9 Soil temperature

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9.1 Introduction

Soil temperature affects many physical, chemical and biological processes in the top soil, for instance the surface energy balance, soil hydraulic properties, decomposition rate of solutes and growth rate of roots. Currently SWAP uses the soil temperatures only to adjust the solute decomposition rate, but other temperature relations may readily be included. SWAP calculates the soil temperatures either analytically or numerically. In the following sections the heat flow equations and the applied analytical and numerical solutions are discussed.

9.2 Temperature conductance equation

Commonly, heat flow by radiation, convection and conduction is modeled by the conduction equation alone. According to De Vries (1975), the rate of heat transfer by water vapour diffusion is small and proportional to the temperature gradient. Therefore, such diffusion might be taken into account by slightly increasing the soil thermal diffusivity. This approach is followed in SWAP as well. Apparent thermal properties rather than real thermal properties are assumed to account for both conductive and non-conductive heat flow.

The one-dimensional soil heat flux, q_{heat} (J cm^{-2} d^{-1}), is described as:

$$q_{\text{heat}} = -\lambda_{\text{heat}} \frac{\partial T}{\partial z} \qquad (9.1)$$

where λ_{heat} is the thermal conductivity (J cm^{-1} °C^{-1} d^{-1}) and T is the soil temperature (°C).

Conservation of energy results in:

$$C_{\text{heat}} \frac{\partial T}{\partial t} = \frac{-\partial q_{\text{heat}}}{\partial z} \qquad (9.2)$$

where C_{heat} is the soil heat capacity (J cm^{-3} °C^{-1}).

Combination of Eq. (9.1) and (9.2) yields the differential equation for soil heat flow:

$$C_{\text{heat}} \frac{\partial T}{\partial t} = \frac{\partial \left(\lambda_{\text{heat}} \frac{\partial T}{\partial z} \right)}{\partial z} \qquad (9.3)$$

9.3 Analytical solution (sinus wave)

If the values of λ and C_h are considered to be constant with depth and time, the soil thermal diffusivity D_{heat} (cm^2 d^{-1}) can be defined as:

$$D_{heat} = \frac{\lambda_{heat}}{C_{heat}} \tag{9.4}$$

and Eq. (9.3) simplifies to:

$$\frac{\partial T}{\partial t} = D_{heat} \frac{\partial^2 T}{\partial z^2} \tag{9.5}$$

This partial differential equation can be solved for simple boundary conditions, assuming D_{heat} constant or very simple functions for D_{heat} (Van Wijk, 1966; Feddes, 1971; Wesseling, 1987). A commonly used top boundary condition is a sinusoidally varying soil surface temperature during the year:

$$T(0,t) = T_{mean} + T_{ampl} \sin\left(\tfrac{1}{2}\pi + \omega\left(t - t_{max}\right)\right) \tag{9.6}$$

where T_{mean} is the mean yearly temperature (°C), T_{ampl} is the wave amplitude (°C), $\omega = 2\pi / \tau$ is the angular frequency, where τ is the period of the wave (365 d), t is time (d) starting January 1st and t_{max} equals t when the temperature reaches its maximum. In case of a semi-infinite soil profile with constant D_{heat} and subject to the top boundary condition according to Eq. (9.6), the solution to Eq. (9.5) is:

$$T(z,t) = T_{mean} + T_{ampl} \, e^{\frac{z}{d_{temp}}} \sin\left(\tfrac{1}{2}\pi + \omega\left(t - t_{max}\right) + \frac{z}{d_{temp}}\right) \tag{9.7}$$

where d_{temp} is the damping depth (cm), which equals:

$$d_{temp} = \sqrt{\frac{2 D_{heat}}{\omega}} \tag{9.8}$$

Model input

Variable	*Code*	*Description*
	SWHEA	Switch for simulation of heat transport
	SWCALT	Switch for method: 1 = analytical method, 2 = numerical method
T_{ampl}	TAMPLI	Amplitude of annual temperature wave at soil surface (°C)
T_{mean}	TMEAN	Mean annual temperature at soil surface (°C)
t_{max}	TIMREF	Time in the year with top of sine temperature wave (d)
d_{temp}	DDAMP	Damping depth of temperature wave in soil (cm)

9.4 Numerical solution

In reality, λ_{heat} and C_{heat} depend on the soil moisture content and vary with time and depth. Also the soil surface temperature will deviate from a sinus wave. Therefore higher accuracy can be reached by numerical solution of the heat flow equation. Numerical discretization of Eq. (9.3) is achieved in a similar way as the discretization of the water flow equation (Eq. (2.3)). SWAP employs a fully implicit finite difference scheme as described by Wesseling (1998). The soil heat flow equation is written as:

$$C_i^{j+1}\left(T_i^{j+1}-T_i^j\right)=\frac{\Delta t^j}{\Delta z_i}\left[\lambda_{i-½}^{j+½}\frac{T_{i-1}^{j+1}-T_i^{j+1}}{\Delta z_{\mathrm{u}}}-\lambda_{i+½}^{j+½}\frac{T_i^{j+1}-T_{i+1}^{j+1}}{\Delta z_\ell}\right] \tag{9.9}$$

where superscript j denotes the time level, subscript i is the node number, $\Delta z_{\mathrm{u}} = z_{\mathrm{i+1}} - z_{\mathrm{i}}$ and $\Delta z_{\mathrm{l}} = z_{\mathrm{i}} - z_{\mathrm{i+1}}$ (see Figure 3). The coefficients C_{heat} and λ_{heat} are not affected by the temperature, which makes Eq. (9.9) linear.

Both volumetric heat capacity and thermal conductivity depend on the soil composition. The volumetric heat capacity is calculated as weighted mean of the heat capacities of the individual components (De Vries, 1963):

$$C_{\mathrm{heat}} = f_{\mathrm{sand}}C_{\mathrm{sand}} + f_{\mathrm{clay}}C_{\mathrm{clay}} + f_{\mathrm{organic}}C_{\mathrm{organic}} + \theta C_{\mathrm{water}} + f_{\mathrm{air}}C_{\mathrm{air}} \tag{9.10}$$

where f and C on the right hand side of Eq. (9.10) are respectively the volume fraction (cm^3 cm^{-3}) and volumetric heat capacity (J cm^{-3} °C^{-1}) of each component. Table 4 gives values of the volumetric heat capacity for the different soil components.

Table 4 Volumetric heat capacity and thermal conductivity of the soil components.

Component	Volumetric heat capacity (J cm^{-3} °C^{-1})	Thermal conductivity (J cm^{-1} °C^{-1} d^{-1})
Sand	2.128	7603
Clay	2.385	2523
Organic	2.496	216
Water	4.180	492
Air (20°C)	1.212	22

In order to calculate C_{heat} (and λ_{heat}) in De Vries model, we need to input the percentage (by volume) of sand and clay, denoted VP_{sand} and VP_{clay} respectively. VP_{sand} and VP_{clay} are taken as percentages of the total *solid* soil matter and may differ for each soil layer. The total volume fraction of solid matter is given by:

$$\theta_{\mathrm{solid}} = 1 - \theta_{\mathrm{sat}} \tag{9.11}$$

where θ_{sat} is the saturated volumetric water content. The volume fraction of air is equal to the saturated minus the actual water content:

$$f_{\mathrm{air}} = \theta_{\mathrm{sat}} - \theta \tag{9.12}$$

f_{sand}, f_{clay} and f_{organic} are then calculated by:

$$f_{\mathrm{sand}} = \frac{VP_{\mathrm{sand}}}{100}\theta_{\mathrm{solid}} \tag{9.13}$$

$$f_{\mathrm{clay}} = \frac{VP_{\mathrm{clay}}}{100}\theta_{\mathrm{solid}} \tag{9.14}$$

$$f_{\text{organic}} = \theta_{\text{solid}} - f_{\text{sand}} - f_{\text{clay}} \qquad (9.15)$$

where Eq. (9.15) assumes that solid matter that is not sand or clay, is organic.

As shown in Table 4, the thermal conductivities of the various soil components differ very markedly. Hence the space-average thermal conductivity of a soil depends upon its mineral composition and organic matter content, as well as the volume fractions of water and air. Since the thermal conductivity of air is very much smaller than that of water or solid matter, a high air content (or low water content) corresponds to a low thermal conductivity. The components which affect thermal conductivity λ_{heat} are the same as those which affect the volumetric heat capacity C_{heat}, but the measure of their effect is different so that the variation in λ_{heat} is much greater than of C_{heat}. In the normal range of soil wetness experienced in the field, C_{heat} may undergo a threefold or fourfold change, whereas the corresponding change in λ_{heat} may be hundredfold or more. One complicating factor is that, unlike heat capacity, thermal conductivity is sensitive not merely to the volume composition of a soil but also to the sizes, shapes, and spatial arrangements of the soil particles (Hillel, 1980). SWAP employs the method of De Vries (1975) as applied by Ten Berge (1986) to calculate the thermal conductivity. A clear description of the method is given in Ashby et al. (1996). The method requires no extra input data.

At the soil surface the daily average air temperature T_{avg} is used as boundary condition. At the bottom of the soil profile SWAP assumes $q_{\text{heat}} = 0.0$.

Application of Eq. (9.9) to each node and including the boundary conditions at the top and bottom of the soil profile, results in a tri-diagonal system of equations, as shown in Annex G. SWAP solves the equations with *LU*-decomposition for tridiagonal systems (Press et al., 1989).

Model input

Variable	*Code*	*Description*
	SWCALT	Switch for method: 1 = analytical method, 2 = numerical method
T_i	TSOIL	initial temperature as function of soil depth ZH (^{o}C)
Specify for each soil layer:		
VP_{sand}	PSAND	Sand fraction in soil layer (g g^{-1} mineral parts)
VP_{silt}	PSILT	Silt fraction in soil layer (g g^{-1} mineral part)
VP_{clay}	PCLAY	Clay fraction in soil layer (g g^{-1} mineral part)
VP_{organic}	ORGMAT	Organic fraction in soil layer (g g^{-1} dry soil)