CODE OF PRACTICE FOR
THE DESIGN OF OPEN WATERCOURSES
AND ANCILLARY STRUCTURES
CODE OF PRACTICE FOR
THE DESIGN OF OPEN WATERCOURSES
AND ANCILLARY STRUCTURES

DRAWN UP BY A WORKING PARTY ON OPEN WATERCOURSES
CONSTITUTED BY THE ROYAL NETHERLANDS INSTITUTE OF ENGINEERS
AND THE ROYAL NETHERLANDS SOCIETY OF AGRICULTURAL SCIENCE

ROYAL VAN GORCUM Ltd. / ASSEN / THE NETHERLANDS 1964
H. VEEENMAN & ZONEN Ltd. / WAGENINGEN / THE NETHERLANDS
International Institute for Land Reclamation and Improvement
Institut International pour l'Amélioration et la Mise en valeur des Terres
Internationales Institut für Landgewinnung und Kulturtechnik
Instituto Internacional de Rescate y Mejoramiento técnico de Tierras

P.O. BOX 45 / WAGENINGEN / THE NETHERLANDS
FOREWORD

During recent years, The Netherlands have undertaken an increasing number of reclamation and land improvement works. The engineers who had to design these works or check these designs felt the need for the collection and assimilation of the various principles for the design of open canals for drainage into a code of practice. In order to meet these wishes from the profession, the Section for Soil Conservation and Improvement of the Royal Netherlands Institute of Engineers and the Section for Land and Water use of the Royal Netherlands Society of Agricultural Science decided, in 1954, to commission a Working Party to carry out studies that might lead to such a code of practice. This Working Party consisted of engineers working in this field employed by government departments, by consulting engineers and by land improvement societies, as well as of representatives from the Technological University at Delft and the Agricultural University at Wageningen.

The draft resulting from the work of this Working Party was discussed in a joint meeting in November 1956 of the Sections mentioned above. The Code of Practice was thereafter published in the Dutch language. A collection of nomographs to
facilitate the design of open canals was made by the Working Party and published separately in four languages (English, French, German and Dutch) under the name of 'Graphs for the calculation of open watercourses and structures built in them' (publisher: Royal Van Gorcum Ltd., Assen, The Netherlands).

A number of foreign engineers studying in the reclamation branch of the International Course in Hydraulic Engineering at Delft, The Netherlands, and in the Post-graduate Training Centre on Land Drainage at Wageningen (Neth.), as well as some foreign engineers who were using the graphs, asked for the Code of Practice to be translated into the English language.

The present results refer to conditions found in The Netherlands, especially in its lower areas. These regions have, characteristically, very flat slopes and a moderate rainfall and evaporation, whereas supply of irrigation water is only needed incidentally.

Thus, this Code of Practice will, in many regions of the world, need to be adapted to local conditions. Nevertheless, the above-mentioned Sections decided to publish an English translation as requested.

The reasons for this decision were as follows: there are several other regions of the world where engineers could benefit directly from the use of this code of practice with regard to their own designs. In order to make the present edition more useful some parts of the original which refer more specifically to conditions found only in The Netherlands, were omitted. For the same reason small additions have been made in other places.

The editors hope that this edition in the English language will further the international exchange of experience between professional colleagues.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1</td>
<td>9</td>
<td>Introduction</td>
</tr>
<tr>
<td>Section 2</td>
<td>10</td>
<td>General design of watercourses</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1. Drawing up plans and calculations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2. Water level in the channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.3. Hydraulic gradient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4. Velocity of flow</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.5. Curves in channels</td>
</tr>
<tr>
<td>Section 3</td>
<td>14</td>
<td>Cross-section of watercourses</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.1. Introduction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.2. Minimum cross-section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.2.1. Arable land</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.2.2. Grassland</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.3. Shape of the cross-section, Inclination of the side-slopes</td>
</tr>
<tr>
<td>Section 4</td>
<td>17</td>
<td>Calculation of the cross-section, the hydraulic gradient or the discharge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.1. Introduction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.2. Formulae to be used</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3. Lined and non-vegetated channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3.1. Formulae to be used</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3.2. Bed roughness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3.3. Thickness of the boundary layer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4. Vegetated channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4.1. Introduction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4.2. Formulae to be used</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4.3. Bed-roughness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4.4. Rules for the bed-roughness of vegetated channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.5. Calculations for non-vegetated channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.6. Calculations for vegetated channels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.7. Checks necessary</td>
</tr>
<tr>
<td>Page</td>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>24</td>
<td>Section 5</td>
<td>Structures: culverts and bridges</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.1. Influence on the watercourse</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.2. Dimensions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.3. Shapes of the cross-sections. Minimum sizes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.4. Calculation of culverts and bridges</td>
</tr>
<tr>
<td>28</td>
<td>Section 6</td>
<td>Structures: weirs</td>
</tr>
<tr>
<td>30</td>
<td>Note 1</td>
<td>Drawing up a plan for the construction or improvement of watercourses</td>
</tr>
<tr>
<td>34</td>
<td>Note 2</td>
<td>The choice of hydraulic gradient</td>
</tr>
<tr>
<td>38</td>
<td>Note 3</td>
<td>Theoretical basis</td>
</tr>
<tr>
<td>54</td>
<td>Note 4</td>
<td>Minimum cross-sections of ditches and small watercourses</td>
</tr>
<tr>
<td>56</td>
<td>Note 5</td>
<td>Measurements for determining the coefficient of bed-roughness</td>
</tr>
<tr>
<td>64</td>
<td>Note 6</td>
<td>The influence of the choice of the coefficient of bed-roughness on the design and the water levels subsequently occurring</td>
</tr>
<tr>
<td>68</td>
<td>Note 7</td>
<td>Calculation for the construction of the backwater curve for non-uniform flow</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

The Code provides directions for determining the shape and dimensions of channels and structures such as culverts, bridges and small weirs, for which the location and the quantity to be conveyed are considered given. In assessing the accuracy required for the calculation, it should be borne in mind that in general the quantities to be conveyed are known only approximately.

With respect to structures, only hydraulic problems are dealt with, no directions on constructional aspects being provided.

The Code does not relate to the design and calculation of longer closed conduits (e.g. tile-drains, sewers).

Neither does it relate to the design of the siting of channels nor to their arrangement within the drainage or water-supply area. Furthermore, it gives no indication regarding the determination of the quantities to be conveyed through the channels.

It is impossible to give binding and explicit rules for every case that may occur. For this reason, even if this code is followed, the designs will have to be made by qualified engineers.
2. GENERAL DESIGN OF WATERCOURSES

2.1. DRAWING UP PLANS AND CALCULATIONS

When dealing with a region of some extent, care should be taken that in site-plans and corresponding tables and formulae, the subdivisions of the area, the channel reaches, and the structures are represented systematically and conveniently. The representation must be able to accommodate alterations.

In many cases – particularly where the treatment of sloping areas is involved – it is advisable to draw longitudinal sections of the channels in question. These drawings should mention heights, gradients, discharges, location and description of cross-sections etc.

Calculations should be made systematically and clearly, and to this effect the use of standard forms is recommended.

2.2. WATER LEVEL IN THE CHANNELS

Before the plans are worked out, it must be decided what water levels in the channels are essential or are most desirable. A distinction should be made in this
General design

connection between channels for supply and channels for discharge of water. In the former, the water level at tapping points should be such that water in the fields forming part of the area to be supplied with water by the channel concerned can be brought up to and maintained at the necessary level for the irrigation method to be applied.

In discharge channels, the water level is determined by the drainage-depth. This drainage-depth is the difference between site elevation and the most desirable water level in the channels, and is determined by the nature of the soil and the requirements set by agriculture. This optimum water level is defined with respect to a datum plane and is known as 'polder level'.

It will frequently prove impossible to establish a single polder level for regions of some extent. This may be caused by the differing requirements with respect to the drainage or by differences in elevation in the site. Such regions are then subdivided into sections, each with its own level.

As a rule, a higher polder level is aimed at during the summer months than in winter time. Winter levels determine the plan for water-discharge, summer levels the nature and size of the structures necessary to keep the water at the required level for as long as possible.

If the same channels are used for both the discharge and the supply of water, the winter level is determined by the requirements of water-discharge and the summer level by the requirements of water-supply.

2.3. Hydraulic gradient

By hydraulic gradient is meant the fall in a channel per unit length; the fall is the difference in height of the water surface at two places in a channel.

In calculating the dimensions of watercourses it is usual to proceed from the flow (quantity to be supplied or discharged per unit time \( Q \)) and an assumed value for the hydraulic gradient \( S \).

Decisive in establishing the gradient in supply canals is the water level occurring at the tapping point in the watercourse from which the water is taken and the level needed to irrigate adjoining fields. In this connection attention should be paid to requirements which agriculture sets with respect to the maximum permissible variation in the optimum water level. These requirements will be influenced by the nature of the soil\(^1\).

\(^1\) If irrigation water is to be pumped up, an attempt will be made to obtain small gradients in order to restrict the delivery head. Velocities will then be small and cross-sections of the conduits will be large. In this case a comparison of costs will be decisive in determining the gradient.
To determine the gradient in discharge channels it is usual to proceed on the one hand from the permissible excess of the polder level at the upstream end of the watercourse and on the other hand from the permissible depression of the polder level near the outfall. The above excess depends on considerations of soil-type and vegetable-physiology, whilst the assumed design discharge and storage capacity also exert an influence. Neither this excess nor the depression of the polder level can be determined exactly. In establishing the permissible depression, the capacity and delivery head of the pumping station are important. Alternatively the dimensions of the sluice and low water levels downstream may have to be taken into account.

In sloping areas, the hydraulic gradient is often determined by the permissible velocity (see Section 2.4.).

In order to limit the amount of earth-moving it is usually desirable to choose the highest permissible value of the hydraulic gradient.

In cases where the side-slopes of the channels are very sensitive to changes in the water level it may be desirable to choose a hydraulic gradient as small as possible (see Note 2).

The gradient is sometimes calculated to a higher degree of accuracy than is justified by the accuracies inherent in the value assumed for the discharge and in the method of constructing the channel.

This is especially true when it appears that the bed of the channel should be nearly horizontal.

In practice the bed is then made horizontal.

When there is a sharp fall near the outfall, the calculation of the water level with the formula of uniform flow is no longer justified. In such cases see Note 7.

2.4: VELOCITY OF FLOW

With high discharges which occur regularly, the velocity of flow should remain below a critical value.

This is the speed at which the material of the bed and banks is not quite set in motion.

In polder areas (see Section 2.2.), the said speed will generally not be attained with the gradients available.

The critical speed for the various types of soil encountered in the Netherlands may be taken as:
Cohesive heavy soil (clay, loam, loess) 0.6 -0.8 m/sec.
Sandy clay and cohesive sandy soil 0.3 -0.6 m/sec.
Fine sand 0.15-0.3 m/sec.
Coarse sand 0.2 -0.5 m/sec.
Stiff peat 0.3 -0.6 m/sec.
Soft peat 0.15-0.3 m/sec.

The upper limit of the above values may be assumed when the channel banks are vegetated or protected in some other way. This also applies to deep channels.

The velocity must be moderated whenever the stability of the side-slope is reduced by seepage.

If the channel is allowed at least sometimes to discharge solid matter a higher value may be taken for the velocity because of the quantity of solid matter to be transported.

(The physical magnitude which determines whether the bed-particles are to remain at rest is the shear stress; this varies with the square of the velocity: see Note 3).

In choosing the maximum velocity, the inclination of the side-slopes should be taken into account (see Section 3.3.).

2.5. CURVES IN CHANNELS

In a curved reach of a channel the danger of scour of the bed and banks is greater than in a straight reach with an equal velocity. If the velocity in the straight reach therefore approaches the critical value, either the bend will have to be given a large radius, or the cross-section will have to be enlarged or the side-slope which is exposed to attack will have to be protected. At the same time (and this is usually more important than the behaviour of the current in the bend) the situation in the channel immediately adjacent to the bend should be considered. It may be useful to provide a revetment and to insert transition curves (see also Note 3).
3. CROSS-SECTION OF WATERCOURSES

3.1. INTRODUCTION

For practical reasons a minimum cross-section is maintained in small channels. It should of course be checked to what extent this section is adequate. The dimensions of larger channels are determined by a calculation which allows for a normal condition of maintenance which is suited to the circumstances. In addition to calculating the watercourse for design winter discharge, whereby the normal maintenance condition is considered, it may be advisable to make a calculation for a design summer flow with a less favourable maintenance condition or a somewhat degenerate section.

3.2. MINIMUM CROSS-SECTION

The following is applicable to water-carrying drainage ditches (field and road ditches) in the Netherlands and regions with approximately the same conditions of topography, climate, agriculture and agricultural economy. A distinction is made between arable land and grassland. The transition from minimum cross-
section to calculated cross-section should preferably be introduced at a conspicuous point (e.g. field boundary) (see Note 4).

3.2.1. Arable land

In arable land with tile-drains, the bed lies at least 0.2 to 0.4 m below the invert of the drain outfall, which in turn usually lies 0.8 to 1.2 m below ground level. In arable land without tile-drains, the bed lies 0.2 to 0.4 m below the required groundwater level with the design discharge. A longitudinal bed-gradient is introduced only in sloping sites or long plots; in this case, the smallest depth mentioned above applies to the upstream end. In other cases the bed is made horizontal. The width of the bed is 0.5 m, or may go up to 1 m depending on the method of construction. In general, ditches with bed-widths of 0.5 m suffice for areas with a maximum of 15 to 20 ha; ditches with bed-widths of 1 m suffice for areas with a maximum of 20 to 30 ha. The inclination of the side-slopes is 1 vertical: \( p \) horizontal, where \( p \) depends on the nature of the soil (see Section 3.3), and varies from \( 3/4 \) to 2. For dyke and road ditches the inclination of the side-slopes on the dyke or road side is 1 : \((p + 1/2)\).

3.2.2. Grassland

The bed lies at least 0.5 m below the optimum ditch water level in dry periods. A minimum dimension applies to the width at the water surface which generally depends on local usage of cattle holding or maintenance procedures. For the inclination of the side-slopes see Section 3.2.1.

3.3. Shape of the cross-section. Inclination of the side-slopes

A trapezoidal cross-section is chosen in simple cases. Experience has taught that except in very tenacious ground, a trapezoidal section with rather steep side-slopes does not last. The side-slope assumes a curved shape, the lower part showing a flatter gradient. Such small deformations prove to have little influence on the hydraulic gradient and depth of water in the channel. For larger channels, a composite cross-section showing banks with a single or double dog-leg is recommended. The inclination then increases with the height above the bed.
The following slopes are recommended with the types of soil (such as occur in the Netherlands) and the critical speeds shown (no transport of material, see Section 2.4.):

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Velocity m/sec</th>
<th>$\rho$</th>
<th>Where side-slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive heavy soil (clay, loam, loess)</td>
<td>0.6 -0.8</td>
<td>3/4 -2</td>
<td></td>
</tr>
<tr>
<td>Sandy clay and cohesive sandy soil</td>
<td>0.3 -0.6</td>
<td>1 1/2 - 2</td>
<td>1/2</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.15-0.3</td>
<td>2-4, sometimes more</td>
<td></td>
</tr>
<tr>
<td>Coarse sand</td>
<td>0.2 -0.5</td>
<td>1 1/2 - 3</td>
<td></td>
</tr>
<tr>
<td>Stiff peat</td>
<td>0.3 -0.6</td>
<td>1-2</td>
<td></td>
</tr>
<tr>
<td>Soft peat</td>
<td>0.15-0.3</td>
<td>2-4, sometimes more</td>
<td></td>
</tr>
</tbody>
</table>

1) $\rho = 3/4$ only for the minimum cross section (see Section 3.2).

In selecting an inclination for the side slopes it should be borne in mind that:

1. the deeper the channel, the flatter its side-slopes should be
2. the side-slope is designed flatter if there is seepage towards the channel in special cases, advice on the mechanical properties of the soil should be sought
3. the side-slope is designed more steeply if there is vegetation on the bed (for example in ditches that frequently stand dry)
4. the side slope is designed more steeply when the bank is protected. (This may be necessary if, for example, wave-attack is to be expected)
5. the side slope is designed flatter if there is a possibility of cattle trampling on it (see Section 3.2.2.)
6. the side-slope is designed flatter if the ditch borders on a roadway having practically no verge, or if allowance has to be made for heavy machines driving alongside the ditch.
4. Calculation of the Cross-Section, the Hydraulic Gradient or the Discharge

4.1. Introduction

It should be ascertained whether the situation that occurs can be treated with the methods of calculation that apply to steady uniform flow of turbulent type motion with turbulent flow. The motion is steady and the flow uniform when in the reach of channel under consideration, the velocity $v$, the cross-sectional area of flow $A$ and the hydraulic gradient $S$ are everywhere the same and do not change with time.

No indications are provided here for unsteady or non-uniform flow. The influence of the presence of structures in the channel on the motion of water is discussed under Section 5.1.

Non-turbulent motion occurs only in channels with a very small cross-section, with low velocities or with viscous liquids; in watercourses such as are encountered in practice, it will never occur.

In lined channels (flumes) and in channels with a variable cross-section, the nature of the motion will have to be given due consideration.
4.2. Formulae to be used

Of the three magnitudes: cross-section, hydraulic gradient and discharge, two are known (or assumed), the third is calculated.

De Chézy’s formula is employed:

\[ v = C \sqrt{RS} \]  

(1)

Further: \( Q = vA \), or: \( Q = AC \sqrt{RS} \)

(2)

where \( v = \) average velocity of flow (m/sec)
\( C = \) a coefficient (m\(^{1/2}\)/sec)
\( R = \) hydraulic radius of the cross-section (m)
\( S = \) gradient of the channel (dimensionless)
\( Q = \) discharge (m\(^3\)/sec)
\( A = \) cross-sectional area of flow (m\(^2\))

The coefficient of De Chézy \( C \) can be calculated from the following:

a. a logarithmic formula after White and Colebrook.

\[ C + 18 \log \frac{6R}{a + \delta/7} \]

(3)

where \( a = \) a length, characteristic of the bed-roughness (m)
\( \delta = \) thickness of the boundary layer (m)

Distinction is made between channels with hydraulically rough bed, for which the thickness of the boundary layer \( \delta \) is negligible compared with the roughness \( a (\delta \approx 0) \) and those with a hydraulically smooth bed for which the opposite is true \( (a \approx \delta) \).

b. the exponential formula of Manning:

\[ C = k_M \cdot R^{1/6} \]

(4)

where \( k_M = \) a coefficient related to the bed-roughness (m\(^{1/3}\) sec).

Equation (1) now acquires the form:

\[ v = k_M \cdot R^{2/3} S^{1/2} \]

(5)

In the literature, in addition to \( k_M \), the reciprocal value \( n = 1/k_M \) is often encountered (see Note 3).
4.3. Lined and non-vegetated channels

4.3.1. Formulae to be used

The above mentioned formulae (2) and (3) are used here:

\[ Q = AC \sqrt{RS} \]

and

\[ C = 18 \log \frac{6R}{a + \delta/7} \]

The exponential formula (4) is applied less frequently in this case (see Note 3).

4.3.2. Bed roughness

\( a \) is defined as the radius of spheres which, imagined as covering the bed if they were attached to the bed, would cause the same frictional resistance as the bed in question. For non-vegetated and lined beds, the following table (6) shows values of \( a \) expressed in mm:

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulders or stones</td>
<td>1/2 - 1 x diameter of stones, depending on the spacing</td>
</tr>
<tr>
<td>Unrippled bed of sand</td>
<td>1/2 x diameter of the grains</td>
</tr>
<tr>
<td>Rippled bed of sand</td>
<td>3/4 - 1 x height of ripples</td>
</tr>
<tr>
<td>Smooth earth bed, very clean</td>
<td>10 - 25</td>
</tr>
<tr>
<td>Gravel</td>
<td>5 - 25</td>
</tr>
<tr>
<td>Set stone</td>
<td>2.5 - 10</td>
</tr>
<tr>
<td>Masonry and brickwork</td>
<td>0.25 - 2.5</td>
</tr>
<tr>
<td>Concrete (depending on smoothness)</td>
<td>1 - 10</td>
</tr>
<tr>
<td>Concrete (plastered)</td>
<td>0.1 - 0.25</td>
</tr>
<tr>
<td>Concrete (spun)</td>
<td>0.05</td>
</tr>
<tr>
<td>Steel (severely rusted)</td>
<td>0.5 - 1</td>
</tr>
<tr>
<td>Steel (rivetted)</td>
<td>0.25 - 1</td>
</tr>
<tr>
<td>Steel (welded or with countersunk heads)</td>
<td>0.05 - 0.25</td>
</tr>
<tr>
<td>Steel (asphalted)</td>
<td>0.01 - 0.0255</td>
</tr>
<tr>
<td>Asbestos cement</td>
<td>0.25 - 1</td>
</tr>
<tr>
<td>Wood (rough)</td>
<td>0.25 - 1</td>
</tr>
<tr>
<td>Wood (planed)</td>
<td>0.1 - 0.25</td>
</tr>
</tbody>
</table>
4.3.3. **Thickness of the boundary layer**

This is found from the formula:

\[ \delta = \frac{v}{\sqrt{gRS}} \]  

(7)

where \( g \) = acceleration due to gravity (9.8 m/s²/sec)
\( \nu \) = kinematic viscosity. This varies with the temperature of the water as follows:

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>1.8</td>
<td>1.3</td>
<td>1.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The boundary-layer exerts its influence only if \( \delta > \frac{1}{3} a \).
This occurs with low velocities and smooth beds (small \( a \)).
In earth channels \( \delta \) may be neglected.

4.4. **Vegetated channels**

4.4.1. **Introduction**

The banks and bed are often covered with vegetation. Floating weeds may also be present and this influences the size of the hydraulic radius. However, the calculation is made using the unrestricted cross-section, that is the clean or excavated section, and the restrictions are expressed by including their effect in the value for the roughness coefficient. In this case, the use of \( k_M \) is recommended.

4.4.2. **Formulae to be used**

Here the formula to be used is thus:

\[ Q = A k_M R^{2/3} S^{1/2} \]  

(from (2) and (5) of 4.2.).

4.4.3. **Bed-roughness**

In judging the roughness of existing channels, use may be made of the following values, which have been derived from measurements in small watercourses (depth < 0.8 m):
Description

Very clean channel (transition from non-vegetated to vegetated). In principle, bed and side-slopes entirely clean.

Here and there a single reed clump, other vegetation or some algae may occur. Down to a few cm below the water surface some grass may grow on the side-slopes or hang in the water.

Clean channel. Bottom and sideslopes with thin vegetation (a few cm) or weeds in places, but with many bare patches. Few reeds.

Lightly vegetated channel. Light, transparent but continuous vegetation on bed and side-slopes, in which water-thyme usually occurs and some reeds and algae. Occasionally with current channels in the bed and vegetation.

Moderately vegetated channel. Thick vegetation on bed and side-slopes. Current channels in the vegetation, which usually consists largely of water-thyme. Bed visible only in places through the vegetation.

Fairly heavily vegetated channel. Cross-section largely overgrown, here and there up to the surface. Sometimes a continuous fringe of reeds along each side. Water-thyme and algae.

Very heavily vegetated channel. Fringes of reeds or rushes along the sides. Very dense overgrowth of water plants in the centre of the cross-section.

Within each class, the higher values of \( k_M \) apply where the cross-section is more regular and no appreciable variations occur in the dimensions of the cross-section, lower values of \( k_M \) apply for irregular sections which show fairly marked variations in size. (see Notes 5 and 6).

4.4.4. Rules for the bed-roughness of vegetated channels

In the calculation, it is usual to proceed from a design winter discharge and a normal condition of maintenance: where necessary (see Section 3.1.) the calculation is repeated for a design summer discharge and a less favourable condition of maintenance.

Channels in soils poor in nutrients (e.g. sand, oligotrophic peat) or containing toxic components (e.g. acid-sulphate, saline or alkali soils) or in soils of a physically unfavourable structure (e.g. boulder clay) usually acquire vegetation less rapidly than those in fertile soils of a loamy or clayey nature or those through which ‘fertile’ water flows.
This difference should be expressed in the design roughness. Allowance may also have to be made for the fact that water which contains few nutrients or is noxious (for example owing to salt or refuse matter) hinders the growth of vegetation in the conduits. 

In dealing with supply canals it will have to be decided whether or not it is justified to associate the design discharge with the ‘normal’ condition of maintenance.

In watercourses with vegetated (or partially vegetated) side-slopes and a bed that is almost without vegetation, the roughness of the various components of the wetted perimeter varies greatly; in this case an average value of $k_M$ should be taken.

This consideration, as well as the fact that shallow channels acquire vegetation more rapidly than deep ones is reason for making $k_M$ dependent on water-depth in normally maintained channels.

In the following Table 8, recommended values for $k_M$ are given for various sizes of channel and types of soil. For intermediate conditions, the $k_M$ value can be suitably adjusted.

<table>
<thead>
<tr>
<th>I. Small channels (water-depth &lt; 0.8 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. soil poor in nutrients or otherwise unfertile</td>
</tr>
<tr>
<td>b. heavy soil or ‘fertile’ water</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II. Medium-sized channels (water-depth 0.7-1.7 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. soil poor in nutrients or otherwise unfertile</td>
</tr>
<tr>
<td>b. fertile soil or ‘fertile’ water</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III. Large channels (water-depth &gt; 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 - 50</td>
</tr>
</tbody>
</table>

1) It is often preferable to calculate large channels as non-vegetated (see Section 4.2.)

4.5. Calculations for non-vegetated channels

The formula: $Q = AC \sqrt{RS}$ ((1) and (2)) is applied in conjunction with:

$$C = 18 \log \frac{6R}{a + \delta/7}$$

If the cross-section and gradient are known (or assumed) and the discharge is required:
Calculations

\( \alpha \) is taken from Table 6, see Section 4.3.2;
\( \delta \) is calculated where necessary from (7), see Section 4.3.3;
\( C \) is calculated from (3), see Section 4.2., or read off from Graph 1 in Note 3;
\( Q \) is calculated from (2), see Section 4.2.

If cross-section and discharge are known (or assumed) and the gradient is required then \( \delta \) is put at zero.
If necessary, after \( S \) has been found, the calculation is repeated with an improved value for \( \delta \).
If gradient and discharge are known (or assumed) and the cross-section is required, then \( R \) must first be estimated in order to be able to determine \( C \).

4.6. CALCULATIONS FOR VEGETATED CHANNELS

The formula (5) is applied; see Section 4.2.
\( C \) is not calculated, but the relation between \( S, Q, k_M, b \) (bed-width) and \( d \) (water depth) together with \( A \) (cross-sectional area of flow) and \( v \) can be read off from graphs\(^1\).

4.7. CHECKS NECESSARY

After completing a calculation according to the given directions it should be checked:
1. whether the flow is sufficiently uniform; (for non-uniform flow, see Note 7);
2. whether the calculation should be repeated for a lower discharge and a less favourable condition of maintenance, see Table 8;
3. whether there are grounds for repeating the calculation for a degenerate cross-section.

\(^1\) See the collection of graphs with information on use which is published separately.
5. STRUCTURES: CULVERTS AND BRIDGES

5.1. INFLUENCE ON THE WATERCOURSE

Every structure gives rise to a certain backwater effect in the watercourse: it should be determined to what extent this is permissible in view of the level of land in the area.

Upstream of the structure, the backwater curve approaches the hydraulic gradient (for uniform flow) in the (undisturbed) watercourse asymptotically. At a certain distance from the weir the backwater effect is so small that it can be neglected for practical purposes. Very roughly, this distance, i.e. the length of the backwater curve, can be approximated by a rule of thumb:

\[ L = \frac{2z}{S} \]  

(9)

where

- \( L \) = length of the backwater curve (m)
- \( z \) = rise in water level caused by the structure (m)
- \( s \) = hydraulic gradient in undisturbed watercourse

It is recommended that not more than half of the drop is used in the structures.
It needs hardly be mentioned that the magnitude of the permissible rise is also dependent on economic factors; the smaller the permissible rise in level, the greater the cross-sectional area required for the flow.
In case of great rises of the water level, see Note 7.
The exit velocity must be determined, taking into consideration possible scour of the bed and banks of the watercourse (see Section 3.3.); it may be economical to carry out protection works. Improvement of the flow at entry and exit by erecting wing-walls is generally expensive, but sometimes has to be considered.

5.2. Dimensions

The width of the structure should be chosen such that the cross-sectional area of flow keeps within the wetted cross-section of the channel unless this would cause the rise in water level or the velocity of flow to become unacceptably high. In the latter case, the channel will have to be enlarged in the neighbourhood of the structure.

With respect to the elevation, the following recommendations are made:

a) If the height exceeds the water depth by more than 0.10 m, the difference should be equally divided i.e. the invert of the culvert should be laid a distance below the bed of the channel equal to the height of the soffit of the culvert above the water surface;

b) If the height does not exceed the water depth by more than 0.10 m the invert of the culvert should be at the same level as the bed of the channel;

c) In exceptional cases, if the height of the culvert is less than the depth of water in the watercourse (in syphons, for example) the culvert should be laid entirely under water.

5.3. Shapes of the Cross-sections. Minimum sizes

For small culverts numerous cross-sections are available: round, egg-shaped, elliptical, compound or rectangular. The first two are economical in relatively deep watercourses. The costs of the various alternatives should be compared. As a general rule the diameter of a circular section should not be less than 0.40 m; under roads and embankments it is recommended that the diameter should be not less than 0.50 m.
5.4. **Calculation of Culverts and Bridges**

For these the following formula holds:

\[ z = (\zeta_n + \xi_f + \xi_x) \frac{vo^2}{2g} \]  

or  

\[ v = \mu \sqrt{2gz} \]  

and  

\[ Q = vA \]  

where  

\[ \mu = \frac{1}{\sqrt{\zeta_n + \xi_f + \xi_x}} \]  

In the formulae:  

- \( z \) = loss in head caused by the structure (m)  
- \( v \) = velocity of flow in culvert (m/sec)  
- \( \zeta_n \) = entry-loss coefficient (dimensionless)  
- \( \xi_f \) = friction-loss coefficient  
- \( \xi_x \) = exit-loss coefficient  
- \( Q \) = discharge m³/sec  
- \( A \) = wetted cross-section of culvert (m²)

The values of the loss coefficients may be chosen as follows:

a. Entry loss caused by extraction at entry:

- for rectangular boundary: \( \zeta_n = 0.5 \)  
- for rounded opening \( \zeta_n = 0.2 \)

b. Friction loss is determined by  

\[ \xi_f = \frac{2gL}{C^2R} \]  

where  

- \( l \) = length of the culvert (m)  
- \( C \) = DeChézy’s coefficient, to be calculated from formula (3), see Section 4.2.  
- \( R \) = hydraulic radius of the culvert (m).

c. Exit loss coefficient \( \xi_x \) may be put as \((1 - a)^2\) where \( a \) is the ratio of the wetted cross section in the culvert or bridge to that in the watercourse immediately downstream.

For commonly occurring culverts of trade material the following values of \( \mu \) can be substituted in equation (11):

- for very short culverts \( \mu = 0.8 \)  
- for culverts up to 20-30 m \( \mu = 0.7 \)
Culverts and bridges

The calculation from (11) and (12) can be assisted by a graph. For simple cases the above leads to the equation:

\[ Q = \mu A \sqrt{2gz} \] (15)

where \( Q \) = discharge through the culvert or bridge (m³ / sec).

\( \mu \) = a coefficient

\( z \) = head-difference (m)

Where the friction loss may be neglected (in short culverts and bridges, the following values for \( \mu \) (formulae 11 and 15) may be taken as approximately correct.

<table>
<thead>
<tr>
<th>Bed-level at structure</th>
<th>Bed-level at structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same as that in channel</td>
<td>Higher than that in channel</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sides</th>
<th>( \mu )</th>
<th>Sill</th>
<th>Sides</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>0.80</td>
<td>Rectangular</td>
<td>Rectangular</td>
<td>0.72</td>
</tr>
<tr>
<td>Rounded</td>
<td>0.90</td>
<td>Rounded</td>
<td>Rectangular</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rounded</td>
<td>Rounded</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Calculation may be by means of a graph.

For less simple cases, where the approach velocity may not be neglected or intermediate piers are present, handbooks of hydraulics should be consulted or advice from a hydraulics laboratory should be sought.

1) See the collection of graphs published separately.
6. STRUCTURES: WEIRS

To complete this code, a short summary of the calculation of weirs is included.

For the situation of the weirs, indications are provided by the longitudinal section (see Section 2.1.).
No directions are given here to assist in the choice of type and the construction.

The backwater curve can be determined by the methods of calculation applicable to non-uniform flow. These are not dealt with in this Code. In practice, formula 9 gives an approximation; see Section 5.1. and also Note 7.

The flow over a weir with horizontal crest can be in principle calculated for simple cases from the formula: \( Q = 1.7 \, m \, b \, h^{3/2} \),

where \( Q = \) flow over the weir (m\(^3\)/sec.)
\( m = \) a coefficient, depending on the shape of the weir structure; for somewhat rounded crest and wing-walls the value 1.1 may be assumed
\( b = \) width of the weir (m)
\( h = \) upstream water level with respect to the crest of the weir.
Calculation may be by means of a graph\(^1\).

For less simple cases (e.g. approach velocity not negligible, broad-crested weir = standing wave flume, drowned weir, side-contraction, sharp crests), handbooks of hydraulics should be consulted or advice from a hydraulics laboratory should be sought.

\(^1\) See the collection of graphs published separately.
NOTE 1

DRAWING UP A PLAN FOR THE CONSTRUCTION OR IMPROVEMENT OF WATERCOURSES

In drawing up plans for drainage or water-supply, it is advisable to work according to a lucid system.

For improvements in drainage, the boundaries of the area draining to the outfall structure are first of all established. When there is more than one outfall structure, each area draining relative to a structure is denoted separately.

Inside each area, a separate notation is applied to each subsidiary catchment whose excess water discharges into the watercourses lying in the area. These component catchments are bounded by the natural and artificial watersheds found in the area. The natural watersheds are determined by the geohydrology of the area, artificial ones are due to the presence of roads, embankments etc.

In cases where several outfall structures are present it is advisable to leave some room for additions between the last symbol in the first area and the first symbol in a subsequent area, in connection with possible later extensions which can occur especially with schemes for land re-allocation.
Draw up a plan

The watercourses through which the run-off has to be discharged are provided with a symbol at the upstream end as well as the downstream end, providing there is space available on the drawing. It is recommended that a different denotation should be used for watercourses which are used exclusively for water-supply. Watercourses which are used both for drainage and supply receive the denotation applicable to the drainage system, but in addition, provided there is space available on the drawing, the denotation for the supply system is also represented. This denotation with symbols proceeds right up to the last subdivision which is supplied with water.

The reach of channel immediately upstream of the outfall is considered as the lowest reach of the main channel. All adjacent reaches of channel up to an upstream end point are also taken as belonging to the main water channel. The greatest possible length of channel should be chosen unless the special situation of a certain watercourse or a channel having its own particular name makes it preferable to consider such a watercourse as the main water channel. Only one main water channel is designated even if several watercourses meet at the outfall.

The various symbols for the subdivisions of the area and the watercourses, like those for the structures and cross-sections to be mentioned later, always follow a downstream sequence.

The denotation is continued down to where a tributary channel flows into the main channel. Upstream of the confluence the same rules hold for the tributary as have been described for the main water channel. Every branch is treated in the same fashion. When the main water channel has been reached again, the denotation is continued in the same sequence.

As a rule it is advisable to supply structures which bound upon the channel with a designation different from those which lie in the channel.

The points in the watercourses at which cross-sections have been drawn are also denoted. A cross-section is drawn such that the righthand side of the drawing represents the right bank, looking downstream.

Especially in areas with irregular contours it may be advisable to use longitudinal sections, provided that here too the system of denotation which has been adopted is continued. These sections are especially useful in areas where the required drainage-depth is obtained by backing-up the water profile by means of weirs. In Appendix 2 is to be found a worked example of an arbitrary longitudinal section in which an attempt has been made to present all known data.

With respect to the data for the calculation of the dimensions of watercourses and structures, it is recommended that these should be presented in a separate table.
Appendix 1. Plan for a drainage scheme
### Number of the cross-section

| Number of the cross-section | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **Distances**               |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Left bank                   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Right bank                  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

### Existing bed level of the channel reach

| Existing bed level of the channel reach |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

### Existing direct level of the structure or earth level of bridge

| Existing direct level of the structure or earth level of bridge |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

### Condition

<table>
<thead>
<tr>
<th>Condition</th>
<th>g, good</th>
<th>p, poor</th>
<th>p, poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Number of the channel reach

| Number of the channel reach | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **Total area draining to the channel reach in ha** |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **Hydraulic gradient**      |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **Water depth**             |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **Bed width**               |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **Average velocity in m/sec.** |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **Erosion water level**     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

### Notes

*To be filled in as necessary: sortif level of bridge, width of weir, level of crest with respect to Datum level, water level on the crest with respect to Datum level*
In general the same rules hold when drawing up a plan for water-supply although the sequence of symbols is reversed. Thus it is necessary in designing an irrigation scheme first to determine the area of land requiring water. From the size of this area and the amount of water required in mm/day can be found the total flow of water required in m³/sec.

APPENDIX 1 shows a water-management scheme based on a system usual in the Netherlands. APPENDIX 2 shows how the system is represented in a longitudinal section.
NOTE 2

THE CHOICE OF HYDRAULIC GRADIENT

In any design it first has to be established what drainage-depth is normally desirable. This is determined by agricultural factors (choice of crop, soil profile etc.). This means that in a flat area a horizontal polder level is the most desirable. In order for water to flow however, a certain gradient is required. In the design, the design discharge is fixed for each reach of the channel and then a certain gradient and a value for the bed-roughness is assumed. From this a certain cross-section for the conduit may be deduced. How big should this hydraulic gradient be? In choosing the hydraulic gradient consideration has to be given to the following:

1. the permissible velocities of flow;
2. the permissible decrease in the drainage-depth at the upstream end of the main channel;
3. the permissible increase in the drainage-depth at the outfall of the main channel;
4. the variations in the water levels;
5. the earth-moving and the area of land taken up;
6. the cross-sections of the existing channels.

1. *The permissible velocities of flow*

   Proceeding from the critical velocities, the permissible hydraulic gradient decreases with increasing discharge.
   In sloping areas the critical velocity can be decisive in the choice of the gradient. In flat areas it will only be in exceptional cases that consideration has to be given to the critical velocities. In particular these cases will occur when discharges are high and when the soils are non-cohesive.

2. *The permissible decrease in the drainage-depth*

   The drainage-depth is usually determined for normal winter or summer conditions.
   In the case of drainage, the design discharge is usually a discharge which occurs infrequently and only for short periods.
   For these short periods a certain decrease in the drainage-depth is permissible. This decrease is dependent upon several factors which are not yet accurately known. There are indications that it may be allowed to reach 10-20% of the optimum drainage-depth.
   It is evident that the hydraulic gradient increases with increasing water level chosen at the upstream end.
   In the case of water supply a decrease occurs for longer periods but then the drainage depth is less so that in this case 10-20% can also be allowed.
   In connection with the amount of earth-moving, the greatest possible gradient is desirable (see Section 5).

3. *The permissible increase in the drainage-depth*

   At the outfall a certain increase is permissible in the chosen drainage-depth. The size of this increase is dependent, among other things, upon the construction and running costs of the outfall structure (e.g. running costs of the pumping station).
   In the winter the increase will hardly be determined by agricultural factors. Caution should however be exercised in peaty and sandy areas.
   In cases where agricultural factors are of no importance, the increase is determined
by the construction and maintenance costs of the channels compared with those of the outfall structure. A greater drainage-depth makes greater hydraulic gradients possible and this almost always leads to a smaller amount of earth-moving and thus lower construction costs (see also Section 5).

In addition to the costs of construction, maintenance costs should also be taken into consideration.

4. The variations in the water levels

When there is no discharge, the water surface is in theory horizontal. The amount of variation in the water levels increases with increasing value chosen for the hydraulic gradient corresponding to design flow.

In supply channels, agricultural interests are served by small variations and therefore by a low hydraulic gradient. In discharge channels, special attention must be paid to effect of variations in water level on the side-slopes.

5. The amount of earth-moving and the area taken up

Theoretically there exists a certain hydraulic gradient for which the amount of earth-moving is a minimum.

If the critical velocities and the permissible departures from the drainage-depth are taken as design criteria a gradient is usually arrived at which is smaller than that corresponding to the minimum amount of earth-moving.

In practice it may therefore be said that the total gradient should be chosen as great as possible. Moreover a channel with a high gradient occupies less surface area than one with low gradient.

6. The cross-sections of the existing channels

If an existing conduit already has a big cross-section, the design will naturally be adapted to suit. A correct choice for the gradient will help to achieve this.

Conclusions

1. To limit earth-works the highest permissible value for the gradient should usually be chosen.
2. In hilly areas this gradient is limited by the critical velocities. In flat areas the velocity of flow is hardly ever a design factor in the choice of gradient.
3. In connection with Section 1 it is advisable to proceed from the highest permissible departures from the optimum drainage-depth.

4. The permissible departures are usually absolute values so that long channels need a flatter gradient and in short channels a steeper gradient is permissible.
NOTE 3

THEORETICAL BASIS

1. Introduction

In order to satisfy the conditions of uniform flow, the forces acting on the flowing water must be in equilibrium.

Gravity supplies a component force acting in the direction of flow. If the slope of the bed and of the water surface which runs parallel is $S$, the component of gravity is $G \sin S \approx G.S$.

Friction acts in a direction opposite to that of flow. The frictional resistance depends on the roughness and the size of the surface over which the water flows. The size of the surface is dependent on the shape of the cross-section and the occurrence of obstacles in the watercourse; the roughness is dependent on the smoothness of the bed and the nature of the obstacles. Here of course there is a big difference between a vegetated and non-vegetated cross-section. If vegetation is present, the leaves and stems are in fact additional surfaces over which the water sweeps. The continual variation in the vegetation makes it impossible to
express this surface in the dimensions of area, so that here too the surface of the
bed and banks is used but with a suitably estimated roughness coefficient.
In channels with no vegetation the bed-roughness may be equated to that of a
bed covered with spheres of a certain diameter. This diameter is then the quantity
in which the roughness of a non-vegetated cross-section is expressed.
The starting point for the theoretical approach to the problem is the condition
for equilibrium in a 1 m long element of the flow considered.

\[ \frac{\rho gAS}{\tau} = \frac{\rho gA}{P} S = \rho gRS \]

where \( \rho \) = density of the water
\( g \) = acceleration due to gravity
\( \tau \) = shear stress

In the following treatment the quantity \( \sqrt{\tau/\rho} \) will frequently occur. This
quantity has the dimensions of a velocity and is therefore known as the shear
velocity \( v_f \). \(^1\)

Thus \( v_f = \sqrt{\tau/\rho} = \sqrt{\rho RS} \)

In what follows we shall treat successively the velocity distribution in the
cross-section, the formulae for discharge and average velocity, the calculation of
channels with vegetation, the maximum permissible velocity in watercourses
with no bed transport and the bends in a watercourse.

\(^1\) For an elementary description of this theory, see for example W. KAUFMANN, Fluid
2. The distribution of velocity over the cross-section

In laminar flow the velocity gradient $\frac{dv}{dz}$ is directly proportional to the magnitude of the shear stress. The coefficient of the proportionality is the dynamic viscosity.

In turbulent flow the relation between velocity gradient and shear stress is less simple. Physical considerations lead to the relation:

$$\tau = \rho (xz)^2 (\frac{dv}{dz})^2$$

So here the value of $\tau$ is dependent on the distance $z$ from the nearest boundary and the density, $\rho$, of the fluid. The viscosity is now of no consequence. The value of the number $x$ does not depend on the properties of the fluid; it is approximately equal to 0.4.

By integration an expression can be found for the velocity distribution over the cross-section. The calculation proceeds as follows:

$$\frac{dv}{dz} = \frac{1}{xz} \sqrt{\frac{\tau}{\rho}} = \frac{v_f}{xz}$$

$$v = \frac{v_f}{x} \ln z + \text{constant}$$

The velocity distribution in the section appears logarithmic. In experiments too, a logarithmic curve is always found, but it is also found that this curve does not intercept the line $v = 0$ at the boundary but at a small distance $z_o$ from the boundary.

In the immediate proximity of the bed and banks the velocity distribution is no longer logarithmic. There, it depends on the nature of the surface of bed and banks.

By substituting $z = z_o$ for $v = 0$, the integration constant can be eliminated from the last equation.

The velocity distribution then becomes:

$$v_z = \frac{v_f}{x} \ln \frac{z}{z_o}$$
3. Formulae for discharge and average velocity

By integration of the latter formulae over the cross-section of a channel a formula for the magnitude of the discharge is obtained. In this way a different formula is obtained for every shape of cross-section. From the above diagram it appears that in turbulent flow the velocity is more evenly distributed over the cross-section than in laminar flow. It might therefore be expected that the influence of the shape of the cross-section on the magnitude of the discharge is less in the turbulent condition.

In order to verify this, the integration is carried out for the two most extreme possibilities, that is for the very wide channel with uniform depth and for the closed circular pipe.

a. Very wide rectangular channel

Consider a strip of unit width:
\[ q = \frac{h}{x} \int_{\xi}^{\eta} \frac{v_f}{\xi} \ln \frac{z}{z_0} \, dz = \frac{v_f}{\xi} \left[ h \ln \frac{h}{z_0} - h \right] = \]
\[ = \frac{v_f h}{\xi} \ln \frac{h}{z_0} \]
\[ \nu = \frac{q}{h} = \frac{v_f}{\xi} \frac{h}{\xi} \frac{h}{\xi} \frac{h}{\xi} \frac{0.37 R}{z_0} = \]
\[ = \frac{\sqrt{g R S}}{\xi} 0.37 R \ln \frac{z_0}{z_0} = \]
\[ = \frac{\sqrt{g}}{\xi} 2.3 \log \left( \frac{0.37 R}{z_0} \right) \sqrt{RS} = \]
\[ = 18 \log \left( \frac{0.37 R}{z_0} \right) \sqrt{RS} \]

So here we have the well known formula of De Chézy, \( \nu = C \sqrt{RS} \), where \( C = 18 \log \frac{0.37 R}{z_0} \)

b. **Conduit with regular polygonal cross-section**

The circular pipe also comes under this heading, since a circle can be considered as a polygon with an infinite number of sides.

Consider one of the isosceles triangles of which the polygon consists:

\[ \text{Fig. 4} \]
Theoretical basis

\[Q = \int_{0}^{h} v_{f} \ln \frac{z}{z_{o}} \frac{h - z}{h} b dz =\]

\[= \frac{b v_{f}}{k} \left[ \int_{0}^{h} \ln \frac{z}{z_{o}} dz - \frac{1}{h} \int_{0}^{h} z \ln \frac{z}{z_{o}} dz \right] =\]

\[= \frac{b v_{f}}{k} \left[ h \left( \ln \frac{h}{z_{o}} - 1 \right) - \frac{1}{2} h \left( \ln \frac{h}{z_{o}} - 1/2 \right) \right] =\]

\[= \frac{h b v_{f}}{2k} \left[ \ln \frac{h}{z_{o}} - 3/2 \right] = \frac{h b v_{f}}{2k} \ln \frac{0.22 h}{z_{o}}\]

\[v = \frac{Q}{1/2 \ h b} = \frac{v_{f}}{k} \ln \frac{0.22 h}{z_{o}} = \frac{v_{f}}{k} \ln \frac{0.44 R}{z_{o}} =\]

\[= 2.3 \frac{\sqrt{g}}{k} \log \frac{0.44 R}{z_{o}} \sqrt{RS} =\]

\[= 18 \log \frac{0.44 R}{z_{o}} \sqrt{RS}\]

Here too therefore we find the formula of De Chézy:

\[v = C \sqrt{RS}\]

where now

\[C = 18 \log \frac{0.44 R}{z_{o}}\]

Hence it appears that the shape of the cross-section has little influence on the value of \(C\). The shapes of cross-sections occurring in practice lie in between the extremes calculated. When one formula is required which holds with reasonable accuracy for all types of cross-section then the best way is to take an average which lies between the two. The following is such a formula:

\[C = 18 \log \frac{0.40 R}{z_{o}}\]

From experiments it is apparent that \(z_{o}\) depends both on the size of the protruberances on the bed \(a\) and on the thickness of the laminar boundary layer \(\delta\).
Physical considerations give for the thickness of the boundary layer:

\[ \delta = \frac{12\nu}{v_f} = \frac{12\nu}{\sqrt{g} RS} \]

According to White & Colebrook, the magnitude of \( z_0 \) is approximately equal to:

\[ \frac{1}{15} a + \frac{1}{105} \delta \]

When this is substituted, we obtain the simplified formula:

\[ C = 18 \log \frac{6R}{a + \delta/7} \]

Where \( \delta/7 \) is small in relation to \( a \), the bed is described as relatively rough and the formula becomes:

\[ C = 18 \log \frac{6R}{a} \]

Where \( a \) is small in relation to \( 1/7 \delta \), the bed is described as relatively smooth and the formula becomes:

\[ C = 18 \log \frac{42R}{\delta} \]

In the transitional region, the logarithmic formula should be used in its complete form. The values of \( C \) can be read off from the accompanying Graph 1.

4. Calculation of vegetated channels

It was already noted in the introduction that in vegetated channels importance must be attached to phenomena quite different from that of friction along the bed particles.

The vegetation influences the roughness of the side-slopes and the cross-sectional area of flow to such an extent that exact calculations can hardly be put in hand.

The conclusions which are founded on a theoretical basis should therefore be considered very critically.

Every theory which is propounded for vegetated channels seems to founder on the irregular nature of the vegetation. Within a limited area the milieu of the plants is usually uniform. The ratio between the plant dimensions and the water depth
Theoretical basis

Graph 1
will vary little for different watercourses in such an area, provided that these are relatively shallow.

The bed-roughness \( a \) in the logarithmic formula is directly proportional to the dimensions of the plant and in watercourses with varying depth it will thus have different values. The observations which have been carried out show that the ratio between the dimensions of the plants and the dimensions of the watercourse can be characterised better by the coefficient \( (k_M) \) in the formula of Manning mentioned below.

The observations mentioned were performed in several small vegetated channels under the auspices of the Civil Engineering Department of the Agricultural University at Wageningen, The Netherlands. The results of these measurements clearly showed that the relation between the vegetation and the theoretically assumed particle diameter in the logarithmic formula was scarcely tenable. In view of these measurements and numerous investigations abroad, for vegetated channels we have to be content with the empirical formula of Manning.

Moreover in practice it appears necessary that nomographs should be available for the calculation of open watercourses. In such nomographs it is required that the discharge may be read off directly from the bed-slope and the dimensions of the channel, but it appears that it is impossible to construct such a graph for the logarithmic formula. One would have to work with two nomographs simultaneously.

For the use of the exponential formula of Manning it is found that the production of nomographs is quite straightforward. This is another reason for employing this formula.

In view of the above considerations the working party chose the Manning formula for the calculation of vegetated conduits:

\[
v = k_M R^{2/3} S^{1/2}
\]

so that the \( C \) in De Chézy's formula \( (v = C \sqrt{RS}) \) becomes:

\[
C = k_M R^{1/6}
\]

For further discussion of the value of \( k_M \) and the measurements performed, reference should be made to Section 4.4. in the Code.

5. Limitation of the Maximum Permissible Water Velocity in a Channel with No Bed Transport

a. Very wide channel with loose sand bed

The particles lying on the bed must remain at rest. This is the case when the shear
stress exerted by the water on the bed is less than the shear stress which is needed to put the particles in motion.

The shear stress exerted by the water on the bed is:

\[ \tau = \rho g RS = \frac{\rho g}{C^2} \]

The force necessary to move one spherical particle of diameter \( d \) is:

\[ F = f \frac{\pi d^3}{6} (\rho_z - \rho) g \]

where \( f \) is a ‘friction coefficient’ and \( \rho_z \) is the density of the material. Let \( \phi \) be the portion of the bed surface occupied by loose particles. Then the number of loose particles per unit area of the surface is:

\[ \frac{4\phi}{\pi d^2} \]

The force needed per unit area of the bed surface to bring particles into motion is then

\[ \tau_c = \frac{2}{3} f \phi (\rho_z - \rho) gd \]

The magnitude of the constant \( 2/3 f \phi \) may be derived from the many measurements which have been carried out. According to Lane\(^1\) a value may be assumed (allowing for a coefficient of uncertainty) from 0.05 to 0.065 or an average of 0.06. The formula then becomes:

\[ \tau_c = 0.06 (\rho_z - \rho) gd \]

The diameter \( d \) of the particles corresponds to a sieve aperture which retains 25% of the material by weight.

b. Channel with trapezoidal cross-section in coarse loose sand

For the very wide channel we were able to suppose that the shear stress was uniformly distributed over the whole width of the bed.

\(^1\) E. W. Lane, Proceedings American Society Civil Engineers, Sept. 1953, Vol. 79, Separate no. 280 (Fig. 3).
For a narrower trapezium-shaped cross-section this is no longer the case. O. J. Olsen and Q. L. Florey, collaborators with Lane, investigated the distribution of the shear stresses over the bed and side-slopes of trapezoidal cross-sections. Their results can be represented graphically in a simple manner.

The largest shear stresses along bed and side-slopes are given by:

$$\tau = IgdS$$

where $d$ is the water depth and $I'$ is a constant which for a certain shape of cross-section may be found from Graph 2.

The force needed to move the particles along the bed again follows from the formula for $\tau_c$. In loose coarse material the force needed to move the particles along the side-slopes is not determined only by the shear stress exerted by the flowing water. In addition, the component of gravity acting in the plane of the side-slopes now plays a part. Depending on the gradient $\alpha$ of the side-slopes and on the angle of internal friction of the soil ($\Theta$), the particle on the side-slope will be set in motion by a smaller shear stress than would be needed for a bed-particle.

For the sake of simplicity, the gradient of the side-slopes is allowed for by introducing a coefficient $k$. This is equal to the ratio between the shear stress necessary to set in motion the particles of the side-slope and the shear stress necessary to move the bed-particles.

Fig. 5

The critical shear stress for the side-slopes thus becomes:

$$\tau_c = 0.06 \; k \; (\rho_e - \rho) \; gd$$

In order to calculate $k$ we proceed from an angle of internal friction $\Theta$ and side-slopes inclined at an angle $\alpha$. The friction along the bed which is necessary to set the particle in motion is then:

$$F_b = G \; tg \; \Theta$$
On the side-slope is exerted a total frictional force: \( F_t = G \cos \alpha \tan \Theta \); this can be thought of as resolved into a gravity component which makes the particle roll downwards \( F_g = G \sin \alpha \) and a frictional force \( F \) in the direction of flow which is determined by the formula:

\[
F = \sqrt{F_t^2 - F_g^2}
\]

\[
F = k F_b = \sqrt{G^2 \cos^2 \alpha \tan^2 \Theta - G^2 \sin^2 \alpha}
\]

\[
k G \tan \Theta = \sqrt{G^2 \cos^2 \alpha \tan^2 \Theta - G^2 \sin^2 \alpha}
\]

\[
k = \cos \alpha \sqrt{1 - \frac{\tan^2 \alpha}{\tan^2 \Theta}}
\]

c. Channel in cohesive soil

The shear stress exerted by the water on the bed and side-slopes can for this case again be calculated from the formula \( \tau = g \Delta S \). The graphs of Olsen and Florey can prove very serviceable here (see Graph 2).

The critical shear stress which is necessary to set in motion the material of the sides and bed is however no longer given by the simple formula for \( \tau_c \). In this case we have to refer to empirical values for the critical shear stress.

Owing to the greater cohesion shown by the soil, it is here no longer necessary to introduce an extra coefficient to allow for the side-slopes.

d. Channel in fine sandy bed

This case stands more or less intermediate between the cases mentioned under \( b \) and \( c \). If the channel has just been put into service case \( b \) will occur.

If water has previously been flowing through the channel, it is quite probable that the fine particles have already been bound together by transported silt. In this event, case \( c \) is present. Seepage out of the channel works conducive to this bonding whilst seepage towards the channel hinders it. Especially in this case a soil mechanics investigation will often be necessary.
6. BENDS IN WATERCOURSES

Where bends are present in open watercourses, in addition to the velocity in the direction of flow, there is also a component perpendicular to this direction. The bed friction will therefore also have a component perpendicular to the direction of flow.

In the bend therefore, the total shear stress will be greater than in a straight reach of the flow.

In the design of bends in open watercourses one of the following two principles may serve as basis:

a. In the bend a somewhat greater bed-shear may be accepted than in a straight part of the watercourse.

The same cross-section is preserved in the bend as in a straight reach. The shear stress therefore increases and the consequent greater likelihood of material transport is tolerated to a certain degree.

b. The cross-section is increased in the bend by an amount such that the bed-shear in the bend is no greater than in a straight section of the watercourse.

For a further theoretical treatment of the above principles reference should be made to the article by Ir. L. Van Bendegom in ‘De Ingenieur’ of 24 Jan. 1947 entitled: ‘Enige beschouwingen over riviermorphologie en rivierverbetering’ (Some considerations on river morphology and river improvement).

Proceeding from the equations derived in the above article, the cases a and b may be treated more fully.

a. If the cross-section in the bend is not increased, a minimum radius has to be calculated such that a permissible augmented shear stress is not exceeded.

For this we use the following symbols:

\[ F = \text{total shear stress} \]
\[ F_x = \text{shear stress in the straight reach of channel} \]
\[ B = \frac{F}{F_x} \]
\[ C = \text{coefficient in De Chézy’s formula} \]
\[ d = \text{water depth} \]
\[ r = \text{radius of the bend} \]
\[ g = \text{acceleration due to gravity} \]
\[ p = \text{percentage by which the shear stress increases in the bend} \]
According to the treatment of Van Bendegom:

\[ B = \sqrt{1 + \frac{0.0392 C^4 d^2}{r^2 g^2}} \]

By equating \( B \) with \( 1 + \rho \) we get:

\[ r = \frac{0.039 C^2 d}{g} \frac{1}{\sqrt{\rho^2 + 2\rho}} \]
From Fig. 6 it is seen that if the bed shear in the bend may exceed that in a straight reach by only a small amount, the minimum radius becomes quite large (if $\phi \to 0$, then $\frac{1}{\sqrt{\phi^2 + 2\phi}} \to \infty$).

Example: $d = 2$ m, $C = 40$ m$^{1/2}$/sec. A permissible shear stress increase of 1% ($\phi = 0.01$) gives a value of $r \approx 88$ m as a minimum.

b. If the shear stress is not allowed to increase in the bend, the increase in cross-section is calculated as follows:

The average velocity in the bend is $v'$. If we assume that the increase in cross-section at the bend is not large, this allows the same value of $C$ to be kept and then ($\tau$ is proportional to $v^2$):

$$\frac{g v'^2}{C^2} = \frac{g v^2}{B C^2}$$

or

$$v' = \frac{v}{\sqrt{B}}$$

Thus if care is taken that in the increased cross-section the average velocity is equal to or less than the normal average velocity divided by $\sqrt{B}$, then the bed-shear in the bend will not exceed that in the normal straight reach.

This is true only in so far as the increase in section is not so great that the value of $C$ essentially alters.

In this treatment, wide watercourses have been assumed, so that $R$ has been equated to $d$.

Attention must always be paid to the transition zone from curved to straight channel. Owing to the current, serious erosion of the bed and side-slopes can occur, so that extra provisions should always be considered here.
NOTE 4

MINIMUM CROSS-SECTIONS OF DITCHES AND SMALL WATERCOURSES

In designing drainage and irrigation schemes, not only does consideration have to be made of the channel dimensions which have been calculated, but also of those which have been determined on practical grounds. All parts of a certain area should have a connection with the main watercourse such that the discharge or supply of water will be satisfactory.

The dimensions of minimum cross-sections should be made dependent on the method of construction, weed-growth and consequent systems of cleansing. Once these dimensions have been fixed, it can be found what area may be drained or supplied by these minimum sections.

The ditches in catchments smaller than the area found do not then need to be calculated. The ditches in catchments bigger than this area should be calculated.

Ditches with no discharging function are not considered here because factors other than drainage or supply then play a part.

For circumstances in The Netherlands, Graph 3 shows the relation between the dimensions of minimum ditches and the drainage area.
Graph 3. Relation between the dimensions of small ditches and the area to be drained
NOTE 5

MEASUREMENTS FOR DETERMINING THE COEFFICIENT OF BED-ROUGHNESS

If, in a straight channel, a steady, uniform condition of flow is present, then by measurement of the gradient, the discharge and the wetted section, the $C$ can be calculated and hence the coefficients of bed roughness which occur in the various formulae which exist for $C$.

Such measurements and calculations are available from various investigators e.g. Bazin, Ramser, Scobey, Ree and Palmer. On this subject the following remarks may suffice.

The investigations of Bazin\(^1\) were chiefly related to non-vegetated channels; nevertheless his formula used to be employed quite generally in the Netherlands.

The great number of measurements performed under the direction of Ramser\(^2\)

\(^1\) H. Bazin, Étude d'une nouvelle formule pour calculer le débit des canaux découverts. Annales des Ponts et Chaussées 1897.

Coefficient of bed-roughness still forms the most important source of information on the subject of the bed-roughness coefficient. These measurements, totalling something more than 560, were carried out in vegetated channels and by means of photographs and a description the conditions in each channel were defined.

Similarly, very many data can be taken from the 175 measurements in vegetated channels which were performed under the direction of SCOBEY.

The tables of $k_M$ values found in the various hydraulics books are founded for a large part on the above mentioned measurements of RAMSER and SCOBEY. All these $k_M$ values must be regarded as a total resistance coefficient, since both investigators expressed in this coefficient not only the resistance due to bed-roughness but also all other resistances in the channels.

Further, the measurements of Ree and PALMER may be mentioned, conducted in shallow ditches in which the bed and side-slopes were covered with different types of grass. These experiments were aimed at acquiring knowledge of the degree of erosion, the permissible velocity and the resistance for each separate type of grass.

The great difficulty in such measurements is to describe objectively the total resistance of the ditch concerned, which often varies from place to place.

In various countries investigations are still being performed into this question. As examples, the publications of Cowan and Ree may be mentioned.

Cowan attempted to resolve the total resistance into 6 components.

REE proposed that the height of the vegetation relative to the water depth should be taken into consideration.

There is no doubt that in the future many publications on this subject may still be expected.

In the period 1953-1956 the departments of Civil Engineering and Agronomy of the Agricultural University, Wageningen have conducted, under the leadership of J.M. GEENSE, a number of measurements in open earth channels. The aim of these measurements was to determine the total resistance in ditches with

various degrees of vegetation and the investigation of the increase in this resistance when a winter-vegetation changes to a summer-vegetation.

In total 49 small channels were measured, all with a hydraulic radius less than 0.4 m. The measurements were carried out principally during winter so that usually no heavy vegetation was present in the ditches. The ditches were straight with approximately parallel flow and usually had a fairly constant cross-section; the length of the experimental reach was as a rule 200 m. The discharge was measured at two points, the wetted section at three or more points, and the hydraulic gradient was determined every 20 m by double levelling of the water surface. Two photographs were taken of every ditch and all the factors which together give rise to the total resistance were noted. The most important measuring instruments were: a levelling instrument, a staff fitted with a water-level needle, a wooden and later an aluminium gauging bridge. The discharge was measured with the small Ott current meter.

From these data, C was determined for each channel and hence $k_M$, $a$ and $\alpha$ were calculated using the formula of Manning, the logarithmic formula and the formula of Bazin.

In addition, the Department of the Waterstaat concerned with stream gauging supplied the results of 17 measurements for inspection by the working party. These were measurements in various small streams in The Netherlands with a hydraulic radius between 0.12 and 0.48 m, depths of water which mainly lay between 0.15 and 0.55 m, and the width of the water surface in the various streams (Geul, Gulp, Oostrumse Beek and Rode Beek) almost always lay between 3 and 8 m.

In general the $k_M$ values recommended in the Code are based on the measurements of the Agricultural University; for a summary of the results of these measurements reference may be made to Section 4.4.3. of the Code.

From the measurements of the Waterstaat in the Geul and the Gulp, average $k_M$ values were found which usually lay between $k_M = 22$ and $k_M = 38$. These however apply to very shallow cross-sections which were not constant, a winding course and bed and banks covered with very coarse gravel. Although the sections were clean, the above-mentioned factors naturally resulted in a fairly high resistance. In the Oostrumse Beek, as a result of scour channels and the very shallow cross-section, average $k_M$ values were found of between 21 and 55.5.

Much less variation in resistance was met with in the Rode Beek and the Bornse Beek, entirely clean channels with a constant cross-section and with banks which consist wholly or partly of concrete. Here $a$ values were determined which lay between 1.25 and 4 mm.
Ramser recommended $k_M = 25$ for small channels with a fertile bed which leads to rapid growth of vegetation, with a very small dry-water flow and with annual cleansing of the channels. The Code gives the same value for $k_M$ for winter-discharge in small channels with a fertile bed.

In irrigation channels the vegetation can increase very sharply in a short time; in an irrigation channel in the North East Polder 10 days after cleansing $k_M$ had already fallen to 18.3.

In the accompanying photographs and descriptions, the results of a few measurements are reproduced; in the given values for $a$, $k_M$, and $C$ are included all the resistances which occurred in the channels concerned.

Extending the number of measurements in small channels further has little to commend it. However, it is urgently required that a better insight is gained into the total resistance of medium and large channels in the Netherlands.
Fig. 7

\[ a = 0.0023 \text{ m} \]
\[ k_M = 45 \text{ m}^{1/3}/\text{sec} \]
\[ C = 42 \text{ m}^{1/3}/\text{sec} \]

Cross-section: completely regular and constant.
Vegetation: none. Side-slopes covered with concrete slabs.
On bed a thin layer of mud.
Extremely clean cross-section.
Soil type: humous sand.
Fig. 8
\[ a = 0.018 \text{ m} \]
\[ k_M = 42 \text{ m}^{1/3}/\text{sec} \]
\[ C = 28.9 \text{ m}^{1/3}/\text{sec} \]
Cross-section: regular and constant.
Vegetation: almost absent.
Very clean cross-section.
Soil type: sand.

Fig. 9
\[ a = 0.06 \text{ m} \]
\[ k_M = 34 \text{ m}^{1/3}/\text{sec} \]
\[ C = 27.8 \text{ m}^{1/3}/\text{sec} \]
Cross-section: regular and constant
Vegetation: bed and side-slopes here and there covered with algae.
Clean cross-section.
Soil type: sand.
Fig. 10
\[ a = 0.10 \text{ m} \]
\[ k_M = 24 \text{ m}^{1/3}/\text{sec} \]
\[ C = 16.8 \text{ m}^{1/4}/\text{sec} \]
Cross-section: approximately regular and constant.
Vegetation: moderate.
Soil type: sand.

Fig. 11
\[ a = 0.25 \text{ m} \]
\[ k_M = 16 \text{ m}^{1/3}/\text{sec} \]
\[ C = 11.85 \text{ m}^{1/4}/\text{sec} \]
Cross-section: semi-irregular.
Vegetation: moderate to heavy.
Soil type: sand.
Fig. 12

\[ \sigma = 0.40 \text{ m} \]
\[ k_M = 8 \text{ m}^{1/3} / \text{sec} \]
\[ C = 5.8 \text{ m}^{1/4} / \text{sec} \]

Cross-section: irregular.
Vegetation: very heavy.
Soil type: sand.
NOTE 6

THE INFLUENCE OF THE CHOICE OF THE COEFFICIENT OF BED-ROUGHNESS ON THE DESIGN AND THE WATER LEVELS SUBSEQUENTLY OCCURRING

In Manning's formula: \( v = k_M R^{2/3} S^{1/2} \), which is used for the calculation of vegetated channels, the bed-roughness coefficient \( k_M \) occurs which represents the resistance which the flow experiences in the channel as a result of the roughness of bed and banks. It should be noted here that in the calculation of actual channels, the value chosen for \( k_M \) expresses not only the resistance due to the rugosity of the bed but also numerous other resistances due to differences between successive cross-sections, grass hanging in the water, reed growth (grid resistance) etc. In such cases therefore, \( k_M \) must be regarded more as a coefficient of total resistance.

If Manning's formula is written in the form \( Q/k_M = AR^{2/3} S^{1/2} \), it is seen that in a channel to which a certain gradient \( S \) has to be assigned, the assumed value of \( k_M \) exerts an influence on the design wetted cross-section equivalent to that exerted by the assumed discharge. In other words, if \( k_M \) is assumed \( n \times \) too small a wetted
Influence of choice of coefficient.

cross-section is designed which is the same size as it would have been if the design discharge had been assumed \( n \times \) too big.

Graph no. 4 shows the cross-sectional area of flow with various \( k_M \) values for the usual dimensions of various cross-sections and with gradients of 10 cm/km and 25 cm/km.

Graph no. 5 shows the rise in water level (equilibrium depth) for various cross-sections if the actual value of \( k_M \) subsequently falls below the assumed design values, which in this graph have been put at \( k_M = 40 \) for large sections and \( k_M = 30 \) for smaller sections. For example if a channel with a water depth of 0.80 m, a bed-width of 1.20 m, side-slopes 1:1.5 is assumed to have \( k_M = 30 \) in calculation, then Graph 5 indicates that the water level in the channel will rise 0.17 m if, owing to weeds etc., \( k_M \) has fallen to 20.

It can be proved that this rise is independent of the gradient of the channel.

Corresponding graphs can be drawn in which the \( k_M \)-scale is replaced by a scale for the design discharge.
Graph 4: Relation between A and Km for various discharges and with S = 10 cm/km and S = 25 cm/km, determined by Manning’s Formula: \( V = \frac{A}{n} \sqrt{Km} \).
Influence of choice of coefficient

Graph 5

\[ \Delta d \] is dependent only on the shape of the wetted cross-section and the change in \( k_M \) with a different gradient \( S \) and therefore a different \( Q \), but with the same wetted cross-section, \( \Delta d \) remains the same for the same change in \( k_M \).
NOTE 7

CALCULATION FOR THE CONSTRUCTION OF THE BACKWATER CURVE FOR NON-UNIFORM FLOW

Differentiation is made between:

A. elevation of the normal flood-profile by an obstruction in the channel (e.g. culvert, weir);
B. lowering of the normal flood-profile by reduction of the resistance (e.g. outfall into a reservoir or watercourse with a lower water level).

A. ELEVATION OF THE NORMAL FLOOD-PROFILE, KNOWN AS THE BACKWATER CURVE

Fig. 13 shows a schematic example in which the line a-b would be the normal flood-profile if no weir is present (uniform flow which is defined by the normal depth $y_n$).
The following calculation of prismatic watercourses has been performed according to the method given by Boris A. Bakhmeteff, for the theoretical basis reference should be made to this work\(^1\).

1. The data needed for the calculation with the usual symbols are:
   - the discharge \( Q \) in m\(^3\)/sec
   - the bed-slope \( S \)
   - the bed width \( b \) in m
   - the gradient of the side-slopes (1 vertical to \( p \) horizontal)
   - the Manning coefficient for bed-roughness \( k_M \)
   - the width of the water surface \( W \) in m.

Initially a number of water depths are assumed, starting out with the water depth \( y_0 \) at the weir, see Fig. 14.

---

2. We require to calculate

a. the normal depth $y_n$ at uniform flow.
   For this we use Manning's formula $Q = k_M A R^{2/3} S^{1/2}$.
   For rapid calculation reference should be made to the collection of graphs.

b. the cross-sectional area of flow $A$ for $y_n$ and $y_o$;

c. the wetted perimeter $P$, also for $y_n$ and $y_o$

d. the hydraulic radius $R = A/P$ for $y_n$ and $y_o$ and the average value

e. $\eta = y/y_n = \frac{\text{local backed-up depth}}{\text{normal depth}}$

f. the hydraulic exponent $n$

$$Q = A k_M R^{2/3} S^{1/2}$$

$$S = \frac{Q^2}{k_M^2 A^2 R^{4/3}} = \frac{Q^2}{K^2}$$

$$K = \sqrt{\gamma y_n} \quad (K = \text{conveyance})$$

$$\gamma = \text{a constant}$$

$$\frac{K_o^2}{K_n^2} = \frac{\gamma y_o^n}{\gamma y_n^n} = \frac{(y_o)^n}{(y_n)^n} = \frac{A_o^2 R_o^{4/3}}{A_n^2 R_n^{4/3}}$$

For several cross-sections, the value of $n$ belonging to a certain $b/y_n$ and $\eta$ have been plotted in Graph no. 8 (it has been assumed that $k_M$ is constant for the entire channel between $y_o$ and $y_n$).

g. the average coefficient of resistance $C$ from the formula $C = k_M R^{1/6}$ (see Graph no. 6 for this)

h. $B(\eta)$ for various values found for $\eta$ and $n$; see Graph 7. In using this graph attention should be paid to the correct manner of reading off. With a backed-up flood-profile $\eta$ is always greater than 1, whilst with a drawn-down flood-profile $\eta$ is smaller than 1.

j. a factor $\beta$ from the formula $\beta = \frac{C^2 WS}{gP}$.

All the data are now known which are needed for substitution in the formula

$$X = \eta - (1 - \beta) B(\eta).$$

For every water level a value for $X$ may be calculated. If the water levels are
Calculation for construction of backwater curve

\[ C = k_m \times R^{1/6} \]
known for two points, the distance between them (in m) can be calculated from the formula:

\[ L = (X_1 - X_2) \frac{y_n}{S} \]

\[ X_2 = \eta_2 - (1 - \beta)B(\eta_2) \]

\[ \eta_2 = \frac{y_2}{y_n} \]

\[ X_1 = \eta_1 - (1 - \beta)B(\eta_1) \]

\[ \eta_1 = \frac{y_1}{y_n} \]

\[ \eta_2 < \eta_1 \]

If the distances from the weir are known of the several points with given water depth, then the backwater curve can be constructed.

**B. Drawn-down of the normal flood-profile due to reduction of the resistance**

Naturally, the formulae and calculations of B. A. Bakhmeteff hold likewise for lowering of the water surface, but with the difference that \( \eta = y/y_n \) is now smaller than 1, as shown schematically in Fig. 15.
Graph 7. Graph to assist the determination of the highwater profile for non-parallel flow.
Calculation for construction of backwater curve

If in an actual case \( n \) and \( \eta \) have been calculated, the \( B(\eta) \) is found by using Graph 7. Whereas \( A, L = (X_2 - X_1)\frac{y_n}{S} \) with a lowering from the equilibrium depth \( \eta < 1 \), we now have:

\[
L = (X_2 - X_1)\frac{y_n}{S} \\
X_2 = \eta_2 - (1 - \beta) B(\eta_2) \\
\eta_2 = \frac{y_2}{y_n} \\
X_1 = \eta_1 - (1 - \beta) B(\eta_1) \\
\eta_1 = \frac{y_1}{y_n} \\
\eta_2 > \eta_1
\]

The starting point is the calculated water depth at the downstream end of the channel, and subsequently the distance to this point can be calculated for several different depths of water in the manner described above.

**Worked Example of the Calculation of a Backwater Curve as Described Under A**

Given is a weir immediately upstream of which the water depth \( y = 1.80 \text{ m} \) (see Fig. 16, point A). We require to calculate the distances from the weir of the points at which the depths of water are 1.78, 1.76, 1.74, 1.72, 1.70, 1.65, 1.60, 1.55, 1.50, 1.40, 1.30 and 1.20.
The data to be used in the example are as follows:

\[ Q = 2.50 \text{ m}^3/\text{sec} \]
\[ S = 0.0005 \text{ (i.e. 50 cm fall per kilometer)} \]
\[ b = 3.00 \text{ m} \]
\[ p = 1 \]
\[ k_M = 30 \]
\[ y_n = 1.11 \text{ m} \]
\[ g = 9.81 \text{ m/sec}^2 \]

Whatever the backwater curve to be constructed, every calculation of the distances commences with the calculation of the \( n \)-value.

If this \( n \) cannot be determined from Graph no. 8, it must be calculated for \( y_n \) and \( y_o \) from \( A \) and \( R \).

For:

\[ y_n = 1.11 \text{ m} \]
\[ A = (b + \rho y) y = 4.562 \text{ m}^2 \]
\[ P = b + 2y\sqrt{1 + \rho^2} = 6.139 \text{ m} \]
\[ R = \frac{A}{P} = \frac{4.562}{6.139} = 0.743 \text{ m} \]
\[ A^2 = 20.812 \]
\[ R^{4/3} = 0.673 \]
\[ A^2R^{4/3} = 14.006 \]

For:

\[ y_o = 1.80 \text{ m} \]
\[ A = 8.640 \text{ m}^2 \]
\[ P = 3 + 2 \times 1.8\sqrt{2} = 8.092 \text{ m} \]
\[ R = \frac{8.640}{8.092} = 1.068 \text{ m} \]
\[ A^2 = 74.650 \]
\[ R^{4/3} = 1.091 \]
\[ A^2R^{4/3} = 81.443 \]
\[ \eta = \frac{y_o}{y_n} = 1.622 \]
Calculation for construction of backwater curve

\[ \eta^n = \frac{A^2 R^{4/3}(y_o)}{A^2 R^{4/3}(y_n)} = \frac{81.443}{14.006} = 5.815 \]
\[ n' = 3.6 \]

The reading from Graph 8 for \( \eta = 1.6 \) gives the same result.

Now \( C \) can be determined, using the average \( R \) value.

average \( R = 0.91 \)
\( C = k_M R^{1/6} = 29.5 \) (take \( C = 30 \))

Reading from Graph 6 gives the same result.

The calculation of the required points can now proceed:

\[ y_o = 1.80 \text{ m} \]
\[ B(\eta) = 0.119 \] (Graph no. 7)
\[ \beta = \frac{C^2 SW}{gP} = \frac{30^2 \times 0.0005 \times 6.6}{9.81 \times 8.092} = 0.04 \]
\[ X_1 = \eta - (1 - \beta) \times B(\eta) \]
\[ X_1 = 1.62 - (1 - 0.04) \times 0.119 = 1.506 \text{ m} \]
\[ L = 0 \]
\[ y = 1.78 \text{ m} \]
\[ \eta = \frac{1.78}{1.11} = 1.604 \]
\[ B(\eta) = 0.122 \]
\[ \beta = 0.04 \] (\( W \) and \( P \) have been altered here, \( P = 8.034 \))
\[ X_2 = 1.60 - (1 - 0.04) \times 0.122 = 1.483 \text{ m} \]
\[ L = (X_1 - X_2) \frac{y_n}{S} = 0.023 \times \frac{1.11}{0.0005} = 51 \text{ m} \]

Obviously, for every given water level an \( X \) value can be calculated. Deducted from the \( X \)-value at the weir and multiplied by the constant factor (see p. 76)
\( y_n/S \left( = \frac{1.11}{0.0005} = 22.209 \right) \), this yields the distance from the weir to the point with the given water level. In the following summary, all the results of calculation applicable to the several water depths have been tabulated.

<table>
<thead>
<tr>
<th>( y )</th>
<th>( A )</th>
<th>( P )</th>
<th>( R )</th>
<th>( C )</th>
<th>( n )</th>
<th>( \eta )</th>
<th>( B(\eta) )</th>
<th>( \beta )</th>
<th>( X )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m(^2)</td>
<td>m</td>
<td>m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.80</td>
<td>8.640</td>
<td>8.092</td>
<td>1.068</td>
<td>30</td>
<td>3.6</td>
<td>1.622</td>
<td>0.119</td>
<td>0.04</td>
<td>1.506</td>
<td>0</td>
</tr>
<tr>
<td>1.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.11</td>
<td>4.562</td>
<td>6.139</td>
<td>0.743</td>
<td>30</td>
<td>3.6</td>
<td>1.081</td>
<td>0.535</td>
<td>0.04</td>
<td>0.567</td>
<td>2092</td>
</tr>
</tbody>
</table>

-- The backwater curve

Now that it has been calculated at what distance from the weir each of the given water depths is attained, the profile of the backwater curve is also known. Setting out the water depths against the corresponding distances yields the backwater curve, as shown in Fig. 17.
Calculation for construction of backwater curve

The above worked example demonstrates that the calculation of a backwater curve in a prismatic channel demands rather laborious calculation.

Fortunately, further investigation of this method of calculation shows that it is possible to introduce a considerable simplification without seriously detracting from the accuracy.

The value of $\beta$ varies with the bed-gradient $S$. In regions with moderate bed-gradients, the value of $\beta$ is frequently small. The inaccuracy then introduced by neglecting $\beta$ is smaller than the tolerances that are generally allowed in calculating open watercourses.

It is advisable for greater bed-gradients to assess the order of magnitude of the error introduced by this neglect.

Bakhmeteff gives in his book the values for $\beta - B$ that can be used for the simpler calculation where $\beta$ is neglected. Ven Te Chow has in his more recent book\(^1\) given a simplified method of calculation that can often be used with advantage.

An approximation to the nearest 1% is permissible in most cases.

STEP-BY-STEP APPROACH

This method has the advantage that those acquainted with the technique of backwater calculation can see clearly what they are doing. The disadvantage is however that a continuous step-by-step operation is necessary for which more time is required when calculating a certain water level at a given distance from the weir. This calculation is given in Fig. 18.

![Diagram](image)

Fig. 18

S is small, so we can put $L_0 = L$

$$y + \frac{v_1^2}{2g} + \Delta H = SL + y + \Delta y + \frac{v_2^2}{2g}$$

$$\Delta H - SL = \Delta y + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \Delta y + \Delta \frac{v^2}{2g}$$

$$\frac{\Delta H}{L - S} = \frac{\Delta y}{L} + \frac{2g}{L}$$
\[
\frac{\Delta H}{L} = \text{energy gradient; this is caused only by frictional losses so that we can call it the friction-gradient:}
\]

\[
S_w = \frac{\Delta H}{L} = \frac{v^2}{C^2R}
\]

The average values of \(v, C\) and \(R\) have now to be calculated, reach for reach, in a table for example, as performed above:

\[
Ay + A \frac{v^2}{2g} \quad \text{or} \quad S_w - S = \frac{A \frac{v^2}{2g}}{L}
\]

\(Ay\) can be assumed and \(L\) then calculated from the equation:

\[
L = \frac{Ay + A \frac{v^2}{2g}}{S_w - S}
\]

For \(S_w\), the average value of \(\frac{v^2}{C^2R}\) over the reach may be assumed.

With low gradients, velocities are often low also and in certain cases \(A \frac{v^2}{2g}\) can be neglected. In most cases an investigation will be needed to check whether this neglect is permissible.