter the Von Neumann scene. Although we do not want to question the importance of these phenomena, we do think that a proper treatment lies still far beyond what applied economics can actually cope with.

Rejecting macro-economic growth models for the reasons stated above does not automatically lead to the choice of the Von Neumann model. Both the dynamic Leontief input-output model and the multi-period linear programming model are also disaggregated, multi-sectoral linear models of an economy. As Bròdy [13] shows, both can even be written as Von Neumann models.

However, as a growth model, we think, the Von Neumann model has some clear advantages over these alternatives also.

The main drawbacks of the dynamic Leontief model are that it does not allow for joint production and that the technology matrices must be square. Consequences of the latter are that only one technology level can be taken into account and that no 'regime-switches' can take place.

If the Von Neumann model is compared with a multi-period linear programming model, the following differences can be noticed: first, because of its smaller size, the Von Neumann model is computationally much easier to handle. Secondly, it has become clear in the empirical part of the study that because of the non-linearity of the growth factor,the Von Neumann model is less liable to large shocks if coefficients are marginally changed. Thirdly, the Von Neumann model yields an endogenous interest factor. This interest factor plays a role in determining the cost of a process. Fourthly, the Von Neumann model describes an equilibrium state in which balanced growth is on its maximum. Finally, the result can be summarized in one number (growth factor). The outcome of a multi-period linear programming model will, on the other hand, given the somewhat arbitrary manner by which the objective function has to be formulated, be of a much more 'ad-hoc' character.

1.3 PLAN OF THE STUDY

In chapter 2 the Von Neumann model in its original form is discussed. It appears that the original model has a number of limitations which makes an application to a real world economy less interesting. Therefore the model is adjusted in a number of ways. These adjustments are the subject of chapter 3.

Chapter 4 is devoted to computational procedures for solving the model. Because existing procedures are, when the matrix is decomposable, not satisfactory, an alternative algorithm is designed and discussed.

Chapter 5 provides some background information on Bangladesh.

In chapter 6 the model is empirically elaborated. First the model structure is explicitly formulated, then a brief account of the data is given. The chapter ends with an outline of the model software.

Chapter 7 is devoted to the model results. Model outcomes resulting from alternative scenarios are discussed and compared with each other.

The study concludes with chapter 8. Here the main findings are briefly summarized and discussed in the light of the original objectives.

Chapter 2

THE CLOSED MODEL

2.1 PLAN OF THE CHAPTER

The Von Neumann model describes an economy in a state of balanced growth. In such an economy supply and demand of all goods increase at the same rate. The price system corresponding with balanced growth is such that all processes carried out yield the same profit rate or rate of return, while processes not carried out do not yield higher profits.

Except for some minor modifications, the discussion in this chapter will be limited to the Von Neumann model in its original form. We shall not bother too much about model characteristics that are, in a real world context, rather unrealistic. A discussion on the latter is postponed to chapter 3.

The plan of the chapter is as follows: we start with a more or less informal, mainly verbal discussion of the different model components. First the physical side will be discussed. Sections 2.2.1 and 2.2.2 introduce goods and processes, and growth respectively. Special assumptions on the technology matrices are discussed in section 2.2.3, while section 2.2.4 has the 'dual' side of the model as a topic, i.e. here prices, profits and the interest factor enter the scene.

The remainder of the chapter is devoted to a more formal discussion of the model. In section 2.3.1 the model is described by means of five axioms, each axiom being a mathematical formulation of an hypothesis about the state of the economic system described by the model. In section 2.3.2 an example illustrates the axioms. In 1956 Kemeny, Morgenstern and Thompson [39] reformulated the model in game-theoretic terms. By doing so, the original five axioms could be reduced to three. In section 2.3.3 we discuss this reformulation. The subject-matter of sections 2.4.1 - 2.4.4 is the existence of economic equilibria. It is proven here that the model has at least one and at most min (m,n) equilibrium solutions, where min (m,n) must be read as the minimum of the number of processes (m) and the number of goods (n) belonging to the economy. Because parts of the existence proof will also play a role in chapter 4 where a new algorithm is presented with which the model can be solved, the discussion is rather detailed.

2.2 BASIC ELEMENTS OF THE MODEL

2.2.1 Goods and processes

Goods and processes are the central elements in a Von Neumann model. Goods are of two different kinds: first, there are unproduced goods or raw materials. For example, iron ore deposit, uncultivated land, game, rain, etc. Second, there are produced goods as for example paddy, cultivated land, fertilizer, etc. Several kinds of services also belong to this category. Only produced goods enter the model; unproduced goods (raw materials) are left out. They are assumed to be non-restrictive in the production process and are therefore considered to be free. When some organized effort takes place to make a raw material productive, it is no longer free. Thus crude oil is in the Von Neumann model a free good as long as it stays in the earth. However, as soon as it has been pumped up it is not free anymore but it has instead become a produced good. The economic relevance of this assumption and its implication for modeling a real world situation will be discussed in section 3.5.1. We define a process as a connected series of actions to be taken to transform inputs into outputs. For example, we can call the transformation of land services, draught power, manure, human labour and seed into paddy and straw, a process. Because every process can be broken down into a number of other processes at a lower level of aggregation, the definition has an element of undeterminacy. Therefore, as in any modeling exercise, at the outset of a real world application, one has to start with a definition of what one considers a process. It is not necessary to make any assumptions on either the number of (produced) goods (n) or the number of processes (m), except that both n and m are finite and fixed integers. In a real world situation

$m>n$ will usually apply, i.e. the number of available processes will exceed the number of goods because most goods can be produced by several alternative production processes. A process i is represented by two vectors: an input vector $A^i(=a_{i1}, a_{i2}, ..., a_{in})$ and an output vector $B^i(=b_{i1}, b_{i2}, .., b_{in})$. The input coefficient a_{ij} indicates the amount of good j which is used up in process i if the process operates at unit intensity. Similarly b_{ij} denotes the output of good j from process i if the process operates at unit intensity. Naturally, both a_{ij} and b_{ij} are real non-negative numbers. Because more than one element of vector B^i may be positive, the model can handle joint production, i.e. multiple outputs may occur. Joint production allows to treat capital goods at different stages of wear and tear as qualitatively different goods instead of as one, single, slowly evaporating good. Thus the process of spinning in a Von Neumann world needs raw cotton and a spinning wheel of age t as inputs and yields yarn and a spinning wheel of age t+1 as outputs. In theoretical work this property has been heavily exploited because it permits to get around the aggregation aspect of the capital problem of the neo-classical growth models[2].

Von Neumann considers labour needed in the production processes as a raw material. As a consequence it stays out of the model. The necessities of life are however taken into account. They are treated as part of the input vector ([70], p. 240).

The model assumes constant returns to scale. If all elements of the input vector A^i are multiplied by a scale factor α, all elements of the output vector B^i have to be multiplied by the same factor. Thus if we describe process i as

$$(a_{i1}, a_{i2}, ..., a_{in}) \rightarrow (b_{i1}, b_{i2}, ..., b_{in})$$

then the process described as

$$(\alpha a_{i1}, \alpha a_{i2}, ..., \alpha a_{in}) \rightarrow (\alpha b_{i1}, \alpha b_{i2}, ..., \alpha b_{in})$$

---

[2] See Jones ([37], p. 127 - p.145) for a brief discussion of the neo-classical controversy.

belongs also to the technology of the economy. It should be clear that constant returns to scale are not incompatible with the principle of diminishing returns. The former refers to a situation where all inputs are increased by the same factor while in case of the latter, at least one input is kept constant. If the principle of diminishing returns applies to a certain production process, different input-output combinations can enter the model as different processes. For example, suppose the relationship between milk production per animal (M/A) and feed intake per animal (F/A) obeys the formula:

$$M/A = \sqrt{F/A}$$

where $1 \leq F/A \leq 10$ is the relevant domain. In this example, which has constant returns to scale, one could consider the following processes as part of the technology:

	Input				Output		
	Cows	feed	milk		Cows	feed	milk
Process 1	(1 ,	1 ,	0)	→	(1 ,	0 ,	1.00)
" 2	(1 ,	4 ,	0)	→	(1 ,	0 ,	2.00)
" 3	(1 ,	7 ,	0)	→	(1 ,	0 ,	2.65)
" 4	(1 ,	10 ,	0)	→	(1 ,	0 ,	3.16)

Of course a different number of points could be considered as processes also.

Good j may be measured in any convenient unit, as long as the same goods are expressed in the same unit in all processes. The technology of the economy can now be summarized by an input matrix A and an output matrix B as follows:

$$A = \begin{bmatrix} a_{11} & \cdots\cdots & a_{1n} \\ a_{i1} & \cdot\cdot\; a_{ij} \;\cdot\cdot & a_{in} \\ a_{m1} & \cdots\cdots & a_{mn} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & \cdots\cdots & b_{1n} \\ b_{i1} & \cdot\cdot\; b_{ij} \;\cdot\cdot & b_{in} \\ b_{m1} & \cdots\cdots & b_{mn} \end{bmatrix}$$

Both matrices A and B are of size mxn, and one can interpret them as the sum total of the technical and organizational knowledge available to a country, a firm, or, more generally, an economy. Contrary to the convention in linear programming, processes (activities)[3] are denoted by rows and goods by columns.

The operation of the economy is expressed by means of an intensity vector $x=(x_1, x_2, ..., x_m)$, where x_i is a real non-negative number indicating the level upon which process i operates. For the time being and because the constant returns to scale assumption applies, only the relative levels of activity are considered and hence we may normalize x so that $\Sigma x_i = 1$. Vector x is a row vector. A special case occurs if m = n and matrix B is the identity matrix, i.e. the matrix for which all elements along the principal diagonal are unity and all other elements are zero. In that case the model has a Leontief input-output structure and shows some similarity with the models of Marx and Sraffa[4].

2.2.2 Growth

The time period of the model can be of an arbitrary length. It must however be the same for all processes of the economy. Because of the relative freedom in choosing the level of aggregation of a process, this is not a very restrictive requirement. In modeling a real world economy the time period will usually be a year. If we assume that we start with a bundle of goods at the beginning of period t and that we end with another bundle at the end of period t, the physical production change can be represented, given the above defined symbols, as follows:

$$\begin{array}{ccc} xA & \longrightarrow & x\,B \\ \text{beginning of period} & & \text{end of period} \end{array}$$

---

[3] The terms activities and processes will be used as synonyms.

[4] See Abraham Frois and Berrebi [1], Morishima [64] and Schefold [83] for a discussion on the similarities and differences between the models of Marx, Sraffa and Von Neumann; Marx and Von Neumann; and Sraffa and Von Neumann, respectively.

In this expression xA is a row vector, showing the total physical amounts of goods j ($j = 1,2,\ldots,n$) that are needed as inputs if the economy operates according to intensities x. For example:

$$\sum_{i=1}^{m} x_i a_{ij}$$

is the total amount of good j used up in the different processes i. Likewise xB is a row vector showing the total amounts of goods j produced by the m activities of the economy. For good j total output amounts to

$$\sum_{i=1}^{m} x_i b_{ij}$$

If we divide the first component of xB by the first component of xA, the second component of xB by the second component of xA, etc., until the n-th component, we get a set of ratios α_j($j = 1,2,\ldots,n$) which can be considered as the relative expansion or growth factors of goods j during the period. It will be clear that the value of α_j is, given matrices A and B, a function of vector x. Thus:

$$\alpha_j(x) = \frac{\Sigma_i\ x_i\ b_{ij}}{\Sigma_i\ x_i\ a_{ij}}$$

Because both xB and xA are expressed in physical quantities, we can look upon α_j as the technical growth factor (or as one + the technological growth rate) of good j.

Depending on its value, α_j will fall into one of the following four categories:

(a) α_j is undefined
This is the case if both $\Sigma_i\ x_i\ b_{ij} = 0$ and $\Sigma_i\ x_i\ a_{ij} = 0$ which means that good j is not produced and not used up by any process of the economy.

(b) $\alpha_j = \infty$
This will happen if $\Sigma_i\ x_i\ b_{ij} > 0$ and $\Sigma_i\ x_i\ a_{ij} = 0$ which means that when the economy is in operation, good j is produced by at least one process but is not used up by any process of the economy.

(c) $\alpha_j = 0$

The reverse of (b) namely $\Sigma_i\ x_i\ b_{ij} = 0$ and $\Sigma_i\ x_i\ a_{ij} > 0$. Good j is not produced but is used up by at least one process. Such an economy is of course not viable in the long run.

(d) $0 < \alpha_j < \infty$

As we will see the most interesting case. Here both $\Sigma_i\ x_i\ b_{ij} > 0$ and $\Sigma_i\ x_i\ a_{ij} > 0$.

2.2.3 Assumptions on technology matrices A and B

An intensity vector x which yields a maximum for α_1 need not necessarily do so for α_2. An example will elucidate this. Consider an economy consisting of two goods and two processes for which the input (A) and output (B) matrices look as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 \\ 1 & 2.5 \end{bmatrix}$$

The growth factor for good 1 is:

$$\alpha_1 = \frac{4x_1 + x_2}{x_1 + x_2}$$

It can easily be checked that α_1 is maximum at x = (1,0). The corresponding growth factor for good 2 at these intensities is 1, i.e. good 2 does not grow at all: the output just equals the input. In case no extra stock of good 2 exists, the growth rate of good 1 can only be sustained for one period. Of course, this is a direct consequence of the fact that in our example both goods are needed as input for the production of each other. However, also more subtle examples are conceivable where goods are needed as inputs for the production of each other in an indirect way. Thus, if good a is needed for the production of b, while b is required for the production of c and c serves as input for d, the growth of a can limit the growth of d. In most economic models these interdependencies are ignored by aggregating over the goods. To what consequences this can lead can nicely be shown with the example. If we aggregate good 1 and good 2 by adding them up, the economy can grow at a maximum rate of 150 per cent [= (the growth factor - 1) x 100], while, as one can verify, the maximum growth rate sustainable in the long term according to the disaggregated form is only 100 per cent.

A Von Neumann model can in principle handle these problems adequately because:

(1) goods and processes enter the model as they 'physically appear' in the real world; and

(2) in analyzing the growth factor all goods and processes are taken into account simultaneously.

To facilitate the analysis, Von Neumann ([70], p. 243) and later on Kemeny, Morgenstern and Thompson[5] ([39], p.118) made some particular assumptions with regard to these dependencies which we will discuss successively. Von Neumann made the assumption that every process either uses or produces a positive amount of each good of the economy. Or, in symbols:

$$a_{ij} + b_{ij} > 0 \quad \text{for all } i \text{ and } j \tag{2.1}$$

From a purely technical point of view this is a very unrealistic assumption. In a statistical sense it is not: as long as numbers a_{ij} or b_{ij} that have to be added are small enough, they can be interpreted as observation errors. Von Neumann needed the assumption to prevent that the economy 'might break up into disconnected parts' ([70], p. 243). And, as he says, 'since the a_{ij}, b_{ij} may be arbitrarily small this restriction (i.e. assumption 2.1) is not very far-reaching. . . .' ([70], p. 243). Of course, there is nothing against making use of an unrealistic assumption in case its implications are not very far-reaching. However, as we will show by an example, Von Neumann's assumption can have important consequences. Consider an economy with the following technology matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

As can easily be seen, the maximum growth factors for the two goods are 4 and 1 respectively. Because $a_{12} + b_{12} = 0$ and $a_{21} + b_{21} = 0$, the economy

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[5] Abbreviated as KMT.

does not satisfy assumption 2.1 (or Von Neumann's assumption on technology). Let us therefore add an amount ε, $0 < \varepsilon << 1$, to the two zeros of matrix B so that Von Neumann's assumption is satisfied. The technology of the economy is now as follows:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & \varepsilon \\ \varepsilon & 1 \end{bmatrix}$$

The adjusted economy has only one (maximum) growth factor which is an increasing function of ε and goes assymptotically to 4 as ε falls to zero[6]. Thus, because $\varepsilon > 0$, the 'new' growth factor is always greater than 4. We could also have satisfied the Von Neumann assumption on technology in another way, that is, by adding small positive amounts to the zero elements of matrix A. In that case the technology matrices would be changed into

$$A = \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

Now the maximum growth factor is a decreasing function of ε. If ε falls to zero, the growth factor rises asymptotically to 1 but remains, for any positive value of ε, smaller than 1. Thus, some arbitrarily small changes in the technology matrices result in a drastic decrease of the growth factor: from more than 4 to less than 1. One could object that the example is small and artificial. Still there is, on a priori grounds, little reason to assume that a straightforward application of the Von Neumann assumption on technology on real world data will have less dramatic consequences for the growth factor. By introducing trade, so that (small) quantities of goods can be imported and/or exported, and initial endowments the model becomes less sensitive to variations in coefficient values near zero (see chapter 3 and the model results in chapter 7).

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[6] Theorem 2.4 (section 2.4.3) proves the existence of at least one maximum growth factor. Theorem 4.1 proves that for an economy satisfying assumption 2.1 at most one maximum growth factor exists. In Appendix A a method is given for finding the growth factor of 2 x 2 economies.

In their article on the generalization of the Von Neumann model, Kemeny, Morgenstern and Thompson [39] replaced assumption 2.1 by the intuitively more plausible conditions:

$$a_{ij} > 0 \quad \text{for at least one } j \text{ per } i \tag{2.2}$$

$$b_{ij} > 0 \quad \text{for at least one } i \text{ per } j \tag{2.3}$$

Assumption 2.3 states that every good (commodity) can be produced by at least one process. Its plausibility follows from the fact that in the Von Neumann model only produced goods appear. So, if the output of a commodity would be zero in all activities, the good would in a closed system disappear after one period of production and this part of the system would not be sustainable. Turning next to assumption 2.2 which says that every process consumes some (produced!) inputs. Morgenstern and Thompson motivate the plausibility of this assumption by asserting that 'there is no known physical process that produces outputs without any physical inputs' ([69], p. 28). However, because raw materials in their original state do not enter the technology matrices A and B, $a_{ij} = 0$ for all j for some i, does not necessarily mean that something is produced from nothing but only that something is produced without using any produced goods. This is quite another matter. Nevertheless, we consider assumption 2.2 plausible and economically meaningful. Its economic significance follows from the fact that a good 'produced' from raw materials only[7], can also be considered a raw material and thus can be left out of the technology matrices. Assumption 2.2 is therefore by no means an assumption on technology following from a basic physical principle but it is a criterion for distinguishing purely physical and biological processes from economic processes. This point is best illustrated by an example.

Paddy growing on a farm needs produced goods as inputs, for example seed, fertilizer, irrigation, labour (which is produced through food intake), etc.

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[7] For a correct understanding of the argument it must be kept in mind that if we are speaking here and in the following of 'raw materials', goods are meant that have been unaffected by purposeful human intervention. Thus, rain is a raw material while irrigation water is a produced good. According to this definition '. . . a good produced from raw materials only' is properly speaking a contradiction in terms.

So the input vector has some elements $a_{ij} > 0$. Therefore, although the process has physical and biological aspects, it is also an economic process. The growth of paddy under natural conditions on the other hand does not need any produced goods as inputs. Thus $a_{ij} = 0$ for all j and therefore the process can be considered as being purely biologically and not economic[8].

2.2.4 Prices, profits and the interest rate

So far, the discussion has been limited to the physical side of the economy. Von Neumann introduced, in a 'dual' way, also a price system which will be outlined now. Each good j is assigned a price y_j, where y_j is a real non-negative number. The set of all prices is indicated by a (column) vector $y = (y_1, y_2, \ldots, y_n)'$. Prices serve as a unit of account only and not as a store of value. Therefore only relative prices are of interest and we may proceed from a normalized price vector y satisfying $\Sigma_j y_j = 1$. Symbolically, value changes during one time period can now be described as follows:

$$\underset{\text{beginning of period}}{Ay} \xrightarrow{\qquad} \underset{\text{end of period}}{By}$$

In this scheme Ay is a column vector showing for each process the total value of goods j (j=1,2,...,n) which are needed as inputs to run the process at unit intensity. For example, element i of Ay can be calculated as:

$$\sum_{j=1}^{n} a_{ij}y_j$$

which is the total value of the inputs needed to run process i at unit intensity. Likewise By is a column vector, element i showing the total value of the output of process i, if run at unit intensity. Thus:

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[8] The gathering of paddy grown under natural conditions is of course again an economic process.

$$\sum_{j=1}^{n} b_{ij}y_j$$

is the output value of process i if $x_i = 1$.

If we divide each component of By by its corresponding component of Ay, the resulting set of ratios β_i $(i=1,2,\ldots,m)$ can be considered as the relative profitability factors of processes i. For example, the profitability factor of the first process equals

$$\beta_1(y) = \frac{\Sigma_j b_{1j}y_j}{\Sigma_j a_{1j}y_j}$$

Related to the profitability or interest factor is the profit or interest rate which is equal to the profitability or interest factor minus one, that is $(\beta-1)$. y_j, a_{ij} and b_{ij} are real non-negative numbers. Therefore, as for α_j, we can distinguish four possible cases regarding the value of β_i.

(a) β_i is undefined

This occurs if both $\Sigma_j b_{ij}y_j = 0$ and $\Sigma_j a_{ij}y_j = 0$. That is, if

(i) nothing of value is produced; and

(ii) nothing of value is used up in process i[9].

An uninteresting case from an economic point of view: all goods involved in the process are free, thus no economic problem exist.

(b) $\beta_i = \infty$

This will happen if $\Sigma_j b_{ij}y_j > 0$ and $\Sigma_j a_{ij}y_j = 0$. Or, process i produces something of value but does not need anything valuable as input.

(c) $\beta_i = 0$

The reverse of (b), i.e. $\Sigma_j b_{ij}y_j = 0$ and $\Sigma_j a_{ij}y_j > 0$. Nothing of value is produced while something of value is needed as input.

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[9] When it is stated that nothing of value is used or produced, it does not necessarily mean that nothing is used up or produced at all. For it is also possible that a process uses or produces only goods which are free, i.e. goods which have a zero price in the economy.

(d) $0 < \beta_i < \infty$

The most interesting case if we study a real world situation. Here both $\Sigma_j b_{ij} y_i > 0$ and $\Sigma_j a_{ij} y_j > 0$. Something of value is used up and produced by process i.

2.3 THE CLOSED MODEL: AN AXIOMATIC DESCRIPTION

2.3.1 Axioms of the closed model

After providing the building blocks, we will proceed to an axiomatic description of the model. An axiom must be understood here as a mathematical formulation of an hypothesis about the structure of the economy. Taken together, the axioms form the model. The Von Neumann model consists of five axioms. Together they describe an economy in equilibrium, i.e. they describe the whole economic system in a state where physical balances are consistent with laws of nature and where all non-zero processes yield the same rate of profit. Although the equilibrium state applies to all periods, it is not a stationary state: the system can expand at a uniform rate. Following Champernowne ([16],p. 11) one could call such a state a quasi-stationary state, although Von Neumann did not use this term. In such a quasi-stationary state the structure of the economic system does not change over time, i.e. the intensity vector x, the price vector y, the expansion factor α and the interest factor β are the same for all periods. Only the scale of the economy changes. Therefore, to study the model one can confine oneself to one period. And because it will usually be clear from the context which period is meant, we shall only use the time subscript when leaving it out may lead to confusion. Let us now state and briefly discuss the axioms:[10]

Axiom 1: $xB \geqq \alpha xA$

According to this axiom the outputs of one period (xB) must equal or exceed the inputs of the succeeding period. The axiom says something about the growth, the origin and the destination of the goods.

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[10] The axioms as they are presented here, are taken from Morgenstern and Thompson ([69], p. 23-24).

i) Growth

The growth of the physical production through time can be described by the following scheme:

$$\underset{t}{\underline{xA \longrightarrow xB}} \geq \underset{t+1}{\underline{\alpha xA \longrightarrow \alpha xB}} \geq \underset{t+2}{\underline{\alpha^2 xA \longrightarrow \alpha^2 xB}} \geq \underset{t+3}{\underline{\alpha^3 xA \longrightarrow \alpha^3 xB}} \geq \ldots\text{etc.}$$

Because the intensity vector x is normalized ($\Sigma_i x_i = 1$), the elements of vector xA and xB are normalized quantities of goods. Therefore if we consider the inequalities separately, each side may be divided by α (or α^t) which means that axiom 1 serves for all periods. According to the axiom all goods increase by at least (α-1) x 100 per cent per period. It follows that the slowest growing good(s) determine(s) the maximum feasible growth rate of the economy. As far as all goods are needed in the production of each other, this is a realistic assumption. If the coherence among goods is not so rigorous one can doubt the reality content of the assumption. However, as we will see below (section 2.4.4) in that case different blocks (sets of processes), each having its own growth factor, can be distinguished in the economy.

ii) Origin

Goods used as inputs in period t+1 have to be taken from the outputs of period t. This means that no goods from the rest of the world can enter the economy, i.e. the model is closed.

iii) Destination

An output good is either used as input in next period or disposed of. This choice between alternatives implies the introduction of an actor who can make the decision to dispose of goods.

Axiom 2: $x(B - \alpha A)y = 0$

In words: the value of the output of period t minus the value of the input in period t+1 is zero. In essence two requirements regarding the price structure of the model are stated by this axiom:

i) Because x, B, α, A and $y \geqq 0$, goods for which in axiom 1 the strict inequality holds have a zero-price. Thus axioms 1 and 2 imply free disposal.

ii) Only goods that are not over-produced can (but need not) have a positive price. Since $\Sigma_j y_j = 1$ at least one produced good must have a positive price. It also follows that when the economy is in equilibrium not all goods can be in surplus.

Axiom 3: $By \leqq \beta Ay$

According to this axiom the output value of each process is less than or equal to the capitalized input value. Axiom 3 makes the economy profitless at an interest rate $(\beta-1)$. Because $(\beta-1)$ can be interpreted as the profit rate, one could also say that in equilibrium no super-profits can be earned. In symmetry with the growth of the physical production, the value changes which take place through time can be described by the following scheme:

$$\underset{t}{\underline{Ay \longrightarrow}} By \leqq \underset{t+1}{\underline{\beta Ay \longrightarrow}} \beta By \leqq \underset{t+2}{\underline{\beta^2 Ay \longrightarrow}} \beta^2 By \leqq \underset{t+3}{\underline{\beta^3 Ay \longrightarrow}} \beta^3 By \leqq \ldots \text{etc.}$$

Because y is normalized, the elements of Ay and By are normalized values. Therefore, if we consider the inequalities separately, each side may be divided by β (or β^t) which leads for each period to axiom 3.

Axiom 4: $x(B - \beta A)y = 0$

This axiom can best be understood in connection with axiom 3. If in axiom 3 for process i the strict inequality applies, it can only be carried out with a loss. Axiom 4 says that in that case the intensity of the process will be zero. The plausibility of this is easily understood. Processes which make a loss can, if no subsidies are paid, only exist a short time, i.e. only when the economy is out of equilibrium. Axiom 4 also says that in an equilibrium state only processes that do not make a loss can (but not necessarily need) have a positive intensity.

Axiom 5: $xBy > 0$

Or: something of value must be produced. The economic meaning of this axiom is trivial. In the original formulation of the model [70] Von Neumann did not need axiom 5. As we will see in chapter 4, under the assumption of a Von Neumann technology[11], solutions to the model under axioms 1-4 are unique and will, except for some extreme cases[12], always satisfy axiom 5. However, if the, from an economic point of view, more realistic KMT-assumptions on technology apply[13], solutions to the model under axioms 1-4 need not necessarily be unique. Moreover, if more than one solution exists only a few are economically meaningful in the sense that axiom 5 is satisfied. In section 2.4.4 we will come back to this point.

This leads us to the following definition of an equilibrium solution:

> An equilibrium solution of an economy, represented by technology matrices A and B, can be defined as a quadruple $Q = \{x, y, \alpha, \beta \mid x \geq 0, y \geq 0, \alpha \geq 0, \beta \geq 0, \Sigma x_i = 1 \text{ and } \Sigma y_i = 1\}$ satisfying axioms 1-5 (KMT-technology) or axioms 1-4 (Von Neumann technology).

If not explicitly stated we shall assume in the following that the KMT-assumptions on technology apply.

2.3.2 An example

Consider a simplified economy consisting of two processes and two goods. Product 1 is a capital good and product 2 is a consumption good. In process 1 capital goods are produced: one unit of the capital good and four units of the consumption good are needed as input to produce three units of the capital good as output. In process 2 consumption goods are produced: three units of the capital good and one unit of the consumption good yield two units of the capital good and four units of the consumption good. So the input matrix A and the output matrix B look as follows:

---

[11] Recall that in a Von Neumann technology $a_{ij} + b_{ij} > 0$ for all i and j.

[12] For example, if B=0, then xBy = 0.

[13] Recall that in a KMT-technology:
- $a_{ij} > 0$ for at least one j per i, and
- $b_{ij} > 0$ for at least one i per j.

$$\begin{array}{lcc} & \text{A} & \text{B} \\ & \begin{array}{cc}\text{Capital good} & \text{Consumption good}\end{array} & \begin{array}{cc}\text{Capital good} & \text{Consumption good}\end{array} \\ \begin{array}{l}\text{Process 1}\\ \text{Process 2}\end{array} & \begin{bmatrix}1 & 4\\ 3 & 1\end{bmatrix} & \begin{bmatrix}3 & 0\\ 2 & 4\end{bmatrix}\end{array}$$

As can easily be verified, matrices A and B satisfy both the Von Neumann and the KMT-assumptions on technology. Observe that in process 2 joint production takes place. Solving the model consists of finding vectors x and y and scalars α and β that obey axioms 1-5. Because $n = m = 2$, both vectors x and y have two elements and because x as well as y is normalized, one can express x_2 and y_2 as $(1-x_1)$ and $(1-y_1)$, respectively.

Starting now with axiom 1 we get:

$$3x_1 + 2(1-x_1) \geq \alpha[x_1 + 3(1-x_1)] \tag{2.4}$$

and

$$4(1-x_1) \geq \alpha[4x_1 + (1-x_1)] \tag{2.5}$$

Rearranging (2.4) and (2.5) yields:

$$\alpha \leq \frac{2 + x_1}{3 - 2x_1} \tag{2.6}$$

and

$$\alpha \leq \frac{4-4x_1}{3x_1 + 1} \tag{2.7}$$

Figure 2.1 shows the feasable area for α. The shaded area contains all technological expansion factors. The maximum feasible α can be calculated as 1.091 which corresponds with $x_1 = .4$ and $x_2 = .6$.

Then considering axiom 3, we can write:

$$3y_1 \leq \beta[y_1 + 4(1-y_1)] \tag{2.8}$$

and

$$2y_1 + 4(1-y_1) \leq \beta[3y_1 + (1-y_1)] \tag{2.9}$$

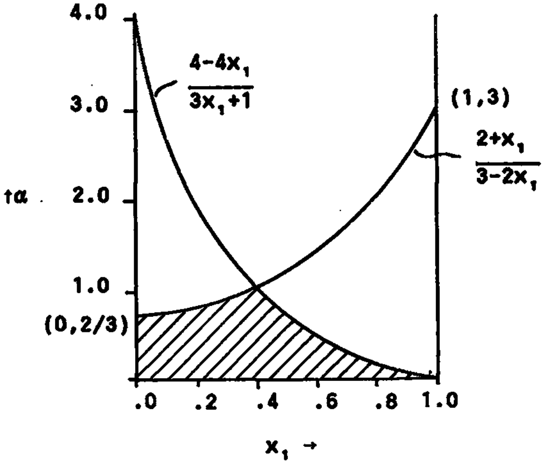

Figure 2.1

Relation between growth factor and process intensities ($x_2 = 1-x_1$)

Rearranging (2.8) and (2.9) results in:

$$\beta \geq \frac{3y_1}{4-3y_1} \tag{2.10}$$

and

$$\beta \geq \frac{4-2y_1}{1+2y_1} \tag{2.11}$$

In figure 2.2 both (2.10) and (2.11) are shown. The shaded area contains all β's satisfying axiom 3. The minimum interest factor appears to be as high as the maximum growth factor[14], namely 1.091. The prices y_1 and y_2 belonging to this minimum are .6957 and .3043 respectively. If we substitute α_{max}, β_{min} and the corresponding price and intensity vector into the remaining axioms (2, 4 and 5) this leads to:

Axiom 2: $x(B - \alpha A)y = 0 \quad \rightarrow$

$$(.4 \ \ .6) \left[\begin{pmatrix} 3 & 0 \\ 2 & 4 \end{pmatrix} - 1.091 \begin{pmatrix} 1 & 4 \\ 3 & 1 \end{pmatrix} \right] \begin{pmatrix} .6957 \\ .3043 \end{pmatrix} = 0$$

---

[14] In section 2.3.3 we shall show that this is no coincidence.

Axiom 4: $x(B - \beta A)y = 0$

Because $\beta = \alpha$, it follows from axiom 2 that axiom 4 is also satisfied.

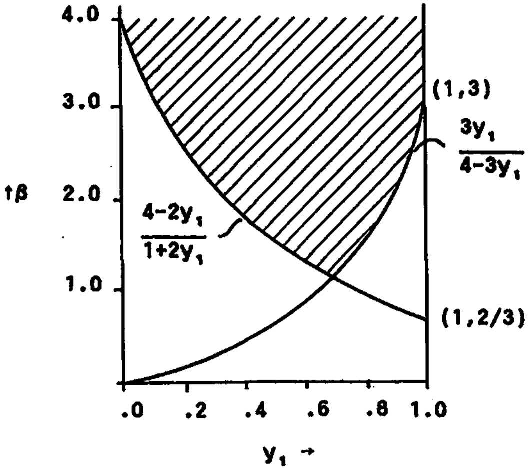

Figure 2.2

Relation between interest factor and prices ($y_2 = 1-y_1$)

Axiom 5: $xBy > 0$

$$(.4\ .6)\begin{bmatrix}3 & 0\\2 & 4\end{bmatrix}\begin{pmatrix}.6957\\.3043\end{pmatrix} = 2.4$$

Because all axioms are satisfied it can be concluded that an equilibrium solution has been found. The equilibrium growth rate of the economy is 9.1 per cent which is at the same time the maximum proportional growth rate. The corresponding interest rate turns out to be also 9.1 per cent which is at the same time the minimum interest rate at which no price system is conceivable by which both processes can earn a super-profit, i.e. a profit exceeding the interest rate (see figure 2.2).

Suppose that the economy has initial endowments in the right proportions, that is in proportions $\bar{x} A^1 : \bar{x} A^2$ [15]. Say there are 1,000 units of the capital good and 1,000 units of the consumption good. At the end of period 1, it possesses 1,091 of each. After two production cycles the economy has grown to 1,190 units of the capital good and 1,190 units of the consumption good etc. The way along which an economy moves through time is called the growth path of the economy. The Von Neumann growth path is called the turnpike. From the above it follows that a growth path is properly speaking not a path but a series of discrete points (Koopmans [50], p. 361).

2.3.3 The model as a two-person zero-sum game

Von Neumann presented his model for the first time as early as 1932[16]. The model remained however practically unnoticed for many years, partly because of the unaccessibility of the mathematical arguments to many economists. A first and major step towards a better understanding of the model was taken by Kemeny, Morgenstern and Thompson in 1956. In their article 'A generalization of the Von Neumann model of an expanding economy' [39], the authors introduced, among other things, matrix game theory as a mathematical tool for analyzing the model. This tool appeared to be very fruitful. Especially in proving the existence of economic and non-economic solutions. Because of the close relationship between linear games and linear programming, it also opened the way to the construction of algorithms for finding solutions to the model. Therefore we shall, in this section, restate the model in game-theoretic terms. For easy reference, basic concepts and relevant theorems of matrix game theory have been summarized in appendix A.

Recall that an equilibrium solution of the Von Neumann model consists of a quadruple:

---

[15] In section 3.4 we discuss the situation where initial endowments are not in the right proportions.

[16] See footnote 1.

$$Q = \{x, y, \alpha, \beta \mid x \geqq 0, y \geqq 0, \alpha \geqq 0, \beta \geqq 0, \Sigma x_i = 1 \text{ and } \Sigma y_j = 1\}$$

satisfying the following five axioms:

$$xB \geqq \alpha xA \tag{2.12}$$

$$x(B - \alpha A)y = 0 \tag{2.13}$$

$$By \leqq \beta Ay \tag{2.14}$$

$$x(B - \beta A)y = 0 \tag{2.15}$$

$$xBy > 0 \tag{2.16}$$

For restating the problem in game-theoretic terms, we need the following theorem which is due to KMT ([39], p. 119).

Theorem 2.1

If (x, y, α, β) is a solution to (2.12 - 2.16), then $\alpha = \beta = xBy/xAy$ applies.

Proof:

From (2.13) and (2.15) it follows that $xBy = \alpha xAy = \beta xAy$. Because $xBy > 0$ (2.16), and both $\alpha \geqq 0$ and $\beta \geqq 0$, xAy is also > 0. Hence $\alpha = \beta = xBy/xAy$. □

The economic meaning of this theorem is that, in equilibrium, the expansion factor equals the interest factor. Theorem 2.1 allows us to substitute α for β in (2.14) and (2.15). Having done this, (2.13) and (2.15) become identical. If we further postmultiply both sides of inequality (2.12) by y and we premultiply both sides of inequality (2.14) by x, we have

$$xBy \geqq \alpha xAy \tag{2.17}$$

and

$$xBy \leqq \alpha xAy \tag{2.18}$$

It follows from (2.17) and (2.18) that xBy must be equal to αxAy. And thus, if (2.12) and (2.14) are satisfied, axioms (2.13) and (2.15) are also satisfied. This leaves us with only three of the original five axioms, namely:

$$xB \geq \alpha xA$$ (2.19

$$By \leq \alpha Ay$$ (2.20

and

$$xBy > 0$$ (2.21

Next we define $M_\alpha = (B - \alpha A)$; (2.19) - (2.21) can then be rewritten as:

$$xM_\alpha \geq 0$$ (2.22

$$M_\alpha y \leq 0$$ (2.23

$$xBy > 0$$ (2.24

From theorem 2.1 it follows that (2.22) and (2.23) are equivalent to (2.12) and (2.14). If we consider M_α as the pay-off matrix of the fair game[17] M_α, we know from appendix A that optimal x- and y- strategies satisfy the inequalities (2.22) and (2.23). If we further refer to $v(M_\alpha)$ as the value of the game M_α which is equal to $\tilde{x}M_\alpha\tilde{y}$, were $\tilde{x}$ and $\tilde{y}$ are optimal strategies (see appendix A), the problem of finding a solution of the Von Neumann model can be expressed in game-theoretic terms as follows ([39], p. 120):

Given non-negative mxn matrices A and B such that the value of $-A$ $(=v(-A))$, is negative and the value of $B(=v(B))$ is positive[18]; set $M_\alpha = (B-\alpha A)$ and find α so that $v(M_\alpha) = 0$; then find a pair of x- and y- strategies such that $xBy > 0$ and such that x is optimal for the maximizing player and y is optimal for the minimizing player.

An α for which $v(M_\alpha) = 0$ is called an allowable α. If $(\hat{x},\hat{y})$ is a pair of optimal strategies for M_α such that $\hat{x}B\hat{y} > 0$, we call it an economic solution to the game M_α. If $(\bar{x},\bar{y})$ is a pair of optimal strategies for M_α such that $\bar{x}B\bar{y} = 0$, then we call $(\bar{x},\bar{y})$ non-economic solutions to the game M_α. In principle, we are only interested in economic solutions.

---

[17] If the value of a game is zero, one speaks of a fair game (see appendix A).

[18] $v(-A) < 0$ and $v(B) > 0$ follow from the KMT-conditions on technology. For a proof, see theorem 2.2.

To illustrate the game-theoretic interpretation we shall recalculate the example of section (2.3.2):

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 2 & 4 \end{bmatrix}$$

$$\rightarrow \quad M_\alpha = \begin{bmatrix} 3-\alpha & -4\alpha \\ 2-3\alpha & 4-\alpha \end{bmatrix} \tag{2.25}$$

Applying formula A-16 (appendix A) yields:

$$-11\alpha^2 + \alpha + 12 = 0$$

with solutions $\alpha_1 = -1$ and $\alpha_2 = 1.091$. It is clear that $\alpha_1 = -1$ has no economic meaning; the only relevant solution is $\alpha_2 = 1.091$. Substituting this value in (2.25) yields:

$$M = \begin{bmatrix} 1.909 & -4.364 \\ -1.273 & 2.909 \end{bmatrix}$$

The optimal x- and y- strategies can now be calculated by means of formulas A-12 and A-14 (appendix A) which results in $(x_1, x_2) = (.4, .6)$ and $(y_1, y_2) = (.6957, .3043)$. It can be verified that these strategies are in accordance to the solutions found in section 2.3.2.

2.4 THE EXISTENCE OF ECONOMIC EQUILIBRIA

2.4.1 Introduction

For an analytical model of an economic system to be of relevance to real world situations, at least two requirements must be met:

(1) The model must be an acceptable abstraction from reality; and

(2) The model must be free from internal contradictions.

Ad (1): It is difficult to judge objectively how far the first requirement is met. Reality is complex and in many respects so little transparant that an economic model can only reflect some principal traits of it. To what extent the Von Neumann model has caught these principal traits must be judged in the light of existing economic theory on which the individual axioms (hypotheses) are based. As far as economic theory is not unequivocal in its judgement, acceptance of the model is also determined by personal taste. A discussion on the extent to which we consider the Von Neumann model an acceptable framework to analyze reality will be postponed until chapter 3.

Ad (2): The second requirement follows naturally from the first. If a model is not free of internal contradictions it cannot be an acceptable abstraction from reality. Why then make a distinction between (1) and (2)? One could say that the first requirement refers to the individual axioms while the second requirement is primarily concerned with the consistency of the axioms in relation to each other. For example, if we would add an axiom according to which overproduced goods have to be stored, the free disposal assumption (axiom 3) would be contradicted. As a consequence the model should have to be rejected, despite the acceptability of the individual axioms. Whether a model contains any contradictions, can, amongst others, be found out by an existence analysis. If the model possesses a solution, the abstract economy described by the model can exist in a logical sense. In case of the Von Neumann model this means that an economy can have maximum balanced growth and at the same time a price system which is consistent with profit maximizing behaviour.

The importance of an existence proof will be clear by now. In the course of the years several authors have given existence proofs for the Von Neumann model. The original proof by Von Neumann is quite complicated, it depends on a generalization of Brouwer's fixed point theorem. More elementary proofs were given by Loomis in 1946 [51] and Georgescu-Roegen in 1951 [27]. However, all these proofs contain the Von Neumann assumption on technology which is from an economic point of view unattractive. In 1956 Kemeny [38] and Thompson [97] presented proofs in which this bothersome assumption was relaxed. Instead the much more acceptable KMT-assumptions on technology were introduced. In Thompson's proof extensive use is made of game theory. Later on a number of non-game-theoretic proofs appeared, all under the

KMT-assumptions on technology, viz. by Gale [24], Howe [25] and Moeschlin and Bol [60]. Although the proofs by Thompson, Gale, Howe and Moeschlin and Bol may seem to be quite distinct from each other, a close inspection reveals that they have much in common too. All start with proving the existence of an α for which

$$xM_\alpha \geqq 0 \tag{2.26}$$

and

$$M_\alpha y \leqq 0 \tag{2.27}$$

Then they show that the α-domain for which both (2.26) and (2.27) applies, is closed and bounded. And finally they prove that for α_{max} and/or α_{min} an x and y vector must exist for which:

$$xBy > 0 \tag{2.28}$$

The existence proof we are going to discuss will proceed along the same lines. It consists of two parts. First we shall discuss the function $v(M_\alpha)$ for $\alpha \geqq 0$. It will be shown that $v(M_\alpha)$ is a continuous non-increasing function of α. Because properties of $v(M_\alpha)$ play a crucial role in chapter 4 where a constructive algorithm is presented for solving the model, $v(M_\alpha)$ will be discussed in a rather detailed fashion. In the second part of the proof it will be shown that for some values of α, vectors x and y must exist for which (2.28) applies. Although our proof is heavily based on the aforesaid, it has a new element in it also, i.e. in proving that for at least one α satisfying (2.26) and (2.27), (2.28) applies, use is made of the strong theorem of the alternative.

2.4.2 The value of the game M_α

Theorem 2.2:

Given non-negative matrices A and B. Set $M_\alpha = B - \alpha A$. The value of the game M_α can be summarized as follows:

(a) $v(B) = v(M_0) > 0$;
(b) $v(-A) < 0$;
(c) $v(M_\alpha)$ is a continuous function of α;
(d) $v(M_\alpha)$ is non-increasing;
(e) $v(M_\alpha) = 0$ for some $\alpha > 0$; and
(f) $S = \{\alpha > 0 \mid v(M_\alpha) = 0\}$ is closed and bounded.

Proof:[19]

(a) $v(B) = v(M_0) > 0$.

Let $\hat{x}$ and $\hat{y}$ be optimal strategies to the game $B = M_0$. Let $\bar{x} = (1/m, 1/m, \ldots, 1/m)$. Since B has at least one strictly positive element in each column (each good can be produced; KMT-condition on technology), $\bar{x}B > 0$. The truth of (a) follows now from a result of elementary matrix game theory according to which $\hat{x}B \geqq \bar{x}B$ (appendix A). Thus also $v(B) = v(M_0) = \hat{x}B\hat{y} \geqq \bar{x}B\hat{y} > 0$. □

(b) $v(-A) < 0$.

Since A has at least one strictly positive entry in each row (each process needs some produced inputs; KMT-condition on technology), $(-A)\bar{y} < 0$ for $\bar{y} = (1/n, \ldots, 1/n)'$. Given $\hat{x}$ and $\hat{y}$ being optimal strategies to the game $v(-A)$, we may write $v(-A) = \hat{x}(-A)\hat{y} \leqq \hat{x}(-A)\bar{y} < 0$. □

(c) $v(M_\alpha)$ is a continuous function of α.

Let $\hat{x}$ and $\hat{y}$ solve $v(M_{\hat{\alpha}})$; similarly, let $\bar{x}$ and $\bar{y}$ solve $v(M_{\bar{\alpha}})$. Then we have (Morgenstern and Thompson [69], p. 232):

$$\begin{aligned} v(M_{\hat{\alpha}}) &= \hat{x}\, M_{\hat{\alpha}}\, \hat{y} \leqq \hat{x}\, M_{\hat{\alpha}}\, \bar{y} \\ &= \hat{x}\, M_{\bar{\alpha}}\, \bar{y} + (\bar{\alpha} - \hat{\alpha})\, \hat{x}A\bar{y} \\ &\leqq \bar{x}\, M_{\bar{\alpha}}\, \bar{y} + (\bar{\alpha} - \hat{\alpha})\, \hat{x}A\bar{y} \\ &\leqq v(M_{\bar{\alpha}}) + (\bar{\alpha} - \hat{\alpha})\, a \end{aligned}$$

where a is the largest element of A.

---

[19] In proving (a)-(f) we assume that matrices A and B satisfy the KMT-conditions on technology. The assumption is only necessary for proving steps (a), (b) and (e).

Hence,

$$v(M_{\hat{\alpha}}) - v(M_{\bar{\alpha}}) \leqq (\bar{\alpha} - \hat{\alpha})\, a \qquad (2.29)$$

In the same way it can be proven that

$$v(M_{\bar{\alpha}}) - v(M_{\hat{\alpha}}) \leqq (\hat{\alpha} - \bar{\alpha})\, a \qquad (2.30)$$

Because of (2.29) and (2.30) one may write:

$$|\, v(M_{\hat{\alpha}}) - v(M_{\bar{\alpha}}) \,| \leqq |\, \bar{\alpha} - \hat{\alpha} \,|\, a$$

from which the continuity of $v(M_{\alpha})$ follows. □

(d) $v(M_{\alpha})$ is non-increasing.

Let $\bar{\alpha} < \hat{\alpha}$ and $(\bar{x}, \bar{y})$ and $(\hat{x}, \hat{y})$ be optimal solutions to $M_{\bar{\alpha}}$, and $M_{\hat{\alpha}}$, respectively. Then

$$\hat{x}(M_{\bar{\alpha}} - M_{\hat{\alpha}})\, \bar{y} \geqq 0 \qquad (2.31)$$

This can be seen as follows: (2.31) is equivalent to:

$$\hat{x}((B - \bar{\alpha}A) - ((B - \bar{\alpha}A) - (\hat{\alpha} - \bar{\alpha})A))\, \bar{y} \geqq 0 \qquad (2.32)$$

Because $\hat{x}$, $(\hat{\alpha} - \bar{\alpha})$, A and $\bar{y} \geqq 0$, (2.32) and thus (2.31) also hold. Now the proof proceeds as follows (see Moeschlin [58], p. 15):

$$v(M_{\bar{\alpha}}) = \bar{x}M_{\bar{\alpha}}\bar{y} \geqq \hat{x}M_{\bar{\alpha}}\bar{y} = \hat{x}(M_{\bar{\alpha}} - M_{\hat{\alpha}})\,\bar{y} + \hat{x}M_{\hat{\alpha}}\bar{y} \geqq \hat{x}M_{\hat{\alpha}}\bar{y} \geqq \hat{x}M_{\hat{\alpha}}\hat{y} = v(M_{\hat{\alpha}})$$

Thus, also $v(M_{\bar{\alpha}}) \geqq v(M_{\hat{\alpha}})$ for all $\bar{\alpha} < \hat{\alpha}$. □

(e) $v(M_{\alpha}) = 0$ for some $\alpha > 0$.

Because $v(M_0) = v(B) > 0$ and $v(M_{\alpha})$ is continuous, it is according to Bolzano's theorem[20], sufficient to prove that there exists an $\alpha > 0$ for which $v(M_{\alpha}) < 0$. A proof of Moeschlin is used to demonstrate this ([58], p. 15):

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[20] The theorem of Bolzano says that if $f(x)$ is a continuous function of x on the interval $[a, b]$ and if $f(a) \times f(b) < 0$, then there exists an $x \in (a,b)$ such that $f(x) = 0$ ([81], p. 58).

Let $\hat{x}$ be an optimal solution to $M_{\hat{\alpha}}$, $\overline{y}$ be an (n x 1)-vector $\overline{y} = (1/n,1/n,\ldots 1/n)'$ and e be an (n x 1)-vector consisting of all ones. Then:

$$v(M_{\hat{\alpha}}) \leqq \hat{x}(B - \hat{\alpha}A)\overline{y} = \hat{x}B\overline{y} - \frac{\hat{\alpha}}{n}\hat{x}Ae \qquad (2.33)$$

The term $\hat{x}B\overline{y}$ is bounded from above by the largest element b_{ij} of B. Because A has at least one strictly positive element in each row j (KMT-condition on technology):

$$\hat{x}Ae \geqq \min_i A^i e > 0$$

Thus, by choosing a proper value for α, $v(M_{\hat{\alpha}})$ in (2.33) can be made as small as desirable. Because $\hat{x}B\overline{y}$, $\hat{x}Ae$ and $n > 0$, for negative values of (2.33), α must be strictly positive. □

(a) - (e) enable us to sketch a general picture of $v(M_\alpha)$. This is done in figure 2.3.

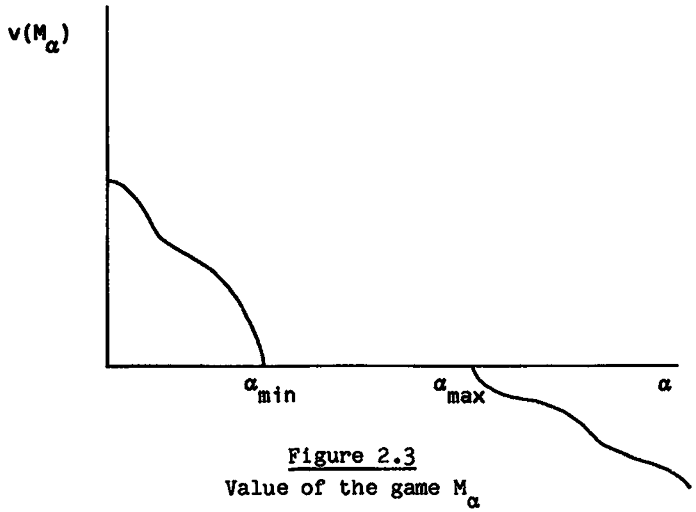

Figure 2.3
Value of the game M_α

(f) $S = \{\alpha > 0 \mid v(M_\alpha) = 0\}$ is closed and bounded.

Because $v(M_\alpha)$ is non-increasing and continuous and there exist non-negative values of α for which $v(M_\alpha) > 0$ and $v(M_\alpha) < 0$, the set $S = \{\alpha > 0 \mid v(M_\alpha) = 0\}$ is bounded by α_{min} and α_{max}. The continuity of M_α

implies that S is closed, which completes the proof of the last part of theorem 2.2. □

Turning back to the axioms of the model, theorem 2.2 states that axioms 1-4 can always be satisfied. To what extent axiom 5 can be met will now be investigated.

2.4.3 Solutions for which $xBy > 0$

Not all α's in the domain $\alpha_{min} \lesseqgtr \alpha \lesseqgtr \alpha_{max}$ are economically meaningful in the sense that pairs (x,y) exist, such that $xBy > 0$. A small example will illustrate this. Consider an economy with the following A and B matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 2 & 2 \end{bmatrix}$$

Given the formulas of appendix A, the game-values for different α's can be calculated (see figure 2.4).

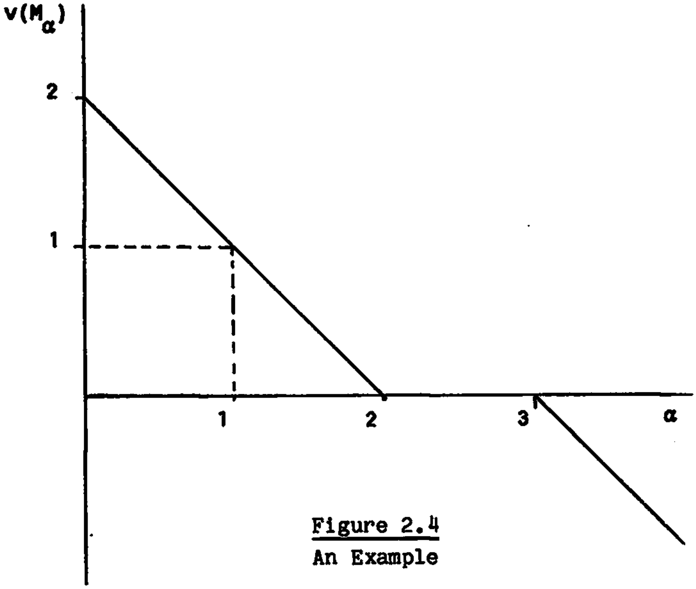

Figure 2.4
An Example

The corresponding (x,y)-strategies are[21]:

i) $\alpha = 2$ $\quad x = (\lambda, 1-\lambda) \quad (0 \leq \lambda \leq 1)$

$\quad y = (0,1)'$

ii) $2<\alpha<3$ $\quad x = (1,0)$

$\quad y = (0,1)'$

iii) $\alpha = 3$ $\quad x = (1,0)$

$\quad y = (1-\lambda, \lambda)' \quad (0 \leq \lambda \leq 1)$

All solutions satisfy (2.22) and (2.23), i.e. the first four axioms. It can easily be checked however that the fifth, i.e. $xBy > 0$ does not hold in all cases. If $\alpha = 2$, $xBy = 2 - 2\lambda$; if $2 < \alpha < 3$, $xBy = 0$; and if $\alpha = 3$, $xBy = 3 - 3\lambda$. Thus economic solutions only exist for $\alpha = 2$, $\lambda \neq 1$ and $\alpha = 3$, $\lambda \neq 1$.

Next we shall prove for the general case that always an α exists which in addition to the first four axioms satisfies $xBy > 0$. For this proof we need the strong theorem of the alternative. Because this theorem and the existence proof make use of the concept of a central solution, we first introduce this concept:

Let $\bar{X}$ and $\bar{Y}$ be the sets of all optimal x- and y-strategies. Then the extreme points of $\bar{X}$ and $\bar{Y}$ are called basic solutions (see Gaver and Thompson [25], p. 124). According to the fundamental theorem on matrix game theory (see appendix A), each matrix game has a finite number of basic solutions. Central solutions can now be defined as positive convex combinations of basic solutions. Or as:

$$x^c = \sum_i \lambda_i x^i, \quad \lambda_i > 0, \quad \sum_i \lambda_i = 1$$

and

$$y^c = \sum_j \gamma_j y^j, \quad \gamma_j > 0, \quad \sum_j \gamma_j = 1$$

where

x^c, y^c are central solutions, and

x^i, y^j are basic solutions, $(i=1,2,\ldots,k)$,

$(j = 1,2,\ldots,\ell)$.

---

[21] Because the x- and y-strategies result from solving a linear programming model, they need not necessarily be unique (see e.g. Simonnard [86]).

Thus the existence of central solutions follows directly from the existence of basic solutions. Now the strong theorem of the alternative can be stated as follows:

Theorem 2.3 (Strong theorem of the alternative)

Let M be an m x n matrix game with value v, and let (x^c, y^c) be any pair of central solutions; then the following statements are true:

(a) $x_i^c > 0$, if and only if $M^i y^c = v$

(b) $y_j^c > 0$, if and only if $x^c M^j = v$

Statement (a) is equivalent to

(a.1) $x_i^c = 0$, if and only if $M^i y^c < v$, and statement (b) is equivalent to

(b.1) $y_j^c = 0$, if and only if $x^c M^j > v$

For a proof of this theorem, see the references mentioned in appendix A. Now we are able to prove the existence of economic solutions.

Theorem 2.4 (Existence theorem)

If the KMT-conditions on technology hold, then there exists at least one economic triple (x, y, α) for which:

$$xM_\alpha \geqq 0 \tag{2.34}$$

$$M_\alpha y \leqq 0 \tag{2.35}$$

$$xBy > 0 \tag{2.36}$$

Proof:

The existence of a triple that satisfies (2.34) and (2.35) has already been established in section (2.4.2). The results of that section are schematically summarized in figure 2.3. We proceed from this figure by taking the allowable solution triple (α_{min}, x^c, y^c) where α_{min} is the minimum α for which (2.34) and (2.35) applies and (x^c, y^c) is an arbitrary pair of central strategies.

Next we rearrange vectors x^c and y^c in such a way that[22]

$$x^c = (x_1, x_2) \quad \text{where} \quad x_1 > 0 \text{ and } x_2 = 0;$$

and

$$y^c = (y_1, y_2) \quad \text{where } y_1 > 0 \text{ and } y_2 = 0.$$

Then M_α ($= B-\alpha_{min}A$) is partitioned in accordance with the rearrangements of x^c and y^c. Now we can write:

$$(x_1, x_2) \begin{bmatrix} M^{11} & M^{12} \\ M^{21} & M^{22} \end{bmatrix} \begin{pmatrix} y^1 \\ y^2 \end{pmatrix} = 0 \qquad (2.37$$

or

$$x_1M^{11}y_1 + x_2M^{21}y_1 + x_1M^{12}y_2 + x_2M^{22}y_2 = 0 \qquad (2.38$$

Because both x_2 and y_2 consist of all zeros, the second, third and fourth terms of (2.38) are also zero. Thus (2.38) becomes

$$x_1 \; M^{11} \; y_1 = x_1 \; (B^{11} - \alpha_{min} \; A^{11}) \; y_1 = 0 \qquad (2.39$$

Let us assume that contrary to (2.36), $x^c \; B \; y^c = 0$. Then also $x_1 \; B^{11} \; y_1 = 0$. Because both x_1, y_1 and $\alpha_{min} > 0$ (theorem 2.2), it follows that

$$B^{11} = A^{11} = 0 \qquad (2.40$$

From theorem 2.3 we know that $x_2 = 0$ implies $M^i \; y^c < 0$, where M^i are the rows corresponding to x_2. Therefore we can state

$$M^{21} \; y_1 + M^{22} \; y_2 < 0 \qquad (2.41$$

because $y_2 = 0$, this becomes

$$(B^{21} - \alpha_{min} \; A^{21}) \; y_1 < 0 \qquad (2.42$$

---

[22] For notational convenience we leave out the c-superscript and the α-subscript in the rearranged x^c and y^c vectors and the partitioned M_α-matrix respectively.

From this and (2.39) and (2.40) it follows that we can lower α_{min} without violating (2.35). This contradicts our choice of α. Therefore $x^c B y^c \neq 0$ and because $x^c B y^c$ is non-negative it must be positive which completes the proof for α_{min}. □

By means of a similar argument it can be proven that for α_{max}, $x^c B y^c$ must also be positive: starting from (2.38), (2.39) becomes

$$x_1 M^{11} y_1 = x_1(B^{11} - \alpha_{max} A^{11})y_1 = 0 \tag{2.43}$$

If we again assume that $x^c B y^c = 0$, then also $x_1 B_{11} y_1 = 0$ and for the same reasons as above

$$B^{11} = A^{11} = 0 \tag{2.44}$$

From theorem 2.4 it follows that

$$x_1(B^{21} - \alpha_{max} A^{21}) > 0 \tag{2.45}$$

(2.43) - (2.45) imply that α_{max} can be increased without violating (2.34). This contradicts our choice of α and thus $x^c B y^c$ must be strictly positive which completes the proof. □

2.4.4 Multiple solutions

In case $\alpha_{min} < \alpha_{max}$, the number of economic α's is not necessarily confined to α_{min} and α_{max}. Consider for example an economy with the following technology matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It can easily be verified that the economy has three different growth factors that satisfy the axioms, viz. $\alpha = 4(=\alpha_{max})$, $\alpha = 2$ and $\alpha = 1(=\alpha_{min})$.

In general there exists an upper bound on the maximum number of economic growth factors. In proving this use is made of central x and y solutions. We first prove that if an α is economic, i.e. has associated with it a set $E = (x,y)$ such that $xBy > 0$, then $(x^c, y^c) \in E$.

Theorem 2.5

If the triple (x,y,α) is an economic solution to M_α, then the triple (x^c, y^c, α), (x^c, y^c) being central solutions to M_α, is also an economic solution to M_α.

Proof

From the definitions of x^c and y^c it follows that

$$x^c M_\alpha \geqq 0$$

and

$$M_\alpha y^c \leqq 0$$

Suppose, contrary to the theorem, that $x^c B y^c = 0$. Because $x By > 0$ this would mean that there is either an element of x^c or of y^c zero where the corresponding element of x respectively y is strictly positive. However, this contradicts the definition of a central solution. Thus if $x By > 0$ then always $x^c B y^c > 0$. □

The next theorem places an upper bound on the maximum number of growth factors. The proof of it is, except for some minor modifications, identical to the one of Moeschlin ([58], p. 32).

Theorem 2.6

The maximum number of economic growth factors does not exceed min (m,n), where m is the number of processes and n is the number of goods of the economy.

Proof

Let (α, x^c, y^c) be an economic triple, x^c and y^c being central. Because $x^cBy^c > 0$, there exists an index value j such that

$$x^c B^j > 0 \quad \text{and} \quad y_j^c > 0 \tag{2.46}$$

Also, because $v(M_\alpha) = x^c(B - \alpha A)y^c = 0$, and $x^c(B - \alpha A) \geqq 0$, (2.46) and the necessity part of the strong theorem of the alternative imply for this j

$$x^c M_\alpha^j = 0 \tag{2.47}$$

Because $\alpha > 0$ and (2.46) and (2.47):

$$x^c A^j > 0$$

Let $(\bar{\alpha}, \bar{x}, \bar{y})$ be another economic triple, $\bar{x}$ and $\bar{y}$ again being central and $0 < \bar{\alpha} < \alpha$. Then again, there exists an index value h for which

$$\bar{x}B^h > 0 \quad \text{and} \quad \bar{y}_h > 0 \tag{2.48}$$

Now the following string of (in-)equalities applies:

$$x^c M_{\bar{\alpha}} = x^c(B - \bar{\alpha}A) = x^c(B - \alpha A + \alpha A - \bar{\alpha}A) = x^c M_\alpha + (\alpha - \bar{\alpha})x^c A \geqq 0 \tag{2.49}$$

Because $(\alpha - \bar{\alpha})x^c A^j > 0$ and $x^c M_\alpha \geqq 0$ we have

$$x^c M_{\bar{\alpha}}^j > 0 \tag{2.50}$$

Suppose that $j = h$, then according to (2.50), $x^c M_{\bar{\alpha}}^j = x^c M_{\bar{\alpha}}^h > 0$.

But then $\bar{y}_h = 0$ which contradicts (2.48). Therefore $j \neq h$ which means that if (2.46) holds for an α, it cannot hold for another α. And because (2.46) holds for at least one good per α, the maximum number of α's, i.e. economic growth factors, cannot exceed the number of goods (= columns) of the economy. A same argument can be used to prove that the number of economic

α's can not exceed m either, i.e. the number of rows of M. This completes the proof of the theorem. □

Summing up, it was shown in sections (2.4.1) - (2.4.4) that:

(a) The model possesses always an economic solution.

(b) If more than one economic α exists for which $v(M_\alpha) = 0$, then both α_{min} and α_{max} yield economic solutions.

(c) There are at most min (m,n) economic solutions to the model.

In figure 2.5 finally, possible solution triples (α,x,y) are schematically summarized. If a triple (α_k,x^k,y^k) satisfies $x^kBy^k>0$ and at the same time the economy contains more than one economic α, the set of processes corresponding to non-zero elements of x^k can be considered as a sub-economy. As will be shown in chapter 4, a unique sub-economy can be associated with each economic α[23].

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[23] For a detailed characterization of sub-economies, see e.g. KMT [39], p. 125-p. 128); Moeschlin ([58], p. 60-p. 66); or Förstner ([21], p. 96-p 114).

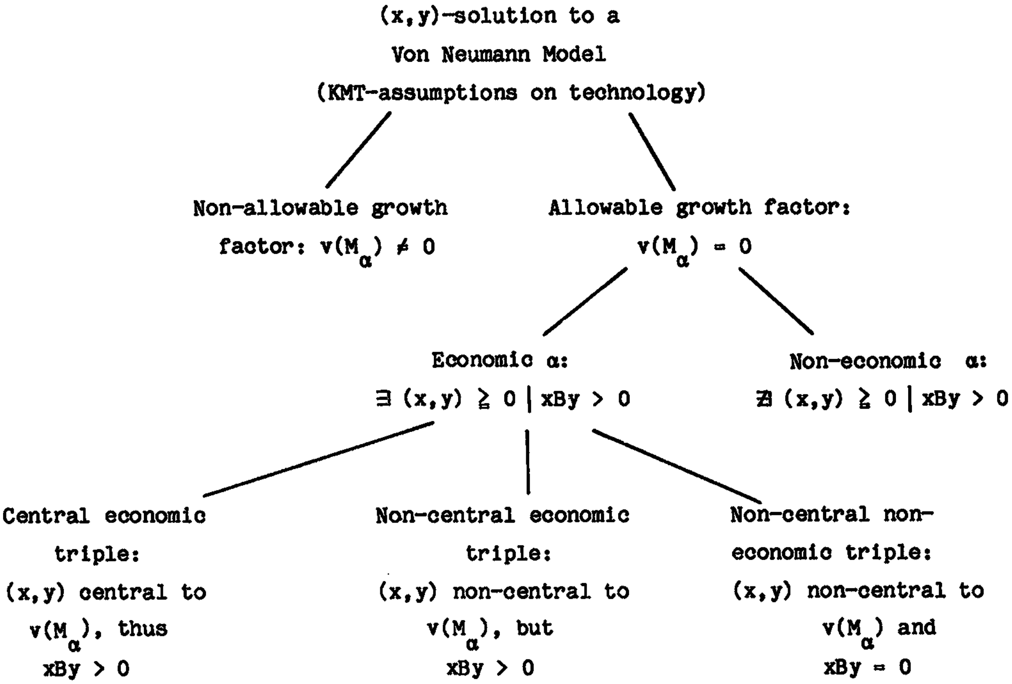

Figure 2.5

Solutions of the Von Neumann model

Chapter 3

EXTENSIONS OF THE MODEL

3.1 INTRODUCTION

An analytical model of an expanding economy, well formulated as it may be, is not the same as a theory of a specific real world situation. More specifically: the Von Neumann model as presented in chapter 2 is not necessarily an acceptable theory about the way a real world economy can develop through time. This is only the case if the model contains the principal traits of the particular economy under study. To what extent the Von Neumann model can be considered an acceptable abstraction of a real world situation is ultimately a matter of taste. However, to obtain a foothold for one's subjective criteria the 'state of the art' in economic modeling may serve as a point of reference. This procedure leads to the following (incomplete) list of objections against the Von Neumann model as discussed in chapter 2:

(i) The model is closed, i.e. no contacts with the outside world are possible.

(ii) Consumption is only allowed at a subsistence level; moreover, consumption is exogenously determined, i.e. independently of the prices generated by the model.

(iii) Land, labour and (other) raw materials are assumed to be available in unlimited quantities, they are therefore not explicitly taken into account.

(iv) All goods are supposed to have the same uniform growth rate, which implies among others that a per capita growth of consumption is not possible.

(v) Monetary phenomena are neglected.

(vi) Initial endowments are assumed to be available in balanced growth proportions.

(vii) The model is deterministic.

(viii) No individual agents (social classes) with budget restrictions are distinguished.

The purpose of this chapter is to remove some of these objections so that the relevance of the model for a real world economy increases. First, the model is made open (section 3.2), both import and export of tradeable goods are allowed for. After a discussion of the Morgenstern-Thompson open model, an alternative way to deal with trade in the model is proposed. Then, in section 3.3, consumption and labour are more explicitly introduced, although consumption remains determined price-exogenously. In section 3.4 the initial endowments enter the model as upper bounds on (groups of) activities. As a consequence the concept of proportional growth in its 'pure form' is departed from.

A more realistic treatment of raw materials and land is proposed in section 3.5. As will be discussed extensively, a proper treatment of nontradeable non-augmenting scarce goods as land requires aggregation. Aggregation is however a much wider phenomenon in a Von Neumann world. Therefore the effects of aggregation are discussed in a more general way in section 3.6.

By aggregating goods and processes some unbalanced aspects of growth come again to the front. A third 'unbalanced' aspect is discussed in section 3.7. Here the assumption of uniform growth of all consumption goods is relaxed. As a consequence production and consumption of a good j need not necessarily grow at the same rate anymore. We shall see that in some cases trade will act as an adjusting variable and regime-switches can be identified.

3.2 THE OPEN MODEL

3.2.1 The Morgenstern-Thompson open economy

In the discussion of chapter 2, it is assumed that the economy does not have any contacts with other economies. That is, if the model describes a firm, 100 per cent self-sufficiency is supposed to exist. And when the model describes a national economy, the closedness implies that no international trade may occur. Although closed systems may be of interest in a historical context, they hardly exist nowadays. Therefore Morgenstern and Thompson (MT) ([66], [67], [68], and [69]), have generalized the closed model so that contacts with the outside world can take place. An assumption regarding these contacts is that world market prices, or in case a firm is being described national market prices, are unaffected by the economy under consideration. The open model of MT can be stated by seven axioms and five assumptions. Because the MT-open model serves only as the starting point for a slightly different formulation in section 3.2.2, our discussion here will be brief. For details the reader is referred to the original articles and also to Moeschlin ([58] and [59]).

Axiom (01)[24]: $xB + w^i - w^e = \alpha xA$
where w^i and w^e are the imports from respectively the exports to the outside world. Both w^i and w^e are 1 x n row vectors. According to the axiom the outputs of period t must, after correction for the flows from (w^i) and to (w^e) the outside world, be sufficient for, or to be more precise, equal to the inputs needed in the next period.

Axiom (02): $By + z^n - z^p = \beta Ay$
where z^n and z^p are m x 1 column vectors. The elements z_i^n and z_i^p denote the loss, respectively the profit of process i. Thus, in words the axiom says that the value of the output of process i in period t plus the loss respectively minus the profit on the process must be equal to the capitalized input value.

Axiom (03): $w^e\, p^e = w^i\, p^i$
where p^e and p^i are n x 1 column vectors denoting the given export (p^e) and import (p^i) prices. Thus, the total value of the exports must be equal to the total value of the imports. Axiom (03) can be looked at as the balance of trade condition.

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[24] The axioms and assumptions as they are stated here are taken from MT ([69], p. 49 - p. 70).

Axiom (04): $t^p z^p = t^n z^n$

where t^p and t^n are exogenously set upper (t^p) respectively lower bounds (t^n) on the processes x_i. t^p and t^n are 1 x m row vectors. According to the axiom the sum total of losses of processes i that stand at their lower bound is equal to the sum total of profits of processes i that stand at their upper bound. Because it can be proven that only the former can suffer a loss while only the latter can make a profit, the axiom also says that the sum total of profits must be equal to the sum total of losses. The motivation of the axiom is that in the long run unprofitable industries can be sustained only at the expense of profitable industries.

Axiom (05): $xBy > 0$

Axiom (05) is identical to axiom 5 of the closed model (see section 2.3.1). It says that the economy must produce something of value.

Axiom (06): $t^n \leqq x \leqq t^p$

Processes x_i must take place within exogenous bounds t_i^n respectively t_i^p.

Finally we have:

Axiom (07): $p^e \leqq y \leqq p^i$

Internal prices must lie between the export and import prices. This is plausible if one supposes that the government refrains from border intervention.

A solution to the model is defined as a set of non-negative vectors (x, w^e, w^i, y, z^n, z^p) and non-negative scalars (α, β) that satisfy axioms (01) - (07) for given non-negative matrices A and B and a set of non-negative vectors (t^n, t^p, p^e, p^i). If the solution satisfies $w^e p^e > 0$, then the economy is open, i.e. then there is exchange with the outside world, if on the other hand $w^e p^e = 0$, then the economy is closed, i.e. then the economy does not have any contacts with the outside world (see MT [69], p. 62 - p. 63).To ensure that a solution to the seven axioms always exists, Morgenstern and Thompson made a number of assumptions with regard to the data of the model:

Assumption (01): Each process needs some (produced) goods as input.
Assumption (02): The economy can produce each good.

Assumptions (01) and (02) are easily recognized as the KMT-conditions on technology. Their plausibility for the closed model was extensively discussed in chapter 2. However, as we shall see in the next section, in case of the open model, assumption (02) is restrictive.

Assumption (03): $0 \leqq p^e \leqq p^i$, or the export prices cannot exceed the import prices.
Assumption (04): $0 \leqq t^n \leqq t^p$, or the lower bound of a process cannot be higher than the upper bound.

Both assumptions (03) and (04) are logical necessities if axioms (06) and (07) have to be satisfied. In order to force an economy to have international trade, Morgenstern and Thompson assumed further that:

Assumption (05): $t^n A p^e > 0$, that is

> "When the economy evaluates its goods at the lowest possible prices p^e, then even when operating at its minimum intensities the economy must have a positive demand to input at least one good which has a positive export price" (MT [69], p. 70).

MT prove that given assumptions (01) - (04), at least one solution exists. If in addition assumption (05) is satisfied the solution also obeys $w^e p^e > 0$, which means that international trade will take place. From the axioms it is easily seen that the growth factor depends, for given A and B matrices, on the upper and lower bounds of the processes and the export and import prices. Thus,

$$\alpha = \alpha(t^p, t^n, p^e, p^i)$$

The growth factor is a non-increasing function of t^n and a non-decreasing function of t^p. The higher t^n and t^p, the tighter respectively the looser the constraints on the system. Morgenstern and Thompson prove that there exists an upper (α_{max}) and a lower (α_{min}) bound on the growth factor. And if t^n and t^p are suitably set all $\alpha \in \{\alpha \mid \alpha_{min} \leq \alpha \leq \alpha_{max}\}$ can be realized.

3.2.2 The open model in a closed form: an alternative formulation

The open model as formulated by Morgenstern and Thompson is a more adequate description of reality than the closed one. There are however, two (minor) problems that have to be solved before an empirical application makes sense.

The first problem refers to assumption (02). According to this assumption all goods must be produceable by the economy. An implication of this is that only goods can be imported that also can be produced by the economy. This is a somewhat restrictive and rather unrealistic assumption. In reality one often sees that a reason for a country to import goods is exactly because it cannot produce them itself. The second problem is that the MT-open model enables the economy to import and export all goods. This is not very realistic either. In a real world economy a number of goods are non-tradeable. In the context of Bangladesh one can think of straw, manure, services of a rickshaw driver, etc. Both problems can be overcome relatively easily. The first problem could be bypassed by formulating one (or more) aggregated foreign goods which can be produced internally only through a very inefficient technology. The second problem could be circumvented by putting the import and export price of a non-tradeable good very high, respectively very low or to introduce trade activities with very low upper bounds. We have, however, chosen for a different solution that does not need such artifices. The alternative model formulation to which this solution leads, has in addition two other advantages.

First, the introduction of upper and lower bounds is not needed. One can consider these bounds as a substitution of the general normalization of processes ($\Sigma x = 1$) by a specific one ($t_i^n \leq x_i \leq t_i^p$). We postpone the specification of bounds to section 3.4. An advantage of this postponement is that one avoids the possibility of confusing effects resulting from opening the model with effects resulting from the introduction of bounds.

An example of such a confusion can be found on p. 7 of MT [69] where the authors suggest that as a consequence of the opening of the model a range of growth factors is possible. However, as they show themselves, the range is only possible by deliberately changing the upper and lower bounds on x, i.e. the bounds on the process levels!

Secondly, the model structure of the alternative formulation is identical to the structure of the closed model of chapter 2, so that all results of that chapter carry over to the open model in the form stated here.

On the other hand, we have to admit that we do not only win. In the linear programming model that can be derived from the MT-open model (see MT [69], p. 68), all contacts with the outside world appear in the objective function. Besides an increase in 'model-insight', this also facilitates the introduction of tariffs and indirect taxes.

Central in the alternative formulation we propose, are

(a) the treatment of import and export of goods as 'normal' processes, and

(b) the treatment of an international currency, which functions as a unit of account, as a good.

The export of good j requires a unit of good j as input and yields p_j^e internationally accepted units of account (iau) as output. The import of good j on the other hand, needs p_j^i iau's as input and yields a unit of good j as output[25]. As a consequence the original A and B matrices are extended with a number of import and export activities. Contrary to the MT-open model only goods that can be traded need to be considered. The subset of goods that can be imported need not necessarily be confined to goods belonging to the original economy, also entirely new goods can be added. It does, of course, only make sense to consider 'new' goods if at the same time 'new' real production processes are added which need these exotic goods. For example, if a farm is modeled, the introduction of a fertilizer buying

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[25] In case a firm is modeled, one needs to read buying and selling instead of importing and exporting respectively. And instead of internationally accepted units of account, nationally accepted units of account are relevant.

activity makes only sense if crop growing processes that need fertilizer are part of the technology matrices.

The extended matrices $\tilde{A}$ and $\tilde{B}$ are schematically shown in figure 3.1. Four types of goods can be distinguished:

(a) non-tradeable goods J_1,

(b) tradeable goods J_2 that can be produced by the economy,

(c) tradeable goods J_3 that cannot be produced by the economy, and

(d) a unit of account, say the US$, or in case a firm is modeled, the national currency which is for Bangladesh the Taka.

As can be seen in the figure, processes have been subdivided into five different types:

(a) processes I_1 belonging to the original closed economy,

(b) processes I_2 which can be carried out as a consequence of the opening of the economy: they need goods J_3 as input,

(c) processes I_3 describing the import of tradeable goods J_2,

(d) processes I_4 describing the import of tradeable goods J_3, and finally

(e) processes I_5 describing the export of tradeable goods J_2.

Goods: / Processes:		Non-tradeables J_1	National tradeables J_2	Exotic tradeables J_3	Unit of account J_4
Domestic processes	I_1	B_{11}	B_{12}	0	0
Processes that need exotic goods	I_2	B_{21}	B_{22}	0	0
Imports of tradeables J_2	I_3	0	B_{32}	0	0
Imports of tradeables J_3	I_4	0	0	B_{43}	0
Exports of tradeables	I_5	0	0	0	B_{54}

Output Matrix $\tilde{B}$

Goods: / Processes:		Non-tradeables J_1	National tradeables J_2	Exotic tradeables J_3	Unit of account J_4
Domestic processes	I_1	A_{11}	A_{12}	0	0
Processes that need exotic goods	I_2	A_{21}	A_{22}	A_{23}	0
Imports of tradeables J_2	I_3	0	0	0	A_{34}
Imports of tradeables J_3	I_4	0	0	0	A_{44}
Exports of tradeables	I_5	0	A_{52}	0	0

Input Matrix $\tilde{A}$

Figure 3.1

Extended technology matrices: The open model

Sub-matrices B_{11}, B_{12}, A_{11} and A_{12} form the original closed economy, i.e. they all are non-negative and satisfy the KMT-conditions on technology. The sub-matrices defining the 'new' real production processes I_2 are non-negative, in addition each process of this category has by definition at least one strictly positive element on the part of the input vector belonging to sub-matrix A_{23}. Sub-matrices B_{32}, B_{43} and A_{52} are identity matrices. Complementary to import is a demand for iau's, say US$. Complementary to export is a supply of iau's. Demand and supply of the US$ in this sense are demand and supply for purchasing power and not for the US$ as a currency in the sense of a store of value. Vectors B_{54}, A_{34} and A_{44} denote the export and import prices in the iau. Because we assume that internationally tradeable goods are not free, i.e. expressed in the iau their price is strictly positive, B_{54}, A_{34} and A_{44} are also strictly positive.

The model axioms and assumptions we propose are the same as for the closed model[26], namely:

Axiom (r1) $x(\tilde{B} - \alpha \tilde{A}) \geq 0$

Axiom (r2) $(\tilde{B} - \alpha \tilde{A})y \leq 0$

Axiom (r3) $x\tilde{B}y > 0$

where for $\tilde{B}$ and $\tilde{A}$ it is assumed that the KMT-conditions on technology apply. Given the above discussion, it can easily be verified in figure 3.1 that this is indeed the case: each column of matrix $\tilde{B}$ and each row of matrix $\tilde{A}$ have at least one strictly positive element. A solution to the reformulated open model is now defined as a triple (α, x, y) satisfying axioms (r1)-(r3). The following theorem will cause no difficulties:

Theorem 3.1

The reformulated open model has at least one and at most min (m,n) economic solutions, where m is the number of rows and n is the number of columns of the technology matrices.

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[26] Because of theorem 2.1, we proceed from $\alpha = \beta$ and reduce the original five to three axioms.

The theorem is a direct consequence of theorems 2.4 and 2.6 (see section 2.4).

Before investigating the model in more detail, we shall first give an illustration of it by means of a small example.

An example

Consider an economy with the following technology matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

At first we assume that no contacts with the outside world exist. Thus the economy is closed. The maximum growth factor of the economy can then be calculated as $1\frac{1}{2}$. The corresponding price and intensity vectors are y = (6/7, 1/7)', respectively x = (1/2, 1/2). Thus in equilibrium both processes are carried out at the same level and the first good is six times as expensive as the second good.

Next we assume that the economy can have contacts with the outside world, i.e. we make the model open. The import and export prices expressed in iau's are stated in the following matrix:

$$\begin{array}{c} \\ p_e \\ p_i \end{array} \begin{array}{cc} \text{good 1} & \text{good 2} \\ \left[\begin{matrix} 1 \\ 2 \end{matrix}\right. & \left.\begin{matrix} 4 \\ 5 \end{matrix}\right] \end{array}$$

where p_e is the export price and p_i is the import price. Thus if the economy wants to import good 2, it needs five units purchasing power, . . . etc. The original A and B matrices can now be extended to $\tilde{A}$ and $\tilde{B}$.

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \tilde{B} = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

or,

$$\tilde{M}_\alpha = \begin{array}{cccl} \text{good 1} & \text{good 2} & \text{unit of account} & \\ 1-\alpha & 3 & 0 & \text{process 1} \\ 2-\alpha & -2\alpha & 0 & \text{process 2} \\ 1 & 0 & -2\alpha & \text{import good 1} \\ 0 & 1 & -5\alpha & \text{import good 2} \\ -\alpha & 0 & 1 & \text{export good 1} \\ 0 & -\alpha & 4 & \text{export good 2} \end{array}$$

The new growth factor, i.e. the growth factor of $\tilde{M}_\alpha$, is 2.219, which means that as a consequence of international trade the economy can grow 71.9 per cent faster. The increased growth is coupled with a drastic change in the structure of the economy. The new intensity vector $x = (.28, 0, .34, 0, 0, .38)$. The economy specializes in process 1. As a result the production of good 2 increases while the production of good 1 lags behind. To balance the two, the economy will export good 2 and import good 1. One could say that the economy has a comparative advantage in the production of good 2. The price structure also changes, the equilibrium $y = (.61, .25, .14)'$. It can be verified that at these prices processes 2, 4 and 5 can only be carried out with a loss while the processes that are actually carried out (1, 3 and 6) just yield enough to pay for β times the value of the inputs. A consequence of international trade is that the price of good 2 becomes relatively more expensive. The price of the third good can be considered as the relative price (relative to the prices of the domestic goods) of one unit of purchasing power on the international market.

Through the introduction of international trade it becomes possible to express all tradeable goods in a common denominator, i.e. the iau. As a consequence a higher growth rate is possible. In sections 3.5.4 and 3.6 we shall discuss the consequences of aggregating goods and processes in more detail.

We now proceed to investigate solutions to the open model. Therefore we start with writing out the axioms (r1) - (r3). Referring to figure 3.1 and denoting x_i and y_j as intensities of processes I_i and prices of goods J_j respectively, we obtain:

Axiom (r1)

(a) The production of non-tradeable goods in t cannot be less than the quantity needed in t+1:

$$x_1 B_{11} + x_2 B_{21} \geqq \alpha (x_1 A_{11} + x_2 A_{21})$$

(b) The production and the imports of non-exotic tradeables in t must be sufficient for the exports and the inputs in t+1:

$$x_1 B_{12} + x_2 B_{22} + x_3 B_{32} \geqq \alpha (x_1 A_{12} + x_2 A_{22} + x_5 A_{52})$$

(c) The inputs of exotic goods in t+1 can only come from the imports in t, because:

$$x_4 B_{43} \geqq \alpha x_2 A_{23}$$

(d) The export value in international prices in t may not be less than the import value in international prices in t+1. One can interpret (d) as the balance of trade condition:

$$x_5 B_{54} \geqq \alpha(x_3 A_{34} + x_4 A_{44})$$

Axiom (r2)

No super-profits can be earned on real production processes and import and export activities. Recall that sub-matrices B_{32}, B_{43} and A_{52} are identity matrices and that sub-matrices (veqtors) A_{34}, A_{44} and B_{54} denote import and export prices in iau's (e.g. US$). (a) and (b) refer to real production processes and (c), (d) and (e) to trade activities.

(a) $B_{11} y_1 + B_{12} y_2 \leqq \beta$[27] $A_{11} y_1 + A_{12} y_2)$ and

(b) $B_{21} y_1 + B_{22} y_2 \leqq \beta(A_{21} y_1 + A_{22} y_2 + A_{23} y_3)$

(c) $B_{32} y_2 \leqq \beta A_{34} y_4$

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[27] Because all results from the closed model apply to the open model also, we have substituted β for α.

(d) $B_{43}\ y_3 \leqq \beta\ A_{44}\ y_4$ and

(e) $B_{54}\ y_4 \leqq \beta\ A_{52}\ y_2$

Axiom (r3)

Something of value must be produced. For:

$$x_1 B_{11} y_1 + x_2 B_{21} y_1 + x_1 B_{21} y_2 + x_2 B_{22} y_2 + x_3 B_{32} y_2 + x_4\ B_{43} y_3 + x_5\ B_{52} y_4 > 0$$

Next we shall prove two theorems by which some properties of equilibrium solutions are stated. The first one tells something about prices while the second theorem refers to some properties of the growth rates.

Theorem 3.2

Given an open economy described by matrices $\tilde{A}$ and $\tilde{B}$ (see figure 3.1), then:

(a) A structural surplus on the balance of trade makes the international currency as a unit of account a free good in the economy.

(b) If the (shadow) price of the iau (y_4), which can be interpreted as the normalized internal value of an iau, is zero, then all tradeables have a zero-price.

(c) If all goods are tradeable the normalized value of the unit of account must be strictly positive.

(d) If the price of an iau is strictly positive, then all tradeables have a strictly positive price.

Proof

(a) A structural surplus on the balance of trade means that for axiom (r1)-d the strict inequality applies which in its turn implies that y_4, i.e. the internal value of a unit of purchasing power on the international market, will be zero.

(b) Because B_{32} and B_{43} consist of identity matrices, it directly follows from axioms (r2)-c and (r2)-d that $y_4 = 0$ implies $y_2 = 0$ and $y_3 = 0$, i.e. if the unit of account has a zero internal price then all tradeables have a zero internal price.

(c) Because in a solution some goods must have a strictly positive price, the shadow price of y_4 must be strictly positive if all goods are tradeable.

(d) A_{52} is an identity matrix. Thus it follows from axiom (r2)-e that $y_4 > 0$, implies $y_2 > 0$. □

Theorem 3.3

(a) The minimum growth factor of an economy with trade (open economy) can never be less than the minimum growth factor of the same economy without trade (closed economy).

(b) An economy has at most one open solution with a strictly positive price for the unit of account.

(c) If all goods are tradeable, an open economy has exactly one growth factor.

Proof

(a) Let α^c_{min}, α^o_{min} and $y^o = (y_1, y_2, y_3, y_4)'$ be the minimum growth factor of the closed economy, the minimum growth factor of the open economy and the equilibrium price vector of the open economy, respectively. Referring to figure 3.1 we may write

$$(B_{11} - \alpha^o_{min} A_{11})y_1 + (B_{12} - \alpha^o_{min} A_{12})y_2 \leqq 0 \tag{3.1}$$

From theorem 3.2 it follows that if $y_4 > 0$ then $y_2 > 0$ and if $y_4 = 0$ then at least one element of $y_1 \neq 0$, since $y_2, y_3 = 0$ and $\Sigma y_j = 1$. Thus whatever the value of y_4, $(y_1, y_2) \neq 0$ and (y_1, y_2) can, after normalization, be considered as a column strategy to the game:

$$[B_{11} - \alpha^o_{min} A_{11}, B_{12} - \alpha^o_{min} A_{12}]$$

Elementary matrix game theory tells us that because of (3.1) the value of this game is non-positive. But because α^c_{min} is the minimum α for which $v(B_{11} - \alpha A_{11}, B_{12} - \alpha A_{12}) = 0$, and because $v(M_\alpha)$ is non-increasing, $\alpha^o_{min} \geqq \alpha^c_{min}$.

(b) For the proof of this part of the theorem we need the following lemma:

Lemma 3.1:

Processes that use and/or produce goods that have a strictly positive price in the economy with growth factor α, will have a zero intensity if α increases.

For a proof, see section 4.3.3 .

Now the proof of part (b) proceeds as follows. Given the minimum growth factor α^o_{min}, according to the lemma all processes that require or yield good J_4 (iau) will have a zero intensity if α^o_{min} increases. Thus I_3, I_4 and $I_5=0$ (see figure (3.1)). Because no goods J_3 will be imported, it can immediately be seen from axiom (r1)-c that $x_2 = 0$ also. The resulting economy consists of the original closed economy, which cannot yield an open solution, i.e. a solution including non-zero trade activities.

(c) According to theorem 3.2-b, $y_4 = 0$ implies $y_2 = 0$ and $y_3 = 0$. Because all goods are tradeable the set J_1 is empty. And since in a solution some goods must have a strictly positive price, $y_4 > 0$ for α_{min} and because of theorem 3.2-d, $y_2 > 0$ also. We have just proven, under (b) above, that given $y_4 > 0$, for each $\alpha > \alpha_{min}$, processes $x_2 = 0$, $x_3 = 0$, $x_4 = 0$ and $x_5 = 0$. And because processes x_1 use also some tradeable goods, $y_2 > 0$ means, according to lemma (3.1), that for all $\alpha > \alpha_{min}$, $x_1 = 0$ also. This contradicts $\Sigma x_i=1$, hence α_{min} is at the same time α_{max}, by which the uniqueness of α_{min} is proven. □

The result of part (c) of theorem 3.3 is similar to a result of Moeschlin ([58], p. 123 - p. 124) according to which the MT-open model has in its original formulation[28] exactly one solution!

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[28] The MT-open model discussed in section 3.2.1 differs from the original MT-open model in that the latter did not need the KMT-conditions on technology but required in addition to $t^n A p_i > 0$ also $t^n B p_e > 0$ (see MT [66], p. 449).

We end this section with some remarks on the differences and similarities between the MT-open model and the alternative formulation. A close look reveals that there are basically three differences:

(i) Contrary to the alternative formulation, the MT-open model contains tradeable goods only.

(ii) Instead of a general normalization rule the MT-open model contains explicit upper and lower bounds on real production activities.

(iii) In the MT-open model trade activities do not take time, while in the alternative formulation trade activities last, as real production activities, one period.

If the MT-open model is adjusted for these differences where an adjustment for (i) and (ii) is more a matter of cosmetics, while an adjustment for (iii) has more to do with principle, it can be verified that the solutions of the two models are identical.

3.3 BALANCED GROWTH OF CONSUMPTION

In the original Von Neumann model consumption is treated as a necessary input for production. As such it is part of the A matrix. All income in excess of necessities of life are assumed to be invested (Von Neumann [70], p. 240). One could interpret such an economy as a slave economy. To paraphrase Marx on a saying about classical economists: "In a Von Neumann economy, the proletarian is but a machine for the production of surplus value; on the other hand, the capitalist is in such an economy only a machine for the conversion of this surplus value into additional capital". The maximum balanced growth-minimum consumption model, as one could typify the original Von Neumann model, can serve as a point of reference against which the effects of higher consumption levels on the growth rate can be evaluated. The way this 'extra-consumption' can be incorporated is the subject matter of a number of studies (e.g. Frisch [22], Los [52], Malinvaud [54], Morishima [61] and Morgenstern and Thompson [65]. Morishima's and Morgenstern and Thompson's models lie most in line of our model discussion sofar. However, as we have explained in more detail elsewhere (Stolwijk [93]), both alternatives are not very attractive for our purposes. Morgenstern and Thompson's model does not always yield meaningful economic results. And the problem with Morishima's model is that the coefficients of the A-matrix become price dependent. Although this is interesting from a theoretical point of view, computationally the model becomes much more difficult to handle and we would have to leave the model framework discussed sofar[29]. For these reasons we do not follow these lines. Instead, we propose a simple alternative way to deal with consumption. This is done in two steps. In this section we slightly adjust Von Neumann's original model so that consumption comes more explicitly to the fore. In this set-up consumption grows in a balanced way, i.e. if the economy grows at a rate $(\alpha-1)$, consumption of each good increases also at a rate $(\alpha-1)$. Thus, income elasticities of 1 are assumed for all goods. In section 3.7 we relax this assumption and consumption behaviour will be introduced with other income elasticities. By doing this the balanced growth concept is abandoned. This second step is therefore discussed under another heading.

To start with step 1, we define two categories of goods, n_1 and n_2. n_1 are capital and intermediate goods; n_2 are consumption goods. Because the model does, in principle, not make a difference between capital and intermediate goods we will also speak of non-consumption goods. It will be clear that in general $n_1 \cap n_2 \neq 0$, for many goods play a role both as a consumption and as an intermediate good. Further, each good belongs to at least one category. Thus $n_1 \cup n_2 = n$. Input matrix A contains only goods n_1, i.e. capital and intermediate goods. In addition we define an m x n matrix C, row i denoting the quantities of goods j consumed if process i is carried out at unit intensity. Determination of C can in principle be done in many ways. As a start it seems not too unreasonable to assume that consumption is proportional to the quantity of labour involved in the process (if carried out at unit intensity). Because consumption per unit of labour equals total consumption divided by total labour input, we may write:

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[29] In Keyzer [45] the step from exogenous A and B matrices to price endogenous A and B matrices is explicitly made and discussed.

$$C = \ell c$$

where

$$\ell = (\ell_1, \ell_2 \; . \; . \; . \; . \; \ell_m)'$$

with

ℓ_i = labour quantity required to carry out process i at unit intensity

and

$$c = (c_1, c_2 \; . \; . \; . \; . \; c_n)$$

with

$$c_j = \frac{\bar{\bar{c}}_j}{L}$$

where

$\bar{\bar{c}}_j$ = total consumption of good j and
L = total labour used by the economy.

If we assume that wages are paid at the end of the period, the axioms can be stated as follows[30]:

Axiom (c1): $x(B - \alpha(A+C) \geqq 0$
Physical balances must fit.
Axiom (c2): $x(B - \alpha(A+C)y = 0$
Overproduced goods have a zero-price.
Axiom (c3): $(B - \beta A)y - \beta \ell y^W \leqq 0$
where y^W is the wage-rate.
No super-profits can be made.
Axiom (c4): $x(B-\beta A)y - \beta x \ell y^W = 0$
Processes that yield a loss are carried out with zero intensity.
Axiom (c5): $xBy > 0$
Something of value must be produced.

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[30] As was shown in the preceding section there is in our set-up mathematically no difference between the closed and open model. Therefore the axioms apply both to closed and open economies.

Given the results of chapter 2, a solution to the model is easily found. We start with defining $M_\alpha = B-\alpha(A+C)$. Next we look for the minimum α for which $v(M_\alpha) = 0$. In chapter 2 it was shown that such α exists and that the corresponding x- and y-strategies satisfy axioms (c1), (c2) and (c5). Moreover, if we set $\beta=\alpha$ and $y^w = cy$ it can be verified that axioms (c3) and (c4) are also satisfied. Thus in equilibrium the wage-rate equals the value of the corresponding basket of consumption goods.

Intuitively it will be plausible that α and β are non-increasing functions of the volume of consumption. The higher the values c_{ij} of C, the lower the growth and interest rate. Given theorem 2.2 and some results by Mills on marginal values of matrix games and linear programs [57], this can easily be proven. However, because the model will only play a minor role in the empirical part of our study, we refrain from doing that. The 'minor role' is a consequence of the severe shortcomings of the model. Apart from the unit income elasticities, the model does not allow an increase in per capita consumption. In the context of growth this is too unrealistic a starting point in empirical work. To overcome these drawbacks the model will be modified. The modification requires the introduction of some non-balanced growth aspects. However, before this will be done, some further model properties which are unrealistic will be discussed and 'removed', also implying an abandoning of the 'pure' balanced growth concept. The consumption model of this section can be considered as the most complete balanced growth version. It is the reference model in the discussions in the remainder of this chapter.

3.4 INITIAL ENDOWMENTS

3.4.1 Motivation

Until now we have disregarded the initial capital, land and labour endowments of the economy. This omission is characteristic for nearly all studies on the Von Neumann model; it stems from the model assumption that capital goods are available in the right proportions. Although this assumption has been criticized (see e.g. Champernowne [16]), much theoretical work on the model, such as the work on turnpike growth, can be interpreted as a justification of it. Results of most turnpike studies[31] point towards the fact that if the planning horizon is sufficiently far away, the fastest way from an initial endowment vector to a final one makes temporarily use (or stays in a close neighbourhood) of the turnpike. In terms of figure 3.2: the fastest road from a to b is not via the broken line but via the seemingly longer turnpike.

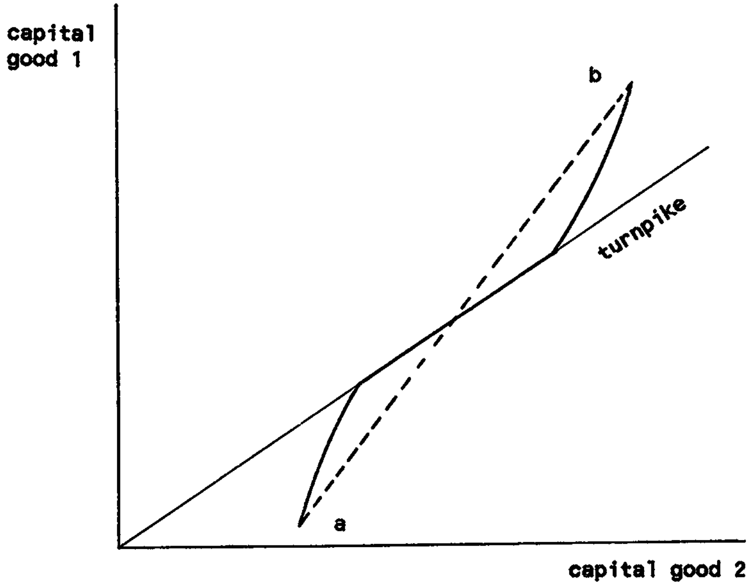

Figure 3.2

Illustration of a turnpike theorem

Two comments are in place here:

First, turnpike theorems have only been proven for Von Neumann models that have a Leontief input-output structure. Morishima [62] shows that if joint production and non-square matrices are allowed for, large oscillations around the turnpike tend to be more efficient than the turnpike itself.

Secondly, how far is 'sufficiently far away' and how long does it take to reach the turnpike? If for example 'sufficiently far away' is more than a

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[31] See for example Dorfman, Samuelson and Solow [18], Makarov and Rubinov [53], Morishima [62] and [63] or Radner [78].

hundred years and it takes ten years to reach the turnpike, turnpike theorems are not very relevant for empirical work: changes in technology and external prices will have affected the original outcomes too much in the meantime. Empirical work on Japan by Tsukui and Murakami [102] shows that the economy moves very quickly to the neighbourhood of the turnpike. The model of Tsukui and Murakami is however very small (9 x 9), i.e. their economy is highly aggregated. Moreover they use Leontief input-output matrices. In conclusion one might say that (i) from a theoretical point of view there is a priori little justification to disregard the composition of the initial endowments and (ii) the empirical findings by Tsukui and Murakami do not lead to a change of that position.

For these reasons but also as a consequence of some of our model results on Bangladesh when initial endowments are disregarded (see chapter 7), we shall adjust the model through incorporating initial endowments. This is done by introducing capacity constraints in the form of upper bounds on (sets of) activities. To ensure that a non-zero solution for x results, at least one non-zero lower bound is required. For this we chose total labour use in a reference year. It is assumed that at least L units of labour have to be employed. The shadow price on this lower bound can be given an interesting economic and institutional interpretation as will become clear below and in chapter 7.

3.4.2 Capacity constraints

The model with capacity constraints will be presented in two steps. First, the individual axioms are introduced and motivated. Then the model is reformulated such that the structure becomes identical to the structure of the original closed model. The advantage of this approach is that all results of chapter 2 carry over to the model with capacity constraints.

The capacity constraints (or initial endowments) are defined in absolute levels. As a consequence, process intensities x_i are, in the first step, also expressed at their 'real world' levels. Because the reformulation in the second step essentially consists of a normalization of these absolute levels, this does not apply to the reformulated model (compare the MT-open model versus the open model in the closed form).

As we shall explain below, the model outcomes do apply to one period only. Therefore the variables have a time-subscript. The model can be stated by seven axioms:

Axiom (e1): $x_t(B-\alpha_t(A+C)) \geq 0$
Physical balances must fit.

Axiom (e2): $x_tE \leq u_t$
where u is an 1 x p row vector of exogenously given production capacities available at the start of the period and E is an m x p matrix, element E_{ij} referring to the quantities of capacity j required per unit of process i. It will be clear that $u_j > 0$ for some j must apply.

Axiom (e3): $x_t\ell \geq L_t$
where ℓ is an m x 1 column vector, element ℓ_i denoting the quantity of labour required to carry out process i at unit intensity and L is a lower bound on total labour that has to find employment in the economy.

Axiom (e4): $x_t(B-\alpha_t(A+C))y_t = 0$
Overproduced goods have a zero-price.

Axiom (e5): $(B-\beta_tA)y_t - \beta\ell y_t^w - \beta_tEy_t^m \leq 0$
with elements y_j^m of vector y^m as the rent on each unit of capacity u_j and y^w as the wage-rate. Without the last term on the left-hand side the axiom is identical to axiom (c3) of the consumption model in the preceding section. The meaning of the latter is clear: the value of the output of process i may, in equilibrium, not exceed the capitalized sum of the value of the intermediate goods and the labour costs. At the risk of belabouring the obvious, we recall that capital goods are treated as intermediate goods so that depreciation costs are automatically taken into account. The difference between axioms (e5) and (c3) is that in (e5) processes are also charged for a competitive rent on the capacity they use. These capacity rents can be interpreted as scarcity premiums on the capital goods.

Elements y_j^m can, in equilibrium, only be positive if the corresponding capacity u_j is fully utilized.

Axiom (e6): $x_t((B-\beta_t A)y_t - \beta_t \ell y_t^w - \beta E y_t^m) = 0$
Thus, processes that yield a loss are not carried out.

Axiom (e7): $x_t B y_t > 0$
Something of value must be produced.

By introducing bounds on production capacities and labour, we have moved away from the balanced growth concept, albeit by a modest step. With respect to capacities u_j this can be seen as follows: the growth factor α does not apply to the endowments available but only to the endowments required in the equilibrium solution. Thus, if the economy has k units of capacity (capital good) u_j at its disposal but only a fraction, v, $0 \leq v \leq 1$, is actually used, the growth factor α_j applies to the used capacity only. Growth of the labour force L on the other hand is treated as an exogenous variable. An equilibrium solution of the model is defined as a sextuple:

$$Q_t = \{x_t, y_t, y_t^w, y_t^m, \alpha_t, \beta_t | \, x_t, y_t, y_t^w, y_t^m, \alpha_t \text{ and } \beta_t \geq 0 \text{ and } \Sigma x \text{ and } \Sigma y > 0\}$$

that for given A, B, C, E, u_t, ℓ and L_t satisfies the axioms (e1)-(e7). After a solution is found for period t, u_t and L_t are updated i.e. $u_{t+1,j} = \alpha_t u_{t,j}^1 + u_{t,j}^2$ where u^1 and u^2 refer to the used and unused capacity respectively and where it is assumed that unused capacity can be mothballed, and $L_{t+1} = \bar{\bar{\delta}} L_t$ where $\bar{\bar{\delta}}$ is an exogenously given growth factor of the labour force.

It is not self-evident that the set of axioms has a solution. For example, if L_t is too high relative to u_t, axioms (e2) and (e3) cannot be satisfied at the same time. To avoid such infeasibilities, but also to return to the original Von Neumann framework, the model is normalized. The normalization essentially consists of an extension of matrices A(+C) and B with E, ℓ, u and L. It can be compared with the step from the MT-open model to the open model in the closed form in section 3.2.2.

$$\begin{array}{c|c|c|c|} & y & y^m & y^L \\ \hline x & B & 0 & \ell \\ \hline x_n & 0 & u & 0 \\ \hline \end{array}$$

output matrix $\tilde{B}$

$$\begin{array}{|c|c|c} y & y^m & y^L \\ \hline A+C & E & 0 \\ \hline 0 & 0 & L \\ \hline \end{array}$$

input matrix $\tilde{A}$

Figure 3.3
Extended technology matrices:
model with initial endowments

The extended matrices $\tilde{A}$ and $\tilde{B}$ are schematically shown in figure 3.3. The interpretation of ℓ and E in the output and input matrix respectively, will cause no difficulties. 'Activity' x_n must be understood in the sense that capacities u_j can only be used if a quantity of labour L is 'put to work'. For $\tilde{B}$ and $\tilde{A}$ the KMT-conditions on technology apply. Thus we may state:

Theorem 3.4

> The reformulated model (figure 3.3) has at least one and at most min (m,n) economic solutions where m is the number of rows and n is the number of columns of the technology matrices.

The theorem is a direct consequence of theorems (2.4) and (2.6) (section 2.4). To compare the reformulated model with the seven axioms above, we shall write the model out in the standard Von Neumann axioms. Referring to figure 3.3 we get:

Axiom (n1): $xB - \alpha x(A+C) \geqq 0$

Axiom (n2): $x_n u - \alpha xE \geqq 0$

Axiom (n3): $x\ell - \alpha x_n L \geqq 0$

Axiom (n4): $By + \ell y^L - \beta(A+C)y - \beta E y^m \leqq 0$

Axiom (n5): $uy^m - \beta L y^L \leqq 0$

Axiom (n6): $xBy + x_n u y^m + x\ell y^L > 0$

Moreover, $\Sigma x=1$ and $\Sigma y=1$. In a solution x_n is either zero or strictly positive. In case $x_n=0$, it follows from (n2) that no capacities are used at all. Instead, only intermediate products are used. In order to scale the model intensities to the 'real world', all intensities have to be multiplied then by a factor $\psi = \frac{L}{x\ell}$. In case $x\ell=0$ also, i.e. if only processes that do not use labour appear with a non-zero intensity in a solution, no scaling at all is possible. We do not investigate these empirically uninteresting cases further. Suppose, on the other hand, that $x_n \neq 0$. If we set $x_n=1$ and scale x accordingly, the equivalence between axioms (e1) and (n1) is easily recognized, (n2) and (n3) differ by a factor α from (e2) and (e3). Because of this factor, the normalized version has, contrary to the original set of axioms, always a solution. Before (n4) is compared with (e5), we define the consumption matrix (as in section 3.3) as:

$$C = \ell c$$

where c can, in principle, be any non-negative row vector. If we further set

$$y^w = cy - \frac{1}{\beta} y^L$$

or

$$y^L = \beta cy - \beta y^w$$

where y^w is the wage-rate, (n4) can be written as:

$$By - \beta Cy - \beta \ell y^w - \beta(A+C)y - \beta E y^m \leqq 0 \tag{3.2}$$

which is exactly axiom (e5) above. Given the results of section 2.4 and the just proven equivalence between (n4) and (e5), it directly follows that a solution of (n1) - (n6) also satisfies (e4) and (e6). Moreover, from the weak theorem of the alternative (see appendix A), it can easily be proven that (n5) has, apart from a factor $\alpha(=\beta)$ its pendant in axioms (e1) - (e7) too, i.e. (n5) is the dual of (e2) and (e3). Finally, although (e7) and (n6) slightly differ from each other, their similarity will need no explanation.

The non-balanced aspects reveal themselves again via u and L. In each period elements u_j and L have to be adjusted according to the rules stated earlier. It may be noted that through (n3) the uninteresting case $x=0$, $x_n=1$

is ruled out. It will be intuitively clear that the growth factors of the balanced model are equal or larger than the growth factors of the model with endowments. More formally this is stated through the following theorem:

Theorem 3.5:

Referring to figure 3.3, let $v_1 = v(M_\alpha) = v(B-\alpha(A+C))$ be the game-value of the model without endowments and $v_2 = v(\tilde{M}_\alpha) = v(\tilde{B}-\alpha\tilde{A})$ the game-value of the model including endowments. Then $v_1 \geqq v_2$.

Proof:

Let $\bar{x}$ be an optimal strategy of the game $M_{\bar{\alpha}}$. Let $\bar{d}$ be the corresponding game-value v_1. Then,

$$\bar{x}M_{\bar{\alpha}} \geqq \bar{d}$$

If $v_2 > v_1$, then

$$(x, x_n)\begin{bmatrix} M_{\bar{\alpha}} & -\bar{\alpha}E & \ell \\ 0 & 0 & -\bar{\alpha}L \end{bmatrix} > \bar{d}$$

and thus

$$xM_{\bar{\alpha}} > \bar{d} \qquad (3.3)$$

Eq. (3.3) contradicts the optimality of $\bar{x}$. Thus $v_1 \geqq v_2$ must apply. □

The difference between α-balanced and α-unbalanced, i.e. α corresponding to M_α and α corresponding to $\tilde{M}_\alpha$, can be interpreted as the 'costs' of unbalanced growth relative to balanced growth. The reformulation permits, however, also a balanced interpretation of $\tilde{M}_\alpha$. If u and L are kept constant, the model outcomes apply to all periods and the economy finds itself automatically on the turnpike! One could speak of balanced growth of an 'unbalanced' economy. u and L may be treated as fixed parameters as long as one may assume that the labour force grows at the same rate as the overall growth of the economy and non-augmentable resources are not exhausted. Interesting analyses can then be done with the model. For example, by running the model for different values of L, effects of different lengths of the working week or labour participation rates etc., on the growth rate and the capacity utilization levels could be investigated. Or, by adding, deleting or constraining certain activities, consequences of specific technologies that are e.g. labour intensive or environmentally attractive, on growth and structure of the economy could be evaluated. In the context of Dutch agriculture one could think of adding manure processing activities and setting lower bounds on them, in order to trace the effects of abating water and soil pollution by manure. If such 'artificial' (institutional) constraints are binding, the shadow price gives an indication of the subsidy required to make the process attractive. If the model is used in this manner, its role lies more in the field of *technology assessment*. As such it can contribute to answering questions regarding the costs of control of technology, not only in terms of money, but also in terms of growth and employment.

For a number of reasons we shall, for Bangladesh, not concentrate too much on this interesting line of analysis. First, because L is in Bangladesh not only exogenous to the model but, in the medium term, also to society, 'playing' with L does not make so much sense. Second, because of the overwhelming importance of the non-augmentable resource land, an investigation of growth potential cannot do without a yearly update of relevant elements of u. Finally, somewhat generalized, one could say that for a poor country like Bangladesh, an optimal technology is a technology that contribute most to (balanced) growth. Moreover, stimulating and stopping specific technologies through a complicated policy of subsidies lies beyond the capacity of the government.

To end this section, we shall dwell for a moment on another aspect of the model. If we multiply each term of (3.2) by x and substitute α for β, we may write:

$$\underset{\substack{\uparrow\\ \text{output}\\ \text{value}}}{xBy} = \underset{\substack{\uparrow\\ \text{input}\\ \text{costs}\\ \text{(incl.}\\ \text{depreci-}\\ \text{ation)}}}{xAy} + \underset{\substack{\uparrow\\ \text{value of net}\\ \text{investment =}\\ \text{interest on}\\ \text{fixed and}\\ \text{intermediate}\\ \text{goods}}}{(\alpha-1)xAy} + \underbrace{\underset{\substack{\uparrow\\ \text{total}\\ \text{wage-}\\ \text{bill}}}{\alpha x\ell y^{w}} + \underset{\substack{\uparrow\\ \text{rent}\\ \text{on cap-}\\ \text{acities}}}{xEy^{m}}}_{\text{= value of consumption}} \qquad (3.4)$$

Eq. (3.4) indicates how the output value is distributed among the different cost categories. If $y^m = 0$, i.e. if no scarcity premium can be earned on capacities (which is the case if capital goods are available in the 'right' proportions) value of consumption, but also actual consumption is proportional to labour input (= wage labour + labour of the employer). However, if $y^m \neq 0$, some capacities earn an extra rent at the expense of the wage-rate. The latter follows directly from axiom (n5). Thus, by taking endowments into account a functional income distribution enters the model. It is also easily seen that the wage-rate is a non-increasing function of the quantity of labour that has to be employed into the economy. For: the higher L, the higher y^L, and the lower y^w. But a high value for y^L implies also a high value for y^m(n5). The economic interpretation is clear: if many labourers have to find work in the economy the wage-rate drops; on the other hand a big supply of labour makes the available capital goods (production capacity) relatively scarce which leads to high scarticy premiums. It will be needless to say that in addition to a lower bound on labour and upper bounds on capacity, an upper bound on labour and lower bounds on capacity can also be introduced. The interpretation is analogous.

3.5 RAW MATERIALS AND LAND

3.5.1 Raw materials

Von Neumann proceeds from the assumption that raw materials are available in unlimited quantities. They are assumed to be free and have as a consequence a zero-price. They can therefore be left out of the model. The latter does not mean that raw materials do not play a role in the production process. On the contrary, they are important in determining the input-output coefficients, be it that their role is of an implicit nature. We take sunshine as an example. Although it is not an explicit model variable in the crop-growing processes, it clearly influences the value of the input-output coefficients. For a raw material as sunshine, which is not really tradeable and which is for all practical purposes not in a state of near-depletion, a treatment in this way is satisfactory. However, there are many raw materials which are of a fundamentally different nature. Consider for example, crude oil. An important difference between the raw materials crude oil and sunshine is that the former can, in principle, be easily converted into a 'Von Neumann good', viz. by pumping it up, while the latter cannot. A consequence of this is that sunshine is a datum for the economy while crude oil is a variable. Because crude oil is also a scarce good, a treatment as Von Neumann proposes is highly unsatisfactory. We propose therefore to treat tradeable scarce raw materials in the model in the same way as produced goods. As far as a country or a firm does not own a stock of the raw material as a natural resource, this will be obvious. If in that case the country or the firm wants to make use of the raw material, it has to be bought and there is no principal difference between the purchase of, say, fertilizer and crude oil. As explained in section 2.2, crude oil is in that stage already a produced good. Labour and capital have been invested to pump it up. For the purchaser this is not relevant. For him the activity of buying oil needs an amount of takas or dollars as input, while it yields an amount of oil as output.

But how to treat this type of raw material if a country owns an unexploited stock of it? If the stock is, for all practical purposes infinitely large, it is sufficient to bring only the costs of production into account. However, many stocks of tradeable raw materials are finite. In that case it would be unsatisfactory to bring only the costs of production into account, because growth takes place at the cost of uncharged scarce raw materials. An alternative would be to allow for export and import of the raw material so that the international price will function as a guideline. However, many objections can be raised against this procedure. The valuation of a raw material is ultimately a subjective matter. It depends among other things on the thriftiness of the country, the confidence one has in technical changes in future, the extent a country feels itself responsible for future generations, etc. The model does not give answers to these aspects of the valuation problem. It can at most evaluate the effects on the growth rate and structure of the economy by setting different upper bounds on the production capacity.

3.5.2 The problem with land

Within the set of raw materials a distinctive place must be reserved for land. Its character differs both from raw materials such as sunshine and crude oil. A difference with sunshine is that land cannot be used in its natural state. To make land productive, it has first to be cleared or, to say it in another way, it has first to be transformed into a produced good. As a produced good the role of land is explicit. It can, unlike sunshine, not be 'hidden' in the technical coefficients of other variables. To that extent it falls in the same category as crude oil. In other aspects it differs however fundamentally from a raw material such as crude oil. Besides being a so-called renewable resource it is a more fundamental difference in the Von Neumann context that land is not internationally traded. When a country possesses a wealth of uncleared potentially arable land this is not important. For most countries and certainly for Bangladesh, this is however not the case. When all land is cleared, the non-tradeability of land implies that the produced good 'cleared land' cannot increase; it can neither be produced nor be bought. Since cleared land is a necessary input for all crop-growing activities, balanced growth of that sub-sector, and, as far as the crop-sector provides other sectors with necessary inputs, also of these sub-sectors, is not possible. Because land services are a renewable resource only a stationary state is possible. Yet one sees in the real world many instances of growth in crop production, despite the fact that no uncleared land is available. Fertilizer is introduced, land is upgraded through irrigation, etc. By means of a small example we will discuss and illustrate the way these phenomena can be dealt with in a Von Neumann world.

Example

The example describes different stages of development of an agricultural economy. Only variables that are of interest in the context of the discussion in this section are taken into account.

We start with a very primitive economy which has a bounty of uncleared land. The economy can be described by three activities: land clearing, paddy production and subsistence consumption of paddy. The latter yielding labour as output. The technology matrices are specified as follows:

Case A: Traditional paddy production, uncleared land available

Input matrix

$$\begin{array}{l|ccc|}
 & \text{cleared land} & \text{labour} & \text{paddy} \\
\text{land clearing} & 0 & 1 & 0 \\
\text{paddy production} & 1 & 1 & 0 \\
\text{subsistence consumption} & 0 & 0 & 1
\end{array}$$

Ouput matrix

$$\begin{array}{l|ccc|}
 & \text{cleared land} & \text{labour} & \text{paddy} \\
\text{land clearing} & 1 & 0 & 0 \\
\text{paddy production} & 1 & 0 & 1.0202 \\
\text{subsistence consumption} & 0 & 1.0408 & 0
\end{array}$$

It can be verified that the equilibrium growth (and interest) factor amounts to 2.02 per cent. The corresponding intensity and price vectors are:

$$x = (.010, \quad .495, \quad .495)$$

and

$$y = (.336, \quad .329, \quad .336)'$$

Thus the economy grows in equilibrium at a rate of 2.02 per cent. This is at the same time the rate at which the population may grow. The bulk of the activities consists of paddy production and subsistence consumption, only $.01/(.01+.495) = .0198$ per cent of the labour force is allocated to clearance of new land. The growth can of course not take place for ever. After a certain period of time the economy will hit the land bound, i.e. the economy will have run out of uncleared land. This means that the first process cannot be carried out anymore and the technology matrices become:

Case B: Traditional paddy production, no uncleared land available

Input matrix

$$\begin{array}{l|ccc|} & \text{cleared land} & \text{labour} & \text{paddy} \\ \text{paddy production} & 1 & 1 & 0 \\ \text{subsistence consumption} & 0 & 0 & 1 \end{array}$$

Ouput matrix

$$\begin{array}{l|ccc|} & \text{cleared land} & \text{labour} & \text{paddy} \\ \text{paddy production} & 1 & 0 & 1.0202 \\ \text{subsistence consumption} & 0 & 1.0408 & 0 \end{array}$$

Because cleared land cannot be produced anymore, the maximum growth rate drops to zero. The corresponding intensity and price vectors can be calculated as

$$x = (.5,\ .5)$$

and

$$y = (1,\ 0,\ 0)'$$

In equilibrium only cleared land has a positive price, while both labour and paddy are 'overproduced'. Labour because the part of the labour traditionally allocated to land clearing is not needed anymore and paddy because this labour does not need be rewarded in the system. The further development of the system depends entirely on the growth of the population. The 2.02 per cent surplus of paddy will appear as luxury consumption in the consumption matrix of the class that owns the land. If this class fails however to stabilize their number, this luxury consumption will soon be part of the input vector of the subsistence consumption process. Part of the labourers, i.e. the ones who cleared the land, will face a shortage of food because they have lost their source of income. In general one can say that the Malthusian bogey becomes reality if the population keeps on growing. By introducing initial endowments as described in section 3.4.2, one can illustrate such a process. In a real world situation the size of the population will ultimately be checked by famine and starvation. The only 'solutions' for a national economy are, apart from migration, either to stay in a stationary state or to introduce new and more productive technologies. Of course, reality is not so rigid. In practice the existing technologies can be marginally changed and refined so that population growth can go on for a while[32]. Ultimately, however, the combination of a growing population under a given technology is a self-defeating process, i.e. it will result in famine and starvation. Let us therefore investigate the consequences of the latter alternative: suppose that a new technology is introduced, say that the possibility of irrigation is discovered. As a consequence, two new processes are added to the technology matrices. First, the process of transforming unirrigated land into unirrigated land and second, the process of paddy growing under irrigated conditions. Let us assume that the following input-output matrices are now relevant to the economy of our example:

Case C: Introduction of a new technology

Input Matrix

$$\begin{array}{l|cccc|}
 & \text{unirri-gated land} & \text{irri-gated land} & \text{labour} & \text{paddy} \\
\hline
\text{paddy (rainfed)} & 1 & 0 & 1 & 0 \\
\text{paddy (irrigated)} & 0 & 1 & 2 & 0 \\
\text{investment in irrigation} & 1 & 0 & 1 & 0 \\
\text{subsistence consumption} & 0 & 0 & 0 & 1 \\
\end{array}$$

---

[32] For the process of paddy growing in our example, one could think of an increase in the input of labour which will certainly be accompanied by an increase of paddy production. The phenomenon of a situation which has reached a seemingly definitive form but nonetheless fails either to stabilize or to transform to a new pattern is baptized by Goldenweiser 'involution' (see Geertz ([26], p. 79-p. 80), who uses this concept to analyze the development of the Javanese economy).

Output Matrix

$$\begin{array}{l|cccc|}
 & \text{unirrigated land} & \text{irrigated land} & \text{labour} & \text{paddy} \\
\text{paddy (rainfed)} & 1 & 0 & 0 & 1.0202 \\
\text{paddy (irrigated)} & 0 & 1 & 0 & 2.4410 \\
\text{investment in irrigation} & 0 & 1 & 0 & 0 \\
\text{subsistence consumption} & 0 & 0 & 1.0408 & 0
\end{array}$$

It can be verified that the model is irreducible, i.e. the production of each good requires the input of all others, be it directly or indirectly. As a consequence, the model has only one growth factor (Gale [23], p. 315) which can be calculated as:

$$\alpha = \beta = 1$$

The corresponding intensity and price vectors are:

$$x = (.2, .2, .0, .6)$$
$$y = (1.0, .0, .0, .0)'$$

Thus in equilibrium, the economy finds itself in a stationary state. Only unirrigated land has a positive price. And except for investment in irrigation, all processes can be carried out without yielding a loss. Given the state of the economy a highly unsatisfactory solution. For, despite the high pressure on land and the relative surplus of labour, investment in irrigation is not attractive enough to be carried out. The situation is a consequence of the requirement for balanced growth. Because the produced good 'unirrigated land' cannot increase, the whole economy cannot grow.

From an economic point of view it is not realistic to maintain the balanced growth requirement in this case. An interesting question is if we can deal with the problem without sacrificing the essentials of the Von Neumann model. An early answer to this question was given in 1945/46 by Champernowne. According to him "in a world where the scarcity of non-augmentable resources exerts a major influence on the productive system, Von Neumann's model ceases to be so interesting" ([16], p. 17). We think Champernowne's conclusion is too rapidly stated, it is, in our opinion, a too mechanical interpretation of the model. In two ways the Von Neumann model can be of help as an analytical tool to understand and describe what is going on in situations as sketched by the example.

First, the model can be used to understand how and how fast a new technology eliminates an old one. Second, one can, by aggregating over goods, define the problem away. Although the latter does not sound too orthodox, aggregation is so inherent to economic modeling that we need not worry about it too much. On the contrary we would say, because aggregation enters so explicitly the scene in a Von Neumann world, it helps in understanding what exactly one is doing. Both points will be illustrated in next subsections through a further elaboration of the example. Because of the importance of aggregation in the empirical part of our study, we shall devote a more formal discussion to this problem in section 3.6.

3.5.3 Competition among technologies

Because the Von Neumann model clearly distinguishes alternative technologies, we think it can throw some light on some aspects of the speed by which new technologies drive out old ones. In the economy of the example of the preceding sub-section there is, at first, only a finite amount of uncleared land. We define this situation as technology level zero. Traditional paddy growing, i.e. on unirrigated land, is called technology level one. Introduction of this technology can only take place at the cost of the available amount of uncleared land. As we have seen above, as long as the latter is freely available, traditional paddy growing can earn a rent of 2.02 per cent, which is at the same time the maximum equilibrium growth rate of the system. One could however, also imagine an institutional structure where uncleared land, instead of being free, is owned by some actor who does not cultivate the land himself and that the cultivating actor has to purchase each unit of uncleared land that is taken into cultivation. If we suppose that payment proceeds in kind, e.g. in the form of paddy, the paddy growing process in case A must get an extra coefficient in the input vector. The resulting decrease of the growth factor is shown in figure 3.4.

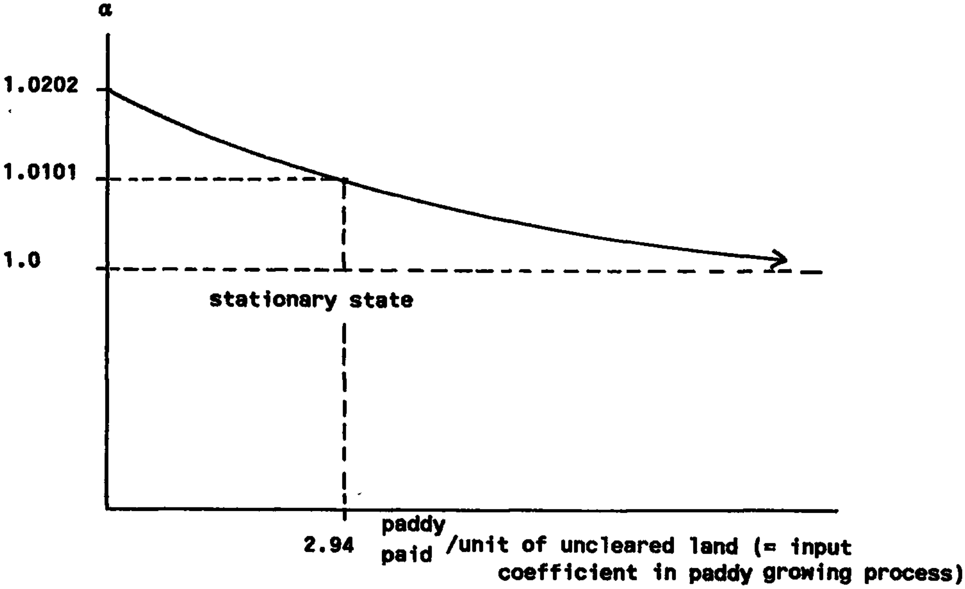

Figure 3.4

Relation between growth rate and 'price' per unit of uncleared land

As can be seen in the figure, the higher the 'price' the lower the growth rate. If for example, the price is 2.94 units of paddy, the growth rate is halved. The economy stops growing as the price of uncleared land, which is the rent for the owner of the uncleared land, goes to infinity. In that case no uncleared land is bought by the paddy growing actors. Thus in general one could say that the growth of technology level one depends on the degree in which representatives of this level have to compensate representatives of the zero technology level.

The introduction of irrigated paddy, i.e. of technology level two, can be interpreted in a similar way. The role played by uncleared land in case of technology level one is now played by unirrigated land. Thus one could say that unirrigated land is the raw material for technology level two. And the latter can only grow at the expense of technology level one. Let us therefore first look at technology level two in isolation and consider unirrigated land as a raw material. Starting from the technology matrices described under case C, the relevant input-output matrices can be obtained by deleting both the first process and the first good. Thus:

Case D: Paddy growing under irrigated conditions

Input Matrix

$$\begin{array}{l|ccc|} & \text{irrigated land} & \text{labour} & \text{paddy} \\ \text{irrigated paddy growing} & 1 & 2 & 0 \\ \text{investment in irrigation} & 0 & 1 & 0 \\ \text{subsistence consumption} & 0 & 0 & 1 \end{array}$$

Ouput Matrix

$$\begin{array}{l|ccc|} & \text{irrigated land} & \text{labour} & \text{paddy} \\ \text{irrigated paddy growing} & 1 & 0 & 2.441 \\ \text{investment in irrigation} & 1 & 0 & 0 \\ \text{subsistence consumption} & 0 & 1.0408 & 0 \end{array}$$

The equilibrium growth factor can be calculated as:

$\alpha = \beta = 1.1$

The corresponding intensity and price vectors are:

$x = (.30, .03, .67)$

$y = (.3611, .3283, .3106)'$

Thus one can say that the new technology can drive out the old one at a rate of 10 per cent a year if no compensation has to be paid. As long as the land belongs to the same actor this assumption is not too unrealistic. If, however the technology is attributed to a new actor, the growth rate of the sub-sector, (which is not a sub-economy in the KMT-sense), will be lower than 10 per cent because the investment in irrigation process requires some additional input to compensate the owner of the unirrigated land. Thus the growth rate of technology level two will lie somewhere between zero and ten per cent. But since level two can only grow at the cost of level one, the growth rate of the latter will be negative if the growth rate of the former is positive. The overall growth rate of the economy will lie somewhere between the growth rates of level one and level two. Since the absolute importance of the two technologies will change over the years, the overall growth rate will be different each year. The situation will continue until all rainfed land is irrigated. Then the economy comes again into a stationary state. And renewed growth is only possible by introducing a new technology. Let us assume that a fertilizer/irrigation technology is introduced next. If the latter has a positive growth rate it can pay a rent to technology level two and the fertilizer/irrigation technology can drive out the irrigated paddy technology, etc. Thus, by looking at the different technologies in isolation and by dropping the balanced growth requirement for some goods, the model can, in principle, explain why certain institutional structures lead to higher growth rates than others.

3.5.4 Aggregation of land types

The use of the Von Neumann model, as in section 3.5.3, falls within Hicks' definition of pure economic theory ([34], chapter 1), i.e. the model leads to a statement about the real world while no use needs to be made of any empirical datum at all. An empirical elaboration of a national economy along the lines of section 3.5.3 would indeed be very hard to substantiate. Analyzing and quantifying unbalanced growth as a consequence of power relations which are on their turn a reflection of all kinds of institutional arrangements is not an easy matter within the Von Neumann framework. It is also not the focus of our study. In this section we shall propose an alternative 'solution' for the 'problem of land', or, more generally, the problem of non-tradeable non-augmentable resources. We start by claiming that the problem is, properly speaking, a very general one and does not only occur to non-tradeable non-augmentable resources. For whatever the economy, all its goods are unique. And because reproduction implies cardinality, reproduction is only possible if one classifies different variables in the same category. Thus, one always has to generalize with regard to qualitatively different goods. If the principle of balanced (= proportional) growth is pushed to the extreme, growth is always impossible. Instead, one can speak of qualitative change only. Because this is not a very workable starting point for an economist studying economic growth, a usual way out in both theoretical and empirical work is to proceed from the assumption that there are only a limited number of different goods in the economy under study. Aggregation of goods implies full substitution and in principle it is only allowed if the implicit substitution assumption is not too rigorous in the context of the problem at hand. As Kogelschatz [48] and Filimon [20] show, aggregation leaves the growth factor of a Von Neumann model only under very stringent conditions unaffected.

Turning back to the problem of land, a way out must also be found through aggregation. Both acceptable from a theoretical point of view and workable in an empirical sense, is to give up the distinction between cleared land on the one hand and irrigated land on the other. Instead, both landtypes can be aggregated into a newly defined good which we call 'capacity to grow paddy'. Because analyzing an economy on its balanced growth possibilities allows, in terms of policy relevance, a looser interpretation than proportional growth of both cleared and irrigated land, this aggregation procedure poses, from a theoretical point of view no difficulty, especially if it is realized that differences in input structure are still taken into account. Empirically aggregation is acceptable if proper weights can be found to add the goods. In case of our example, the actual production capacities could be used.

There is however, still one problem which needs some attention. It can be explained by working out the example. We proceed from case C above. Unirrigated and irrigated land are aggregated according to their respective capacities to grow paddy. It can easily be checked that according to that criterion one unit of irrigated land equals 2.4 units of unirrigated land. The following input-output matrices are now relevant to the economy of our example:

Case E: Aggregating landtypes

Input Matrix

$$\begin{array}{l|ccc|}
 & \text{Capacity to grow paddy} & \text{Labour} & \text{Paddy} \\
\text{paddy (rainfed)} & 1 & 1 & 0 \\
\text{paddy (irrigated)} & 2.4 & 2 & 0 \\
\text{investment in irrigation} & 1 & 1 & 1 \\
\text{subsistence consumption} & 0 & 0 & 1
\end{array}$$

Output Matrix

$$\begin{array}{l|ccc|}
 & \text{Capacity to grow paddy} & \text{Labour} & \text{Paddy} \\
\text{paddy (rainfed)} & 1 & 0 & 1.0202 \\
\text{paddy (irrigated)} & 2.4 & 0 & 2.4410 \\
\text{investment in irrigation} & 2.4 & 0 & 0 \\
\text{subsistence consumption} & 0 & 1.0408 & 0
\end{array}$$

The model has only one growth factor which can be calculated as:

$$\alpha = \beta = 1.085$$

The corresponding intensity and price vectors are:

$$x = (.0,\ .294,\ .046,\ .660)$$
$$y = (.296,\ .359,\ .345)'$$

In equilibrium the economy can grow at a rate of 8.5%, which is slightly below the growth potential of the irrigation sector but well above the stationary state solution of the overall economy (respectively case D and case C, in section 3.5.2).

The problem referred to earlier becomes clear if one looks at the intensity vector: in the equilibrium solution only irrigated paddy is grown. Of course this is an unsatisfactory situation. The cause is not difficult to find. The difference between growing rainfed paddy and irrigated paddy with regard to input requirements has both a qualitative and a quantitative aspect. By aggregating away differences in landtypes, only the quantitative differences remain. As irrigated paddy appears to be more efficient per unit of paddy growing capacity than rainfed paddy, only the former has a strictly positive intensity in the solution.

To avoid such unrealistic solutions there exist two alternatives. First, upper and lower bounds can be introduced on activities. For example, if the economy has ten units of unirrigated land and one unit of irrigated land at its disposal, $x_1 \leqq 10$ and $x_2 \leqq 1$ can be added to the model. To generate a non-zero solution for x also a lower bound has to be introduced, for this labour availability could be used. This has been explained in section 3.4.

Another way to tackle the problem is to aggregate all processes in which the aggregated good appears, either as input or as output. As weights one could use the (maximum) intensities in a base year solution. Aggregation of processes is in a sense the opposite of aggregation of goods. As we shall show in a more formal way in the next section, a consequence of the latter is that the growth rate increases (or stays at the same level) while the aggregation of processes leads to a lower (or equal) growth rate. Aggregation of processes is, in principle, only allowed if one may assume that the relevant processes are carried out in fixed proportions, i.e. no substitution is allowed. A discussion on the consequences of such an aggregation can best take place by continuing our example.

We proceed from the input-output matrices of case E above. Suppose that the ratio in which rainfed and irrigated paddy growing can take place is 10 to 1. We use these numbers to aggregate the two paddy growing processes. After proper normalization we get the following input-output matrices:

Case F: Aggregating processes

Input Matrix

$$\begin{array}{l|ccc|}
 & \text{Capacity to grow paddy} & \text{Labour} & \text{Paddy} \\
\text{paddy growing} & 1.127 & 1.091 & 0 \\
\text{investment in irrigation} & 1 & 1 & 0 \\
\text{subsistence consumption} & 0 & 0 & 1
\end{array}$$

Output Matrix

$$\begin{array}{l|ccc|}
 & \text{Capacity to grow paddy} & \text{Labour} & \text{Paddy} \\
\text{paddy growing} & 1.127 & 0 & 1.1465 \\
\text{investment in irrigation} & 2.4 & 0 & 0 \\
\text{subsistence consumption} & 0 & 1.0408 & 0
\end{array}$$

The model has only one growth factor which can be calculated as:

$$\alpha = \beta = 1.033$$

The corresponding intensity and price vectors are:

$x = (.468, .013, .519)$

$y = (.2735, .3618, .3647)$

Thus the equilibrium growth rate is 3.3% per period. Compared with the 8.5% in case E, this is a firm decrease.

If instead of aggregating the paddy growing processes, upper bounds on x_1 and x_2 and a lower bound on labour use are introduced, the former in the ratio 10 to 1 and the latter according to the solution of case E, the resulting growth factor and intensities are the same. Prices do however differ: a positive shadow price appears for the upper bound of process 2 (irrigated paddy growing) and a negative shadow price appears for labour.

However, regardless which procedure we follow, i.e. either we aggregate processes or we introduce bounds, the balanced growth path is left.

In case of process aggregation this can be seen as follows: After one period the quantity of irrigated land is increased while the quantity of unirrigated land has decreased. As a consequence the weights by which the processes are aggregated have to be changed. Thus the A and B matrices in period t+1 differ from the matrices in period t. And the model solution will differ also. More or less the same argument applies in case bounds are introduced. Referring to the example, the upper bound on x_1 decreases with the amount with which x_2 increases.

3.6 A THEOREM ON AGGREGATION

As was shown in the preceding section the problem of non-tradeable non-augmentable scarce resources can be tackled by aggregating these goods with nearby substitutes. It was also explained why aggregation of goods (and processes) will, in general, not be limited to this kind of goods. Because of their great number, aggregation of qualitatively different goods and processes has to take place in all empirical national or sectoral modeling work in order to keep the models manageable. According to Zauberman e.g., in the Sovjet Union the list of commodities - the so-called 'nomenclature' - contains around 15 million items ([114], p. 9). If for all practical purposes goods can be considered as substitutes, there is, in principle, no objection why they would not be treated as such. In the Von Neumann context it is clear that aggregation affects the growth factor(s): aggregation results in a change in the A and B matrices and there is a priori little reason to assume that such a change will not result in a different growth factor.

Although one cannot say in general anything on the size of the change, something can be said on the direction of the change of the growth factor.

Theorem 3.4

(a) Process aggregation results in (an) unchanged or (a) lower growth rate(s).

(b) Goods aggregation results in (an) unchanged or (a) higher growth rate(s).

Proof (see MT [69], p. 91):
We start with defining a p x m ($1 \leq p \leq m$) process aggregating matrix P and an n x q ($1 \leq q \leq n$) goods aggregating matrix Q.

It will be clear that P has one positive entry per column and at least one positive entry per row; on the other hand Q will have one positive entry per row and at least one positive entry per column. Next we look at the effect on α of process aggregation. Suppose the triple $(\bar{x}, \bar{y}, \bar{\alpha})$ is a Von Neumann solution to M_{α}. Then:

$$v(M_{\bar{\alpha}}) = \max_i M^i_{\bar{\alpha}} \bar{y} = 0 \quad (i=1,2,\ldots,m)$$

where $M^i_{\bar{\alpha}}$) is the i-th element of $M_{\bar{\alpha}}\bar{y}$. Process aggregation results in a matrix PM_{α}. It is easy to see that since $M^i_{\bar{\alpha}} \bar{y} \leq 0$ (eq. 2.23) and $P \geq 0$, $PM^i_{\bar{\alpha}} \bar{y} \leq 0$.
And

$$v(PM_{\bar{c}}) \leq \max_i PM^i_{\bar{\alpha}} y \leq \max_i PM^i_{\bar{\alpha}} \bar{y} \leq 0$$

by which the first part of the theorem is proven. The proof of part (b) is similar:

$$v(M_{\bar{\alpha}}) = \min_j \bar{x} M^j_{\bar{\alpha}} = 0 \quad (j=1,2,\ldots,n)$$

where $xM^j_{\bar{\alpha}}$ is the j-th element of $\bar{x} M_{\bar{\alpha}}$. Goods aggregation results in a matrix $M_{\alpha}Q$. Now we can state:

$$v(M_{\bar{\alpha}} Q) = \min_j x M^j_{\bar{\alpha}} Q \geq \min_j \bar{x} M^j_{\bar{\alpha}} Q \geq 0$$

which is a proof of the second part of the theorem. □

3.7 NON-BALANCED GROWTH IN CONSUMPTION AND REGIME-SWITCHES

3.7.1 Non-balanced growth in consumption

As a consequence of the introduction of initial endowments and the allowance of aggregation, the tight restrictions of balanced growth have been relaxed to a certain extent. In this section we shall discuss a third 'unbalanced' model-adjustment, viz. the possibility of different growth rates for different consumption goods.

A major limitation of the consumption model of section 3.3 is that consumption of all goods increases at the same uniform rate $(\alpha-1)$. Thus, if $\bar{\bar{c}}_{jt}$ is total consumption of good j in period t:

$$\bar{\bar{c}}_{j,t+1} = \alpha\bar{\bar{c}}_{jt}$$

for all j and all t. We propose an adjustment on three points:

(i) to take income elasticities[33] into account,

(ii) to introduce population growth explicitly into the model, and

(iii) to allow for an overall growth rate of consumption deviating from $(\alpha-1)$.

As a starting point we assume that expenditures per capita (per household or per unit of labour) on good j can be described by a linear expenditure system (les):

$$p_j c_j = \mu_j(y-\Sigma p_k\bar{\bar{c}}_k) + p_j\bar{\bar{c}}_j \qquad (3.5)$$

where

c = consumption

$\bar{\bar{c}}$ = committed consumption

p = price

y = total expenditure

μ = marginal budget share $(\mu>0,\ \sum_j \mu_j = 1)$

According to (3.5) expenditures on good j can be decomposed into two components: the first part is spent on a committed quantity of good j; the

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[33] In this section 'income' stands for total expenditures on consumption.

second part is a fraction μ_j of the so-called 'super-numerary income' which is the income above the amount required for financing all committed quantities[34]. If we divide (3.5) by the price p_j we get:

$$c_j = \frac{\mu_j y}{p_j} - \frac{\Sigma p_k \bar{\bar{c}}_k}{p_j} + \bar{\bar{c}}_j \tag{3.6}$$

As has been discussed in section 3.3, consumption is assumed to be unaffected by changing prices. This rigid assumption is necessary to stay within the computationally attractive Von Neumann framework[35]. Thus, volume of consumption of good j can be split in a fixed part and a part linearly dependent on income, or

$$c_j = k_j y + \bar{\bar{d}}_j \tag{3.7}$$

where $k_j y$ and $\bar{\bar{d}}_j$ stand for the first and the second and third term of the right-hand side of (3.6) respectively. It will be clear that d_j can be both positive, negative or zero, while $k_j y$ will always be strictly positive. If (3.7) is multiplied by P, i.e. total population, one gets the consumptive demand for good j by the whole economy:

$$C_j = k_j Y + \bar{\bar{D}}_j \tag{3.8}$$

If total expenditures grow at a rate α while there is no growth in population the dynamic form of (3.8) is

$$C_{jt} = \alpha^t k_j \, Y_o + \bar{\bar{D}}_j \tag{3.9}$$

Depending on the sign of $\bar{\bar{D}}_j$, consumption of good j grows faster or slower than overall consumption. If $k_j Y$ is divided by total labour availability, a matrix $\tilde{C}$ analogous to matrix C in section 3.3 can be constructed which can be incorporated into the model structure. Vector $\bar{\bar{d}}$, i.e. the vector with

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[34] See Barten [7] or Deaton [17] for details on the linear expenditure system.

[35] See also footnote 29 of section 3.3.

elements $\bar{\bar{D}}_j$ appears on the right-hand side of axiom 1. The physical side of the economy can now be described as follows[36]:

$$\begin{aligned} x(B-\alpha(A+\tilde{C})) &\geqq \bar{\bar{d}} \\ xE &\leqq u \\ L^{\ell} \leqq x\ell &\leqq L^{u} \end{aligned} \tag{3.10}$$

Vector $\bar{\bar{d}}$ guarantees a non-zero x-solution. The lower bound on labour can therefore, in principle, be left out. An (α,x)-solution of (3.10) for period t will not necessarily satisfy for period t+1. The latter is a direct consequence of keeping $\bar{\bar{d}}$ exogenous. A discussion how to deal with this problem will be postponed to the end of this section.

A drawback of the treatment of consumption hitherto is the assumption of zero population growth. The introduction of non-zero growth can however be done in a straightforward manner. Suppose population grows at a rate $(\delta-1)$. If total expenditures increase at a rate $(\alpha-1)$, per capita growth is α/δ. Thus, the dynamic form of (3.7) becomes:

$$c_{jt} = (\frac{\alpha}{\delta})^{t}\ k_j y_o + \bar{\bar{d}}_j \tag{3.11}$$

Multiplied by total population $(=\delta^{t}P)$ yields:

$$C_{jt} = \alpha^{t} k_j Y_o + \delta^{t}\bar{\bar{D}}_j \tag{3.12}$$

Incorporation of (3.12) into (3.10) implies that each period $\bar{\bar{d}}$ has to be multiplied by δ.

3.7.2 A deviating overall growth rate of consumption

In the discussion so far we have implicitly assumed that there is no difference between the growth rate of total expenditures on consumption and the overall growth rate of the economy, i.e. both are $(\alpha-1)$. It is

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[36] The complete set of axioms will be presented after all 'consumption-adjustments' have been discussed.

interesting to allow for different growth rates. Suppose growth rates of total consumption ($\alpha'-1$) and investment (incl. intermediate goods) ($\alpha-1$) are in the proportion $\bar{\bar{\mu}}$. Then,

$$\alpha' = \bar{\bar{\mu}}(\alpha-1) + 1 \qquad \text{(3.13}$$

Substituting (3.13) into (3.12) yields:

$$C_{jt} = [\bar{\bar{\mu}}(\alpha-1)+1]^{t}k_jY_o + \delta^{t}\bar{\bar{D}}_j \qquad \text{(3.14}$$

If we limit the analysis to a comparison between period t=1 and period t=0, we may write (leaving out the time-subscript):

$$C_j = \alpha\bar{\bar{\mu}}k_jY + (1-\bar{\bar{\mu}})k_jY + \delta\bar{\bar{D}}_j \qquad \text{(3.15}$$

which can be incorporated in (3.10). The expression for $\bar{C}$ can be derived from $\alpha\bar{\bar{\mu}}k_jY$ as in section 3.3; the expression for $\bar{\bar{d}}$ is extended with the second term of the right-hand side of (3.15). Thus,

$$\bar{\bar{d}}^{1} = \begin{bmatrix} \bar{\bar{d}}_1 \\ \bar{\bar{d}}_2 \\ | \\ | \\ | \\ \bar{\bar{d}}_p \end{bmatrix} = \begin{bmatrix} (1-\bar{\bar{\mu}})k_1Y + \delta\bar{\bar{D}}_1 \\ (1-\bar{\bar{\mu}})k_2Y + \delta\bar{\bar{D}}_2 \\ | \\ | \\ | \\ (1-\bar{\bar{\mu}})k_pY + \delta\bar{\bar{D}}_p \end{bmatrix}$$

An example:

We illustrate the above by means of an example of a two-good economy. In terms of equation (3.15) we assume the following parameter values: $Y_o=10$, $\bar{D}_1=10$, $\bar{D}_2=-3$, $k_1=.4$, $k_2=.5$ and $\alpha=1.2$.

Table 3.1 contains the resulting per capita consumption levels of three scenarios. In scenario 1 it is assumed that there is no population growth, in scenarios 2 and 3 on the other hand, population growth is set at 5% per period. Furthermore, scenarios 1 and 2 proceed from the assumption that expenditures grow at the same rate as the overall growth rate of the economy while in scenario 3, $\overline{\overline{\mu}}$ is set at .4.

TABLE 3.1: Per capita consumption

	Scenario 1		Scenario 2		Scenario 3	
	good		good		good	
Period	1	2	1	2	1	2
0	14.0	2.0	14.0	2.00	14.0	2.0
1	14.8	3.2	14.6	2.70	14.1	2.2
2	15.8	4.2	15.2	3.53	14.2	2.5
3	16.9	5.6	15.6	4.46	14.4	2.8
4	18.3	7.4	16.0	5.53	14.5	3.0
5	20.0	9.4	16.6	6.78	14.6	3.3

The differences in consumption levels are striking. For good 1, per capita growth over the five periods varies from 4.3% in scenario 3, to 42.8% in scenario 1. For good 2 these percentages are, for the same scenarios, 65% and 370% respectively. Given the above, the interpretation of these differences will need no further explanation.

3.7.3 The full model with consumption

The complete set of axioms can now be stated as follows:

Axiom (cc1): $x_t(B-\alpha_t(A+\overline{\overline{\mu}}\tilde{C}_t)) \geqq \delta^t\overline{\overline{d}}$

Axiom (cc2): $x_tE \leqq u_t$

Axiom (cc3): $x_t\ell \geqq L_t$

Axiom (cc4): $(x_t(B-\alpha_t(A+\overline{\overline{\mu}}\,\tilde{C}_t)) - \delta^t\overline{\overline{d}})y_t = 0$

Axiom (cc5): $(B-\beta_tA)y_t - \beta_t\ell y_t^W - Ey_t^m \leqq 0$

Axiom (cc6): $x_t((B-\beta_tA)y_t - \beta_t\ell y_t^W - Ey^m{}_t) = 0$

Axiom (cc7): $x_tBy_t > 0$

An equilibrium solution to the model is defined as a sextuple:

$$Q = \{x,y,y^W,y^m\alpha,\beta \mid x\geqq 0,\ y\geqq 0,\ \alpha>0,\ \beta>0,\ \Sigma x>0,\ \Sigma y>0\}$$

that for given A, B, $\tilde{C}$, u, ℓ, L, δ, $\overline{\overline{d}}$ and $\overline{\overline{\mu}}$ satisfies the axioms. Because x, y, y^W, y^m,α and β are the only endogenous variables we have not left the Von Neumann framework. However, because $\tilde{C}$, $\overline{\overline{d}}$, L and u have to be updated each period, the solution applies to one period only.

It is not self-evident that the set of axioms has a solution. For example if $\delta^t\overline{\overline{d}}$ is set too high relative to u_t, axioms (cc1) and (cc2) cannot be satisfied at the same time. To avoid such infeasibilities we normalize the model. As was the case with the model with capacity constraints (section 3.4.2), the normalization implies a slight modification, i.e. α_tL_t,α_tx_tE and $\alpha_t\delta^t\overline{\overline{d}}$ must be substituted for L_t, x_tE and $\delta^t\overline{\overline{d}}$ in the above. The normalization is schematically shown in figure 3.5. A solution is guaranteed by theorem 3.4. If we take $\tilde{C} = \ell\overline{\overline{c}}$, where $\overline{\overline{c}}$ may be any non-negative row-vector, the expression for the wage-rate becomes

$$y^W = \overline{\overline{\mu}}\,\overline{\overline{c}}y - \frac{1}{\beta}\,y^L$$

Because of the similarity between the full model with consumption and the model with capacity constraints, we refer to section 3.4.2 for further details.

$$\begin{array}{c|c|c|c|}
 & y & y^m & y^L \\ \hline
x & B & 0 & \ell \\ \hline
x_n & 0 & u & 0 \\ \hline
\end{array}
\qquad
\begin{array}{c|c|c|c|}
 & y & y^m & y^L \\ \hline
x & A + \bar{\bar{\mu}}\,\tilde{C} & E & 0 \\ \hline
x_n & \delta\bar{\bar{d}} & 0 & L \\ \hline
\end{array}$$

output matrix $\tilde{B}$ input matrix $\tilde{A}$

Figure 3.5
Normalized full model with consumption

3.7.4 Regime-switches

In the empirical part of our study we shall mainly use the full model with consumption. However, as has been explained above, the solution applies to one period only. This is a severe drawback. A possible way out is to solve the model on a period-by-period base. Thus, after solving it for t=1, the relevant parameters are adjusted and the model is solved for t=2, etc. In this way we could sketch a semi-balanced development path of the economy through time. Under alternative assumptions regarding international prices, population growth, increases in efficiency, alternative technologies, etc. interesting comparisons among potential growth paths can be made.

We have only partly followed this most advisable line. The reasons for this are twofold. On the one hand it would mean a very substantial amount of extra work. Adjusting the parameters between the years will take a lot of bookkeeping and programming. On the other hand solving the model sequentially for a number of periods, say 10, would be rather expensive computationally[37].

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[37] We have to admit, however, that the implementation of the algorithm (see chapters 4 and 6) has probably not been done in the most efficient way.

A second, more modest procedure to overcome the drawback is to update only when important changes occur and to use the solution for t=1 to approximate the solution for t=2. Identification of important changes can be done as follows. If we start from an equilibrium solution for period t, the following commodity balance applies for a non-overproduced good:

$$\text{production}_t - \text{intermediate demand}_t - \text{consumption}_t = \text{net export}_t$$

In case of balanced growth, the same equation would hold in period t=2, except that the actual numbers would have to be multiplied by a factor α. Thus,

$$\alpha(\text{production}_t - \text{intermediate demand}_t - \text{consumption}_t) = \alpha \text{ net export}_t$$

In case consumption increases at a rate $\Pi \neq \alpha$, the following balance can be derived for t=2 from the solution for t=1:

$$\alpha(\text{production}_t - \text{intermediate demand}_t) - \pi \text{ consumption} = k_1 \text{ net export}_t$$

where

$$k_1 = \frac{\alpha \text{ net export}_t + (\alpha - \pi) \text{ consumption}_t}{\text{net export}_t}$$

Thus trade activities are treated as the adjusting variables. For t=3 we get:

$$\alpha^2 \, (\text{production}_t - \text{intermediate demand}_t) -$$
$$\pi^2 \text{ consumption}_t = k_2 \text{ net export}_t$$

where

$$k_2 = \frac{\alpha^2 \text{ net export}_t + (\alpha^2 - \pi^2) \text{ consumption}_t}{\text{net export}_t}$$

etc.

In this way diagrams as in figure 3.6 can be constructed and regime-switches can be identified. Such switches are interesting because if a country changes from exporter to importer, or vice versa, the relevant price for the good changes drastically[38]. At these points the extrapolation does not apply any more and it becomes necessary to solve the model again, of course after adjustment (extrapolation) of the relevant parameters.

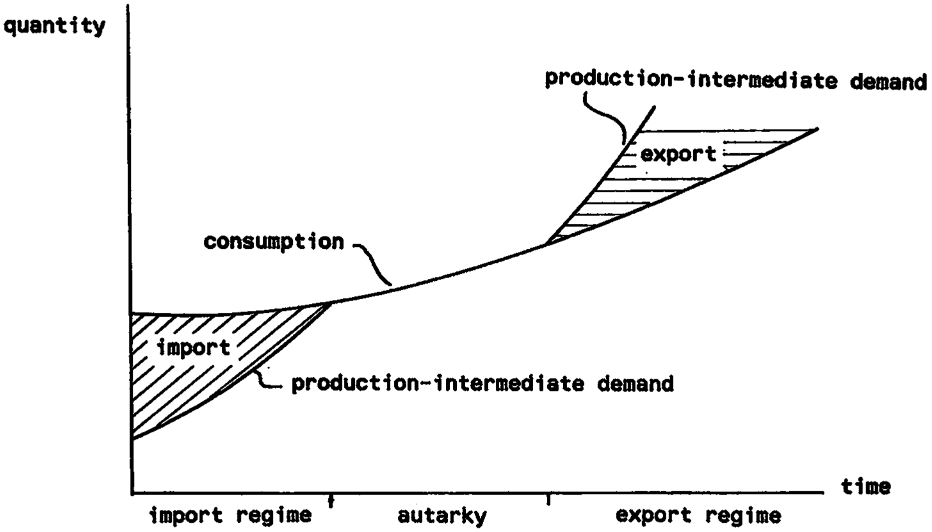

Figure 3.6
Regime-switches

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[38] E.g. the quotient borderprice import/export price export for rice in Bangladesh is estimated at 2.21 in 1985, (UNDP [103], volume II).

Chapter 4

COMPUTING THE GROWTH FACTORS OF THE MODEL

4.1 INTRODUCTION

Theorem 2.4 established the existence of at least one equilibrium growth factor to the model. In the subsequent theorem 2.6 it was shown that there is also an upper bound to the number of growth factors. The proofs of both theorems are non-constructive, i.e. although they prove the existence of at least one and at most min (m,n) growth factors, they do not tell us how we can find them. For small problems this does not lead to difficulties: by some ad-hoc 'trial and error' method the solutions can usually be found. However, if one wants to apply the model to a real world situation, its size readily becomes so large that such 'trial and error' methods do not suffice anymore. Instead one needs an efficient computational procedure to solve the model. Such procedures (algorithms) are the subject of this chapter.

In the course of time a number of algorithms have been developed by different authors. The restatement of the model in game-theoretic terms (section 2.3.3) and the similarity between solving matrix games and linear programming models (see appendix A) particularly stimulated the search for methods to find the growth (and interest) factors.

The algorithms thus far available can be divided into three categories:

(a) Algorithms that find only one growth factor. The main members of this category are:

    (i) The algorithm of Burley [14]. In 1971 Burley described a procedure for calculating the growth factor of the model in the original Von Neumann formulation.

  - (ii) As a competitor to the HTW-algorithm (see hereafter), Bose and Bose [11] presented in 1972 an alternative algorithm for calculating the maximum growth factor of the model.
  - (iii In 1974 Robinson [80] developed a method for finding the unique growth factor of the so-called 'irreducible' Von Neumann model[39].

(b) Algorithms that find both the maximum and minimum growth factor:
  - (i) In 1967 Hamburger, Thompson and Weil (HTW) [30], developed a simple bisection method through which both α_{min} and α_{max} can be found.

(c) Algorithms that find all growth factors irrespective of the uniqueness. Two interesting members of this category are:
  - (i) The algorithm of Weil [108]. By combining the HTW-algorithm with a procedure to decompose the matrix M_α, Weil showed a way to find all growth factors.
  - (ii) In 1974 Thompson [98] presented a new algorithm which also employs the HTW-technique and which makes, in addition, extensive use of the concept of a central solution to linear programming models.

Because it finds both α_{min} and α_{max} and is, in addition, relatively easy to implement, the HTW-bisection method is superior to the algorithms of Burley, and Bose and Bose and Robinson. Therefore we shall not discuss the algorithms mentioned under (a). For the empirical part of the study we need an algorithm that finds all growth factors. However, neither the algorithm of Weil nor the algorithm of Thompson appears to be appropriate. Both are, in our opinion, too cumbersome to apply. The former because of the complicated decomposition procedure, the latter because of the difficulties of finding central solutions to large linear programming models. Therefore we shall in section 4.3 propose an alternative procedure which does not suffer

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[39] A Von Neumann model is said to be irreducible if it satisfies Gale's requirements on technology. The latter is the case if each process uses/produces each good, either directly or indirectly. For details see Gale ([23], p. 315).

from these drawbacks and which also finds all growth factors of the model. In section 4.3.7 the working of the algorithm will be illustrated by a small example.

4.2 CURRENT ALGORITHMS

4.2.1 A simple bisection procedure

The HTW-algorithm finds both α_{min} and α_{max} for which $v(M_\alpha) = 0$. From the existence theorem (theorem 2.4) it is known that these α's are both economic, i.e. for these α's (not necessarily unique) pairs (x,y) exist such that xBy > 0. For easy reference the general form of $v(M_\alpha)$, already shown in figure 2.3, is repeated in figure 4.1. For any α, $v(M_\alpha)$ can be found by solving a linear programming problem (see appendix A). In order to find the minimum and maximum growth rates HTW [31] propose a simple search-procedure. The algorithm starts with choosing an α_ℓ for which $v(M_\ell) > 0$[40] and an α_u for which $v(M_u) < 0$. Because $v(M_\alpha)$ is non-increasing, both α_{min} and α_{max} are contained in the interval $[\alpha_\ell, \alpha_u]$.

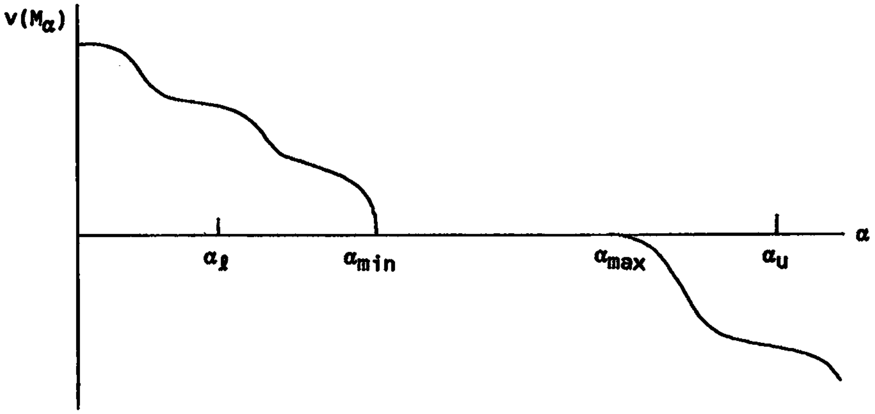

Figure 4.1
Illustration of the HTW-algorithm

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[40] For notational convenience we shall often write $v(M_k)$ instead of $v(M_{\alpha_k})$.

It is easily seen now how the procedure works. By taking the average of α_u and α_ℓ and solving the model for the resulting $\tilde{\alpha}$, the interval containing $\alpha_{min}(\alpha_{max})$ is halved. After substituting $\tilde{\alpha}$ for $\alpha_\ell(\alpha_u)$, the procedure is repeated, etc. The algorithm stops if the length of the interval is smaller than ε, where ε is some prescribed small number. Both the algorithms of Weil and Thompson, which will be discussed in the remainder of this section, and the alternative procedure proposed in section 4.3 make extensive use of the HTW-bisection technique, which is in essence a very old and wellknown technique.

Before the HTW-algorithm can be used in practice, one additional problem has to be solved. Depending on the initial estimates of α_ℓ and α_u and the size of ε, solving a real world application will require, say 8 to 20 iterations (see also chapter 7). This means that 8 to 20 different linear programming models have to be formulated. It will be clear (at least for everyone having some experience in constructing linear programming models), that if the model contains more than, say, 10 goods and 20 processes, it becomes, for all practical purposes, too time-consuming to formulate these new problems 'by hand'. In other words, the HTW-algorithm can in practice only be applied if a matrix generator is at hand by which the linear program is automatically adjusted each time a new α has to be tested. We shall discuss this matter in more detail in chapter 6.

4.2.2 Weil's algorithm

In [108] Weil describes a method to find all economic growth factors. His algorithm combines the HTW-algorithm with a decomposition procedure. Because the HTW-technique has already been discussed in the preceding section, we shall concentrate here on the decomposition part. The algorithm consists of the following three steps:

a) Decompose M_α into p disjoint sets of activities $I_1, I_2, \ldots, I_p$.
b) Use the HTW-algorithm to find α_{min} and α_{max} of each subset $H_i = (I_1, \cup \ldots\ldots \cup I_i)$ for $i=1,\ldots,p$.
c) Test all thus found α's in the whole economy.

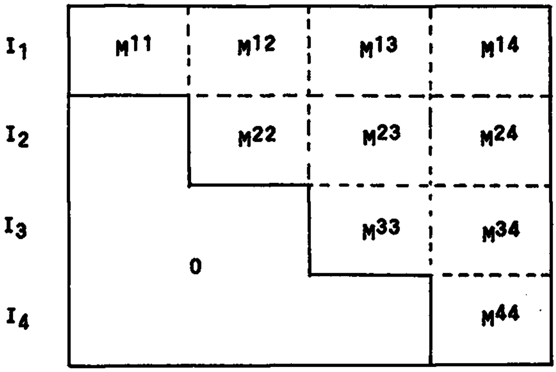
<u>Ad (a)</u>: Matrix M_α is said to be decomposable if the rows and columns can be simultaneously permuted such that M_α is, irrespective of the value of α, of the form:

$$\begin{bmatrix} M_\alpha^{11} & M_\alpha^{12} \\ 0 & M_\alpha^{22} \end{bmatrix} \tag{4.1}$$

If one of the (secondary) diagonal blocks M^{ij} is decomposable,(4.1) can be refined and changed into:

$$\begin{bmatrix} M_\alpha^{11} & M_\alpha^{12} & M_\alpha^{13} \\ 0 & M_\alpha^{22} & M_\alpha^{23} \\ 0 & 0 & M_\alpha^{33} \end{bmatrix}$$

Thus, in theory, there may be many ways to decompose M_α. For the algorithm only the so-called 'canonical form' is of interest. The latter is defined as the, not necessarily unique, matrix of rearranged goods and processes for which the (secondary) diagonal block M_α^{11} is as large as possible without being decomposable, M_α^{22} is as large as possible given M_α^{11} but is itself also not decomposable, and so on (see Weil [107], p. 264). By the canonical form the processes of the economy are divided into disjoint subsets I_i, $i=(1,2,\ldots,p)$, where the processes in I_i correspond to the processes of the diagonal block M^{ii} (see figure 4.2).



<u>Figure 4.2</u>
Canonical form of M_α

Weil refers to two procedures to decompose M_α, one based on graph-theoretic techniques, elaborated in Weil [107] and Kemeny, Snell and Thompson [40], and one described in Weil and Kettler [109] which is based on a well-known technique of Steward for analyzing the structure of large systems of equations [89].

Since we are not going to use the algorithm, we do not discuss the procedures; for these we refer to the above articles.

Ad (b): To all H_i the HTW-algorithm is applied. Thus, referring to figure 4.2, $\alpha_{\min}$ and $\alpha_{\max}$ are successively calculated of I_1, $(I_1 \cup I_2)$, $(I_{15} \cup I_2 \cup I_3)$ and $(I_1 \cup I_2 \cup I_3 \cup I_4)$. This procedure results in at most 2P different growth factors. According to a theorem of Weil ([108], p. 279) all economic growth factors of the original economy are contained in the set $\overline{P} \leq 2P$ found in this manner. However the converse is not necessarily true, i.e. each growth factor in $\overline{P}$ need not be economic in M_α. Consider, for example, the following decomposable economy:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 5 & 6 \\ 3 & 0 & 0 \end{bmatrix} \tag{4.2}$$

The canonical form of (4.2) is:

$$M_\alpha = \left[\begin{array}{ccc} 5-\alpha & 6-\alpha & 1-\alpha \\ 0 & 4-\alpha & 2-\alpha \\ 0 & 0 & 3-\alpha \end{array}\right] \begin{array}{l}]\, I_1 \\ ]\, I_2 \\ ]\, I_3 \end{array}$$

Carrying out step 2 of the algorithm results in three different growth factors, viz. $\alpha_1 = 1$ in I_1, $\alpha_2 = 2$ in $I_1 \cup I_2$ and $\alpha_3 = 3$ in $I_1 \cup I_2 \cup I_3$. However, only one of them is economic in M_α, viz. α_3, therefore step (c) is needed.

Ad (c): In this step all growth factors belonging to $\overline{P}$ are tested in the whole economy. For the example above, $v(M_1)$ and $v(M_2)$ turn out to be strictly positive which means that growth factor candidates $\alpha_1 = 1$ and

α_2=2 have to be dropped. Only α_3=3 survives the test. Thus, α_1=1 and α_2=2 are economically only feasible if process 1, respectively processes 1 and 2 are looked at in isolation.

4.2.3 The algorithm of Thompson

In 1974 Thompson [98] proposed an ingenious but rather complex algorithm. It can be considered as an extension of the HTW-technique: by utilizing the concept of a central solution to linear programming problems not only α_{min} and α_{max} but also all intermediate economic growth factors can be found. The algorithm leans on two lemmas (lemma 3 and lemma 4 in [98]) according to which a non-economic α contains a strictly positive left and right neighbourhood that do not contain any economic α. Figure 4.3 shows this property graphically. It follows directly from the fact that the maximum number of growth factors cannot exceed min (m,n). Therefore the set of economic α's is by definition disjoint.

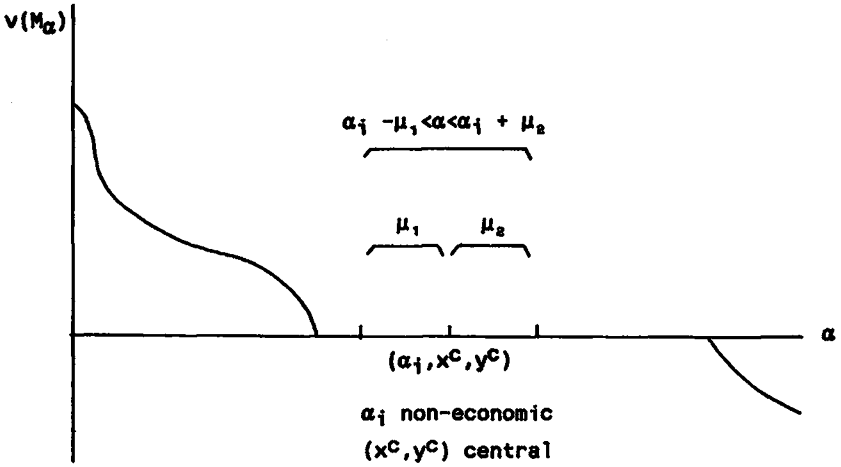


Figure 4.3
Non-economic neighbourhood of α_i

Thus given α_i non-economic, the domain $S = \{\alpha \mid \alpha_i - \mu_1 < \alpha < \alpha_i + \mu_2\}$ does not contain any economic α, where μ_1 and μ_2 are determined by:

$$\mu_1 = \{\max \mu \mid x^c M_{\alpha_i+\mu} \gtrsim 0, \mu \gtrsim 0\}$$

and

$$\mu_2 = \{\max \mu \mid M_{\alpha_i-\mu} y^c \lesssim 0, \mu \gtrsim 0\}$$

The algorithm starts as the HTW-algorithm, i.e. it starts with non-negative numbers α_ℓ and α_u such that $v(M_\ell) > 0$ and $v(M_u) < 0$. As was proven by theorem 2.2, such α's exist. And because of the non-increasing property of $v(M_\alpha)$, the domain $\delta_o = \{\alpha \mid \alpha_\ell \lesssim \alpha \lesssim \alpha_u\}$ contains all economic growth factors α_i. The domain δ_o is now systematically made smaller by determining neighbourhoods μ_1 and μ_2 that cannot contain any economic α's. Figure 4.4 illustrates the search-procedure.

Given α_ℓ and α_u, μ_1^o and μ_2^o can be determined and thus the search domain can be reduced from δ^o to δ^1. Now α_3 is defined as $\frac{\alpha_1+\alpha_2}{2}$, and the central triple (α_3, x_3^c, y_3^c) is computed. In case $v(M_3) < 0$ respectively $v(M_3) > 0$, δ^1 is reduced as before, otherwise, as is assumed in the figure, δ^1 is split up in a domain δ^2 and δ^3, etc. Each step the search domain becomes smaller. The procedure culminates in the finding of all economic α's. For details and proofs be referred to the original article.

The weakness of the algorithm is that for each α, central x- and y-strategies have to be found which is by no means a trivial matter, especially if the algorithm is applied to larger models. This drawback was the main motive for our search for another algorithm which will be discussed in next section. It will be shown that this new algorithm has also some other advantages over the ones discussed sofar.

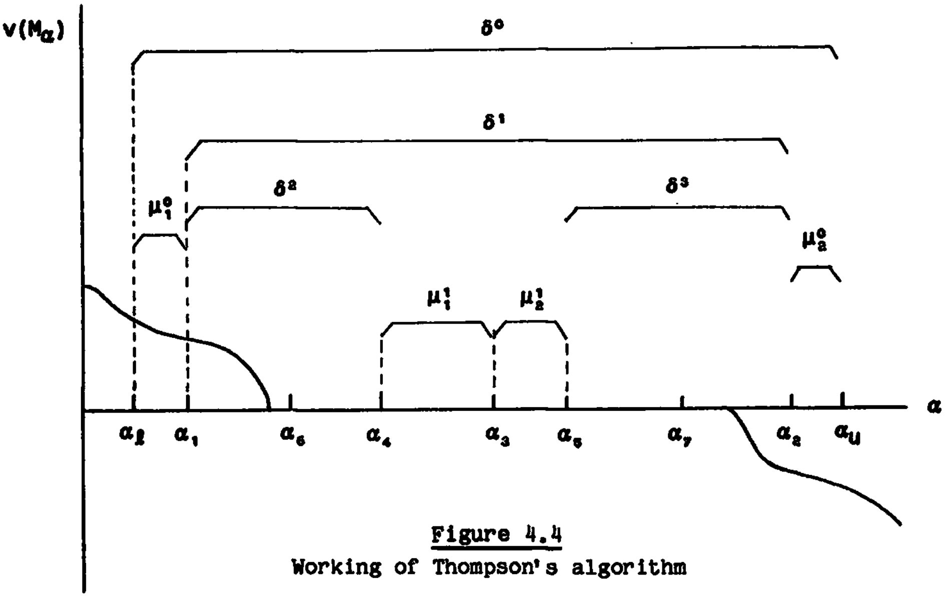


Figure 4.4
Working of Thompson's algorithm

4.3 AN ALTERNATIVE ALGORITHM FOR SOLVING THE VON NEUMANN MODEL

4.3.1 Motive

The algorithms of Weil and Thompson find all growth factors. Both are however cumbersome to apply. In addition to a fairly complicated decomposition procedure, the HTW-technique has to be applied at least twice in Weil's algorithm to find one economic α. And although in case of Thompson's, M_α does not need to be decomposed[41] each step requires a central x and y solution. Especially when the model becomes larger and the coefficients lie within a wide range of values, one easily runs into numerical problems during the search for such a solution[42]. To avoid these

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[41] Thompson's algorithm yields the (not necessarily canonical) decomposition as a by-product, as does as a matter of fact the algorithm discussed here.

[42] In [98] Thompson describes a graph-theoretic search-procedure to find all solutions to a linear programming problem. It is however in his description not at all clear how this method has to be implemented and how numerical stability can be ensured since the graph-theoretic concepts essentially apply to integer programming problems.

difficulties we have developed an alternative procedure for finding all growth factors. The algorithm is, in our opinion, more efficient and easier to apply to small as well as large models than the algorithms of Weil and Thompson.

4.3.2 An overview of the algorithm

The algorithm essentially consists of a search for α^k_{min} of a sequence of economies M^1_α , M^2_α , ..., M^s_α, ..., M^r_α , where each M^s_α corresponds with a sub-economy of the original economy, i.e. M^1_α is the biggest sub-economy (= the original economy), M^2_α is the biggest but one sub-economy, ... and M^r_α is the smallest sub-economy. The search-procedure is schematically shown in figure 4.5. The algorithm consists of two stages (see figure 4.6).

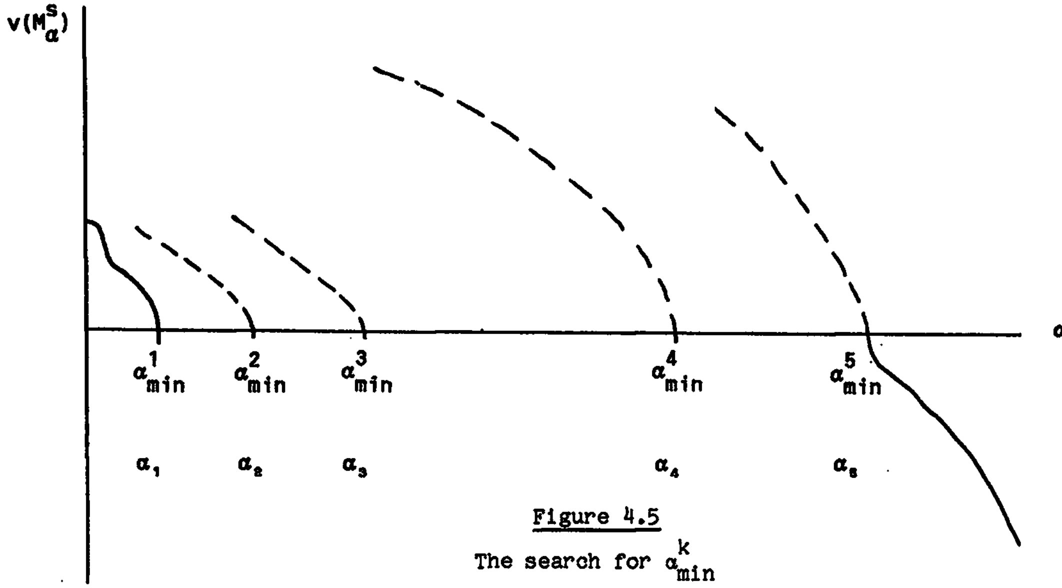


Figure 4.5

The search for α^k_{min}

In the first stage the minimum growth factor α^k_{min} is determined. For this we shall use the HTW-technique, i.e. a sequence of linear programming models is solved. Then the second stage is entered. Here the linear programming model is changed. Some rows (at least one) are relaxed, i.e. a large value is added to the right-hand side so that the constraints concerned will never be binding. Moreover, some processes (at least one) are penalized, i.e. a high penalty is put in the objective so that the processes (activities) concerned will never be chosen in an optimal solution. The adjusted model is defined as M_{α}^{k+1}. Now we go back to stage one where α_{min}^{k+1} of M_{α}^{k+1} is calculated, . . . etc. The algorithm stops if all activities are penalized. But as we shall see below, sometimes it can stop earlier. Before presenting the different steps of the algorithm more explicitly (section 4.3.6), we shall discuss, in sections 4.3.3 and 4.3.4, the transition from M_{α}^{k} to M_{α}^{k+1}. In section 4.3.5 we prove that the procedure really works and in section 4.3.7 the algorithm will be illustrated by a small example.

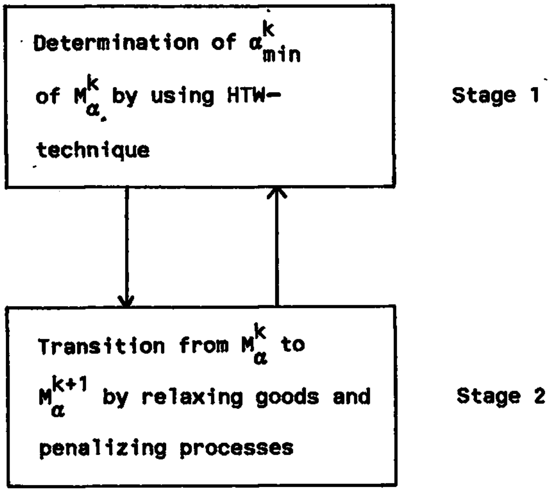


Figure 4.6
A scheme of the algorithm

4.3.3 A rule for determining relevant goods and processes in M_{α}^{k+1}

Suppose that an economy has r economic growth factors $\alpha_1<\alpha_2<. . . \alpha_s<. . . \alpha_r$. In this section we shall prove that in each sub-economy M_s a particular sub-set of goods G_s is relevant. By this we mean that only processes that use/produce these goods can appear at a non-zero level in the equilibrium solution. Further,

$$G_1 \supset G_2 \supset . . . G_s \supset . . . G_r$$

and G_s contains at least one element (good) which is not contained in G_{s+1}.

We use a game-theoretic lemma to determine which goods (and processes) are relevant in M^{k+1} relative to M^k. Kemeny, Morgenstern and Thompson use the same lemma in order to prove the existence of an economic solution. However, by giving an economic interpretation to some intermediate results of the proof of the lemma, it can serve our purposes. In addition, an interpretation in economic terms results also in simpler proofs for theorem 2.6 about the maximum number of economic growth factors and the uniqueness of a solution of the model in its original form (Von Neumann conditions on technology). The lemma reads as follows:

Lemma 4.1

> If α_1 and α_2 ($\alpha_1 < \alpha_2$) are two distinct allowable values of α (this only implies that $v(M_{\alpha_1}) = 0$ and $v(M_{\alpha_2}) = 0$, and not necessarily $xBy > 0$), then $v(M_\alpha) = 0$ for all α in the interval $\alpha_1 \leq \alpha \leq \alpha_2$. Moreover, if x^2 is optimal in M_{α_2} and y^1 is optimal in M_{α_1}, then the pair (x^2, y^1) is optimal in M_α for all α in this interval.

The first part of the lemma has already been proven in section 2.4.2 (theorem 2.2). It is a direct consequence of the continuity and non-increasing property of $v(M_\alpha)$. Therefore we need to prove the second part only. The proof of the latter results however also in an alternative proof of the first part.

Proof (KMT [39], p.120):

Let x^2 be an optimal strategy for the maximizing player in the game M_2; then $x^2M_2 \geq 0$. If α is any number less than α_2, we have:

$$x^2M_\alpha = x^2(B-\alpha A) = x^2(B-\alpha_2A + \alpha_2A - \alpha A) = x^2(B-\alpha_2A) + x^2(\alpha_2-\alpha)A \geq 0 \quad (4.3)$$

Hence $v(M_\alpha) \geqq 0$.
Similarly, let y^1 be optimal for the minimizing player in M_1; then $M_1y^1 \leqq 0$. If α is any number greater than α_1, we have:

$$M_\alpha y^1 = (B-\alpha A)y^1 = (B-\alpha^1 A + \alpha^1 A - \alpha A)y^1 = (B-\alpha_1 A)y^1 + (\alpha_1-\alpha)Ay^1 \leqq 0 \tag{4.4}$$

Hence $v(M_\alpha) \leqq 0$.
The inequalities (4.3) and (4.4) imply that $v(M_\alpha) = 0$ and also show that (x^2,y^1) are optimal strategies in the game M_α for $\alpha_2 \geqq \alpha \geqq \alpha_1$. This concludes the proof of the lemma. □

For our purposes we will have a closer look at both (4.3) and (4.4). Consider an economy with r growth factors. Let us arrange the α's in a sequence of increasing magnitude (see also figure (4.7)):

$$\alpha_1 < \alpha_2 < \alpha_3 \dots < \alpha_s \dots < \alpha_r$$

Next we select two α's, α_s and α_j respectively; both α_s and α_j may be any (economic) growth factor except that $\alpha_s < \alpha_j$ must apply. Let (x^s, y^s) and (x^j, y^j) be corresponding optimal intensity and price vectors. If we substitute in (4.3) x^j for x^2, α_s for α and α_j for α_2, the equation becomes:

$$x^j(B - \alpha_s A) = x^j(B - \alpha_j A) + x^j(\alpha_j - \alpha_s)A \geqq 0 \tag{4.5}$$

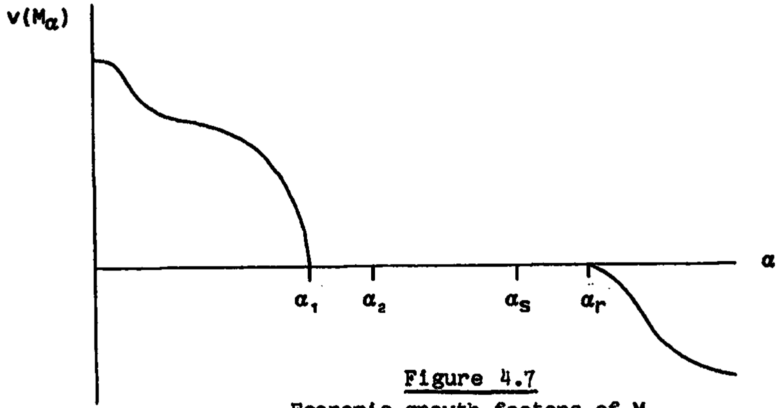


Figure 4.7
Economic growth factors of M_α

Because $x^j(B-\alpha_jA) \geqq 0$, and $(\alpha_j - \alpha_s) > 0$ (by assumption), strict inequality in (4.5) applies for $x^jA^k > 0$, where A^k is the k-th column (good) of A. That is, for all goods that are or (because x^j is not necessarily central) can be used as inputs in a solution to the α_j-economy. But, according to lemma 4.1:

$$x^j(B - \alpha_sA)y^s = 0 \qquad (4.6$$

Thus, prices of these goods must be zero in the α_s-economy. This leads to the following interesting result:

Lemma 4.2.A:

All goods that are (or can be!) used as inputs in an economy with a higher equilibrium growth rate than α_s must have a zero-price in the α_s-economy.

Given an $\alpha > 0$ it is easy to see that each good used as input must also appear as output. The reverse, however, need not necessarily be true. But then the strict inequality sign applies also in (4.5). Together with (4.6) this implies that such goods must have a zero-price in an economy with a lower growth rate. Thus we can strengthen lemma 4.2.A.

Lemma 4.2.B:

All goods that are (or can be) used and/or produced in an economy with growth rate α_j must have a zero-price in the same economy with a lower growth rate.

Next we substitute in (4.4) y^s for y^1, α_j for α and α_s for α_1. Now (4.4) becomes:

$$(B-\alpha_jA)y^s = (B-\alpha_sA)y^s + (\alpha_s-\alpha_j)Ay^s \leqq 0 \qquad (4.7$$

Because

$$(B-\alpha_sA)y^s \leqq 0 \qquad (4.8$$

and

$$(\alpha_s - \alpha_j) < 0$$

the strict inequality in (4.7) applies anyhow, if $A^{\ell}y^{s} > 0$, where A^{ℓ} is the ℓ-th row (process) of A. That is, for all input goods that have a strictly positive price in the α_s-economy. This leads to a lemma similarly to lemma 4.2.A.

Lemma 4.3.A:

> Processes that use goods that have a strictly positive price in the α_s-economy must have a zero intensity if the economy grows at a higher rate.

From (4.8) it follows that processes that produce goods which have a strictly positive price necessarily must use goods which have a strictly positive price. Therefore lemma 4.3.A can also be formulated as:

Lemma 4.3.B:

> Processes that use or produce goods that have a strictly positive price in the α_s-economy must have a zero intensity if the economy grows at a higher rate.

Although lemma 4.3.B is stronger than lemma 4.3.A, they are in fact quite related. Given lemmas 4.2 and 4.3, we can now construct a general rule for determining the potential candidate goods and processes of the 'next' (sub-)economy, i.e. the subset of goods and corresponding processes that are relevant for M^{k+1}.

Consider an economy M with r $[1 \leqq r \leqq \min(m,n)]$ growth factors $\alpha_1 < \alpha_2 < \ldots, \alpha_r$. Suppose we have an economic solution to this economy, say the, not necessarily central, triple $(\alpha_k, x^k y^k)$. Now we rearrange the goods and processes of the economy according to the scheme of figure 4.8.

In this scheme, processes x_1^k use only goods that have a zero-price. If $k \neq r$, such processes must, because of lemma 4.2, exist. The remainder of the processes are defined as x_2^k. Because (α_k, x^k, y^k) is an economic solution and thus $x^k B y^k > 0$, x_2^k contains at least one strictly positive element. The price vector y^k is partitioned into subvectors y_1^k and y_2^k. Elements of y_1^k are zero, while elements of y_2^k are strictly positive. The columns (goods) of M_k are rearranged accordingly. Again, as a consequence of lemma 4.2, if $k \neq r$, y_1^k must contain at least one element. And because $x^k B y^k > 0$, y_2^k must also contain at least one element.

The rearrangement results in four sub-matrices, viz. M^{11}, M^{12}, M^{21} and M^{22}. It is easy to see that as a direct consequence of the rearrangement rules M^{12} and the corresponding sub-matrices A^{12} and B^{12} must consist of zeros only.

If we raise α_k, the intensity of processes x_2^k will become zero (lemma 4.3). In other words, activities x_2^k will have a zero intensity in all sub-economies M_m with $m>k$. In the next section we shall show how this property can be exploited in order to find all (economic) growth factors.

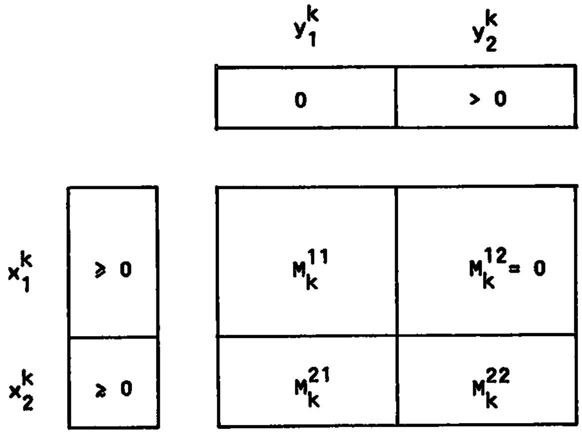


Figure 4.8

M_k after rearrangement

4.3.4 Consequences of relaxing rows and penalizing activities

Suppose that we have an economy M_α and we have found the economic triple (α_1, x^1, y^1) where α_1 is the minimum growth factor of the economy. Referring to figure 4.7 the problem we are faced with is how to find α_2, or more generally, given α_i how do we find α_{i+1}. In this section we shall propose a procedure how this can be done. We start with rearranging goods and processes according to figure 4.8. As will become clear below this rearrangement is only done for explanatory reasons, in 'reality' one can refrain from it. The linear program corresponding with the triple (α_1, x^1, y^1) can now be stated as follows:

$$\begin{array}{llr} & \min -v & \\ \text{s.t.} & & \\ & x_1^1 M_1^{11} + x_2^1 M_1^{21} - ve_1 \geq 0 & \\ & \qquad\qquad x_2^1 M_1^{22} - ve_2 \geq 0 & (4.9) \\ & x_1^1 f_1 + x_2^1 f_2 \qquad\quad = 1 & \\ & x_1^1, x_2^1 \qquad\qquad\qquad \geq 0 & \end{array}$$

where $e = (e_1, e_2)$ is a 1 x n row vector consisting of all ones and $f = (f_1, f_2)'$ is a m x 1 column vector also consisting of all ones. The objective value $\hat{v} = \min -v$ of problem 4.9, which is equal to -1 times the game value of M_1, is, by definition, zero. Further, because $y_2^1 > 0$, it follows from the complementary slackness conditions (see appendix B), that for the second set of constraints the strict equality applies. The procedure we spoke of above consists of two steps:

(i) relax rows j for which $y_j > 0$; and

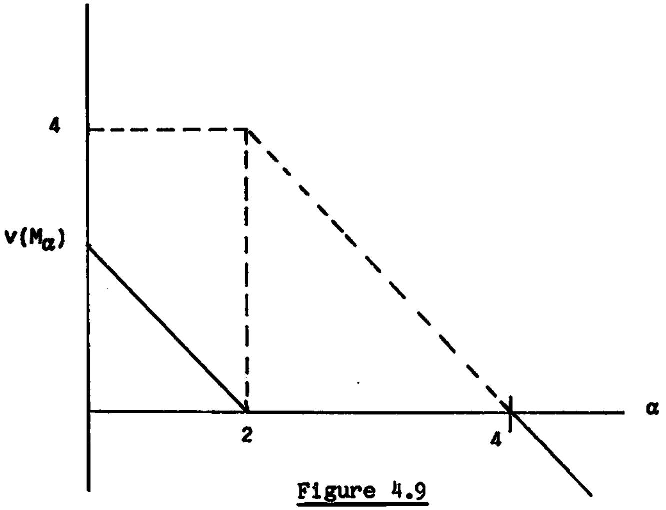
(ii) penalize activities i that use goods j for which $y_j > 0$.

The relaxation consists of adding large negative numbers to the right-hand side of the relevant rows. The penalty consists of adding large positive numbers to the objective coefficients of the activities concerned.

Before we shall investigate the consequences of doing so, we shall first explain the reason for it. Referring to our example (figure 4.8 and problem 4.9), activities x_2^1 are heavily penalized and the second set of constraints is relaxed. From the preceding section it is known that activities x_2^1 will be at a zero level in all economic solutions α_k, k>1. The aim of the penalties is to eliminate these activities from the economy. On the other hand goods for which $y_j > 0$ are most restrictive for growth. By relaxing rows j, they stop being scarce. We illustrate these points by a small example:

$$M_\alpha = \begin{bmatrix} 4-\alpha & 0 \\ 3-\alpha & 2-\alpha \end{bmatrix}$$

$\alpha(=\alpha_{\min})$ and corresponding optimal x-and y-strategies can be calculated as $\alpha_1=2$, $x=(½,½)$ and $y=(0,1)'$ (see also figure 4.9). If activity 2 is penalized and row 2 is relaxed, $\hat{v}$, i.e. the objective value of the corresponding linear program $\tilde{M}_2$ decreases to -4, thus $v(\tilde{M}_2)=4$. Corresponding x- and y-strategies are $x=(1,0)$ and $y=(1,0)'$. In conformity with the results of the preceding section, only activity 1 and good 1 are indeed relevant in the search-procedure for the next economic α. Via the HTW-procedure this factor can be found, its value turns out to be 4, which is, as can easily be checked, an economic growth factor.



<u>Figure 4.9</u>
A small example

In short, the purpose of relaxing rows and penalizing activities is to eliminate goods and processes that are not relevant in the search-procedure any more. A consequence of the relaxation is that we come again in a position $v(M_\alpha)>0$ (see fig. 4.9) so that the HTW-procedure can be applied again in order to find a 'new' $\alpha_{\min}$. A more detailed scheme of the algorithm can now be given (figure 4.10).

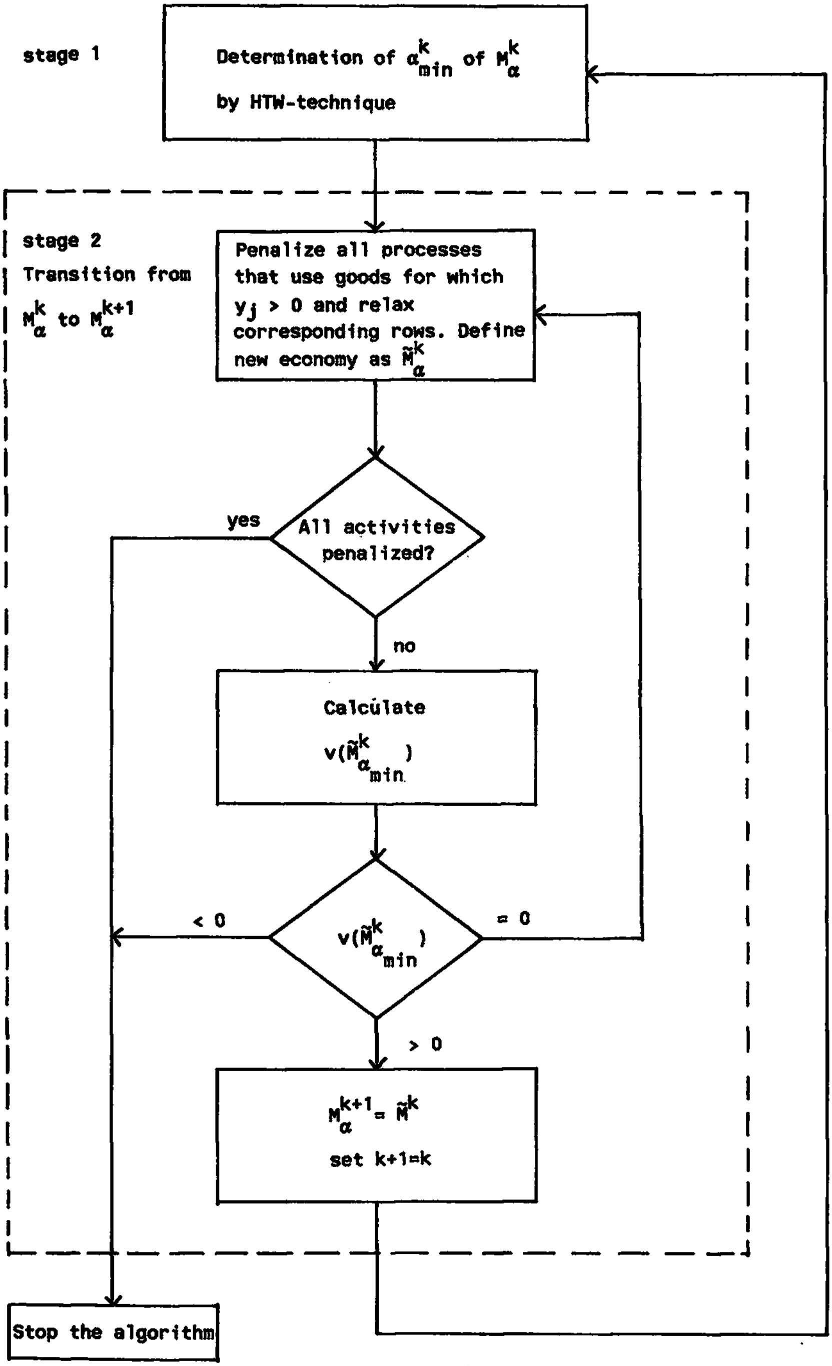


Figure 4.10

A detailed scheme of the algorithm

4.3.5 Proofs of different steps of the algorithm

The proof that the algorithm really works can be broken up into three parts:

Part 1: A proof of the working of the elimination procedure.

Part 2: A proof that the algorithm ends within a finite number of steps, i.e. converges.

Part 3: A proof that in the end we have indeed found all economic growth factors.

Proof of part 1:

To prove part 1 it is sufficient to prove that:

(i) If an economic growth factor has been found (e.g. α_k) and M_α does contain growth factors $\alpha_j > \alpha_k$, penalties and right-hand side relaxations can be given so that goods that were 'binding' in the α_k-economy and activities using these goods will get a zero value in the search-procedure. In other words only relevant goods and processes are taken into account in the search for a next growth factor.

(ii) After goods and processes have been eliminated the HTW-procedure can be applied again.

Part 1-i:

We start from problem (4.9) and define the primal and dual problem after the adjustments have been taken place.

Primal: $\min \; x_2 h - v$

s.t.

$$
\begin{aligned}
x_1 M_k^{11} + x_2 M_k^{21} - ve_1 &\geq 0 \\
x_2 M_k^{22} - ve_2 &\geq -d \\
xf &= 1 \\
x_1, x_2 &\geq 0
\end{aligned}
\qquad (4.10
$$

Dual:

$$\begin{array}{ll} & \max \ - dy_2 + w \\ \text{s.t.} & \\ & M_k^{11} \ y_1 + \qquad\qquad f_1 w \leqq 0 \\ & M_k^{21} \ y_1 + M_k^{22} \ y_2 + f_2 w \leqq h \\ & \qquad\qquad\qquad - ey \ = -1 \\ & y_1, y_s \geqq 0 \end{array} \tag{4.11}$$

where w is an unconstrained variable. The other symbols are defined as before. Because there are no sign constraints on v and w, it is easy to see that both problems have feasible and thus optimal solutions (existence theorem, appendix B). Moreover, for $\alpha \geqq 0$, the maximum value of v can never exceed $\max\limits_{ij}[b_{ij}]$ which must be read as the maximum value of element b_{ij} of output matrix B. Thus, if elements d_r of d are chosen, such that $d_r > \max\limits_{ij}[b_{ij}]$ for all r, the second subset of constraints of the primal can never be binding in an optimal solution. In other words, non-relevant goods can be eliminated. A same line of argument can be used with regard to the penalties on activities. From the dual it follows that variable w can never exceed $\min\limits_{ij}[a_{ij}]$ which must be read as the minimum value of element a_{ij} of input matrix A. Thus if elements h_s of h are chosen such that $h_s > \min\limits_{ij} [a_{ij}]$ for all s, the second subset of the dual can never be binding in an optimal solution. In other words, non-relevant activities can never have a non-zero level in the optimal solution of the primal. □

Part 1-ii:

The HTW-procedure can be applied again if, after rows and columns have been relaxed and penalized respectively, the game-value of the resulting matrix, $v(\tilde{M}_k)$, is strictly positive and, at the same time, if the non-eliminated goods and processes (M_k^{11} in fig. 4.8 and problem 4.9) satisfy the KMT-conditions on technology. In that case all results of theorem 2.2 are fully applicable as can easily be verified.

We start with proving that after the elimination procedure has been properly applied and the economy contains growth factors $\alpha_j > \alpha_k$, $v(\tilde{M}_k) = -\hat{v} > 0$, where $\hat{v}$ is defined as the optimal value of the linear program.

We do not discuss the trivial case in which all activities are penalized: it follows directly from lemma 4.3 that in that case the economy does not contain any economic α. Three cases can in principle be distinguished:

(a) $v(\tilde{M}_k) > 0$

(b) $v(\tilde{M}_k) = 0$

(c) $v(\tilde{M}_k) < 0$

Case (a) needs no explanation; case (b) and (c) are special cases which will be investigated in more detail.

Ad (b): $v(\tilde{M}_k) = 0$

If after the adjustments $v(\tilde{M}_k) = 0$, rows which are binding, i.e. rows corresponding with goods that have a strictly positive shadow price, have to be relaxed and activities using/producing these goods have to be penalized. The reason for this is that activities using/producing these goods will have a zero intensity in case α_k is raised. This can be proven as follows. Let $(\tilde{x},\tilde{y})$ be the x and y solution corresponding with $v(\tilde{M}_k) = 0$. If $\tilde{x}$ and $\tilde{y}$ are partitioned according to (4.10) and (4.11), it follows from (i) that $\tilde{x}=(\tilde{x}_1,0)$ and $\tilde{y}=(\tilde{y}_1,0)$. Thus (4.10) and (4.11) become:

$$\tilde{x}_1 M_k^{11} \geqq 0$$

$$\tilde{x}_1 f = 1$$

and

$$M_k^{11} \tilde{y}_1 \leqq 0$$

$$e\tilde{y}_1 = 1$$

In other words $\tilde{x}$ and $\tilde{y}$ are optimal solutions to the game M_k. Thus, lemmas 4.2 and 4.3 apply, which means that rows and activities can be identified that do not play any role if α_k is raised. □

Economies for which case (b) occurs are characterized by at least one of the following two properties:

(a) The economy contains a sub-economy that can grow at the same growth rate as the original economy. Consider, for example an economy for which:

$$M_\alpha = \begin{bmatrix} 3-\alpha & 0 \\ 3-\alpha & 3-\alpha \end{bmatrix}$$

The triple $\alpha=3$; $x=(\frac{1}{2}, \frac{1}{2})$; $y=(0,1)'$ is economic. Penalizing process 2, relaxing row 2 and solving the problem again yields $v(\tilde{M}_3)=0$, $x=(1,0)$ and $y=(1,0)'$. It can easily be checked that this triple is also economic. The definition of a central solution implies that these instances cannot occur if we look for central strategies instead of just optimal strategies.

(b) The economy contains a subset of goods and processes that can be in equilibrium if run on its own at an $\alpha<\alpha_k$. This α is however non-economic in the whole economy. At the same time the economy does contain a sub-economy that can grow faster than α_k. By way of illustration, consider the following example:

$$M_\alpha = \begin{bmatrix} 1-\alpha & 0 & 0 \\ 0 & 3-\alpha & 0 \\ 8-\alpha & 2-\alpha & 2-\alpha \end{bmatrix}$$

α_{min} and the corresponding optimal strategies can be calculated as $\alpha_{min} = 2$, $x=(1/3, 1/3, 1/3)$ and $y=(0,0,1)'$. Penalizing activity 3, relaxing row 3 and solving the problem again results in $v(\tilde{M}_2)=0$, $x=(0,1,0)$ and $y=(1,0,0)'$. Good 1 has a positive price, thus row 1 has to be relaxed and activity 1 must be penalized. Solving the model again yields $v(\tilde{M}_2)=1$ and after applying the HTW-technique the second growth factor $\alpha=3$ will be found.

Ad (c): $v(\tilde{M}_k) < 0$

We assert that if $v(\tilde{M}_k) < 0$, then the economy cannot contain an $\alpha_j > \alpha_k$. Suppose that the opposite is true and that a triple $(\alpha_1,\hat{x},\hat{y})$ exists which satisfies the axioms. From lemma 4.3 we know that if we partition $\hat{x}$ in the same manner as the x vector in (4.10) and (4.9), i.e. the same elements are put in the same sub-vectors, elements in x_2 must have a zero value. Thus $\hat{x}=(\hat{x}_1,0)$. Substituting $\hat{x}$ for x in (4.10) results in:

$$\begin{aligned} \hat{v}_1 M_k^{11} - ve_1 &\geqq 0 \\ - ve_2 &\geqq -d \\ \hat{x}_1 f &= 1 \end{aligned} \qquad (4.12)$$

Because $\hat{x}$ is economic for $\alpha_j > \alpha_k$ we may write:

$$0 \leqq \hat{x}_1(B^{11}-\alpha_j A^{11}) = \hat{x}_1(B^{11}-(\alpha_j-\alpha_k)A^{11}+\alpha_k A^{11}) \leqq \hat{x}_1(B^{11}-\alpha_k A^{11})$$

Thus, $\hat{x}_1\ M^{11} \geqq 0$ and $v(\tilde{M}_k) = -\tilde{v} \geqq 0$ which contradicts our starting point above. In other words, if $v(\tilde{M}_k) < 0$, then the economy does not contain an $\alpha_j > \alpha_k$ and we can stop the algorithm. □

We shall illustrate the case by a small example.

$$M_\alpha = \begin{bmatrix} 1-\alpha & 0 \\ 3-\alpha & 2-\alpha \end{bmatrix}$$

> α_{min} and corresponding optimal x and y solutions can be calculated as $\alpha_{min}=2$, $x=(\frac{1}{2},\frac{1}{2})$ and $y=(0,1)'$. According to the above stated rules, activity 2 has to be penalized and good 2 relaxed. If the problem is solved again we get $v(\tilde{M})=-1$. Thus the economy does not contain more growth factors. The economic meaning of $v(M_2) < 0$ is that the economy contains a subset of goods and processes which can be in equilibrium when run on its own at a lower growth rate. In the example: process 1 and good 1 with equilibrium growth factor $\alpha=1$ and corresponding intensity and price factor $x=1$ and $y=1$. Within the whole economy, however, the goods are overproduced at that lower growth rate. Moreover, the economy does not contain a sub-economy that can grow faster than the whole economy.

Thus it can be concluded that if the economy contains a growth factor $\alpha_j > \alpha_k$, $v(\tilde{M})_k > 0$ will occur, either at once or via $v(\tilde{M})_k = 0$. Once $v(\tilde{M})_k > 0$, the transition from M_{α}^{k} to M_{α}^{k+1} (see figure 4.10) is completed and $\alpha_{\min}$ of M_{α}^{k+1} is calculated by the HTW-technique. However, we must be certain that an $\alpha_{\min}$ is indeed found in this way. A sufficient proof for this is the proof that theorem 2.2 is applicable. Referring to the linear program (4.9), we have shown above that penalized activities x_2 will always stay at a zero level. Also, rows corresponding with right-hand side coefficients $-d_r$ will never be binding. Thus, theorem 2.2 is applicable if sub-matrix M_k^{11} satisfies the KMT-conditions on technology. Because sub-matrix A_k^{12} corresponding to M_k^{12} (see figure 4.8) consists of all zeros, all rows of sub-matrix M_k^{11} must have at least one strictly positive entry. And because $v(\tilde{M})_k>0$, all columns of B_k^{11} must also contain at least one strictly positive entry. The latter can be seen as follows. Suppose B_k^{11} would have a row ℓ consisting of all zeros. Then there would exist a vector $\tilde{y}=(\tilde{y}_1,0)$ where $y_{1j}=0$ for all $j\neq\ell$ which would make the value of the dual problem (4.11) zero. Thus, $v(\tilde{M})_k$ would be non-positive which contradicts our point of departure. Therefore, each column of B_k^{11} must contain at lease one strictly positive element. □

Proof of part 2:

It is easy to see that the algorithm converges within a finite number of steps. In the algorithm the HTW- and the elimination procedure are successively applied. In section 4.2.1 it was explained that the HTW-procedure stops after a finite number of iterations. Because, thereafter, at least one good (and consequently at least one process) has to be eliminated, after at most min(m,n) (m = number of processes; n = number of goods), the economy does not contain any relevant goods or processes anymore which means that the algorithm has to stop. □

Proof of part 3:

A sufficient proof of part 3 consists of (i) a proof that all α's found are economic in the original economy; and (ii) a proof that no economic α's have been skipped.

Part 3-i:

Suppose we find the triple (α_j,x^j,y^j). We must prove here that α_j is economic in the overall economy. Or:

$$\begin{aligned} x\,M_j^{k+1} &\geqq 0 \\ M_j^{k+1} y &\leqq 0 \qquad\qquad (4.13 \\ xBy &> 0 \end{aligned}$$

must apply. We shall prove that for $x=x^j$ and $y = \frac{y^j+y^k}{2}$, where y^k refers to the y-solution for $\alpha=\alpha_k$ (see figure 4.8) system (4.13) is indeed satisfied. We refer to problems (4.10) and (4.11). From lemma (4.3) and the discussion so far, it follows that the first axiom in (4.13) boils down to:

$$x_1^j\, M_j^{11} \geqq 0$$

which is, in a solution, by definition be satisfied. Because α_j is anyhow economic in M^{11}:

$$x_1^j\, B^{11}\, y_1^j > 0$$

and thus

$$xBy > 0$$

also.

Given the results sofar $y = \frac{y_j+y_k}{2}$ is equivalent with: $y = (\frac{y_1^j}{2}, \frac{y_2^k}{2})$ where elements y_1^j and y_2^k correspond with non-zero elements in y^k (see figure 4.8) and y^j respectively. Because

$$M_k^{12}\, y_2^k \leqq 0$$

and

$$M_k^{22}\, y_2^k \leqq 0$$

it follows that

$$M_j^{12}\, y_2^k / 2 \leqq 0$$

and (4.14

$$M_j^{22}\, y_2^k / 2 \leqq 0$$

also.

From (4.10) it follows that

$$M_j^{11} \; y_1^k \leq 0 \qquad (4.15)$$

If (4.14) and (4.15) are taken together it is easily seen that the second axiom in (4.13) is also satisfied. □

Part 3-ii

For the proof that no economic α's have been skipped we refer to figure (4.11). Suppose α_m has been found by the HTW-procedure. After relaxing/penalizing non-relevant rows and activities $v(\tilde{M}_k)$ turns out to be $\hat{v}$. Further, we assume that, after raising α, the next growth factor found appears to be α_k. We shall prove graphically that no $\alpha_\ell | \alpha_m<\alpha_\ell<\alpha_k$, α_ℓ economic, can exist then. Suppose the opposite is true. Given the results above, it is clear that, if in M_ℓ the same activities and rows as in $\tilde{M}_k$ are penalized and relaxed respectively, the game-value will not be affected, i.e. $v(M_\ell) = v(\tilde{M}_k) = 0$. Because theorem 2.2 is applicable to $v(\tilde{M}_\alpha)$, a decrease of α_ℓ will result in an increase (or staying equal) of $v(\tilde{M}_\alpha)$. Referring to figure (4.11), either $a-\alpha_\ell$, or $b-\alpha_\ell$ or $c-\alpha_\ell$ must apply. It is easy to see that these routes are all impossible: $a-\alpha_\ell$ because the optimal value of the objective of the corresponding linear program is $-\hat{v}$, $b-\alpha_\ell$ because $v(\tilde{M}_\alpha)$ is continuous in α and, finally, $c-\alpha_\ell$ is ruled out because of the single-valuedness of $v(\tilde{M}_\alpha)$ (no bifurcation). In other words, α_ℓ cannot be an economic growth factor and as a consequence, no economic α's can be skipped. □

Because parts 1, 2 and 3 have all been proven, we may conclude that the algorithm works. It may be of interest to compare our algorithm with those of Weil and Thompson. Compared with Weil's algorithm, we need not decompose matrix M_α in order to determine relevant goods and processes in sub-economy α_i. Instead we make use of the resulting shadow prices to eliminate non-relevant goods and processes. A second difference concerns the testing of a resulting growth factor in the overall economy. As we have proven in part 3-i above, we can be assured that each α_{min} is economic in the overall economy also.

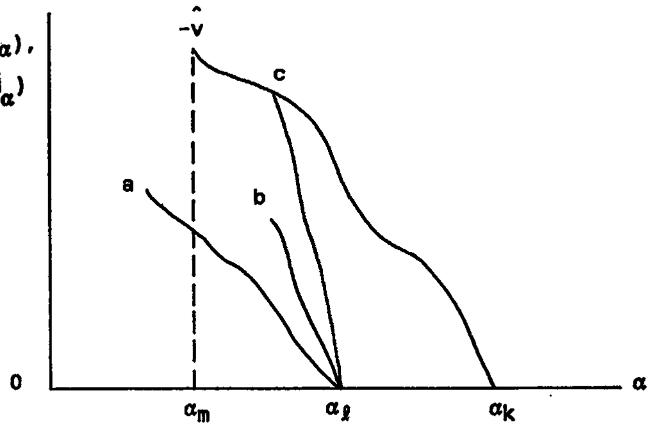


Figure 4.11
An illustration of the proof that no growth factors can be skipped

Central in Thompson's algorithm is that it searches along the axis $v(M_\alpha)=0$, taking, during the procedure, all goods and activities into account. The main difficulty in this procedure is that central solutions are required to determine domains which cannot contain economic growth factors. As explained in section 4.3.1 , it is exactly this difficulty which makes Thompson's algorithm less suitable for practical problems. By making use of lemma (4.3), in our algorithm the search-procedure can be limited to a subset of goods and processes only. The rules for penalizing/relaxing processes/goods have, moreover, as a consequence that the search for a 'new' economic α always starts from a position $v(\tilde{M}_\alpha) > 0$ so that the HTW-bisection method can be applied in a straightforward manner, i.e. without searching for central solutions.

To end this section we shall state two theorems. Although already known (see for example Moeschlin [58], KMT [39] and with regard to theorem 4.2, theorem 2.6), the proofs are simpler.

Theorem 4.1:

In the original Von Neumann model there is at most one economic growth factor.

Proof:

In the original Von Neumann model $a_{ij}+b_{ij}>0$ for each ij. Or each good is either used as input or as output in each process. This means that having found α_k all processes have to be penalized in stage 2 and M_k^{11} (figure 4.8) which contains all economic growth factors except α_k, will be empty. □

Theorem 4.2:

A Von Neumann model with a KMT-technology has at most min (m,n) economic growth factors (m = number of processes; n = number of goods).

Proof:

Each time an economic growth factor has been found processes using goods that have a strictly positive price, have to be penalized. There is, by definition, at least one such good, which is used by at least one process. Thus after maximum min (m,n) steps all processes are penalized and the search-procedure can be stopped. □

4.3.6 The algorithm

Having discussed and proven the various parts of the algorithm, we shall now present the different steps explicitly. A schematized illustration is given in figure. 4.12.

(1) Set k=0, i=0.

(2) Formulate the linear program $M^{k+1} = M^1$ corresponding with the original economy, i.e. without penalties on activities and relaxations of rows.

(3) Choose a non-negative number ℓ so that $v(\tilde{M}_\ell^1) > 0$.

(4) Choose a strictly positive number u so that $v(M_u^1) < 0$.

(5) Let u_1=u, k=k+1 and i=i+1.

(6) If $u_1-\ell < \varepsilon$, where ε is some prescribed small constant, then set

$$\alpha_i = (\frac{u_1+\ell}{2}) \text{ and go to (9)}$$

(α_i is an economic growth factor!).

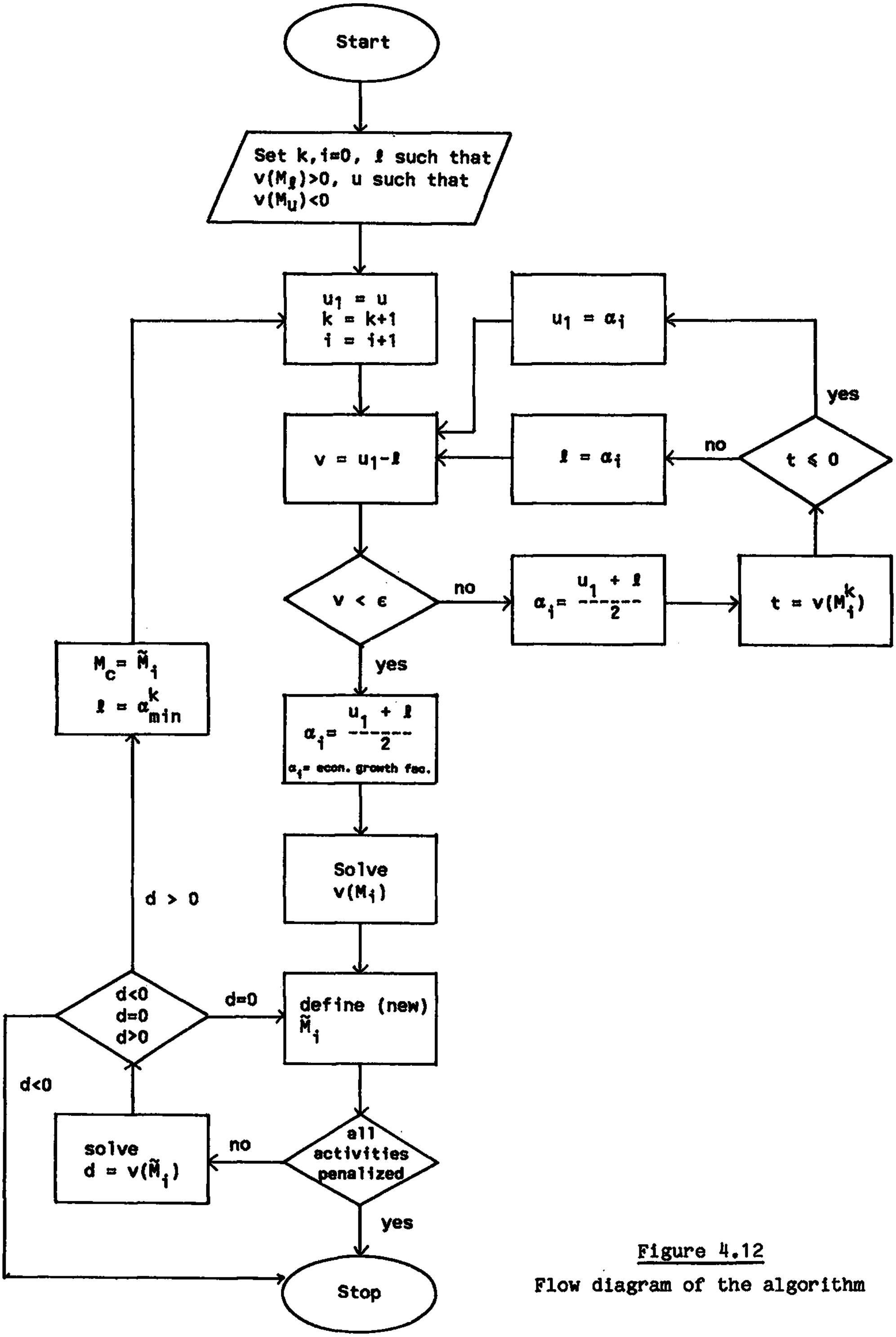


Figure 4.12

Flow diagram of the algorithm

(7) Let $\alpha_i = (\frac{u_1+\ell}{2})$.

(8) Solve the model M_i^k.

- if $v(M_i^k) \leq 0$, replace u_1 by α_i and go to (6).
- if $v(M_i^k) > 0$, replace ℓ by α_i and go to (6).

(9) Solve the model M_i^k. Change the linear program. Penalize activities that use/produce goods that have a strictly positive (shadow-) price. Relax rows corresponding with goods that have a strictly positive (shadow-) price. Define the new model as $\tilde{M}_i^k$. If all activities are penalized, go to (11).

(10) Solve $\tilde{M}_i^k$. $v(\tilde{M}_i^k)=d$.

- if d>0, then $M_i^{k+1} = \tilde{M}_i^k$, replace ℓ by α_i and go to (5).
- if d=0, then, $M_i^k = \tilde{M}_i^k$, go to (9).
- if d<0, go to (11).

(11) End.

4.3.7 An example

To illustrate the working of the algorithm, we shall apply it to a small example. The numbering of the different steps below corresponds with the numbering in section 4.3.6.

Step 1: We start with k=0, i=0.

Step 2: The original economy is:

$$M_\alpha = M_\alpha^1 = \begin{bmatrix} -3\alpha & 0 & 3-\alpha & 1-2\alpha & -5\alpha \\ 3-\alpha & 0 & 0 & 0 & 2-2\alpha \\ 1 & 1-\alpha & 3-\alpha & 1-\alpha & 4-3\alpha \\ 4-2\alpha & 0 & 2-\alpha & 2-\alpha & 1 \\ 0 & 0 & 5-\alpha & 0 & 4-\alpha \end{bmatrix}$$

Step 3: For $\ell=0$, $v(M_\ell^1) > 0$.

Step 4: For $u=8$, $v(M_u^1)<0$.

Step 5: $u_1=u=8$; $k=k+1=1$, $i=i+1=1$.

Step 6: We define $\varepsilon=.00001$, thus $u_1-\ell > \varepsilon$.

Step 7: A new α is defined as: $\bar{\alpha}_1 = \frac{8+0}{2} = 4$.

Step 8: For $\bar{\alpha}=4$, $v(M_4^1) \leqq 0$, thus $4u_1$ is replaced by α_1, i.e. $u_1=4$.

Step 6: $u_1 - \ell = 4-0 > \varepsilon$.

Step 7: $\alpha_1 = \frac{4+0}{2} = 2$.

Step 8: For $\alpha_1 = 2$, $v(M_2^1) \leqq 0$, thus u_1 is replaced by α_i, i.e. $u_1 = 2$.

Step 6: $u_1 - \ell = 2-0 > \varepsilon$

.

.

.

etc. culminating in

Step 6: $u_1 - \ell \leqq \varepsilon$ and $\alpha_1^1 = \frac{u_1+\ell}{2} \approx 1$, which is the minimim economic growth factor of the economy.

Step 9: The corresponding process intensities and prices are:

$x^1 = (1/5, 1/5, 1/5, 1/5, 1/5)$ and

$y^1 = (0, 1, 0, 0, 0)'$ respectively.

Only good 2 has a strictly positive price. Thus the second row of the linear program is relaxed. Because good 2 is used/produced by the third process, this activity gets a high penalty. Because not all activities are penalized we continue with:

Step 10: $v(\tilde{M}_1^1) > 0$. Thus $M_1^2 = \tilde{M}_1^1$ and we start the HTW-procedure again with $\ell=1$.

Step 5: $u_1=8$, $k=2$, $i=2$.

Step 6: $u_1 - \ell = 8-1 > \varepsilon$.

Step 7: $\alpha_2 = \frac{8+1}{2} = 4.5$

Step 8: $v(M^2_{4.5}) \leqq 0$, thus u_1 becomes 4.5.

Step 6: $u_1 - \ell = 4.5-1 > \varepsilon$.

Step 7: $\alpha_2 = \frac{4.5+1}{2} = 2.75$.

Step 8: $v(M^2_{2.75}) \leqq 0$, thus u_1 becomes 2.75

⋮

etc. culminating in

Step 6: $u_1 - \ell \leqq \varepsilon$ and $\alpha_2 = \frac{u_1+\ell}{2} \approx 2$, which is the minimum but one growth factor of the original economy.

Step 9: The corresponding x and y vectors are:

$x^2 = (0,\ 1/3,\ 0,\ 1/3,\ 1/3)$ and

$y^2 = (0,\ 0,\ 0,1,\ 0)'$ respectively

Good 4 has a strictly positive price. This means that processes 1 and 4 get penalized. Further, the fourth row is relaxed. Not all activities are penalized, thus we go on with:

Step 10: $v(\bar{M}^2_2) > 0$, thus $M^3_2 = \bar{M}^2_2$ and $\ell = 2$.

Step 5: $u_1 = 8$, $k = 3$, $i = 3$.

Step 6: $u_1 - \ell = 8-2 > \varepsilon$.

Step 7: $\alpha_3 = \frac{8+2}{2} = 5$.

Step 8: $v(M^3_5) \leqq 0$, thus u_1 becomes 5.

Step 6: $u_1 - \ell = 5-2 > \varepsilon$

Step 7: $\alpha_3 = \frac{5+2}{2} = 3.5.$

Step 8: $v(M^3_{3.5}) \leqq 0$, thus u_1 becomes 3.5.

Step 6: $u_1-\ell = 3.5-2 > \varepsilon.$

Step 7: $\alpha_3 = \frac{3.5+2}{2} = 2.75.$

Step 8: $v(M^3_{2.75}) > 0$, thus ℓ becomes 2.75.

Step 6: $u_1-\ell = 3.5-2.75 > \varepsilon.$

Step 7: $\alpha_3 = \frac{3.5+2.75}{2} = 3.125$

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•
•

etc. culminating in

Step 6: $u_1-\ell \leqq \varepsilon$ and $\alpha_3 = \frac{u_1+\ell}{2} \approx 3$, which is the third growth factor of the economy.

Step 9: The corresponding x and y vectors are:

$x^3 = (0, 1/5, 0, 0, 4/5)$ and

$y^3 = (1, 0, 0, 0, 0)'$ respectively.

Good 1 has a positive price. Thus the first row and the second process are relaxed and penalized, respectively. Except for the fifth process, all activities are penalized. Therefore it is easy to see that $\alpha=4$ is the only growth factor left. The algorithm finds this α in the usual manner, i.e.:

Step 10: $v(\tilde{M}^3_3) > 0$, thus $M^4_3 = \tilde{M}^3_3$, and $\ell=3$.

Step 5: $u_1=8$, $k=4$, $i=4$.

Step 6: $u_1-\ell = 8-4 > \varepsilon.$

Step 7: $\alpha_4 = \frac{8+3}{2} = 5.5.$

Step 8: $v(M^4_{5.5}) \leqq 0$, thus u_1 becomes 5.5

.

.

.

etc. culminating in

Step 6: $u_1 - \ell \leqq \varepsilon$ and $\alpha_4 = \frac{u_1+\ell}{2} \approx 4$, which is the fourth economic growth factor of the economy.

Step 9 : The corresponding x and y vectors are:

$x^4 = (0, 0, 0, 0, 1)$ and

$y^4 = (0, 0, 0, 0, 1)'$ respectively.

After the elimination rules have been applied, it turns out that all processes are penalized.

Step 11: End.

We have found four economic α's, viz. $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 3$ and $\alpha_4 = 4$.

Chapter 5

BANGLADESH: SOME BACKGROUND INFORMATION

5.1 PLAN OF THE CHAPTER

In this chapter we shall provide some background information to the reader who is less acquainted with Bangladesh. A number of topics will be covered; the emphasis will lie on the economic environment.

The chapter is organized as follows: In section 5.2 some basic facts on Bangladesh geography and demography are presented. Thereafter, in section 5.3, a brief sketch will be given of the nation's history. The last section of the chapter, i.e. section 5.4, will be devoted to the Bangladesh economy.

Given the modest length of the chapter, it will need no explanation that the coverage of the topics is both condensed and incomplete. For more information the reader is referred to the references mentioned in the footnotes of the chapter and to other books and articles on Bangladesh from the bibliography at the end of this study.

5.2 GEOGRAPHY AND DEMOGRAPHY[43]

Bangladesh is a deltaic region of about 144,000 square kilometers, i.e. 4.25 times the size of the Netherlands. It is bounded by India on three sides (see map 5.1): in the west, the north and the east; in the southeast there is a small boundary with Birma. In the south lies the Bay of Bengal. The Himalayas are not far from the boundary in the northwest.

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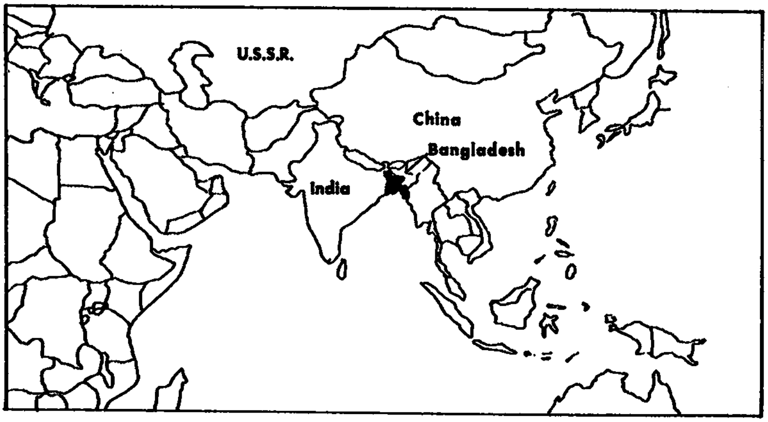
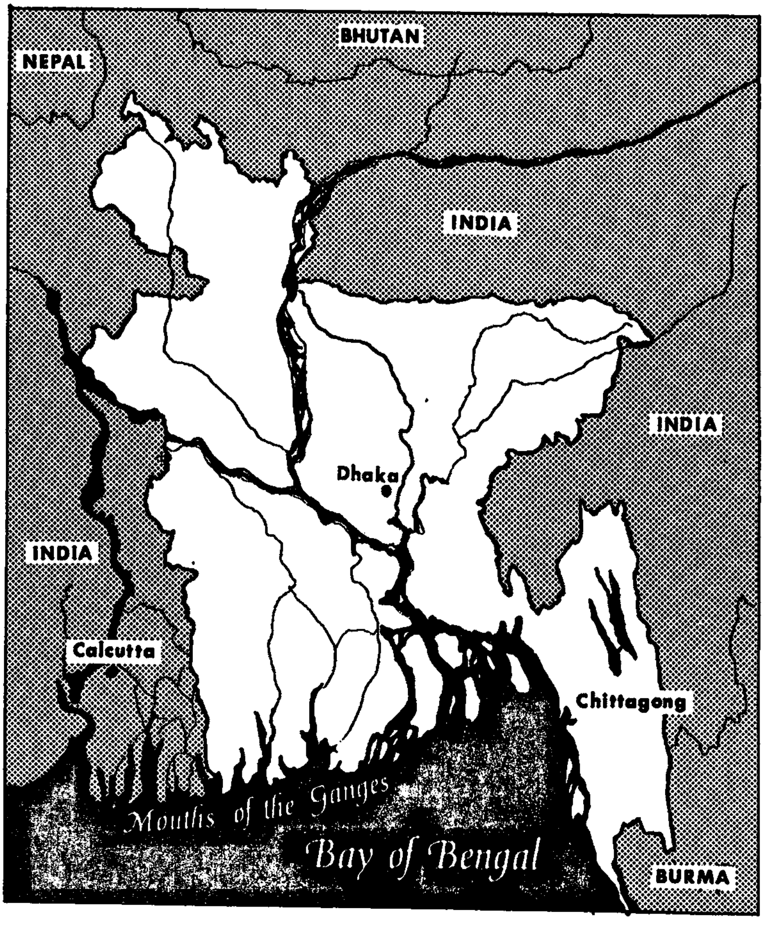
[43] Main sources of information for this section are BBS [9] and Rashid [79].

MAP 5.1

ASIA



BANGLADESH



(The People's Republic of) Bangladesh is a political entity. Geographically it consists of the greater (eastern) part of Bengal and the greater part of the Sylhet district of Assam. Both Bengal and Assam were provinces in former British India. Apart from some small areas in the east and southeast the whole of Bangladesh is made of a vast alluvial plain. This plain is built of alluvial deposits from the three large rivers, the Ganges, the Brahmaputra and the Meghna and from their numerous tributaries. The soil is continuously enriched by heavy silts deposited from the flood waters of these rivers. It is estimated that in an average year about 660 million cubic metres of water flow into the country from India (Rashid [79], p. 55).

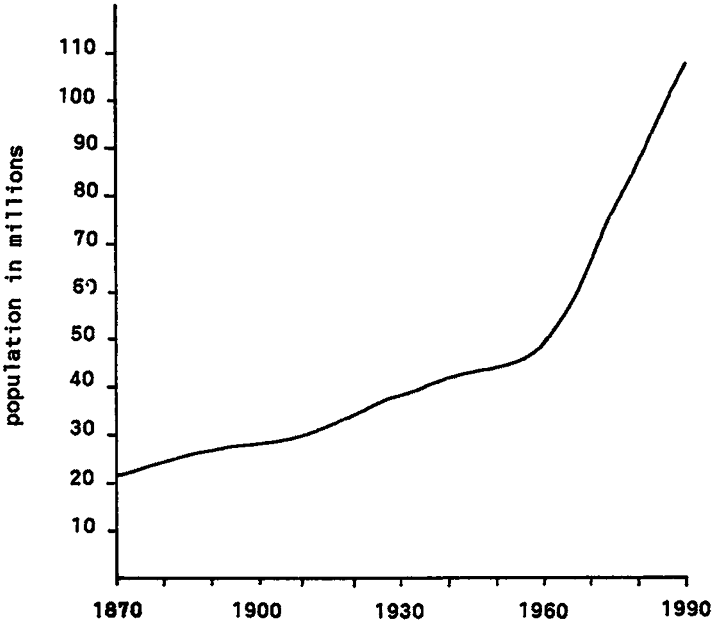
Although more than half of the area lies north of the tropics, the effect of the Himalayan mountain chain is such that the climate is tropical throughout the year. The summers are warm and wet and the winters are dry and moderately cool. The mean minimum temperature varies from 12°C in January to 25°C in July; the mean maximum temperature varies from 25°C to 32°C in the same months. The annual rainfall amounts to 125 cm in the west, 250 cm in the southeast and more than 500 cm in the submontane region of the Assam hills in the north. These rains are often attended by tropical cyclones and floods which sometimes cause extensive damage to man, animal and crop. There are three seasons, viz. winter or rabi, which starts in November and lasts till February, summer or aus beginning in March and ending in May and monsoon or aman which spreads over the remaining months (June-October).

Because of its abundance of water and sunshine, Bangladesh is rich in vegetation. The villages of the country are usually burried in groves of mango, jackfruit, bamboo, coconut and other trees and shrubs. Most of the country is cultivated; less than 9% is covered with forests.

The population of Bangladesh is mainly composed of Muslims (80%); Hindus, Christians and Buddhists form a minority, although with 17% the Hindu minority is a sizable one.

In 1985 the size of the population was estimated at 100.4 million, or at about 700 people per square kilometre. This means that, except for some city states like Hongkong and Singapore, Bangladesh is by far the most densely populated country on earth. In the Netherlands, for example, live 'only' 420 people per square kilometre.

Although population grew throughout the whole of this century, it was not until the fifties that population-growth became really massive. From figure 5.1 it can be seen that it took more than eighty years for the population to double from 22 to 44 million; the doubling from 44 to 88 million took less than thirty years! A characteristic of a fast growing population is the predominance of youth. In Bangladesh 47,5% of the population is below 15 years of age (1981 figure); in the Netherlands only 22.1% belong to this age group (CBS [15]). Most people in Bangladesh live in rural areas. In 1983 approximately 15% lived in cities. Of them 4.5 million lived in Dhaka, the capital and biggest city of the country. Chittagong, the second city counted 1.8 million inhabitants. Although the natural growth rate, defined as the difference between the crude birth rate and the crude death



Sources: BBS [9] and Rashid ([79], p. 496)

Figure 5.1
Population growth in Bangladesh

rate, is slightly higher in rural than in urban areas (.0246 versus .0183), urban population increases at a much higher rate than rural population, due to the migration from rural areas to the cities. As a consequence urban population grew on average at more than 10% per year during the seventies and early-eighties. Because housing and services did not grow at the same pace, conditions for many of these migrants are by all standards very bad.

5.3 A BRIEF HISTORICAL SKETCH[**]

Historians do not know precisely who the original inhabitants were on the Indian subcontinent. It seems that two thousand years B.C., when the Aryans started colonizing most of Northern India, various tribes had already settled. A thousand years later, i.e. a thousand years B.C. the Bang tribe was pushed out of their original home by the Aryan expansionists. The Bang migrated to the southeast and settled in the delta region of the Ganges and the Brahmaputra.

Till the coming of the Muslim rulers in the thirteenth century, the main divisions of Bengal were Varendra (North Bengal), Vanga (South Bengal) and Samatata (most of Eastern Bengal). Early in the sixth century Vanga became independent and dominated nearly the whole of Bengal. This may account for the general acceptance of Vanga-dessa (dessa = country), i.e. Bangladesh as the name of the country. In the eighth century, after a period of instability and political upheaval, Gopala, a Buddhist chief, founded the famous Pala dynasty under which Bengal was to enjoy a long period of prestige and prosperity which lasted from about 750 to 1150 A.D. The period known as the Dark Ages in Europe was one of the brightest in Bangladesh history. The Sena dynasty succeeded the Palas. The Sena kings were orthodox Hindus. They are said to have been cruel and to have persecuted Buddhists. As a result Buddhism nearly disappeared and a strong Brahmanical Hinduism with caste distinctions was introduced.

[**] This section is mainly based on Ahmed ([3], chapter 1) and Rashid ([79], chapters VII and VIII).

At the beginning of the thirteenth century the history of Bengal took a radically new course with the coming of the Muslim conquerers, mostly Turkish tribes from Central Asia. The period from 1338 to 1576 is known as that of an Independent Bengal. It was, however, politically an era of Turkish domination. During that period a mass conversion to Islam took place. In 1526 the sub-continent was invaded by a confederacy of Chagtai Turks which was the beginning of the Moghal Empire. In 1576 Bengal was annexed and became a province. It is said that at that time Bengal was the richest province of the empire and the granary of India.

In the beginning of the eighteenth century the Moghal Empire desintegrated and in 1757 English armed traders were in control of Bengal. The British and the local trading community, who were mostly caste Hindus, cooperated with each other, they jointly exploited the Bengali masses; muslims and non-muslims. The period till 1820 is one of the darkest in the history of the region. The British traders showed a 'spirit of plunder'. A continuing flow of tribute drained the Bengal wealth to the rising capitalism in Britain. Bengal prosperity declined rapidly. The textile industry which was in a flourishing state for several centuries prior to British invasion was compelled to accept very low prices for its goods. In order to protect British textile it is said that thumbs of many Bengal artisans were chopped off (Ahmed [3], p. XXVI). The decline of the textile industry meant also the end of cotton as the main cash crop. Indigo took over, but only after force had been applied. Farmers were compelled to grow indigo on their best land for hardly any remuneration. The discovery of the synthetic dye released the Bengal peasantry from a century of 'grave misery'. In the Moghal period cultivators of the land had to pay a fixed share of the produce through the rent collector to the government. By the Permanent Settlement Act of 1793, the British changed this system. By the Act the rent-collectors were made hereditary landlords. The Act of 1793 introduced feudalism and serfdom. Although attempts were made in 1859 and 1885 to abolish the Act, it lasted till 1952. Within a couple of decades after the Permanent Settlement Act was introduced, most of the old land owning families had been deposed by newcomers who reduced the tenants to serfdom.

During the British era Calcutta gained much in importance as a commercial and industrial centre and Bengal became its natural hinterland. Processing and trading of jute, which became after cotton and indigo the main cash crop, was mainly done in Calcutta.

A notable contribution of the British was the introduction of western science. The seventeenth and eighteenth century were the beginning of a more or less systematic study of Bengal geography, history, architecture, art, flora and fauna, etc. A Bengal intellectual movement came into being which soon showed also political interest. In 1906 the Muslim League was founded in Dhaka, which was until 1947 in the forefront of Indian muslim politics.

Between the two world wars the struggle for independence gained momentum. For some time it looked that Bengal would emerge as an independent country. But the Hindu and Muslim leaders of Bengal could not agree. And after the votes had been counted, two thirds of Bengal together with most of the Sylhet district of Assam became the eastern province of the newly founded Pakistan. The role of Calcutta was, to some extent, taken over by Karachi. Increasing economic disparity between East and West Pakistan on the one hand and political dominance by West Pakistan on the other led to the growth of a movement of independence in East Pakistan. And on 16 December 1971, after a brief but very brutal and bloody war, Bangladesh became an independent state.

5.4 CHARACTERIZATION OF THE ECONOMY

5.4.1 Gross Domestic Product: level and composition

Ever since independence Bangladesh has been one of the poorest countries in the world. Per capita income in 1984/85 was Taka 3,364 (table 5.3) or US$129. This compares with a per capita income of US$820 for Thailand and of US$9,820 for the Netherlands (World Bank [113]). Total GDP at factor costs amounted to Taka 295.7 billion in 1983/84. A breakdown of GDP by main economic sectors is given for some recent years in table 5.1. Although services and industry grew at a faster rate during the period

TABLE 5.1: Gross Domestic Product at Constant Prices of 1983/84 in billion Takas

	1974/75		1977/78		1980/81		1983/84	
	amount	%	amount	%	amount	%	amount	%
Agriculture	116.2	55	137.0	53	142.3	49	156.2	49
Industry	30.2	14	34.7	13	43.6	15	46.1	15
Services	66.5	31	86.8	34	106.0	36	114.2	36
Total GDP m.p.	212.9	100	258.5	100	291.9	100	316.5	100
Net indirect taxes	6.9	3	12.6	5	16.7	6	20.8	7
Total GDP factor costs	206.0	97	245.9	96	275.2	94	295.7	93

SOURCE: Calculated from World Bank ([112], table 2.2).

1974/75 - 1983/84, agriculture is still the dominant sector: even in 1983/84 nearly 50% of GDP was generated in agriculture. Because the main industries process agricultural products (jute, cotton, leather, fish), substantial parts of both industry and services depend on agriculture. Thus, Bangladesh may be characterized as an agricultural oriented country.

Average GDP-growth during 1974/75 - 1983/84 amounts to 4.1% per year. However, two observations are in place. First, this rather high growth rate is partly caused by the choice of the base year. If, for example, one proceeds from 1969/70 as a base year, real GDP-growth is a little less than 3% per year. Secondly, population growth during the seventies and early-eigthies amounted to 2.9% per year, which implies that per capita income growth was only 1.2% per year. Moreover, given the extremely low point of departure one can conclude that the situation of the Bangladesh masses cannot have been improved significantly during the last ten years.

The agricultural character of the economy can also be read from the trade balance (table 5.2): nearly all exports consist of either unprocessed or processed agricultural products. Raw jute and jute manufactures are by far the main items. In 1974/75 and 1983/84 which were generally speaking no particular bad or good years, 76% respectively 58% of all export earnings originated in the jute sector.

TABLE 5.2: Trade balances for 1974/75 and 1983/84 in billion takas of 1983/84

IMPORTS					EXPORTS				
Item	1974/75		1983/84		Item	1974/75		1983/84	
	Amount	% a+c	Amount	% a+c		Amount	% b+c	Amount	% b+c
Rice	3.12	7	1.40	2	Raw Jute	2.48	22	2.92	14
Wheat	13.16	31	8.53	15	Jute manufact.	6.01	54	8.89	43
Crude petroleum + petr. prod.	4.65	11	11.35	19	Leather products	.72	6	2.13	10
Fertilizers	2.58	6	1.87	3	Fish products	.04	0	2.37	12
Cement	.72	2	.92	2	Tea	.52	5	1.72	8
Raw cotton	1.98	5	3.12	5	Textiles (garm.)	.00	0	.90	4
Yarn, textiles	.70	2	1.97	3	Others	1.30	12	1.58	7
Capital goods (incl. transp. equipment)	4.81	11	16.62	28	Total exports	11.07	100	20.51	100
Others	10.25	24	12.93	22	Trade deficit	30.90	297	38.20	186
Total	41.97	100	58.71	100	Total	41.97	379	58.71	286

a) In % of total imports
b) in % of total exports
c) Due to rounding errors sum-totals do not exactly add up to 100

SOURCE: Calculated from World Bank ([112], tables 3.4 and 3.5).

The most striking facts on the import side of the trade balance are the growth of the petroleum bill, the increase in the import of capital goods and the decreasing imports of cereals. In case of petroleum products and cereals this is mainly a matter of prices. In volume terms imports of petroleum and petroleum products have hardly increased, while for cereals the import volume decreased only marginally. The volume of capital goods on the other hand increased by more than 217%.

The trade balance shows, at least in relative terms, a huge deficit: in 1974/75 the value of total imports was nearly four times the value of total exports; and although the difference between imports and exports had shrunk in relative terms in 1983/84, the import-export ratio was still a remarkable 2.86. The deficit is financed from three sources: (workers) remittances from abroad, aid grants and (mainly soft) loans. In the mid-eighties the ratio between these sources was 3/12:5/12:4/12. Because of the very low portion of commercial loans and the long-term of most soft loans, Bangladesh does not belong to the group of countries with an acute debt problem.

An impression of the skewness of the income distribution is given in table 5.

TABLE 5.3: Number of people, income and calory-intake per socio-economic group in 1984/85

Socio-economic group*	Population in 10^6	Per capita income in Takas	Per capita daily calory-intake
Landless farmers	20.13	2246	1632
Small farmers	11.55	2582	1772
Medium famers (tenants)	11.99	2927	1867
Medium farmers (owners)	13.04	3360	2021
Large farmers	10.22	4104	2252
Largest farmers	4.21	5954	2586
Farmers (total)	71.15	3095	1911
Rural informal	10.55	2412	1692
Rural formal	7.04	5448	2536
Urban informal	7.02	2785	1798
Urban informal	4.66	7152	2528
Non-farmers (total)	29.26	3986	2052
Bangladesh (total)	100.41	3364	1953

*Exact definitions of the socio-economic groups can be found in Van Veen et al. [105].

SOURCE: The Macro Model for the Third-Five-Year Plan (UNDP [103], volume II.)

Population is split into ten more or less homogeneous groups. Compared with other countries income distribution is not particularly skew. The richest group, the urban formal, which makes up 4.7% of the total population has an average per capita income of Taka 7,152 which is 3.18 as high as that of an average landless farmer, who belongs to the poorest 20% of the population. The extreme poverty appears most clearly in the last column of the table. Only three out of the ten groups have an average per capita calory-intake equal or above the amount recommended by the World Health Organization.

5.4.2 The agricultural sector

About 75% of agricultural value added originates from crop production. Area under main crops is shown in table 5.4. The most striking characteristic of the table is the predominant role of rice: more than 75% of total cropland is occupied by this crop. Rice varieties are grouped into three seasonal types, viz. aman, aus and boro. Aman is grown during the monsoon period, it is the main rice crop. The crop grows on innundated land and is harvested from November to January when the land is dry. Second in importance is aus rice. It is broadcast between March and May with the northwestern rainfall and is harvested just before the flood from the monsoon (July to August). Boro rice grows during the dry winter season, it is either grown on very low land or on land which has access to irrigation. Until recently the boro crop was the smallest one. Due to rapid expanding irrigation facilities it has since the early-eighties caught up with aus. The increase in irrigation is also responsible for the increase in wheat production. In ten years time (1973/74 - 1983/84) wheat area has more than quadrupled.

The main non-food crop is jute. Jute and aus rice grow on the same soil and during the same time of the year. Consequently there exists strong competition between the two. Other important crops are sugarcane, potatoes, vegetables and fruits. Vegetables and fruits are mainly grown on the homestead area.

Farming methods in Bangladesh are labour intensive and traditional. Practically all power is provided by man and cattle, the latter being used for

TABLE 5.4: Crop production in an average year in the early-eighties

Crops	Area in 10^3 hectares	Average yield per hectare in kgs	Total production in 10^6 kg
Rice	10,400	1,340	13,936
of which: aus	3,112	1,023	3,184
aman	5,936	1,268	7,525
boro	1,352	2,387	3,227
Wheat	520	1,954	1,016
Gram	52	719	37
Khesari	80	746	60
Rape and mustard	188	645	121
Jute	570	1,493	851
Tea	45	903	41
Tobacco	52	931	48
Chillies	76	599	46
Potatoes	109	10,137	1,105
Sweet potatoes	65	10,782	701
Sugarcane	161	44,237	7,122
Fruits	145	9,482	1,375
Vegetables	120	6,625	795
Other crops	497		
Total area	13,080		

SOURCE: Calculated from World Bank ([112], tables 7.5, 7.6 and 7.7).

land preparation, transportation and threshing. Fertilizer use is low, in the mid-eighties on average 75 kg per hectare was used; in China, for example, average use per hectare is about three times as high. In 1977, the year in which the last agricultural census has been performed, average farmsize appeared to be 1.4 ha, which is by all standards very small. According to table 5.5 average farmsize in 1984/85 has even decreased to 1.2 ha. Nearly 80% of all farms (excl. landless farmers) were smaller than 2 hectares (table 5.5). And although 6.1% of the biggest farmers cultivate 21.7% of total crop-land, even the average size of this group is a poor 5.13 ha.

Because of their size Bangladesh farms are mainly family farms. The household supplies labour to the farm, male labour for land cultivation and female labour for working in the kitchen garden, raising chickens,

TABLE 5.5: Farmsize distribution: an estimate for 1984/85

Socio-economic group[a]	Number of farms in 10^3 [b]	Total cultivated area in 10^3 ha[c]	Average farm size in ha
Landless farmers	4,348	33	.01
Small farmers	2,137	639	.30
Medium farmers (tenants)	1,804	1,792	.99
Medium farmers (owners)	1,961	1,879	.96
Large farmers	1,189	2,697	2.27
Largest farmers	382	1,960	5.13
Total	11,821	9,000[d]	.76
Total, excl. landless farmers	7,473	8,967	1.20

[a] Exact definitions of the socio-economic groups can be found in Van Veen et al. [105].

[b] Number of people in 84/85 (table 5.3) divided by average household size.

[c] Total area according to the 1977 agricultural census [8].

[d] Because non-farmers do also cultivate some land, total is slightly less than Bangladesh total.

proccessing rice, (threshing, parboiling, milling), etc. Characteristic for Bangladesh agriculture is that not only main products but also by-products are intensively used. Cow dung is used for either manuring or as energy-source; rice straw is used as cattle feed, building material or energy-source; bran is used as fuel for parboiling rice or as chicken-feed; jute sticks are also used as energy-source or as building material; etc. In short, Bangladesh farmers use to a maximum the possibilities of traditional agriculture.

Growth in crop production mainly originates from better water control, primarily through irrigation and to a lesser extent through drainage. Better water control not only opens the possibility to grow more crops, it also opens the way to introduce more fertilizer responsive high yielding varieties. The livestock sector is hardly developed. Cattle are mainly kept as a source of draught power and wealth. Because of insufficient fodder, the cattle stock is, in general, in a poor condition. Goats are the second type of livestock. In 1984 there were 13 million goats in Bangladesh. They are rarely milked and mainly being kept for their meat and skin. Poultry is third in importance. Although they are large in number (there were 93 million chicken and ducks in 1984) their productivity is low. A chicken year yields on average .6 kg meat and 40 eggs (Stolwijk [90]). Most farms do possess some cattle, goats and chickens.

5.4.3 Industry and services

Taken together, industry and services contribute, from 1980/81 onward, more to GDP than agriculture (see table 5.1). Substantial parts depend, however, on agriculture. This is clearly shown in table 5.6 where a further breakdown in sub-sectors is given. Strong linkages exist between cloth, jute textile, leather, fertilizer, transport and trade on the one hand and agriculture on the other. More than half of the contribution of manufacturing (value added) to GDP takes place in large scale industries.

TABLE 5.6: Sub-sectors within industry and services

	Contribution to value added 1984/85 (%)[a]
Cloth	3.1
Jute textiles	3.0
Leather	1.6
Fertilizer	1.7
Other chemicals	1.8
Cement	.1
Steel and metal products	3.2
Other manufacturing industries	1.6
Building and construction	5.0
Electricity and gas	1.5
Transport	11.0
Housing services	6.2
Health and education	5.7
Public administration	7.7
Trade and other services	46.2
Total	100.0

[a] Calculated from UNDP [103], volume II, p. 12); food industries are included in services.

Their contribution to total industrial employment is, however, only 20% (Koht Norbye [49]). Small scale industries (cottage industries and handlooms) account for the remainder. Thus, although they are less important in terms of value added, they are not in terms of employment.

During the first decade after independence, large scale industries were dominated by the public sector, following the nationalization of almost all large scale enterprises after the war of liberation. However, performance of public enterprises severely lagged behind expectations (World Bank [111], p. 81). Inefficient management and overstaffing resulted in a serious decline in labour productivity. From the early-eighties on significant policy changes have appeared and a substantial number of enterprises have returned to private ownership.

It is generally agreed (see e.g. Koht Norbye ([49]) that available statistics give an inadequate and fragmented picture of Bangladesh manufacturing industry. This applies especially to small scale, cottage and household industries. As a general characterization it can be stated that the great majority of people in small units work with traditional or intermediate technology, at relatively low wages and generating far less value added per worker than workers in the larger industries. Taken together, they supply a large proportion of manufactured goods used in the country. More or less the same characterization applies to the services sector of which even less is known statistically. The figures of table 5.6 must therefore, like most statistical data on the country, primarily be considered as indicative.

Chapter 6

EMPIRICAL ELABORATION

6.1 INTRODUCTION

Before the model(s) of chapter 3 can be applied to the actual economy of Bangladesh, substantial work has still to be done. The following three steps can be distinguished:

(i) Formulation of the model structure;
(ii) Gathering and organization of the data; and
(iii) Developing and implementation of the model software.

Carrying out these steps at a great distance from the country would be, to say the least, an immense task for one person. Fortunately use could be made of the modeling work on Bangladesh done at the Centre for World Food Studies (SOW).

The SOW started its work on Bangladesh in 1980; the author has been involved in it until 1986. The work started with the construction of an economy-wide model in which the emphasis was on food and agriculture (SOW [88]). Development of the model took place under a collaborative arrangement with the Bangladesh Food Policy and Monitoring Unit of the Planning Commission. In October 1984, when the construction of BAM, as the model was baptized, was well on the way, UNDP requested the SOW to assist in the development of a macro-model for the Third Five Year Plan (1985-1990) of Bangladesh. BAM was the starting point in this project; however, in order to satisfy some specific plan needs, which had mainly to do with the sector classification and the project deadline, a number of modifications had to be made. The main ones were: (i) the original two non-agricultural sectors were disaggregated into 25 sectors; (ii) a large-scale linear program (BAM-lp) had to be replaced by an econometric supply module; and (iii) in order to incorporate recent results from the National Water Study (Master

Plan Organization [55]), a land development module was constructed. To distinguish the Macro-Model from BAM, the acronym TFYP (Third Five Year Plan) is used. Because most of the work with regard to BAM and TFYP is documented fairly well[45], the discussion in this chapter can be brief and references will be given for details.

6.2 MODEL STRUCTURE

6.2.1 From BAM-lp to M_α

Starting point in the construction of the M_α(=B-αA)-matrix was the original linear programming tableau of BAM. Figure 6.1 shows a schematic summary of this tableau. The actual size is about 1,100 rows and 1,700 columns; a discussion on its structure can be found in Stolwijk [91]. The transformation of the lp-tableau into Von Neumann matrices was not a straightforward matter, i.e. it was not enough to split the tableau into two matrices B and A respectively where the former contains all output and the latter all input elements. The main modifications that have been made and the motives that lie behind them are:

First, the right-hand side resources of the lp had to be brought into the B matrix; not the actual levels but the output quantities coupled to a specific activity. For example: growing paddy yields in a Von Neumann model, amongst others, paddy and uncultivated land (or crop growing capacity); in the BAM-lp the latter is not an explicit output.

Second, in contrast with the BAM-lp, the Von Neumann model does not distinguish separate socio-economic groups. The reason for this is not that such a distinction does not matter. If, however, the distinction would have been maintained, the model would, because of its size, be very difficult (and expansive) to handle. Moreover, some interpretation problems would have arisen. As has become clear in the theoretical part of our study the

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[45] See SOW [88] for an outline of BAM and Ahmad et al. [2] and UNDP [103] respectively for details on TFYP.

ACTIVITIES			ACTIVITIES											EXCHANGE ACTIVITIES																					
			Crop production			Livestock		Processing		Energy		Subsist. consumpt		Buying inputs					Selling inputs		Selling final prod.		Renting in			Renting out			Investments			Desinvestments		
RECOURCES AND OTHER CONSTRAINTS			No non-farm inputs	+ some fertilizer	+ fertilizer and pesticides	Cows	Chicken	Crops	Animal products	Crop by-products	Manure	Crop products	Animal products	Fertilizer	Pesticides	Energy	Crops	Animal products	Crops	Animal products	Processed crops	Processed animal products	Land	Draught power	Labour	Land	Draught power	Labour	Land	Cattle	Capital	Land	Cattle	Capital
Land		r_1	+	+	+																		-			+			-			+		
Labour		r_2	+	+	+	+	+	+	+																-			+						
Capital		r_3			+										-																-			
Animals		r_4				+	+																							-			+	
Animal draught power		0	+	+	+	-																		-			+							
Main products from crops		0	-	-	-			+									-		+															
By-products from crops		0	-	-	-	+	+	-		+																								
Animal products for processing		0				-	-		+									-		+														
Manure		0				-					+																							
Fertilizer		0												-																				
Energy		0						+	+	-	-	+	+			-																		
Processed products		0						-	-			+	+								+	+												
Cash		r_5												+	+	+	+	+	-	-	-	-	+	+	+	-	-	-	+	+	+	-	-	-
Balances	Crops	0															+		-															
	Animal Products	0																+		-														
	Land rented	0																					+			-								
	Draught power	0																						+			-							
	Labour	0																							+			-						
	Land sold/bought	0																											+			-		
	Cattle	0																												+			-	

Von Neumann model lends itself more for analyzing the structure and properties of technology matrices than for investigating consequences on growth of power relations and all kinds of institutional arrangements[46].

Third, in any growth scenario for Bangladesh, investments in land improvement (irrigation and drainage) and non-agricultural production capacities will play a crucial role. Because the emphasis in our model is on growth, investment activities are much more elaborated than in the BAM-lp.

Fourth, the emphasis in the BAM-lp lies strongly on the agricultural sector. The non-agricultural sector is treated in a highly aggregated manner. In the context of this study it seemed interesting to somewhat restore the balance. Therefore more non-agricultural sectors and goods are distinguished.

The last adjustment we discuss is the most serious one. In the linear programming tableau the duration of the activities varies from a few months to one year. In a Von Neumann model, on the other hand, the time period of all activities must have the same length (see chapter 2). Because the production cycle in agriculture lasts one year, i.e. many products are available once a year only, and the technology matrices have to contain all products, the minimum time period in a Von Neumann model which includes the agricultural sector must be one year. The implications of this, seemingly minor, point are far-reaching. We shall illustrate them by means of a small example:

Consider an economy consisting of three products, viz. labour, cultivated land and paddy, and three processes, viz. the production of cultivated land, paddy and labour respectively. The technology matrices are as follows:

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[46] The difference between 'Von Neumann growth' and actual growth can be interpreted as an indicator for (a part of) the costs of the prevailing institutional structure. The words 'a part of' are inserted because the input-output coefficients themselves are partly a reflection of the institutional structure too.

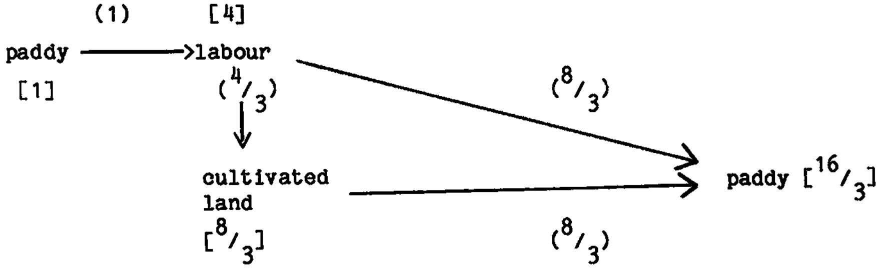
$$\underline{A}$$

$$\begin{array}{clccc} & & \text{Labour} & \begin{array}{c}\text{Cultivated}\\ \text{land}\end{array} & \text{Paddy} \\ 1 & \text{ploughing land} & 1 & 0 & 0 \\ 2 & \text{growing paddy} & 1 & 1 & 0 \\ 3 & \text{subsist. cons.} & 0 & 0 & 1 \end{array}$$

$$\underline{B}$$

$$\begin{array}{clccc} & & \text{Labour} & \begin{array}{c}\text{Cultivated}\\ \text{land}\end{array} & \text{Paddy} \\ 1 & \text{ploughing land} & 0 & 2 & 0 \\ 2 & \text{growing paddy} & 0 & 0 & 2 \\ 3 & \text{subsist. cons.} & 4 & 0 & 0 \end{array}$$

It can be verified that the economy contains one unique equilibrium growth factor. Its value is 2. If the growth period of paddy is one year, then one should be inclined to say that the economy grows at a rate of 100% a year. A detailed inspection of what actually happens in the economy reveals, however, that the growth rate is 433% (growth factor is 16/3)! In figure 6.2 the difference is explained. At the start of the period the economy has one produced good at its disposal, viz. paddy. The paddy is given to the labourers, who in return, plough the uncultivated land and grow paddy on it. At the end of the year 16/3 units of paddy are harvested, which implies a growth rate of (16/3 - 1) x 100% = 433%.



(.) = input

[.] = output

Figure 6.2

Input-output relations in the economy of the example

The seriousness of the problem will be clear. It is remarkable that, as far as we know, no attention at all is paid to it in the literature. We shall briefly discuss three possible ways to handle the problem:

(i) Let us define an lp-activity as a task. As figure 6.1 shows, an output of one task can serve as an input of another. Thus, if ploughing land and transplanting paddy are two tasks, the output of the first (ploughed land) is input for the second (transplanting paddy). If such a connection exists, we say that the tasks are related. By combining related tasks, intermediate goods are netted out; in the example ploughed land is such an intermediate good. Combining related tasks is what we have actually done. That is to say a Von Neumann activity is defined as a combination of related tasks which lasts one period (year).

Although the principle is simple, the procedure has two drawbacks. A first one is that by combining related tasks, intermediate products are netted out. As a consequence the structure of the technology matrices looses much of its richness. It is exactly the possibility to introduce all kinds of intermediate in- and outputs that lends the peculiar charm to a linear programming model (see the discussion on the structure of the BAM-lp [91] for a detailed illustration of this point). A second, and even more serious,drawback is that available technical information is rather inefficiently handled by the procedure. For example, as is shown in Stolwijk ([91], p. 13), each farmgroup in the BAM-lp has 468 potential ways to grow paddy. The number of tasks which actually describes these alternatives is, on the other hand, a mere fifty[47]. The procedure proposed above implies that, in principle, all 468 combinations have to be defined as explicit Von Neumann activities. The seriousness of the drawback will be clear. Because it is for all practical purposes impossible to take 468 paddy growing activities into account, we have to select beforehand and a lot of available information has to be 'thrown to

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[47] The exact number cannot be given because a number of tasks refer to other crop growing activities as well.

the winds'. Because of these drawbacks it seems worthwhile to look for an other solution, i.e. for a solution which can digest available information as efficient as a linear programming model. We have searched in two directions. Although we cannot prove that we are on the right track, the indications are too promising to ignore. We shall discuss them under (ii) and (iii). To avoid possible misunderstanding we stress that the procedure discussed under (i) is the one actually applied. Procedures (ii) and (iii) on the other hand, are merely presented as potentially interesting for further investigation.

(ii) Referring to figure 6.2 we define a step as the road from one product to another (arrow). A sequence of steps (arrows) which together lasts one period (from paddy to paddy) is called a chain. We think the following conjecture is worth further investigation:

$$\hat{\alpha} = \Sigma_i a_i \bar{\alpha}^{t_i} \qquad (a_i \geq 0; \ \Sigma a_i = 1) \qquad (6.1)$$

where

$\hat{\alpha}$ = real growth factor

$\bar{\alpha}$ = growth factor if time sequence and duration of intermediate processes are not taken into account

a_i = the intensity with which chain i is carried out in the optimal solution

t_i = the number of steps in one chain.

In the example $\bar{\alpha}=2$, $i=2$, $a_1={}^2/_3$, $a_2={}^1/_3$, $t_1=2$ and $t_2=3$. From formula (6.1) it follows that for $\bar{\alpha}=1$, which is the case in a linear programming model, $\hat{\alpha}=\bar{\alpha}$. The point is thus not relevant in case of an lp-model. If (6.1) can be proven, the construction of a Von Neumann model can be done analogous to an lp-model, i.e. the time period of a particular activity can freely be chosen.

(iii) A free choice in determining the time period of an activity is also the central element in the third way the problem can possibly be tackled. For expository reasons we divide the model period (say a year) into n sub-periods (say months). At first, activities (tasks, operations) are defined without bothering about differences in length. Then activities that last longer than one month are broken down in sub-activities of one month. By introducing all kinds of intermediate goods this can easily be done. If we denote activities that start in month j as x_j and the corresponding technology matrices as A^j and B^j, the development of the economy within the whole period (year) can be described as follows:

$$\begin{array}{lcl}
\textbf{input} & & \textbf{output} \\
x_1A^1 & \rightarrow & x_1B^1 \\
x_2A^2 & \rightarrow & x_2B^2 \\
 & \vdots & \\
x_{12}A_{12} & \rightarrow & x_{12}B_{12}
\end{array}$$

Given the balanced growth requirements (see chapter 2), a solution vector $\bar{x}=(x_1,x_2,\ldots,x_{12})$ has to satisfy the following constraints:

$$\begin{array}{ll}
x_{12}B^{12} & \geq \alpha x_1A^1 \\
x_1B^1 & \geq x_2A^2 \\
 & \vdots \\
x_{11}B^{11} & \geq x_{12}A^{12}
\end{array} \qquad (6.2)$$

System (6.2) can also be written as:

$$(x_{12},x_1,x_2,\ldots,x_{11}) \begin{bmatrix} B^{12} & 0 & 0 & \cdots & -A^{12} \\ -\alpha A^1 & B^1 & 0 & & 0 \\ 0 & -A^2 & & & \vdots \\ \vdots & & & & \\ 0 & \cdots & & -A^{11} & B^{11} \end{bmatrix} \geq 0$$

Observe that the growth factor α only appears in the first set of inequalities, i.e. α only refers to goods that are available at the beginning and the end of the period! Intermediate goods only have to satisfy a balance condition. In order to guarantee that all goods one disposes of at the beginning of the period are indeed in $\bar{x}_1A^1$ and $\bar{x}_{12}B^{12}$, storage activities have to be defined. For example, if the period starts January 1 and ends December 31 and tractors are only used in April, storage activities have to be introduced to guarantee that the number of tractors keeps pace with the growth of the overall economy. From a theoretical point of view the 'solution' seems attractive. In practice the great many intermediate goods and sub-activities that have to be defined will make the matrix unmanageable. However, although we cannot prove it rigorously, we conjecture that one can do without it and that, instead, one can suffice with the construction of Von Neumann matrices analogous to an lp-matrix, thus by freely choosing the duration of an activity. The search for α takes place in the usual manner, except that α only appears for goods the economy has at its disposal at the start of the period (say January) and in activities which use these goods for the first time. Referring to the example above, this means that only element 3,3 (subs. cons, paddy) is 'relevant' in the search for the balanced growth factor. It can be verified that the procedure indeed results in a growth factor $\alpha = {}^{16}/_{3}$!

A rigorous proof of the procedure would consist of two steps: first, it must be proven that a growth factor will always be found. Second, the resulting growth factor must be equal to the one found according to procedure (i), i.e. the α found from matrix M_α where M_α consists of all possible combinations of related tasks.

6.2.2 Activities and goods

The resulting matrix M_α consists of about 50 rows and 90 columns[48]. A schematic summary of M_α is shown in figure 6.3. Each row corresponds with

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[48] The exact numbers depend on the scenario (see chapter 7).

an activity; each column with a good. Elements of the A matrix are represented by plus-signs; elements of the B matrix by minus-signs[49]. Activities which have entries on the same row are directly related to each other. In case the entry signs are the same, activities compete for the resource. If the sign differs, the activity with a negative sign provides the resource to the ones with a positive sign. In many cases activities are competing for one and complementary for another resource. For example, crop production and livestock activities compete for non-agricultural products and labour but are complementary for feed. A complete list of activities and goods is given in table 6.1.

Main product group Main activity groups		Foreign exchange	Unprocessed crop products	Processed crop products	Livestock products (incl. fish)	Ruminant feed	Non-ruminant feed	Fertilizer/manure	Tradeable non-agricultural product	Non-tradeable non-agricul. product	Crop growing capacity	Livestock	Non-agricultural production capacity	Energy
Crop production (incl. on-farm processing)	input							+	+	+	+	+		+
	output		-	-		-	-	-			-	-		-
Livestock keeping/raising (incl. fishing)	input					+	+		+	+		+		
	output				-			-				-		
Non-agricultural production activities	input		+						+	+			+	+
	output			-					-	-			-	-
Investment in crop growing capacity	input								+	+	+			
	output										-			
Investment in non-agricultural production capacity	input								+	+				
	output											-		
Import	input	+								+				
	output			-				-	-					-
Export	input		+	+	+				+	+				
	output	-												

Figure 6.3

Structure of M_{α} $(=B-\alpha A)$

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[49] Because the structure of the consumption matrix and the labour-requirement vector are trivial, only the B and A matrix are included in the figure.

TABLE 6.1: Goods (rows) and activities (columns) of M_{α} a)

```
1 1
1,0
NEUMANN MATRIX
50,90,1
1      ROW DEFINITIONS FIRST
    1=    C         OBJECTIVE FUNCTION
    2-    MILWHEA   MILLED WHEAT              KG
    3-    MILRICE   MILLED RICE               KG
    4-    VEGEOIL   VEGETABLE OIL             KG
    5-    GURSUGA   SUGAR                     KG
    6-    VEGETAB   VEGETABLES                KG
    7-    FRUITSC   FRUIT                     KG
    8-    POULTRY   POULTRY EGGS              KG PROTEIN
    9-    BOVMEAT   BOVINE MEAT               KG
   10-    INMFISH   FISH                      KG
   11-    DAIRYCG   DAIRY PROD.               KG MILK EQUIV.
   12-    ENERGYT   ENERGY                    MEGA-JOULES
   13-    TRADNOA   TRADEABLE NON-AGR.        1970 $ *100
   14-    NTRADNA   NON-TRAD.NON-AGR.         1970 $ *100
   15-    SLAUCOW   COW FOR SLAUGHTERING      ANIMAL
   16-    COTTONF   COTTON PROC.              METER CLOTH
   17-    JUTERPR   JUTE PROC.                KG
   18-    LEATHER   HIDES PROC.               KG
   20-    FOREIGN   FOREIGN EXCHANGE          TAKA 1984/85
   21-    COTTONU   UNPROCESSED COTTON        KG LINT
   22-    JUTEREU   UNPROCESSED JUTE          KG
   23-    HIDESCU   UNPROCESSED HIDES         KG
   25-    GRCAPKH   KHARIF GROWING CAP.       GROWING CAP.
   26-    GRCAPRA   RABI GROWING CAP.         GROWING CAP.
   28-    FERTCAP   FERTILIZER PROD.CAP.      KG FERT.
   29-    COTTPRC   COTTON PROCESSING CAP.    METER CLOTH
   30-    LEATPRC   HIDE PROCESSING CAP.      KG
   31-    JUTEPRC   JUTE PROCESSING CAP.      KG
   32-    NATRPRC   TRAD.NON-AG.PROC.CAP.     1970 $ CAP.
   33-    NANTRPC   NON-TRAD.NON-AG.PROC.C.   1970 $ CAP.
   34-    OTHAGPC   OTHER AGRIC.PROC.CAP.     KG
   35-    CATAGG1   AGGR.CATTLE FOR DRAUGHT P.AGGR.ANIMAL
   36-    CATAGG2   AGGR.CATTLE FOR MILK/MEAT AGGR.ANIMAL
   37-    YOUNGCO   AGGR.YOUNG COW            AGGR.ANIMAL
   38-    GOAAGGR   AGGR.GOAT                 AGGR.ANIMAL
   39-    CHICKEN   CHICKEN                   CHICKEN
   45-    DRYMATR   DRY MATTER RUMINANTS      KG
   46-    CRUDPRR   CRUDE PROTEIN RUMIN.      GR
   47-    METENER   METABOLIZABLE ENERGY RUM. MEGA-JOULES
   48-    DRYMATN   DRY MATTER NON-RUM.       KG
   49-    CRUDPRN   CRUDE PROTEIN NON-RUM.    GR
   50-    METENEN   METABOLIZABLE ENERGY N.R. MEGA-JOULES
   53-    FERTILI   FERTILIZER                KG N EQUIV.
   54-    PESTICI   PESTICIDE                 1970 $
   55-    GASENER   GAS                       MEGA-JOULES
   56-    CEMENTP   CEMENT                    KG
   57-    GASCAPA   CAP. TO PROD.GAS          1000 CFT.
   58-    ELECAPA   CAP. TO PROD.ELECTRICITY  1000 KWH.
   59-    CEMCAPA   CAP. TO PROD.CEMENT       KG.
   60-    SUMMA1I   SUM.ROW  KHARIF LOCAL TOT.CROP GROW.CAP.
   61-    SUMMA2I   SUM.ROW  RABI LOCAL TOT.  CROP GROW.CAP.
   62-    SUMMA3I   SUM.ROW  CHICKEN          CHICKEN
   63-    SUMMA4I   SUM.ROW  KHARIF HYV.      CROP GROW.CAP.
   64-    SUMMA5I   SUM.ROW  RABI HYV.        CROP GROW.CAP.
   65+    SUMMA6I   SUM.ROW LABOUR            HOURS
   -1=    END ROWS
```

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a) Unlike the notation sofar, in the computer printouts rows correspond to goods and columns to activities.

(table 6.1, continued)

1	1INVKHCAP	INVESTMENT KHARIF CROP GROWING CAPACITY
2	1INVRACAP	INVESTMENT RABI CROP GROWING CAPACITY
3	0CONVRAKH	CONVERSION ACTIVITY (RABI-KHARIF)
5	1INVJUTPC	INVESTMENT JUTE PROCESSING CAPACITY
6	1INVCOTPC	INVESTMENT COTTON PROCESSING CAPACITY
7	1INVLEAPC	INVESTMENT LEATHER PROCESSING CAPACITY
8	1INVOAGPC	INVESTMENT OTHER AGRIC.PROCESSING CAPACITY
9	1INVNTRPC	INVESTMENT NONAG NON-TRADEABLE PROC. CAPACITY
10	1INVTRAPC	INVESTMENT NONAG.TRADEABLE PROC. CAPACITY
11	1INVFERTC	INVESTMENT FERTILIZER PRODUCTION CAPACITY
12	1INVCEMEC	INVESTMENT CEMENT PRODUCTION CAPACITY
13	1INVGASCA	INVESTMENT GAS PRODUCTION CAPACITY
14	1INVELECC	INVESTMENT ELECTRICITY PRODUCTION CAPACITY
15	1PADKHEEE	PADDY GROWING KHARIF HYV.EN-EN-EN (STR,BR,MAN.)
16	1PADKHFFF	PADDY GROWING KHARIF HYV.FE-FE-NI (STR,BR,MAN.)
17	1PADKLEEE	PADDY GROWING KHARIF LOW EN-EN-EN (STR,BR,MAN.)
18	1PADKLEEF	PADDY GROWING KHARIF LOW EN-EN-NI (STR,BR,MAN.)
19	1PADKLEFE	PADDY GROWING KHARIF LOW EN-FE-EN (STR,BR,MAN.)
20	1PADKLEFF	PADDY GROWING KHARIF LOW EN-FE-NI (STR,BR,MAN.)
21	1PADKLFEE	PADDY GROWING KHARIF LOW FE-EN-EN (STR,BR,MAN.)
22	1PADKLFEF	PADDY GROWING KHARIF LOW FE-EN-NI (STR,BR,MAN.)
23	1PADKLFFE	PADDY GROWING KHARIF LOW FE-FE-EN (STR,BR,MAN.)
24	1PADKLFFF	PADDY GROWING KHARIF LOW FE-FE-NI (STR,BR,MAN.)
25	1PADRHEEE	PADDY GROWING RABI HYV. EN-EN-EN (STR,BR,MAN.)
26	1PADRHFFF	PADDY GROWING RABI HYV. FE-FE-NI (STR,BR,MAN.)
27	1PADRLEEE	PADDY GROWING RABI LOW EN-EN-EN (STR,BR,MAN.)
28	1PADRLFFF	PADDY GROWING RABI LOW FE-FE-NI (STR,BR,MAN.)
30	1WHEATHEE	WHEAT GROWING RABI HYV. EN-EN (STR,MAN.)
31	1WHEATHFF	WHEAT GROWING RABI HYV. FE-NI (STR,MAN.)
32	1WHEATLEE	WHEAT GROWING RABI LOW EN-EN (STR,MAN.)
33	1WHEATLFF	WHEAT GROWING RABI LOW FE-NI (STR,MAN.)
34	1JUTEGRHI	JUTE GROWING HIGH YIELD
35	1JUTEGRLO	JUTE GROWING LOW YIELD
36	1SUGAGRHI	SUGARCANE GROWING HIGH YIELD
37	1SUGAGRLO	SUGARCANE GROWING LOW YIELD
38	1POTAGRHI	POTATO GROWING HIGH YIELD
39	1POTAGRLO	POTATO GROWING LOW YIELD
40	1COTTGRHI	COTTON GROWING HIGH YIELD
41	1COTTGRLO	COTTON GROWING LOW YIELD
42	1OILSGRHI	OILSEEDS GROWING HIGH YIELD
43	1OILSGRLO	OILSEEDS GROWING LOW YIELD
44	1PULSGRHI	PULSES GROWING HIGH YIELD
45	1PULSGRLO	PULSES GROWING LOW YIELD
46	1TEANGROW	TEA GROWING
48	1VEGEGRHI	VEGETABLES GROWING HIGH YIELD
49	1VEGEGRLO	VEGETABLES GROWING LOW YIELD
50	1FRUITSHS	FRUIT GROWING HOMESTEAD
53	1FUELWOHS	FUELWOOD GROWING HOMESTEAD
59	1YOUNGCOW	KEEPING YOUNG COWS
60	1MILKMEAT	CATTLE FOR MILK AND MEAT
61	1GOATMEHI	GOAT RAISING
62	1CHICKTRA	CHICKEN KEEPING TRADITIONAL
63	1CHICKMOD	CHICKEN KEEPING MODERN (INTERMEDIATE)
64	1FISHINLA	INLAND FISHERIES
65	1FISHMARI	MARINE FISHERIES
66	1CATTLESL	CATTLE SLAUGHTERING
67	1TRACTCON	TRACTOR POWER CONVERSION
70	1FERTIPRO	FERTILIZER PRODUCTION
71	1COTTPROC	COTTON PROCESSING
72	1JUTEPROC	JUTE PROCESSING
73	1LEATPROC	LEATHER PROCESSING
74	1NOTRAPRO	NON-TRADEABLE PRODUCTION
75	1TRADPROD	TRADEABLE PRODUCTION
76	1CEMEPROD	CEMENT PRODUCTION
77	1GASEPROD	GAS PRODUCTION
78	1ELECPROD	ELECTRICITY PRODUCTION

(table 6.1, continued)

```
 80   1IMPWHEAT    IMPORTING WHEAT
 81   1IMPMIRIC       ,,     MILLED RICE
 82   1IMPSUGAR       ,,     SUGAR
 83   1IMPTRADE       ,,     TRADEABLE
 84   1IMPPESTI       ,,     PESTICIDE
 85   1IMPENERG       ,,     ENERGY
 86   1IMPCOTLI       ,,     COTTON (LINT)
 87   1IMPFERTI       ,,     FERTILIZER
 88   1IMPVEGOI       ,,     VEGETABLE OIL
 89   1IMPCOTCL       ,,     COTTON (CLOTH)
 90   1IMPCEMEN       ,,     CEMENT
 91   1IMPHIDES       ,,     HIDES
 92   1IMPDAIRY       ,,     DAIRY
 93   1IMPMEATB       ,,     BOVINE MEAT
 95   1EXPWHEAT    EXPORTING WHEAT
 96   1EXPMIRIC       ,,     MILLED RICE
 97   1EXPSUGAR       ,,     SUGAR
 98   1EXPTRADE       ,,     TRADEABLE
 99   1EXPRAWJU       ,,     RAW JUTE
100   1EXPPROJU       ,,     PROCESSED JUTE
101   1EXPPROCO       ,,     PROCESSED COTTON
102   1EXPLEATH       ,,     LEATHER
103   1EXPFISHT       ,,     FISH
125   0DUMMYACT    DUMMY ACTIVITY
126   1CONSTANT    CONSUMPTION CONSTANTS
200   0RESOURCE VECTOR
 -1   0END COLUMNS
```

6.3 DATA AND DATA ORGANIZATION

6.3.1 Set-up

A major task in the construction of the matrix consisted of gathering and processing the necessary data to fill the entries of the matrix of figure 6.3. A great variety of data from a wide range of technical fields were needed. Most data have been taken from the data-files underlying the BAM-lp and the macro-model for the third five-year plan. They could, however, in general, not be used without some processing and moulding. To avoid the danger that, as a consequence of such data-processing, the model would be accessible to its constructor only, a set-up analogous to the one followed in constructing the BAM-lp was chosen. This set-up is schematically shown in figure 6.4. We shall briefly discuss the main features of it (for details, see Keyzer, Overbosch and Stolwijk [46]). In the figure a circle and a square represent a data-file and a computer program respectively. The Von Neumann matrix is constructed from a number of data-files. These data-files are transformed, via the interface program, into a so-called interface data-file. The latter serves, together with the

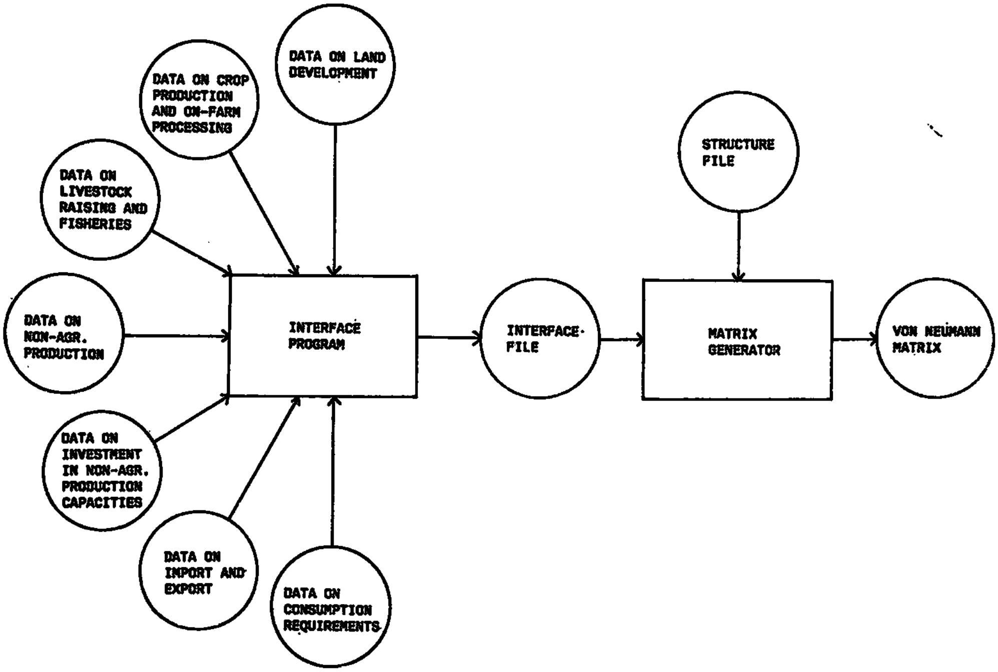

Figure 6.4
From raw data to Von Neumann matrix:
flow chart of data-files and programs

so-called structure file as input for the matrix generator. Output of the matrix generator is the Von Neumann matrix ($=M_\alpha$).

A data-file essentially consists of a number of columns, each of them describing an input-output relation. Columns which are related to each other are grouped together and put into the same table. Tables which refer to the same topic are kept in the same file. By way of illustration an extract from the data-file on land development is shown in table 6.2. The table contains the input-output relations relevant for a drainage project on medium lowland. The project-length is five year and each column refers to the inputs and the outputs relevant for the year concerned. The definitions of row- and column-codes are also part of the data-file. In table 6.2, for example, P2DRML1I stands for: project 2 (P2), drainage (DR), medium lowland (ML), first year of project (1), input (I). Other column-codes can be understood in the same manner. The first four row-codes of the table refer to different types of land. For example LMLKH3D must be read as: medium lowland (LML), kharif season (KH), third year of project output (3), type of flooding 90-180 cm (D). It can be seen in the table that output of land in year t is input of land in year t+1.

TABLE 6.2: Extract from the data-file on land development

```
========================================================================
   3
 PROJECT 2 DRAINING MEDIUM LOWLAND (INPUT)
    8    5
              P2DRML1I    P2DRML2I    P2DRML3I    P2DRML4I    P2DRML5I
                       5           7           9          11          13
LMLKH0D    4     1.00000     0.00000     0.00000     0.00000     0.00000
LMLKH1D    5     0.00000     1.00000     0.00000     0.00000     0.00000
LMLKH2D    6     0.00000     0.00000     1.00000     0.00000     0.00000
LMLKH3D    7     0.00000     0.00000     0.00000     1.00000     0.00000
LMLKH4D    8     0.00000     0.00000     0.00000     0.00000     1.00000
LABOURH   61   160.00000   160.00000   160.00000   160.00000   160.00000
FOREIGN   62     9.30000     9.30000     9.30000     9.30000     9.30000
NONAGTR   63   200.00000   200.00000   200.00000   200.00000   200.00000
C23456789012345678901234567890123456789012345678901234567890123456789012
========================================================================
   4
 PROJECT 2 DRAINING MEDIUM LOWLAND (OUTPUT)
    6    5
              P2DRML1O    P2DRML2O    P2DRML3O    P2DRML4O    P2DRML5O
                       6           8          10          12          14
LMLKH1D    5     1.00000     0.00000     0.00000     0.00000     0.00000
LMLKH2D    6     0.00000     1.00000     0.00000     0.00000     0.00000
LMLKH3D    7     0.00000     0.00000     1.00000     0.00000     0.00000
LMLKH4D    8     0.00000     0.00000     0.00000     1.00000     0.00000
LMLKH5B   10     0.00000     0.00000     0.00000     0.00000      .96000
LNONAGR   64     0.00000     0.00000     0.00000     0.00000      .04000
C23456789012345678901234567890123456789012345678901234567890123456789012
========================================================================
```

Thus, at the beginning of the project one starts with one unit of LMLKHOD and after five years, i.e. at the end of the project one ends up with .96 units of LMLKH5B and .04 units of non-agricultural land. In the meantime labour, non-agricultural tradeable product and foreign exchange are required to keep the project going.

In this way, seven data-files have been constructed:

(i) a data-file on land development
(ii) " " crop production and on farm processing
(iii) " " livestock raising and fisheries
(iv) " " non-agricultural production activities
(v) " " investment in non-agricultural production activities
(vi) " " import and export activities
(vii) " " consumption requirements

Together the files fill 5,900 lines of computer file[50]. From figure 6.4 it can be seen that the next step consists of combining all data-files in order to get, with the aid of the interface program, the so-called interface file. The interface program carries out three kinds of actions:

(a) it tests on consistency within and among data-files and looks for other mistakes,
(b) it carries out basic calculations (matrix, vector, respectively scalar multiplication, division, addition and subtraction), and
(c) it combines all data-files into one file which has the property that all tables can be directly accessed individually, i.e. it merges the data-files into one databank.

Ad (a):
Tests on consistency and other mistakes include checks on format errors and checks on the unique use of codes and abbreviations.

Ad (b):
The actual model-figures will be extracted from the interface file. This extraction is regulated by the structure file (see figure 6.4 and below). For a correct extraction it is necessary that the input-output relations of the interface file be expressed in the proper dimensions. Because the construction of the 'raw' data-files can (and has been) take(n) place in a rather loose way with regard to level of good and activity aggregation and time period of the activities, data-files dimensions are usually not

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[50] A copy of all data-files can be obtained from the author upon request.

consistent with model dimensions. The interface program has a facility to generate 'correct' tables if appropriate commands are given in the original data-file.

Thus the interface file contains all data expressed in the appropriate dimension to fill M_α. The assignment of data to places in the matrix is done by the so-called structure file. This file completely specifies matrix M_α. It consists of three parts:

(a) definitions of rows and columns of the matrix,

(b) an enumeration of correspondences between matrix and interface goods (rows), and

(c) an enumeration of correspondences between matrix and interface activities (columns).

TABLE 6.3: Extracts from the structure file

Row correspondences	Column correspondences
62, 20	9 INVNTRPC
63, 13	301
125, 25	1007
126, 26	-1
152, 53	401
153, 54	1107
160, 22	-1
161, 6	-1
162, 6	-1, 0.0
163, 7	10 INVTRAPC
165, 6	301
190, 39	1008
191, 19	-1
192, 14	401
194, 9	1108
195, 8	-1
196, 8	-1
197, 11	-1, 0.0

Table 6.1 shows part (a) of the structure file. In table 6.3 extracts from part (b) and (c) are shown. 62,20 i.e. the first line of the first column of table 6.3 must be read as: row-code 62 of the interface file corresponds with row-code 20 of the matrix M_α, etc. The second column of table 6.3 contains the correspondences between column codes 9 and 10 of M_α and the interface tables. It says that input-output coefficients for activity 9 (investments in non-tradeable non-agricultural production capacity) are to be found in respectively table 301, column 1007 and table 401, column 1107 of the interface file, etc.

The whole structure file consists of 1690 lines computer file[51]. The interface file and the structure file are inputs to the program that generates the matrix M_α. Table 6.4 contains the full matrix $M_\alpha = B - \alpha(A+C)$ for $\alpha = 1$.

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[51] A copy of the structure file can be obtained from the author upon request.

TABLE 6.4: Matrix M_α

		INVKHCAP	INVRACAP	CONVRAKH	INVJUTPC	INVCOTPC	INVLEAPC	INVOAGPC	INVNTRPC	INVTRAPC
2 MILWHEA	L	3.1286	44.7673	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3 MILRICE	L	12.7313	182.1727	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4 VEGEOIL	L	.3683	5.2698	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5 GURSUGA	L	2.4478	35.0257	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6 VEGETAB	L	1.8502	26.4744	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7 FRUITSC	L	3.8811	55.5343	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8 POULTRY	L	.0729	1.0433	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9 BOVMEAT	L	.9327	13.3455	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10 INMFISH	L	2.3727	33.9516	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11 DAIRYCG	L	6.3591	90.9931	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12 ENERGYT	L	161.2626	2307.5208	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13 TRADNOA	L	447.2434	6553.5827	0.0000	8.5000	1.9800	20.1300	3.0000	28.9200	54.0600
14 NTRADNA	L	211.8729	3031.7087	0.0000	4.0800	1.2200	49.4000	14.0000	177.3300	20.1300
16 COTTONF	L	3.0818	44.0977	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18 LEATHER	L	.0262	.3754	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20 FOREIGN	L	448.3767	10191.2908	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25 GRCAPKH	L	-.4180	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26 GRCAPRA	L	0.0000	-9.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29 COTTPRC	L	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000
30 LEATPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000
31 JUTEPRC	L	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32 NATRPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000
33 NANTRPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000
34 OTHAGPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-20.0000	0.0000	0.0000
60 SUMMA1I	L	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
61 SUMMA2I	L	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
65 SUMMA6I	G	279.0000	3965.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(table 6.4, continued)

			INVFERTC	INVCEMEC	INVGASCA	INVELECC	PADKHEEE	PADKHFFF	PADKLEEE	PADKLEEF	PADKLEFE
2	MILWHEA	L	0.0000	0.0000	0.0000	0.0000	12.3437	12.3437	7.3948	7.3948	7.3948
3	MILRICE	L	0.0000	0.0000	0.0000	0.0000	-891.7894	-891.7894	-381.2879	-381.2879	-381.2879
4	VEGEOIL	L	0.0000	0.0000	0.0000	0.0000	1.4530	1.4530	.8705	.8705	.8705
5	GURSUGA	L	0.0000	0.0000	0.0000	0.0000	9.6577	9.6577	5.7857	5.7857	5.7857
6	VEGETAB	L	0.0000	0.0000	0.0000	0.0000	7.2998	7.2998	4.3731	4.3731	4.3731
7	FRUITSC	L	0.0000	0.0000	0.0000	0.0000	15.3125	15.3125	9.1734	9.1734	9.1734
8	POULTRY	L	0.0000	0.0000	0.0000	0.0000	.2877	.2877	.1723	.1723	.1723
9	BOVMEAT	L	0.0000	0.0000	0.0000	0.0000	3.6797	3.6797	2.2045	2.2045	2.2045
10	INMFISH	L	0.0000	0.0000	0.0000	0.0000	9.3615	9.3615	5.6083	5.6083	5.6083
11	DAIRYCG	L	0.0000	0.0000	0.0000	0.0000	25.0896	25.0896	15.0306	15.0306	15.0306
12	ENERGYT	L	0.0000	0.0000	0.0000	0.0000	-2839.2190	1744.1821	-5285.2271	-985.4939	-1151.6599
13	TRADNOA	L	12.2100	4.3500	49.1400	2945.2000	436.5913	436.5913	263.4574	263.4574	263.4574
14	NTRADNA	L	3.5100	1.6400	38.1200	2759.0600	962.9671	962.9671	535.0456	535.0456	535.0456
15	SLAUCOW	L	0.0000	0.0000	0.0000	0.0000	-.0465	-.0465	-.0465	-.0465	-.0465
16	COTTONF	L	0.0000	0.0000	0.0000	0.0000	12.1591	12.1591	7.2843	7.2843	7.2843
18	LEATHER	L	0.0000	0.0000	0.0000	0.0000	.1035	.1035	.0620	.0620	.0620
20	FOREIGN	L	0.0000	0.0000	0.0000	0.0000	27.5781	27.5781	0.0000	0.0000	0.0000
28	FERTCAP	L	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
35	CATAGG1	L	0.0000	0.0000	0.0000	0.0000	.0288	.0288	.0288	.0288	.0288
45	DRYMATR	L	0.0000	0.0000	0.0000	0.0000	509.9287	-2600.7813	509.9287	509.9287	509.9287
46	CRUDPRR	L	0.0000	0.0000	0.0000	0.0000	14.8253	-94.0496	14.8253	14.8253	14.8253
47	METENER	L	0.0000	0.0000	0.0000	0.0000	3205.6631	-16391.8099	3205.6631	3205.6631	3205.6631
48	DRYMATN	L	0.0000	0.0000	0.0000	0.0000	0.0000	-463.9800	0.0000	0.0000	-202.6200
49	CRUDPRN	L	0.0000	0.0000	0.0000	0.0000	0.0000	-39.4383	0.0000	0.0000	-17.2227
50	METENEN	L	0.0000	0.0000	0.0000	0.0000	0.0000	-4639.8000	0.0000	0.0000	-2026.2000
53	FERTILI	L	0.0000	0.0000	0.0000	0.0000	40.7200	36.8084	13.2500	9.3384	13.2500
54	PESTICI	L	0.0000	0.0000	0.0000	0.0000	.2780	.2780	0.0000	0.0000	0.0000
57	GASCAPA	L	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
58	ELECAPA	L	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
59	CEMCAPA	L	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
60	SUMMA1I	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	.7200	.7200	.7200
63	SUMMA4I	L	0.0000	0.0000	0.0000	0.0000	1.4000	1.4000	0.0000	0.0000	0.0000
65	SUMMA6I	G	0.0000	0.0000	0.0000	0.0000	1183.4000	1183.4000	645.8000	645.8000	645.8000

(table 6.4, continued)

			PADKLEFF	PADKLFEE	PADKLFEF	PADKLFFE	PADKLFFF	PADRHEEE	PADRHFFF	PADRLEEE	PADRLFFF
2	MILWHEA	L	7.3948	7.3948	7.3948	7.3948	7.3948	13.7658	13.7658	9.7839	9.7839
3	MILRICE	L	-381.2879	-381.2879	-381.2879	-381.2879	-381.2879	-1175.4425	-1175.4425	-482.7859	-482.7859
4	VEGEOIL	L	.8705	.8705	.8705	.8705	.8705	1.6204	1.6204	1.1517	1.1517
5	GURSUGA	L	5.7857	5.7857	5.7857	5.7857	5.7857	10.7703	10.7703	7.6549	7.6549
6	VEGETAB	L	4.3731	4.3731	4.3731	4.3731	4.3731	8.1408	8.1408	5.7860	5.7860
7	FRUITSC	L	9.1734	9.1734	9.1734	9.1734	9.1734	17.0766	17.0766	12.1371	12.1371
8	POULTRY	L	.1723	.1723	.1723	.1723	.1723	.3208	.3208	.2280	.2280
9	BOVMEAT	L	2.2045	2.2045	2.2045	2.2045	2.2045	4.1037	4.1037	2.9167	2.9167
10	INMFISH	L	5.6083	5.6083	5.6083	5.6083	5.6083	10.4400	10.4400	7.4202	7.4202
11	DAIRYCG	L	15.0306	15.0306	15.0306	15.0306	15.0306	27.9801	27.9801	19.8867	19.8867
12	ENERGYT	L	-701.8259	131.4961	581.3301	415.1641	864.9981	-621.3946	3207.8993	-770.9219	1118.9520
13	TRADNOA	L	263.4574	263.4574	263.4574	263.4574	263.4574	486.3424	486.3424	347.0393	347.0393
14	NTRADNA	L	535.0456	535.0456	535.0456	535.0456	535.0456	979.2459	979.2459	696.8395	696.8395
15	SLAUCOW	L	-.0465	-.0465	-.0465	-.0465	-.0465	-.0465	-.0465	-.0465	-.0465
16	COTTONF	L	7.2843	7.2843	7.2843	7.2843	7.2843	13.5599	13.5599	9.6376	9.6376
18	LEATHER	L	.0620	.0620	.0620	.0620	.0620	.1154	.1154	.0820	.0820
35	CATAGG1	L	.0288	.0288	.0288	.0288	.0288	.0288	.0288	.0288	.0288
45	DRYMATR	L	509.9287	-889.0213	-889.0213	-889.0213	-889.0213	509.9287	-1749.2713	509.9287	-454.0713
46	CRUDPRR	L	14.8253	-34.1380	-34.1380	-34.1380	-34.1380	14.8253	-64.2467	14.8253	-18.9147
47	METENER	L	3205.6631	-5607.7219	-5607.7219	-5607.7219	-5607.7219	3205.6631	-11027.2969	3205.6631	-2867.5369
48	DRYMATN	L	-202.6200	0.0000	0.0000	-202.6200	-202.6200	0.0000	-606.5400	0.0000	-257.4000
49	CRUDPRN	L	-17.2227	0.0000	0.0000	-17.2227	-17.2227	0.0000	-51.5559	0.0000	-21.8790
50	METENEN	L	-2026.2000	0.0000	0.0000	-2026.2000	-2026.2000	0.0000	-6065.4000	0.0000	-2574.0000
53	FERTILI	L	9.3384	13.2500	9.3384	13.2500	9.3384	50.4000	46.4884	22.0000	18.0884
54	PESTICI	L	0.0000	0.0000	0.0000	0.0000	0.0000	.5725	.5725	0.0000	0.0000
60	SUMMA1I	L	.7200	.7200	.7200	.7200	.7200	0.0000	0.0000	0.0000	0.0000
61	SUMMA2I	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	.7200	.7200
64	SUMMA5I	L	0.0000	0.0000	0.0000	0.0000	0.0000	1.9000	1.9000	0.0000	0.0000
65	SUMMA6I	G	645.8000	645.8000	645.8000	645.8000	645.8000	1267.5000	1267.5000	855.5000	855.5000

(table 6.4, continued)

			WHEATHEE	WHEATHFF	WHEATLEE	WHEATLFF	JUTEGRHI	JUTEGRLO	SUGAGRHI	SUGAGRLO	POTAGRHI
2	MILWHEA	L	-1068.1204	-1068.1204	-415.1649	-415.1649	17.7476	9.6702	13.9364	7.5086	7.5086
3	MILRICE	L	56.4805	56.4805	19.6756	19.6756	72.2209	39.3512	56.7120	30.5550	30.5550
4	VEGEOIL	L	1.6338	1.6338	.5692	.5692	2.0892	1.1383	1.6405	.8839	.8839
5	GURSUGA	L	10.8593	10.8593	3.7830	3.7830	13.8857	7.5659	-2835.2562	-292.5653	5.8747
6	VEGETAB	L	8.2081	8.2081	2.8594	2.8594	10.4956	5.7187	8.2417	4.4404	-3169.9276
7	FRUITSC	L	17.2178	17.2178	5.9980	5.9980	22.0162	11.9960	17.2883	9.3145	9.3145
8	POULTRY	L	.3235	.3235	.1127	.1127	.4136	.2254	.3248	.1750	.1750
9	BOVMEAT	L	4.1376	4.1376	1.4414	1.4414	5.2907	2.8828	4.1546	2.2384	2.2384
10	INMFISH	L	10.5263	10.5263	3.6669	3.6669	13.4598	7.3339	10.5694	5.6945	5.6945
11	DAIRYCG	L	28.2113	28.2113	9.8277	9.8277	36.0735	19.6554	28.3270	15.2619	15.2619
12	ENERGYT	L	210.0901	1765.4194	-600.9533	249.2240	-4477.4014	-2714.5521	-50.5247	175.3367	1437.0302
13	TRADNOA	L	489.6534	489.6534	173.2364	173.2364	626.6490	344.0627	491.3089	266.4340	266.6012
14	NTRADNA	L	982.6932	982.6932	359.6879	359.6879	1234.1630	682.1457	1005.0419	549.7394	545.2412
15	SLAUCOW	L	-.0400	-.0400	-.0400	-.0400	-.0193	-.0193	-.0126	-.0126	-.0132
16	COTTONF	L	13.6720	13.6720	4.7628	4.7628	17.4822	9.5256	13.7280	7.3963	7.3963
18	LEATHER	L	.1164	.1164	.0405	.0405	.1488	.0811	.1169	.0630	.0630
22	JUTEREU	L	0.0000	0.0000	0.0000	0.0000	-1093.0000	-595.0000	0.0000	0.0000	0.0000
34	OTHAGPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	79.0600	8.2900	0.0000
35	CATAGG1	L	.0248	.0248	.0248	.0248	.0407	.0407	.0265	.0265	.0277
45	DRYMATR	L	438.1307	-605.4693	438.1307	24.1307	617.6256	617.6256	402.2318	402.2318	420.1812
46	CRUDPRR	L	12.7379	-23.7881	12.7379	-1.7521	17.9564	17.9564	11.6942	11.6942	12.2161
47	METENER	L	2754.3058	-3820.3742	2754.3058	146.1058	3882.6992	3882.6992	2528.6271	2528.6271	2641.4664
53	FERTILI	L	29.0000	25.6392	0.0000	-3.3609	184.0000	23.0000	96.0000	0.0000	320.0000
54	PESTICI	L	.1450	.1450	0.0000	0.0000	.8000	0.0000	1.0000	0.0000	1.0000
60	SUMMA1I	L	0.0000	0.0000	0.0000	0.0000	0.0000	.8000	0.0000	.7200	0.0000
61	SUMMA2I	L	0.0000	0.0000	.7200	.7200	0.0000	0.0000	0.0000	.7200	0.0000
63	SUMMA4I	L	0.0000	0.0000	0.0000	0.0000	1.4000	0.0000	1.3300	0.0000	0.0000
64	SUMMA5I	L	1.7000	1.7000	0.0000	0.0000	0.0000	0.0000	1.5000	0.0000	1.5000
65	SUMMA6I	G	1222.5000	1222.5000	422.7000	422.7000	1557.6000	854.6000	1227.4000	662.4000	661.7000

(table 6.4, continued)

			POTAGRLO	COTTGRHI	COTTGRLO	OILSGRHI	OILSGRLO	PULSGRHI	PULSGRLO	TEANGROW	VEGEGRHI
2	MILWHEA	L	5.1195	13.9364	6.5416	10.9216	5.9728	5.6883	3.4130	21.4450	11.6611
3	MILRICE	L	20.8330	56.7120	26.6199	44.4437	24.3051	23.1477	13.8886	87.2670	47.4529
4	VEGEOIL	L	.6026	1.6405	.7700	-349.7644	-120.3969	.6696	.4018	2.5244	1.3727
5	GURSUGA	L	4.0055	10.9038	5.1181	8.5450	4.6731	4.4505	2.6703	16.7785	9.1236
6	VEGETAB	L	-437.7610	8.2417	3.8686	6.4588	3.5322	-2150.5860	-589.8496	12.6821	-1349.1039
7	FRUITSC	L	6.3508	17.2883	8.1149	13.5484	7.4093	7.0565	4.2339	26.6029	14.4657
8	POULTRY	L	.1193	.3248	.1525	.2545	.1392	.1326	.0795	.4998	.2718
9	BOVMEAT	L	1.5262	4.1546	1.9501	3.2558	1.7805	1.6957	1.0174	6.3929	3.4763
10	INMFISH	L	3.8826	10.5694	4.9612	8.2830	4.5298	4.3141	2.5884	16.2640	8.8438
11	DAIRYCG	L	10.4058	28.3270	13.2963	22.1991	12.1401	11.5620	6.9372	43.5888	23.7021
12	ENERGYT	L	263.8842	416.5114	1.1854	-1919.6870	-946.0551	1343.2047	175.9228	1105.3816	1651.0696
13	TRADNOA	L	183.0194	492.4797	233.7740	385.5021	212.3682	202.2508	122.6490	-10538.4534	412.8780
14	NTRADNA	L	373.4473	993.5544	477.7639	769.8699	419.7255	404.9625	245.8732	1482.2925	829.4650
15	SLAUCOW	L	-.0132	-.0165	-.0165	-.0115	-.0115	-.0318	-.0318	0.0000	-.0481
16	COTTONF	L	5.0429	13.7280	6.4438	10.7583	5.8834	5.6033	3.3620	21.1243	11.4867
18	LEATHER	L	.0429	.1169	.0549	.0916	.0501	.0477	.0286	.1798	.0978
21	COTTONU	L	0.0000	-650.0000	-175.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
34	OTHAGPC	L	0.0000	0.0000	0.0000	2.5075	.8650	0.0000	0.0000	0.0000	0.0000
35	CATAGG1	L	.0277	.0348	.0348	.0241	.0241	.0197	.0197	0.0000	.0298
45	DRYMATR	L	420.1812	527.8782	527.8782	366.3328	366.3328	-2842.6167	-569.6167	0.0000	527.8782
46	CRUDPRR	L	12.2161	15.3472	15.3472	10.6505	10.6505	-82.4104	-16.4934	0.0000	15.3472
47	METENER	L	2641.4664	3318.5025	3318.5025	2302.9484	2302.9484	-13764.8910	-2399.8910	0.0000	3318.5025
48	DRYMATN	L	0.0000	0.0000	0.0000	-626.8750	-216.2500	0.0000	0.0000	0.0000	0.0000
49	CRUDPRN	L	0.0000	0.0000	0.0000	-203.7344	-70.2813	0.0000	0.0000	0.0000	0.0000
50	METENEN	L	0.0000	0.0000	0.0000	-6018.0000	-2076.0000	0.0000	0.0000	0.0000	0.0000
53	FERTILI	L	25.0000	160.0000	0.0000	65.0000	10.0000	93.3276	-2.6724	60.0000	197.9507
54	PESTICI	L	0.0000	2.0000	0.0000	.5000	0.0000	.5000	0.0000	.5000	1.0000
60	SUMMA1I	L	0.0000	0.0000	.7200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
61	SUMMA2I	L	.7200	0.0000	.7200	0.0000	.8000	0.0000	.7200	0.0000	0.0000
63	SUMMA4I	L	0.0000	1.3300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
64	SUMMA5I	L	0.0000	1.3300	0.0000	2.0000	0.0000	1.5000	0.0000	0.0000	1.5000
65	SUMMA6I	G	447.7000	1265.0000	580.8000	563.6000	522.7000	504.3000	259.3000	1889.0000	1060.8000

(table 6.4, continued)

			VEGEGRLO	FRUITSHS	FUELWOHS	YOUNGCOW	MILKMEAT	GOATMEHI	CHICKTRA	CHICKMOD	FISHINLA
2	MILWHEA	L	9.4995	18.2027	2.6735	1.9909	5.2333	.5688	.2275	.2275	.8533
3	MILRICE	L	38.6567	74.0728	10.8794	8.1017	21.2959	2.3148	.9259	.9259	3.4722
4	VEGEOIL	L	1.1182	2.1427	.3147	.2344	.6160	.0670	.0268	.0268	.1004
5	GURSUGA	L	7.4324	14.2417	2.0918	1.5577	4.0945	.4451	.1780	.1780	.6676
6	VEGETAB	L	-112.3822	10.7647	1.5811	1.1774	3.0948	.3364	.1346	.1346	.5046
7	FRUITSC	L	11.7843	-3077.4193	3.3165	2.4698	6.4919	.7056	.2823	.2823	1.0585
8	POULTRY	L	.2214	.4242	.0623	.0464	.1220	.0133	-.1140	-13.6477	.0199
9	BOVMEAT	L	2.8319	5.4264	.7970	.5935	-21.5259	-5.0288	.0678	.0678	.2544
10	INMFISH	L	7.2045	13.8050	2.0276	1.5099	3.9689	.4314	.1726	.1726	-9.3529
11	DAIRYCG	L	19.3086	36.9985	5.4342	4.0467	-540.8629	1.1562	.4625	.4625	1.7343
12	ENERGYT	L	489.6518	938.2549	-2562.1938	-1054.0392	-1894.8231	-22.3964	11.7282	11.7282	43.9807
13	TRADNOA	L	332.3374	636.8141	93.5321	94.8515	209.5841	24.9404	8.9602	27.9602	79.8507
14	NTRADNA	L	663.3232	1252.7151	191.0550	180.2282	455.5056	71.3223	17.4089	55.4089	157.7835
16	COTTONF	L	9.3575	17.9305	2.6335	1.9611	5.1550	.5603	.2241	.2241	.8405
18	LEATHER	L	.0797	.1526	.0224	.0167	.0439	.0048	.0019	.0019	.0072
23	HIDESCU	L	0.0000	0.0000	0.0000	0.0000	-.7979	-.6855	0.0000	0.0000	0.0000
35	CATAGG1	L	0.0000	0.0000	0.0000	-.5076	0.0000	0.0000	0.0000	0.0000	0.0000
36	CATAGG2	L	0.0000	0.0000	0.0000	-.4384	.5218	0.0000	0.0000	0.0000	0.0000
37	YOUNGCO	L	0.0000	0.0000	0.0000	.8100	-2.4610	0.0000	0.0000	0.0000	0.0000
38	GOAAGGR	L	0.0000	0.0000	0.0000	0.0000	0.0000	-.2200	0.0000	0.0000	0.0000
39	CHICKEN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-.5000	-7.0000	0.0000
45	DRYMATR	L	0.0000	0.0000	0.0000	2393.0400	4684.3000	414.9200	0.0000	0.0000	0.0000
46	CRUDPRR	L	0.0000	0.0000	0.0000	93.6000	159.1100	16.2200	0.0000	0.0000	0.0000
47	METENER	L	0.0000	0.0000	0.0000	16601.7200	30093.5300	2222.8800	0.0000	0.0000	0.0000
48	DRYMATN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	18.0000	270.0000	0.0000
49	CRUDPRN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.0000	46.0000	0.0000
50	METENEN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	143.0000	2150.0000	0.0000
62	SUMMA3I	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	10.0000	0.0000
65	SUMMA6I	G	840.0000	1600.0000	240.0000	178.1000	459.9000	47.9000	18.0000	20.0000	80.0000

(table 6.4, continued)

			FISHMARI	CATTLESL	TRACTCON	FERTIPRO	COTTPROC	JUTEPROC	LEATPROC	NOTRAPRO	TRADPROD
2	MILWHEA	L	.3982	.2275	0.0000	.6826	4.2663	2.8442	9.6702	1.5359	.8533
3	MILRICE	L	1.6203	.9259	0.0000	2.7777	17.3608	11.5739	39.3512	6.2499	3.4722
4	VEGEOIL	L	.0469	.0268	0.0000	.0804	.5022	.3348	1.1383	.1808	.1004
5	GURSUGA	L	.3115	.1780	0.0000	.5341	3.3379	2.2253	7.5659	1.2016	.6676
6	VEGETAB	L	.2355	.1346	0.0000	.4037	2.5230	1.6820	5.7187	.9083	.5046
7	FRUITSC	L	.4940	.2823	0.0000	.8468	5.2923	3.5282	11.9960	1.9052	1.0585
8	POULTRY	L	.0093	.0053	0.0000	.0159	.0994	.0663	.2254	.0358	.0199
9	BOVMEAT	L	.1187	-230.7922	0.0000	.2035	1.2718	.8479	2.8828	.4578	.2544
10	INMFISH	L	-9.6980	.1726	0.0000	.5177	3.2355	2.1570	7.3339	1.1648	.6471
11	DAIRYCG	L	.8093	.4625	0.0000	1.3874	8.6715	5.7810	19.6554	3.1217	1.7343
12	ENERGYT	L	20.5243	11.7282	0.0000	37932.2856	260.3675	378.8373	498.4479	103.3995	116.3578
13	TRADNOA	L	133.9303	7.9602	5000.0000	2452.8805	329.2533	283.5022	2890.3075	243.1312	-575.6493
14	NTRADNA	L	106.9656	15.4089	0.0000	4834.2268	511.9176	415.6117	6088.8799	-856.9897	237.6835
15	SLAUCOW	L	0.0000	3.2980	.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	COTTONF	L	.3922	.2241	0.0000	.6724	-66.7975	3.0016	11.5256	1.7329	1.4605
17	JUTERPR	L	0.0000	0.0000	0.0000	53.0000	0.0000	-99.5000	28.0000	.1200	0.0000
18	LEATHER	L	.0033	.0019	0.0000	.0057	.0358	.0239	-99.9189	.0129	.0072
21	COTTONU	L	0.0000	0.0000	0.0000	0.0000	2.4000	0.0000	0.0000	0.0000	0.0000
22	JUTEREU	L	0.0000	0.0000	0.0000	0.0000	0.0000	108.2000	0.0000	0.0000	0.0000
23	HIDESCU	L	0.0000	-1.9788	0.0000	0.0000	0.0000	0.0000	47.0000	0.0000	0.0000
28	FERTCAP	L	0.0000	0.0000	0.0000	40.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	COTTPRC	L	0.0000	0.0000	0.0000	0.0000	2.0000	0.0000	0.0000	0.0000	0.0000
30	LEATPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.0000	0.0000	0.0000
31	JUTEPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	2.0000	0.0000	0.0000	0.0000
32	NATRPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	.3000
33	NANTRPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	.2000	0.0000
35	CATAGG1	L	0.0000	0.0000	-5.8700	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
53	FERTILI	L	0.0000	0.0000	0.0000	-1000.0000	0.0000	0.0000	0.0000	0.0000	0.0000
55	GASENER	L	0.0000	0.0000	0.0000	36812.1600	0.0000	42.5600	0.0000	19.2774	55.1040
56	CEMENTP	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	4.3800	.3000
65	SUMMA6I	G	35.0000	25.3000	0.0000	62.4000	410.0000	270.0000	841.0000	150.4000	76.3000

(table 6.4, continued)

			CEMEPROD	GASEPROD	ELECPROD	IMPWHEAT	IMPMIRIC	IMPSUGAR	IMPTRADE	IMPPESTI	IMPENERG
2	MILWHEA	L	.1138	.1138	.3982	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	MILRICE	L	.4630	.4630	1.6203	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000
4	VEGEOIL	L	.0134	.0134	.0469	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	GURSUGA	L	.0890	.0890	.3115	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000
6	VEGETAB	L	.0673	.0673	.2355	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	FRUITSC	L	.1411	.1411	.4940	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	POULTRY	L	.0027	.0027	.0093	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	BOVMEAT	L	.0339	.0339	.1187	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	INMFISH	L	.0863	.0863	.3020	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	DAIRYCG	L	.2312	.2312	.8093	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	ENERGYT	L	1289.5271	-44794.1359	-200.5803	0.0000	0.0000	0.0000	0.0000	0.0000	-15.7500
13	TRADNOA	L	113.9801	592.5801	262.9303	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000
14	NTRADNA	L	1068.7045	98.3045	242.7656	.6000	1.4900	2.6600	0.0000	80.5000	1.4900
16	COTTONF	L	.1121	.1121	.3922	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17	JUTERPR	L	7.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	LEATHER	L	.0010	.0010	.0033	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	FOREIGN	L	0.0000	0.0000	0.0000	4.5800	6.8800	5.5200	1.0500	150.0000	6.2500
54	PESTICI	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000
55	GASENER	L	1164.8000	-44800.0000	2103.0464	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
56	CEMENTP	L	-644.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
57	GASCAPA	L	0.0000	3.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
58	ELECAPA	L	0.0000	0.0000	.0300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
59	CEMCAPA	L	20.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
65	SUMMA6I	G	14.0000	4.5000	35.9000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(table 6.4, continued)

		IMPCOTLI	IMPFERTI	IMPVEGOI	IMPCOTCL	IMPCEMEN	IMPHIDES	IMPDAIRY	IMPMEATB	EXPWHEAT
2 MILWHEA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4 VEGEOIL	L	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9 BOVMEAT	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000
11 DAIRYCG	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000
14 NTRADNA	L	.6000	2.8000	8.9900	1.4000	.6600	150.0000	2.7100	12.9600	.9900
16 COTTONF	L	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20 FOREIGN	L	44.8000	12.6000	28.5900	17.2900	1.3900	359.7800	3.4100	36.0000	-2.6500
21 COTTONU	L	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23 HIDESCU	L	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000
53 FERTILI	L	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
56 CEMENTP	L	0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000

(table 6.4, continued)

			EXPMIRIC	EXPSUGAR	EXPTRADE	EXPRAWJU	EXPPROJU	EXPPROCO	EXPLEATH	EXPFISHT	DUMMYACT
1	C	N	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.0000
2	MILWHEA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
3	MILRICE	L	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4	VEGEOIL	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
5	GURSUGA	L	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
6	VEGETAB	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
7	FRUITSC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
8	POULTRY	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
9	BOVMEAT	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
10	INMFISH	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
11	DAIRYCG	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
12	ENERGYT	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
13	TRADNOA	L	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
14	NTRADNA	L	1.9500	1.5600	.1300	1.3000	0.0000	3.3500	84.8700	48.6000	1.0000
15	SLAUCOW	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
16	COTTONF	L	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000
17	JUTERPR	L	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000
18	LEATHER	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000
20	FOREIGN	L	-3.1200	-3.8000	-1.0500	-10.0100	-19.2200	-8.4500	-406.0000	-68.5000	1.0000
21	COTTONU	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
22	JUTEREU	L	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	1.0000
23	HIDESCU	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
25	GRCAPKH	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
26	GRCAPRA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
28	FERTCAP	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
29	COTTPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
30	LEATPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
31	JUTEPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
32	NATRPRC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
33	NANTRPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
34	OTHAGPC	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
35	CATAGG1	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
36	CATAGG2	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
37	YOUNGCO	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
38	GOAAGGR	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
39	CHICKEN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
45	DRYMATR	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
46	CRUDPRR	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
47	METENER	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
48	DRYMATN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
49	CRUDPRN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
50	METENEN	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
53	FERTILI	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
54	PESTICI	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
55	GASENER	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
56	CEMENTP	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
57	GASCAPA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
58	ELECAPA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
59	CEMCAPA	L	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

(table 6.4, continued)

```
                        CONSTANT          RES
 1 C              N       0.0000       0.0000
 2 MILWHEA        L 2520576.0801       0.0000
 3 MILRICE        L10257057.6080       0.0000
 4 VEGEOIL        L   57089.5648       0.0000
 5 GURSUGA        L     246.4804       0.0000
 6 VEGETAB        L  312456.2518       0.0000
 7 FRUITSC        L -220720.7970       0.0000
 8 POULTRY        L   -5185.6603       0.0000
 9 BOVMEAT        L -106863.2087       0.0000
10 INMFISH        L  140780.9844       0.0000
11 DAIRYCG        L -540178.6873       0.0000
12 ENERGYT        L16254544.2330       0.0000
13 TRADNOA        L 2460197.3581       0.0000
14 NTRADNA        L 8275257.2646       0.0000
16 COTTONF        L  207636.9957       0.0000
18 LEATHER        L    2471.6240       0.0000
20 FOREIGN        L       0.0000 47809000.0000
25 GRCAPKH        L    -238.0000       0.0000
26 GRCAPRA        L    -132.0000       0.0000
60 SUMMA1I        L       0.0000    9954.0000
61 SUMMA2I        L       0.0000    2357.0000
62 SUMMA3I        L       0.0000   92707.0000
63 SUMMA4I        L       0.0000    5885.0000
64 SUMMA5I        L       0.0000    6627.0000
65 SUMMA6I        G       0.0000 69200000.0000
```

6.3.2 A brief account of the data

In this section we shall discuss the contents of the seven interface files. Nearly all data have been taken from the data-files underlying the BAM-1p and the macro-model for the third five year plan (TFYP). Therefore the explanation will be brief.

(i) Data on land development

The data-file on land development contains all data of the land development module (ldm) of TFYP. In the ldm a number of land types of different quality are distinguished. Also a number of projects are defined. By carrying out a project, land of, say, type u is transformed into land of, say, type v where v has a greater crop growing capacity than u.

The ldm distinguishes 15 projects altogether and nine land types. For a complete description of the ldm, see Stolwijk [95]. In the Von Neumann model only two projects and two land types are distinguished, one project and land type referring to the kharif season and one project and land type referring to the rabi season. The projects are obtained through aggregating the original projects in such a manner that all original kharif (rabi) land types are used up at the same time. E.g. if the original project 1 refers to draining bottom land, and the original project 2 to draining lowland and undrained bottom and lowland are available in a proportion 1 to 10, the aggregated project consists of $1/_{11}$ x project 1 + $10/_{11}$ x project 2. If two or more projects refer to the same land type, they are assumed to be carried out in the early-eighties proportion. Aggregation of land types is done by using crop growing capacities as weighing factors. For details on the exact aggregation procedure the reader is referred to the interface file.

(ii) Data on crop production and on-farm processing

(a) Crop production activities

Growing a particular crop, say paddy, is carried out through a sequence of tasks. Each task transforms a set of (intermediate) goods into another one. A possible sequence for the case of paddy looks as follows:

	Task	Input	Output
(i)	Land preparation (including ploughing, harrowing, puddling, levelling, bunding and plastering)	Fallow land, labour, draught power	Cultivated land
(ii)	Installation and maintenance of nursery for seedlings	Land, seed, labour	Seedlings
(iii)	Transplanting paddy	Cultivated land, labour, seedlings	Transplanated paddy (TP)
(iv)	Fertilizer application	TP, fertilizer labour	Fertilized TP (FTP)
(v)	Weeding	FTP, labour	Weeded FTP (WFTP)

(continued)

	(Task	Input	Output)
(vi)	Preventing predation of harvestable paddy	WFTP, labour	Harvestable paddy
(vii)	Harvesting of paddy	Harvestable paddy, labour	Harvested paddy, land with crop residuals (LR)
(viii)	Harvesting of crop residuals	LR labour	Straw, fallow land
(ix)	Transportation of straw and harvested paddy	Draught power, straw, harvested paddy, labour	Straw and harvested paddy on homestead
(x)	Threshing	Straw and harvested paddy on homestead, labour	Straw, threshed paddy

Most outputs of a task are input for another one. If one concentrates the whole set of tasks into one activity, these in- and outputs cancel out and the activity becomes a transformation of draught power, fallow land (begin of year), labour and fertilizer into fallow land (end of year), straw and paddy.

Most tasks can be performed in a number of ways. A farmer can, for example, plough his land by using a tractor or animal power; he can either broadcast or transplant his paddy; he can weed once, twice, by hand, by using herbicides, etc.; he can use inorganic fertilizer or organic manure; drainage/irrigation can be applied or not, . . . etc. A particular combination of tasks is called the technology level at which the crop is grown. Technological change means a change of this particular combination. Therefore if one really wants to understand the role of technological change, the different combinations in which the tasks can be performed should be taken explicitly into account. However, if each potential combination of tasks to grow a crop would be treated as a separate activity, the number of activities would grow readily out of bounds. To keep things manageable (see section 6.2.1), we have limited the number of technology levels to two: a low and a high level.

Data to fill up the tables are taken from the lp-data-files 'operat' and 'crop' (see for a description Stolwijk [94]). The contents of 'operat' is based on a study by Van Heemst et al. [32]; the file 'crop' was obtained by aggregating data which were output of a crop model developed in Wageningen. A description of the crop model can be found in Van Keulen en Wolf, eds. [42].

(b) On-farm processing activities

On-farm processing relates to main and by-products. With regard to main products, the data-file distinguishes the following activities: rice parboiling, grain milling, sugarcane processing, oil pressing and cattle and goat slaughtering. Data are taken from the lp-file 'inproce' which in its turn is based on Van Asseldonk [5].

Most agricultural activities yield, apart from a main product, a by-product also. These by-products play a crucial role in the Bangladesh farming system. Their relevance may be established by the facts that (i) about 80 per cent of household energy consumption in Bangladesh is provided by biomass fuels like cow dung, straw, jute sticks, etc. (see Van Asseldonk and Stolwijk [6]), and (ii) rice straw is by far the most important source of animal feed (see World Bank [110]). On-farm processing of by-products refers to the conversion of these by-products into energy, fertilizer or (non-)ruminant feed. Conversion factors are taken from Kennes et al. [41] and Hermans [33].

It is assumed that crop growing and processing take place within the same (model-) year. Therefore, activities had to be aggregated. The principle that 'only realistic combinations must be taken into account' was the main guide-line here.

(iii) Data on livestock and fisheries

The data-file distinguishes four animal types, viz. cattle, goat, chicken and fish. With regard to cattle and goat a further distinction is made in male and female animals, while for fish separate input-output relations are defined for marine and inland fishery. The life-cycles of cattle and goats are split up in periods of one year. Mortality rates and probabilities of off-spring are taken into account. Animals of age t, feed, labour and capital are the main inputs; animals of age t+1, milk, manure and off-spring are the main outputs. Feed requirements are expressed in dry matter, metabolizable energy and crude protein. A distinction is made in ruminant and non-ruminant feed.

Data on feed requirements are taken from Hermans [33], mortality rates and chances of off-spring can be found in Stolwijk [90] and input-output coefficients on fishery are calculated from the agricultural production accounts (BBS [10]). Because livestock data on the data-file are much more detailed than in the M_α-matrix, the file also contains an aggregation procedure.

(iv) Data on non-agricultural production[52]

The lp data-file 'innonag' was the starting point in deriving technical coefficients for the non-agricultural production activities. This file contains a detailed account of the input-output relations with regard to the non-agricultural production acitivities distinguished in the BAM-lp. However, our model is more detailed on the non-agricultural sector. Apart from cloth, jute textiles, leather, tradeable and non-tradeable production activities (as in BAM-lp), separate activities are distinguished for fertilizer, cement, gas and electricity production. Thus, a further disaggregation was necessary. The latter is done along the lines described in Overbosch [72]. Finally, to be consistent with the input-output relations of TFYP, the original coefficients had to be updated.

(v) Data on non-agricultural investment activities

The procedure applied to derive coefficients for the non-agricultural investment activities is essentially the same as the one described above

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[52] In gathering and processing the necessary data, assistance was given by G.B. Overbosch and W.C.M. van Veen.

under (iv): First, relevant BAM-lp activities were further disaggregated. Thereafter, an update took place so that consistency with TFYP was obtained.

(vi) Data on import and export

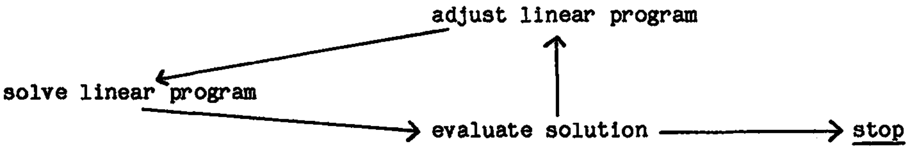
Import and export prices are taken from UNDP ([103], volume II). Because our model abstracts from explicit border interventions (no tariffs, etc.), import and export prices refer to border prices. To take transportation costs into account, importing and exporting products also costs some non-tradeable products[53].

(vii) Data on consumption requirements

The model distinguishes 14 consumption goods, viz. wheat, rice, vegetable oil, sugar, fruit, vegetables, poultry, fish, milk, energy, cotton textile, leather, tradeable non-agricultural product and non-tradeable non-agricultural product. Consumption requirements are expressed in quantities per unit of labour. To take the dynamics of consumption requirements into account (see section 3.7), the data-file also contains income elasticities. Both consumption quantities and income elasticities are taken (calculated) from TFYP[54].

6.4 SKETCH OF THE MODEL SOFTWARE

In chapter 4 it was shown that a growth factor of the model can be found by solving a sequence of linear programs. Schematically the procedure is as follows:



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[53] See footnote 52.

[54] See footnote 52.

Software to actually carry out the procedure was for the greater part already available at the SOW. Therefore, our task could be limited to a minor one. It consisted of the integration of a number of existing jobs into one procedure and the writing of a small program which evaluates the lp-outcome and generates a new estimate for the growth factor[12].

Because of this modest role and because of the fact that documentation on the main parts of the software is already available (see Keyzer, Overbosch and Stolwijk [46] and Overbosch [74]), a sketchy overview of the model software actually used in the computations seems more on place than a detailed discussion[56].

Running the job MINGRO results in the minimum growth factor and the corresponding prices and activity levels. Table 6.5 shows the structure of the job. Four stages are distinguished. In the first stage data for M_α are updated. In the second stage M_α is generated. In stage three the linear program is solved and in stage four the solution is evaluated. This evaluation results either in a new interface file **factor** and a file **estim** or only in a file **estim**. In case the former happens, the job starts again in stage one, if the latter is the case the job stops and the file **estim** contains the minimum growth factor of the model.

MINGRO makes use of four (main) programs, viz. TABLE, TECHIN, MINO1 and ALPHA. Their tasks are briefly explained in the last column of the table. Before MINGRO can be run one has, apart from these programs, to dispose of the combined interface file, the structure file and the files **factor** and **estim**. The first two have been discussed in the preceding section. The file **factor** contains a start value for α and a number of commands for updating all relevant elements (i.e. elements that become part of the A matrix of M_α) of the combined interface file. The file **estim** contains the upper and lower bound of the search domain of α. The lower bound is equal to the start value. In stage four the difference between the upper and lower bound

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[55] In doing this, help was given by M.A. Keyzer and G.B. Overbosch.

[56] Apart from the fact that the author does not know all ins and outs himself.

TABLE 6.5: Structure of MINGRO

Job stage	Name*	Input	Output	Program	Summary description
1	tech	x			Combined interface file
	factor	x			Commands for updating M_α
	TABLE			x	Interface program
	tech		x		Updated comb. interf. file
2	tech	x			
	struc	x			Structure file
	TECHIN			x	Matrix generator
	racor		x		Lp-matrix
3	racor	x			
	struc	x			
	MINO1			x	Lp-algorithm
	racor		x		Lp-matrix + -solution
4a	estim	x			Search domain for α
	racor	x			
	ALPHA			x	Evaluation of growth factor
	estim		x		Updated search domain
	factor		x		Updated commands
4b	estim	x			
	racor	x			
	ALPHA			x	
	estim		x		Resulting growth factor

* - data-file: small letters, not underlined
 - random access file: small letters, underlined
 - (main) program: capital letters.

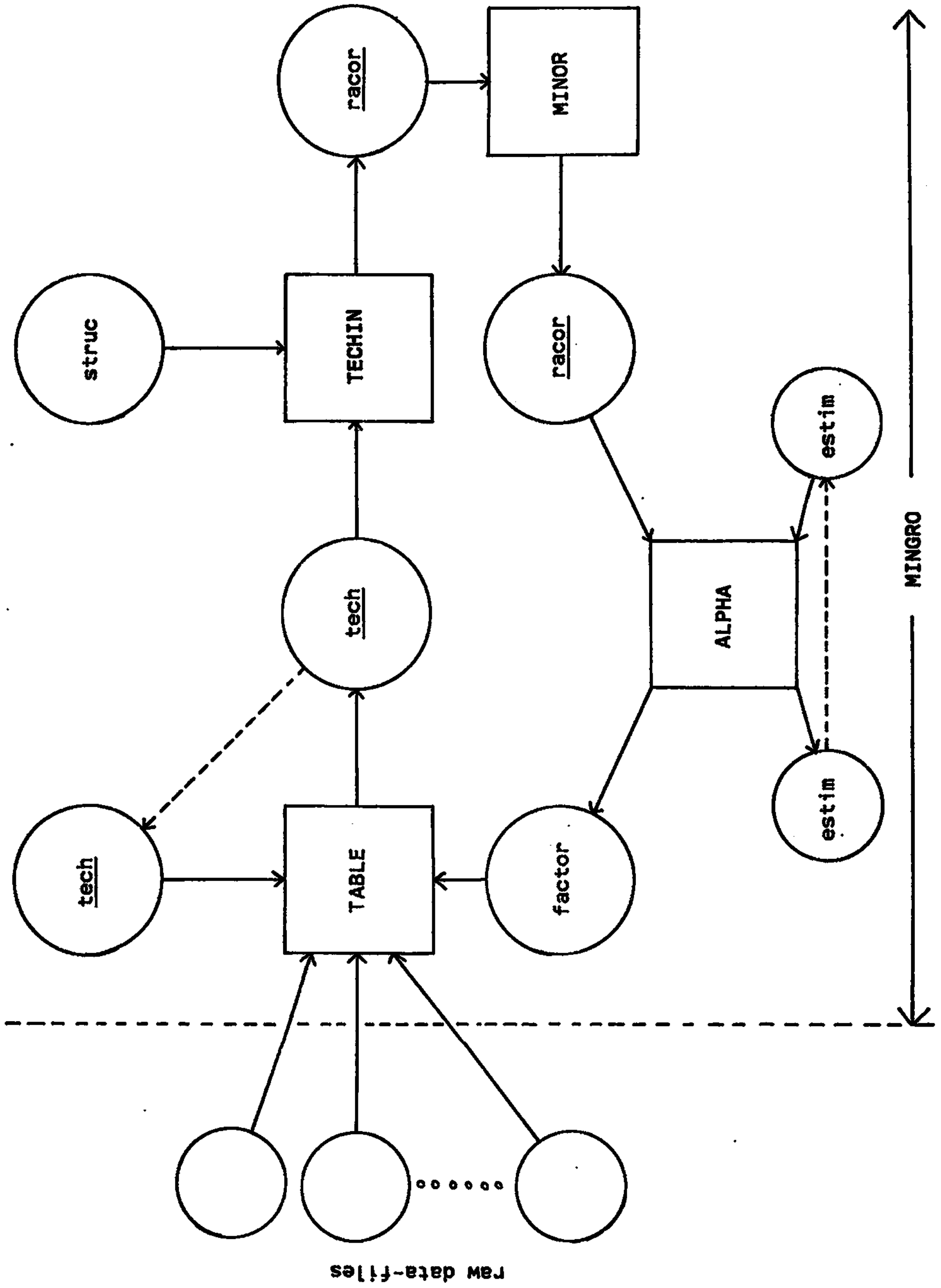
is halved each iteration. If the remaining search domain is smaller than ε, where ε is some prescribed small number, no new file **factor** is created (stage 4b) and the job stops otherwise the whole procedure starts again with a new α. Figure 6.5 shows a flowchart of the job.

The algorithm of chapter 4 is thus not fully implemented. The transition from one sub-economy to another has to be done by hand. The reasons for this are twofold. First, in the applied part of the study (chapter 7) it appeared that sub-economies à la Morgenstern and Thompson (see section 2.4) are no frequently occurring phenomena in the technology matrices of

Bangladesh. Only in the 'straight' application of the model (see section 7.2) some sub-economies could be identified. Second, given the author's limited experience in programming, it would be quite a job to enrich the software with that feature. After having applied the model to Bangladesh we think,moreover, that an extension of the software with a facility to calculate growth factors for a number of years in one run has a much higher priority.

Figure 6.5

Job flowchart



Chapter 7

MODEL RESULTS

7.1 INTRODUCTION

By now we have a theoretical model of an expanding economy (chapter 2); an extended and modified version of it, containing some particular real world characteristics (chapter 3); an algorithm (chapter 4); an empirical elaboration for Bangladesh (chapter 6); and implemented software which permits to perform actual calculations (chapter 6). In short, the set of tools necessary to attain our objectives, as stated in chapter 1, is complete.

In the present chapter we shall discuss some model results. We start with a 'straight' application (section 7.2). By this we mean an application of the model in its original form, i.e. we proceed from the assumption of a uniform growth rate for all goods. The main reasons for disregarding (at first) all non-balanced growth aspects are twofold: on the one hand it illustrates the necessity for the introduction of non-balanced growth aspects. As such it serves as a motivation of the model adjustments of chapter 3. The turnpike outcomes are so remote from the actual performance of the Bangladesh economy that their relevance is rather limited for anyone concerned with potential growth of the overall economy. However, this does not imply that the outcomes are without any practical value at all. Because they show which processes 'survive' at the highest growth rate, the 'pure' Von Neumann model is, as will be shown, among others an interesting tool for evaluating alternative technologies. In sections 7.3 and 7.4 we try to link up with the current state of the Bangladesh economy. Here the model specification is in conformity with section 3.7, i.e. initial endowments and non-balanced growth characteristics are brought in. As explained in chapter 3, non-balanced growth implies that the outcomes apply to one period only. Therefore the model is solved for a sequence of periods: a base-run for ten years and a number of variants for a varying number of years.

7.2 A STRAIGHT APPLICATION

7.2.1 Scenarios

Figure 7.1 summarizes the balanced growth scenarios that will be analysed. The scenario choice is inspired by the discussion in chapter 2 and the first three sections of chapter 3. The number of scenarios is partly suggested by the wish to get some experience with the model (algorithm and interpretation of results).

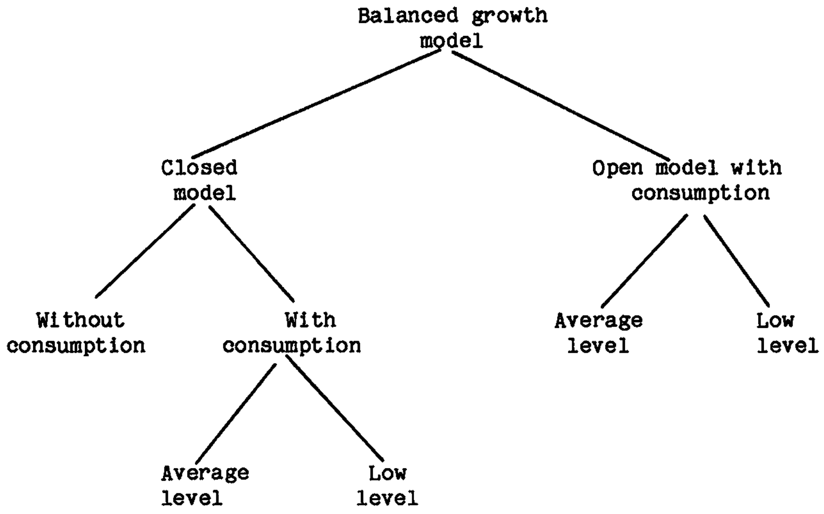


Figure 7.1
Balanced growth scenarios

Although we speak of 'a straight application', this is not completely true: land is treated as described in section 3.5 and is expressed in crop growing capacity. The reason for this will be clear. As explained earlier, when no uncleared land is left, land has to be aggregated in order to avoid zero-growth solutions.

7.2.2 The closed model without consumption

One of the objectives of our study is the testing and evaluation of the Von Neumann model on its empirical usefulness. To achieve this goal it seems wise not only to start with a relatively simple model but also to proceed from a model structure similar to the one found in the mainstream literature on the Von Neumann model. Therefore we start with a closed model under the assumption of zero-consumption. The outcomes throw some light on the question at what rate production capacities can grow at most and in a balanced way if no consumption and no international trade take place. Or, to say it more loosely: how fast can Bangladesh (balanced) grow if consumption is no limiting factor and foreign exchange is maximally restrictive. From a policy point of view the answer to this particular question is not very interesting, but as we have just stressed, policy relevance is not our only criterion.

Although the structure of the technology matrix M_α of the closed model is basically the same as the matrix of figure 6.4, there are some differences:

(i) The closed model does not contain any import and export activities;

(ii) Upper bounds on activities (capacity-constraints) have been deleted;

(iii) There is no lower bound on labour use. A summation row $\Sigma_i x_i = 1$ is added instead, where x_i refers to the level of activity i;

(iv) Consumption levels are set at zero; and

(v) It is assumed that pesticides, tractors and pumps etc. for irrigation and drainage, can be produced locally.

The search range for the algorithm was set at $1.00 \lesssim \alpha \lesssim 1.12$, and the criterion to stop (ε) at 0.001, which implies that the accuracy of the calculated growth factor is .05%. Applying the HTW-procedure yields, after eight iteration-cycles, i.e. after solving eight linear programs, the minimum growth factor ($\alpha_{\min}$). The path to $\alpha_{\min}$ was as follows:

Iteration-cycle	α-value	Objective value linear program
1	$\alpha = 1.00$	1,114
2	$\alpha = 1.06$	674
3	$\alpha = 1.09$	0
4	$\alpha = 1.075$	418
5	$\alpha = 1.0825$	188
6	$\alpha = 1.08625$	0
7	$\alpha = 1.084375$	97
8	$\alpha = 1.0853125$	39

Figure 7.2 shows the route to α_{min}. Only the unbroken line is relevant here. Thus, the minimum growth factor is 1.086. And the balanced growth rate of the economy is 8.6%. Of course, this rather high rate has to be explained from the facts that initial endowments are assumed to be available in the optimal proportions and that no consumption takes place at all. We shall take up both points later on. The objective value in both iteration-cycle three and six is zero. According to the results of chapter 2, this implies that for all α's in the range T bounded by α_6=1.08625 and α_3=1.09, $v(M_\alpha)$=0. Thus, the economy contains at least one sub-economy. Give the numerical results so far, the growth factor of this sub-economy is equal to or larger than 1.09. We use the method of chapter 4 to locate this and eventual other sub-economies.

In the outcomes, female cows and calves are the only goods that have a clear positive price. Some of the other goods have a very small positive price, i.e. a price less than one thousandth and often even less than one millionth of the price of cows and calves. We consider these prices to be zero. We attribute the positiveness to the fact that the objective value of the linear program from which the results are derived, is non-zero (lp of iteration 8). From chapter 4 it is known that if a price is erroneously set to zero, case (b) of part (1-ii) of section 4.3.5 will appear. Female cows and calves rows are 'relaxed' and activities having non-zero elements on these rows are 'penalized'. The resulting economy is solved again. For a good understanding, we like to stress that draught power can also be delivered by tractors, which can, by assumption, be produced by the tradeable sector. Thus, penalizing female cows need not be disastrous for a growth of draught power and the related crop sector.

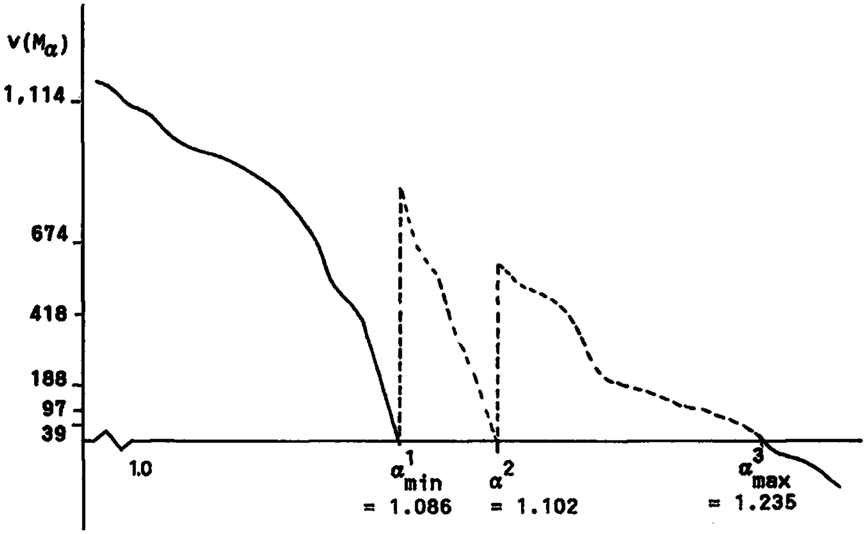


Figure 7.2

Economic solutions of the closed model

We start at the minimum balanced growth factor of the overall economy, i.e. at α=1.086. The objective value appears to be zero. Moreover, only dairy has a positive price. The situation is a nice example of case (b) of part (1-ii) of section 4.3.5: the economy including dairy and excluding female cows and calves can grow at a rate lower than α=1.086 (e.g. at α=1.001). However, this α-value is non-economic in the economy as a whole, thus in the economy including female cows and calves. Dairy has a zero-price in that case. According to the procedure of chapter 4, the dairy-row has to be relaxed. Having done this, the algorithm (HTW-procedure) is applied again and after six iteration-cycles a minimum growth factor α=1.102 is found. Thus, without cows and calves the balanced growth rate rises to 10.2%. Only goats have a clear positive price now. After relaxing rows and penalizing activities which are related to goat raising, the search-procedure is started again. We find, again via an intermediate step, in which the leather sector is penalized, an α-value of 1.235 which is also the maximum growth factor of the economy. The broken lines in figure 7.2 show the route from α^1 to α^2 and from α^2 to α^3. A rough and oversimplified conclusion of these first computations would be that the growth potential inherent to the technology matrix is quite high. Limitations are biological rather than technological in nature.

Because the emphasis is so much on growth of production capacities, it is interesting to look at (relative) levels of investment. These levels have been summarized in table 7.1. To facilitate comparisons between the sub-economies, investment in tradeable capacity is set in all cases at 1,000 while other investment activities are scaled accordingly. For a good understanding of the table it must be kept in mind that (i) the model formulation is such that investment levels refer to gross investments and (ii) because of differences in dimensions, only relative changes between the sub-economies are of interest.

The main conclusion which appears from the table is that if the growth factor increases, investments in non-agricultural production capacities become relatively more important, while investments in drainage and irrigation lag behind. Main exceptions are the tradeable sector and the leather sector. For the latter sector this is obvious: if goats and milk-cows disappear, the leather sector stops being viable. The relative drop of investments in

TABLE 7.1: Relative investment levels in the sub-economies of the closed model with zero-consumption

Investment in sector:	Growth factor		
	$\alpha=1.086$	$\alpha=1.102$	$\alpha=1.235$
Tradeables	1,000	1,000	1,000
Non-tradeables	401	458	821
Jute processing	14	16	28
Cotton/cloth	110	116	158
Leather	3	3	0
Other agricultural products	1	1	1
Fertilizer	2	3	29
Cement	364	419	864
Gas	19	20	27
Electricity	2	3	2
Goats	69	1,419	0
Milk-cows	92	0	0
Calves	222	0	0
Kharif drainage/irrigation	54	36	8
Rabi irrigation	17	11	1

tradeable capacity is related to the lapse in drainage and irrigation investments. Because tradeables form, apart from labour, the main input, backwardness of drainage and irrigation investments have repercussions on the tradeable sector.

It is interesting to compare the model intensities with the actual production pattern of the Bangladesh economy. This can be done by multiplying all intensities by a factor ψ, which is defined as:

$$\psi = \min_{j} \frac{xE^{j}}{k_{j}} \qquad j=1,2...p$$

where

x = intensity vector

E = matrix of capacity requirements, i.e. elements e_{ij} stand for the quantity of capacity j required to carry out process i at unit intensity

k_j = available capacity j.

Without going into details, the results of the scaling can be summarized as follows:

(i) At α=1.086, land (high yielding crop growing capacity) is the limiting factor. Most of the non-agricultural capacity (80-99%) remains unused.

(ii) At α=1.102, livestock (goats) determines the scale. ± 80% of the crop and non-agricultural sectors remain slack.

(iii) At α=1.235, non-agricultural capacities are limiting. About 90% of the crop growing capacity is unused if the economy would behave according to this turnpike.

Case (i) and (iii) are schematically illustrated in figure 7.3. In order to permit representation in two dimensions, case (ii) is left out. In the figure, K stands for the actual production capacities. The conclusion is clear: endowments consistent with turnpike growth, are, according to the model outcomes, too remote from the actual endowments to be of much relevance for statements about growth potential of the overall economy.

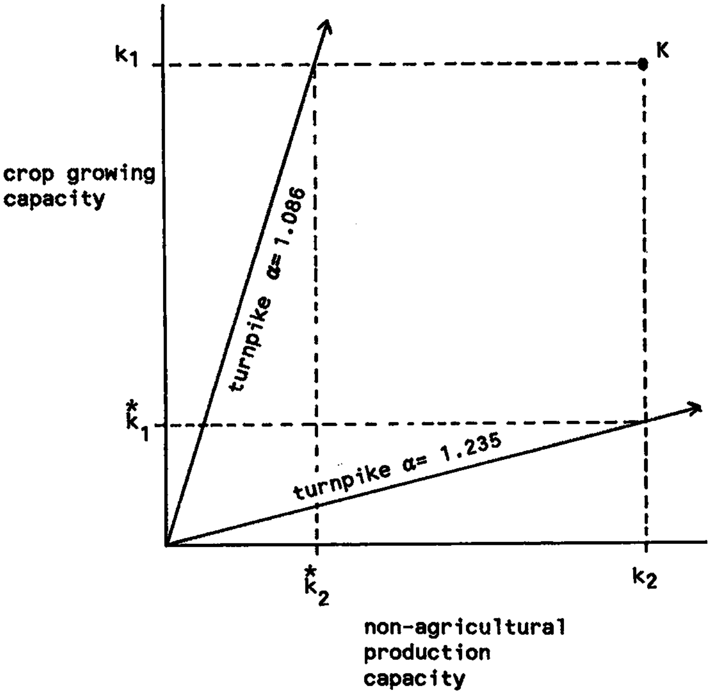


Figure 7.3

Turnpike capacities (k_1, k_2^*, respectively k_1^*, k_2) versus actual capacities (k_1, k_2)

However, because we left out consumption altogether, it is too early to apply this conclusion to the actual Bangladesh economy. For this, but also for other obvious reasons, it is necessary to include consumption into the model.

7.2.3 The closed consumption model

Consumption is included in the form described in section 3.3. Thus, inputs related to consumption are proportional to the quantity of labour involved in the particular process. Consumption per unit of labour is, at first, equated to average consumption in 1984/85. Data on this are in the interface file on consumption (see section 6.3). The inclusion of consumption results in a drastic decrease of the growth factor. The balanced growth rate drops to 4.76 per cent. From the objective values of the linear programs solved in the different iteration-cycles of the algorithm, it follows that the growth factor is unique. This can also be discovered by applying the procedure of chapter 4, i.e. by penalizing all processes that use respectively produce goods that have a non-zero price. In spite of the drastic decrease, a growth rate of 4.76% is rather high, especially if one realizes that we are still dealing with a closed economy. However, for a correct judgement, the results have to be scaled to the 'real economy'. If all intensities are raised at the same rate, goats become the first limiting factor. Scaling the outcomes on goats implies that all intensities have to be multiplied by

$$\frac{\text{actual number of goat raising activities}}{\text{model intensity}} = \frac{5,938,000}{529} = 11,225$$

If all intensities are scaled accordingly, an employment rate of 11.4 per cent results. The corresponding consumption quantities are just enough to provide the same percentage of the population with an average diet. Again, the economy corresponding with turnpike growth is too far away from the actual economy. Except for goats, utilization rates of most capacities are less than 10%. The high intensity of goat raising in the model outcomes, which is the 'cause' of this result, is mainly due to the hide production of these animals. In the closed model, leather production via goat hides appears to be much more efficient than via cattle hides. Over 75 per cent of all leather production originates from goat hides; in reality this percentage has been estimated at nearly 14 (Stolwijk, [90]).

Let us assume now that, either because leather consumption decreases or because the number of goats/cows and/or hide production per goat/cow increases, the number of goats is not the limiting factor any longer. We have to look then for the next bound (limiting factor) to derive a scaling factor. The first one we meet, is the capacity to produce non-agricultural tradeable products. The new scaling factor can be calculated as:

$$\frac{\text{actual capacity}}{\text{model intensity}} = \frac{54{,}740{,}000}{1520.5} = 36{,}001$$

Multiplication of all intensities with this factor results in the corresponding 'turnpike economy'. Although this one is closer to the actual economy than the one scaled on goats, yet only 37.0 per cent of the population can find a living in it, while at the same time nearly all non-drained kharif land remains fallow.

We could go on like this and again make some particular assumption, this time with regard to the production and consumption of the non-agricultural tradeable product. We have not done this, mainly because we do not want to drift away too far from the actual economy. We shall instead open the economy, i.e. we shall allow for international trade. However, we shall first discuss some other outcomes of the closed consumption model and of a variant in which consumption levels are set at 80% of the average. Calory-intake corresponding with this percentage can be considered as the minimum level at which still a normal labour output can be delivered.

A number of goods can be produced in more than one way. In table 7.2 technology choices of the model are summmarized. Except for wheat and jute, the average consumption scenario chooses in all cases a modern production technology. For a good understanding we stress that available production

TABLE 7.2: Technology choices in the closed model

Scenario and technology choice	Average consumption		Zero-consumption	
Product	Modern	Traditional	Modern	Traditional
Rice	x		x	
Wheat		x	x	
Jute		x		x
Sugarcane	x		x	
Cotton	x			x
Oilseeds	x			x
Pulses	x			x
Chicken	x		x	

capacities, are not restrictive in this scenario. Thus, even in a low wage country like Bangladesh, the assumption of no restriction on production capacities which in this special case means no restriction on the landbase, does not lead to a preference for relatively capital extensive and labour intensive farming techniques. In the last two columns of the table, technology choices of the zero-consumption model are summarized. Here the results are less unambiguous, although for rice, sugarcane and chicken the choice for modern technologies remains. To locate the technological switch points of the other products of the table, we have solved the model with consumption levels decreased to 80% of the average. Compared to the average consumption scenario, no 'technology-switches' appear. This result strengthens the earlier conclusion that modern technologies are, at least from an economic point of view, superior to traditional ones, even in a low wage-land abundance environment.

Table 7.3 summarizes relative investment levels. Compared to the zero-consumption model (table 7.1), relative investment levels in sectors mainly producing consumption goods (cloth) are much higher. The sharp rise of the rel-

TABLE 7.3: Relative investment levels in the closed consumption model

	Average consumption scenario	Consumption at 80% of average scenario
	α=1.0476	α=1.0858
Tradeables	1,000	1,000
Non-tradeables	811	818
Jute processing	61	65
Cotton/cloth	945	784
Leather	6	5
Other agricultural products	23	18
Fertilizer	696	723
Cement	637	682
Gas	177	162
Electricity	-	-
Goats	452	236
Milk-cows	17	9
Calves	34	23
Kharif drainage/irrigation	23	24
Rabi irrigation	1	1

ative investment level in fertilizer production capacity is caused by the increased importance of the crop growing sector in the closed consumption model and the superiority of modern (high yielding) varieties.

Relative investment levels in the 'decreased' consumption model do not differ significantly from the average consumption model. Because the model allows substitution of all kinds (among activities), relative levels have not changed in the same direction.

Finally, in table 7.4 relative prices of a number of goods are compared to relative prices of the same goods from TFYP (UNDP [103]) (homestead/ex factory prices). Some differences are striking. The zero-price for vegetables can be explained as follows: output ratios of consumption goods are in the closed consumption model mainly determined by (exogenous) demand, i.e. production equals consumption. Vegetables are an exception. The cause is not difficult to trace. In the model, output of pulses consist of a main product and a by-product. The main product is expressed in kg vegetables; the by-product can be used as a protein-rich livestock feed. Because there

TABLE 7.4: Comparison of relative prices: closed consumption model versus TFYP

	Prices according to:	
	Average consumption scenario	TFYP
Wheat	100	100
Rice	37	157
Vegetable oil	422	499
Sugar	73	86
Vegetables	0	77
Fruit	58	109
Poultry	282	598
Fish	933	397
Dairy	774	132
Beef	0	385
Jute textiles	652	415
Fertilizer	308	82

is a scarcity of crude protein, reflected in the fact that crude protein is the only feed component that has a non-zero shadow price, pulses are, in the model, partly grown for feed purposes only. As a result, vegetables are in over-supply and consequently, have a zero-price. The zero beef price is caused by the same mechanism. Beef[57] is produced as a joint product both by goats and cattle, and it appears that draught power, milk and hides are more determining with respect to the production level than beef. The relatively high price of dairy is partly caused by the scarcity of feed. This scarcity also explains the relatively low price of rice. In general, the following relation applies for a non-zero process:

value of outputs $= \alpha$ value of inputs

For rice and wheat, input quantities, and thus the right-hand side of the equation, do not differ very much. On the output side there is a difference however. The fodder value of the rice by-products (straw + bran) is much higher than that of wheat (straw). In a scarcity situation for fodder, the above equation can only be satisfied then if the price of the main product is relatively low. From an economic point of view this is not implausible. The by-products gain in importance at the expense of the main product. However, because consumption requirements are exogenous in the model, the 'reaction' is too rigid. In reality substitution will appear on the demand side: in the example, wheat will be replaced by rice, poultry and fish by beef, etc.

The drawback can partly be overcome by allowing international trade. The introduction of international trade has, in this respect, two consequences. On the one hand products can be obtained (and sold) in a non-joint way, on the other hand, because international prices are assumed to be independent of Bangladesh demand, international trade introduces upper and lower bounds on prices of tradeable products at the border.

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[57] To avoid misunderstanding: goat meat is included in beef.

7.2.4 The open model

By adding import and export activities, the closed consumption model of the preceding section becomes an open one. Its structure is in conformity with figure 3.1. The open model contains two growth factors, viz. α_{min}=1.086 and α_{max}=1.101. Referring to theorems 3.2 and 3.3, the following properties of the model outcomes will need no further explanation:

(i) At α_{min} all tradeable products have a zero-price.
(ii) At α_{max} all tradeable products have a positive price.
(iii) The price of the international unit of account (iau) is zero in the α_{min}-economy.
(iv) The price of the iau is positive in the α_{max}-economy.
(v) α_{min}-open is higher than α-closed (section 7.2.3), i.e. 1.086>1.048.
(vi) Not all goods of the model are internationally tradeable.

In the α_{min}-economy the livestock sector is again the limiting one. The α_{min}-solution can be considered as the 'closed solution' of the open model. Because of the uninteresting price structure we do not discuss the α_{min}-outcomes any further. The interesting case is, of course, the α_{max}-economy. Compared with the closed consumption model, the growth factor makes an enormous jump: from 4.76 per cent to 10.10 per cent. At the same time, large changes in intensities and prices appear. A striking difference with the closed consumption model is that the non-agricultural tradeable sector nearly disappears in the open model. By way of illustration, investment levels of some important sectors are compared in table 7.5. According to these outcomes Bangladesh is relatively uncompetitive in the tradeable non-agricultural sector. In the open model raw jute is the only export product; tradeable non-agricultural products on the other hand account for the bulk of the imports. The outcomes are consistent with the generally poor performance of the tradeable non-agricultural sector in Bangladesh.

The addition of import and export activities also results in more 'realistic' prices. Compared with the average consumption scenario (see table 7.4) the zero-prices for beef and vegetables have disappeared, while the price for rice has tripled. A growth rate of 10.1 per cent is indeed very high. However, for a correct judgment of this rate, the model intensities must be scaled to the 'real economy'. After applying the

TABLE 7.5: Comparison of relative investment levels

Sector	Closed consumption scenario	Open model
	α=1.0476	α=1.1010
Non-tradeables	1,000	1,000
Tradeables	1,233	0
Kharif drainage/irrigation	28	31
Rabi irrigation	1	1

scaling formula of section 7.2.2, it appears that only a mere 8 per cent of total available labour can be employed in remunerative activities. The upper bound on the export of raw jute (see section 7.3) is the first limiting factor. Gas and marine fish production capacities are the next ones, raising the employment level to 25-30 per cent.

Thus we arrive at the conclusion that although a high growth rate can be obtained, only a small part of the economy will be operational at that rate. An interesting question that comes up then is, to what extent the high growth rate could be extended to a larger part of the economy. The following four ways seem worth investigating in more detail: (i) decrease the average wage-rate; (ii) relax the trade balance; (iii) multiplicate capacities of non-zero activities; and (iv) introduce new technologies. Although we have only done some model-experiments with lower wages, we think the discussion on the model so far permits to say something on the other points also.

Ad (i): To investigate the impact of the wage-rate on the employment level, we have run the model with consumption quantities set at 80 per cent of the average level. Surprisingly, the outcomes did not significantly change. Therefore a more drastic adjustment was tried by lowering the wage-rate and the corresponding consumption quantities to zero. Now the employment level, which can be considered as a proxy for the overall capacity utilization rate, rose to 65 per cent. It is remarkable that even if labour has a zero-price, the model does not generate full-employment. This result is contradictory to a main proposition of the new-classical economists according to which no involuntary unemployment can exist (see e.g. Klamer [47]). The implausibility of this assertion in the context of a Von Neumann model is easily seen: because labour is always, at least partly, a complementary input in the production process, the non-availability or the price of this complementary input can, in principle, cause involuntary unemployment.

As a conclusion we state that by lowering the wage-rate, the scale at which the economy operates can indeed be extended. However, according to the model results, but also because of the already extreme low average wage-rate, the possibilities to actually do this are rather limited.

Ad (ii): If we allow a free movement of capital, the trade balance need not necessarily be satisfied. We can speak of balance of payment then. If the international rate of interest is i, lending and borrowing activities have the following input-output structure:

	Input		**Output**
	iau	**non-trad.**	**iau**
lending	1	k_1	1+i
borrowing	-1	k_2	-(1+i)

We shall first discuss the consequences of adding a lending activity to the model. Although we have not run the model with a lending activity, it follows from the discussion in chapter 3 that two cases can be distinguished, viz. $i \geqq 10.1$ and $i<10.1$. If $i \geqq 10.1$ and we assume, only to keep the argument as simple as possible, that $k_1=0$, then the growth rate of the economy will be i. However, this growth rate will apply to an equal or even smaller part of the economy. At $i \geqq 10.1$ Bangladesh sees its savings flowing a broad and we arrive at a conclusion similar to Pasinetti's who states that

> "if [underdeveloped countries] want to discourage their own savings from flowing abroad, there is no hope they can put in the fact that

> they have a low degree of mechanisation and a low stock of capital goods. They have to speed up their rates of economic growth"

([76], p. 249). If, on the other hand, r<10.1 no lending at all will take place. Because of the negative value of some of the input-output coefficients, the addition of a borrowing activity causes specific problems which we shall not discuss here. Consequences of borrowing on the economy can also be investigated in another way, i.e. by adding a positive number, to the right-hand side of the 'trade balance row'. It is easy to see that the economy can be 'blown up' then. And trade-offs between debt, growth and employment can be studied, both within a year and, assuming that debt plus interest has to be repaid, between consecutive years. If lending and borrowing activities are introduced in such an asymmetric way, it can also be shown that under certain conditions (lack of sufficiently profitable investment opportunities), a combination of capital flight and borrowing can exist, a situation which was typical for many of the nowadays debt-countries in the late-seventies and early-eighties.

Ad (iii): If the outcomes of the open model are looked at, one can wonder whether it would not be possible for Bangladesh to 'multiply' the part of the economy that can grow at a rate $\alpha=1.101$. Unfortunately, things are not as easy as that. As we saw above, the upper bound on jute exports and marine fishing and gas production capacities are the first bounds that are met. With respect to jute exports a multiplication of the upper bound is not possible at all. And although gas production and marine fishing capacities can certainly be expanded, some absolute bounds will soon be met also.

Ad (iv): Instead of attracting foreign capital in terms of iau's, the country can also try to attract foreign investors which take their technologies with them. Studying such a scenario could be done by adding new activities and solving the enlarged model. Although we have not followed this road either, the model outcomes so far permit some speculation on the effects of such a policy. Because of the relatively inefficient infrastructure, indicated by the very low utilization rate at $\alpha=1.101$, it is not very likely that foreign investors will find it attractive to fully integrate such new technologies in the overall economy. Instead free trade zone type of constructions, like the existing one in Chittagong, will probably be preferred. Although such zones have certainly their merits, one must, for the following reasons, not fully pins one's faith to them. First, because such zones are in fact isolated islands, the spillover effects to the rest of the economy will be rather limited. Second, if production in these zones really expands, it is not unlikely that a corresponding expansion of exports is only possible after serious price concessions. Third, too much outward-orientation makes a country vulnerable for protectionistic measures of import countries. The problems Bangladesh has with the export of textiles to the US and Western Europe are, in this connection, worth noting. Finally, foreign investors in such zones are usually not very committed to the country. If, after a number of years, conditions in another country are more attractive, production capacity will be moved without qualms.

7.3 RESULTS OF THE BASE-RUN

7.3.1 Non-balanced growth

The uniform growth model of the preceeding sections revealed some interesting characteristics of the technology matrix of the Bangladesh economy. Yet, the overall results remain too far away from the actual performance of the economy to be of much interest with respect to the growth potential of the Bangladesh economy. As became clear, the main cause of this is that the initial endowments deviate too much from the turnpike endowments. This empirical fact and the model characteristic that consumption growth per capita is not possible, motivated the introduction of non-balanced growth aspects (see chapter 3). The most 'complete' model in this connection can be found in section 3.7. In the present section the results of this particular model specification will be discussed.

By introducing actual endowments and putting a lower bound on labour, one 'forces' the model to make more fully use of existing production capacities. If the bound on labour is effective, an interesting question is how to interpret the shadow price (y^L), in other words, is there a(n) (empirical) justification for this 'coercion'. In section 3.7 it is explained that the higher y^L, the lower the wage-rate (y^W). However, an endogenous decrease of the wage-rate in the dual, does not have consequences for the basket of consumption goods in the primal. In other words, the marginal value of labour is in case $y^L>0$ smaller than the average wage-rate. So far as this is a consequence of the fact that we do distinguish only one type of homogeneous labour, this does not seem implausible. There are, moreover, other reasons why a differing marginal and average wage-rate need not be worrying. In urban areas trade unions and other interest groups contribute to such a situation. In rural areas, and to a lesser extent in urban areas also, the institutional structure dictates that less productive labour is not written off. As Thorp [100] explains, the owner of the land (malik) has to take care of his family (paribar) to which even completely unrelated field labourers may become members. The malik has to decide how the daily food is shared among the paribar. Although the duty to support unproductive labour has a religious base, it can also be interpreted as a survival strategy. Discussions on the institutional structure and power relations in Bangladesh by Brac [12], Huq [36] and Siddiqui [85] endorse the findings of Thorp, albeit in quite different terms.

The model is run for a ten-year period. For a correct assessment of the results we shall start with a discussion of the main assumptions underlying the value of some important model parameters. Later on, the model will also be run under alternative assumptions. The current version will therefore be referred to as the base-run.

7.3.2 Assumptions

Only assumptions regarding non-balanced growth characteristics of the model will be explained. They are discussed under two headings, viz. consumption, and endowments and technology matrix. At the outset we emphasize that we have tried to find a maximum of correspondence with the macro-model for the Third-Five-Year Plan (TFYP; see section 5.1).

Consumption

Incorporating consumption along the lines of section 3.7.1 requires information on:

(i) average consumption per capita/unit of labour;

(ii) a splitting of average consumption into a fixed and a variable part;

(iii) growth rate overall per capita consumption relative to growth rate per capita investments; and

(iv) population growth.

Ad (i): Average consumption per capita is contained in the databank. Multiplied by units of labour per capita yields consumption per labourer

Ad (ii): We proceed from equation (3.12), i.e.

$$c_j = k_j y + \bar{\bar{d}}_j$$

where

c_j = average per capita consumption on good j

y = per capita income (expenditures)

$\bar{\bar{d}}_j$ = fixed part in consumption equation[58]

k_j = parameter

Income elasticities $\bar{\bar{\varepsilon}}_j$ for all j can be drawn from the databank. Estimates for $k_j y$ and $\bar{\bar{d}}$ can now be made as follows:

$$\frac{\Delta c_j}{\Delta y} \cdot \frac{y}{c_j} = \frac{k_j y}{c_j} = \bar{\bar{\varepsilon}}_j$$

Thus,

$$k_j y = \bar{\bar{\varepsilon}}\, c_j$$

and

$$\bar{\bar{d}}_j = c_j - k_j y = c_j(1-\bar{\bar{\varepsilon}}_j)$$

---

[58] To avoid misunderstanding $\bar{\bar{d}}_j$ is not the committed consumption of the linear expenditure system (see section 3.7.1).

Ad (iii: In order to make an estimate of the growth rate of the overall consumption relative to the growth rate of investments we assume that:

(a) investments are equal to national savings;

(b) growth in investments is equal to growth of intermediate goods; and

(c) per capita income is equal to per capita consumption plus per capita investment.

Following UNDP ([103], volume 1, p. 22) we assume that per capita savings increase with about 1.45 per cent for every per cent increase in real per capita income. Or, together with the above assumption:

$$\frac{\Delta I}{\Delta(C+I)} \cdot \frac{C+I}{I} = 1.45 \qquad (7.1)$$

where I and C stand for total per capita expenditures on investments and consumption, respectively. If we write (δ-1) for the population growth rate, then

$$\frac{\Delta I}{I} = (\alpha-\delta) \qquad (7.2)$$

Further, if β is defined as the share of consumable goods in total domestic production, then

$$\frac{\Delta(C+I)}{C+I} \approx \beta\frac{\Delta C}{C} + (\alpha-\beta)\frac{\Delta I}{I} \qquad (7.3)$$

Next we define (γ-δ) as the per capita growth rate of overall consumption. Instead of (7.3) we may write now:

$$\frac{\Delta(C+I)}{C+I} \approx \beta(\gamma-\delta) + (1-\beta)(\alpha-\delta) \qquad (7.4)$$

Substituting (7.4) and (7.2) in (7.1) and rearranging the terms yields the following expression for per capita consumption growth:

$$\frac{C_t}{C_{t-1}}(1+\gamma-\delta) \approx \frac{(1.45\beta-.45)\alpha - (1.45\beta-.45)\delta}{1.45\beta} + 1 \qquad (7.5)$$

Values for δ and β are taken, respectively calculated from TFYP, i.e., δ=1.023 and β=.85. Filled in (7.5) yields:

$$\frac{C_t}{C_{t-1}} \approx .635\alpha+.351 \tag{7.6}$$

<u>Ad (iv)</u>: Growth in consumption originates from two sources. On the one hand per capita income increases. Relevant parameters to take this into account have been derived above. On the other hand consumption inceases because of population growth. In the base-run we proceed from a yearly growth of the population of 2.3 per cent[59]. Thus, apart from effects due to a per capita growth in consumption, both the fixed and the variable part of the consumption equation must increase at a rate of 2.3 per cent per year. This is done in the model through a yearly update of the consumption coefficients and the lower level on labour by this percentage.

Endowments and technology matrix

In general, growth of production capacities is generated by the model and year-to-year adjustments are made accordingly. Other coefficients of the B and A matrices are in principle the same for the whole of the ten-year period.

There are some exceptions to this rule:

- Because actual yields in agriculture are so far below their potential (see UNDP [103], volume 2), it is assumed in the base-run that yields per acre increase at a rate of 1.5 per cent per year. For this a 1.5 per cent increase of fertilizer, pesticide, energy and tradeable inputs are assumed to be necessary.
- Total crop growing capacity need only to grow at a rate of (α-1.015) x 100 per cent per year. Moreover, instead of investing in

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[59] The 2.3 per cent applies only to periods 1-4, for periods 5-7 and 8-10 it is assumed that the growth rate decreases to 2.2 and 2.1 per cent, respectively.

kharif crop growing capacity, extra investments in rabi crop growing capacities are permitted.

- Imports are partly being paid for by exports and partly by grants, loans and foreign remittances. It is assumed that the total of grants, loans and foreign remittances grow at the same rate as total population.
- The model contains upper bounds on total area for tea, sugarcane, oilseeds and cotton. These bounds are assumed to grow at a rate $(\alpha-1)$ if the bound is hit. In case there is slack capacity this growth rate applies to the actual area only. The same rule applies to fish and chicken production capacities.
- Because of the oligopolistic position on the world jute market (see Overbosch [73]) we proceed from the assumption that actual exports of raw jute and jute textiles cannot increase with respect to their level in the mid-eighties[60].
- Conversion of gas into electricity involves a loss of energy. Because the model does not distinguish electricity as a separate good (both appear on the energy row), energy from gas will always be preferred to energy from electricity. Therefore a lower bound is put on electricity production (equal to 75 per cent of capacity).

7.3.3 Base-run outcomes: growth rates

The model has been run for a ten-year period. Because initialization has been done with 1984/85 data, the base-run can be considered as a normative growth scenario for the period 1984/85 - 1994/95, Figure 7.4 shows the computed growth rates $(\alpha-1)$. It should be realized that the α's do not refer to growth of the overall economy. Because consumption growth lags behind, overall growth lies somewhere in between $(\alpha-1)$ and the growth of consumption. Or, if overall growth is defined as π, then

$$\pi \approx (1-\beta)(\alpha-1) + \beta(\delta-1) + \beta(\alpha-\delta)\gamma$$

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[60] It is assumed that Bangladesh has reached the point where the price elasticity of the world market demand for Bangladesh jute is -1. According to G.B. Overbosch (personal communication), this is not too unrealistic an assumption.

where the symbols have the same meaning as in section 7.3.2. Given the model-values for γ and δ and the 'TFYP-value' for β (see section 7.3.2), we may also write

$$\pi \approx - .678 + .686\alpha$$

The bottom line in figure 7.4 shows the π-values corresponding with the values for α. Overall growth varies from a minimum of 3.8 per cent in the first year to a maximum of 4.3 per cent in the fifth to ninth year. On a per capita basis these percentages are 1.5 and 2.2, respectively.

It is interesting to note that the growth rates generated were 'first results', i.e. the model did hardly need any 'ad-hoc' adjustment in the run phase. To the reader who has experience with empirical model-building, this may come as a surprise. Why should such a collection of numbers not result, at first, in growth figures of, say -35 per cent or +120 per cent? We can give two reasons for this which (partly) explain this phenomenon. In the first place all data for the non-agricultural activities originate from an input-output table. With some exaggeration one can say that our model 'contains' an input-output model. Because an input-output model with a positive demand has an eigenvalue >1, and there is a close relationship between a Von Neumann model and an eigensystem (see Thompson and Weil [99] for details), the positiveness of the growth rate can readily be understood. Coefficients and endowments of the agricultural activities on the other hand mainly originate from the BAM-lp. Because a lot of testing has been done with the latter, it is not really surprising that this part of the model did not lead to strange outcomes either.

Turning back to the growth figures. We observe that accumulated over the whole period per capita growth amounts to 21.2 per cent. Total growth of production capacity (α)[61] even amounts to 61 per cent. Compared to Bangladesh historical growth record, the base-run definitely does not show

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[61] If the context cannot give rise to a misunderstanding, we shall often write α instead of $(\alpha-1)$.

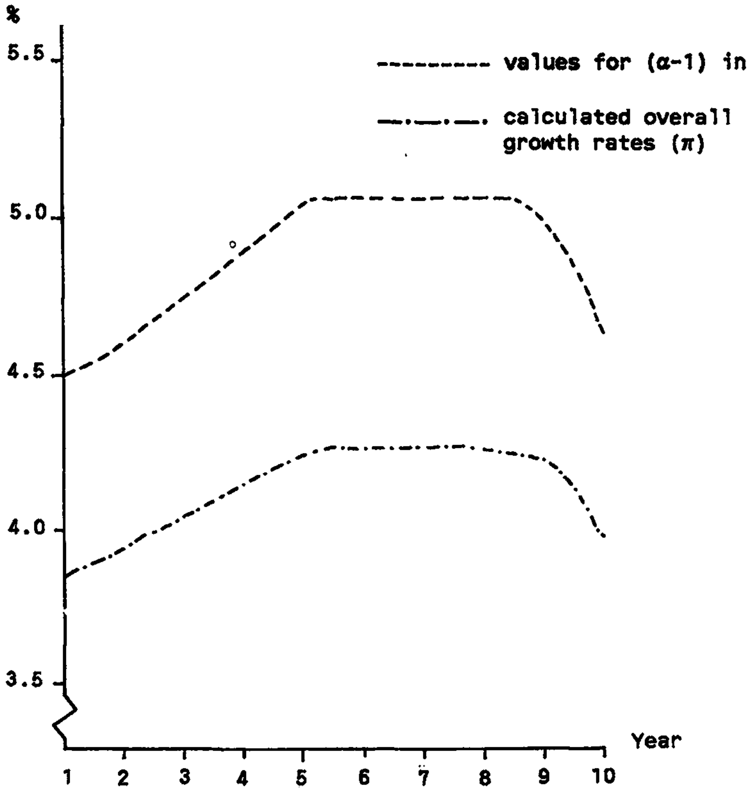

Figure 7.4

Growth rates base-run

a bleak picture[62]. Yet the figure indicates a weak spot also. In the first five years growth increases, α rises from 4.5 per cent in the first year to 5.1 per cent in the fifth year. Thereafter, the growth rate stays at the same level, until the tenth year, in that year α decreases to 4.6 per cent. The turning points in the fifth and tenth year coincide with a 'regime-switch' for rice and wheat, respectively. From the fifth year on Bangladesh

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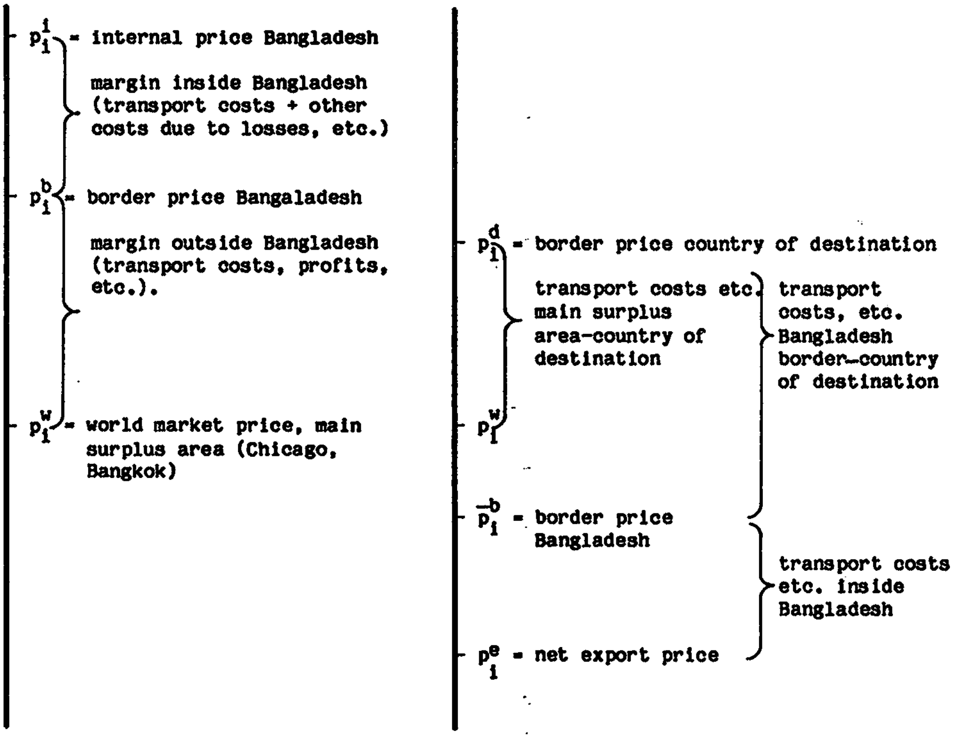
[62] See section 5.4.1 or Paulino [77] who calculated from World Bank figures an average per capita GNP-growth of less than one per cent per year for the period 1961-1980.



Price relations import regime

Price relations export regime

Figure 7.5
Consequences for the price structure of a regime-switch

is, according to the model outcomes, self-sufficient in rice and from the tenth year on no foodgrains at all need to be imported anymore. The implication of this phenomenon for the economy is far-reaching. The domestic price-structure changes drastically. Schematically this is shown in figure 7.5. The model assumes for the consumer perfect substitution between home grown and imported rice and wheat, respectively[63]. Thus, as long as the economy finds itself in an import regime, the competing price for, say, home grown rice is p_i^i, which is equal to the exogenous border price plus a

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[63] But not between rice and wheat!

trade and transportation margin. The volume component of this margin is also exogenous, the price component is not, but moves up and down with the price of the non-agricultural non tradeable product. However low this price may be, p^i_i will lie well above p^w_i, the world market price. Things change if the economy enters an export regime. Instead of p^i_i, p^e_i becomes relevant, assuming again perfect substitution. The difference between p^e_i and p^w_i does not only depend on the margin (transport costs, losses, etc.) within Bangladesh, but also on the distance between Bangladesh and the country of destination relative to the distance between the latter and the main surplus area (Chicago, Bangkok). In the figure it is assumed that the Bangladesh border lies at a greater distance than the main surplus area. This need of course not necessarily be the case. The fact remains, however, that a regime-switch causes a drastic change in the relevant prices. From the technology matrix (see table 6.4) the ratio $p^b_i/\bar{p}^b_i$ can be calculated at 2.21.

In reality the situation is even worse for Bangladesh. Since taste and quality of Bangladesh rice lie too much out of line with what is wanted by the main import countries, there is hardly a market for it outside the country itself. Because of the importance of foodgrains in the Bangladesh economy (see chapter 5), we shall come back to this in more detail in sections 7.4 and 7.5.

Figure 7.4 must be understood as follows: The increase of the growth rate in the first four years is due to the fact that capacities grow faster than consumption. Relatively high investments in t generate a higher growth rate in t+1. This applies as long as no (serious) regime-switches appear. The attainment of self-sufficiency in rice in the fifth year results in a standstill of the increase of growth. Still, things are not too bad at that point. Till the tenth year the economy can grow at a uniform and rather high rate. As we shall see in more detail in next sections, instead of expanding rice production, a vigorous growth of wheat production takes place. Because there appears to be ample scope for substitution of rice for wheat on the production side, the import price for wheat functions as a bottom price for both wheat and rice[8]. In the tenth year self-sufficiency

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[64] However, according to Bangladesh-experts, our model may overestimate substitution possibilities between wheat and rice. In that case, the consequences of a regime-switch for rice are, contrary to our model results, to a lesser extent 'postponed' to the regime-switch for wheat.

in wheat is reached also. As a consequence a further slowdown of the growth rate appears.

A question that may arise is why overall growth in t+1($=\pi_{t+1}$) is not equal to growth of capacities in t($=\alpha_t$). There are at least three reasons for this. First, it must be kept in mind that aid, loans, and remittances from abroad increase only at a rate of 2.1-2.3 per cent per year, which is significantly lower than even the lowest value for α. Second, jute export capacity cannot increase at all. And finally, due to differences in income elasticity of demand, consumption requirements do not increase at a uniform rate for all goods. The effect of the first two factors on the value of π_{t+1} relatively to the value of α_t is negative. The direction of the effect of a changing demand cannot be stated a priori.

Another question that may come up is what happens after the ten-year period. The reason why we have not run the model beyond t=10 is not just because we had to stop somewhere but also because we think that if Bangladesh would have experienced such a long period of relatively high growth, it is reasonable to expect that the structure of the economy would, in terms of infrastructure, productivity, quality of labour force, consumer behaviour, etc. have changed so much that the technology matrices that underlie our model will have become less relevant.

We now proceed with a more detailed discussion of some of the results. We start with a comparison between the model outcomes for the first year and the actual 1984/85 figures.

7.3.4 Model results and actual performance in 1984/85

If one wants to investigate options for economic growth, the existing situation is a natural starting point. In this section we shall compare the first year outcomes of the base-run with the actual performance of the Bangladesh economy in 1984/85. We shall concentrate on the volume accounts. A consistent and complete description of the latter is the social accounting matrix of TFYP. TFYP generates the SAM-data as calibrated outcomes of the first year simulation. Although initial endowments have been introduced and the same data sources have been used in many instances, there are a number of reasons why one should not expect the first year Von Neumann outcomes to be identical to the first year TFYP-outcomes. Of the three reasons we want to mention, one has to do with the theoretical underpinning, one with the mathematical structure and one with the data. We shall briefly explain them.

In the first place, and contrary to TFYP, the Von Neumann model is above all a normative model, primarily designed to answer "what if" type of questions. The most striking difference with TFYP is the complete neglect of a distinction in socio-economic groups. Behaviour is imposed through a (semi-)balanced growth requirement. Connected with this are the absence of taxes, tariffs, quota and other government interventions. As a consequence only by a rare coincidence the Von Neumann model will generate outcomes accurately matching the TFYP actual realizations.

A second and more technical reason why the first year Von Neumann outcomes will not fully agree with the SAM-data, is that the Von Neumann outcomes are, apart from the growth factor, outcomes of a linear programming model. Because a linear program describes a point to set mapping and the algorithm generates an arbitrary corner solution, a linear program solution will, even if the model set-up is non-normative, in general not fully agree with real world realizations.

The last point has to do with the data. As is described in section 6.3.2, the input-output coefficients for the non-agricultural processes are, for the greater part, taken from Overbosch [72]. A further disaggregation of the tradeable sector has been done by using the input-output study of the Planning Commission. Although for TFYP the same basic data sources are used, updating, reconciling and moulding the data for the much more disaggregated non-agricultural sector in TFYP, proceeded in a different way (see [103], volume III). A same type of remark applies to the agricultural processes. As described in section 6.3.2, input-output coefficients are taken from the interface files underlying the BAM-lp. In TFYP on the other hand, the agricultural supply data are of a much more statistical nature.

In short, one can say that although, in general, the same basic data sources have been used, differences in model structure led to differences in data processing. And because updating, refinement, (dis-)aggregation, etc. unavoidably imply some ad-hoc decisions, the technical relations in TFYP and the Von Neumann model are not fully compatible.

With these points in mind one has to look at the figures of table 7.6 in which the comparison between the first year outcomes of the model and the actual performance of the Bangladesh economy in 1984/85 according to TFYP is shown. The model contains one 'ad-hoc' adjustment, viz. rice imports were set at a lower bound of 700,000 mton. Without this bound no rice would be imported at all, more rice and less wheat would be grown and consequently the level of wheat imports would be higher. Thus, as long as Bangladesh is in an import regime for foodgrains, the country appears to be more efficient in rice than in wheat production. To link up with the actual situation, the lower bound was added.

Returning to table 7.6, we claim that, in general, the results are reasonably satisfactory, i.e. the introduction of initial endowments leads to model outcomes that are not too far away from the actual situation of the economy. Compared with the outcomes of the straight application in the earlier sections of this chapter, the threads connecting model and reality are much thicker.

Most percentages that deviate much from 100 refer either to small numbers or can readily be explained. For example, although net imports of dairy, fish and hides differ 85, 115 and 285 per cent respectively from actual realizations, expressed as a percentage of production the differences decrease to 21,4 and 37 per cent respectively. Moreover, as we shall see later, imports of dairy will show a very rapid increase. On the other hand, exports of raw jute were relatively low in 1984/85, so the upper bound on raw jute exports was deliberately set at a higher level.

The high intermediate demand for cloth has mainly to do with the fact that the use of cloth by the cloth sector itself has, contrary to TFYP, not been netted out. The difference in intermediate demand for jute textiles cannot

TABLE 7.6: Volume accounts according to the first year of the base-run: level and percentage of TFYP-outcomes

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	1,493	113	1,797	92	3,290	98	–	.
Rice	('000 mton, milled)	12,685	99	700	99	13,385	99	–	.
Sugar	('000 mton)	356	89	246	115	602	98	–	.
Beef	('000 mton, carc.w.)	129	82	–	.	129	82	–	.
Poultry + eggs	('000 mton, prot.eq.)	13	100	–	.	13	100	–	.
Dairy	('000 mton)	1,030	125	38	15	1,068	98	–	.
Vegetables	('000 mton, stand.)	783	100	–	.	783	100	–	.
Fruit	('000 mton. stand.)	784	100	–	.	784	100	–	.
Fish	('000 mton)	787	101	-60	215	717	96	–	.
Vegetable oils	('000 mton)	66	73	85	58	151	64	–	.
Protein feed	('000 mton, cake)	117	112	–	0	–	.	117	105
Cotton	('000 mton, lint)	15	143	19	38	–	.	34	56
Jute	('000 mton)	1,084	135	-350	135	–	.	734	134
Hides	(mill. pieces)	6	67	3	-385	–	.	8	100
Cloth	(mill. meters)	1,300	135	124	86	990	125	434	136
Jute textiles	('000 mton)	583	112	-518	107	–	0	65	256
Leather	(mill. tk 84/85)	7,226	96	-3,449	173	3,708	68	–	.
Cement	('000 mton)	270	100	397	58	–	.	667	70
Fertilizer	('000 mton, nutr.)	301	100	255	115	–	.	556	107
Tradeable non-ag.	(mill. tk. 84/85)	58,860	x	41,106	111	31,501	x	68,465	x
Non-tradeable non-ag.	(mill. tk. 84/85)	125,990	x	–	.	64,671	x	61,319	x

Notes:

- investments are included in intermediate demand.
- 0 is noted as –, $\frac{0}{0}$ as ., $\frac{0}{>0}$ as 0, and if an entry is not calculated an x is written.

be explained adequately. It is probably more a reflection of input-output coefficients that are set at too high a level than of reality.

Finally, because tradeable and non-tradeable non-agricultural goods in TFYP consist of a very heterogeneous mixture of goods and services for which different kinds of tariffs, taxes and quota regulations apply, we have not tried to compare our results with the base year TFYP-outcomes.

7.3.5 Volume accounts in the base-run

Tables 7.7 - 7.9 show the volume accounts of the base-run. To save space, the accounts are given for three years only, viz. the fourth, the seventh and the tenth year. To facilitate a mutual comparison, all outcomes are expressed as a percentage of the base year outcomes (table 7.6). A first glance at the figures already shows that the balanced growth path has been left. Otherwise the percentages in the three tables would all have the same value. However, defined more loosely the overall picture is a rather balanced one. We shall organize the discussion of the tables under four headings, viz. foodgrains, other crop products, animal products and non-agricultural products.

Foodgrains:

The most remarkable characteristic with regard to foodgrains is that self-sufficiency lies within the reach of Bangladesh. Already from the fifth year on, no imports of rice are necessary. And in the tenth year no imports of foodgrains have to take place at all. In figure 7.6 the growth of production over the whole period is shown. Total growth amounts to slightly more than 50 per cent; growth per year averages 4.7 per cent. Because it is assumed that yield per acre increases with 1.5 per cent per year (see section 7.3.2), 3.2 per cent growth originates from capacity expansion, i.e. from investments in drainage and irrigation. After self-sufficiency in rice is reached, growth of rice production slows down, it just keeps pace with growth of consumer demand. Given the existing world market price, it appears that it is not attractive for Bangladesh to grow rice for the world

TABLE 7.7: Volume accounts according to the fourth year of the base-run: level and percentage of first year-outcomes

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	1,954	131	1,624	90	3,578	109	–	.
Rice	('000 mton, milled)	14,256	112	300	43	14,556	109	–	.
Sugar	('000 mton)	407	114	280	114	687	114	–	.
Beef	('000 mton, carc.w.)	154	119	–	.	154	119	–	.
Poultry + eggs	('000 mton, prot.eq.)	15	115	–	.	15	115	–	.
Dairy	('000 mton)	1,033	100	216	568	1,249	117	–	.
Vegetables	('000 mton, stand.)	866	111	–	.	866	111	–	.
Fruit	('000 mton. stand.)	926	118	–	.	926	118	–	.
Fish	('000 mton)	902	115	-80	133	822	115	–	.
Vegetable oils	('000 mton)	74	112	93	109	167	110	–	.
Protein feed	('000 mton, cake)	131	112	–	.	–	.	131	112
Cotton	('000 mton, lint)	18	120	21	111	–	.	39	115
Jute	('000 mton)	1,124	104	-350	100	–	.	774	105
Hides	(mill. pieces)	7	117	3	100	–	.	10	125
Cloth	(mill. meters)	1,488	114	127	102	1,152	116	463	107
Jute textiles	('000 mton)	614	105	-541	104	–	.	73	112
Leather	(mill. tk 84/85)	8,294	115	-4,029	117	4,265	115	–	.
Cement	('000 mton)	–	0	652	164	–	.	652	98
Fertilizer	('000 mton, nutr.)	345	115	317	124	–	.	662	119
Tradeable non-ag.	(mill. tk. 84/85)	67,349	114	47,420	115	35,770	114	78,999	115
Non-tradeable non-ag.	(mill. tk. 84/85)	144,460	115	–	.	73,187	113	71,273	116

See notes table 7.6.

TABLE 7.8: Volume accounts according to the seventh year of the base-run: level and percentage of first year-outcomes

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	2,798	187	1,091	61	3,889	118	-	.
Rice	('000 mton, milled)	15,823	125	-	0	15,823	118	-	.
Sugar	('000 mton)	494	139	297	121	791	131	-	.
Beef	('000 mton, carc.w.)	166	129	24	x	190	147	-	.
Poultry + eggs	('000 mton, prot.eq.)	18	140	-	.	18	140	-	.
Dairy	('000 mton)	1,032	100	474	1,247	1,506	141	-	.
Vegetables	('000 mton, stand.)	974	124	-	.	974	124	-	.
Fruit	('000 mton. stand.)	1,042	133	-	.	1,042	133	-	.
Fish	('000 mton)	1,046	133	-113	188	933	130	-	.
Vegetable oils	('000 mton)	93	141	97	114	190	126	-	.
Protein feed	('000 mton, cake)	164	141	-	.	-	.	164	141
Cotton	('000 mton, lint)	22	147	23	121	-	.	45	132
Jute	('000 mton)	1,144	106	-350	100	-	.	794	108
Hides	(mill. pieces)	7	117	4	133	-	.	11	138
Cloth	(mill. meters)	1,727	133	119	96	1,264	128	582	134
Jute textiles	('000 mton)	631	108	-550	106	-	.	81	125
Leather	(mill. tk 84/85)	9,622	133	-4,790	139	4,832	130	-	.
Cement	('000 mton)	-	0	757	191	-	.	757	191
Fertilizer	('000 mton, nutr.)	401	133	395	155	-	.	796	143
Tradeable non-ag.	(mill. tk. 84/85)	78,168	133	54,643	133	41,244	131	91,567	134
Non-tradeable non-ag.	(mill. tk. 84/85)	167,512	133	-	.	84,053	130	83,459	136

See notes table 7.6.

TABLE 7.9: Volume accounts according to the tenth year of the base-run: level and percentage of first year-outcomes

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	4,211	282	-	0	4,211	128	-	.
Rice	('000 mton, milled)	17,133	135	-	0	17,133	128	-	.
Sugar	('000 mton)	650	183	256	104	906	150	-	.
Beef	('000 mton, carc.w.)	171	133	61	x	232	180	-	.
Poultry + eggs	('000 mton, prot.eq.)	22	167	.3	x	22	169	-	.
Dairy	('000 mton)	900	87	898	2,363	1,798	168	-	.
Vegetables	('000 mton, stand.)	1,086	139	-	.	1,086	139	-	.
Fruit	('000 mton. stand.)	1,224	156	-	.	1,224	156	-	.
Fish	('000 mton)	1,213	154	-159	265	1,054	147	-	.
Vegetable oils	('000 mton)	108	164	102	120	210	139	-	.
Protein feed	('000 mton, cake)	191	164	-	.	-	.	191	164
Cotton	('000 mton, lint)	25	167	26	137	-	.	51	150
Jute	('000 mton)	1,057	98	-350	100	-	.	707	96
Hides	(mill. pieces)	7	125	6	183	-	.	13	163
Cloth	(mill. meters)	1,989	153	101	81	1,430	144	660	152
Jute textiles	('000 mton)	610	105	-550	106	-	.	60	92
Leather	(mill. tk 84/85)	11,189	155	-5,761	167	5,428	146	-	.
Cement	('000 mton)	-	0	852	215	-	.	852	128
Fertilizer	('000 mton, nutr.)	-	0	894	351	-	.	894	161
Tradeable non-ag.	(mill. tk. 84/85)	90,583	154	57,815	141	47,106	150	101,292	148
Non-tradeable non-ag.	(mill. tk. 84/85)	188,933	150	-	.	95,618	148	93,315	152

See notes table 7.6.

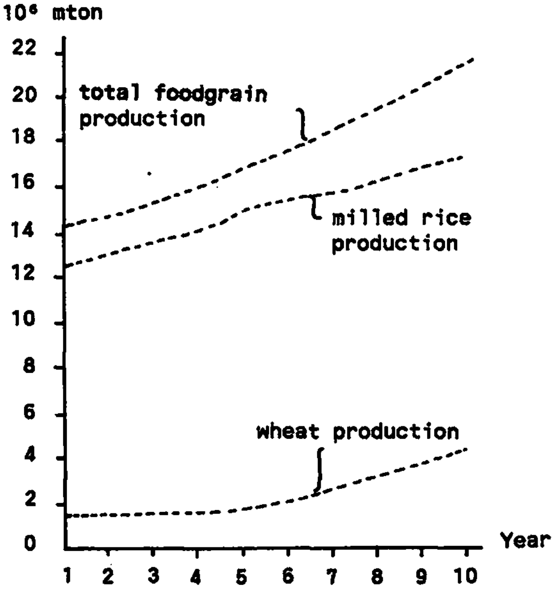

Figure 7.6
Grain production in the base-run

market, instead rice stays in a regime of autarky. Given the importance of rice for the Bangladesh economy and the potential for further production growth, this is clearly a serious matter. The problem has a lot in common with the problems EC agriculture faces: beyond the boundary of self-sufficiency, growth can only go on if it is financially backed by government. Of course, the Bangladesh government is not able to give such support on a large scale. In sections 7.4 and 7.5 we return to this point.

Although growth of rice production slows down from the fifth year onward, growth of wheat production increases. Especially in the rabi season where there is ample scope for substitution on the production side[65]. Expressed

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[65] But see footnote 64.

as a percentage of production in the base year, wheat production in the tenth year is 182 per cent higher. From tables 7.6 - 7.9 average yearly growth rates can be calculated at 9.5, 12.6 and 14.9 per cent in the years 1-4, 4-7 and 7-10 respectively.

Other crop products:

Other crops distinguished in the model are sugarcane (sugar), oilseeds (vegetable oils and protein feed), cotton, jute, vegetables and fruits.

Apart from jute, other crops show roughly speaking the same picture on the production side. A rather strong growth which, except for fruits and vegetables, outpaces growth of consumption and intermediate inputs. Because the model does not allow imports and exports of vegetables and fruits, the autarky regime for these products is in fact imposed. A fact not shown in the tables is that, except for jute, the model outcomes reveal a strong preference for high yielding production methods. Given the results of section 7.2, this is not surprising.

The relative backwardness of jute production is mainly related to the upper bound on exports. It appears that raw jute exports are always at their upper bound. In the first four years jute processing capacity is not sufficient to satisfy both internal and export demand. According to the model it is more efficient then to satisfy domestic demand first. As a consequence the upper bound on exports of jute textiles is not effective during these years. However, even in these years at least 94 per cent of the 'export quotum' is used up.

The slight drop in jute production in the tenth year is caused by the fall in internal demand. The latter, in its turn, is caused by the contraction of the fertilizer industry which is a main intermediate consumer of jute textile.

Animal products:

The model distinguishes five animal products, viz. beef, dairy, poultry and eggs, fish and hides. They are produced by four animal types, viz. cattle, goats, chickens and fish. Apart from fish, production growth of all animal products lags behind growth of consumption or, in case of hides, growth of intermediate inputs. High income elasticities of demand are not the only factor to blame for this phenomenon. A shortage of feed, especially metabolizable energy appears to be a main obstacle for an expansion of the cattle herd and thus of beef, dairy and hide production. According to the model outcomes, growth of the number of male cattle follow draught power requirements. Growth of the number of female cattle on the other hand is much more erratic. In figure 7.7 this is graphically shown. In the first six periods, the number of (aggregated) milk-cows stays more or less at the same level. Then a firm decrease takes place, while in the tenth year the trend changes again. The change in the tenth year can be explained as follows: in the tenth year self-sufficiency in foodgrains is attained. As explained before, it is not attractive for the country to enter an export regime then. As a consequence, growth of foodgrain production is rather

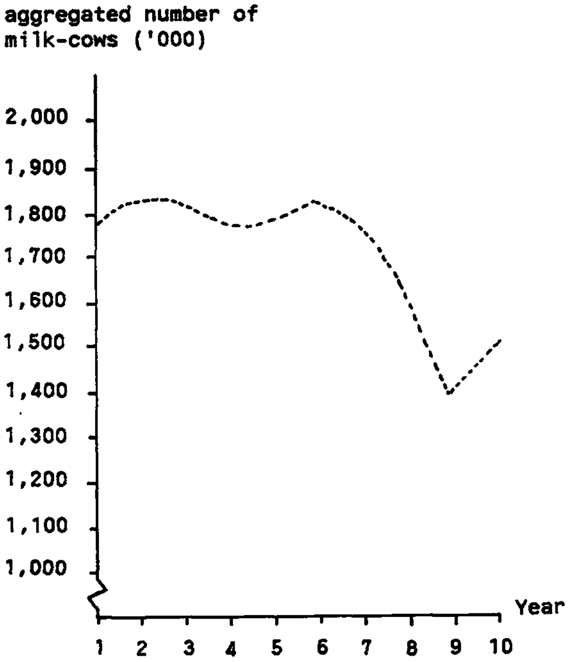

Figure 7.7

Aggregated number of milk-cows in the base-run limited in the tenth year. Sugarcane production takes advantage of this slow down. This is best illustrated by the fact that, despite a rather high income elasticity of demand, imports of sugar drop with nearly 20 per cent in the tenth year. Because sugarcane processing yields an energy-rich by-product (molasses), the 'energy balance' of the model can, as a consequence, be satisfied by using less grain by-products (straw, bran), which can instead be used as animal feed. The latter finds expression in an increase of the aggregated number of milk-cows at the end of the year.

Over the whole period there is, however, a firm decrease of the number of milk-cows. Combined with a high income elasticity of demand this results in a rapid increase of dairy import: from 38,000 mton in the first year to 898,000 mton in the tenth year. Although we acknowledge that it is in a way an open question to what extent Bangladesh is really able to grow according to our normative base-run scenario, the model outcomes with regard to dairy at least provide an illustration of two points: (i) if the Bangladesh economy really starts growing, it can become an interesting market for the EC-dairy producers and (ii) Bangladesh is an outstanding example of a country which can benefit from low world-market prices for dairy products. It may be of interest to note that these findings agree with those in a study on India (Parikh et al. ([75])[66].

In spite of the lagging behind of the growth of the cattle herd, both beef and hide production grow significantly. This is mainly caused by the increase of the goat population. The model outcomes show a yearly growth rate equal to (α-1). Fish production also grows at that rate. Because consumption does not increase so fast, exports of fish become more and more important. One could ask if production capacities and exports of fish can really grow so fast. That does, however, not alter the fact that according to the model outcomes, fish appears to be a very attractive product for the Bangladesh economy.

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[66] A low world-market price for dairy can, on the other hand, be the main obstacle in the development of a domestic dairy sector also. Although the model is well-suited for investigating this matter, it has not been done. Because such an investigation could result in a conclusion diametrically opposite to point (ii), a careful interpretation of (ii) is necessary.

Non-agricultural products

On the basis of the model outcomes the seven non-agricultural production sectors can be divided into three groups. The cloth, the leather, the non-tradeable non-agricultural and the tradeable non-agricultural sector belong to the first, the jute textile sector belongs to the second and the fertilizer and cement sector come in the third group. Members of the first group have in common that they all show high and relatively uniform growth rates equal to about (α-1) per year. Jute textile production shows hardly any growth. This is partly caused by the assumption that export possibilities are exhausted. In addition, sales perspectives for the domestic market appear to be very limited. Exports of jute textiles remain, however, attractive during the whole period. Apart from the first years, the shadow price of the upper bound on the export of jute textiles is positive. Cement and fertilizer have in common that their production levels are non-zero in the first year and zero in the tenth year. Cement production drops to zero already in the second year. Fertilizer production shows an increase in the first eight years, in the ninth year production decreases sharply while only in the tenth year Bangladesh goes out of fertilizer production altogether. Although the quality of the data can certainly be criticized, we think that the outcomes form at least an indication that Bangladesh is relatively inefficient in cement and, to a lesser extent, in fertilizer production. Despite a growth in domestic demand, these are the only sectors for which production is completely abandoned in the tenth year[67].

7.3.6 Consumption levels and calory-intake

Economic growth is, of course, not an objective by itself. It is only a means for increasing prosperity. In the face of the poverty situation in Bangladesh, an increase in prosperity and general well-being should be assessed by looking at consumption levels of the main food and

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[67] To avoid misunderstanding: such an abandoning does not imply that Bangladesh has to give up cement and fertilizer production altogether. As explained in chapter 2, it only means that fixed + variable costs cannot be paid from the proceeds.

TABLE 7.10: Per capita consumption levels

Product	Year			
	1	4	7	10
Non-agricultural tradeable	313.755	332.781	359.456	385.732
Non-agricultural non-tradeable	644.133	680.884	732.552	782.977
Milled rice	133.320	135.420	137.904	140.295
Wheat	32.769	33.287	33.894	34.482
Sugar	6.000	6.391	6.894	7.418
Fish	7.141	7.647	8.131	8.631
Vegetable oils	1.504	1.554	1.656	1.719
Energy	570.000	596.747	630.322	665.971
Beef	1.285	1.433	1.654	1.899
Poultry, eggs	.132	.144	.160	.178
Vegetables	7.803	8.057	8.490	8.891
Fruits	7.809	8.178	9.084	10.027
Dairy	10.637	11.620	13.125	14.726
Cloth	9.860	10.717	11.017	11.707
Leather	36.932	35.678	42.113	44.448

Note: Energy in useful mega-joules.
Other dimensions are, apart from a factor 10^8, the same as in tables 7.6 - 7.9.

non-food products. A summary of the consumption levels according to the base-run scenario is given in table 7.10. The most striking feature of the table is that in spite of the relatively high savings ratio and the high populaton growth, there appears to be room for an increase in per capita consumption levels. Depending on the income elasticities of demand, average consumption per head increases by a minimum 5.2 per cent for rice and wheat, to a maximum of 47.8 per cent for beef.

In figure 7.8 consumption levels are expressed as daily per capita calory-intake. Calory-intake increases from 1928 in the first year to 2089 in the tenth year, a modest growth which is, as compared to the historical record, still significant. Because income elasticities of demand for foodgrains are lower than for other food products, the share of foodgrains decreases, from 84 per cent in the first year to 81 per cent in the tenth year.

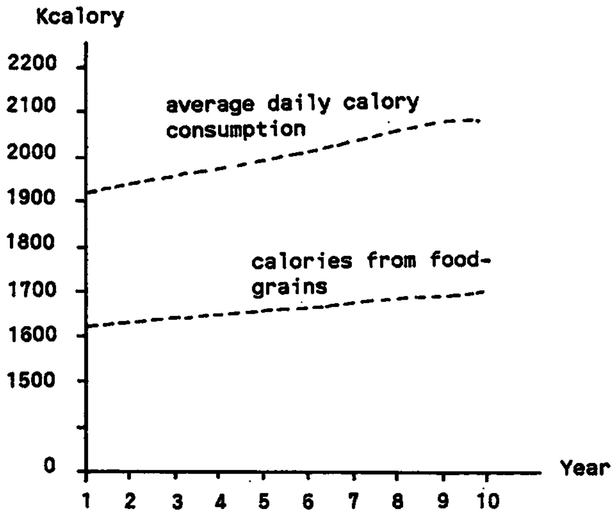


Figure 7.8
Daily per capita calory-consumption and share of foodgrains

7.3.7 Relative prices

Until now we have limited the discussion of the base-run results to the volume side of the model. It seems worthwhile to turn some attention to the prices also. In table 7.11 relative prices of the main products are summarized. All prices are expressed in units of milled rice. A brief inspection of the outcomes reveals that all products, except vegetables, become more expensive relative to rice. The largest shock appears in the tenth year, i.e. in the year in which self-sufficiency in foodgrains is reached. A much smaller shock can be identified in the fourth year. In that year Bangladesh becomes self-sufficient in rice, as a consequence wheat becomes more expensive than rice.

The deteriorating relative prices of rice, wheat and vegetables, the latter two being near substitutes for rice on irrigated rabi land, are a direct consequence of the regime-switch for foodgrains. In terms of rice a unit of tradeable non-agricultural product is in the tenth year 1.95 times more

TABLE 7.11: Relative prices

Product	Year			
	1	4	7	10
Non-agricultural tradeable	.207	.210	.215	.423
Non-agricultural non-tradeable	.062	.066	.067	.240
Milled rice	1.000	1.000	1.000	1.000
Wheat	.986	1.010	1.025	1.031
Sugar	1.319	1.356	1.376	2.932
Fish	9.828	9.857	9.981	12.840
Vegetable oils	6.510	6.678	6.777	13.914
Energy	.005	.017	.022	.074
Beef	.000	.000	8.649	17.957
Poultry, eggs	19.812	21.575	23.370	79.847
Vegetables	.471	.499	.493	.393
Fruits	.034	.037	.036	.393
Dairy	.886	.914	.924	2.098
Cloth	3.664	3.752	3.808	5.031
Leather	.175	.178	.180	.315
Cement	.331	.342	.345	.736
Jute textiles	3.635	3.690	1.019	5.030
Raw jute	.848	.904	.958	1.399
Fertilizer	2.790	2.859	2.903	5.822

Note: Energy in useful mega-joules.
Other dimensions are, apart from a factor 10^8, the same as in tables 7.6 -7.9.

expensive than in the first year. Thus, the fast growing foodgrain sector causes the terms of trade agriculture versus non-agriculture to deteriorate severely. And most, if not all, efficiency gains are reaped by the non-agricultural sector.

Because world-market prices are kept fixed, prices of tradeable products that do not experience a regime-switch (non-agricultural tradeable, sugar, fish, vegetable oils, jute, dairy, cloth, leather, cement, fertilizer) do not change much relative to each other. The import and export prices operate as exogenous bounds. The anomaly with regard to the price of jute textiles in the seventh year has to be understood in the context of the imposed upper bound on export of the product.

Beef and poultry and eggs experience a regime-switch from self-sufficiency to import. In the tenth year more than 26 per cent of total beef consumption and slightly more than 1 per cent of poultry and eggs consumption have to be imported (see table 7.9). The price consequences are enormous and probably somewhat exaggerated. The drawback of the price-independent demand assumption is most clearly shown through these products.

The zero-price for beef in the first and fourth year can be explained as follows: the model does not have a beef export activity. Moreover, beef can only be produced as a joint product. These two facts combined with a price exogenous demand can, outside the import regime, easily result in a zero-price. The zero-price can best be interpreted as a very temporary export regime.

Extreme price rises take also place in the non-tradeable sectors, i.e. price increases for the non-agricultural non-tradeable product, energy and fruit are even more pronounced than for the tradeable non-foodgrain products. This phenomenon can partly be explained by the changing shadow price for labour. In section 3.4 it was explained that by introducing initial endowments and labour, a functional income distribution enters the model. The (negative) shadow price of the labour row implies a negative correction on the wage-rate. Because the average basket of consumption goods per labourer does not change accordingly, it can best be interpreted as a strain on the existing institutional structure.

In figure 7.9 the shadow price of the labour row in terms of rice and tradeable non-agricultural product is shown. From the eighth year on a huge drop of the (absolute) value of the shadow price of the labour row takes place. Although it is difficult to point out exactly what the turning point is, the underlying causes are clear. The relatively fast growth of production capacities and the slowing down of population growth make labour more scarce. As a consequence the 'penalty' on labour decreases. The labour intensive non-tradeable sectors benefit most from this development. A crucial (implicit) assumption is, of course, that labour must be fully substitutable between various sectors. Education activities and measures to remove institutional barriers have to take care of this.

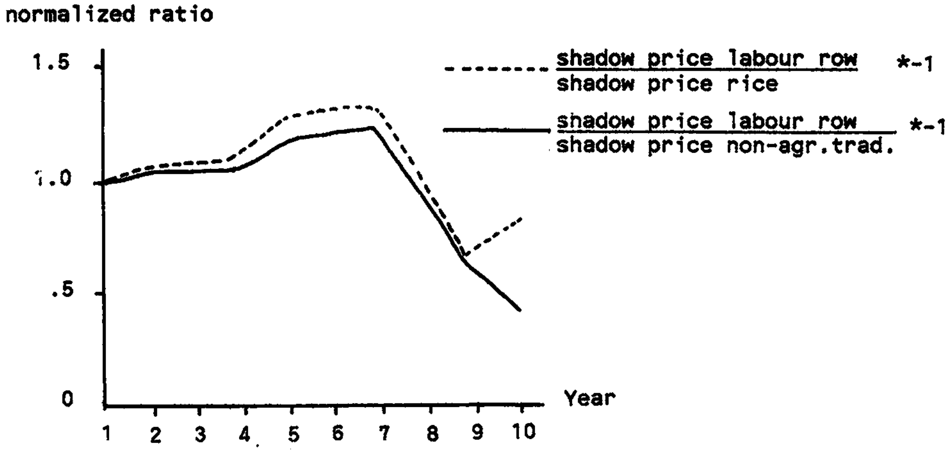


Figure 7.9

Shadow price of labour (x(-1)) in terms of rice
and tradeable non-agricultural product

7.4 ACCELERATED YIELD INCREASE OF RICE

7.4.1 Alternative scenario choice

If crop growing capacities are converted into their area equivalents, it follows that total rice area does not increase at all during the base-run period. On the contrary, in the tenth year total area with rice appears even 2 per cent less than in the first year. The capacity expansion we spoke of in section 7.3.5 must therefore entirely be seen as the result of upgrading (irrigating, draining) the existing rice area. The expansion of the total land base which takes place through irrigation of former non-crop rabi land is fully used for increasing the area with other crops than rice, notably wheat.

On a per acre base, increase in rice production in the model period amounts to 37.7 per cent; growth per year is 3.6 per cent. Given the historical record these figures are quite high. In the period 1975-1984 for example, per acre growth rate amounted to 1.5 per cent only (FAO production yearbook

[19]). By other standards the increase in yield is, however, not particularly high. Countries as diverse as Burma, China and Indonesia realized, according to the FAO production yearbook, average yearly growth rates of 6.1, 4.6 and 4.2 per cent, respectively during the period 1975-1984. Also if we look at absolute levels, yields per acre are not impressive in Bangladesh. Out of all Asian countries in which more than 250,000 acres of rice is grown, only Kampuchea and Thailand had lower yields in 1984. And even if we compare the average yield in the tenth year of the base-run, thus after a nine-year period of sustained relatively high growth, with the average rice yield in the whole of Asia in 1984, it appears that yields in Bangladesh are still significantly lower, i.e. at 86 per cent of the Asian average only.

These figures are at least an indication that there is ample scope for yield increases in the Bangladesh rice sector. Technological data support this notion. According to UNDP ([103], volume II, p. 33) the ratio actual/ potential yield for rice is only .73 in 1984. For high yielding varieties the ratio is even lower. Because of this we think it is not overly optimistic to proceed from a higher yield increase than the 1.5 per cent of the base-run. As alternative we assume that yields of high yielding rice varieties increase with an additional 1.5 per cent in the sixth, seventh and eighth year and with an additional 2.5 per cent in the ninth and tenth year. To realize these increases, huge efforts in the fields of education and extension have to take place.

Intuitively one would expect two (main) consequences of such an acceleration in the increase of rice yields. First, the overall growth rate will probably be raised and second self-sufficiency in foodgrains will most likely be reached in an earlier year. With regard to the first point it is interesting to have an idea of the order of magnitude the growth rate is affected by an extra stimulus of the rice sector. With regard to the second point, the effects of a regime-switch in foodgrains will probably come more explicitly to the fore. To facilitate such a regime-switch, we also assume that from the sixth year on the income elasticity of demand for rice decreases with five per cent points per year, i.e. it decreases from .35 in the fifth to .10 in the tenth year. Given the relatively high level of rice consumption this is certainly not an unrealistic assumption. The income saved is assumed to be spent on tradeable non-agricultural product.

7.4.2 Main results

The assumptions of the preceding sections imply that the model has to be run from the sixth year onward. We have summarized the main results in figure 7.10 and tables 7.12 and 7.13. In interpreting figure 7.10 it must be kept in mind that the α-values do not refer to the overall growth rate of the economy. As explained in section 7.3.3, the overall growth rate is somewhat lower, approximately .8 - .9 per cent. Both level and course of the high rice yield run (hry)outcomes deviate significantly from the base-run outcomes. In the sixth to eighth year growth increases constantly. In the eighth year α is at its top and reaches a value of 5.4 per cent which corresponds, roughly speaking, to an overall growth rate of 4.5 and a per capita growth rate of 2.4 per cent. However, in the ninth and tenth year the economy lands again in the 'regime-switch trap'. Once self-sufficiency in foodgrains is reached the growth rate decreases drastically, even to such an extent that in the tenth year the value for α is at the same (low) level as in the corresponding year of the base-run.

Volume accounts are shown in table 7.12 and 7.13. To save space, only figures for the seventh and tenth year are shown. To facilitate a comparison with the base-run, volumes are also expressed as a percentage of the seventh and tenth year volumes of the base-run, respectively. The overall picture is perhaps not so shocking, since most changes lie even in the tenth year in a 1-2 per cent range, still some interesting structural changes can be detected. The contours of these changes can already be seen in the seventh year. Because they come more clearly to the fore in the tenth year, we limit the discussion to table 7.13.

Yields of high yielding varieties are in the tenth year, on a per acre base, nearly ten per cent higher in the hry-run than in the base-run. Still, it appears unattractive to enter the export regime. Instead food grains stay in the autarky regime. For rice this means that, as a con-

TABLE 7.12: Volume accounts according to the seventh year of the high rice yield run: level and percentage of corresponding year of the base-run

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	3,084	110	806	74	3,890	100	–	.
Rice	('000 mton, milled)	15,811	100	–	.	15,811	100	–	.
Sugar	('000 mton)	495	100	299	101	794	100	–	.
Beef	('000 mton, carc.w.)	165	99	25	104	190	100	–	.
Poultry + eggs	('000 mton, prot.eq.)	18	100	–	.	18	100	–	.
Dairy	('000 mton)	1,009	98	498	105	1,507	100	–	.
Vegetables	('000 mton, stand.)	974	100	–	.	974	100	–	.
Fruit	('000 mton. stand.)	1,043	100	–	.	1,043	100	–	.
Fish	('000 mton)	1,046	100	-111	98	935	100	–	.
Vegetable oils	('000 mton)	93	100	97	100	190	100	–	.
Protein feed	('000 mton, cake)	164	100	–	.	–	.	164	.
Cotton	('000 mton, lint)	22	100	23	.	–	.	45	100
Jute	('000 mton)	1,146	100	-350	100	–	.	796	100
Hides	(mill. pieces)	7	100	4	100	–	.	11	100
Cloth	(mill. meters)	1,727	100	121	102	1,265	100	583	100
Jute textiles	('000 mton)	632	100	-550	100	–	.	82	19
Leather	(mill. tk 84/85)	9,622	100	-4,783	100	4,839	100	–	.
Cement	('000 mton)	–	.	762	101	–	.	762	101
Fertilizer	('000 mton, nutr.)	401	100	404	102	–	.	805	101
Tradeable non-ag.	(mill. tk. 84/85)	78,418	100	55,364	101	41,552	101	92,230	101
Non-tradeable non-ag.	(mill. tk. 84/85)	168,316	100	–	.	84,093	100	84,223	101

See notes table 7.6.

TABLE 7.13: Volume accounts according to the tenth year of the high rice yield run: level and percentage of corresponding year of the base-run

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	4,221	100	–	.	4,221	100	–	.
Rice	('000 mton, milled)	17,007	99	–	.	17,007	99	–	.
Sugar	('000 mton)	913	140	–	0	913	101	–	.
Beef	('000 mton, carc.w.)	158	92	78	128	236	102	–	.
Poultry + eggs	('000 mton, prot.eq.)	22	100	.3	100	22	101	–	.
Dairy	('000 mton)	709	79	1,113	124	1,822	101	–	.
Vegetables	('000 mton, stand.)	1,092	101	–	.	1,092	101	–	.
Fruit	('000 mton. stand.)	1,239	101	–	.	1,239	101	–	.
Fish	('000 mton)	1,222	101	-161	101	1,061	101	–	.
Vegetable oils	('000 mton)	109	101	102	100	211	100	–	.
Protein feed	('000 mton, cake)	193	101	–	.	–	.	193	101
Cotton	('000 mton, lint)	25	100	26	100	–	100	51	100
Jute	('000 mton)	1,068	101	-350 n	100	–	.	718	101
Hides	(mill. pieces)	7	98	6	105	–	.	13	101
Cloth	(mill. meters)	2,016	101	92	91	1,440	101	668	101
Jute textiles	('000 mton)	620	102	-550	100	–	.	70	117
Leather	(mill. tk 84/85)	11,250	101	-5,798	101	5,452	100	–	.
Cement	('000 mton)	–	.	859	101	–	.	859	101
Fertilizer	('000 mton, nutr.)	177	x	727	81	–	.	904	101
Tradeable non-ag.	(mill. tk. 84/85)	90,865	100	60,102	104	48,041	102	102,926	102
Non-tradeable non-ag.	(mill. tk. 84/85)	190,686	101	–	.	96,361	101	94,325	101

See notes table 7.6.

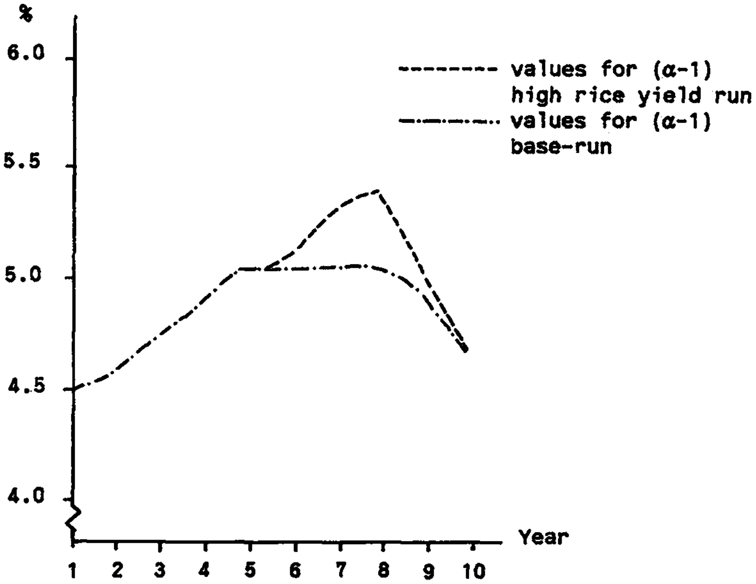


Figure 7.10
Growth rates high rice yield run

sequence of the diminishing income elasticities of demand, production is at an even lower level than in the corresponding year of the base-run.

The higher per acre rice yields are, by assumption, not coupled with higher straw yields. Because there is a shift from relatively high straw yielding local varieties to relatively low straw yielding hyv, total straw production even decreases. The consequences of this are mainly found in the cattle sector. Because straw is the main feed product for cattle, the cattle herd cannot be maintained and consequently production of beef, dairy and hides decrease. Because consumption of these products increase slightly, import quantities rise.

As there is less crop growing capacity required to grow rice, non-rice crops can expand. Sugar profits most of this extra room, even to such an extent that Bangladesh becomes self-sufficient in sugar. It is remarkable that the export regime for sugar is not entered either, although the ratio export border price/import border price for sugar is not such an unfavourable .7.

The most remarkable change in the non-agricultural sector relates to fertilizer production. In contrast to the tenth year of the base-run it does not appear to be attractive to abandon fertilizer production altogether, albeit that less than fifty per cent of total production capacity is used. Apart from livestock products, imports of tradeable non-agricultural products also increase significantly. This is mainly due to the rising income elasticity of demand. Because domestic production capacity hardly differs from the capacity in the corresponding years in the base-run, the extra tradeable goods have to be imported.

Finally we shall have a brief look at some important shadow prices. If we set the shadow price of rice at one and normalize the shadow prices of the tradeable non-agricultural product and the labour row accordingly, we get, for year 10 the following figures:

	base-run	hry-run
Rice	1	1
Tradeable non-agricultural	.423	.530
Labour row	-.263	-.530

What can we conclude from this? In the first place, that rice becomes cheaper in terms of the tradeable non-agricultural product. Thus, the efficiency gains in the high yielding variety rice sector is, at least partly, transferred to the non-agricultural sector. Moreover, the high negative shadow price on labour means a decrease of the economic value of labour (see section 3.4). The cause of this lies in the rice sector. Because high yielding rice requires less labour per unit and total rice production does not increase, the labour saved has to find an outlet elsewhere which has a depressing effect on its value.

As a general conclusion it appears that in a situation of near self-sufficiency and unattractive world market prices, accelerating yield increases in the rice sector is not without danger. It appears that owners of non-agricultural production capacity benefit most of it, wage labourers on the other hand are confronted with both high prices for non-agricultural products and a lowering marginal value of their labour. The fall in the price of rice is insufficient to off-set the latter.

7.5 OVERCOMING THE 'REGIME-SWITCH TRAP' FOR FOODGRAINS

7.5.1 Raising the net export price for foodgrains

The 'regime-switch trap' for foodgrains remains the most gloomy spot in the otherwise rather optimistic scenarios we have sketched so far. Given the overwhelming importance of the foodgrain sector, it comes, of course, not as a surprise that a near standstill on this sector has very negative consequences for the overall growth factor. Although in reality the stage of self-sufficiency has not at all been reached, it does, in view of the country's potential, not seem a luxury to anticipate this situation. The most important question coming up then is, how Bangladesh can raise the net export price for foodgrains. In theory, actions on three fronts can alleviate the situation:

(i) Improve the infrastructure within the country so that the input volume of the non-tradeable non-agricultural product in the export process decreases. In other words, improve the transport and trade system.

(ii) Adapt taste and quality of rice (and wheat) so that the product becomes more attractive to potential importers.

(iii) Try to find markets in as close a neighbourhood as possible.

The effects of these actions are obvious; they all contribute to a higher net export price. With regard to point (iii), the most natural market is of course Calcutta. However, we shall not digress on these particular points because, as such, they lie outside the scope of our study. Supposing that Bangladesh will succeed in raising the net export price along the lines above, an interesting next question that comes up then is what the effect of such a change would be. We have tried to shed some light on this question by running the model under alternative export prices and non-tradeable non-agricultural input volumes.

TABLE 7.14: Alternative assumptions

	Wheat			Rice		
	Old	Alternative 1	Alternative 2	Old	Alternative 1	Alternative 2
Export price	2.65	3.206	4.122	3.12	4.816	6.192
Non-tradeable non-agric. input volume	.99	.495	.495	1.95	.975	.975
Import price	4.58	4.580	4.580	6.88	6.880	6.880

Starting point is year nine of the scenario discussed in the preceding section. In that year the country is, according to the model, self-sufficient both in rice and in wheat. For year ten, two sets of alternative assumptions with regard to the export prices of foodgrains are made. They are summarized in table 7.14. In the first alternative it is assumed that the export price for both wheat and rice stands at 70 per cent of the import price; in the second alternative this percentage is raised to 90. Moreover, in both alternatives we proceed from the assumption that the input volume on non-tradeable non-agricultural products required in the export activities is halved.

7.5.2 Main results

The effects of higher export prices are quite significant. In both alternatives Bangladesh enters, indeed, the export regime for rice. The growth rate which stands at a mere 4.65 per cent in the tenth year of the hry-run (see figure 7.10), increases to 5.01 and 5.25 in alternatives 1 and 2, respectively. Table 7.15 shows the main consequences in volume terms. Volumes of production and imports are expressed in the corresponding volumes of year 10 of the hry-run (see also table 7.14). Because consumption levels remain (practically) unchanged and intermediate inputs (if there are any) form the bottom entries, figures on production and imports reflect most relevant information with respect to volume changes.

TABLE 7.15: Effects of higher export prices of rice and wheat on production and net import volumes

Product	Export price at 70% of import price		Export price at 90% of import price	
	Production	Net import	Production	Net import
Wheat (incl. coarse grains)	100	.	16	x
Rice	112	-	132	-
Sugar	66	x	66	x
Beef	106	87	112	76
Poultry and eggs	100	100	100	100
Dairy	111	93	152	67
Vegetables	100	.	100	.
Fruits	100	.	100	.
Fish	100	100	100	100
Vegetable oils	100	100	100	100
Protein feed	100	.	100	.
Cotton	100	81	100	38
Jute	97	100	98	100
Hides	114	0	114	0
Cloth	90	263	68	602
Jute textiles	98	100	99	100
Leather	60	23	62	27
Cement	.	101	.	105
Fertilizer	0	134	0	141
Tradeable non-ag. prod.	100	99	100	102
Non-tradeable non-ag. prod.	101	.	105	.

Notes:

See notes table 7.6. Volumes are expressed relative to the corresponding volumes of table 7.13, e.g. the entry for rice production in the first column is calculated as $\frac{\text{volume rice production high rice price run, year 10}}{\text{volume rice production high rice yield run, year 10}}$ x 100.

If we concentrate on the first two columns (alternative 1), two types of changes can be identified. On the one hand we see, within agriculture, an increase of the production of rice and livestock products (beef, dairy, and hides) mainly at the expense of sugar. This development is also reflected on the trade balance. Rice exports amount to 2.1 million tons and beef, dairy and hide imports decrease. Opposite to this is an increase of sugar imports. These were zero in year ten of the hry-run (see table 7.13) and amount to more than .3 million tons in alternative 1. The expansion of the livestock sector is mainly due to the extra feed availability from rice by-products. A second development can be detected in the non-agricultural sectors. Apart from the non-tradeable non-agricultural sector which expands marginally as a consequence of the overall increase in trade activities and the tradeable non-agricultural sector which does not change at all, there is a contraction in all non-agricultural sectors. Fertilizer production even disappears altogether, although overall fertilizer consumption increases (cf. table 7.13). Leather production also decreases significantly, in spite of the growth in domestic hide production. It appears that leather production from imported hides becomes unprofitable. The most remarkable result of alternative 2 (third and fourth column of table 7.15) is the regime-switch for wheat. In spite of the higher price (see table 7.14), production decreases drastically. Because consumption stays at the same level, more than 3.5 million tons have to be imported. Rice production on the other hand increases vigorously and more than 5.5 million tons are available for exports. Thus, at the given price structure Bangladesh has clearly a comparative advantage in growing rice. The outcome bears some resemblance to the situation in China in the seventies where rice was also exported in order to finance (cheaper) wheat imports.

The increase in overall trade finds also expression in the production figure of the non-agricultural non-tradeable product. Extra transport and trade result in a production growth of five per cent relative to the self-sufficiency in foodgrains situation of the hry-run.

The relative shadow prices for rice, non-agricultural tradeable and the labour row indicate the effects on income distribution. The following table can be constructed:

	base-run	hry-run	alternative 1	alternative 2
Rice	1	1	1	1
Tradeable non-agricultural	.423	.530	.280	.211
Labour row	-.263	-530	-.249	-.196

Compared to both the base-run and the hry-run, some interesting changes appear. In terms of rice, the tradeable non-agricultural product becomes cheaper. The export price of rice functions as an effective bottom price now. Thus, roughly speaking, the terms of trade agriculture vs. non-agriculture improve in favour of the former.

Even more interesting is the sharp decrease of the 'penalty' on labour. The implication of this is that the marginal value of labour increases. From the above figures it can be seen that this is the case both in terms of rice and tradeable non-agricultural product.

If we summarize the main results of sections 7.4 and 7.5, we think we may conclude that: accelerating rice production in the neighbourhood of self-sufficiency has hardly any effect on the overall growth rate if the extra rice cannot be exported at a 'reasonable price'. It has, however, grave consequences on the functional income distribution. The domestic price of rice and, even more, the marginal value of labour drop drastically relative to the price of the tradeable non-agricultural product. If there is, on the other hand, a foreign market, the picture is completely different. The overall growth rate increases significantly, rice can maintain its price relative to the tradeable non-agricultural product. The marginal value of labour will also be higher. The comparative advantage of growing rice relative to growing wheat may even result in a return to the import regime for wheat so that more rice can be grown and exported.

7.6 A MORE EFFICIENT NON-AGRICULTURAL SECTOR

7.6.1 Introduction

Until now we have concentrated the discussion on the agricultural sector. The reasons for this are twofold: on the one hand the agricultural sector is, by most standards, by far the most important economic sector. On the other hand, the agricultural sector has been elaborated in the model in a much more detailed way than the non-agricultural sector. Consequently, model results are more explicit with regard to agriculture than to non-agriculture where the greater part of production is concentrated in two highly aggregated products, a tradeable and a non-tradeable non-agricultural product, respectively. However, as we saw in chapter 5, the non-agricultural sector is too important to permit a superficial treatment. Therefore we shall in this and the next section discuss some model outcomes which resulted from alternative assumptions with regard to the non-agricultural sector. The development of the non-agricultural sector in the runs so far shows, roughly speaking, the following picture (see also tables 7.6-7.15): production capacities, if fully utilized, increase at a faster rate than labour availability. As a result, relatively inefficient capacities are increasingly unused. In sequence, the capacities to produce cement, fertilizer and, in some runs, leather based on imported hides, and cloth, (also based on imported raw material) lie partly idle. It simply does not pay to perform activities which make use of these capacities at the going interest and wage-rate and the prices of inputs and outputs.

The shift to more productive capacities can be compared with the shift to high yielding varieties in the crop sector, albeit that here no capacity (land) lies idle in any run. Also in another respect there is some asymmetry between the sectors. Crop production does not only increase as a result of this shift but also through a more efficient use of existing capacities, e.g. in the base-run crop yields are assumed to increase with 1.5 per cent per year (see section 7.3). Because it is generally agreed that the non-agricultural sector is rather inefficiently organized (see e.g. Artep [4]) it seems interesting to make a same kind of assumption with regard to the non-agricultural sector. As a start we assume that all non-agricultural processes require each year three per cent less tradeable and non-tradeable non-agricultural product as input to yield the same output. Since non-agricultural investment processes do not need other inputs, the assumption implies a yearly efficiency gain of three per cent for these processes. The input structure of other non-agricultural processes is less specialized (see table 6.4). As a consequence the efficiency gains are here, on balance, less than three per cent.

7.6.2 Results

The model has been run for a seven-year period. The resulting growth rates are shown in figure 7.11. To facilitate a comparison with the

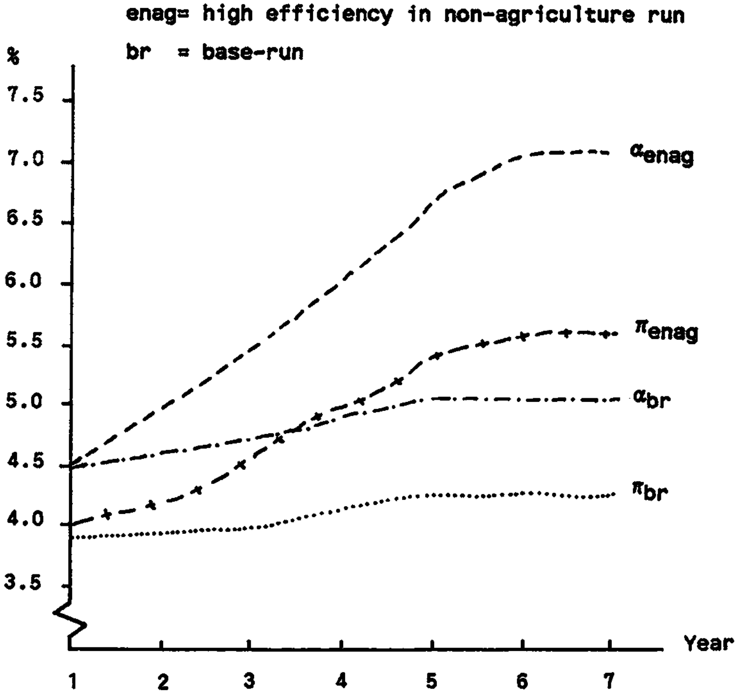


Figure 7.11

Growth rates high efficiency in non-agricultural run: growth rate of capacity (α) and calculated overall growth rate (π)

base-run, the growth rates of the base-run are also shown in the figure. The differences are significant. From the first year on the gap between the efficiency in non-agriculture run (enag-run) and the base-run starts increasing. In the seventh year the growth rates are 7.1 and 5.1 per cent respectively. These percentages refer to growth of capacity. As can be seen in the figure, overall growth rates, which can be approximated with the formula of section 7.3.3, are somewhat lower. In the seventh year of the enag-run the overall growth rate can be calculated as 5.67 per cent, which is in view of Bangladesh historical record, very high. Although we realize that the underlying assumptions are rather optimistic, we do not think they are completely unrealistic.

Some further results are shown in tables 7.16 and 7.17 which contain the volume accounts of the fourth and seventh year. Because some emerging trends in the fourth year appear more pronounced in the seventh year, we shall only discuss table 7.17. A more efficient non-agricultural sector results in an increase of the overall growth rate. The latter is a weighted average of growth in consumption and growth in investments. Consumption of all goods increases relatively to the base-run. Growth in investments is reflected in the production figures, although as table 7.17 shows, not all products have increased at the same rate relative to the seventh year of the base-run. Production of a number of goods is even lower than in the corresponding base-run year. Within agriculture this applies only to dairy cattle.

As a result beef and dairy production show a sharp decrease. The increase in crop growing capacity is mainly used for wheat. The tendencies within the agricultural sector are, given the results of the preceding sections, not very surprising: a preference for self-sufficiency in foodgrains and a lagging behind of the dairy sector.

The picture in the non-agricultural sector is scattered but, given the base-run results, easily understandable. Cloth and tradeable non-agricultural production are on their upper bound; jute is constrained by export demand; leather production appears only attractive in case domestic hides can be used; the production of non-tradeable non-agricultural production is determined by internal demand; cement production is abandoned altogether from the sixth year on; and fertilizer production is in the seventh year below its upper bound for the first time.

Because consumption quantities increase relatively smoothly and intermediate inputs move proportionately with production, the main adjustments take place via international trade; it is there that we see the most sharp changes.

In the agricultural sector we see a sharp decrease in wheat imports; beef and dairy imports on the other hand are more than 100 per cent higher than in the corresponding year of the base-run. In the non-agricultural sector

TABLE 7.16: Volume accounts of the fourth year of the high efficiency in non-agriculture run: level and percentage of fourth year volume of the base-run

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	1,858	95	1,729	106	3,587	100	-	.
Rice	('000 mton, milled)	14,364	101	230	77	14,595	100	-	.
Sugar	('000 mton)	403	99	291	104	694	101	-	.
Beef	('000 mton, carc.w.)	149	97	8	x	157	102	-	.
Poultry + eggs	('000 mton, prot.eq.)	15	100	-	.	15	101	-	.
Dairy	('000 mton)	952	92	317	147	1,269	102	-	.
Vegetables	('000 mton, stand.)	872	101	-	.	872	101	-	.
Fruit	('000 mton. stand.)	939	101	-	.	939	101	-	.
Fish	('000 mton)	904	100	-75	94	829	101	-	.
Vegetable oils	('000 mton)	77	104	91	98	168	101	-	.
Protein feed	('000 mton, cake)	136	104	-	.	-	.	136	104
Cotton	('000 mton, lint)	19	104	21	103	-	.	40	103
Jute	('000 mton)	1,161	103	-350	100	-	.	811	105
Hides	(mill. pieces)	6	86	4	115	-	.	10	101
Cloth	(mill. meters)	1,504	101	142	112	1,162	101	484	105
Jute textiles	('000 mton)	633	103	-550	102	-	.	83	114
Leather	(mill. tk 84/85)	8,327	100	-3,992	99	4,335	101	-	.
Cement	('000 mton)	282	x	497	76	-	.	779	119
Fertilizer	('000 mton, nutr.)	348	101	338	107	-	.	686	104
Tradeable non-ag.	(mill. tk. 84/85)	68,028	101	45,161	95	36,125	101	77,064	98
Non-tradeable non-ag.	(mill. tk. 84/85)	147,349	102	-	.	73,872	101	73,477	103

See notes table 7.6.

TABLE 7.17: Volume accounts of the seventh year of the high efficiency in non-agriculture run: level and percentage of seventh year volume of the base-run

Product		Production		Net import		Consumption		Intermediate inputs	
		Level	%	Level	%	Level	%	Level	%
Wheat+coarse grains	('000 mton)	3,478	124	479	44	3,957	102	-	.
Rice	('000 mton, milled)	16,101	102	-	.	16,101	102	-	.
Sugar	('000 mton)	524	106	327	110	851	107	-	.
Beef	('000 mton, carc.w.)	146	87	62	258	208	109	-	.
Poultry + eggs	('000 mton, prot.eq.)	19	106	.5	x	20	106	-	.
Dairy	('000 mton)	698	68	951	201	1,649	109	-	.
Vegetables	('000 mton, stand.)	1,012	104	-	.	1,012	104	-	.
Fruit	('000 mton. stand.)	1,130	108	-	.	1,130	108	-	.
Fish	('000 mton)	1,108	106	-117	104	991	105	-	.
Vegetable oils	('000 mton)	103	111	96	93	199	105	-	.
Protein feed	('000 mton, cake)	182	111	-	.	-	.	182	111
Cotton	('000 mton, lint)	23	106	26	113	-	.	49	109
Jute	('000 mton)	1,161	101	-350	100	-	.	811	102
Hides	(mill. pieces)	7	100	-	0	-	.	7	64
Cloth	(mill. meters)	1,829	106	136	114	1,337	106	628	108
Jute textiles	('000 mton)	627	99	-550	100	-	.	77	95
Leather	(mill. tk 84/85)	5,812	60	-777	16	5,035	104	-	.
Cement	('000 mton)	-	0	797	105	-	.	797	105
Fertilizer	('000 mton, nutr.)	150	37	690	175	-	.	840	106
Tradeable non-ag.	(mill. tk. 84/85)	82,890	106	46,498	85	43,884	106	85,504	93
Non-tradeable non-ag.	(mill. tk. 84/85)	172,983	103	-	.	88,424	105	84,559	101

See notes table 7.6.

large changes can be registered in the leather, fertilizer and tradeable non-agricultural trade. The decrease in leather exports is mainly a reflection of the stop in hide imports. The decrease in fertilizer production is more than compensated by the increase in imports. On balance fertilizer use is six per cent higher compared to the corresponding year in the base-run.

Imports of tradeable non-agricultural products are about fifteen per cent lower which is, in value terms the biggest change on the trade balance. The general picture which emerges from this scenario is highly optimistic: a high and balanced growth. This picture is even strengthened if one looks at the relative shadow prices for some main products. The following table gives an idea of the changes in functional (labour vs. non-labour) and sectoral (agriculture vs. non-agriculture) incomes:

	Relative shadow prices in the seventh year:	
	Base-run	Enag-run
Rice	1	1
Tradeable non-agricultural products	.226	.219
Labour row	-.435	-.183

From these figures it can be concluded that a more efficient use of inputs in the non-agricultural sector results in a slight improvement of the income distribution agriculture vs. non-agriculture. The cause of this can easily be understood. Because the tradeable non-agricultural product is more efficiently produced, its price will, in a competitive environment, drop compared to the price of rice. However, because both rice and the tradeable non-agricultural product do not change of regime, the relative price movement is limited.

The 'penalty' on labour changes more drastically. The implication of this is an improvement in the functional income distribution labour vs. non-labour. The high growth rates of production capacities (see figure 7.11) is the main cause of the 'scarcity' of labour.

As a motivation for some alternative assumptions in the following sections, we shall end this section with two critical comments. In the first place one can question the feasibility of such a long period of increasing input efficiency in all non-agricultural processes. Especially in the non-tradeable non-agricultural sector such an assumption seems too optimistic. Because of the importance of this sector (e.g. it can be calculated that, depending on the run and period, 25-30 per cent of total employment is provided by the non-tradeable sector), it seems interesting to investigate the consequences of a less optimistic assumption with respect to the non-tradeable sector.

The second comment has to do with regime-switches. If the columns referring to trade of tables 7.8 and 7.17 are compared with each other, one can detect only one (minor) regime-switch, i.e. in the enag-run Bangladesh becomes self-sufficient in hides. We think that, given the changes in production structure that certainly underly such a long period of efficiency gains, one may in 'reality' expect more switches. The composition of imports and exports will probably change drastically. The fact that this phenomenon does hardly occur in our results, has certainly to do with the level of aggregation of the non-agricultural sector. The tradeable non-agricultural product consists of a collection of all kinds of different goods, which are, as has been explained in chapter 3, implicitly treated as complete substitutes. We realize that by doing this, a number of interesting issues are neglected. The fact that most economic models suffer much more from this type of aggregation problem than ours is only poor comfort. To slightly cure the drawback, we shall in section 7.8, somewhat artificially, introduce a regime-switch in the non-agricultural sector. We shall see there that the consequences of such a switch can, again, be far-reaching.

7.7 A LESS OPTIMISTIC ASSUMPTION ON THE NON-TRADEABLE NON-AGRICULTURAL SECTOR

7.7.1 No efficiency gains from the fifth year on

To meet the first point of the criticism with which we ended the preceding section, we have run the model with a less optimistic assumption on the non-tradeable non-agricultural sector. Instead of assuming an on-going process of efficiency gains, we proceed from the assumption that from the fifth year on, no efficiency gains can be attained anymore in the non-tradeable non-agricultural sector. We shall refer to this run as to the etag-run and compare the results with those of the enag-run of the preceding section.

7.7.2 Some results

It will be clear that the results for the first four years do not differ. For the years five to seven we found the following growth factors:

	α-etag	α-enag
year 5	6.6	6.8
year 6	6.7	7.1
year 7	6.6	7.1

The effect on α is, not surprisingly, negative. In the seventh year the difference is .5 per cent; although not dramatic, the effect is certainly significant.

More detailed results are shown in table 7.18. Because consumption volumes are marginally lower than in the enag-run and intermediate inputs change about proportionately with production, only figures on production and net import volumes are shown. Apart from the sharp increase of fertilizer production, most significant changes appear on the trade balance. The increase in fertilizer production is a reflection of a slightly smaller overall production capacity and an equal labour volume. The resulting lower import volume is compensated by an increase of imports of wheat and tradeable non-agricultural products.

The most interesting changes appear on the 'price side' of the model. If we compare some important shadow prices of the etag-run with the same prices of the enag-run, we get the following picture:

TABLE 7.18: Production and import volumes in the seventh year of the high efficiency in tradeable non-agriculture run: level and percentage of base-run and enag-run volumes.

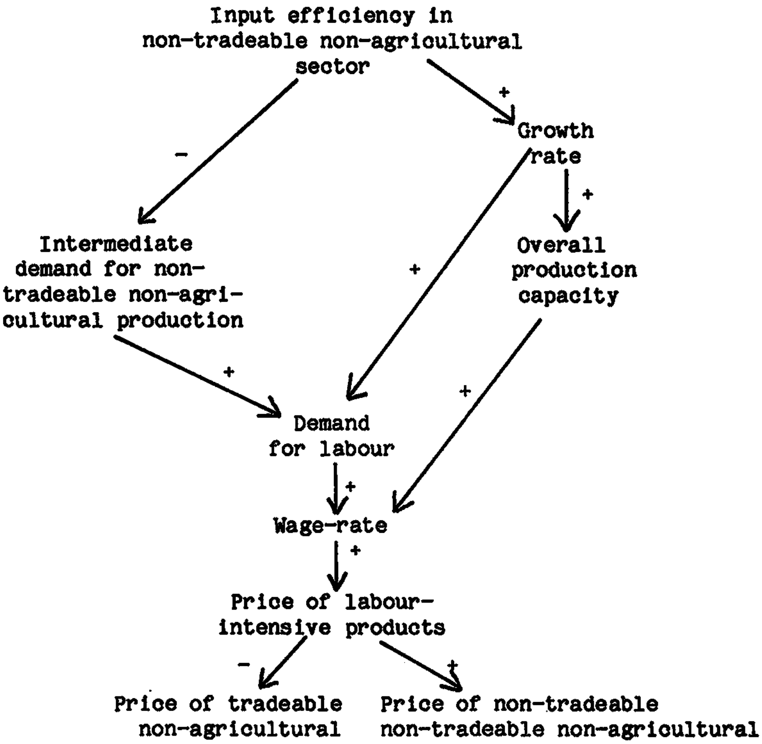
Product	Production			Net imports		
	Level	% of base-run	% of enag-run	Level	% of base-run	% of enag-run
Wheat	3,388	121	97	560	51	117
Rice	16,065	102	100	-	.	.
Sugar	522	106	100	321	108	98
Beef	148	89	101	59	246	95
Poultry + eggs	20	100	100	.4	x	80
Dairy	704	68	101	919	104	97
Vegetables	1,004	103	99	-	.	.
Fruits	1,112	107	99	-	.	.
Fish	1,104	106	100	-120	106	103
Vegetable oils	99	106	96	95	99	99
Protein feed	175	106	96	-	.	.
Cotton	23	106	100	26	113	100
Jute	1,164	102	99	-350	100	100
Hides	7	100	100	-	0	.
Cloth	1,823	106	100	128	108	94
Jute textiles	628	100	100	-550	100	100
Leather	5,685	59	98	-817	17	105
Cement	0	0	.	796	105	100
Fertilizer	240	60	160	591	150	86
Tradeable non-ag. production	80,486	103	97	48,623	89	105
Non-tradeable non-ag. production	173,174	103	100	-	.	.

See notes table 7.6.

	Etag-run	Enag-run
Rice	1	1
Tradeable non-agricultural	.218	.219
Non-tradeable non-agricultural	.127	.123
Labour row	-.179	-.183

The main conclusion that emerges from these figures is that a lagging behind of input efficiency in the non-tradeable non-agricultural sector leads to a relatively higher price of the non-tradeable non-agricultural product and to a higher wage-rate. The latter is remarkable because of the decrease in the overall production capacity/labour ratio.

Figure 7.12 shows how the chain of 'causation' works. The effect of the lower growth rate is slightly more than off-set by the effect of the (small) increase in intermediate demand for non-tradeable non-agricultural product.



$\xrightarrow{+}$ an increase in the variable on the left-hand side of the arrow causes, ceteris paribus, an increase* in the variable on the right-hand and a decrease a decrease*

$\xrightarrow{-}$ an increase causes, ceteris paribus, a decrease*, a decrease causes an incease*

Figure 7.12
Main impacts of a change in input efficiency in the non-tradeable non-agricultural sector on product prices

*or leaves unchanged

7.8 A REGIME-SWITCH FOR CLOTH

7.8.1 Problems in exporting industrial products

In spite of a lack of reliable data, there appears to be enough evidence that the structure of most industries within the Bangladesh manufacturing sector has, in terms of production, efficiency and labour, only marginally changed in recent years (see e.g. Artep [4], chapter 6). There are many reasons why the manufacturing sector has not been the engine of economic growth. Some industries are supply constrained and suffer from lack of capital, lack of skilled labour, inefficient use of inputs, low level of development of infrastructure etc. Others are demand constrained and suffer from low effective domestic demand, limited export possibilities etc.

In the preceding two sections we have mainly concentrated on the supply constraining factors. As a general conclusion it appeared that an improvement of these factors can have a significant effect on the overall growth rate. It will be clear that tremendous efforts in the fields of education and organization are required to realize the efficiency assumptions. The results also showed that, apart from jute and jute textiles, no demand constraints were hit in the manufacturing sector, i.e. no (major) regime-switches appeared. As explained, this result is probably mainly due to the high level of aggregation of the tradeable sector in the model. As a consequence the scope for import substitution is strongly exaggerated while problems to enter an export regime are 'aggregated away'. For at least three reasons these problems will, in general, be rather large for Bangladesh. First, trade-ties within the region are not strongly developed. In the second place, there are hardly transnational companies within the country under whose wings small companies could explore export possibilities. And thirdly, the level of development of the infrastructure within the country is rather low. A consequence of the first two factors is, in terms of figure 7.5, that the ratio $p_i^b/\bar{p}_i^b$ is generally very high; the third factor results, in addition, in high margins within the country itself.

A detailed investigation of the effects of these factors for different industrial products within our model framework would require a further disaggregation of the tradeable non-agricultural product. Given the limited scope of our study, we have left this undone. Instead we shall only illustrate the point by introducing, somewhat artificially, a regime-switch for a tradeable non-agricultural product.

Because, according to most of our model results, Bangladesh is on the verge of self-sufficiency with regard to cloth production, while in reality the country actually exports cloth, analysing a regime-switch for cloth appears to make sense. This is done by reducing consumer demand for cloth by 30 per cent. To compensate the effect on the overall consumption, consumption of the tradeable non-agricultural product is raised accordingly. Other assumptions are similar to these of the etag-run of the preceding section. Because we only want to illustrate the particular point under discussion, we have solved the model under the low cloth consumption assumption (lcc-run) for one period only. The last (seventh) year of the etag-run serves as a reference period.

7.8.2 Main results

An effect on the overall growth rate can hardly be identified. Because, compared to the etag-run, only a (relatively) marginal change in consumption is assumed, this will not come as a surprise. Of course, things can be different if the model is run for a number of subsequent periods.

The most interesting effects appear in the production and trade sphere. Table 7.19 shows the most noticeable changes. It can be seen in this table that the decrease in cloth consumption results, ceteris paribus, in an autarky regime for cloth. This result does not necessarily contradict the actual situation where one could recently notice a strong rise in export of some textile products. Not only because in our model different cotton-textile products are aggregated into one product, but also because we proceed from an average wage-rate. A lower wage-rate could, in principle, result in an export regime for cloth (see also section 7.2.4).

Turning back to table 7.19, one can also see a drop in the production level of cloth. Compared with the etag-run, cloth production decreases with 21 per

TABLE 7.19: Main effects of lower cloth consumption, compensated by higher tradeable non-agricultural product consumption, on production and net import volumes

Product	Production	Net import
Cotton	100	62
Cloth	79	0
Leather	178	611
Fertilizer	176	69
Tradeable non-agricultural products	100	115

Note: Volumes are expressed relative to the corresponding volumes of table 7.18, e.g. the entry for cloth production is calculated as:
$\frac{\text{volume cloth production lcc-run}}{\text{volume cloth production etag-run}} \times 100.$

cent. Because of the production decrease, less raw material (cotton) is needed also. It appears most efficient to adjust via a lowering import volume and to keep domestic cotton production unchanged. Tradeable non-agricultural production capacity was already fully utilized in the reference run. An increase in demand will therefore result in an increase in imports.

The changes with regard to trade and production of fertilizer and leather can best be explained by looking at the employment effects of the alternative assumptions. Because cloth production is a rather labour intensive activity, a decrease in production will result either in unemployment or in a drop of the wage-rate so that it becomes attractive to use less profitable production capacity. The lower bound on labour sees to that the latter happens, hence the expansion of leather (based on imported hides) and fertilizer production. Because domestic consumption of these goods hardly changes, the volume accounts are balanced via an adjustment in international trade. Leather exports increase and fertilizer imports decrease. As was the case for foodgrains, the difficulty to enter the export regime has also consequences for the functional income distribution. Compared with the reference run, the shadow price of labour decreases; in terms of rice from -.179 to -.337.

The border price for export of cloth is 49 per cent of the border price for imports (see table 6.4). It does not seem unreasonable that due to an improvement in quality and an easier access to foreign markets, the gap between import and export price can be narrowed to, say, 80 per cent. In addition, one can imagine, that the costs from factory gate to border can be halved. We have run the model under these assumptions. As a result cloth entered, indeed, the export regime. A side effect was a change in the functional income distribution in favour of labour. It is interesting to note that the assumption with regard to a decrease in the costs between ex-factory gate and border was crucial. In the absence of this assumption, cloth stayed in the autarky regime. This result is, in our opinion, not without practical value. It somewhat relativates the blessings of a small scale village oriented industrial policy. Although attractive as long as a country is in an import regime, problems arise once self-sufficiency is reached and increased production has to be sold on the world market. Because of the relative large margin between factory gate and border, it will be difficult to compete internationally.

Chapter 8

A SUMMARY OF THE MAIN FINDINGS AND SOME FINAL COMMENTS

At the start of our study we had two objectives in mind. In the first place, we wanted to investigate the growth potential of the Bangladesh economy. In the second place we were aimed at an evaluation of the Von Neumann model on its empirical usefulness. Although chapters 2 to 7 provide a detailed account of the extent in which we have been successful in achieving the objectives, it seems useful to end our study with a brief summary of the main findings.

To start with the Von Neumann model, we think that the preceding chapters have amply shown that this model is, indeed, an interesting tool to analyze reality. However, as has extensively been discussed theoretically in chapter 3 and shown empirically in chapter 7, only after a number of model adjustments are made. Balanced growth in the original Von Neumann sense, i.e. under a complete neglect of the composition of the initial endowments and an unchanging per capita consumption, is too strict a concept to be of much relevance for analyzing growth possibilities of a real world economy.

This finding does not mean that the model, in its original form, has no empirical value at all. As we showed in section 7.2, it can, among other things, play a role as a tool for technology assessment on a more micro level. For example, in a laboratory situation where one has to decide which technology package should be introduced 'in the field', the Von Neumann framework can be used for getting a first idea about the superiority of one package above another. Precisely because potential clients will usually not have the disposal of endowments in the same proportions, an 'ideal endowment outcome' can function as an interesting benchmark. The superiority of the high yielding variety technology which was found in section 7.2 is, in this connection, an illustrative result. Besides being valuable as a tool for technology assessment, an empirical elaboration of the model in its original form appeared to be interesting in another sense also. As was shown in section 7.2, the economy of Bangladesh behaves at a great distance from the turnpike. The computed growth rates can be obtained by a small part of the economy only. The implication of this is not without practical value. The simultaneous existence of sectors in an economy with different growth rates results in a drain of resources to the sectors which can grow fastest. In an open economy where capital can move freely over the border while labour has to stay home, this can lead to an outflow of capital while at the same time a large part of domestic capacities (e.g. labour) remain idle. According to the results of section 7.2.4 the break-even point for Bangladesh in this respect is 10.1 per cent. This interesting issue could be analyzed further by adding foreign investment activities explicitly to the model. Consequences of a free movement of labour could be analyzed in an analogous manner. Be it that this is less relevant in practice: discussions on liberalizing world trade usually refer to everything under the sun but labour. Because we had to be selective, an elaboration on these points have been left undone.

To become a tool for analyzing growth potential, two main adjustments appeared necessary. First, initial endowments had to be taken into account; and second, a more realistic description of consumption had to be allowed for.

The application of the adjusted model on Bangladesh data yields, to our best knowledge, meaningful results. However, meaningful results are, although important, not the only criterion to judge the empirical usefulness of an analytical instrument. The theoretical underpinning is at least as important. For this we refer to chapters 1 to 3. Because the model adjustments could all be done within the original Von Neumann framework, the theoretical attractiveness of the model was preserved, i.e. equilibrium-states of (very) disaggregated economies where the number of processes is not necessarily equal to the number of goods and where joint production is allowed, remain the subject of analysis. The adjustment of the model had a number of side-effects.

First, the model solution applies, in contrast to the original model, to one period only. A number of model parameters has to be updated between the periods. Theoretically this means that the balanced growth path, in the strict sense of the word, is left. In practice it also means a tremendous amount of extra work. We shall come back to this point below.

A second side-effect consists of the introduction of a functional income distribution. Shadow prices on production capacities and labour give an indication about the relative scarcity of capital and labour.

Finally, the introduction of initial endowments can also be seen as the addition of an extra process which uses labour as input and yields initial capital endowments as output. The model can be interpreted then as an original Von Neumann model. A difference is that the economy finds itself automatically on the turnpike. The model interpreted in this way can function as a tool for technology assessment on a more macro level. As explained in chapter 3, using the model in this way seems more interesting for a highly industrialized country like the Netherlands than for a poor country like Bangladesh.

Before the adjusted model could be applied to a real world situation, in casu to Bangladesh, a computational procedure (algorithm) was required in order to be able to solve it. Most algorithms we found in literature find only one or two solutions (growth factors). Because the model can, in principle, have many solutions, already after a first screening these were excluded. Moreover, after a thorough investigation, the remaining two appeared to be not appropriate either. Therefore an alternative algorithm was developed. Because the multiple solution case appeared of rare occurrance in the empirical part, i.e. only in the 'straight' application the model contained more than one growth factor (see section 7.2), the role of the newly developed algorithm was a modest one outside chapter 4. Before turning to the main findings with respect to growth possibilities of the overall economy, we shall first say a few words on some of the shortcomings and other imperfections of our study.

The first point we want to mention has to do with the (nearly) complete absence of price feedbacks, (government-) policies and other distortions (tariffs, monopolies, risk, etc.). The usefulness of the model would be greatly enhanced if such feedbacks and distortions were introduced. In a more general context this issue is theoretically discussed in Keyzer [45]. This paper also contains an algorithm with which real world applications can be solved efficiently. An empirical elaboration along this line appears very promising to us.

The second point we want to mention has to do with the software used in doing the actual calculations. As explained in chapter 6, the 'Von Neumann software' mainly consists of a combination of existing SOW-software. Although we never had any (real) problem with the software as such, some limitations significantly constrained our empirical work. These limitations were of two kinds. On the one hand the update between two periods had to be done by hand. This was not only very time consuming but also an ever recurring source of mistakes. The other limitation had to do with processing the results. Because no data management facility was linked to the software, all table entries had to be calculated by hand. This was, again, a very time consuming procedure. Moreover, the lack of such a facility had also a restrictive influence on the type of results which are actually presented. If more real world applications will be done with the model, a linkage with a data management facility like DRUKKER (see for a description Keyzer [43]), is highly recommended.

The above software problems can relatively easily be overcome. A more serious point which is extensively discussed in section 6.2.1, has to do with the time-length of the model activities. The requirement that all activities must start and end respectively on the same date, severely limits the amount of technical data that can explicitly be handled by the model in practice. Some suggestions (conjectures) were done for solving this problem. However, they could not be proven rigorously. Because of the seriousness and the consequences a solution can have for the software, further work on the problem deserves a high priority.

The last point refers to the application to Bangladesh. The empirical elaboration has been done in a somewhat unbalanced way, i.e. the non-agricultural sector is treated on a much more aggregated level than the agricultural sector. As a consequence a lot of interesting relations remain under the table. In section 7.8 we have speculated about the occurrence of regime-switches in the tradeable sector but one can also think of an explicit modeling of educational processes or an inclusion of the pipeline structure of major investment projects, etc. We are convinced that by a further disaggregation the practical value of the model will be greatly enhanced.

Improving the model along these lines is clearly within the scope of the work we have done sofar. With respect to other shortcomings, e.g. the lack of monetary phenomena, the absence of stochastical elements, the impossibility of input substitution within a process, etc., this is to a far lesser extent the case. We shall now proceed with a discussion of the results of the model with endowments.

Sections 7.3-7.8 reveal that Bangladesh certainly has a potential to grow. Growth figures in all scenarios were significantly higher than the (assumed) growth rate of the population. In spite of a high overall savings (investment) rate, there appeared to be also room for an increase in per capita consumption.

Of course, we are aware that this rather optimistic outcome has partly to do with the underlying assumptions. In this respect the Von Neumann model does not differ from other economic models. And we must indeed admit, that under less optimistic assumptions, much lower growth figures are generated. For example, if yields do not increase in agriculture and population grows slightly faster while at the same time foreign aid does not increase at all, the resulting overall growth rate does not even match the growth rate of population.

The reason why we have not presented such a 'doomsday' scenario, and certainly not centered the discussion around it, is not only because we had to be selective, but also because we do not want to join the more popular thinking on Bangladesh[68]. Because we are convinced that the assumptions

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[68] The saying of Dr. H. Kissinger on Bangladesh as being the basket case of the world is most famous in this respect. On the other hand assumptions underlying plans made by the Bangladesh government itself are especially known for their over-ambitious targets.

underlying the runs of chapter 7, although rather optimistic, are not unrealistic altogether, we think that our choice is justified[69].

Turning back to the results. The sources of growth can be subdivided into two categories. On the one hand production increases as a consequence of a relatively fast increase of production capacity, a process strengthened by the high savings (investments) ratio. On the other hand existing capacity in agriculture produces higher yields, while, in some runs, capacity in the non-agricultural sector is more efficiently used.

Domestic resource mobilization will be the central element of any development strategy. The two sources of growth are a reflection of two aspects of such a strategy: capacity expansion being the result of land development and capital accumulation on the one hand and increases in yield and efficiency resulting from a qualitative improvement of the labour force on the other. Land development mainly means drainage and irrigation. As a consequence more crops can be harvested with higher yields. At the same time the variability in yields will be reduced. In the non-agricultural sector capital accumulation has mainly to do with investments in machines, buildings and infrastructural works. As the model shows, there is, in principle, ample scope to follow this road. Improving the quality of the labour force, which is necessary to make optimal use of existing and newly developed production capacity will require huge efforts in the fields of education, extension and training. Although these types of investments are not modeled explicitly, the results give at least an indication that such investments could pay off handsomely.

Thus, the process of resource mobilization can indeed result in a period of high balanced growth. However, there are also some clouds on the horizon. The most important one we discussed appears once the country achieves self-sufficiency in foodgrains. Because at existing prices it is, according to the model, not attractive to export foodgrains, self-sufficiency will be coupled with a drastic decrease of the domestic price of foodgrains, the overall growth rate and the terms of trade agriculture vs. non-agriculture.

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[69] The fact that our assumptions are, roughly speaking, consistent with those underlying TFYP (see section 7.3), while the latter resulted from intensive consultations and discussions between and with Bangladesh experts, may be a further evidence of this statement.

To overcome the 'regime-switch trap' actions on three fronts seem necessary. The infrastructure within the country has to be improved, taste and quality of rice have to be adapted to standards of potential customers, and markets must be found in as close a neighbourhood as possible. The model results show that if Bangladesh is successful with respect to these actions, the period of high balanced growth can go on.

In the non-agricultural sector we did not come across such shocks. However, as explained, we do not think that the situation is fundamentally different here. A further disaggregation would certainly reveal 'regime-switch shocks' once production of non-agricultural goods really starts growing.

To investigate this matter in more detail, a model experiment was done with cloth consumption drastically reduced. Again it was found how difficult it is to enter the export regime. An improvement of the domestic infrastructure turned out to be crucial. This finding leads to the conclusion that a policy directed at the development of small scale village oriented industries is not without danger. Although attractive as long as the country is in an import regime, problems arise once self-sufficiency is reached and increased production has to be sold on the world market.

In summary: a more efficient production is a necessary condition for Bangladesh to enter a long period of high growth. It is on the other hand not a sufficient one. Problems arise once regime-switches are met. The resulting change in the price-structure can have devastating effects on the growth rate. And although through effective demand management policies, prices can (eventually) be kept temporarily from falling at excessively low levels (see also UNDP [104]), in the long run efficient domestic production has to be accompanied by a favourable situation at the international market (higher prices, easy access to foreign markets, etc.).

Thus, as a general conclusion we may state that a process of domestic resource mobilization can significantly raise the growth rate of the Bangladesh economy. However, a real success can only be achieved in a 'friendly' international environment. Like most general statements made by the social sciences this overall conclusion may sound somewhat trivial. We hope, however, to have shown that, in the context of Bangladesh, the statement has a clear empirical content.

Appendix A

TWO-PERSON ZERO-SUM GAMES

A.1 INTRODUCTION

Throughout this study extensive use is made of game theory as a mathematical tool for analyzing the Von Neumann model. For the sake of completeness we shall summarize some basic concepts and relevant theorems from this theory which are used at different locations in the text.

More extensive expositions can be found in many books. For example: Gale [23], Gaver and Thompson [25], Thie [96] and Von Neumann and Morgenstern [71]. Our discussion, which will be restricted to 'two-person zero-sum games' is mainly based on these references.

A.2 DEFINITIONS AND BASIC CONCEPTS

Matrix game theory is a special branch of game theory in which it is assumed that:

(i) there are two players;

(ii) each player has a complete knowledge of all actions available to himself and his opponent;

(iii) each player has knowledge of the results of the conflicts associated with any given selection of actions; and

(iv) each player acts rationally to maximize his expected gains.

The information critical to a matrix game can be represented by means of a so-called 'pay-off matrix' showing the pay-offs to each of the two players given alternative possible pairs of courses of action (strategies). If the interests of the two players are diametrically opposed in the sense that what one player wins, the other player looses, the game is a zero-sum game. If we speak in the following of a (matrix)game, we tacitly assume that a zero-sum game is meant.

$$\begin{array}{ll|ccccccccc|}
 & & \multicolumn{9}{l}{\text{Player C}} \\
 & & 1 & 2 & 3 & . & . & . & . & . & n \\ \hline
 & 1 & m_{11} & m_{12} & m_{13} & . & . & . & . & . & m_{1n} \\
 & 2 & m_{21} & m_{22} & m_{23} & . & . & . & . & . & m_{2n} \\
 & 3 & m_{31} & m_{32} & m_{33} & . & . & . & . & . & m_{3n} \\
\text{Player R} & . & . & . & . & . & . & . & . & . & . \\
 & . & . & . & . & . & . & . & . & . & . \\
 & . & . & . & . & . & . & . & . & . & . \\
 & m & m_{m1} & m_{m2} & m_{m3} & . & . & . & . & . & m_{mn} \\ \hline
\end{array}$$

Figure A-1
Pay-off matrix M

In order to describe such a game we define the two players as R and C. Player R controls the rows of the matrix and Player C controls the columns. We suppose the pay-off matrix M is m x n (see figure A-1).

Playing the game consists of choosing a strategy by both the row and column player, called the row and the column strategy, respectively. A row strategy is an m-component row vector x, satisfying

$$x \geq 0 \text{ and } xf = 1 \tag{A-1}$$

where f is a column vector consisting of all ones. Similarly a column strategy is an n-component column vector y satisfying

$$y \geq 0 \text{ and } ey = 1 \tag{A-2}$$

where e is a row vector consisting of all ones. Notice that a row strategy x puts a probability distribution on the rows of M while a column strategy puts a probability distribution on the columns of M.

A distinction can be made between two types of strategies:

(1) Pure strategies: a pure strategy is a strategy in which one component of the x (and y) vector takes the value 1. Consequently, the values of the other components are all zero.

(2) Mixed strategies: in a mixed strategy, no component of the x (and y) vector has a value 1. From this it follows that a mixed strategy has at least two components of the x (and y) vector > 0.

Strategies x and y are optimal for the matrix game M if the following inequalities are satisfied:

$$xM \geqq ve \tag{A-3}$$

and

$$My \leqq fv \tag{A-4}$$

where x, y, e, f and M are defined as above and v is a scalar. It is easy to see that from (A-3) and (A-4) it follows that

$$xMy = v \tag{A-5}$$

Optimal x's and y's are solutions to the game. The corresponding v is the value of the game. If $v \geqq 0$, then C pays R the amount v; if $v < 0$, then R pays C the amount $|v|$. Because both R and C play rationally, i.e. both maximize their expected gains, R will choose an x-strategy that maximizes v under the assumption that C will choose an y-strategy that minimizes v; on the other hand C will choose an y-strategy that minimizes v under the assumption that R will choose an x-strategy that maximizes v. Thus, if X and Y are defined as the respective sets of row and column strategies and $\bar{x}$ and $\bar{y}$ are optimal strategies, then for each $x \in X$:

$$xM\bar{y} \leqq \bar{x}M\bar{y} \tag{A-6}$$

and for each $y \in Y$:

$$\bar{x}My \geqq \bar{x}M\bar{y} \tag{A-7}$$

These results are extensively used in chapter 2.

If v=0 the game is said to be fair and if a game can be solved in pure strategies, the game is strictly determined. The value of a strictly determined game is equal to $\bar{m}_{ij}$ which is the smallest entry of row i and at th same time the largest entry of column j. The value $\bar{m}_{ij}$ is also called the saddle value of M.

A fundamental theorem of matrix game theory is the *existence theore* This theorem states that for any mxn matrix game M, there exist a unique value v and (not necessarily unique) optimal strategies x and y that satisfy conditions (A-1) - (A-4). Moreover, the set of optimal strategies for each player is a bounded polyhedral convex set.

Other important theoretical results about solutions to matrix games are t weak and strong theorem of the alternative. The *weak theorem of the alternative* states that if M is an mxn matrix game, v=v(M) is the game value and x' and y' are optimal strategies then

$$x'_i > 0 \quad \text{implies} \quad M^i y' = v$$

and

$$y'_j > 0 \quad \text{implies} \quad x' M^j = v$$

or, if x'_i is positive, then the y'-strategy must satisfy the i-th constra of (A-3) as an equality; and if y'_j is positive, then the x'-strategy must satisfy the j-th constraint of (A.4) as an equality. These results can be strengthened through the strong theorem of the alternative. Because this theorem makes use of the concept of a *central solution*, we first introduce this concept: Let $\bar{X}$ and $\bar{Y}$ be the sets of all optimal x- and y-strategies. Then the extreme points of $\bar{X}$ and $\bar{Y}$ are called *basic solutions*. Each set has a finite number of basic solutions. Central solutio can now be defined as positive convex combinations of basic solutions. Or as:

$$x^c = \sum_i \lambda_i x^i, \ \lambda_i > 0, \ \sum_i \lambda_i = 1$$

and

$$y^c = \sum_j \gamma y^j, \ \gamma_j > 0, \ \sum_i \gamma_j = 1$$

where

x^c, y^c are central solutions, and

x^i, y^j are basic solutions, $(i=1,2,\ldots,k)$,

$(j=1,2,\ldots,\ell)$.

The strong theorem of the alternative can now be stated as follows: Let M be an mxn matrix game with value v, and let (x^c,y^c) be any pair of central solutions; then the following statements are true:

(a) $x_i^c > 0$, if and only if $M^i y^c = v$

(b) $y_j^c > 0$, if and only if $x^c M^j = v$

Statement (a) is equivalent to:

(a.1) $x_i^c = 0$ if and only if $M^i y^c < v$

and statement (b) is equivalent to

(b.1) $y_j^c = 0$ if and only if $x^c M^j > v$

For proofs of the above results, see the references mentioned in the introduction.

A.3 COMPUTATIONAL TECHNIQUES

A.3.1 General

The problem of determining optimal strategies and the value of a matarix game is equivalent to solving a linear programming model. This can be seen as follows:

Consider the mxn matrix game M. The row player wants to make the value of the game as large as possible, i.e. his behaviour can be described by the following linear programming model:

$$\begin{aligned} &\text{Max } v' \\ \text{s.t. } &xM - v'e \geqq 0 \\ &xf = 1 \\ &x \geqq 0 \end{aligned} \qquad (A-8)$$

It can be checked that the constraints of (A-8) are equivalent to (A-1) and (A-3). The variable v' is unconstrained. To prove that (A-8) has a solution, we take for x an arbitrary 1xm probability vector, then v' can always be chosen in such way that $xM-v'e \geqq 0$. In addition, because $v'e \leqq M$ and $xM \leqq \max [m_{ij}]$[70], it follows that (A-8) has a feasible solution which is bounded from above. Let $\bar{v}$ be the maximum value of v' in (A-8). In order to show that $\bar{v}$ is the value of the game, we next define the dual problem of (A-8). Therefore, we introduce the dual variables $y=(y_1,y_2,\dots,y_n)'$ and v''. Using the rules for constructing dual problems, we get:

$$\begin{aligned} &\text{Min } v'' \\ \text{s.t. } &My-fv'' \leqq 0 \\ &ey = 1 \\ &y \geqq 0 \end{aligned} \qquad (A-9)$$

Here, v'' is an unconstrained variable; M, f, e and y are defined as above. It is easy to see that the constraints of (A-9) correspond with (A-2) and (A-4). To prove that (A-9) has a solution, we take for y and arbitrary nx1 probability vector, then v'' can always be chosen such that $My-fv'' \leqq 0$. In addition, because $fv'' \geqq My$ and $My \geqq \min[m_{ij}]$, it follows that (A-9) has a feasible solution which is bounded from below. From the duality theorem (Appendix B) it follows that at the optimum, the values of the objective functions of (A-8) and (A-9) are equal. Or:

$$\text{Max } v' = \text{Min } v'' = \bar{v} \qquad (A-10)$$

---

[70] Max $[m_{ij}]$ (respectively min $[m_{ij}]$) stands for the maximum (respectively minimum) element m_{ij} of M.

A.3.2 2 x 2 games

Throughout the text we frequently illustrate our argument by means of small examples, often square and of order 2 x 2. Solutions to them can be given by simple formulas which will be derived below. For a full discussion of solution methods to the general problem be referred to Chapter 4.

Consider the game with pay-off matrix:

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Determining the optimal x-strategy, consists of solving the following problem:

$$\begin{aligned} & \text{Max } v \\ \text{s.t. } & m_{11}x_1 + m_{21}x_2 \geq v \\ & m_{12}x_1 + m_{22}x_2 \geq v \\ & x_1 + x_2 = 1 \\ & x_2, x_1 \geq 0 \end{aligned} \qquad \text{(A-11)}$$

If we substitute x_2 for $(1-x_2)$, the following graph can be constructed:

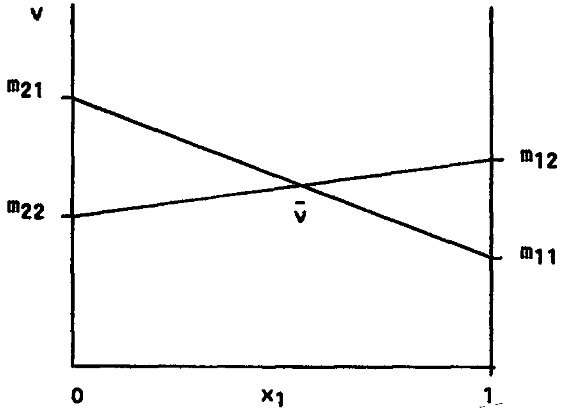

Figure A-2

Graphical solution of 2 x 2 games

Two cases can be distinguished:

Case 1: The two lines do not intersect

From the figure, it can be concluded that in this case

either

$m_{11} \leq m_{12}$ and $m_{21} \leqq m_{22}$ (a)

or

$m_{12} < m_{11}$ and $m_{22} < m_{21}$ (b)

It is easy to see that the game can be solved for x by a pure strategy, i.e.

$x = (1,0)$ if:
either (a) applies and $m_{11} \geqq m_{21}$ (a1)
or (b) applies and $m_{12} \geqq m_{22}$ (b1)

$x = (0,1)$ if:
either (a) applies and $m_{21} \geqq m_{11}$ (a2)

or (b) applies and $m_{22} \geqq m_{12}$[71] (b2)

The corresponding values of the game are m_{11}(a1), m_{12}(b1), m_{21}(a2) and m_{22}(b2) respectively. In general, if the game can be solved for x by a pure strategy, the value $\bar{v}$ will be equal to $\tilde{m}_{ij}$, where $\tilde{m}_{ij}$ is the smallest entry of row i and at the same time te largest entry of column j. It can be verified that for such an $\tilde{m}_{ij}$ the pure y-strategy with an element $y_j=1$ satisfies both (A-2) and (A-4) and thus is a solution of the game.

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[71] In case the equality signs apply, convex combinations are also possible.

Case 2: The two lines intersect

Now:

$$\bar{v} = m_{11}x_1 + m_{21}(1-x_1) = m_{12}x_1 + m_{22}(1-x_1)$$

$$\Rightarrow \quad x_1 = \frac{m_{22} - m_{21}}{m_{11} + m_{22} - m_{12} - m_{21}}$$

and because $x_2 = (1-x_1)$: (A-12)

$$x_2 = \frac{m_{11} - m_{12}}{m_{11} + m_{22} - m_{12} - m_{21}}$$

and

$$\bar{v} = \frac{m_{11}m_{22} - m_{12}m_{21}}{m_{11} + m_{22} - m_{12} - m_{21}} \qquad \text{(A-13)}$$

Along the same lines $y = (y_1, y_2)$ can be calculated as:

$$y_1 = \frac{m_{22} - m_{12}}{m_{11} + m_{22} - m_{12} - m_{22}}$$

and (A-14)

$$y_2 = \frac{m_{11} - m_{21}}{m_{11} + m_{22} - m_{12} - m_{22}}$$

A.3.3 Computing the growth factor of 2 x 2 matrices

We end with a formula to determine all α's for which $M_\alpha = 0$ and $xBy > 0$. We start with defining the following non-negative matrices[72]:

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[72] We assume the KMT-conditions on technology apply.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

From these and a parameter α, we form M_α:

$$M_\alpha = (B-\alpha A) = \begin{pmatrix} b_{11}-\alpha a_{11} & b_{12}-\alpha a_{12} \\ b_{21}-\alpha a_{21} & b_{22}-\alpha a_{22} \end{pmatrix} \qquad \text{(A-1}$$

We want a solution to M_α only for values of α such that $v(M_\alpha) = 0$ and $xBy > 0$. From theorem 2.6 we know that the maximum number of such α's is 2. Together with the proof of the existence theorem it follows that in case there are more α's for which $v(M_\alpha)=0$, only α_{min} and α_{max} satisfy the additional condition that x and y strategies exist such that $xBy > 0$.

Again two cases can be distinguished (see Morgenstern and Thompson [69], p. 228-229):

Case (i): An α exist such that $v(M_\alpha)=0$ and M_α is strictly determined

This can only happen if an α is chosen such that at least one entry of M_α becomes (or stays) zero and at the same time a zero element of M_α is simultaneously the smallest entry of row i and the largest entry of column j. One thing and another is a direct consequence of case (1) above. For a 2x2 matrix it can easily be verified if and for what value(s) of α this will happen.

Case (ii): An α exist such that $v(M_\alpha)=0$ and M_α is not strictly determined

In this case (A-13) applies. From figure (A-2) it can be checked that in a non-strictly determined game $m_{11} + m_{22} - m_{12} - m_{21}$ can never be zero. Therefore, the value of such a game will be zero if and only if the value of the nominator of (A-13) is zero, that is, if the determinant of M_α(A-15) is zero.

Or:

$$\alpha^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} - \alpha \left(\begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix} \right) + \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \tag{A-16}$$

From this quadratic equation α can be determined.

Appendix B
LINEAR PROGRAMMING: SOME BASIC RESULTS

B.1 INTRODUCTION

In all algorithms discussed in chapter 4, extensive use is made of linear programming. Although the theory of linear programming is well-known and described in many textbooks (see e.g. Gaver and Thompson [25] or Simonnard [86]), we shall, for easy reference, summarize the basic results of this theory. The exposition in this appendix is based on Morgenstern and Thompson ([69], p. 233-p.238).

B.2 BASIC CONCEPTS AND THEOREMS

Consider the standard form of a pair of primal and dual linear programming models:

$$\begin{array}{llll} \text{primal:} & \text{minimize } xb = g & \text{dual:} & \text{maximize } cy = h \\ & \text{s.t.} \quad xM \geq c & & \text{s.t.} \quad My \leq b \\ & \qquad\quad x \geq 0 & & \qquad\quad y \geq 0 \end{array} \tag{B-1}$$

where M is an mxn matrix, x is 1xm, y is nx1, c=1xn, b=mx1 and g and h are scalars. Vectors x and y will be said to be feasible if they satisfy their respective sets of constraints.

The inequality constraints $x \geq 0$ and $y \geq 0$ are automatically imposed by the steps of the (revised) simplex method. By introducing an mx1 vector z and an 1xn vector w of slack variables, the inequalities $xM \geq c$ and $My \leq b$ can be changed into equalities:

$$\begin{array}{llll} \text{primal:} & \text{minimize } xb = g & \text{dual:} & \text{maximize } cy = h \\ & \text{s.t.} \quad xM-w = c & & \text{s.t.} \quad My+z = b \\ & \qquad x,w \geq 0 & & \qquad y,z \geq 0 \end{array} \qquad \text{(B-2)}$$

From (B-2) Tucker's duality equation can be derived:

$$\begin{aligned} g-h &= xb-cy \\ &= x(My+z) - (xM-w)y \end{aligned}$$

or

$$g-h = xz+wy \qquad \text{(B-3)}$$

If x,y,w and z are feasible solutions to (B-2), they satisfy the constraints of the respective problems, i.e. x,y,w and $z \geq 0$ applies. Applying this result to (B-3) yields:

$$g \geq h \qquad \text{(B-4)}$$

If g' and h' are defined as the optimum values of (B-3), we may write:

$$g \geq g' \geq h \qquad \text{(B-5)}$$

and

$$g \geq h' \geq h \qquad \text{(B-6)}$$

In words: (B-5) says that any feasible solution to the minimizing problem gives an upper bound to the optimum value of the maximizing problem, and (B-6) says that any feasible solution to the maximizing problem provides a lower bound to the optimum value of the minimizing problem.

The duality theorem can be considered as the keystone of linear programming. According to the theorem, the maximum problem of (B-1) has a feasible solution y' such that $cy' = \max cy$, if, and only if, the minimum problem has a feasible solution x' such that $x'b = \min xb$. Moreover, the equality $cy' = x'b$ holds, if, and only if, y' and x' are optimum solutions to their respective problems.

Thus, the duality theorem states that one of the problems in (B-1) has a finite optimum solution if, and only if, the other one also has and further that their respective objective values are equal.

The existence theorem follows directly from the duality theorem. It says that both the dual programs (B-1) have optimal solutions x' and y' if, and only if, they both have feasible solutions x and y.

B.3. COMPLEMENTARY SLACKNESS

As in the case of matrix games, there is a connection between positive components of solution vectors and the equality of the corresponding constraint. In linear programming this is called the complementary slackness condition. From Tucker's duality relation (B-3) we can derive the following. Let x', w', y' and z' be optimal solutions to (B-2). By the duality theorem

$$g' = x'b = h' = cy'$$

so that (B-3) implies

$$x'z' + w'y' = 0.$$

Since all vectors are non-negative

$$x'z' = 0 \tag{B-7}$$

and

$$w'y' = 0 \tag{B-8}$$

From (B-7) and non-negativity, we can derive the first complementary slackness conditions:

$$x'_i > 0 \text{ implies } z'_i = b_i - M^i y' = 0$$

Similarly, from (B-8) and non-negativity, we can derive the second complementary slackness conditions:

$$y'_j > 0 \text{ implies } w'_j = xM^j - c_j = 0.$$

These conditions can be strengthened to if, and only if, statements by going to the concept of central solutions to linear programming problems. Because of the similarity with the strong theorem of the alternative in matrix game theory, we refer for this to appendix A.

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INDEX

SAMENVATTING

Opties voor economische groei in Bangladesh: een toepassing van het Von Neumann model.

Met dit proefschrift staan ons twee doelstellingen voor ogen. In de eerste plaats willen we de economische groeimogelijkheden van Bangladesh analyseren. We willen dit doen met behulp van het Von Neumann groeimodel. Bij ons weten bestaan er geen serieuze empirische toepassingen van dit model, d.w.z. toepassingen op niet-vierkante matrices waarin samengestelde produktie is toegestaan. Een tweede doelstelling bestaat daarom uit een evaluatie van het Von Neumann model op zijn empirische bruikbaarheid.

De a priori aantrekkelijkheid van het Von Neumann model zit hem met name in de volgende eigenschappen:

(1) De nadruk in het model ligt vooral op de wijze waarop de economie van tijdstip 1 verbonden is met die van tijdstip 2.

(2) Het model beschrijft een situatie in evenwicht. Hiermee bedoelen we dat de volumebalansen kloppen en, tegelijkertijd, de gegenereerde prijsstructuur consistent is met rationeel gedrag.

(3) Het beschreven evenwicht is (semi-)dynamisch en de groei vindt gelijkmatig (uniform) over de sectoren plaats. De groei van de economie kan bovendien in een enkel getal worden samengevat.

(4) In tegenstelling tot de traditionele macro-economische groeimodellen staat het Von Neumann model een zeer grote mate van disaggregatie toe. Dit geldt zowel ten aanzien van de goederenclassificatie als ten aanzien van de onderscheiden produktietechnieken.

(5) Anders dan bij een dynamisch Leontief input-output model behoeven de matrices niet vierkant te zijn. Dat wil zeggen: het model staat competitie tussen verschillende technologieën die hetzelfde produkt voortbrengen toe. Bovendien is in een Von Neumann model samengestelde produktie toegestaan. Als gevolg van deze eigenschappen kan in het

model, in theorie althans, een schier onbeperkte hoeveelheid technische (ingenieurs-)informatie worden opgenomen.

(6) Wordt het Von Neumann model vergeleken met een multi-periode lineair programmeringsmodel dan vallen de volgende voordelen op: (i) Door de geringere omvang is het Von Neumann model veel beter hanteerbaar. (ii) De non-lineairiteit van de groeifaktor vermindert het risico van 'klapperen', met andere woorden, het risico op grote veranderingen in de uitkomsten als gevolg van marginale veranderingen in de modelcoefficiënten/parameters is kleiner. (iii) Het Von Neumann model genereert een endogene interestfactor. (iv) Het Von Neumann model beschrijft een situatie met een maximale evenwichtige groei; de uitkomst van een multi-periode lineair programmeringsmodel is, gegeven de enigszins arbitraire wijze waarvoor de doelstellingsfunctie moet worden geformuleerd, meer 'ad-hoc' van karakter.

In de studie wordt vertrokken van de modelformulering zoals deze in de literatuur wordt aangetroffen. We baseren ons hierbij op een uit 1956 daterend artikel van Kemeny, Morgenstern en Thompson [39]. In dit artikel wordt het oorspronkelijke model op een belangrijk punt geamendeerd. Daarnaast vindt een herformulering in spel-theoretische termen plaats.

De wijziging betreft de veronderstelling van Von Neumann dat ieder produkt dat tot de economie behoort, een rol speelt in alle produktieprocessen, hetzij als input, hetzij als output. Kemeny, Morgenstern en Thompson vervangen deze veronderstelling door de meer plausibele eisen dat (i) ieder proces minimaal een positieve hoeveelheid van één goed als input nodig heeft, en (ii) alle inputgoederen door de economie kunnen worden voortgebracht. De herformulering in spel-theoretische termen heeft als voordeel dat de wiskundige toegankelijkheid er door wordt vergroot.

Het model bestaat uit vijf axioma's die tezamen een economie in evenwicht beschrijven. Onder een axioma verstaan we een wiskundige formulering van een hypothese over de structuur van het economische systeem. De interne consistentie van de axioma's kan worden aangetoond met behulp van het existentiebewijs. Een reden waarom we hier nogal veel aandacht aan besteden, is dat zo'n bewijs veel leert over de wiskundige eigenschappen van het model. Bij de bestudering van oplossingstechnieken is grondige kennis van deze eigenschappen een belangrijke vereiste.

Tegen het Von Neumann model kunnen een aantal bezwaren worden aangevoerd. Hoewel die bezwaren in de theoretische specificatie van het model besloten liggen, werd de ernst van sommige ervan pas echt duidelijk bij de toepassing op Bangladesh. Op een aantal punten wordt het model daarom gewijzigd.

Een eerste wijziging betreft de introduktie van internationale handel. Vertrekpunt hier is het open model van Morgenstern en Thompson. Aan dit model blijken enige (kleine) bezwaren te kleven. En hoewel deze bezwaren door middel van kleine aanpassingen van het Morgenstern-Thompson open model zijn te ondervangen, kiezen we voor een enigszins andere modelformulering. Een belangrijke eigenschap van de alternatieve formulering is dat ze qua wiskundige structuur niet verschilt van het oorspronkelijke model. Alle resultaten met betrekking tot existentie en andere wiskundige eigenschappen blijven dus van toepassing.

Een tweede wijziging betreft de wijze waarop consumptie is opgenomen. In het oorspronkelijke Von Neumann model is consumptie een noodzakelijke input voor de produktie; verondersteld wordt dat alle produktie boven een biologisch minimum consumptieniveau geïnvesteerd wordt. In de literatuur zijn een aantal alternatieve formuleringen te vinden. De meest interessante is die van Morishima [63] waar de consumptieniveau's een functie zijn van de endogene prijzen. Omdat hiermee het Von Neumann raamwerk verlaten wordt, met alle nadelige gevolgen van dien, nemen we de formulering van Morishima niet over. In plaats daarvan wijzigen we het oorspronkelijke model op de volgende twee (kleine) punten: (i) het consumptieniveau mag afwijken van het biologische minimum en (ii) in de axioma's die de financiële rekeningen beschrijven wordt arbeid expliciet opgenomen. Het resulterende open consumptiemodel is qua mathematische structuur en eigenschappen nog volledig in overeenstemming met het oorspronkelijkez model. Om een drietal redenen is het in onze opvatting echter nog steeds niet echt geschikt als instrument om potentiële groei van een economie mee te analyseren. In de eerste plaats staat het model geen groei van de consumptie per hoofd, juist een belangrijk aspect van ieder economisch groeiproces, toe. Verder wordt de initiële samenstelling van de kapitaalgoederenvoorraad 'optimaal', dat wil zeggen in overeenstemming met 'turnpike-groei', verondersteld. Omdat de numerieke uitwerking van het model onder andere opleverde dat de Bangladeshe economie zich tamelijk ver van de turnpike bevindt, is dit een ernstige tekortkoming. Ten slotte is de Von Neumann veronderstelling dat grondstoffen en land in onbeperkte mate voorhanden zijn onacceptabel in een model van een grondstoffen- en land-arm land als Bangladesh.

Door het inbrengen van een les-vraagstelsel, een vector met de initiële kapitaalgoederenvoorraad, alsmede door land uit te drukken in termen van produktiecapaciteit wordt aan deze bezwaren tegemoetgekomen. Een belangrijk gevolg van deze wijzigingen is wel dat het model geen economie meer beschrijft waarin alle goederen met uniforme snelheid groeien. Afhankelijk van spaargedrag, inkomenselasticiteit, mogelijkheden voor afzet in het buitenland e.d. zal de groei van goed i boven dan wel onder het gemiddelde liggen.

Met de modelwijzigingen wordt ook een functionele inkomensverdeling geïntroduceerd. Bovendien lijkt het gewijzigde model interessante toepassingsmogelijkheden te bezitten als instrument voor 'technology assessment' op een meer macro-niveau.

Voor de modelbouw houden de wijzigingen in dat een aantal modelcoëfficiënten iedere periode aangepast dient te worden en dat, als gevolg hiervan, de modeloplossing slechts geldig is voor één periode.

Voordat een empirische uitwerking zin heeft, dienen we over een procedure (algoritme) te beschikken waarmee het model kan worden opgelost. Daarom worden bestaande algoritmes geïnventariseerd en beoordeeld. De beoordeling heeft betrekking op een eventuele bruikbaarheid bij de numerieke uitwerking van het model. Hierbij spelen twee eisen een rol. In de eerste plaats hoort het algoritme alle groeifactoren te vinden. In de tweede plaats dient de implementatie zo eenvoudig mogelijk te zijn en dient het geen problemen op te leveren bij toepassingen op iets grotere modellen.

Alleen de algoritmes van Weil en Thompson blijken aan de eerste eis te voldoen. Beide hebben echter nadelen bij een concrete toepassing op wat grotere modellen. Het algoritme van Weil heeft als nadelen dat de matrix M_α eerst ontleed moet worden en dat er relatief veel lineair programmeringsmodellen moeten worden opgelost alvorens er een groeifactor gevonden is. Deze laatste eigenschap maakt de toepassing van het algoritme op grotere modellen relatief duur. De ontledingsprocedure is niet nodig bij het algoritme van Thompson. Het centrale element in dit algoritme is de bepaling van de centrale oplossing van een lineair programmeringsmodel. Het vinden van zo'n centrale oplossing is echter, als het om iets grotere modellen gaat tenminste, allesbehalve een triviale aangelegenheid. Hiervoor zijn namelijk alle hoekoplossingen van het lineair programmeringsmodel nodig. Gegeven de numerieke onnauwkeurigheid waarmee lineair programmeringsoplossingen behept zijn, is de zoektocht naar alle hoekoplossingen, praktisch gesproken, een ondoenlijke zaak als het veelvuldige toepassingen op iets grotere modellen betreft.

Om die reden is een alternatief algoritme ontwikkeld dat niet aan de genoemde bezwaren lijdt. Het alternatieve algoritme is gebaseerd op een meer economische interpretatie van een bestaand theorema van Kemeny, Morgenstern en Thompson. Kern van het algoritme is dat met behulp van de modeloplossing α_1 vastgesteld wordt welke goederen en processen in het vervolg van de zoektocht naar groeifactoren geen rol zullen spelen. Door middel van 'straffen' en 'vrij maken' worden deze processen en goederen als het ware geëlimineerd.

Om de lezer die onbekend is met Bangladesh een referentiekader te verschaffen voor bij de beschrijving van de empirische uitwerking en de bespreking van de modelresultaten bevat de studie enige achtergrondinformatie over het land. Beknopt worden de geografie en demografie, de historie en, iets uitgebreider, de economie van het land besproken.

Bij de empirische uitwerking kunnen drie stappen worden onderscheiden: (i) de formulering van de modelstructuur, (ii) het verzamelen en organiseren van benodigde data, en (iii) de ontwikkeling en implementatie van de software.

Bij de concrete uitvoering van deze stappen is gebruik gemaakt van het vele werk dat met betrekking tot Bangladesh door de Stichting Onderzoek Wereldvoedselvoorziening (SOW) is verricht. Zo is bij de formulering van de modelstructuur vertrokken van een bestaand lineair programmeringsmodel van het land (BAM-lp). Verder kon vóor de benodigde data enerzijds geput worden uit de databank waarop de BAM-lp is gebaseerd en kon anderzijds uitgebreid gebruik worden gemaakt van de informatie uit het macro-model dat ten grondslag ligt aan het derde vijf-jaren plan voor Bangladesh (TFYP). Ook de gebruikte software is bijna in zijn geheel afkomstig van de SOW. De 'ontwikkeling en implementatie' beperkte zich tot het schrijven van een klein programma en het herschikken van aanwezige programmatuur.

Met het model zijn een aantal varianten doorgerekend. Bij de bepaling van de scenario's is in eerste instantie de discussie in de meer theoretische hoofdstukken gevolgd. We hadden hier twee redenen voor, een pragmatische en een logische. De pragmatische betreft het feit dat we geen enkele ervaring met het draaien van het model hadden. Er bestond om die reden een voorkeur om met een zo'n eenvoudig mogelijke modelspecificatie te beginnen en alle onevenwichtigheids-complicaties in eerste instantie buiten de deur te houden. Anderzijds vormden de uitkomsten van de zogenaamde 'rechttoe-rechtaan' toepassingen juist één van de belangrijkste motieven om een aantal van de besproken modelaanpassingen door te voeren.

De resultaten van de 'rechttoe-rechtaan' toepassingen zijn in de context van groeipotenties van de gehele economie niet bijster interessant. Toch leveren ze enige anderszins interessante conclusies op. In de volgende punten worden deze kort samengevat.

(i) De Bangladeshe economie beweegt zich op een grote afstand van de turnpike.

(ii) De zogeheten 'high yielding varieties' zijn economisch superieur aan de lokale variëteiten, zelfs als de landrestrictie niet geldt.

(iii) Bij een volledig vrij kapitaalverkeer loopt Bangladesh het gevaar dat de investeringen vooral in het buitenland zullen plaatsvinden. Binnenslands zal zich een 'duale' ontwikkeling voltrekken met enerzijds een klein snelgroeiend modern deel en anderzijds een groot stagnerend en deels 'slack' liggend traditioneel deel.

Vervolgens wordt het model gedraaid inclusief de voorgestelde wijzigingen. Dat wil zeggen, er wordt in het model rekening gehouden met de initiële kapitaalgoederenvoorraad en met spaar- en consumptiegedragsparameters die niet consistent zijn met uniforme groei. Met het model is een aantal scenario's doorgerekend, een basisscenario over een periode van tien jaar en een aantal varianten daarop die elk een kortere tijdsperiode omspannen. Het basisscenario kan worden beschouwd als een normatief groeiscenario voor Bangladesh voor de periode 1984/85-1994/95. Volgens dit scenario zijn groeipercentages van rond de 4 procent haalbaar. Omdat de vooronderstellingen die ten grondslag liggen aan de basisrun zo veel als mogelijk aansluiten bij die van TFYP en de laatste na intensief overleg tussen en met Bangladesh experts zijn vastgesteld, zijn de uitkomsten van de basisrun zeker niet onrealistisch. Hoewel de uitkomsten in het algemeen een relatief rooskleurig beeld laten zien, doemt er ook een tamelijk groot probleem op. Dat probleem onstaat op het moment dat Bangladesh het zelfvoorzieningsregime voor granen binnentreedt. Vanaf dat moment blijkt er een sterke terugval in de groei op te treden. De oorzaak hiervan is dat er voor de rijst- en tarweproducent een drastische verandering in prijsstructuur optreedt. De relevante prijs is niet meer gelijk aan de som wereldmarktprijs plus marge (transport, verliezen e.d.), doch ligt beduidend lager. Bij export zal ze gelijk zijn aan het verschil van die twee componenten. Omdat Bangladesh nogal geïsoleerd gelegen is ten opzichte van belangrijke overschot- en tekortgebieden (de handelsactiviteiten zijn slechts marginaal op het naburige Calcutta gericht) terwijl bovendien de binnenlandse infrastructuur betrekkelijk slecht ontwikkeld is, zijn de marges nogal groot en dientengevolge het prijseffect bij een regimes-verandering.

Een nadere analyse naar gevolgen van de regimes-verandering leert dat te veel nadruk op de rijst- en tarwesector in een wereld met relatief lage prijzen bepaald niet zonder gevaar is. Een drastische terugval in de groei en een afvloeiing van de produktiviteitswinst naar de niet-landbouwsector blijken de belangrijkste gevolgen. Worden de boeren daarentegen met hoge externe prijzen geconfronteerd, bij voorbeeld als gevolg van een verbeterde infrastructuur, een kortere handelsweg (levering aan Calcutta) en/of hogere wereldmarktprijzen, dan kunnen de hoge groeicijfers voortduren. Omdat de produktiecapaciteit in die situatie sneller groeit dan het arbeidsaanbod, kunnen ook de loonarbeiders hiervan profiteren.

Naast scenario's die vooral gericht zijn op de ontwikkeling van de landbouw, zijn er ook enige scenario's gedraaid waarbij de nadruk vooral op de niet-landbouwsector ligt. Optimistischer veronderstellingen met betrekking tot produktiviteitstoenames blijken al snel een significant effect te hebben op de groei van de gehele economie. Ook hier komt duidelijk naar voren hoe belangrijk een snelle groei van de produktiecapaciteit is voor een betere beloning van de factor arbeid.

In de niet-landbouwsector treden nauwelijks regimes-veranderingen op. Omdat het vermoeden gewettigd lijkt, dat dit meer het gevolg is van het nogal hoge aggregatieniveau van deze sector dan van het feit dat regimes-veranderingen niet van belang zouden zijn voor de niet-landbouw, is een scenario met een enigszins kunstmatige regimes-verandering voor textiel gedraaid. Wederom blijkt hoe moeilijk het voor Bangladesh is een exportregime binnen te treden. Volgens de uitkomsten kan in dit geval de binnenlandse marge prohibitief zijn. De conclusie dringt zich dan ook op dat een industriebeleid gericht op kleinschalige dorpsprojecten tot grote problemen kan leiden indien een land meer dan zelfvoorzienend dreigt te worden.

CURRICULUM VITAE

Herman Stolwijk werd op 6 november 1948 te Leiden geboren. Na, in Boskoop, achtereenvolgens de Christelijke Lagere Tuinbouwschool, de Rijks Middelbare Tuinbouwschool, en de Rijks Hogere School voor Tuin- en Landschapsinrichting te hebben doorlopen, werd in 1973 de studie Economie aan de Landbouwhogeschool te Wageningen aangevangen. In 1977 werd met lof het doctoraal diploma behaald, met als hoofdvakken de Algemene Agrarische Economie en de Wiskundige Statistiek, en als bijvak de Agrarische Bedrijfseconomie.

In mei 1977 trad hij in dienst bij de Stichting Onderzoek Wereldvoedselvoorziening (SOW) te Amsterdam. Hier werkte hij mee aan de constructie van modellen voor Thailand en Bangladesh. Dit proefschrift kan worden beschouwd als een 'bij-produkt' van de modelwerkzaamheden aan Bangladesh.

In 1985 werd de SOW verruild voor het Centraal Planbureau te Den Haag. Als hoofd van de afdeling Landbouw en Agrarische Grondstoffen is hij hier verantwoordelijk voor ramingen en andere onderzoekswerkzaamheden op het gebied van de Nederlandse landbouw en de voedingsmiddelen- en de dranken- en tabaksproduktenindustrie.

STELLINGEN

1. In tegenstelling tot het optimaliseringsmodel dat de WRR gebruikt om de middellange termijn groeimogelijkheden van de Nederlandse economie te onderzoeken, heeft het Von Neumann model de volgende eigenschappen:

   (i) De relaties binnen het model hebben een duidelijke economische, technologische en/of institutionele betekenis. Er verschijnen geen niet of moeilijk te interpreteren ad hoc restricties als 'dei ex machina' op het (model-)toneel.

   (ii) Het model staat de specificatie van meerdere produktietechnieken toe. Binnen de projectieperiode kunnen endogene wisselingen van produktietechniek optreden.

   (iii) Het niveau van een (produktie-)activiteit in de modeluitkomsten wordt bepaald door haar relatieve economische aantrekkelijkheid; en niet door de bijdrage die geleverd wordt aan een doelfunctie die een zeer ongeloofwaardig gedrag van de achterliggende economische agenten impliceert.

   (iv) Het handelsregime van een goed (export, import, autarkie) is endogeen en kan binnen de projectieperiode wisselen.

   Onder andere om deze redenen verdient het Von Neumann model meer aandacht als instrument waarmee de middellange termijn groeimogelijkheden van een economie kunnen worden geanalyseerd.

   WRR (1987): 'Ruimte voor groei'. Rapport 29, Staatsuitgeverij, 's-Gravenhage.
   Dit proefschrift.

2. De 'revolutie' die de hypothese van de rationele verwachtingen in de macro-economie heeft teweeg gebracht, doet nogal onevenwichtig aan als men beziet met welk een gemak er binnen diezelfde macro-economie geabstraheerd wordt van allerlei zeer relevante kenmerken van de economische werkelijkheid. Het feit dat er binnen de macro-economische modelbouw überhaupt geen sprake is van expliciet gedefinieerde economische agenten is in dit verband wel het meest in het oog springend.

   David K.H. Begg (1984): 'The rational expectations revolution in macro-economics'. Philip Allan Publishers, Oxford.

3. De door Veerman gebruikte argumenten in zijn kritiek op de 'gangbare (landbouw-)economische theorie' zijn deels onjuist; in zoverre ze juist zijn, zijn ze niet bruikbaar en bovendien inconsistent met zijn conclusies. De door hem bepleite 'grondige bezinning op de uitgangspunten van de landbouweconomie' kan daarom maar beter achterwege blijven.

   C.P. Veerman (1987): 'Over landbouweconomie en landbouwpolitiek'. Inaugurele rede, KUB, Tilburg.

4. De economische wetenschap wordt gekenmerkt door de (bijna) onmogelijkheid tot het doen van experimenten onder gecontroleerde externe omstandigheden en door een in het algemeen zeer matige kwaliteit van het beschikbare datamateriaal. Bij discriminatie en beoordeling van model en modeluitkomsten dienen daarom modelconsistentie en theoretische onderbouwing een belangrijker rol te spelen dan t-waarden, r-kwadraten en ex-post voorspelkracht.

5. Het ene algemene evenwichtsmodel is het andere niet.

6. Met behulp van een serieus empirisch onderbouwd algemeen evenwichtsmodel zou de discussie over het in Nederland te voeren sociaal-economische beleid op een hoger plan kunnen worden gebracht.

7. De bewering van Rutten dat de door hem geformuleerde 'vaste vuistregels' meer houvast bieden voor de algemeen-economische politiek dan 'modelresultaten', is vooral interessant als illustratie van het gemak waarmee men binnen de economische wetenschap politieke knollen voor economische citroenen kan verkopen.

   F.W. Rutten (1987): 'Economische wetenschap en economisch beleid'. Economenblad, jaargang 9, nr. 6.

8. Argumenten die pleiten voor een vrij internationaal verkeer van goederen en kapitaal zijn evenzeer van toepassing op een vrij internationaal verkeer van arbeid. Het is een bedenkelijke vorm van opportunisme dat met betrekking tot arbeid de vrijhandelsargumenten zo goed als volledig genegeerd worden.

9. Het EG-landbouwbeleid stimuleert de consumptie van plantaardige vetten ten koste van die van dierlijke. Het positieve effect hiervan op de volksgezondheid speelt in de huidige discussie over dit beleid ten onrechte geen rol van betekenis.

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in Bangladesh
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