Similarity analysis of subsurface flow response of hillslopes with complex geometry

A. Berne, R. Uijlenhoet, and P. A. Troch

Hydrology and Quantitative Water Management Group, University of Wageningen, Wageningen, Netherlands

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[1] The matter of the efficient and parsimonious parameterization of hillslope subsurface flow remains an important issue in catchment hydrological studies (Brutsaert, 1995). Insights into the influence of the shape and hydraulic characteristics of hillslopes is required to further our understanding and our ability to model catchment hydrological processes. Recently, Troch et al. (2003) introduced the hillslope-storage Boussinesq (HSB) equation to describe subsurface flow and saturation along geometrically complex hillslopes. The HSB equation can be linearized and further reduced to an advection-diffusion equation for subsurface flow in hillslopes with constant bedrock slopes and exponential width functions. This paper presents a dimensional analysis of the latter equation in order to study the moments of the characteristic response function (CRF), corresponding to the free drainage of this type of hillslope. These moments, in a dimensionless form, can be expressed as functions of a similarity parameter, hereafter called the hillslope Péclet number, and a group of dimensionless numbers accounting for the effects of the boundary and initial conditions. The analytical expressions for the first four central CRF moments are derived for two types of initial conditions. The analysis of their respective influences shows that the hillslope Péclet number is an efficient similarity parameter to describe the hillslope subsurface flow response. Moreover, comparison between the CRF moments predicted by means of our similarity analysis and empirical moments derived from outflow measurements for different types of laboratory hillslopes shows good agreement.

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1. Introduction

[2] Landscape geomorphology (hillslope geometry) and pedology (soil properties) influence the hydrological response of catchments. Thus clear insight into the effect of the shape and hydraulic characteristics of landscape elements is required to further our understanding of and our ability to model catchment hydrological processes. For some time, research has focused on identifying and quantifying hillslope processes as a first step toward assessment of (sub)catchment response. In hilly catchments the importance of subsurface flow processes in generating variable source areas was first addressed by *Dunne and Black* [1970] and *Freeze* [1972a, 1972b]. The idea that landscape structure is a dominant control of the hydrological behavior, and that hydrological models therefore should take this structure explicitly into account has already a long tradition in hydrology. For example, *Beven and Kirkby* [1979] showed how geomorphometric parameters can be used to describe the hydrological behavior at a given position within the landscape, while *Rodríguez-Iturbe and Valdés* [1979] showed how the shape of a catchment unit hydrograph can be explained from the structure of the channel network. However, the role of geomorphological characteristics of individual hillslopes and their effect on runoff generation has received less attention.

[3] There is hence a need for quantifying the hillslope hydrological processes and for the development of appropriate models to describe these processes. Several models have been developed over the past 30 years. The most complete models involve numerically solving the three-dimensional Richard's equation for complex hillslope geometries [*Paniconi and Wood*, 1993; *Bronstert*, 1994]. To overcome difficulties associated with 3-D models (parameterization, computational demands), a series of low-dimensional hillslope models has recently been developed [*Fan and Bras*, 1998; *Troch et al.*, 2003]. These models are called hillslope-storage dynamics models and are able to treat the three-dimensional structure of hillslopes in a simple way, resulting in a significant reduction of model complexity. *Troch et al.* [2003] and *Hilberts et al.* [2004] demonstrated that (numerical) solutions of the 1D hillslope-storage Boussinesq (HSB) equation account explicitly for plan shape (by means of the hillslope width function) and profile curvature (local bedrock slope angle and hillslope soil depth function) of the hillslope. Recently, *Troch et al.* [2004] presented an analytical solution of the linearized HSB equation for exponential hillslope width functions. Analytical solutions like these provide essential insights in the functioning of hillslopes and may form the basis of hillslope similarity analysis [*Brutsaert*, 1994].

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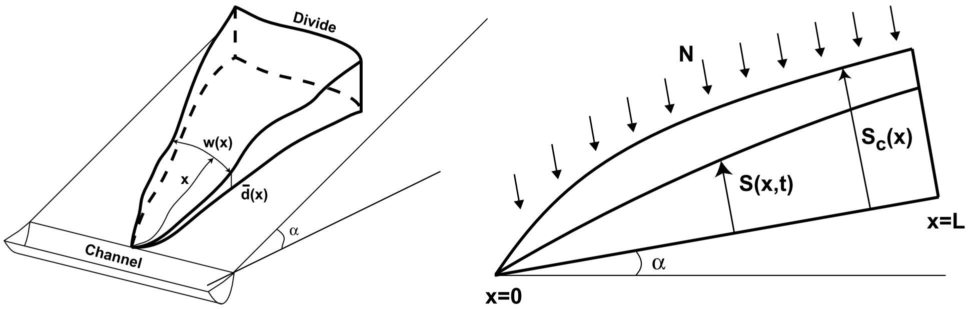

Figure 1. Hillslope conceptualization and definition of the storage (from *Troch et al.* [2003]).

[4] The search for hydrological similarity indices to classify catchments based on soil, topography, vegetation and climate characteristics has been a very active research topic over the past decades [e.g., *Hebson and Wood*, 1986; *Sivapalan et al.*, 1987, 1990; *Saleem and Salvucci*, 2004], but definitive conclusions on similarity behavior of landscapes, based on similarity of dominant hillslope processes, have not yet been achieved. This is mainly explained by the lack of analytical relations between the flow processes and the landscape characteristics [*Aryal et al.*, 2002]. Analytical solutions for subsurface flow in (complex) hillslopes provide a link between the physics of the subsurface flow processes and the hillslope geometric and hydraulic properties, and therefore are useful tools to understand landscape hydrological similarity. In this paper, we focus on groundwater flow and other hydrological processes (e.g., macropore or overland flow) are not considered.

[5] *Brutsaert* [1994] derived an analytical solution to a linearized Boussinesq equation to study the hillslope subsurface flow unit response, corresponding to the free drainage of an unconfined aquifer. The motivation for his work was to provide a direct link between the underlying physical mechanisms of hillslope subsurface flow and the general linear theory of catchment hydrology [*Dooge*, 1973]. The analytical approach provides a powerful framework to analyze the influence of the different characteristics (hydraulic and geometric) of the hillslope on the shape of its hydrological response. In linear systems theory, the unit response function of a spatially lumped system (e.g., a linear reservoir) completely describes its dynamics. For a spatially distributed linear system with specified boundary conditions, the characteristic response is also influenced by the initial conditions [e.g., *Brutsaert*, 1995; *Chapman* 1995]: the way a given volume of water is initially distributed within an aquifer of finite length will influence its drainage. To describe the subsurface flow response of a hillslope, we define the characteristic response function (CRF hereafter) as the free drainage discharge normalized by the total outflow volume (for given initial and boundary conditions). The normalization allows to compare different hillslopes.

[6] The main objective of our paper is to link subsurface flow dynamics to the geometric and hydraulic characteristics of the hillslope, by means of a similarity analysis. The hillslope hydrological response will be studied through the moments of the CRF. Since we are interested in deriving explicit relations between hillslope aquifer properties and the characteristic response, we seek to separate the effects of the boundary and initial conditions and the effects of the hillslope geometric and hydraulic properties on the moments of the CRF. A dimensional analysis of a linearized HSB equation and the obtained hydrological similarity parameter are presented in section 2. The derivation of the analytical expressions of the CRF moments for two different types of initial conditions is given in section 3. In section 4, the dimensionless moments of the CRF are analyzed and compared with empirical moments estimated from laboratory hillslope outflow measurements. Finally our conclusions are presented in section 5.

2. Dimensional Analysis

2.1. General Formulation

[7] Our starting point is equation (16) of *Troch et al.* [2003], which describes the evolution of the saturated soil water storage $S = fw\bar{h}$ (where f is the drainable porosity, w is the hillslope width function and $\bar{h}$ is the average groundwater table height, perpendicularly to the bedrock, over the width) along a hillslope with an exponential width function $w(x) = ce^{ax}$ (see Figure 1):

$$\frac{\partial S}{\partial t} = K\frac{\partial^2 S}{\partial x^2} + U\frac{\partial S}{\partial x} + Nw \tag{1}$$

with

$$K = \frac{kpD\cos\alpha}{f}$$
$$U = \frac{k\sin\alpha}{f} - aK$$

where x is the distance from the outlet of the hillslope, t is time, k is the hydraulic conductivity, D is the soil depth, α is the slope of the bedrock, p is a linearization parameter and N is the recharge to the groundwater table. The main assumptions for the validity of (1) are a shallow soil mantle, streamlines parallel to an impervious bedrock, a negligible influence of the unsaturated zone, a constant slope angle and a uniform hydraulic conductivity. These are common assumptions in hillslope hydrology [*Brutsaert*, 1994] and we are convinced that analytical solutions of (1) provide useful insights into the hydrological response of individual hillslopes. Equation (1) is the classical linear nonstationary advection-diffusion equation for which analytical solutions can be derived given suitable boundary and initial conditions as well as a suitable recharge rate. In the following, advection and diffusion refer to the transport of water due to total head gradients and should not be confused with advection and diffusion in solute transport. The integrated discharge Q over the hillslope width is

$$Q = -K\frac{\partial S}{\partial x} - US \tag{2}$$

Hereafter, we shall consider Q to be negative and U to be positive, so that the flow is toward the outlet. The signs are defined by the x axis orientation (see Figure 1). The assumption that U must be positive leads to the following geometric constraint:

$$\tan\alpha > apD \tag{3}$$

For divergent hillslopes ($a < 0$) this inequality is always true, so there is no constraint on a, α or pD. On the contrary, for convergent hillslopes ($a > 0$) the constraint on the degree of convergence is $a < \tan\alpha/(pD)$.

[8] We consider the free drainage of a hillslope, i.e., $N = 0$, given specific initial conditions (see section 3). In the Laplace domain, the partial differential equation (1) becomes an ordinary differential equation (ODE):

$$K\frac{\partial^2 \widetilde{S}}{\partial x^2} + U\frac{\partial \widetilde{S}}{\partial x} - s\widetilde{S} = -S_0 \tag{4}$$

where $\widetilde{S}$ is the Laplace transform of S (see equation (A2) in appendix A), s is the Laplace variable and S_0 denotes the assumed initial condition. From (4) it is possible to derive an analytical expression for $\widetilde{S}$ and hence for $\widetilde{Q}$, the Laplace transform of Q. As $\widetilde{Q}$ (normalized by the total input volume) is the moment generating function of the CRF [e.g., *Brutsaert*, 1994], we analyze the CRF through its moments to avoid the difficult transformation back to the time domain [e.g., *Troch et al.*, 2004].

[9] In order to derive similarity parameters for the CRF, a dimensional analysis is conducted. We have to define characteristic values for the dimensions involved in (4), i.e., length and time. We use half of the hillslope length ($\frac{L}{2}$) to normalize the length dimension. From (1), we define the characteristic diffusive time from the middle of the hillslope as $\tau_K = L^2/(4K)$. We use τ_K to normalize the time dimension (see section 2.2). The dimensionless form of (4) reads

$$\frac{\partial^2 \widetilde{S}^*}{\partial x^{*2}} + \frac{UL}{2K}\frac{\partial \widetilde{S}^*}{\partial x^*} - s^*\widetilde{S}^* = -S_0^* \tag{5}$$

where the asterisk denotes a dimensionless variable. We can derive a general formulation for the dimensionless CRF moments (see Appendix A):

$$m_n^* = \phi_n\left(\frac{UL}{2K}, \pi^*\right) \tag{6}$$

where m_n^* denotes the dimensionless nth-order moment of the CRF, ϕ_n is a function of dimensionless variables and π^* represents a set of dimensionless variables linked to the boundary and initial conditions. The dimensionless central moments are then given by:

$$\mu_n^* = \sum_{k=0}^{k=n} (-1)^{n-k} \binom{n}{k} m_k^* \, {m_1^*}^{n-k} \tag{7}$$

where $\binom{n}{k}$ represents the binomial coefficient. Please note that in the following, the first dimensionless central moment is defined around zero. The dimensional moments m_n and μ_n are obtained by multiplying m_n^* and μ_n^* by τ_K^n. Equations (6) and (7) provide dimensionless expressions for the CRF moments and therefore can be used to perform a similarity analysis. A dimensional analysis can also be conducted by means of Buckingham's pi theorem [*Buckingham*, 1914]. However, such an analysis is independent of the form of (1) and will not allow to separate in an effective and objective manner the effects of the hillslope geometric and hydraulic properties from effects of the boundary and initial conditions and is therefore not conducted here.

2.2. Similarity Parameter: The Hillslope Péclet Number

[10] From (6) we see that the dimensionless CRF moments depend on the dimensionless number $UL/(2K)$. Together with the π^* terms, this dimensionless number defines the normalized hillslope hydrological response. We therefore propose to use $UL/(2K)$ as the subsurface flow similarity parameter for complex hillslopes. This number can be interpreted as the ratio between the characteristic diffusive time and the characteristic advective time, defined for the middle of the hillslope [e.g., *Kirchner et al.* , 2001], and therefore is called hereafter the hillslope Péclet number for subsurface flow:

$$\mathrm{Pe} = \frac{\tau_K}{\tau_U} = \frac{\frac{(L/2)^2}{K}}{\frac{(L/2)}{U}} = \frac{UL}{2K} = \left(\frac{L}{2pD}\right)\tan\alpha - \left(\frac{aL}{2}\right) \tag{8}$$

where the characteristic advective time is

$$\tau_U = \frac{L}{2U} = \frac{Lf}{2k(\sin\alpha - apD\cos\alpha)} \tag{9}$$

and the characteristic diffusive time is

$$\tau_K = \frac{L^2}{4K} = \left(\frac{fL}{2k}\right)\left(\frac{L}{2pD}\right)\left(\frac{1}{\cos\alpha}\right) \tag{10}$$

The characteristic diffusive time τ_K is used to normalize the time dimension because it does not approach infinity when Pe approaches 0, as opposed to τ_U. Equation (3) guarantees that τ_U, and hence Pe, is always defined and positive. From (8) we can see that Pe is a function of three independent dimensionless numbers: $L/(2pD)$, $\tan\alpha$ and $aL/2$; $L/(2pD)$ represents the ratio of the half length and the average depth of the aquifer, and $\tan\alpha$ represents the slope of the bedrock. Their product characterizes the vertical geometry of the aquifer, while $aL/2$ characterizes the planar geometry of the aquifer. When $[L/(2pD) \times \tan\alpha]$ decreases or $aL/2$ increases, Pe decreases. This means that the storage gradients become stronger and therefore the contribution of the diffusion term in (1) increases. Note that for a uniform hillslope ($a = 0$), (8) reduces to the dimensionless parameter given by *Brutsaert* [1994, equation [28]].

[11] In addition to the influence of the initial and boundary conditions (see section 4), the dimensionless number Pe defines the hydrological similarity between hillslopes with respect to their characteristic response.

3. Analytical Expressions for the Dimensionless CRF Moments

[12] In section 2 a general formulation for the moments of the CRF has been derived. This section is devoted to

the derivation of analytical expressions for these moments for two different types of initial conditions. In this manner, explicit relations between the hydraulic and geometric properties of the hillslope and its subsurface flow response are obtained. We have to solve (5) to derive an analytical expression for the dimensionless Laplace transform of the discharge $\widetilde{Q}^*$ and use it as the CRF moment generating function. The first step is to define the boundary and initial conditions that will be used to solve the differential equation.

3.1. Boundary Conditions

[13] Equation (5) will be solved with the following boundary conditions commonly used in hillslope hydrology [e.g., *Brutsaert*, 1994; *Verhoest and Troch*, 2000; *Troch et al.*, 2004]: (1) assuming the groundwater table height at the outlet to be small in comparison to the mean groundwater table height along the hillslope, we impose the storage to be zero at the outlet; (2) the uphill boundary of the hillslope coincides with the catchment divide, and we assume the flow through the divide to be zero. In a dimensionless form and in the Laplace domain, these boundary conditions are expressed as

$$\begin{aligned} &\widetilde{S}^* = 0 && x^* = 0 \quad \forall t \geq 0 \\ &\frac{\partial \widetilde{S}^*}{\partial x^*} + \mathrm{Pe}\widetilde{S}^* = 0 && x^* = 2 \quad \forall t \geq 0 \end{aligned} \tag{11}$$

It must be noted that these boundary conditions do not require other numbers than Pe to be described in a dimensionless way. Therefore π^* will only depend on the initial condition in this case.

3.2. Initial Condition 1

[14] The first type of initial condition corresponds to a storage profile defined as a fraction γ of the storage capacity $S_c = fwD$ [*Troch et al.*, 2004]:

$$S_0(x) = \gamma D f w(x) = \left(\frac{L}{2}\right)^2 \frac{4\gamma D f c}{L^2}\, e^{ax} \qquad \forall x \in [0, L] \tag{12}$$

where $\gamma \in [0, 1]$ is a factor defining the initial groundwater table height as a fraction of the soil depth D. If the soil depth is constant along the hillslope, then the initial groundwater table height (γD) is also constant along the hillslope. The dimensionless initial storage profile is given by

$$S_0^*(x^*) = \frac{4\gamma D f c}{L^2}\, e^{\frac{aL}{2}x^*} \tag{13}$$

Therefore the parameter set π_0^*, representing the initial condition, is $\left\{\frac{4\gamma Dfc}{L^2}; \frac{aL}{2}\right\}$. However, $\frac{4\gamma Dfc}{L^2}$ is a proportionality factor for S_0^*. Because of the linearity of (2) and (4), it will also be a proportionality factor for $\widetilde{Q}^*$. Hence this factor will disappear in the expression for the dimensionless CRF moments and as a consequence π_0^* reduces to $\pi^* = \left\{\frac{aL}{2}\right\}$. It must be noted that this initial storage profile does not satisfy the imposed boundary condition at the outlet for $t = 0$.

[15] The obtained expression for the dimensionless discharge in the Laplace domain is (see section B1)

$$\begin{aligned} \widetilde{Q}^*(s^*, x^*) = &-\frac{8\gamma Dfc}{L^2}\frac{1}{(aL(aL+2\mathrm{Pe}) - 4s^*)}\left\{-e^{\frac{aL}{2}x^*}(aL+2\mathrm{Pe})\right. \\ &+ \frac{4s^*\left[e^{(d-b)L}e^{\frac{L(d+b)}{2}x^*} - e^{(d+b)L}e^{\frac{L(d-b)}{2}x^*}\right]}{L[(b-d)e^{(d+b)L} + (b+d)e^{(d-b)L}]} \\ &\left.+ \frac{(aL+2\mathrm{Pe})Le^{aL}\left[(b-d)e^{\frac{L(d+b)}{2}x^*} + (b+d)e^{\frac{L(d-b)}{2}x^*}\right]}{L[(b-d)e^{(d+b)L} + (b+d)e^{(d-b)L}]}\right\} \end{aligned} \tag{14}$$

where $dL = -\mathrm{Pe}$ and $bL = \sqrt{Pe^2 + 4s^*}$. The dimensionless total initial volume uphill of the outlet ($x^* = 0$) is

$$V^* = -\widetilde{Q}^*(0,0) = \frac{8\gamma Dfc}{aL^3}\left(e^{aL} - 1\right) \tag{15}$$

Integrating (13) between $x^* = 0$ and $x^* = 2$ yields the same expression. Taking the first derivative of (14) with respect to s^* for $s^* = 0$ at the outlet and normalizing by (15) yields the first dimensionless CRF moment (i.e., the normalized mean response time):

$$\begin{aligned} m_1^* = &\frac{4}{aL(aL+2\mathrm{Pe})(e^{aL}-1)} \times \left\{1 + \frac{aL}{2\mathrm{Pe}}\left(1 - e^{-2\mathrm{Pe}}\right)\right. \\ &+ e^{aL}\left[aL - 1 + \frac{aL}{2\mathrm{Pe}}\left(aL - 1 - \frac{aL}{2\mathrm{Pe}}\right)\right] \\ &\left.+ aL\left(1 + \frac{aL}{2\mathrm{Pe}}\right)\frac{e^{aL-2\mathrm{Pe}}}{2\mathrm{Pe}}\right\} \end{aligned} \tag{16}$$

For a uniform hillslope ($a = 0$), this reduces to:

$$m_1^* = \frac{\left[2\mathrm{Pe}^2 - 1 + (2\mathrm{Pe}+1)e^{-2\mathrm{Pe}}\right]}{2\mathrm{Pe}^3} \tag{17}$$

Equation (17) is consistent with the expression given by *Brutsaert* [1994, equation [24]]. Higher-order moments can be derived in a similar manner, however their expressions are too lengthy to be given here.

3.3. Initial Condition 2

[16] The second type of initial condition corresponds to a storage profile derived from the steady state solution of (1) for a given recharge (similar to that of *Verhoest and Troch* [2000]). This guarantees that the initial storage profile is consistent with the governing flow equation and with the boundary conditions. The steady state solution is given by

$$S_0(x) = \left(\frac{L}{2}\right)^2 \frac{N_0 c}{K}\frac{2}{aL}\left\{\frac{e^{aL}}{\frac{UL}{2K}}\left(1 - e^{-\frac{Ux}{K}}\right) + \frac{1}{\frac{aL}{2} + \frac{UL}{2K}}\left(e^{-\frac{Ux}{K}} - e^{ax}\right)\right\} \qquad \forall x \in [0, L] \tag{18}$$

where N_0 is the recharge such that the maximum groundwater table height just reaches the ground surface (see Appendix C). The dimensionless initial storage profile is given by

$$S_0^*(x^*) = \frac{N_0 c}{K}\frac{2}{aL}\left\{\frac{e^{aL}}{\mathrm{Pe}}\left(1 - e^{-\mathrm{Pe}x^*}\right) + \frac{1}{\frac{aL}{2} + \mathrm{Pe}}\left(e^{-\mathrm{Pe}x^*} - e^{\frac{aL}{2}x^*}\right)\right\} \tag{19}$$

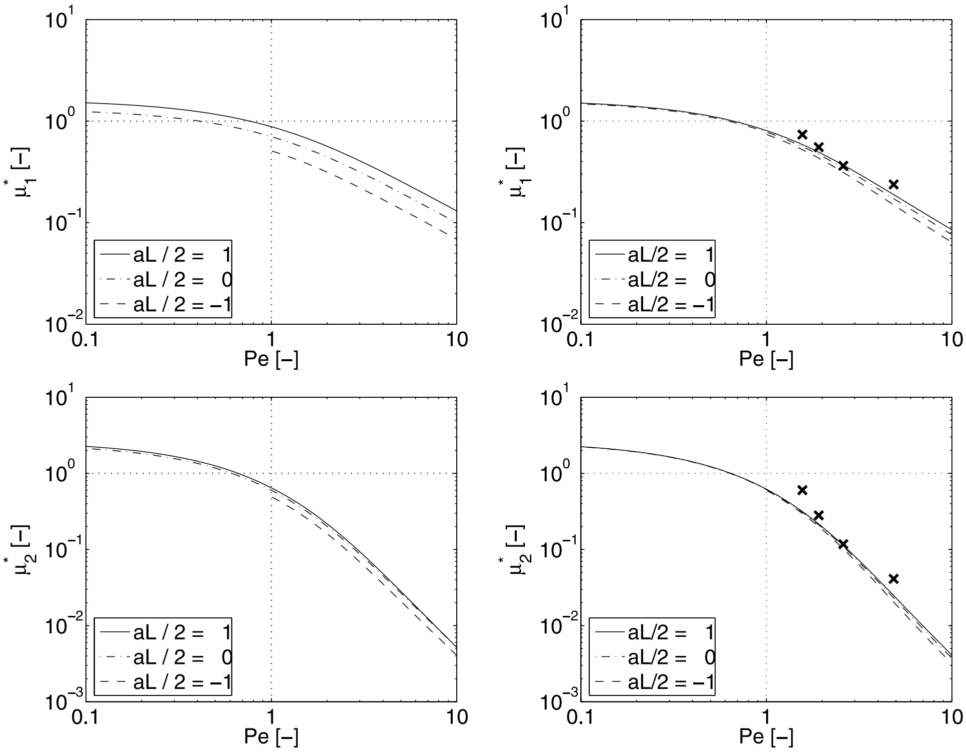

Figure 2. First and second dimensionless CRF central moments plotted as functions of Pe for (left) the first type of initial condition and (right) the second type of initial condition. The vertical dotted line indicates Pe = 1, and the horizontal dotted line indicates $\mu_n^* = 1$. The crosses indicate the results from the laboratory outflow experiments.

The dimensionless parameter set π_0^* is $\{\frac{N_0c}{K}; \frac{aL}{2}; \mathrm{Pe}\}$. $\frac{N_0c}{K}$ is a proportionality factor for S_0^* and hence for ϕ. Pe appears in π_0^* because the initial condition is the solution of the steady state differential equation, with the same geometric constraints, but it does not yield an additional dimensionless number for $\widetilde{S}^*$ because the expression for $\widetilde{S}^*$ already contains Pe (appendix A). This leads to $\pi^* = \{\frac{aL}{2}\}$. However, the fact that the two types of initial conditions produce the same parameter set π^* does not mean that the characteristic responses will be the same, because the functions ϕ_n will be different in general.

[17] The obtained expression for the dimensionless discharge in the Laplace domain is (see section B2)

$$\widetilde{Q}^*(s^*,x^*) = -\frac{8N_0c}{K}\frac{1}{4s^*[aL(aL+2\mathrm{Pe})-4s^*]} \cdot \left\{ \frac{e^{aL}[aL(aL+2\mathrm{Pe})-4s^*]+4s^*e^{\frac{aL}{2}x^*}}{aL} - \frac{4s^*\left[e^{(d-b)L}e^{\frac{L(d+b)}{2}x^*} - e^{(d+b)L}e^{\frac{L(d-b)}{2}x^*}\right]}{L[(b-d)e^{(d+b)L}+(b+d)e^{(d-b)L}]} - \frac{e^{aL}(aL+2\mathrm{Pe})L\left[(b-d)e^{\frac{L(d+b)}{2}x^*}+(b+d)e^{\frac{L(d-b)}{2}x^*}\right]}{L[(b-d)e^{(d+b)L}+(b+d)e^{(d-b)L}]}\right\} \tag{20}$$

The dimensionless total initial volume uphill of the outlet is

$$V^* = -\widetilde{Q}^*(0,0) = \frac{8N_0c}{aLK}\left\{\frac{e^{aL}}{2\mathrm{Pe}}\left[1-\frac{1}{2\mathrm{Pe}}\left(1-e^{-2\mathrm{Pe}}\right)\right] - \frac{1}{aL+2\mathrm{Pe}}\left[\frac{1}{aL}\left(e^{aL}-1\right)-\frac{1}{2\mathrm{Pe}}\left(1-e^{-2\mathrm{Pe}}\right)\right]\right\} \tag{21}$$

This expression can also be obtained by integrating (19) between $x^* = 0$ and $x^* = 2$. The analytical expressions for the first and higher order dimensionless CRF moments can be derived in a similar manner as in section 3.2. However, their expressions are too lengthy to be given here.

4. Discussion

4.1. General Behavior of CRF Moments as Functions of Pe

[18] In this section, we analyze the dimensionless moments of the CRF derived from the two types of initial conditions. As discussed before, the hillslope Péclet number is a similarity parameter for subsurface flow along complex hillslopes. In Figures 2 and 3, we present the first four dimensionless CRF central moments as function of Pe. A double-logarithmic scale is used to improve the visual inspection for advection dominated and diffusion dominated responses. The analytical solutions derived in the Laplace domain show that the dimensionless number linked to both types of initial conditions is $aL/2$. Consequently, in order to analyze the influence of the initial condition, the moments are also plotted for three different values of $aL/2$ corresponding to a convergent ($aL/2 = +1$), a uniform ($aL/2 = 0$) and a divergent ($aL/2 = -1$) hillslope, respectively.

[19] From Figures 2 and 3 it is clear that the curves representing the relation between the dimensionless central moments and Pe are similar for the two types of initial conditions and the investigated values of $aL/2$. This indicates that Pe is an efficient similarity parameter to define the hillslope subsurface flow response. The shape of the curves

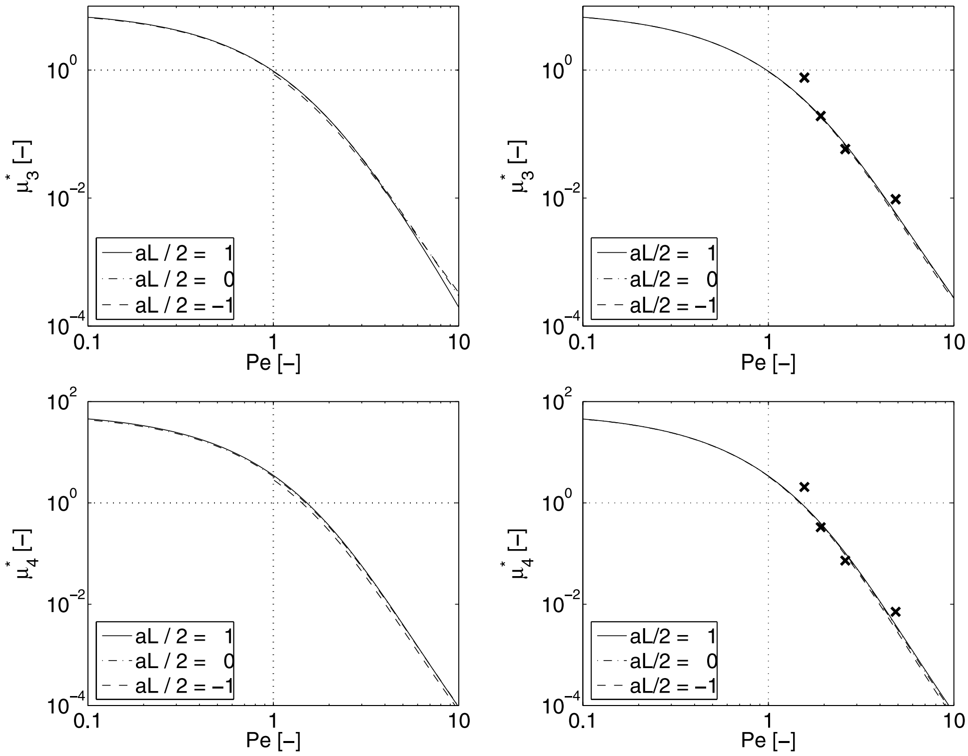

Figure 3. Third and fourth dimensionless CRF central moments plotted as functions of Pe for (left) the first type of initial condition and (right) the second type of initial condition. The vertical dotted line indicates Pe = 1, and the horizontal dotted line indicates $\mu_n^* = 1$. The crosses indicate the results from the laboratory outflow experiments.

for a given moment suggests different asymptotic behavior for the pure diffusion (Pe $\rightarrow$ 0) and pure advection (Pe $\rightarrow +\infty$) case. When both processes are more or less in balance (Pe $\sim$ 1), there is a transition zone. It is interesting to observe that these curves are almost overlapping for distinct values of $aL/2$, except for the first moment and the first type of initial condition. This shows that, in this case, the function ϕ_1 is sensitive to $aL/2$.

[20] For both types of initial conditions, the evolution of the first four dimensionless CRF moments with Pe is similar. When Pe increases and therefore the contribution of the diffusion term becomes less important, the dimensionless moments decrease and the higher the order of the moment, the faster the decrease. We can also observe in Figures 2 and 3 that at Pe values close to 1, the dimensionless central moments assume values around 1, hence the timescales of the moments are of the order of the characteristic times of the diffusive and advective processes.

[21] We can further study the influence of the separate hillslope parameters on the CRF moments. First we focus on the hydraulic parameters. As Pe and the π^* set from the studied types of initial conditions are independent of the hydraulic parameters f and k, the dimensionless central moments of the CRF are not affected by the variations of the hydraulic parameters. Equation (10) implies that τ_K is proportional to f and to $1/k$. Therefore the dimensional central moments μ_n are proportional to f^n and to k^{-n}.

[22] Next, we study the influence of the geometric parameters. As explained in section 2.2, Pe is function of three dimensionless numbers linked to the bedrock slope ($\tan\alpha$), to the aquifer length/depth ratio ($L/(2pD)$), and to the plan shape ($aL/2$). Equation (8) shows that Pe increases with $L/(2pD)$ ($\tan\alpha$ respectively). Hence μ_n^* decreases when the hillslope soil mantle becomes thinner (steeper respectively). When the effect of the initial condition is limited (in particular for the second type), the influence of the plan shape can be directly deduced from Figures 2 and 3. When $aL/2$ increases, Pe decreases and therefore μ_n^* increases. As τ_K is independent of $aL/2$, μ_n increases when $aL/2$ increases, keeping all other parameters fixed.

4.2. Comparison With Experimental Data

[23] The similarity analysis described above can be tested with outflow measurements for different hillslope types during free drainage experiments. *Hilberts et al.* [2005] report such data from hillslope experiments conducted at the Hydraulics Laboratory of the Hydrology and Quantitative Water Management group at Wageningen University. During these experiments 6 hillslope configurations were used: 2 plan shapes (linearly convergent and linearly divergent) and 3 bedrock slopes (5%, 10%, and 15%). Each hillslope was brought to steady state by means of a rainfall generator applying a constant rain rate. The boundary conditions during these experiments correspond to those assumed in the derivation of the analytical expressions for the CRF moments (section 3.1). The initial conditions compare to our second type of initial condition: a steady state saturated storage profile corresponding to a constant recharge N. When the constant rain rate was stopped, the outflow and the saturated storage changes were measured with 10 min time intervals. For more details about this experiment we refer to *Hilberts et al.* [2005].

Table 1. Parameter Values for the Experimental Hillslopes[a]

Parameter	Convergent ($\tan\alpha = 0.15$)	Divergent			Units
		$\tan\alpha = 0.05$	$\tan\alpha = 0.10$	$\tan\alpha = 0.15$	
Soil depth D	0.48	0.44	0.44	0.44	m
Drainable porosity f	0.23	0.18	0.26	0.31	
Width at the outlet c	0.5	2.5	2.5	2.5	m
Degree of convergence a	0.31	−0.185	−0.185	−0.185	m^{-1}
Initial recharge rate N_0	21.4	17.7	25.9	31.7	mm h^{-1}
Linearization parameter p	0.16	0.34	0.50	0.50	
Hillslope Péclet number Pe	4.86	1.56	1.92	2.60	
Characteristic diffusive time τ_K	16.1	6.5	6.5	7.7	h

[a]Based on the work of *Hilberts et al.* [2005].

[24] The length L of the hillslopes was 6 m. The hydraulic conductivity k of the sandy soil used in the experiments was estimated, on soil cores, at 40 m day^{-1}. The drainable porosity values were taken from *Hilberts et al.* [2005], who computed them as the ratio between the total outflow volume and the total soil volume (pore volume plus solid matrix). For the exponential width function, the parameter c was taken as the outlet width and the degree of convergence a was adjusted in order to preserve the experimental hillslope area, $A = c(e^{aL} - 1)/a$. The initial recharge rate N_0 was derived from the measured outflow at $t = 0$ (Appendix C and equation (C4)). The linearization parameter p was treated as a fitting parameter [e.g., *Brutsaert*, 1994]. For each hillslope, a value of p was derived (numerically) such that the theoretical total outflow volume given in (21) was equal to the measured total outflow volume. However, for the convergent hillslopes with 5% and 10% slopes, the obtained p values were not realistic ($p > 1$). Therefore we could not derive the experimental CRF moments in these two cases. Our estimation of p is sensitive to uncertainties affecting the hillslope characteristics (in particular the width function and the hydraulic properties) and the recharge applied to reach the steady state. On the basis of these geometric and hydraulic parameters the hillslope Peclet number (Pe) and the characteristic diffusive time (τ_K) were computed (Table 1).

[25] The range of Pe values is from 1.56 to 4.86 (in the moderate advective regime) and the range of τ_K values is from 6.5 to 16.1 hours. After normalizing the observed hydrographs during each free drainage experiment, the first four dimensionless empirical moments of the CRF were calculated and plotted in Figures 2 and 3. Note the close agreement with the theoretical dimensionless moments. The functional dependence of these moments on Pe is well preserved for all four moments. The difference between the empirical and theoretical moments is small, especially if we consider the effect of measurement errors, the effect of linearization of the governing dynamic equation, the imposed exponential plan shape (which in reality is linear), and the uncertainties related to the determination of p. This illustrates that the proposed similarity parameter Pe allows, at least to first order, to predict the CRF of hillslopes with complex geometry.

[26] As the linearization parameter p influences Pe and τ_K, its estimation is an important question. We estimate the value of p by matching the total outflow volume, but other optimization strategies are possible (fitting the experimental storage profile for example). The issue of the estimation of p is subject of ongoing research.

5. Conclusions

[27] This paper reports the results of an analytical similarity study of subsurface flow response along complex hillslopes. Our similarity analysis is based on an exact analytical solution of an advection-diffusion equation, derived from a linearized form of the governing equation, in the Laplace domain. This solution is employed as the moment generating function of the characteristic response function in order to derive analytical expressions for the moments as functions of the hydraulic and geometric hillslope properties.

[28] By means of a dimensional analysis, we show that the effects of the hillslope properties and those of the boundary and initial conditions can be separated. In a dimensionless formulation, one similarity parameter is sufficient to describe the characteristic subsurface flow response, apart from the influence of the boundary and initial condition. This number depends only on the geometric characteristics of the hillslope and is referred to as the hillslope Péclet number. It accounts for the relative importance of the diffusion and advection terms.

[29] Given fixed boundary conditions, we demonstrated the consistency of the global behavior of the CRF moments for both types of initial conditions. The initial condition has a significant influence on the low-order moments, when the initial volume of water is uniformly distributed over the hillslope (first type of initial condition). On the contrary, the initial condition has a limited impact when the initial storage profile corresponds to a steady state storage profile (second type of initial condition). Therefore, in this case, the hillslope Péclet number almost completely describes the dimensionless CRF moments. Because we consider a spatially distributed system, the position within the hillslope is important and many different initial conditions can be defined. Therefore care should be taken to use the CRF obtained from an arbitrary initial condition in a convolution operation to compute the hillslope subsurface flow response during time-varying recharge.

[30] Outflow measurements from an experimental hillslope in four different configurations offered the opportunity to test our approach. The confrontation of the theoretical and empirical values of the dimensionless

moments of the CRF shows that the derived analytical expressions provide the relevant order of magnitude.

[31] The validity of our results is restricted to (1) the validity domain of the HSB equation (shallow soil mantle, streamlines parallel to the impervious bedrock, negligible influence of the unsaturated zone and absence of overland flow), (2) the validity domain of the linearization (constant slope angle, uniform hydraulic parameters and storage profile close to the mean profile), and (3) the considered boundary conditions.

[32] Further research is being carried out to validate the hillslope Péclet number as hillslope subsurface flow similarity parameter by confrontation with other experimental data. As previously mentioned, the estimation of the linearization parameter is an important issue for the applicability of the approach and hence must be studied. Finally, our analysis has been conducted at the hillslope scale and the scaling from hillslopes to catchments deserves further investigation.

Appendix A: Derivation of the Dimensionless Equation

[33] The dimensionless initial storage profile is given by

$$S_0^*(x^*, \pi_0^*) = \left(\frac{L}{2}\right)^{-2} S_0 \tag{A1}$$

where $\pi_0^* = \{\pi_1^*, ..\pi_{n_0}^*\}$ denotes the set of dimensionless parameters required to describe S_0^*. Now we define the dimensionless Laplace transform of the storage as

$$\widetilde{S}^* = \left(\frac{L}{2}\right)^{-4} K\widetilde{S} = \left(\frac{L}{2}\right)^{-4} K \int_0^\infty e^{-st} S(x,t)\, dt \tag{A2}$$

Introducing these variables in (4) yields

$$\frac{\partial^2 \widetilde{S}^*}{\partial x^{*2}} + \frac{UL}{2K}\frac{\partial \widetilde{S}^*}{\partial x^*} - s^*\widetilde{S}^* = -S_0^* \tag{A3}$$

which implies that

$$\widetilde{S}^* = \psi\left(s^*, x^*, \frac{UL}{2K}, \pi_\psi^*\right) \tag{A4}$$

where ψ is a function of dimensionless variables, and π_ψ^* denotes the set of parameters π_i^* independent of $\frac{UL}{2K}$ and the dimensionless parameters required to describe the boundary conditions. Let $\widetilde{Q}$ be the Laplace transform of the discharge and let us write (2) in the Laplace domain:

$$\widetilde{Q} = -K\frac{\partial \widetilde{S}}{\partial x} - U\widetilde{S} \tag{A5}$$

Similarly, we define the dimensionless Laplace transform of the discharge $\widetilde{Q}^*$ as

$$\widetilde{Q}^* = \left(\frac{L}{2}\right)^{-3} \widetilde{Q} \tag{A6}$$

So (A5) becomes

$$\widetilde{Q}^* = -\frac{\partial \widetilde{S}^*}{\partial x^*} - \frac{UL}{2K}\widetilde{S}^* \tag{A7}$$

Combining (A4) and (A7) yields

$$\widetilde{Q}^* = -\phi\left(s^*, x^*, \frac{UL}{2K}, \pi_\psi^*\right) \tag{A8}$$

where ϕ is a positive function of dimensionless variables. At this stage, we have a general formulation for the dimensionless Laplace transform of the discharge. The dimensionless total volume of water initially stored in the hillslope uphill of x is given by:

$$V^*(x^*) = -\left[\widetilde{Q}^*\right]_{s^*=0} = \left[\phi\left(s^*, x^*, \frac{UL}{2K}, \pi_\psi^*\right)\right]_{s^*=0} \tag{A9}$$

Because $\widetilde{Q}^*$ is negative, the dimensionless CRF is obtained by normalizing $-\widetilde{Q}^*$ by the dimensionless total volume at the outlet ($x^* = 0$). As the Laplace transform of the CRF is its moment generating function, we can derive a general formulation for the dimensionless CRF moments:

$$m_n^* = (-1)^{n+1}\frac{1}{V^*(0)}\left[\frac{\partial^n \widetilde{Q}^*}{\partial s^{*n}}\right]_{s^*=0} = \phi_n\left(\frac{UL}{2K}, \pi^*\right) \tag{A10}$$

where m_n^* denotes the dimensionless nth-order moment and π^* represents the subset of dimensionless parameters from π_ψ^* that remain after the normalization. ϕ_n is defined as

$$\phi_n = \frac{(-1)^n}{V^*(0)}\left[\frac{\partial^n \phi}{\partial s^{*n}}\left(s^*, x^*, \frac{UL}{2K}, \pi_\psi^*\right)\right]_{\{s^*, x^*\}=\{0,0\}} \tag{A11}$$

According to (A9) and (A11), any proportionality factor of ϕ will disappear in ϕ_n. This property is applied in section 3.

Appendix B: Derivation of an Analytical Solution in the Laplace Domain

[34] For a given s, (4) is a linear ordinary differential equation (ODE) with constant coefficients and nonhomogeneous forcing term. The solution of such an ODE is given as the sum of the solution ($\widetilde{S}_h$) of the corresponding homogeneous ODE and a particular solution ($\widetilde{S}_p$). The general solution of the homogeneous equation is:

$$\widetilde{S}_h(s,x) = C_1 e^{(d+b)x} + C_2 e^{(d-b)x} \tag{B1}$$

where

$$d = -\frac{U}{2K}$$
$$b = \sqrt{d^2 + \frac{s}{K}}$$

C_1 and C_2 are constants and their values are chosen such as to satisfy the boundary conditions. Both (C_1, C_2) and the particular solution (derived using the method of variation of parameters [*Boyce and DiPrima*, 1977]) depend on the assumed initial condition, through the forcing term S_0.

B1. Initial Condition 1

[35] The particular solution is

$$\widetilde{S}_p = -\frac{\gamma Dfc}{Ka^2 + Ua - s} e^{ax} \quad \text{(B2)}$$

So the general solution is

$$\widetilde{S}(s,x) = C_1 e^{(d+b)x} + C_2 e^{(d-b)x} - \frac{\gamma Dfc}{Ka^2 + Ua - s} e^{ax} \quad \text{(B3)}$$

where C_1 and C_2 are calculated in order to satisfy the boundary conditions (11):

$$C_1 = \frac{\gamma Dfc}{Ka^2 + Ua - s} \frac{e^{aL}(a - 2d) + e^{(d-b)L}(d + b)}{(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}}$$
$$C_2 = \frac{\gamma Dfc}{Ka^2 + Ua - s} \frac{e^{(d+b)L}(b - d) - e^{aL}(a - 2d)}{(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}} \quad \text{(B4)}$$

Using (2) we can derive the expression for the discharge dynamics in the Laplace domain:

$$\widetilde{Q}(s,x) = -L^3 \frac{\gamma Dfc}{L^2} \frac{1}{\left(aL(aL + 2\mathrm{Pe}) - s\frac{L^2}{K}\right)} \left\{ -e^{ax}(aL + 2\mathrm{Pe}) + \frac{s\frac{L^2}{K}\left[e^{(d-b)L}e^{(d+b)x} - e^{(d+b)L}e^{(d-b)x}\right]}{L[(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}]} + \frac{(aL + 2\mathrm{Pe})Le^{aL}\left[(b - d)e^{(d+b)x} + (b + d)e^{(d-b)x}\right]}{L[(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}]} \right\} \quad \text{(B5)}$$

As $dL = -\mathrm{Pe}$ and $bL = \sqrt{Pe^2 + 4s^*}$, (B5) can be expressed in the form of (A8).

B2. Initial Condition 2

[36] The particular solution of (4) is derived using to the initial storage profile given in (C1):

$$\widetilde{S}_p = \frac{N_0 c}{a}\left[\frac{e^{aL}}{sU}\left(1 - e^{-\frac{U}{K}x}\right) + \frac{1}{(Ka + U)}\left(\frac{e^{-\frac{U}{K}x}}{s} + \frac{e^{ax}}{Ka^2 + Ua - s}\right)\right] \quad \text{(B6)}$$

So the general solution is

$$\widetilde{S}(s,x) = \frac{N_0 c}{a}\left[\frac{e^{aL}}{sU}\left(1 - e^{-\frac{U}{K}x}\right) + \frac{1}{(Ka + U)} \cdot \left(\frac{e^{-\frac{U}{K}x}}{s} + \frac{e^{ax}}{Ka^2 + Ua - s}\right)\right] + C_1 e^{(d+b)x} + C_2 e^{(d-b)x} \quad \text{(B7)}$$

and C_1 and C_2 are

$$C_1 = -\frac{N_0 c}{s(Ka^2 + Ua - s)} \frac{e^{aL}\left(a + \frac{U}{K}\right) + e^{(d-b)L}(d + b)}{(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}}$$
$$C_2 = -\frac{N_0 c}{s(Ka^2 + Ua - s)} \frac{e^{(d+b)L}(b - d) - e^{aL}\left(a + \frac{U}{K}\right)}{(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}} \quad \text{(B8)}$$

Similarly, the expression for the discharge in the Laplace domain is derived using (2):

$$\widetilde{Q}(s,x) = -L^3 \frac{N_0 c}{K} \frac{1}{s\frac{L^2}{K}\left[aL(aL + 2\mathrm{Pe}) - s\frac{L^2}{K}\right]} \cdot \left\{ \frac{e^{aL}\left[aL(aL + 2\mathrm{Pe}) - s\frac{L^2}{K}\right] + s\frac{L^2}{K}e^{aL\frac{x}{L}}}{aL} - \frac{e^{aL}(aL + 2\mathrm{Pe})L\left[(b - d)e^{L(d+b)\frac{x}{L}} + (b + d)e^{L(d-b)\frac{x}{L}}\right]}{L[(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}]} - \frac{s\frac{L^2}{K}\left[e^{(d-b)L}e^{L(d+b)\frac{x}{L}} - e^{(d+b)L}e^{L(d-b)\frac{x}{L}}\right]}{L[(b - d)e^{(d+b)L} + (b + d)e^{(d-b)L}]} \right\} \quad \text{(B9)}$$

Equation (B9) can also be expressed in the form of (A8).

Appendix C: Derivation of the Initial Steady State Solution

[37] We derive the steady state solution of (1) for a given recharge N:

$$S_0 = \frac{Nc}{a}\left[\frac{e^{aL}}{U}\left(1 - e^{-\frac{U}{K}x}\right) + \frac{1}{(Ka + U)}\left(e^{-\frac{U}{K}x} - e^{ax}\right)\right] \quad \text{(C1)}$$

In order to define a unique initial condition, we have to fix a value for N. We use the recharge such that the maximum mean (over the hillslope width) groundwater table height $\overline{h}_m$ along the hillslope just reaches the ground surface. We must first calculate the maximum mean groundwater table height as a function of N and then determine N_0 such that $\overline{h}_m = D$.

[38] According to the definition of the storage S, the mean groundwater table height is

$$\overline{h} = \frac{S}{fw} = \frac{Ne^{-ax}}{af}\left[\frac{e^{aL}}{U}\left(1 - e^{-\frac{U}{K}x}\right) + \frac{1}{Ka + U}\left(e^{-\frac{U}{K}x} - e^{ax}\right)\right] \quad \text{(C2)}$$

We want to calculate the x-coordinate x_m where the mean groundwater table height is maximum. Solving $\overline{h}'(x_m) = 0$, where the prime denotes a derivative with respect to x, yields

$$x_m = \frac{K}{U}\ln\left[1 + \frac{U}{Ka}\left(1 - e^{-aL}\right)\right] \quad \text{(C3)}$$

For uniform hillslope ($a = 0$), this reduces to $x_m = \frac{K}{U}\ln(1 + \frac{UL}{K})$, which is consistent with *Verhoest and Troch* [2000, equation [21]]. It is easy to check that $x = x_m$ corresponds to a maximum. To derive a simple expression for $\overline{h}(x_m)$, we must note that the discharge expression is simple in the steady state. Integrating the continuity equation leads to

$$Q(x) = -\int_x^L Nw(u)\,du = \frac{Nc}{a}\left(e^{ax} - e^{aL}\right) \quad \text{(C4)}$$

From (C2), we can also write

$$\overline{h}' = \frac{e^{-ax}}{cf}(S' - aS)$$

So

$$S'(x_m) = aS(x_m) \tag{C5}$$

Introducing (C5) in (2) yields

$$S(x_m) = -\frac{Q(x_m)}{aK + U} \tag{C6}$$

So we obtain the following expression for the maximum mean groundwater table height:

$$\overline{h}(x_m) = \frac{N}{fa(aK+U)}\left\{e^{aL}\left[1+\frac{U}{aK}\left(1-e^{-aL}\right)\right]^{-\frac{aK}{U}}-1\right\} \tag{C7}$$

This expression can also be obtained by substituting (C3) directly into (C2). We can finally derive the expression of the recharge N_0 that leads to the maximum mean groundwater table height equal to D:

$$N_0 = \frac{faD(aK+U)}{e^{aL}\left[1+\frac{U}{aK}\left(1-e^{-aL}\right)\right]^{-\frac{aK}{U}}-1} \tag{C8}$$

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A. Berne, P. A. Troch, and R. Uijlenhoet, Hydrology and Quantitative Water Management Group, Wageningen University, Nieuwe Kanaal 11, NL-6709 PA Wageningen, Netherlands. (alexis.berne@wur.nl)