VELOCITY, TEMPERATURE AND SCALAR LENGTH SCALES IN THE CBL: OBSERVATIONS AND THEORY

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1. INTRODUCTION

The convective boundary layer (CBL) is in horizontally homogeneous and quasi-stationary conditions characterized by the velocity scale w_* and the CBL height h. Turbulent processes are assumed to be governed by the CBL turn-over timescale h/w_*, and integral length scales of turbulent statistics are of the order h. In a real atmosphere mesoscale phenomena are assumed to be irrelevant for a timespan of a few hours only, and moreover, separated by a 'spectral gap' from the CBL turbulence. Though this general view of turbulent structure of the ABL is widespread, it is difficult to find experimental evidence for it. In fact, aircraft measurements of the CBL (e.g., GATE and ASTEX, see Jonker et al. (1999)) did not confirm the existence of the spectral gap.

Jonker et al. (1999) performed a Large Eddy Simulation of the CBL and evaluated statistics of vertical velocity, potential temperature and a passive scalar. One turn-over time after the start of the simulation the length scale associated with the vertical velocity became stationary (approximately equal to h), confirming the general view that h is the only relevant length scale in the CBL. However, length scales associated with potential temperature and the passive scalar were considerably larger than h, and appeared to be steadily increasing, flattening off slightly only towards the end of the simulation. Contrary to vertical velocity, temperature and passive scalar variance kept growing during the simulation, suggesting that these turbulence variables are governed by larger timescales than the turn-over time h/w_*.

In their numerical experiment the turbulence was driven by buoyancy only. The boundary-layer height increased slightly from 700 to 800 m. Passive tracers were introduced at the top (top-down diffusion) and at the surface (bottom-up diffusion). Typical runs lasted 40 turn-over times. Arbitrary concentration fields were inferred from linear superpositions of top-down and bottom-up simulations. The magnitude of the length-scale appeared to be dependent on the ratio between the the entrainment flux and the surface flux.

The findings of Jonker et al. (1999) initiated an experimental verification in a laboratory environment and a subsequent analysis based on the Reynolds equations which will be reported here.

2. LENGTH SCALES

The general definition of an integral length scale is given by

$$L = \int_0^\infty \rho\left(r\right) dr. \tag{1}$$

where $\rho\left(r\right)$ is the normalized one dimensional spatial autocorrelation function of, for example, the concentration field defined as

$$\rho\left(r\right) = \frac{\overline{c\left(\boldsymbol{x}\right) \cdot c\left(\boldsymbol{x}+\boldsymbol{r}\right)}}{\overline{c\left(\boldsymbol{x}\right)^2}}. \tag{2}$$

The autocorrelation function is a measure of the correlation between the values of the variable c at location $\boldsymbol{x}$ and $\boldsymbol{x}+\boldsymbol{r}$ and is, due to the horizontal homogeneity and anisotropy, a function of the distance r only. The associated power spectrum $E(k)$ is given by

$$E\left(k\right) = \int_{-\infty}^{+\infty} \rho\left(r\right) e^{ik\cdot r}\, dr. \tag{3}$$

The determination of length scales from (peaks in) power spectra or from the shape or integral of correlation functions is an art in itself. Often the location of the spectral peak is uncertain, or the correlation function does not converge properly. Jonker et al. (1999)

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uses for example

$$\overline{k_c} = \frac{\int_0^\infty k E(k)\, dk}{\int_0^\infty E(k)\, dk}, \tag{4}$$

and the length-scale follows from $L \approx \frac{1}{k_c}$. This method tends to average out fluctuations that naturally occur in the spectra. Here, we shall use the shape of the correlation function between $r = 0$ and the first zero-crossing, as explained by Durand et al. (1974).

3. OBSERVATIONS

3.1 The convection tank

The experiments were carried out in a 3.2 m long, 1.6 m wide and 0.7 m deep saline convection tank (Hibberd (1996)). The aspect ratio of the tank is considered sufficient to distinguish the horizontal dimension from the boundary layer depth (typically $\sim$ 20 cm). The four sides and the bottom of the tank are constructed of 15 mm thick glass to permit visualization of the convection experiments. The tank can be filled with a variable salt water mixture which 'mimicks' the CBL upside down. The top side of the tank is covered with a 'source tray' containing salt water which leaks into the boundary layer through a porous membrane at prescribed rates, simulating the convection process. Salinity probes can be towed through the tank in horizontal and vertical directions.

An Argon-ion laser points at a rotating mirror which generates a thin horizontal sheet of laser light at arbitrary levels in the CBL. Fluorescent dye patterns visualize actual horizontal concentration fields, which are recorded (at 25 Hz) by a video camera underneath the tank and stored on tape.

3.2 The experiments

The experiment begins by putting the source tray on top of the fresh water boundary layer and draining the tank at a very low rate. This releases the salty source tray mixture and initiates the convection.

Dye can be added to the tray solution (bottom-up diffusion) and, during the tank filling process, in the stable entrainment layer, to simulate top-down diffusion. 11 experiments of $\sim$ one hour were carried out with varying (dye) entrainment ratios. From the salt concentration measurements mean (vertical) profiles were determined which revealed a realistic CBL structure consisting of a steadily growing (from 18 to 26 cm) well-mixed layer, capped by an inversion. Also the vertical profiles of salt concentration variance were determined and were found to be in agreement with similar atmospheric data. Dye concentration fields were collected at 6 levels (from .2h to 1.2h) with a resolution of 600 x 350 pixels which covers an area of approximately 90 x 60 cm.

3.3 Preliminary results

Mean vertical variance profiles of the passive scalar fields were determined. We found that the variance of the scalar generally increased with time, whereas the variance of the salinity ('potential temperature') quickly reached steady state, in accordance with Jonker et al. (1999). We noted further that there is a significant difference in magnitude of the (dimensionless) variance among the different experiments. While the variance ranges to approximately 5 for the bottom-up experiments, the variance for the top-down runs typically reaches a value of 50. This is only in qualitative agreement with the LES results of Jonker et al. (1999), who showed that the variance of the top-down scalar was roughly only 5 times bigger than that of the bottom-up scalar.

The variance profiles for the bottom-up experiments are quite similar in shape and magnitude to aircraft measurement of humidity during the AMTEX, Limagne and Beauce experiments (Lenschow and Agee (1976), Tuzet et al. (1983)).

There are some indications that for scalar flux-ratios of $\sim$ -0.25 to -0.5 the variance increases at a slower rate. The slower progress appears more clearly at lower heights in the boundary layer suggesting that the increase in length scale is also height-dependent. More specifically, for the bottom-up experiments there is no apparent height dependence in the length scale. It is almost constant throughout the boundary layer with a typical value of $2.3h$. The length scale for the top-down experiment on the other hand shows a stronger height dependence. The maximum value of the average length scale is attained close to the surface, with a value of approximately $3.4h$.

Also Jonker et al. (1999) found lower values in mid-boundary-layer, at a ratio of approximately -0.5. They also observed that the length scale is height-dependent. For a bottom-up scalar, the length scale should increase with height, whereas for the top-down scalar the opposite was found.

The difference in behaviour between the scalar and the salinity variance is quite convincing: as soon as the tank reaches a quasi-steady state after a couple of turn-over times, the variance of the salinity does not change significantly anymore, whereas the variance of the passive scalar keeps increasing.

Spectra of the passive scalar have a significant different shape than those from the salinity data. Large-scale fluctuations in the scalar fields dominate the spectrum, while the salinity spectra tend to have their maximum energy at smaller wavelengths. Another observation is that when time passes, the gradual increase in variance can mostly be attributed to an increase of the low-wavelength part of the scalar spectra .

4. THEORY

The evidence that the concentration variance is ever increasing suggests that the timescale associated with concentration variance dynamics is appreciably larger than that of velocity. (the turn-over time in the CBL, $\approx h/w_*$). This requires a more detailed study of the associated governing dynamical equations.

4.1 The Reynolds averaged equations for scalar length scales and variances

In an incompressible flow ($\nabla.\tilde{\boldsymbol{u}} = 0$), the conservation equation of a passive scalar is

$$\frac{\partial \tilde{c}}{\partial t} + \tilde{\boldsymbol{u}}\nabla \tilde{c} = \kappa \Delta \tilde{c}, \tag{5}$$

where κ is the molecular diffusivity.

We denote the actual concentration and velocity field by $\tilde{c}$ and $\tilde{\boldsymbol{u}}$, respectively, and make a decomposition into mean (capitals) and fluctuating (lower case) quantities, $\tilde{c} = C + c$, $\tilde{\boldsymbol{u}} = \boldsymbol{U} + \boldsymbol{u}$. The equation for the fluctuating concentration, c, is then

$$\frac{\partial c}{\partial t} + U.\nabla c = -\boldsymbol{u}.\nabla c - \boldsymbol{u}\nabla C + \nabla.\overline{\boldsymbol{u}c} + \kappa \Delta c. \tag{6}$$

From this equation we may construct a rate equation for $\overline{c(\boldsymbol{x},t)c(\boldsymbol{x'},t)}$, where $\boldsymbol{x'}$ is an arbitrary other location in a horizontal plane at level z, thus $\boldsymbol{x} = \boldsymbol{x}(x,y,z)$ and $\boldsymbol{x'} = \boldsymbol{x'}(x',y',z)$:

$$\frac{\partial \overline{cc'}}{\partial t} = -2\overline{wc'}\frac{\partial C}{\partial z} - 2\kappa \overline{\frac{\partial c}{\partial z}\frac{\partial c'}{\partial z}}. \tag{7}$$

Primed variables depend on primed independent variables (x' and y'). In equation 7, all variables (apart from $\partial C/\partial z$ which depends on z and t only) depend on r, z and t, where r is the magnitude of the vector which connects $\boldsymbol{x}$ and $\boldsymbol{x'}$. The derivation is analogous to the one for the velocity correlation tensor, as is for instance discussed in Batchelor (1953), leading to the Von Karman-Howarth equation. From equation 7 it is now possible to construct a governing equation for the length-scale, using its definition (1):

$$\frac{\partial L\overline{c^2}}{\partial t} = -2\alpha\phi\frac{\partial C}{\partial z}L - \frac{2}{3}\,\epsilon_c\, L. \tag{8}$$

The flux term is shortly written as $\phi = \overline{wc}$.

In the derivation of (8) it is further assumed that $\int_0^\infty \overline{wc'}\,dr \sim \phi L_\phi)$, and that $\kappa\,\overline{(\frac{\partial c}{\partial z}\frac{\partial c'}{\partial z})} = \frac{1}{3}\,\epsilon_c\,\rho_{cc'}$, where $\rho_{cc'(r)}$ is a correlation function with integral scale $\propto L$. The dissipation rate is defined as $\epsilon_c = \kappa\,\overline{(\frac{\partial c}{\partial x_j})^2}$ and α is defined by

$$\alpha \equiv \frac{\int_0^\infty \overline{wc'}\,dr}{\phi\, L}, \tag{9}$$

and is assumed to be approximately constant. LES results indicate that α varies between 0.7 and 0.8. Using the same approximations, we can also formulate the (well-known) expression for the variance as:

$$\frac{\partial \overline{c^2}}{\partial t} = -2\phi\frac{\partial C}{\partial z} - 2\,\epsilon_c. \tag{10}$$

This set of equations (equation 8 and 10) is the starting point of the analysis of the dynamics of a passive scalar in the CBL.

First we note that the equation for the variance (10) contains a production term ($-2\phi\frac{\partial C}{\partial z}$) which is absent in the corresponding equations for the velocity variance, since all terms $\frac{\partial U_i}{\partial x_j}$, are zero. This allows, contrary to the velocity variances, the passive scalar variance to grow.

It should be emphasized that the passive scalar production term is positive when the flux is down-gradient. However, in a region where the flux is counter-gradient, the production term is negative, which, according to (10), inhibits the increase of the variance (the variance may still grow, however, due to turbulent transport terms, which are neglected here).

Further, in a quasi steady situation the production term is approximately constant (in time), since it is the product of the (approximately stationary) flux and mean vertical gradient. As a consequence the dissipation timescale is the governing timescale in equation 10, and in view of the experimental and numerical evidence, it must be considerably larger than h/w_* in most conditions. The standard parameterization for the dissipation is

$$\epsilon_c = k_\epsilon \overline{c^2}/\tau, \tag{11}$$

where τ is the turn-over time scale h/w_* and k_ϵ a constant of order one. This would imply that according to (10) the scalar variance reaches steady state after a couple of turn-over times, clearly in contradiction with the observations and LES results. Therefore, guided by the above set of equations, we propose to parameterize the scalar dissipation as

$$\epsilon_c = k_\epsilon \overline{c^2} L / w_*, \tag{12}$$

with k_ϵ a constant of order unity. With this modified parameterization, we have solved for $\overline{c^2}$ and L from equation 8 and 10. For the production term we have adopted an expression originating from countergradient theory (van Dop and Verver (2001)). We were able to obtain preliminary results which are very similar to those in Jonker et al. (1999) and also to the experimental results presented here. The top-down variance is overestimated by a factor of 2 compared to the results ofJonker et al. (1999), but is in good agreement with the experimental results of the bottom-up variance. However, in order to produce these results we had to put α equal to 0.45, a value which is appreciably lower than estimates based on the LES concentration fields. We have to look more closely both into the LES simulations and the approximations used in the derivation of equation 8 and 10 to resolve this issue.

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