The use of time declining discount rates in climate change projects

Student: Phan Dang Thu Ha
Study programme: Environmental Sciences
Registration number: 871019651060

Thesis Environmental Economics and Natural Resources
ENR 80436

Supervisors:
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Abstract

This thesis studies the issue of time declining discounting in climate change projects evaluation in two different contexts: deterministic and uncertain world. First, an overview of standard discounting is introduced in the second chapter to help explaining why this approach is not appropriate in long term project evaluation. It then comes to the central focus on the concept of time declining discounting, which is sequentially presented in chapter 3 and chapter 4 by investigating current literature relating to the identified topic. In chapter 3, we rationalize the use of time declining discounting in a deterministic world in the light of two main theories developed by Sterner (1994), and Weitzman (1994). Although the two approaches are different from each other, they both consider the environmental aspects as factors making the consumption discount rates declining over time. In chapter 4, we investigate time declining discounting in case of an uncertain world with the focus on uncertain discount rates and uncertain economic growth. Assuming the discount rate is uncertain, we adopt the theory studied by Weitzman (1998), and Newell and Pizer (2003) to explain why discount rates could be declining over time. When the future economic growth is unknown from today's perspective, we employ Gollier's (2002) theory about precautionary effect to legitimate the decline of consumption discount rates over time. Chapter 5 presents the application of time declining discounting in climate change policy and its implications. Chapter 6 concludes.

Key words: standard discounting, time declining discounting, inter-generational projects, deterministic world, uncertainty, consumption discount rates.

Chapter 1 Introduction

1.1. Background on current approaches in discounting

Experts in project appraisal surely are very familiar with the concept of discounting when practicing Cost Benefit Analysis (CBA). Since "the evaluation by CBA is very sensitive to the discount rate" (Jouini et al., 2010, pp.831), the issue of how to choose a sensible discount rate in project evaluation has always been controversial. Over the past few years, great debates have erupted over discounting, stimulated by the economic analysis of climate change (Hepburn, 2009). There is indeed a general agreement in the essential of using discounting in economics but a disagreement in the choice of discounting methods, e.g. constant or time declining. According to Weitzman (2001, pp.261), "the discount-rate problem is rooted in fundamental differences of opinion, which are unlikely to go away soon". Different points of view have been raised not only between economists and non-economists but also amongst economists (Pearce et al., 2003; Dasgupta, 2008).

Regarding the use of a time invariant discount rate (called standard discounting), it has been critically discussed whether the rate should be formulated endogenously based on theoretical foundation or be exogenously taken from the observed market. New discounting approaches have been developed as well. For instance, Broome (1992) and Dasgupta et al. (1999) criticize the use of positive discount rates and call for the use of a zero rate. Henderson and Bateman (1995), Laibson (1997), Cropper et al. (1998), and Farmer and Geanakoplos (2009) suggest using hyperbolic discounting to convert future value to present one based on observations of individual choice, namely Behavioural Economics. Reaching the same conclusion recommending the use of time variant discount rate, Weitzman (1998, 2001), Gollier (2002) and Newell & Pizer (2003) however have their argumentation rooted in the aspect of uncertainty. In the meantime Sterner (1994), Weitzman (1994), and Groom et al. (2005) have proved that time declining discounting is still plausible even in a deterministic world.

As can be seen, till now decision makers have in their hands various means to deal with discounting but which one is the most suitable and is largely advocated by CBA experts is still an open question. Philibert (2003) for example criticizes that the use of a high discount rate will lead to a "play" against the environment and future generations while using a low one implies more sacrifices for present generations, although future generations may be richer. When working on climate change projects, decision makers have to face an unavoidable trade-off between paying the costs of mitigation today or doing nothing now and losing the income in future (due to climate change consequences). Both decisions lead to a loss in income but at different points in time, i.e. today or in the future.

1.2 Problem definition

Since the 1950's there have been serious discussions about what should be the appropriate discount rate applied in public investments, especially when it relates to inter-generational issues (Arrow, 1995). For short term projects, using a constant positive discount rate is reasonable since it effectively reflects individual preference for inter-temporal consumption as well as risk aversion. However whether the rate is endogenously derived from the Ramsey formula or exogenously taken from the market interest rate is still in debate. Things change problematically when we move to the case of long term project appraisal. Because applying a constant and positive discount rate can make far distant future values become negligible (Broome, 1992; Harvey, 1994; Cropper and Laibson, 1998; Dasgupta et al., 1999). According to Perman et al. (2003), the value of a discount rate is very important when dealing with inter-temporal projects as using an incorrect rate to discount future value can rigorously affect the outcomes of decisions made today.

In case of intra-generational or short-term projects, discounting using a positive constant rate seems reasonable as it reflects the marginal rate of return to capital in investments. However, the problem becomes visible once ones move to projects spreading across generations. As stated by Atkinson (2009, pp.1) "Nowhere are the theoretical and empirical challenges of discounting more evident than in the context of public policies with very long-run consequences, such as climate change". Only a small difference in the choice of discount rates can make a great differentiation in the outcomes of policy assessment. So what are the reasons making such a problem?

First, long term projects or climate change policies "have a distant future feature" as their time horizon is much longer than conventional ones (Pearce et al., 2003). For example, the costs of climate change are expected to occur over two or three centuries, even to indefinite future (Arrow, 1995, Gollier, 2002a). As reported by Conceição et al. (2007, pp.1) greenhouse gases (GHGs) emission is an irreversible process of which "today's emission will stay in the atmosphere for centuries". The second important feature of climate change projects results from the first one that the long term horizon leads to great uncertainty in future. As choosing discount rate inherently depends on the state of the economy, such uncertainties will considerably drive the discount rate (Weitzman, 1998, Weitzman, 2001; Gollier, 2002a; Newell and Pizer, 2003). For instance, the benefit of climate change projects is predicted to be visible after two or three centuries though the cost incurs today. Since today people are uncertain about the way our far distant descendants value future commodities and amenities, discounting future values will become very complicated (Dasgupta et al., 1999).

Given the inappropriateness of standard discounting in inter-temporal valuation, this thesis specifically focuses on the rationale of time declining discounting and the applicability of this innovative approach. Since time declining discounting is considered a solution to the problem of standard discounting, there is therefore a need for having a systematic study about such issue, in terms of theoretical background and the numeric values of recommended discount rates as well.

1.3 Thesis objectives and research questions

Given the problem identified in previous sessions, this research aims to study the rationale of using *time declining discount rates* (hereafter DDRs) in climate change project[1] evaluation. Currently time declining discounting approaches are deeply studied both in theoretical basis and in numeric values of the discount rate. In this thesis, the overall goal is to help strengthening the theory of discounting and the precision of today's decision making as well.

Key objectives of this research are:

(i) To rationalize DDRs in a *deterministic* world

(ii) To rationalize DDRs in an *uncertain* world w.r.t uncertainty in discount rates and in economic growth

(iii) To illustrate the implications of DDRs in climate change policies by using the DICE model of Nordhaus, 2007

Based on these three main objectives, research questions and sub-questions are formulated as followed:

1. What is the rationale of standard discounting?
    - 1.1. What are the reasons for discounting?
    - 1.2. Prescriptive or Descriptive approach?
    - 1.3. Why standard discounting is no longer appropriate to climate change projects?
2. Why could the consumption discount rate be declining over time in a *deterministic* world?
    - 2.1. How does the change in consumption growth affect the social discount rate?
    - 2.2. How does the increase in environmental investment affect the social discount rate?
3. How does *uncertainty* have effect on long-term discount rates?
    - 3.1. How does uncertainty in discount rates lead to DDRs?
    - 3.2. How does uncertainty in economic growth rates lead to DDRs?
4. What are the DDRs schemes used for application? What are the implications of applying DDRs in climate change policy?

1.4 Methodology

Given the objectives and research questions mentioned above, this thesis employs two methods, namely *literature study* and *environmental economic modelling*. *Literature study* is needed because it helps gaining fundamental insights about the nature of discounting, especially the underlying meanings of different discounting approaches. For instance, to answer question 1, a brief overview of discounting concept is synthesized from current discussions of discounting, especially in the context of climate change. To deal with questions 2, publications relating to time declining discounting in *a deterministic world* are researched. The study "*Discounting in a world of limited growth*" by Sterner (1994) supports sub-question 2.1. Weitzman also contributes one study about that issue, known as "*On the "Environmental" Discount Rate*" published in 1994. Questions 3.1 about the effect of *uncertain discount*

[1] *Projects mentioned are small scaled and at a national level. The issues of international cooperative projects with large scale are beyond the scope of this thesis.*

rates will be answered by investigating articles of Weitzman (1998, 2001) and Newell & Pizer (2003), entitled "*Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate*", "*Gamma discounting*", and "*Discounting the distant future: how much do uncertain rates increase valuations?*", respectively. Similarly, the relation between *uncertainty in economic growth* and social discount rates (question 3.2) is examined by surveying one publication of (Gollier, 2002a), "*Discounting an uncertain future"*. Paper "*Declining Discount Rates: The Long and the Short of it*" by Groom et al. (2005) supports both the two main research questions (2 and 3).

With regards to question 4, *environmental economic modelling*, namely the DICE model by Nordhaus (2007) is applied. This model is required since it helps providing the future values of climate change damages cost from which present values are obtained by using discounting. To clearly illustrate and visualize the effect of DDRs in outcome's implications, different discounting schemes are employed in this session.

1.4 Road map of the thesis

After the chapter of Introduction, the main content of this thesis will be structured as followed:

Chapter 2 Rationale of standard discounting

Chapter 3 Time declining discount rates used in long term projects in a deterministic world

Chapter 4 Time declining discount rates used in long term projects in an uncertain world

Chapter 5 Applying time declining discount rates based on the DICE model

Chapter 6 Conclusions

Chapter 2 Rationale of standard discounting

2.1 Overview of discounting

2.1.1 What is discounting?

Discounting is used in economic theory in order to convert future value to present value, hence making them comparable. In economic analysis, discounting is considered one of the most crucial steps in CBA (Boardman et al., 2010). According to Gollier and Weitzman (2010, pp.1) "the concept of discounting is central to economics, since it allows effects occurring at different future times to be compared by converting each future dollar into a common currency of equivalent present dollars". The word "discounting" itself implies that future value is attached less weight or 'cheaper' than the present one. How much future values are cheaper then present ones depend greatly on a concept called "discount rates".

There are two types of discount rates, namely *utility discount rate* and *consumption discount rate* which respectively correspond to *time discounting* and *goods discounting* (Nordhaus, 1993). Time discounting represents the way people discount future well-being (utility) with "ρ" is the utility discount rate or the pure rate of time preference. Goods discounting relates to the trade-off of having consumption of goods and services today or making investment to have more consumption in future, reflected by the consumption discount rate "r". As noted by Groom et al. (2005, pp.448) "the utility discount rate [...] is the appropriate discount rate for costs and benefits that are measured in utility" while consumption discount rate is used "when costs and benefits are measured in consumption equivalents". The relation between utility discounting and goods discounting is well described in the Ramsey formula[2]. In reality, analysts often use consumption discount rate or "social discount rate" in project appraisal as it relates directly to changes in consumption of goods and services. Standard discounting refers to the use of a single constant consumption discount rate to convert future values to present values.

2.1.2 Why discounting?

Two main streams of reasons for discounting have been discussed, namely the *time preference* and the *wealth effect*. The first argumentation is based on the observation that humans are impatient and are uncertain about life chance so they prefer enjoying their utility in the present rather than in the future. In other words, there is an opportunity cost of postponing consumption today to the future. The second reason is based on the expectation that future generations will be much richer than today's ones. Thanks to technological progress and productive capital accumulation, future people will have chance to enjoy a higher level of consumption. Therefore, rising consumption provides a second justification for discounting future costs and benefits at a positive rate (Dasgupta, 2008). If the utility function is concave

[2] *This issue will be discussed in detail in "2.2.1. Prescriptive approach and the Ramsey formula"*

with respect to (hereafter w.r.t) consumption and future people are really better off than current people then utility will increase together with consumption but with a diminishing rate. Simply put, an increment of consumption is less important to a rich person than to a poor person (Heal, 2009). So even if we are unwilling to discount future values, the wealth effect still rationalizes the act of discounting (Portney and Weyant, 1999).

2.1.3 Discounting in a two-period model

Consider a two-period model, where period 0 denotes present and period 1 denotes the future. The utility level in period 0 and 1 are U_0 and U_1, respectively. Utility is a function of consumption and is time invariant. The inter-temporal social welfare is a function of utility:

$$W = W(U_0, U_1) \tag{2.1}$$

Social welfare is presented by an inter-temporal sum of utility of a representative agent and is assumed to be time-separable. Hence equation (1) can also be written as:

$$W = \varphi_0 U_0 + \varphi_1 U_1 \tag{2.2}$$

where φ_0 and φ_1 are the utility discount factors, i.e. weights attached to the utility in each period and W is the sum of discounted utility (Groom et al., 2005).

Since people have pure rate of time preference, set $\varphi_0 = 1$ and $\varphi_1 = \frac{1}{1+\rho}$ where ρ is the *utility discount rate*. Equation (2.2) becomes:

$$W(U_0, U_1) = U_0 + \frac{1}{1+\rho} U_1 \tag{2.3}$$

The *consumption discount rate* is defined as "the rate at which the value of a small increment of consumption falls as its date of receive is delayed" (Perman et al., 2003, Weikard and Zhu, 2005). This rate can be written as (Perman et al., 2003):

$$r = -\frac{dC_1}{dC_0} - 1 \tag{2.4}$$

where C_0 and C_1 denote consumption levels in period 0 and 1, respectively.

Put another way, the consumption discount rate equals the slope of welfare indifferent curve, i.e. marginal rate of utility substitution (MRUS), minus 1. The slope of welfare indifferent curve or the MRUS measures the rate at which one is indifferent between substituting today consumption for future consumption while holding the level of inter-temporal welfare unchanged (Perman et al., 2003; Boardman et al., 2010)

Apply the consumption discount rate to calculate the present value of future net benefits of projects, we obtain:

in a two period model:

$$NPV = B_0 + \frac{B_1}{1+r}$$

in a discrete multi-period model:

$NPV = \sum_0^t D_t B_t$ (2.5)

where NPV is the Net Present Value,

D_t is the consumption discount factor, i.e. $D_t = \frac{1}{(1+r)^t}$, (2.6)

B_t is the net benefits at time t,

r is the (constant) consumption discount rate.

2.2 Prescriptive or Descriptive approach?

These two basic approaches in determining discount rates were first named and distinguished by Arrow et al. in an annual report of the Intergovernmental Panel of Climate Change (IPCC), 1995. The *prescriptive* approach is based on "ethical principles" about the way we 'should' weigh the well-being of different generations, motivated by the question *"How (ethically) should impacts on future generations be valued?"*. The *descriptive* approach is based on observations from market interest rates, beginning with the question *"What choices involving trade-offs across time do people actually make?"*. Apparently, using the prescriptive approach will result in a lower rate than in descriptive one (Arrow, 1999). In a perfect economy without tax, transaction costs and externalities the two rates derived from two approaches are identical (Dasgupta et al., 1999; Perman et al., 2003; Philibert, 2003).

2.2.1 Prescriptive approach and the Ramsey formula

The most fundamentally theoretical basis of discounting in Prescriptive approach is the Ramsey formula. By applying the optimal growth model[3], it is shown that in a deterministic world with a stationary population and a perfectly functioning market, the consumption discount rate is defined as:

$r = \rho + \eta g$ (2.7)

where r is the consumption discount rate, representing the social rate of return to capital,

ρ is the utility discount rate or pure rate of time preference,

η is the elasticity of marginal utility w.r.t. consumption, a measure of relative effect of a change in consumption on welfare. It is the rate at which marginal utility falls as consumption rises (Heal, 2009)), defined as:

$\eta = -\frac{(\partial^2 U/\partial C^2)C}{(\partial U/\partial C)} = -\frac{U''C}{U'}$ and $\eta > 0$ (2.8)

g is the percentage consumption growth rate (measured by average growth rate of national income, i.e. GDP.), defined as:

$g = \frac{\partial C}{\partial t}/C$ and $g > 0$ (2.9)

Equation (2.7) shows that the consumption discount rate equals the pure rate of time preference plus the elasticity of marginal utility multiplied by the per capita consumption growth rate. As can be realized, using the optimal growth model means that social discount rate is viewed as "an endogenous

[3] *for more detail, see Annex 1*

outcome of a dynamic general equilibrium model" (Newell and Pizer, 2003, pp.55). Also "the Ramsey formula has been derived under the assumption of homogeneous agents" (Jouini et al., 2010) without considering the differentiation between the rich and the poor.

The Ramsey formula explicitly sets out the two main reasons of discounting, of which 'ρ' captures the human impatience or "myopia" while 'ηg' represents the belief that future consumers will be better off than today's consumers (Arrow et al., 1996b). Put differently, the first component of the RHS[4] in equation (2.7) corresponds to the *impatience effect* while the second one stands for *the wealth effect*, referred as "time discounting" and "growth discounting" respectively.

Notably, there are some interrelated conditions under which a positive and constant consumption discount rate (r) is derived from the Ramsey formula:

- Utility discount rate (ρ) is constant over time.
- Consumption is expected to increase ($\frac{\partial C}{\partial t} > 0$) and growth rate of consumption stays unchanged in time, i.e. $g = const$.
- Agents are assumed to be risk averse in terms of consumption, captured by the notion of *Arrow-Pratt relative risk aversion (θ)*. In the Ramsey formula, this concept is mathematically equal to η. Since η is defined to be constant (elasticity of marginal utility is independent of consumption), agents are treated as if they display constant relative risk aversion (CRRA[5]). See box 1, pp.9 for detail on the underlying implications of η.
- Utility function is concave w.r.t consumption, i.e. $\frac{\partial U}{\partial C} > 0$ and $\frac{\partial^2 U}{\partial C^2} < 0$

It has been proven that both power and logarithmic utility function satisfy the condition of CRRA. Utility function is given as below:

$$U(C) = \begin{cases} \frac{C^{1-\theta}}{1-\theta} \text{ for } \theta > 0 \text{ and } \theta \neq 1 & \text{(power function)} \\ \ln C \text{ for } \theta = 1 & \text{(logarithmic function)} \end{cases}$$

- Utility function is time additively separable, i.e. $U(C_1, C_2) = U(C_1) + U(C_2)$

The Ramsey formula does function very well in distinguishing between time discounting and goods discounting (Nordhaus, 1993). However, determining the value of these parameters is not a simple task and turns out to be controversial in practice. Since the values of ρ, η and g are not readily available, "the value to be assigned to them is purely an ethical question" (Perman et al., 2003).

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[4] *Right hand side*

[5] *Given θ is the Arrow- Pratt relative risk aversion, CRRA means $\frac{\partial \theta}{\partial C} = 0$ or $\theta = -\frac{U''C}{U'} = const$*

Decreasing (or increasing) relative risk aversion, i.e. DRRA (or IRRA) means $\frac{\partial \theta}{\partial C} < 0$ (or $\frac{\partial \theta}{\partial C} > 0$)

For constant (decreasing or increasing) relative risk averse function, the optimal proportion of wealth invested in risky assets would also be constant (increasing or decreasing) as his wealth increases (Liu, 1988, pp.78).

First, plugging a numerical value to ρ is truly a hard judgement as "its choice is a decision on the relative weights of different generations of human beings" (Heal, 2009, pp.280). For instance, Cline (1992), and Heal (2009) argue that pure rate of time preferences should be equal to zero because a positive utility discount rate "is ethically indefensible" and "difficult to see any reason for valuing future people differently from present people just because of their futurity". In contrast, Arrow (1995, pp.7) believes "a zero time preference implies unacceptably large saving rates" and suggested to use ρ at 1%. Nordhaus (1994) even uses a 3% utility discount rate in his paper "*Managing the global commons: The economics of climate change*".

Moreover, having ρ equals zero does not mean consumption discount rate is equal to zero too. There is still *the wealth effect* in equation (2.7), i.e. ηg. As mentioned by Philibert (2003), "the productive nature of the economy legitimates discounting" and makes consumption discount rate positive. Approximating the elasticity of marginal utility gives η ranging from 1 to 2 while the consumption growth rate g is estimated to be at 4% per annum, which is likely the case in developed countries. Some economists even reason out the circumstance when consumption discount rate could be negative as a result of declining consumption. According to them, there exist environmental limits to growth that would cause the collapse of the world economic system (Perman et al., 2003). Under such circumstances, having negative economic growth rates is possible. Dasgupta et al. (1999) stress that unless economic activities cause no harms to the environment, the social discount rate could be non-negative.

Criticism of Prescriptive approach

First, as discounting is based on the assumption that future people will be better off than today's people, it turns out to be irrelevant if they are in fact not going to be better off. The reason might be due to climate change's consequences in the future (Cline, 1999; Philibert, 2003).

Secondly, using 'prescriptive' discount rates might lead to economic inefficiency if it prescribes lower discount rates for climate-change investments compared to those in conventional investments (Nordhaus, 1997). Because "in doing so we would be forgoing better alternative investments" (Arrow et al., 1996, pp.132).

Box 1. ***The triple role of elasticity of marginal utility of consumption (η)***

According to Conceição et al. (2007) and Atkinson et al. (2009), the elasticity of marginal utility of consumption simultaneously reflects preferences for:

(i) Risk aversion towards consumption fluctuations in future

(ii) Inter-generational (or time) inequality aversion, and

(iii) Intra-generational (or spatial) inequality aversion

Regarding the first meaning, the level of η determines the attitude of today's people respective to changes in their future consumption. A high value of η means "individual is not willing to allow his consumption to vary over time" (Conceição et al., 2007, pp.24). In contrast, a low η implies the fact that people are more tolerant to changes in temporal consumption. Besides, the parameter $1/\eta$ also represents the "elasticity of marginal substitution of intern-temporal consumption". A higher η and thus a lower $1/\eta$ means people are less willing to substitute their today consumption for future.

The second role of η reflects the level at which today's people are concerned about the unevenly distribution of consumption between today and in the future. It is rooted from a general idea that people in future will be much better off than today ones, so it is unfair if present poorer people have to sacrifice their welfare for future richer ones. A high η means people care a lot about such issue and according to their opinion, a same level of consumption is worthier to them than to future generations.

The last concept of η relates to the preferences of people in terms of inequality in consumption distribution between the rich and the poor at the same point in time.

Disentangle the three concepts of η, the case of climate change

According to Atkinson et al. (2009), the correlations between attitudes towards risk, intra-generational and inter- generational inequality are weak. Take climate change policy as the case to illustrate that finding. Since impacts of climate change are predicted to be severe and have most of its effect on poor countries, calling for urgent actions to deal with this problem is being made all around the world. As a result, using a lower social discount rate to estimate climate change's damages is in need. Given ρ and g unchanged, a lower discount rate implies lower value of η. However, decreasing η has "possibly divergent effects" (Atkinson et al., 2009, pp.4).

From the perspective of *inter-generational inequality aversion*, a lower η means present people care less about such an inequality and think more for future generations, despite the fact that they are the ones who pay the cost of taking action today while future richer descendants will benefit from that. However the same argument does not hold in the light of risk aversion and intra-generational inequality aversion. Think about *risk aversion*, a lower η implies today's people care less about risk in their consumption, they are more risky. If so, why should they spend lots of their money in mitigating climate change's impacts? Regarding the last term, if urgent actions are to be made right now, it should imply a higher level of aversion to *intra-generational inequality* w.r.t consumption. As "climate change is predicted to inflict far more damage to the tropics (the poor world) than to the temperate zone (the rich world) […] urgent action is needed to avoid the increase in inequality between the poor and the rich" (Conceição et al., 2007, pp.18). In that sense, η should be higher.

2.2.2 Descriptive approach

This second approach attains discount rates by observing economic behaviour of people at real market. The way people discount future value is represented by the marginal rate of time preference (*MRTP*) - the rate that people are willing to trade present consumption for future consumption (Harrison, 2010). For instance, if you are indifferent between having $100 today and $110 next year, your *MRTP* will be 10% per annum. In the economy, this rate is well reflected by the market interest rates through which borrowing and lending are undertaken.

There are two types of market interest rates with respect to consumers and investors (Broome, 1992; Arrow et al., 1996b; Harrison, 2010). Investor interest rates refer to the private rate of transformation between investment today and in the future, also known as the private rate of return on capital, denoted by δ. The investor interest rate captures the effect of resource allocation between public and private projects in the sense that public projects poses the opportunity cost of forgone benefits gained by private ones. Consumer interest rates are reflected by the after-tax rate of return to savings, denoted by 'i'. In case of perfect market, the market interest rate offered to consumers and investors will be the same and equal to the social rate of return to capital, i.e. "$\delta = i = r$" (Harrison, 2010).

Criticism of descriptive approach

The strength of this approach is that the rates are observed in real market hence are able to reflect the economic behaviour of people. However, it exhibits several weaknesses as well**.** First, this approach only reflects the preferences of present individuals but not the future ones (Broome, 1992; Boardman et al., 2010). Put differently, the differentiation between intra-generational and inter-generational perspective makes discounting problematic when projects spread over long term periods. Even in terms of intra-generation, Dasgupta et al. (1999) raise the question whether individual preferences can be correctly generalized to public preferences.

Secondly it turns out that people's actual time preferences appear inconsistent, in terms of both time and choices. For instance, many people may lend and borrow money at the same time with different rates of interest or gains are more discounted than losses (Laibson, 1997; Boardman et al., 2010). By observing individual economic behaviour, Laibson (1997) and Cropper et al. (1998) have shown that the discount rate people assigned to future costs and benefits seems declining as time goes further, with larger discount rates to near term returns than to returns in the distant future. So using a unique constant interest rate to discount values at different points in time is not reasonable in that sense.

Thirdly, the market interest rates offered to consumers and investors are not the same in reality. In an imperfect economy with cooperative tax and income tax, the rates levied on these two agents will be diverged from each other and normally "when an investor borrows money from banks, the interest rate he has to pay is higher than the interest rate received by a consumer who lends money" (Broome, 1992). For instance, suppose the consumer's market interest rate is 6% then an investor will decide to invest his money in a project only if he yields a rate of return to investment at least at 6% from that project. However the income tax levied on him is 25% and the cooperative tax levied on the firm is 50%. So that the project must yield an after-tax rate of return to capital at 8%[6], and at 16%[7] before-tax which is clearly higher than the consumer's market interest rate.

2.2.3 When are the discount rates obtained from two approaches (not) equivalent?

In a world without market imperfections like tax, externalities, and transaction costs one would write $\delta = i = r$, which means the private rate of return to capital, the consumer market interest rate and the social rate of return to capital are equivalent to each other. When government policies are imperfect, however, a "wedge" between "δ" and "r" appears, making the first rate higher than the second one (Dasgupta, 2001). Since private projects often ignore "*externalities*" or "*public bad*" caused by their economic activities, they tend to have a higher rate of return to capital than the social one (Dasgupta et al., 1999). Determination of the appropriate discount rate in the presence of market distortion is a complicated and controversial issue reflecting diverging views on the role of public and private investments in the economy (Philibert, 2003). Seen from the perspective of private firms, society might be under-investing when applying prescriptive approach because the rates associated tend to be smaller than the descriptive ones, i.e. "$r < \delta$". So a public project with a rate of return to capital higher than "r" can still be qualified although its rate is lower than private's one.

2.3 Current use of discount rates in practice

"Current discounting practices in governments vary enormously", say Boardman et al. (2010, pp.268). In research by Zerbe and Dively (1990), it is revealed that only 43 percent of U.S municipalities use discounting in evaluating projects. The rates vary from country to country and from department to department. Discount rates in developing countries are higher than in developed ones, 8-12% and 3-7% respectively (Harrison, 2010). It is because the economy in those poorer countries is expected to grow more rapidly in the coming decades than in richer ones (Schelling, 1999). For different economic sectors, the U.S Environmental Protection Agency proposed to use a specific discount rate in environmental projects, say 2-3% which is significantly lower than the rate applied to conventional projects, i.e. 7%.

Recently, economists have devoted considerable time and effort to discuss three influential papers of three well-known experts, i.e. Cline (1992), Nordhaus (1994) and Stern et al. (2006). In these articles, the discount rates derived from the Ramsey formula were used to calculate the costs and benefits of CO_2 emission reduction on a global scale. Amongst the three authors, Stern's rate is the lowest, i.e. 1.4 percent

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[6] *as* $6\% * \frac{100\%}{(100\%-25\%)} = 8\%$

[7] *as* $8\% * \frac{100\%}{(100\%-50\%)} = 16\%$

while Nordhaus uses a relatively high discount rate at 4.3 percent, see Table 1. The implications drawn from their results hence are not the same.

"…Cline (1992) and Stern (2006) have recommended that the world spends substantial sums today to tame climate change, while Nordhaus (1994) has recommended a far more gradualist investment policy – (Dasgupta, 2008, pp. 11,12)"

Table 1. Discount rates used in Cline (1992), Nordhaus (2004) and Stern et al. (2006) (Dasgupta, 2008)

Author	**Social discount rate ($r = \rho + \eta g$)**	**Utility discount rate (ρ)**	**Elasticity of marginal utility (η)**	**Consumption's growth rate (g)**
Cline 1992	**2.05%**	**0.0%**	1.5	~ 1.37%
Nordhaus 1994	**4.3%**	**3.0%**	1.0	1.3%
Stern 2006	**1.4%**	**0.1%**	1.0	1.3%

As can be seen from the table, the utility discount rate employed by Nordhaus (1994) is significantly higher than the ones used by Cline (1992) and Stern et al. (2006).

Noticeably, there has been an emergence of using non-constant discount rates. Gollier (2009) notes that "there is a tendency among decision makers to choose a discount rate for the very long term that is smaller than the one used to discount cash flows occurring in the short term". In the 2003 Green Book[8], the British Treasury recommends using sliding scale discount rates with the rates declining by periods. Based on the prescriptive approach, they come up with the rates ranging from 3.5% to 1% as time goes by (see Table 2). In the short term (0-30 years) the values for ρ, η and g recommended by the Treasury's Green Book are: ρ = 1.5; η = 1; g = 2 (Metroeconomica, 2004). Applying the Ramsey formula, i.e. equation (2.7), a social discount rate of 3.5 percent is derived. Similarly, since 2005, the French public institutions are required to use a 4% rate per year to discount cash flows up to 30 years, and to use a 2% rate for longer horizons (Gollier, 2009).

Table 2. Discount rates used by the UK Government (Guo et al., 2006)

Period of years	0-30	31-75	76-125	126-200	201-300	301+
Discount rate	3.5%	3.0%	2.5%	2.0%	1.5%	1.0%

2.4 Standard discounting and inter-temporal projects

Although economists are in a general agreement that standard CBA, including discounting is useful in appraising different projects, it does matter when applied in (extremely) long-term projects

[8] *The Green Book is an official guidance to Ministries on the appraisal of investments and policies.*

(Arrow et al., 1996b). Increasing effect of exponential discounting over time is the main reason making standard discounting inappropriate. When applying the standard discounting to inter-generational projects, "it turns out that the deep future part of the project didn't much matter" and "what happens a few centuries from now hardly counts at all" (Weizman, 1999, pp.23). Gollier (2002b, pp.464) stresses that "discounting far distant costs and benefits at the same rate for the shorter terms is equivalent to ignoring these long-term effects". Because of the very long time involved in climate change policies, the choice of a discount rate powerfully affects the net present value of alternative policies, and thus recommendations that emerge from climate change analysis (Arrow et al., 1996b). For example, at a discount rate of 7% a damage cost of $1 billion will have a present value of $1.15 million[9] if it occurs after 100 years and $1,300 after 200 years.

As mentioned above, one of the two main reasons to discount future value is because people in the future are assumed to be richer than us. However that might not hold in case of climate change (Philibert, 2003). The impact of climate change is predicted to be enormous, long lasting, at global scale and its distribution across time and space is unevenly (Conceição et al., 2007). Poor countries are foreseen the most vulnerable to climate change consequences because of their low coping capacity and adaptive capacity. And – "given the depth of the North South divide, developing country people in the future may well be still poorer than current developed country people", says Philibert (2003, pp.4). Therefore, the wealth effect is no longer a plausible reason for discounting.

Regarding the use of a constant discount rate, some economists have questioned the use of a fixed positive rate of return on capital in inter-temporal discounting. For example, Cline (1999) suspects the credibility of an inter-generational promise that each generation will transfer part of its wealth to the followings with a constant rate in order to tackle climate change's impacts. Arrow et al. (1996) hold the same point of view that the commitment of a fund for future greenhouse victims is simply not credible.

[9] *Apply equation (2.6) $1.15 million is equivalent to* $\frac{10^9}{(1+0.07)^{100}}$

Chapter 3 Time declining discount rates used in long term projects in a deterministic world

This chapter presents argumentations in favour of using time declining discounting (DDRs) given a deterministic world. The word "deterministic" refers to a situation where the future socio-economic prospect of the world is projected. Regarding the issue of discounting under such circumstance, there are number of deterministic situations in which the discount rate will be declining by time (Groom et al., 2005). Within the scope of this thesis, two economic theories relating to DDRs in a deterministic world are discussed. Interestingly, these two discounting approaches were parallel developed in the same year, 1994. The first is about the relation between long term discount rates and the change in consumption growth rate (g). The second one developed by Weitzman (1994) approaches that issue by integrating the idea of environmental concern into discounting theory. Following sub-chapters will give more explanations.

3.1 Time declining discounting in relation with consumption growth rate

To examine the changing pattern of the social discount rate over time, we take the first derivative of equation (2.7) with respect to time t, and obtain:

$$\dot{r} = \dot{p} + \dot{\eta}g + \eta\dot{g}$$

It is assumed that the pure rate of time preference (ρ) and elasticity of marginal utility (η) are constant over time, $\dot{p} = 0$ and $\dot{\eta} = 0$. Then we reach:

$$\dot{r} = \eta\dot{g} \tag{3.1}$$

Equation (3.1) founds the key argumentation favouring the use of declining discount rates as it describes cases where $\dot{r}$ is negative. It is clearly seen that under the given assumption $\dot{r} < 0$ if and only if $\dot{g} < 0$. In words, *decreasing in the growth rate of consumption will lead to decline in discount rates*.

3.1.1 Logistic economic growth and time declining discount rates by Sterner (1994)

Sterner (1994) studies the issue of DDRs by employing a theory called "*The logistic economic growth*". The idea of logistic economic growth is based on the theory of logistically biological growth, that there are limitations prohibiting the infinite evolution of ecosystems (Sterner, 1994; Weitzman, 1999; Perman et al., 2003). They include diseases, lack of food and habitat, predation etc. Economic development is not an exception since there are also several factors limiting its expansion like population growth, natural resources scarcity and environmental crisis, international conflicts and wars. However it is normally the prospective scenario viewed by ecologists. Economists, to some certain extent, appear to be more optimistic as they believe substitution and technological progress will help humans to come over those kinds of constraints on economic development, as argued by Weizman (1999). Under that circumstance, one should question such economists about the substitutability of essential natural resources since some economic activities cannot be maintained in the absence of them.

Back to Sterner's study (1994), this author bases his argumentations on the opinion of ecologists and applies the theory of limited biological growth to the situation of long term economic development. According to him, given that the economy has a logistic growth, consumption will increase over time to a finite upper bound and stay fixed at that point. The level at which consumption becomes steady is defined as the "final equilibrium consumption level" or the *carrying capacity of consumption*. Growth of consumption in that case is "density-dependent" (Perman et al., 2003, pp.558). Interestingly enough, it recalls the concept of the carrying capacity of ecosystems in light of biology.

The consumption growth function (in continuous time) then is defined as:

$$\frac{\partial C}{\partial t} = \dot{C} = \beta\left(1 - \frac{C}{C_0}\right)C \qquad (3.2)$$

where $\beta > 0$ is the intrinsic growth rate of consumption, $\beta = const$

C_0 is the carrying capacity of consumption.

According to equation (2.9), the growth rate of consumption (g) is written as:

$$g = \frac{\dot{C}}{C} = \beta - \frac{\beta}{C_0}C \qquad (3.3)$$

The carrying capacity of consumption, C_0 is determined as the level at which consumption becomes stationary over time. At $C = C_0$, the growth rate (g) is equal to zero and the consumption discount rate is equal to the utility discount rate.

Taking the first derivative of equation (3.3) w.r.t time, we have:

$$\dot{g} = -\frac{\beta}{C_0}\dot{C} \qquad (3.4)$$

It can be seen from equation (3.4) that *if consumption increases over time* ($\dot{C} > 0$) *its growth rate will accordingly be declining* ($\dot{g} < 0$).

3.1.2 Declining economic growth rate as a result of climate change

Another school of thought supporting the case of decreasing economic growth rate ($\dot{g} < 0$) is based on political point of view. That economic development might be slowed down due to rise in significant investments in climate change mitigation and adaptation. Once climate change is surely proven to cause severe impacts on human society, society might establish more stringent and urgent actions to deal with climate change consequences. As the (financial) resources used for such public investments are taken from both private and public sectors, reduction in consumption will come about as a result. Apparently, in order to "feed" those costly and long term projects society needs to increase national saving rate. Using economic based instruments is one of the most common methods. For instance, society might be able to drive the consumption rate to be lower by proposing a higher tax system (Nordhaus, 1999). However it appears to be more relevant in the context of developed countries rather than in poor ones where affording a great budget for environmental projects is hardly affordable.

The argumentation mentioned above justifies DDRs as a result of responsibly taking action today to reduce climate change consequences in the future. However, situations with DDRs are still likely even with worse results if present people do nothing. Dasgupta (2001) argues that as growing consumption provides a plausible reason why we should use positive discount rates, declining consumption due to climate change impacts might legitimate the use of negative discount rates. To illustrate, Dasgupta (2008) supposes that global warming's consequences will lead to a negative consumption growth rate at -1% per year. Plug $\rho = 0$ (Ramsey's opinion), $\eta = 1.5$, and $g = -1\%$ into equation (2.7) we have $r = -1.5\%$ per year, which is negative.

3.2. Environmental discount rates by Weitzman (1994)

3.2.1 Environmental expenditure as a component of production output

According to Weitzman (1994), concerns for the environment nowadays have become greater. To motivate this assumption, it is argued that as income becomes higher, people will care more about the environment by investing more in environmental protection. The reason motivating higher income people to hold a higher priority to the environment is "as levels of income rise, environmental effects become increasingly more important" (Weitzman, 1994, pp.200). Weitzman's argumentation is similar to a renowned theory about the relation between economic growth (expressed in income) and environmental consequences, namely "the environmental Kuznets curve (EKC)", see Box 2.

Box 2. A glance at the environmental Kuznets curve

The EKC graphically illustrates the relationship between income and environmental damages as an inverted U-shape. At the beginning of economic development, there is a positive correlation between income (in terms of GDP) and environmental impacts in which increases in income will raise the level of environmental degradation due to exploitation of natural resources and pollutant emissions. However, after reaching a certain high level of income (called the turning point), income continue increasing but environmental problems decline. This can be explained that after the turning point, production of leading industrial sectors becomes cleaner, valuation of people about the environment is higher, and institutions of environmental regulation become more effective (Dasgupta et al., 2002). The hypothesis appears to behave well in the context of short term environmental problems with local scale while climate change is however a global problem with long term and intergenerational consequences (Arrow et al., 1996a).

Environmental expenditure is introduced by Weitzman as a component in the function of production output, together with investment and consumption. In this case, the social discount rate derived is called "*the environmental social discount rate*"[10]. As reported by Weitzman, spending on environmental projects

[10] *In reality, there is a wedge between social and private rate of return on capital due to the generating of externalities like environmental damages as a by-product of economic activities.*

are now considerably increasing and taking up part of economic output (expressed in national income, GDP). Therefore, the production output function is formulated as below:

$$Y(t) = I(t) + C(t) + \psi(t) \tag{3.5}$$

where $Y(t)$ is output of economic activities, and is a also a function of capital $K(t){:}Y(t) = F(K(t))$

$I(t)$ is investment,

$C(t)$ is conventional consumption,

$\psi(t)$ is environmental expenditure.

Equation (3.5) can be interpreted as output can be used for investment, consumption or environmental expenditure.

3.2.2 The (environmental) social discount rate

Environmental damage like pollution, degradation is considered a side-effect of economic development. To lessen the impacts of environmental damages, ones should invest more in environmental improvement as compensation to nature (Weitzman, 1994). Therefore, the output function is captured not only by conventional consumption and investment but also in environment spending, see equation (3.5).

(Environmental) social discount rate is smaller than private discount rate

Conceptually, the private discount rate is represented by the marginal productivity of capital, written as:

$$i = \frac{\partial Y}{\partial K}$$

of which the RHS of equation implies marginal output w.r.t capital or marginal rate of return to capital.

As mentioned before, externalities cause a wedge between private rate of return to capital and the social one. Hence, in the presence of environmental damage which is undoubtedly a negative externality, the social rate is diverged from the private rate as below:

$$r = \frac{\partial Y}{\partial K} - \psi_Y \frac{\partial Y}{\partial K} = i(1 - \psi_Y) \tag{3.6}$$

where $\psi_Y = \frac{\partial \psi}{\partial Y}$ is the marginal environmental investment w.r.t output and $\psi_Y > 0$ by definition, i.e. expenditure on the environment increases together with growth in output.

In that sense, ψ_Y is defined as "the proportion of which private rate of return should be diminished" to assure that the level of environmental damage is kept at a constantly sustainable level (Weitzman, 1994, pp.206). The underlying implication of equation (3.9) is the actual marginal rate of return to capital equals the private rate of return less the rate of increase in environmental expenditure to offset the environmental impacts of increased economic activities. Since ψ_Y is positive, the social rate is undoubtedly smaller than the private rate. Introducing the concept of environmental expenditure into the output function helps lowering the social discount rate.

(Environmental) social discount rate is declining over time

The relation between environmental damages and environmental expenditure is captured by Weitzman as follow:

$$\frac{D}{Y} = G\left(\frac{\psi}{Y}\right) \tag{3.7}$$

where D is the level of environmental damage.

Equation (3.7) shows that level of environmental damages per unit of output (LHS) is a function of level of environmental expenditure per unit of output (RHS).

The task assigned in this case is given the fluctuation in output (Y), one needs to keep D at a constantly sustainable level, says $\overline{D}$. To reach the goal, it is required that any damages to the environment (D) must be offset by an increase in environmental expenditure (ψ).

With $D = \overline{D}$, equation (3.7) equals:

$$\overline{D} = Y\, G\left(\frac{\psi}{Y}\right) \tag{3.8}$$

Assume that environmental expenditure is a function of output, i.e. $\psi = \psi(Y)$, take the first derivatives of equation (3.8) we obtain:

$$\frac{\partial \overline{D}}{\partial Y} = 0 = G\left(\frac{\psi}{Y}\right) + Y\, G_{\frac{\psi}{Y}}\left(\frac{\psi_Y}{Y} - \frac{\psi}{Y^2}\right) \tag{3.9}$$

where $G_{\frac{\psi}{Y}}$ is the first derivative of G w.r.t $\frac{\psi}{Y}$, written in short as G'.

It is assumed that $G' < 0$, which implies that as expenditure in environmental protection increases, environmental damage will be reduced.

From equation (3.9), the marginal environmental expenditure w.r.t output (ψ_Y) can be written as:

$$\psi_Y = \frac{\psi}{Y} - \frac{G}{G'} \tag{3.10}$$

Define $Z = \frac{\psi}{Y} > 0$ representing the fraction of output (i.e. expressed in % of total income) spent on environmental improvement, and

$E = -Z\frac{G'}{G}$ and $E > 0$ as the elasticity (measured in percentage) of environmental improvement w.r.t environmental expenditure.

Given Z = 1% then $E = -\frac{G'}{G}$ which is the rate of reduction in environmental damage due to increase in environmental investment. As explained by Weitzman (1994, pp.203), "*E* represents the percentage by which degradation declines due to a 1% increase in environmental spending". The higher is the value of *E*, the easier in reducing environmental damage.

Substitute *E* and *Z* into the RHS of equation (3.10), we have:

$$\psi_Y = Z + \frac{Z}{E} = Z\left(1 + \frac{1}{E}\right) \tag{3.11}$$

Plug the RHS of equation (3.11) into the RHS of equation (3.6), we obtain:

$$r = i\left(1 - Z\left(1 + \frac{1}{E}\right)\right) \quad (3.12)$$

Notably, r becomes non-positive as ψ_Y is equal to or greater than 1. Having $r \leq 0$ implies two extremes. First it might be the case that society over-care about the environment then output is spent mostly on environmental protection (Z ≈ 100%). Under such circumstance, environmental goods would become extremely luxurious. Another explanation is based on the hypothesis that environmental catastrophes occur then all output is used for tackling the impacts. A similar argumentation holds if elasticity of environmental improvement, E, is extraordinarily small or it is very difficult to reduce the consequences of environmental damages.

Investigating the changing pattern of r depending on E and Z

First, we look at the case where *Z* is constant over time, say 2% of the national income. Then the social rate will be driven only by the elasticity *E*. If the elasticity is relatively high (which means it is easy and effective to improve the environment), *r* will be closer to the private rate. In contrast, a low value of *E* indicates the difficulty in "cleaning up" the environment probably due to the irreversibility of the impacts. In that sense, there will be a larger gap between *r* and *i*.

The next case examines the situation where *E* is unchanged while *Z* is increasing over time. As noted by Weitzman, this setting is more realistic than the first one mentioned above. It can be seen from equation (3.12) that given a constant *E,* an increase in environmental spending (*Z*) will result in a decline in social discount rates. In his research, Weitzman has studied historical data on environmental expenditure of the U.S to examine the inter-temporal development of Z. As can be seen from Figure 1, from 1972 to 2000, the cost of paying for pollution control in the U.S has risen gradually. However Weitzman's assumption appears to be not applicable in developing countries where output from economic development will be mostly spent on alleviating poverty and heightening up their living standard. Calling for using a great part of their hard-earn money on environmental protection is still a far-away mission in such countries.

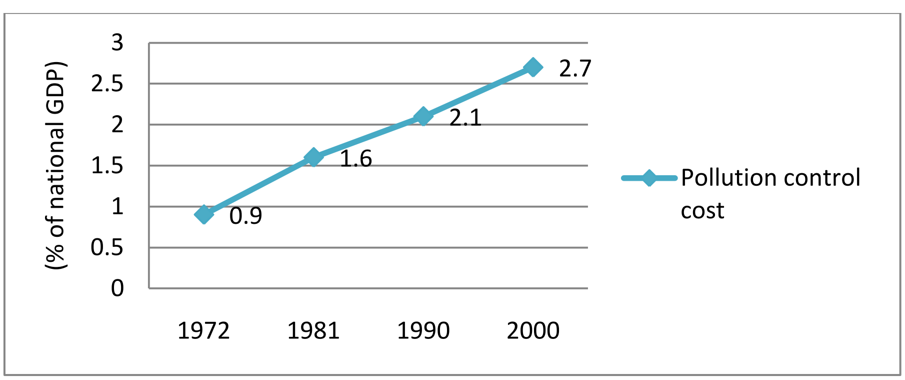

Figure 1. Rise in U.S pollution control cost from 1972 to 2000. Data source: (Weitzman, 1994)

3.3 Concluding remarks

So far we have discussed about the rationale of time declining discounting in case of a deterministic world. It is justified by three main theories: logistic economic growth by Sterner (1994), increasing environmental concern by Weitzman (1994), and climate change effects on consumption growth rate. Sterner (1994) treats economic growth as if it has the same characteristics like ecosystems, i.e. there are inherent constraints on evolution. According to him, it is the carrying capacity of consumption that lessens the growth rate (g) hence making the social discount rate decline as time goes further.

Weitzman (1994) bases his argumentation on the idea that in the presence of negative externalities like environmental damages, there is a gap between private and social rate of return to capital. If society spends more money on environmental improvement, the social discount rate will be declining as a result.

The third school of thought is rooted in the prediction of climate change impacts. That regardless of the way society reacts to climate change (taking action or doing nothing), it is likely that climate change will induce a reduction in consumption growth due to its disastrous and long term impacts. However, effects of decline in economic growth under "doing nothing" option seem to be more serious and long-lasting than in case of "taking action".

Chapter 4 Time declining discount rates used in long term projects in an uncertain world

This chapter investigates the relationship between time declining discounting and uncertainty. Two studied aspects of uncertainty are uncertain discount rates and uncertain economic growth. Given the small scale of projects, the effect of uncertainty on the discount rates is considered significant. Pizer (1999) stresses that "analyses which ignore uncertainty can lead to inefficient policy recommendations". Indeed, uncertainties in far distant future have been viewed as a problem making prediction of the discount rates more difficult. "The farther we look into the future, the greater is the inherent uncertainty as to the future growth rate of the economy and future market of interest" (Boardman et al., 2010, pp.263). As noted by Gollier and Weitzman (2010) there is enormous uncertainty and controversy about choosing an appropriate rate of return for discounting distant-future events, like climate change. According to them, this issue is a fundamental point in applying CBA in case of inter-generational projects. A survey by Bazelon and Smetters (2001) has shown that uncertainties in the future are currently studied from three different dimensions relating to (i) the discount rate itself (Weitzman; Newell & Pizer), (ii) to future economic growth (Gollier) and (iii) to the life chance (Kula). Since Kula approaches the issue of uncertainty from a different aspect, only the first two types of uncertainty are examined, namely uncertain discount rates and uncertain economic growth. Gollier (2002a) considers these two ways are complementary and both lead to time declining discount rate. The following sup-chapters will help clarifying that conclusion by providing an inclusive understanding of these studies.

4.1 The effect of uncertain future discount rates on discounting

In this part, we sequentially study the two well-known approaches capturing the effect of uncertainty in discount rates, developed by Weitzman (1998) and Newell & Pizer (2003). Both come up with a conclusion of time declining discount rate though Weitzman suggests a lower initial rate and a faster falling rate than Newell & Pizer (Boardman et al., 2010).

4.1.1 Theoretical basis in the two studies

Averaged discount rate or averaged discount factor?

Given the discount factor function w.r.t discount rate $\left(D_t = \frac{1}{(1+r)^t}\right)$, we should notice that the relation between these two concepts is not linear hence averaging the discount rate does not bring the same results as doing so to the discount factor. Since future consumption values are directly discounted by the consumption discount factor, Weitzman (1998) and Newell and Pizer (2003) argue that "the variable should be possibility-averaged over various uncertain states of the world is not the discount rates *r* but the discount factors D_t". To illustrate this statement, let's consider a simple example of determining the *NPV* of $1,000 after t = 200 years, with p_1 = 50% chance of having consumption discount rate r_1 at 1% and p_2 = 50% chance of r_2 at 7%.

Commonly, the first approach is to compute the *averaged (expected) consumption discount rate* as $\bar{r} = p_1 r_1 + p_2 r_2 = 4\%$. Apply equation (2.6) we obtain the corresponding discount factor D_t which is equal to 3.92 x 10^{-4}. Plugging D_t = 3.92 x 10^{-4}, B_t = \$1,000 into equation (2.5) gives us the *NPV* after 200 years equalling to 40 cents, which counts almost nothing.

However, there is another way to compute the *NPV* by averaging the discount factor D_t instead of the discount rate *r*. Apply equation (2.6):

with $r_1 = 1\%$, we obtain $D_1 = 0.14$, $p_1 = 50\%$.

with $r_2 = 7\%$, we obtain D_2 = 1.33 x 10^{-6}, $p_2 = 50\%$.

The *averaged (expected) consumption discount factor* then is: $\bar{D} = p_1 D_1 + p_2 D_2 = 0.07$. The corresponding discount rate *r* is 1.34%. It is clear that the effective discount rate in this case is lower than the 4% rate derived from the first approach. Weitzman (1998) and Boardman et. al (2010) suggest that the discount rate obtained from averaging discount factors is the accurate rate that CBA analysts should use. Apply equation (2.5), the *NPV* will be: $NPV_{(2)}$ = \$70 which is 175 times higher than the $NPV_{(1)}$ (40cents). The example given above can be graphically demonstrated as in Figure 2.

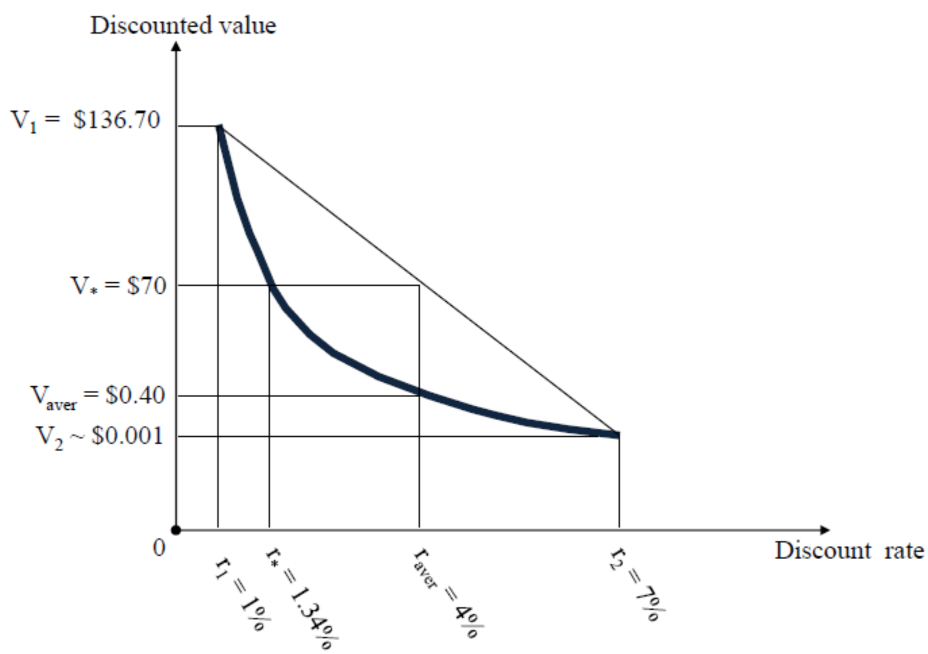

Figure 2. The relation between discount rate and discounted value

As can be seen from the figure, discounted value is a convex function of discount rate (Newell and Pizer, 2003). As a result, using the averaged expected discount rate (r_{aver} = 4%) results in a lower present value than using the rate obtained from averaging the discount factor (r_* = 1.34%). Since r_* = 1.34% is closer to r_1 = 1% "the expected discount factor corresponds to the minimum discount rate having any positive possibility" (Weizman, 1999, pp.29).

Risk-neutral agents and effective social discount rates

We first make a short tour around the issue of risk preferences. An agent's attitude toward risk is characterized by his or her preference between a risky consumption, say a lottery ticket and a deterministic bundle of consumption which brings the same utility compared to the expected utility of the risky one. A risk-averse agent would rather have the deterministic bundle of consumption rather than the lottery ticket. In contrast, a risk lover would like the lottery ticket better. A risk neutral agent would be indifferent between the two options. In terms of utility function, an agent is risk averse (neutral/lover) if his or her utility function is concave (linear/convex) (Perman et al., 2003).

Usually people are assumed to be risk averse, which is proven by the fact that people do buy insurance (Ostaszewski and Binmore, 1993). However, in Weitzman's and Newell & Pizer's studies, agents are assumed to be risk-neutral. Put differently, these authors are not concerned about the issue of risk preferences in their study (Gollier, 2002a) and there is no cost of risk bearing to risk neutral individuals (Perman et al., 2003).

Since agents are risk-neutral, their utility function is linear w.r.t consumption level and the certainty equivalent value is equal to the expected value (Perman et al., 2003). Applying this implication to the case of discounting means the *CEDF* (*Certainty Equivalent Discount Factor*) is identical to the *EDF* (*Expected Discount Factor*). The effective social discount rate at each point of time therefore is the *CEDR* (*Certainty Equivalent Discount Rate*) which will be derived from the *CEDF*.

In (Weitzman, 1998), the *CEDF* at time *t* is defined as:

$$CEDF = EDF = A(t) = \sum_0^i p_i a_i(t) = E[exp(-\int_0^t r_i(s)ds)] \qquad (4.1)$$

where $A(t)$ is the CEDF at time t

r_i is the discount rate in scenario i

p_i is the possibility of having discount rate r_i at time t

a_i is the corresponding discount factor in scenario i

In (Newell and Pizer, 2003), the *CEDF* at time *t* is defined as:

$$CEDF = EDF = A(t) = E[exp(-\sum_1^t r_t)] \qquad (4.2)$$

The *CEDR* is defined as:

$$R(t) = -\frac{\dot{A}(t)}{A(t)} \qquad (4.3)$$

The *CEDR* is instantaneous in the sense that it measures the period-to-period rate of change of the discount factor (Groom et al. 2005). Notably, both Weitzman and Newell & Pizer use the concept of the *CEDR* defined in equation (4.3) to form the fundamental argumentations in their research.

4.1.2 Gamma discounting by Weitzman (1998)

According to Weitzman (1998) there are important reasons leading to uncertainty in the discount rate, categorized as economic, environmental, technological and social factors. From this point of view, uncertainty relates to capital accumulation, the state of the environment, the pace of technical progress, and states of international relation. All of these factors result in unpredictable discount rates. Since time horizon is a factor making the magnitude of uncertainty greater, Weitzman's research objective is to reveal the relation between time horizon and uncertain discount rates.

In Weitzman's study, the world's timeline is viewed as two periods, called "near future" and "distant future" of which the discount rate is known in the first period and unknown in the second one. Weitzman particularly shows his interest in the distant future context where people are uncertain about the discount rates. In terms of near future period, it is assumed that "discount rates in the short-term should be equal to the present interest rates".

It is presumed that there are different possible scenarios capturing the state of our world in the future, ranging from 1, 2, 3…, *i*; of which each scenario is assigned a possibility to become true p_i ($p_i > 0$ and $\sum p_i = 1$). Each scenario has a predicted discount rate at different point of time, denoted by $r_i(t)$ - the (instantaneous) discount rate of scenario *i* at time *t*. As noted by Newell and Pizer (2003) "here, uncertainty represents a current lack of consensus about the correct discount rate for all future time periods".

The rate of discounting in this case is fixed but uncertain[11]. The consumption discount rate is established as below:

when t is small, $r_i(t)$ is very close to today's interest rate,

when t goes to infinity, $r_i(t)$ approaches a limiting rate denoted by r_i^*:

$$\lim_{t\to\infty} r_i(t) = r_i^* \qquad (4.4)$$

The corresponding discount factor at time *t* is denoted by $a_i(t)$. The *CEDR* is defined as *R(t)* in equation (4.3). Plug $A(t) = \sum_0^i p_i a_i(t)$ into equation (4.3) , *R(t)* becomes:

$$R(t) = \frac{\sum p_i r_i(t) a_i(t)}{\sum p_i a_i(t)} = \sum w_i(t) r_i(t) \qquad (4.5)$$

where $w_i(t)$ is the weight attached to the discount rate $r_i(t)$ of each scenario, defined as:

$$\begin{cases} w_i(t) = \dfrac{p_i a_i(t)}{\sum p_i a_i(t)} \\ \sum_0^i w_i(t) = 1 \end{cases}$$

[11] *One should keep in mind that once the real discount rate is revealed, the rate therefore will be kept constant for ever (Gollier, 2002a).*

Taking the first derivative of *R(t)* w.r.t. time, we obtain:

$$\frac{\partial R(t)}{\partial t} = \dot{R}(t) = -\sum w_i(t)(r_i - R(t))^2 \qquad (4.6)$$

which is clearly negative.

Equation (4.6) shows us the first important finding of Weitzman: *"the CEDR is declining over time"*.

The following part will help determining how far the *CEDR* will decline to.

Apply equation (4.4), the *CEDR* in far distant future is expressed as:

$$\lim_{t\to\infty} R(t) = R^* \qquad (4.7)$$

Plug *R(t)* built in equation (4.5) into the LHS equation (4.7), we have:

$$\lim_{t\to\infty} R(t) = \lim_{t\to\infty}(\sum w_i(t)r_i(t)) = \lim_{t\to\infty}(w_1(t)r_1(t)) + \lim_{t\to\infty}(w_2(t)r_2(t)) + \dots + \lim_{t\to\infty}(w_i(t)r_i(t)) \qquad (4.8)$$

Suppose that scenario 1 (i = 1) has the lowest possible discount rate when $t \rightarrow \infty$, hence:

$$\min\{r_i^*\} = r_1^* = \lim_{t\to\infty} r_1(t) \qquad (4.9)$$

Since: $$\begin{cases} \lim_{t\to\infty} \frac{w_i(t)}{w_1(t)} = 0 \\ \lim_{t\to\infty} w_1(t) = 1 \end{cases} \longrightarrow \lim_{t\to\infty} w_i(t) = 0 \text{ for } i \neq 1 \qquad (4.10)$$

Plug the results derived in (4.9) and (4.10) into the LHS of equation (4.8) we see that:

$$\lim_{t\to\infty} R(t) = \lim_{t\to\infty}(w_1(t)r_1(t)) = \lim_{t\to\infty} w_1(t) \text{ x } \lim_{t\to\infty} r_1(t) = r_1^* \qquad (4.11)$$

Equation (4.11) tells us that *"as time goes to infinity, the effective discount rate will decrease to the minimum possible rate"*.

Recommended discount rates

The findings of Weitzman's research is developed further in (Weitzman, 2001). In order to establish the probability distribution of discount rate, Weitzman conducted two surveys[12] asking respondents to choose only one possible constant discount rate used in climate change CBA. The first one was sent to over 2,000 professional Ph.D level economists while the second one was aimed to survey opinions of 50 leading experts in discounting.

The key results of the surveys are: (i) the answers of respondents about their estimated interest rate exhibited a gamma probability distribution and (ii) interestingly enough, the results obtained from two different surveys were close to each other. The mean of the gamma distribution is α = 4% and the standard deviation is β = 3%.

[12] *In Weitzman (2001) the operative part of the questionnaire asked the subjects to reply with their "professionally considered gut feeling" to the following re-quest: Taking all relevant considerations into account, what real interest rate do you think should be used to discount over time the (expected) benefits and (expected) costs of projects being proposed to mitigate the possible effects of global climate change?*

The *CEDR* is given as:

$$r(t) = \frac{\alpha}{1+t\frac{\beta^2}{\alpha}} \tag{4.12}$$

Plugging α = 4% and β = 3% into equation (4.12) we obtain:

$$r(t)= \frac{4}{100+2.25t} \tag{4.13}$$

Based on equation (4.13), Weitzman provides a set of specific recommended discount rates ranging from 4% to 0% respective to different time horizons of projects, see Table 3.

Table 3. Weitzman's recommended discount rates (Bazelon and Smetters, 2001)

Time Period (years)	**Recommended discount rate[13] (*r*)**	**Growth rate of consumption (*g*)**
1 to 5	4%	2.67
6 to 25	3%	2.00
26 to 75	2%	1.33
76 to 300	1%	0.67
More than 300	0%	0.00

Pitfalls of the approach

As mentioned in Weitzman's article, the discount rates become uncertain in the distant future. However how should we define "the distant future" is not clearly given by Weitzman. According to Groom et al. (2005), a case could be made for having constant present discount rate up to 30-40 years which is equivalent to the maturity of government bonds. Beyond that length of time, the future rates will become unknown.

4.1.3 Modelling the interest rate's behaviour by Newell and Pizer (2003)

While Weitzman believes that uncertainty is a result of a current lack of consensus about the correct discount rate, Newell & Pizer (2003, pp.54) assume that "there is a reasonable consensus about the correct discount rate today based on market rates, but this rate is likely to change over time". Motivated by the idea that past discount rate are well connected to future ones, these two authors study historical data of the U.S interest rates in order to forecast the changing pattern of future discount rates. In their paper, they employ the assumption that "the discount rates are *uncertain* and *highly persistent"* and prove this argumentation by investigating historical data. These economists believe that uncertainty in the discount rate begins immediately in present, which is different from Weitzman's opinion.

[13] *given that* ρ = 0; η = 1.5

In their research, Newell and Pizer use modelling to deduce the time series behaviour of discount rates. The objective is to "provide a transparent connection between historical data and forecast values" (Newell and Pizer, 2003, pp.56). Since the discount rate is uncertain but persistent, its two components are determined as "a permanent" and "a random" component, denoted by η and ε respectively:

$$r_t = \eta + \varepsilon_t \quad (4.14)$$

both η and ε_t are uncertain.

The meaning of these two components is that they exhibit the main characteristics of the discount rates: uncertain and changing over time but in a persistent manner. The permanent component η has a normal distribution with mean $\bar{\eta}$ and variance σ_η^2 , of which the variance measures the degree of uncertain in the mean $\bar{\eta}$.

The random component ε_t is related to shocks to the discount rate, with mean $\overline{\varepsilon_t} =$ zero and is defined as:

$$\varepsilon_t = p\, \varepsilon_{t-1} + \xi_t \quad (4.15)$$

where ξ_t is the deviation from $\bar{\eta}$. ξ_t has a normal distribution with mean zero and variance σ_ξ^2

p describes the persistence degree of deviations from $\bar{\eta}$ and $p \in (0,1)$. $p = 1$ means "the interest rate can persistently deviate from the mean rate, staying consistently above or below it for many periods" while $p = 0$ means "a period of abnormally high interest rates may be followed by rates above or below the mean with equal probability" (Newell and Pizer, 2003, pp.57).

From equation (4.14) and (4.15), the discount rate is written as:

$$r_t = \eta + p\, \varepsilon_{t-1} + \xi_t \quad (4.16)$$

Since η and ε_t are assumed to be independent:

$$\overline{r_t} = \bar{\eta} + \overline{\varepsilon_t} = \bar{\eta}$$

or the mean of the discount rate is $\bar{\eta}$.

Similarly to Weitzman's assumption, Newell and Pizer assume agents to be risk-neutral hence making the *CEDF* equalling the *EDF*. Substitute the RHS of equation (4.16) in to equation (4.2), we attain the *CEDF* as follow:

$$A(t) = E[exp(-\sum(\eta + p\, \varepsilon_{t-1} + \xi_t))] \quad (4.17)$$

Apply *A(t)* built in equation (4.17) into equation (4.3), the *CEDR*, i.e. *R(t)* becomes:

$$R(t) = \bar{\eta} - t\, \sigma_\eta^2 - \sigma_\xi^2\, \Omega(p,t) \quad (4.18)$$

Equation (4.18) is the key result of Newell and Pizer's research since it illustrates the three important terms forming the *CEDR, R(t).* The first term is the mean discount rate $\bar{\eta}$ and it has a positive correlation to *R(t),* obviously. Both the second and the last term have a negative correlation with the *CEDR*. The second term tells us that as time goes further and the mean rate is more uncertain, *R(t)* will become smaller. The last term captures the relation between *R(t)* and (i) the uncertainty in deviation from $\bar{\eta}$ (σ_ξ^2) and (ii) the persistence of deviations (*p*). In short, the core finding of this approach is *the effective social discount rate becomes lower as the time horizon becomes longer, the more uncertain is the mean rate and the greater is the persistence of deviations from the mean rate*. As noted by Newell and Pizer (2003, pp.59) "the degree of persistence in discount rate fluctuations turns out to be a critical component of what drives the CEDR down over time". For example, with p = 0.96 the discount rate declines only 0.3% after 100 years while with $p = 1.0$ the result is 1%, other things being equal[14].

Recommended discount rates

In order to deduce numerical values for the effective discount rate, Newell and Pizer employ two models investigating historical data, namely the random walk and the mean-reverting model[15]. Running these models helps determining parameters shown in the RHS of equation (4.18). *R(t)* therefore will be derived. Under the random walk model, the certainty-equivalent rate declines continuously from 4% to 2% after 100 years, 1% after 200 years, and 0.5% after 300 years. For 400 years, the present value increases by a factor of over 40,000 compared to conventional discounting. Certainty-equivalent rates for the mean-reverting model, on the other hand, remain above 3% for next 200 years, declining to 2% only after 300 years and 1% after 400 years. The result of their study is then applied in calculating the costs of GHGs emission. The finding is that "incorporating discount rate's uncertainty almost doubles the expected present value of benefits from one ton of carbon mitigation[16]" (Newell and Pizer, 2003, pp.55).

Pitfalls of the approach

First the important assumption in (Newell and Pizer, 2003) that the past is informative to the future is then criticized by other scientists. According to Gollier and Weitzman (2010, pp.2) "there is no deep reason of principle that allows us to extrapolate past rates of return on capital into the distant future" and that assumption "is not based on any underlying theory that would confidently allow projecting the past far into the future". Moreover, many fundamental factors drive the change of discount rate are not extrapolatable, for example the unknown future pace of technological progress.

Secondly, the selection of models also makes differences in the conclusion then is sensitive to the outcomes of the research. For example, Groom et al. (2005) argue that applying Newell and Pizer's approach to the UK context shows that the mean-reverting model is more appropriate than the random walk model. This result is undoubtedly reverse to the case applied in the U.S.

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[14] $\bar{\eta}$ = 4% , σ_n = 0.52% and σ_ξ = 0.23% *(Newell and Pizer, 2003, pp.59)*

[15] *According to the two authors, the result of the random walk model is more accurate than the mean-reverting since it gives a greater degree of persistence in the discount rate than the second one.*

[16] *The time horizon is 400 years and the initial rate is 4%. The result is corresponded to the Random Walk model.*

4.2 The effect of uncertain economic growth rate on discounting by Gollier (2002)

4.2.1 The divergence of opinions in predicting future economic growth

As shown in the Ramsey formula, the social discount rate depends significantly on the growth rate of the economy (*g)*. However, predicting future economic growth is considered a difficult task with "potentially enormous errors" especially when the projects are long run like climate change policies (Gollier, 2002a, pp.150). "How might the future look like?" actually is an unanswerable question. In fact, human history has shown that before the industrial revolution in Western countries, there had been lots of economic slumps. In the scientist community, there have been two main schools of thought concerning that question (Bazelon and Smetters, 2001). For instance, some base on the concept of "technical progress" and "future productivity" to convince the possibility of higher discount rates in far future. In contrast, Dasgupta et al. employ the theory of "Limits to growth" to support the use of a lower rate in the future. Their main argumentation is that the exhaustion of (essential) natural resources and impacts of climate change will overcome technological advances and lead to declining consumption growth rates over time. Given the largely divergence in individual opinion, Gollier's research aims to find out the actual behaviour of discount rates under the circumstance of uncertain growth. The outcome is well presented in his 2002 article titled "*Discounting an uncertain future*".

4.2.2 Precautionary effect as the third component of the social discount rate

Let's first revisit the Ramsey formula, i.e. equation (2.7). In the light of this model, in a deterministic world there are two main effects governing the discount rate, called the time preference (ρ) and the wealth effect *(*ηg*)*. Based on this traditional argumentation, Gollier founds his study by combining the Ramsey's theory with "the precautionary effect", a concept of saving behaviour, see Box 3. Accordingly, under uncertain economic growth the discount rate is captured by three (but not two) main components: the pure rate of time preference (ρ), the wealth effect (ηg) and the precautionary effect (P). Following Ramsey's opinion that every generation should be treated the same, Gollier assumes utility discount rate to be zero. The consumption discount rate therefore is governed by the last two effects which "act in opposition to one another in determining the discount rate" (Conceição et al., 2007, pp.11). While the wealth effect increases the rate as a result of diminishing marginal utility of consumption, the precautionary effect however goes inversely.

In his study, Gollier aims to find out the balance between the wealth and the precautionary effect in case of uncertain economic growth. Based on that finding, the changing pattern of the discount rate in such context is revealed. The magnitude of the wealth effect depends on the rate of consumption growth while the precautionary effect is amplified by the degree of uncertainty and how prudent we are. Especially, increase in the degree of uncertainty is correlated to the time horizon as the further time goes, the more uncertain is the growth rate (due to the accumulation of period to period growth risks).

Box 3. What is the precautionary effect?

The precautionary effect emerges when people are prudent, which means "*his willingness to save increases in the face of an increase in his future income risk*" (Gollier, 2002a, pp.151). Leland (1968, pp.465) describes this effect "as the extra saving caused by future income being random rather than determinate". According to Groom et al. (2005, pp.466) "the desire to engage in more precautionary saving in the face of uncertain income growth [...] is dependent upon the convexity of marginal utility". Mathematically, it means the third derivative of utility function is positive, i.e. $U'''(C) > 0$. Moreover, given a concave utility function of consumption, $U'''(C) > 0$ is a necessary condition of decreasing absolute risk aversion, i.e. DARA (Nelson, 2008). Coefficient of Arrow-Pratt Absolute Risk Aversion (ARA) is defined as:

$$\text{ARA} = \frac{-U''}{U'} \quad \text{and} \quad \frac{\partial ARA}{\partial C} = \frac{(U'')^2 - U'''U'}{(U')^2}$$

Agents are DARA if and only if $\frac{\partial ARA}{\partial C} < 0$. As U' is always assumed to be positive, U''' must be positive to satisfy the condition of DARA (Leland, 1968).

4.2.3 When does the precautionary effect outweigh the wealth effect?

Before dive into the main findings, it is worth mentioning some pre-conditions in Gollier's approach as followed:

(i) Agents are assumed to be risk averse in terms of consumption, expressed by the term η in the Ramsey formula.

(ii) Utility function is concave w.r.t consumption and its marginal utility function is convex w.r.t consumption, i.e. $\frac{\partial^2 U}{\partial C^2} < 0$ and $\frac{\partial^3 U}{\partial C^3} > 0$,

(iii) Utility function is time additively separable.

Under such conditions, two key results in Gollier's (2002) study are:

(1) If agents are prudent, the discount rate under uncertainty will be lower than the rate without uncertainty.

(2) Under uncertainty, the precautionary effect will dominate the wealth effect if agents exhibit decreasing relative risk aversion and there is no risk of recession.

Gollier's (2002) proof of the two above findings can be summarized as below.

Finding (1). Under the case of uncertainty, consider a two date model with *t* and *t+1* as today and the future. The consumption level corresponded to date *t* and *t+1* are C_t and $\tilde{C}_{t+1}$, respectively. Seen from *t*, $\tilde{C}_{t+1}$ is uncertain. The consumption growth from *t* to *t+1* is unknown, denoted by $\tilde{g}_{t+1}$:

$$\tilde{g}_{t+1} = \frac{\tilde{C}_{t+1}}{C_t} - 1$$

which equals:

$$\tilde{C}_{t+1} = C_t(1 + \tilde{g}_{t+1})$$

The inter-temporal welfare is the weighted sum of utility at time *t* and *t+1*, as:

$$W(C_t, \tilde{C}_{t+1}) = U(C_t) + \beta.E[U(\tilde{C}_{t+1})]$$

where β is the utility discount factor, $\beta = \frac{1}{1+\rho}$

$E[u(\tilde{C}_{t+1})]$ is the expected utility at time *t+1*.

Maximizing $W(C_t, \tilde{C}_{t+1})$ gives us the short-term social discount rate at time *t+1* (r_{t+1}) as:

$$r_{t+1} = \frac{U'(C_t)}{\beta.E[U'(\tilde{C}_{t+1})]} - 1 \quad (4.19)$$

with r_{t+1} is the risk-free discount rate.

In the absence of uncertainty, given a sure consumption level (say *m*) equalling $E[\tilde{C}_{t+1}]$ at *t+1*, the social discount rate denoted by r_C, is defined as:

$$r_C = \frac{U'(C_t)}{\beta.U'(m)} - 1 = \frac{U'(C_t)}{\beta.U'(E[\tilde{C}_{t+1}])} - 1 \quad (4.20)$$

If marginal utility of consumption is convex w.r.t consumption, then we have:

$$E[U'(\tilde{C}_{t+1})] > U'(E[\tilde{C}_{t+1}]) \quad (4.21)$$

That means the expected marginal utility of consumption is greater than the marginal utility of expected consumption**.** As can be seen from Figure 3, the expected consumption (*E[C]*) is higher than the certainty equivalent consumption ($C_{equivalent}$), implying agents are risk averse.

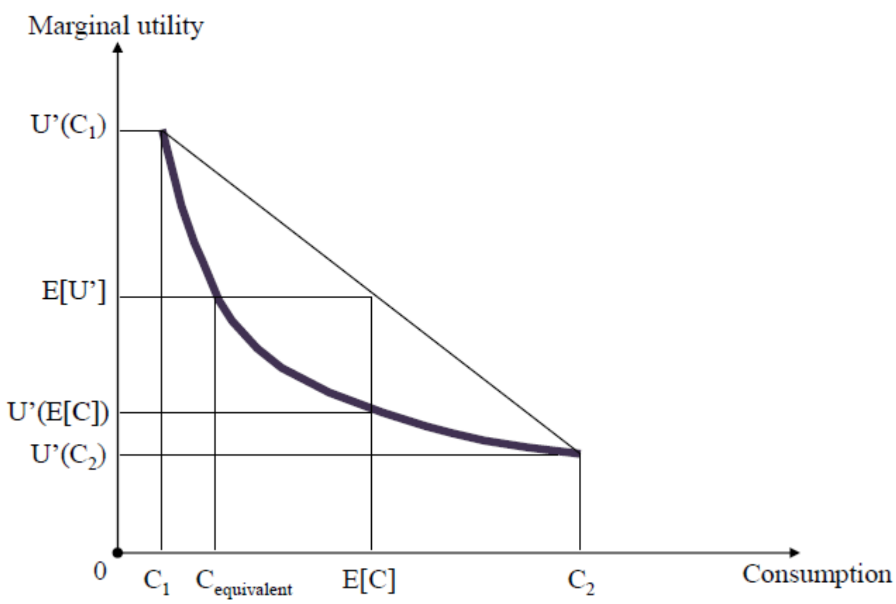

Figure 3. The relation between marginal utility and consumption in the presence of precautionary effect

From equation (4.19), (4.20) and (4.21) it is clear that $r_{t+1} < r_C$. Put another way, if agents are prudent, uncertainty in growth rate will induce society to select a smaller discount rate than the one under the case of no uncertainty.

The short-term social discount rate r_{t+1} is then expressed in terms of three effects by modifying equation (4.19), as:

$$r_{t+1} = \underbrace{\left[\frac{1}{\beta} - 1\right]}_{\text{Impatience }(\rho)} + \underbrace{E[\tilde{g}_{t+1}]\theta}_{\text{Wealth effect}} - \underbrace{0.5Var(\tilde{g}_{t+1})\theta P}_{\text{Precautionary effect}} \tag{4.22}$$

where $Var(\tilde{C}_{t+1})$ is the variance of the growth rate of consumption,

θ is a measure of relative risk aversion, defined as: $\theta = -\frac{U''C}{U'} > 0$

P is a measure of relative prudence, defined as: $P = -\frac{U'''C}{U''} > 0$

As can be seen from equation (4.22), the social discount rate is measured by three terms: (i) the impatience of agents; (ii) the wealth effect and (iii) the precautionary effect. When $Var(\tilde{C}_{t+1}) = 0$, i.e. there is no uncertainty, we are back to the Ramsey formula. In that case, the precautionary effect no longer exists and the discount rate is dependent upon the time discount rate and the wealth effect.Under the circumstances of uncertainty, given $\rho = const$ the changing pattern of the discount rate are merely subject to the balance between the wealth and the precautionary effect (Groom et al., 2005). It is clear from equation (4.22) that the second term is positively correlated to the discount rate while the last term is negatively correlated. Put other words, the precautionary effect lowers the effective discount rate if there exists uncertainty in growth rate.

Finding (2). Substitute $\tilde{C}_{t+1}$ by $C_t(1 + \tilde{g}_{t+1})$, equation (4.19) is equivalent to:

$$r_{t+1} + 1 = \frac{U'(C)}{\beta.E[U'(\tilde{C}_{t+1})]} = \frac{U'(C_t)}{\beta.E[U'(C_t(1+\tilde{g}_{t+1}))]} \tag{4.23}$$

The RHS of equation (4.23) reflects the ratio between the current marginal utility and the discounted future expected marginal utility (Groom et al., 2005).

Generalize equation (4.23) into multi-period model, the long-term discount rate, r_{t+n} is derived as:

$$(r_{t+n} + 1)^n = \frac{U'(C_t)}{\beta.E[U'(C_t \prod_1^n (1+\tilde{g}_{t+n}))]} \tag{4.24}$$

Gollier has shown that when agents exhibit DRRA and there is no risk of recession[17], the long-term discount rate will be smaller than the short-term one and be decreasing over time. Atkinson and Brandolini (2006) hold the same idea as they argue that η shouldn't be constant w.r.t consumption. Instead η should first rise as income rises, then fall. In fact, there are empirical evidence showing that individual

[17] *That means the uncertain growth rate is positive almost surely (Gollier, 2002a)*

RRA is now declining hence supporting Gollier's findings. For instance, by studying consumption data, Ogaki and Zhang (1999) find that relative risk aversion is decreasing in developing countries. Other research conducted by Kessler and Wolft (1991) show the same trend of relative risk aversion in developed countries as well. When individuals have CRRA then the wealth effect and the precautionary effect compensate one another. In that case we are back to the conventional situation with a constant rate over time.

Recommended discount rates

For short term projects, one should use a "risk free rate that is observable on financial markets". Between 50 and 100 years, discount rates lower than 5% should be applied and then gradually decline to around 1.5% for benefits and costs generated after 200 years.

Pitfalls of the approach

The main weakness in Gollier's approach is that it is inherently complex and difficult to validate the results. One critical condition in Gollier's finding is agents are DRRA. Though it has been proven to be true both in developed and developing nations, one can still question the form of utility function as DRRA results in a different utility function (see Box 4), compared to the case in the Ramsey formula. To test and determine the appropriate sets of DRRA utility function in near future are not a simple task and require numerous further researches.

Box 4. DRRA utility function

In his paper, Gollier (2002b) has shown two types of utility function resulting in DRRA.

The first one is the classical *Hyperbolic Absolute Risk Aversion* (HARA) utility function:

$$U(C) = \frac{(C-k)^{1-\gamma}}{1-\gamma} \text{ with } C \geq k \geq 0 \text{ and } \gamma \geq 0$$

where γ is a non-negative constant,

k is defined as "some minimum subsistence level of consumption" (Gollier, 2002a, pp.159).

The utility function above yields:

$$U'(C) = (C-k)^{-\gamma}$$

For those functions, RRA is written as:

$$\theta = \frac{-CU''}{U'} = \frac{\gamma C}{C-k}$$

and $\frac{\partial\theta}{\partial C} = \frac{-\gamma k}{(C-k)^2} < 0$ or DRRA.

The second example is the case of *One-Switch* utility function (linear plus power):

$$U(C) = aC + \frac{C^{1-b}}{1-b} \text{ with } a, b \text{ are positive constants}$$

Marginal utility of consumption then is defined as:

$$U'(C) = a + C^{-b}$$

which yields:

$$\theta = \frac{-CU''}{U'} = \frac{b}{1+aC^b} \text{ which has } \frac{\partial\theta}{\partial C} = \frac{-ab^2C^{b-1}}{(1+aC^b)^2} < 0 \text{ or DRRA.}$$

4.3 Concluding remarks

Under the circumstance of *uncertainty*, issue of DDRs is examined in terms of uncertain discount rates and uncertain economic growth rates. Although these two concepts are studied separately, there is in fact a relation between them as the uncertainty in the discount rate is partly resulted from the uncertainty in economic growth (Weitzman, 1998; Gollier 2002). The Ramsey formula does explain the dependence of consumption discount rate upon the economic growth.

Risk preference plays an important role in determining the changing pattern of the discount rate. In the three studied research, Weitzman (1998) and Newell and Pizer (2003) assume agents to be *risk neutral* while Gollier (2002a) considers them to be *risk averse*. As risk preferences certainly determine the form of utility function (Ostaszewski and Binmore, 1993), there is differentiation between Gollier's utility function and the ones from Weitzman's and Newell and Pizer's approach, i.e. concave versus linear. Linear utility function w.r.t consumption means the marginal utility of consumption is constant and elasticity of marginal utility equalling zero. Mathematically, $\partial U/\partial C = constant$ and $\partial^2 U/\partial C^2 = 0$. In this case, the wealth effect in the Ramsey formula is cancelled out and the social discount rate depends only on the pure rate of time preference, i.e. $r = \rho$ (Heal, 2005). However, the debate around the value of ρ is still ongoing and there is no sign that a general agreement about that issue will be made soon. Regarding Gollier's study, his assumption about risk-averse agents seems to be more appropriate in reflecting everyday experience (Perman et al., 2003; Groom et al, 2005). For this type of individuals, there exists a cost of risk bearing.

Both Weitzman (1998) and Newell and Pizer (2003) recommend using *averaged discount factor* instead of *averaged discount rate*. According to these economists, using averaged discount factors implies a higher *NPV* of future values hence making them more important from today's perspective. Given the uncertainty in the discount rate, the 'weight' or the importance of discount factors obtained from higher discount rates will be diminished due to the exponential effect of standard discounting. Therefore only the discount factor of lower discount rates is influential and directly determines the value of the averaged discount factor. In case of standard discounting all these values will be "much less important now because their expected value is so severely shrunk by the power of compound interest at high rate".

Gollier (2002a) proves the legitimacy of DDRs by bringing the concept of *precautionary effect* into his study. As shown in session 4.2, integrating the precautionary effect to the Ramsey formula will mathematically help lowering the long term discount rates as time goes by. Considering the interrelation between prudent agents and saving rates, uncertainty and time horizon gives us explanations for the decline of social discount rates over time. Since uncertainty of future consumption is accumulating by time, the further we look into the future, the more uncertain we are. Given agents are *prudent*; in the face of increasing uncertainty about their future they tend to save more money. Clearly, rise in saving will lead to less consumption hence making the discount rate lower.

Chapter 5 Applying time declining discount rates based on the DICE model

In this chapter, a number of discounting schemes are employed to calculate the present value of *climate change damages cost* (denoted by *Damage(t))* on a global scale, in a period of 300 years. To obtain the current (or future) value of *Damage(t)*, we use the DICE model of Nordhaus (2007). For the discounting step, three different schemes of discount rates are used, i.e. standard discounting (at the rate of 3%, constant over time), Weitzman's rates and the ones recommended by the UK Government.

5.1 A brief introduction about the DICE model

5.1.1 Objective and key components

The Dynamic Integrated Model of Climate and the Economy (in short DICE) is a model developed by Williams Nordhaus since 1994, aiming to calculate the optimal path of economic development in relation with GHGs emission reduction. Being considered an answer to the optimization problem, the focal point of this model is to maximize the sum of discounted social welfare in a finite time period, subject to *environmental* and *economic constraint*. The underlying meaning of the DICE model is to incorporate "both the dynamics of GHGs emissions and the impacts and economic costs of policies to curb emissions" (Nordhaus, 1993, pp.28). Until now "the models have constantly evolved over time, benefiting from revised estimates from the natural sciences as well as structural improvements[18]" (Murphy, 2008, pp.4) but "the basic modeling philosophy remains unchanged: to incorporate the latest economic and scientific knowledge and to capture the major elements of the economics of climate change" (Nordhaus, 2007, pp.6).

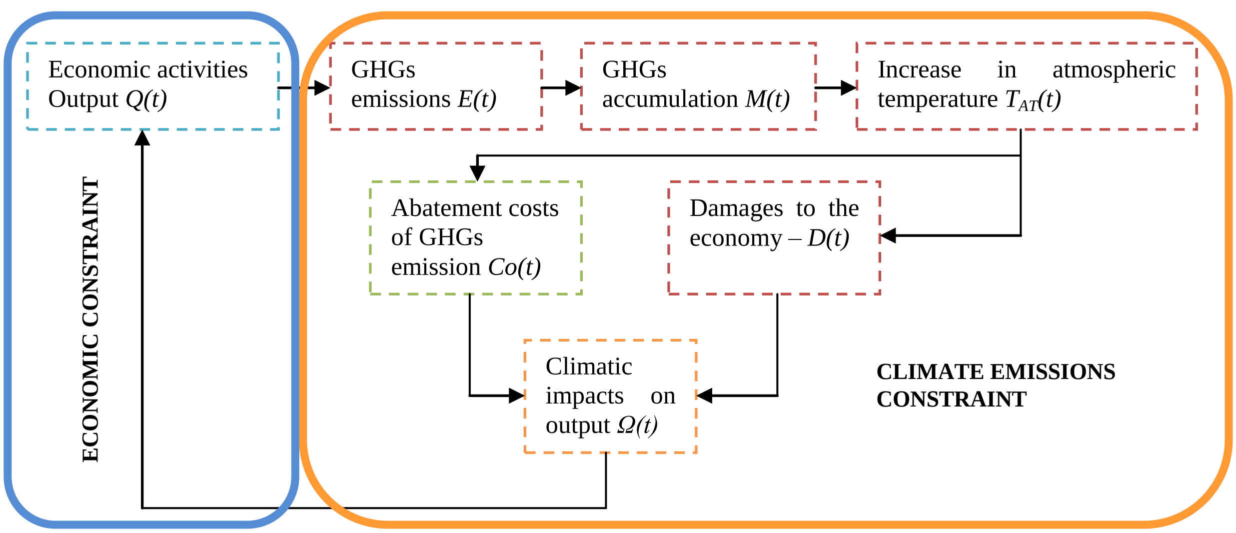

Figure 4. Overview of the DICE model

[18] *In this thesis, we use the version of the DICE model updated in July 2008. This version is theoretically similar to the older versions except update in data and inclusion of new variables. Notably, there are new values assigned to the elasticity of marginal utility of consumption (η=2) and the utility discount rate (ρ=1.5%). The values used for the older versions are η=1 and ρ=3.0%. All calculations are run on a global scale, with the starting year of 2005 (year 0) and on 10-year time-steps.*

Figure 4 describes the basic DICE's components and the interrelations between them. In the model, Nordhaus mutually integrates economic development and the environment into all steps, of which each has effects on one another. Economic activities and global warming are strongly linked to each other by the emissions and accumulation of GHGs in the atmosphere. The cost of GHGs abatement and the damages of GHGs emissions to the economy are considered a feedback of climate change to human well-being. Accumulation of GHGs is a negative natural capital in the output function, together with other positive capitals like labour, technology and natural resources stock. For key equations, see Table 4 with explanation included.

Table 4. Key equations of the DICE model

	Key equations	Explanations
Model's Objective	$\max \sum_0^t \frac{U(t)}{(1+\rho)^t}$ where *U(t)* is the utility function w.r.t consumption, $U(t) = U[c(t),P(t)] = P(t)\frac{c(t)^{1-\theta}}{1-\theta}$ with *c(t)* is per capita consumption and *P(t)* is total population; ρ is the utility discount rate. subject to: Economic constraint and, Climate-Emissions Constraint	The objective in the DICE model is to maximize the sum of discounted utility of consumption in a certain period, given constraint in the economy and in climate-emissions. Per capita consumption is total consumption divided by total population, i.e. $c(t) = \frac{C(t)}{P(t)}$ where *C(t)* is total consumption.
Economic constraint	$\boldsymbol{Q(t) = \Omega(t)\, A(t)\, K(t)^{\gamma}\, P(t)^{1-\gamma}}$ where *Q(t)* is net output, expressed in income; *Ω(t)* refers to negative impacts of climate change on output; *A(t)* is technological parameter; $K(t)$ is capital stock; $P(t)$ is labour, which is proportional to population; γ is the constant elasticity of output w.r.t capital stock and labour, i.e. $\gamma = 0.3$	*Economic constraint* is understood as the constraint on output of production, i.e. technology, labour, capital and climatic impacts. Output is expressed in a Cobb-Douglas production function.
	$\boldsymbol{K(t) = (1-\delta_K)K(t-1) + I(t)}$ where *K(t)* is capital stock at time *t*, *I(t)* is investment, δ_K is the rate of depreciation of capital stock.	Stock of capital is the sum of capital remained from previous periods and current investment.
Climate-emissions constraint	Climate-emissions constraint consists of several sub-components, sequentially introduced in the following equations.	*Climate-emissions constraint* describes the relationship between economic activities and climate change. Without effective environmental policies, economic development will lead to

		degradation of environmental quality due to its GHGs emission.
	$E(t) = [1-\mu(t)]\ \sigma(t)\ Q(t)$ where $E(t)$ is GHGs emissions from economic activities; $\mu(t)$ is the emissions-control rate, representing attempts to reduce GHGs emissions; $\sigma(t)$ is the ratio of uncontrolled GHGs to output $Q(t)$.	Emissions of GHGs from economic activities are affected by social ability to control GHGs emissions[19].
	$M_{AT}(t) = E(t) + b_1 * M_{AT}(t-1) + b_2 * Mu(t-1)$ where $M_{AT}(t)$ is the GHGs concentration in the atmosphere, $Mu(t)$ is the GHGs concentration in shallow oceans, b_1 is the rate of irremovable GHGs in the atmosphere from the last period, $b_1 = 0.810712$; b_2 is the rate of irremovable GHGs in shallow oceans from the last period, $b_2 = 0.097213$;	Accumulation of GHGs in the atmosphere is a sum of current GHGs emissions and the remained GHGs concentration in the atmosphere and in shallow oceans from previous periods.
	$F(t) = \eta\{\log [M_{AT}(t) / M_{AT}(1750)]\} + F_{ex}(t)$ where $F(t)$ and F_{ex} is total and exogenous radiative forcing, respectively, η is the temperature-forcing parameter, $T_{AT}(t) = T_{AT}(t-1) + \xi_1 \{F(t) - \xi_2 T_{AT}(t-1) - \xi_3 [T_{AT}(t-1) - T_{LO}(t-1)]\}$ where $T_{AT}(t)$ is global atmospheric temperature, $T_{LO}(t)$ is global deep oceans temperature, ξ_1, ξ_2, ξ_3 is parameters of climate equations.	Accumulations of GHGs lead to global warming through increases in radiative forcing ($F(t)$). Then higher radiative forcing warms the atmospheric layer, which then warms the shallow ocean, gradually warming the deep ocean (Nordhaus, 2007, pp.38).
	$D(t) = 1 + a_1 * T_{AT}(t) + a_2 * T_{AT}(t)^2$ where $D(t)$ is physical damage of climate change per unit of gross output, a_1 is the linear damage coefficient, $a_1 = 0.00$ a_2 is the quadratic damage coefficient, $a_2 = 0.0028388$	Physical damage of climate change depends on how rapidly the global atmospheric temperature increases.

[19] *Modifying the RHS of this equation, we obtain:*

$$E(t) = \sigma(t)\ Q(t) - \mu(t)\ \sigma(t)\ Q(t)$$

which means total GHGs emissions is equal to the uncontrolled GHGs emissions less the controlled ones.

	$Co(t) = cost1(t)\mu(t)^{cost2}$ where *Co(t)* is abatement cost of GHGs emissions per unit of gross output, *cost1(t)* is abatement cost function with intercept *cost1(0)* = 0.03, *cost2* is the exponential parameter of abatement cost function, *cost2* = 2.15	Abatement cost of GHGs emission depends on $\mu(t)$-the emissions-control factor and abatement cost function, *cost1(t).*
	$\Omega(t) = \frac{1 - Co(t)}{1 + D(t)}$	The impact of climate change on output depends on the cost of GHGs emission reduction *Co(t),* and the loss of income due to increased temperature *D(t).* The impact does not seriously affect output (or *Ω(t)* is not small) if the cost of GHGs abatement is small or/and the damage of climate change is not significant.
	$Damage(t) = A(t)\, K(t)^{\gamma}\, P(t)^{1-\gamma}\left(1 - \frac{1}{1+D(t)}\right)$ where *Damage(t)* is the cost of climate change damages, measured in billions of \$.	The damages cost function assumes that costs of climate change damages are proportional to gross world output (Nordhaus, 2007, pp.35).

5.1.2 No control versus Optimal policy

In the DICE model, the question of optimization is examined in two different scenarios, namely "No control" (or Baseline) and "Optimal policy". As explained by Nordhaus (2007, pp.34) "the Baseline or No control case […] represents the outcome of market and policy factors as they currently exist" and "no significant emissions reductions are imposed" (pp.16). Simply put, it refers to the case in which society does not react to climate change impacts at all. Hence major economic and environmental variables are projected as "business as usual". Optimal policy, in contrast, represents the case in which "all countries join together to reduce GHG emissions in a fashion that is efficient across industries, countries, and time" (Nordhaus, 2007, pp.16). One might think about it as the situation in which issues of climate change mitigation and adaptation are precisely considered, with a optimal attempt to reduce climate change's consequences.

5.2 The NPV of climate change damages

5.2.1 Three schemes of discounting

Two schemes of time declining discount rates applied in this chapter are the UK Greenbook's discount rates (mentioned in session 2.3) and Weitzman's rates (in 4.1.2). As explained in the GreenBook, the reason motivating the use of time declining discount rates (DDRs) in UK is mainly due to uncertainty in the future, both in terms of discount rates and economic growth. Meanwhile DDRs developed by Weitzman is considered an outcome of uncertainty in the discount rates themselves. We then compare the results obtained from these two schemes to the ones from standard discounting with a constant rate r = 3%. The aim is to reflect the significant differences between the NPV obtained from DDRs and the one using a time invariant rate. A modified version of the rates mentioned in the two approaches[20] is shown in Table 5. To know how we calculate the discount factor in case of time variant rates, one can have a look at Table 6.

Table 5. Three discounting schemes used for calculating the NPV of climate change damages cost

Standard Discounting		**Weitzman's scheme**		**UK GreenBook's scheme**	
Periods	Consumption discount rates	Periods	Consumption discount rates	Periods	Consumption discount rates
1- 300 (years)	3.0%	The first 20 years (t = 1-20)	3.5%	The first 30 years (t = 1-30)	3.5%
		Next 50 years (t = 21-70)	2.0%	Next 40 years (t = 31-70)	3.0%
		Next 230 years (t = 71-300)	1.0%	Next 50 years (t = 71-120)	2.5%
				Next 80 years (t = 121-200)	2.0%
				Next 100 years (t = 201-300)	1.5%

Table 6. Formulas for calculating discount factors

Periods of years	**Corresponded consumption discount rates**	**Corresponded consumption discount factors at *t* (D_t)**
T_1 = from year *0* to year *m*	r_1	$D_t = \frac{1}{(1+r_1)^t}$
T_2 = from year *m*+1 to year *m*+*n*	r_2	$D_t = \frac{1}{(1+r_1)^m} \cdot \frac{1}{(1+r_2)^{(t-m)}}$
T_3 = from year *m*+*n*+1 to year *m*+*n*+*p*	r_3	$D_t = \frac{1}{(1+r_1)^m} \cdot \frac{1}{(1+r_2)^n} \cdot \frac{1}{(1+r_3)^{t-m-n}}$

[20] *Approximating the years (in both schemes) and the discount rates (in Weitzman's approach) to make these schemes compatible with the '10 year step' mechanism in the DICE model.*

5.2.2 Results

No control and Optimal Policy

After running the DICE model, current value of climate change damages cost (*Damage(t)*) at different points in time is obtained. The current value is then converted to NPV by multiplying by the consumption discount factor (D_t) mentioned in Table 6. Data in detail of current and present value of damages cost in three discounting schemes are shown in Annex 2.

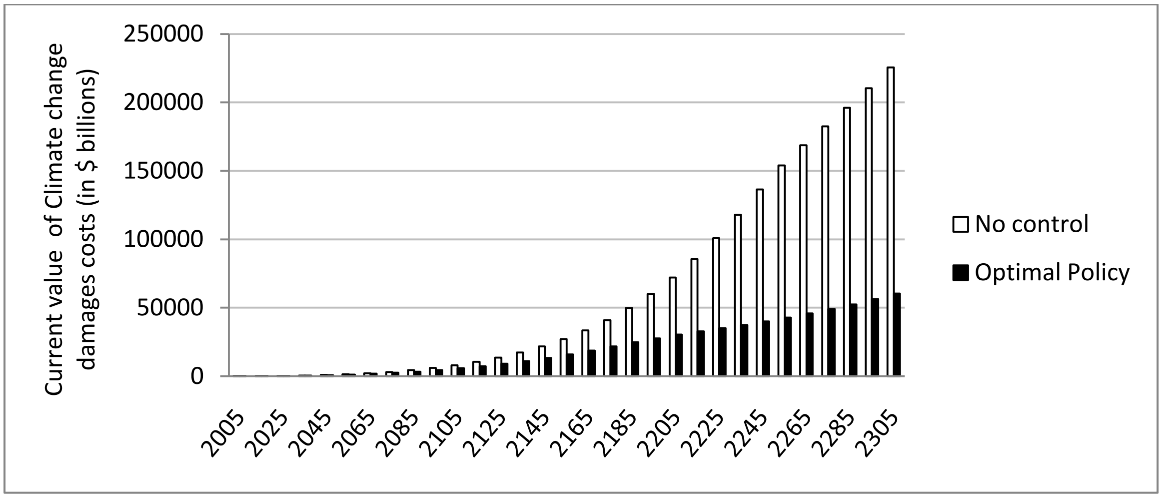

Figure 5. The zero-discounting value of climate change damages cost in case of No control and Optimal Policy

First, it is clear from Figure 5 that without discounting, the monetary value of climate change damages cost in case of No control is always higher than in Optimal Policy because emission of GHGs in the first case is undoubtedly greater[21] than the second one. Another important remark is *future* damages cost rises over time with a considerably higher pace in case of No control, compared to Optimal policy.

However, due to the effect of discounting, whether the gap between the *present* values obtained from No control and Optimal policy, is significant or not critically depends on the way we discount future values. Figure 6 helps proving that assumption (however, one should note that the scale used in Figure 6.c is different from 6.a and 6.b). From Figure 6.a it is clearly seen that under standard discounting, the benefits of taking Optimal Policy over No control is not significant. On average, the damages cost in Optimal policy is just $82 billion lower than the one in No control. In the last century, the difference between “doing nothing” and “taking action” is declining by time, from $112.92 billion in 2205 to $23.27 billion in 2305. However, results change when we use time declining discounting, see figure 6.b, 6.c. To be detail, under UK’s and Weitzman’s discounting, the gap between No control and Optimal policy is $182 and $1,418 billion on average, which is more than two and 17 times greater than the one in standard

[21] *More GHGs emission in No control makes the physical damage, D(t) in equation (5.7), greater than the one in case of Optimal policy. As defined in equation (5.10), damages cost of climate change will also be higher accordingly.*

discounting, respectively. Especially, the gaps in these two discounting schemes become larger as time goes by and just slightly decline at the end.

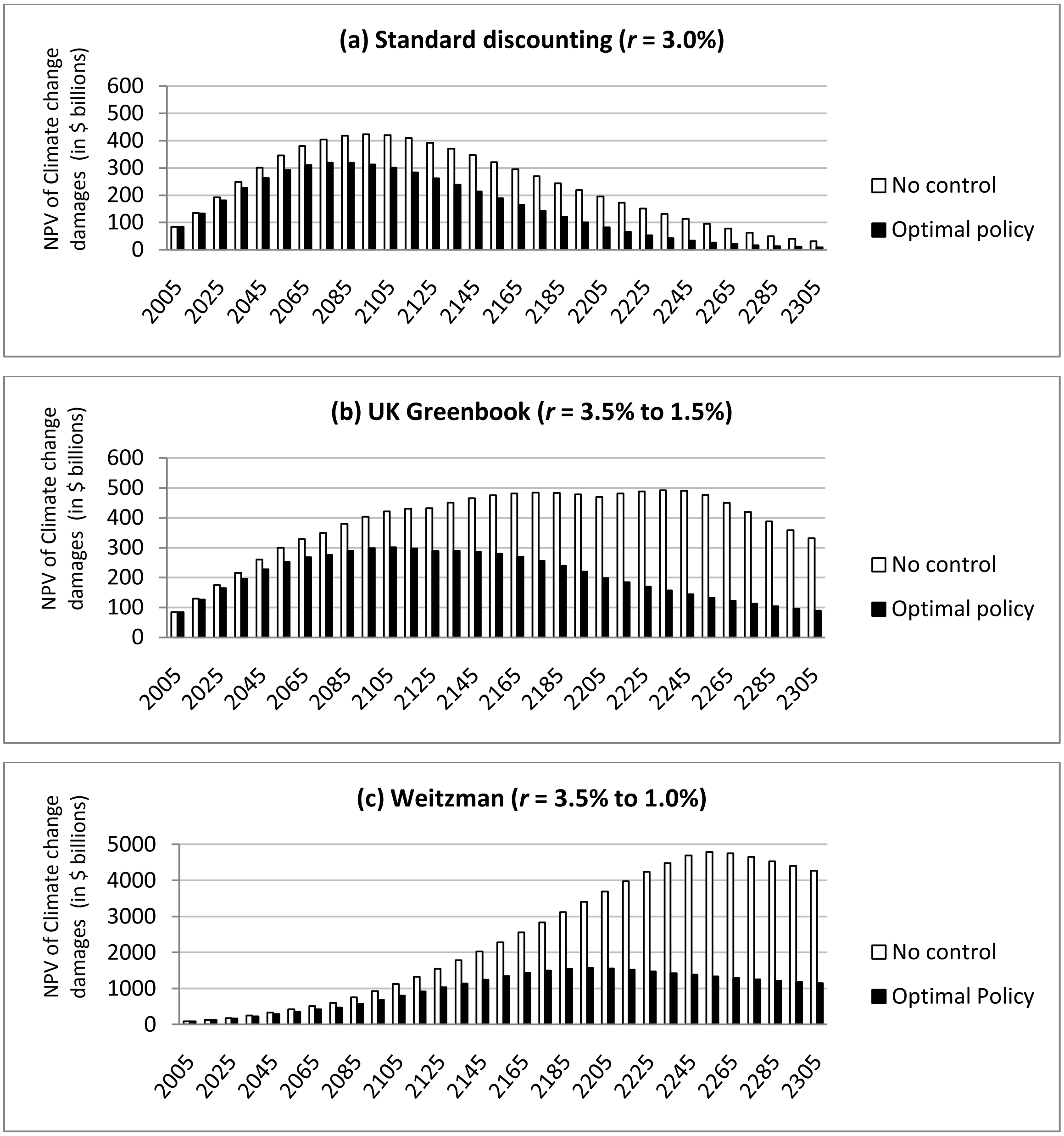

Figure 6. NPV of the difference in damages cost obtained from No control and Optimal scenario, using standard discounting, UK Greenbook's & Weitzman's rates

All points mentioned above do justify the important role of discounting in determining the NPV of future values. It is clearly seen from Figure 7 that the Weitzman's and UK's discount factor remain to be significant until the end of the 300 year period although the former is more powerful than the latter. Notably, it is clear that Weitzman's and UK Greenbook's scheme have the same discount factors at the beginning (with r = 3.5% in 20 first years) but there is then a large divergence between them. It is because after 70 years (2075), Weitzman suggests using a discount rate at only 1% while the rate recommended by

UK Greenbook is varied from 2.5% to 1.5% for different periods of time. Regarding the case of standard discounting, the discount factor becomes very small after 150 years and its value is rapidly declining over time due to the exponential effect. In the last 50 years (since 2255 through 2305), its averaged discount factor is around 7 and 74 times smaller than the ones in UK and Weitzman's discounting, respectively. For more detail data, Annex 4 gives the numerical values of discount factor respective to each specific discounting scheme.

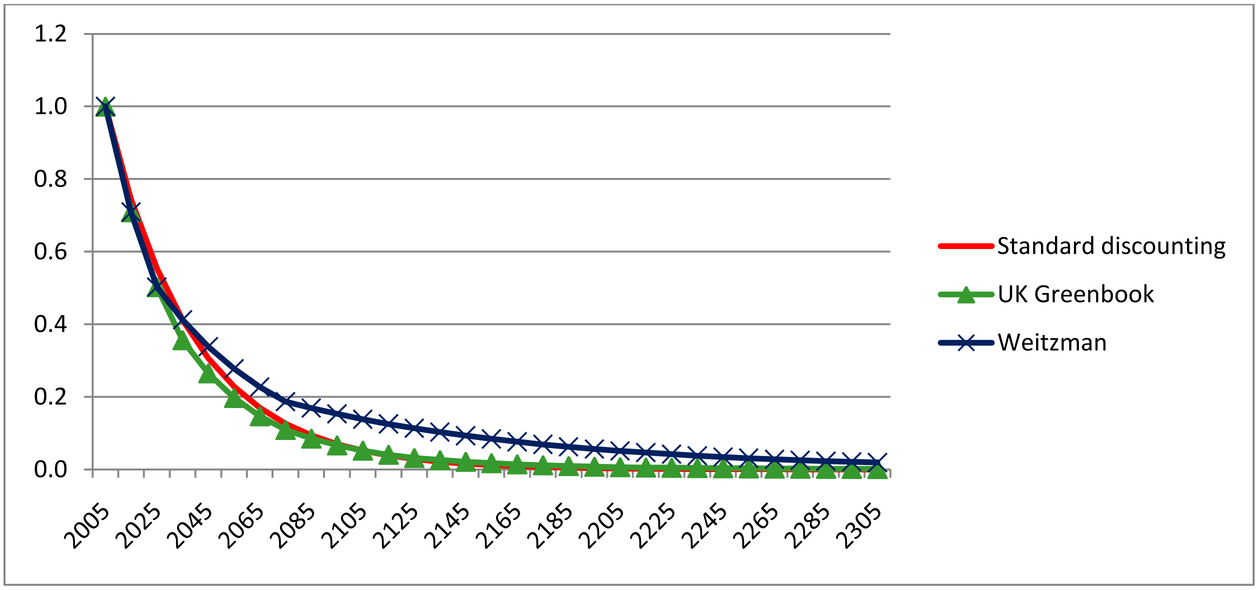

Figure 7. Comparison of discount factors under standard, UK Greenbook's and Weitzman's rates

A closer look at the case of Optimal Policy

In this session, we specifically focus on the present values obtained from the case of Optimal policy. Overall, value obtained from Weitzman's discount rates is the highest amongst three employed discounting schemes at almost all points in time, see figure below.

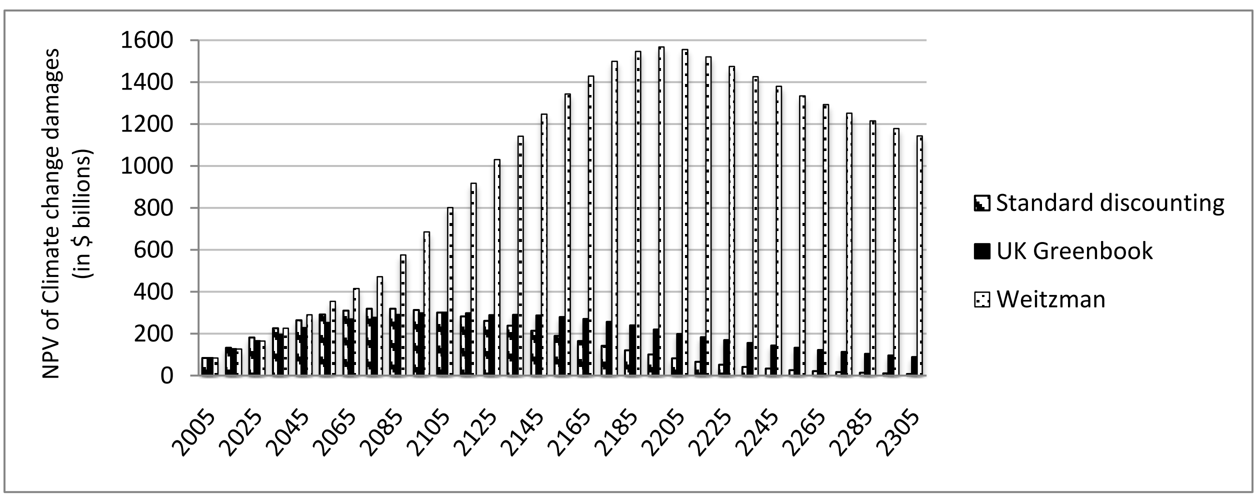

Figure 8. *Comparison the NPV of Climate change damages under standard, UK Greenbook's and Weitzman's Discount rates* - *The case of Optimal Policy.*

Throughout the first 50 years, the NPV of damages cost using three approaches is relatively equal to each other. The period of next 70 years (2055-2125) witnesses a rapid increase in the NPV of damages in Weitzman's, from $355 to $1,000 billion in 2055 and 2125 respectively. Damages costs in standard and UK Greenbook's discounting still keep being equivalent, stay at around $270 billion in 2125, which is 4 times lower compared to Weitzman's cost at the same point in time.

In the next 80 years (2125 through 2205), damages cost derived from Weitzman's discounting continues increasing sharply, making the gap between Weitzman's present values and the ones attained from standard and UK's rates larger. In 2205, the NPV of climate change damages in Weitzman's is correspondingly 20 and 8 times greater than the ones in standard's and UK's discounting. Interestingly, since the year 2125, there is a rising divergence between the NPVs under standard's and UK's discount rates in which UK's values are all the time higher than standard's ones. To prove, since 2125 to 2205, the gap between these two schemes has risen from $27 to $116 billion.

In the last 100 years (2205-2305), all present values got from the three schemes are falling down but with different pace. Standard discounting poses the most significant decline with its NPV damages cost down by 10 times since 2205 till 2305 while the movement goes smoothly under Weitzman's and UK's scheme (about 1.5 and 2.2 times lower, compared to its own values). Notably, in the final year (2305), there is a huge gap between the NPV under Weitzman's and the two other discounting schemes. That the NPV attained from Weitzman's is 135 and 13 times higher than the ones got from standard's and UK's discount rates, respectively. One should recall the fact that in the first 50 years, they are identical to each other.

5.2.3 Some implications

First of all, using DDRs in most cases makes the benefits of "taking action" over "doing nothing" much higher than in case of standard discounting. "It follows that policies on climate change are more likely to pass a CBA" (Guo et al., 2006, pp.214).

Secondly, one should keep in mind that although a relatively low discount rate (at 3%) is applied in standard discounting, the result obtained from the comparison is still significant and meaningful. So if we employ a discount rate which is as high as normal market interest, say at 7%, the difference will be much more remarkable.

Thirdly, the relative equality in the damages cost of climate change between No control and Optimal policy during the first 50 years reveals the nature of this climatic phenomenon, see Figure 5 and 6. That climate change is a long term and inter-temporal process of which its consequences are only be noticed and sensed after centuries. Although we take actions to tackle climate change's impacts right now, the benefits might not be seen in short term but after generations. It is one of the most important points that today's people should take into account when think about climate change related issues.

Chapter 6 Conclusions

In this thesis, the concept of time declining discounting is introduced and explained in light of different theoretical approaches, given the context of long term projects. It is concluded that there are certain theories that can be used to legitimate the use of DDRs both in deterministic and uncertain circumstances. In the field of CBA, using DDRs help resolving a number of inherent weaknesses of the conventional 'standard discounting' approach. As said by Jouini et al. (2010, pp.834) "It is clear that using a declining discount rate could make an important contribution towards the goal of sustainable development. But what formal justifications exist for using a declining discount rate?". To deliver a comprehensive answer to his question, we investigate the issue of time declining discounting from the perspective of uncertainty, called DDRs in a *deterministic* and an *uncertain* world.

Assuming the future world is *deterministic*, declining discounting can be rationalized by employing the theories of "logistic economic growth", "increasing environmental expenditure" and "climate change impacts on economic development". The "logistic economic growth" employs the concept of 'carrying capacity of consumption' as a reason for decline in the consumption growth rate (g). As a result, the social discount rates decline over time. Weitzman (1994) bases his argumentation on the key idea that if society spends more money on environmental improvement, the social discount rate will be lower. The third school of thought directly points out the negative impacts of climate change as an influential constraint on economic expansion.

In a world with *uncertainty*, DDRs are studied in two different cases relating to uncertainty in discount rates and in economic growth. Though still having some limitations, each study gives valuable contribution to the theory of DDRs to some certain extent. For instance, using the averaged discount factor to derive the effective consumption discount rate is considered a major finding to the practice of discounting. Regarding Gollier's approach, DDRs due to the precautionary effect is expected to contribute a good reason for declining discounting though it is still unsure whether agents have DRRA utility function.

In chapter 5, we carry out a small 'experiment' in which three different discounting schemes are parallel used to obtain the NPV of climate change damages cost. Overall, the NPV of damages cost is the highest in Weitzman's scheme, followed by the UK's and standard's one, respectively. Although in the first 50 years, the NPV are relatively equivalent amongst the three approaches, there is a significantly growing difference between them as time goes further. Moreover, applying DDRs make the benefit of 'taking action' over 'doing nothing' more visible under UK's and Weitzman's discounting. On average, the gap between No control and Optimal policy is more than two and 17 times greater than the one in standard discounting, respectively.

To conclude, think about the issue of climate change policies is not just to think of the benefits of your own countries and for your own people. It is, however, about the equality in the well-being of global humanity both in present and in the future. For example, rich countries are expected to lead the mission as they pay the cost of climate change mitigation while the benefits of their actions might mostly be enjoyed by people in poor countries. Therefore when facing climate change, the discount rate should reflect the willingness to transfer the wealth from rich to poor countries and from current to far distant future generations.

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Annex 1. Optimal growth model and the Ramsey formula

Proof. Consider a simple two-period model as mentioned in 2.1. The function of inter-temporal social welfare hence is:

$$W(U_0, U_1) = U_0 + \frac{1}{1+\rho} U_1 = U(C_0) + \frac{1}{1+\rho} U(C_1) \qquad (1)$$

The objective of the optimal growth model is to maximize the inter-temporal social welfare, i.e. max $W(U_0, U_1)$ over two periods subject to the constraint:

$$Q_0(K_0) - (K_1 - K_0) = C_0 \qquad (2a)$$

$$Q_1(K_1) - (K_2 - K_1) = C_1 \qquad (2b)$$

where Q_t is the output obtained,
K_t is the capital stock at the beginning of the period,
C_t is the level of consumption,
All parameters are corresponded to period t in time.

Equations (2a) and (2b) say that output Q_t can be either used for investment K_t or consumption C_t
Maximizing equation (1) under the conditions of (2a) and (2b) shows that:

$$r = -\frac{dC_1}{dC_0} - 1 = \frac{(1+\rho)U'(c_0)}{U'(c_1)} - 1 = \frac{U'(c_0)}{\frac{1}{1+\rho}U'(c_1)} - 1 \qquad (3)$$

Working on the case of continuous time, maximizing inter-temporal social welfare:

$$W = \int_{t=0}^{t=\infty} U_t e^{-\rho t}\, dt \qquad (4)$$

subject to the constraint:

$$\dot{K}_t = Q(K_t) - C_t \qquad (5)$$

gives the consumption discount rate equal to:

$$r = \rho - \frac{(\partial^2 U/\partial C^2)}{(\partial U/\partial C)}\frac{\partial C}{\partial t} \qquad (6)$$

Define $\eta = -\frac{(\partial^2 U/\partial C^2)C}{(\partial U/\partial C)}$ is the elasticity of marginal utility and $g = \frac{\partial C}{\partial t}/C$ is the growth rate of consumption, equations (6) can be written as:

$$r = \rho + \eta g \qquad (7)$$

Annex 2

Table 7. Current value and NPV of damages cost obtained from three approaches (in billions of $).

Years	Current value		NPV using standard discounting		NPV by UK Greenbook's rates		NPV by Weitzman's rates	
	No control	Optimal Policy	No control	Optimal Policy	No control	Optimal Policy	No control	Optimal Policy
2005	84	84	84.00	84.00	84.00	84.00	84.00	84.00
2015	182	178	135.43	132.45	129.02	126.19	129.02	126.19
2025	347	328	192.13	181.61	174.39	164.84	174.39	164.84
2035	605	549	249.25	226.18	215.55	195.60	249.43	226.34
2045	983	860	301.35	263.64	260.60	227.99	332.46	290.86
2055	1516	1279	**345.81**	**291.75**	**299.05**	**252.30**	**420.62**	**354.86**
2065	2242	1827	380.54	310.10	329.09	268.17	510.30	415.84
2075	3204	2526	404.66	319.03	349.94	275.89	598.24	471.65
2085	4457	3401	418.86	319.62	380.28	290.18	753.38	574.88
2095	6061	4479	423.83	313.21	403.99	298.54	927.47	685.39
2105	8089	5787	**420.89**	**301.11**	**421.19**	**301.33**	**1120.57**	**801.67**
2115	10579	7313	409.59	283.14	430.32	297.47	1326.70	917.12
2125	13618	9080	392.33	261.59	432.73	288.53	1546.07	1030.86
2135	17309	11104	371.05	238.03	451.21	289.46	1778.99	1141.25
2145	21763	13394	347.14	213.65	465.40	286.43	2024.92	1246.23
2155	27107	15946	321.73	189.26	475.54	279.74	2283.26	1343.16
2165	33475	18734	295.64	165.45	481.75	269.61	2552.59	1428.54
2175	41013	21702	269.52	142.62	484.19	256.21	2831.19	1498.12
2185	49867	24748	243.84	121.02	482.96	239.68	3116.35	1546.58
2195	60181	27704	218.97	100.80	478.14	220.11	3404.70	1567.34
2205	72085	30376	**195.16**	**82.24**	**469.83**	**197.98**	**3691.91**	**1555.74**
2215	85675	32778	172.60	66.03	481.16	184.08	3972.34	1519.76
2225	100986	35120	151.38	52.65	488.69	169.95	4238.77	1474.12
2235	117950	37530	131.56	41.86	491.82	156.49	4481.90	1426.08
2245	136329	40087	113.15	33.27	489.82	144.03	4689.64	1378.97
2255	153861	42842	95.02	26.46	476.34	132.64	4791.44	1334.16
2265	168625	45828	77.49	21.06	449.83	122.25	4753.85	1291.98
2275	182410	49061	62.37	16.78	419.29	112.77	4655.41	1252.12
2285	196117	52554	49.90	13.37	388.44	104.09	4531.18	1214.23
2295	210347	56311	39.82	10.66	358.99	96.10	4399.65	1177.81
2305	225493	60337	**31.77**	**8.50**	**331.61**	**88.73**	**4269.74**	**1142.49**

Annex 3

Table 8. The gap in the NPV of damages cost between No control and Optimal policy

Years	Difference in the NPV of Climate change damages cost between No control and Optimal Policy (in $ billions)		
	Standard discounting	**UK Greenbook's rates**	**Weitzman's rates**
2005	0.00	0.00	0.00
2015	2.98	2.84	2.84
2025	10.52	9.55	9.55
2035	23.07	19.95	23.09
2045	37.71	32.61	41.60
2055	**54.06**	**46.75**	**65.76**
2065	70.44	60.91	94.46
2075	85.63	74.05	126.59
2085	99.24	90.10	178.50
2095	110.63	105.45	242.08
2105	**119.78**	**119.86**	**318.90**
2115	126.45	132.85	409.59
2125	130.74	144.20	515.21
2135	133.02	161.75	637.74
2145	133.49	178.97	778.69
2155	132.47	195.80	940.11
2165	130.19	212.14	1124.06
2175	126.90	227.98	1333.07
2185	122.83	243.28	1569.77
2195	118.17	258.03	1837.37
2205	**112.92**	**271.85**	**2136.17**
2215	106.56	297.07	2452.58
2225	98.74	318.74	2764.65
2235	89.70	335.33	3055.83
2245	79.88	345.79	3310.67
2255	68.56	343.71	3457.28
2265	56.43	327.58	3461.87
2275	45.60	306.52	3403.29
2285	36.53	284.35	3316.95
2295	29.16	262.89	3221.84
2305	**23.27**	**242.88**	**3127.25**

Annex 4

Table 9. Discount factors obtained from three approaches

Years	Discount factor		
	Standard discounting	**UK Greenbook's rates**	**Weitzman's rates**
2005	1.0000	1.0000	1.0000
2015	0.7441	0.7089	0.7089
2025	0.5537	0.5026	0.5026
2035	0.4120	0.3563	0.4123
2045	0.3066	0.2651	0.3382
2055	**0.2281**	**0.1973**	**0.2775**
2065	0.1697	0.1468	0.2276
2075	0.1263	0.1092	0.1867
2085	0.0940	0.0853	0.1690
2095	0.0699	0.0667	0.1530
2105	**0.0520**	**0.0521**	**0.1385**
2115	0.0387	0.0407	0.1254
2125	0.0288	0.0318	0.1135
2135	0.0214	0.0261	0.1028
2145	0.0160	0.0214	0.0930
2155	0.0119	0.0175	0.0842
2165	0.0088	0.0144	0.0763
2175	0.0066	0.0118	0.0690
2185	0.0049	0.0097	0.0625
2195	0.0036	0.0079	0.0566
2205	**0.0027**	**0.0065**	**0.0512**
2215	0.0020	0.0056	0.0464
2225	0.0015	0.0048	0.0420
2235	0.0011	0.0042	0.0380
2245	0.0008	0.0036	0.0344
2255	0.0006	0.0031	0.0311
2265	0.0005	0.0027	0.0282
2275	0.0003	0.0023	0.0255
2285	0.0003	0.0020	0.0231
2295	0.0002	0.0017	0.0209
2305	**0.0001**	**0.0015**	**0.0189**