Simulating the effect of capillary flux on the soil water balance in a stochastic ecohydrological framework

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Groundwater uptake can play a major role in the survival of vegetation in semiarid areas, but this has not yet been included in an earlier developed ecohydrological stochastic framework. In this paper we provide a piecewise linear equation which includes capillary fluxes from shallow groundwater in the loss function of the ecohydrological stochastic model. The results indicate that this model is able to simulate the capillary fluxes, and the model also reflects the impact of the fluxes on the soil moisture balance. In addition, the results are analytically tractable and allow calculation of the probability density functions of soil water saturation and water stress for different groundwater depths below the root zone.


1. Introduction

Recently, Rodríguez-Iurbe and Porporato [2004] developed a framework for the stochastic modeling of the soil water balance and have coupled this to a colonization and competition model to describe the dynamics of semiarid (savanna) vegetation systems, i.e., systems which are primarily controlled by water. Their stochastic framework currently does not include any groundwater interaction. However, groundwater can be an important driver of vegetation growth and occurrence in many semiarid areas, particularly in relation to groundwater-dependent ecosystems in riparian areas [i.e., Mensforth et al., 1994; Thorburn and Walker, 1994; Walker et al., 1993]. This raises the question, which groundwater levels lead to important contributions of groundwater to the soil water balance?

There are two possible mechanisms for the interaction between groundwater and vegetation: (1) part of the root mass interacts with the groundwater and water is taken up directly, and (2) capillary fluxes cause water to move into the root zone after which it is taken up by the vegetation. Which of the two dominates is not clear from the literature, but in this study we will concentrate on the capillary fluxes. One way to investigate the importance of the different processes is to describe the process of groundwater interaction using a model, i.e., adapting the stochastic framework to include groundwater uptake.

Evaporation and related capillary flow from a relatively shallow groundwater table has been studied for some time [e.g., Gardner, 1958; Philip, 1957]. The conceptual model of the system is a homogeneous soil with a root zone to a depth Z, and a groundwater table at a depth Z below the soil surface (Figure 1). Evaporation and rainfall occur at the soil surface and affect mainly the water storage in the root zone. No hysteresis occurs and the hydraulic relationships are generally of an exponential or linear form [Salvucci, 1993]. Drainage from the soil store reaches the water table instantaneously, and the soil water profile below the root zone has reached steady state. This means that the fluctuations in the groundwater table occur at a much larger time scale than the fluctuations in the climatic drivers (i.e., years versus days and weeks). With some exceptions (such as transmission losses from a river on a highly permeable bed), this is reasonable for semiarid systems. Thus, we also assume that the groundwater table is at a constant level throughout the period of study, and this will be discussed further in section 3.

The steady flux of water in the root zone can be described with the Darcy equation:

\[ q = K(h) \left( \frac{dh}{dz} - 1 \right), \]  

(1)

where \( q \) is the flux (L T⁻¹), \( K(h) \) is the hydraulic conductivity function, and \( dh/dz \) is the potential gradient. For a steady flux, (1) can be integrated to lead to

\[ Z = \int_0^{h(Z=0)} \frac{dh}{1 + q/K(h)}. \]  

(2)

Equation (2) describes the maximum height for which a designated capillary flux \( q \) can be supplied for particular soil hydraulic properties and dryness at the soil surface.

Full complex analytical solutions for equation (2) can be derived, assuming specific forms of the hydraulic conductivity function [Warrick, 1988]. However, the solutions are not practical for implementation in analytical models.

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since they cannot be explicitly solved for \( q \), except in the case of \( q \leq K_s \), the saturated hydraulic conductivity, and cannot be inverted to provide \( h(q, Z) \). Approximate models that do not have these disadvantages have therefore been developed [Eagleson, 1978; Salvucci, 1993]. However, both are approximate models for equation (2), and while allowing an analytical description, they cannot be easily included into the stochastic framework [Rodriguez-Iturbe and Porporato, 2004].

[7] The main aim of this paper is, with an adapted stochastic framework, to study the importance of the capillary fluxes from groundwater to replenish the evapotranspiration demand of the vegetation and to study the resulting changes in the soil water balance in semiarid areas. To achieve this aim, we develop a new piecewise linear function that matches the illustration in Figure 1 and \( q \) derived from equation (2) for describing groundwater uptake that fits within the stochastic framework [Rodriguez-Iturbe and Porporato, 2004] and allows calculation of probability density functions of the annual soil saturation.

[8] Similar to a recent paper by Ridolfi et al. [2008], but approaching the same issue from a different direction, this paper is a first step toward a more complete stochastic model of groundwater surface water interaction.

2. Methods

2.1. Background Theory

[9] The previously derived loss function in the stochastic framework (henceforth denoted the “RI model”) is defined as [Rodriguez-Iturbe and Porporato, 2004] (Figure 2)

\[
\rho(s) = \begin{cases} 
\frac{\eta_w (s - s_h)}{(s_w - s_h)} & s_h < s \leq s_w \\
\eta_w + (\eta - \eta_w) \frac{(s - s_w)}{(s^* - s_w)} & s_w < s \leq s^* \\
\eta + m e^{(s - s^*)} - 1 & s^* < s \leq s_c \\
\end{cases}
\]

\[
\eta = \frac{E_{\text{max}}}{\phi Z_r}, \\
\eta_w = \frac{E_w}{\phi Z_r}, \\
m = \frac{K_s}{\phi Z_r} e^{[(1 - s_c) - 1]}. 
\]

Here \( s \) is the soil saturation (0–1), \( \phi \) is the porosity, \( Z_r \) is the root zone depth normalized versions of \( E_{\text{max}} \) and \( E_w \) respectively [Rodriguez-Iturbe and Porporato, 2004]. The model uses a piecewise linear formulation to enable an analytical solution for the saturation probability density function [Laio et al., 2001; Rodriguez-Iturbe et al., 1999], and the new groundwater uptake function within this framework should therefore be defined along similar lines.

[10] The climate in the RI model is defined by the parameters \( \lambda \) and \( \gamma \), which arise from the Poisson distributed rainfall [Laio et al., 2001; Rodriguez-Iturbe and Porporato, 2004]. The parameter \( \lambda \) is equal to \( \lambda e^{-\Delta/Z_r} \) where \( \Delta \) is the interception depth (cm), \( \phi \) is the mean storm depth, and \( \lambda \) is the mean time between rainstorms [Laio et al., 2001; Rodriguez-Iturbe and Porporato, 2004]. The parameter \( \gamma \) is equal to \( \phi Z_r / \alpha \) or, equivalently, \( 1/\gamma \) is the root zone weighted mean storm depth.

2.2. A Simplified Capillary Flux Model That Fits Into the Ecological Framework

[11] We aim to identify a function for the total losses from the stored soil water (in the root zone) as a function of the soil saturation \( s \). This means following equation (3) and with the total of capillary and drainage fluxes defined as \( \eta_{\text{total}} \), we seek \( \eta_{\text{new}} = \eta_{\text{total}} - \eta_{\text{total}} \) as a function of \( s \), where \( \eta_{\text{total}} \) is evapotranspiration, which matches \( q \) from equation (2). Rather than searching for an analytical solution of equation (2) that is based on “first” principles, we approached the problem from a different end in view of the mathematical complexity of the former.

[12] We solve equation (2) through optimization using “optim” in R [R Development Core Team, 2007]. This algorithm uses the “Nelder Mead” simplex algorithm to find an optimal solution [R Development Core Team, 2007]. The combinations of \( q(s, z) \) must be described with a piecewise linear function that fits to these curves and is appropriate for the stochastic framework. This desired
The boundaries of such a function need to be
\[
q_{\text{lim}} = s \left( \frac{dh}{dz} = 1 = \left( \frac{Z - Z_c}{h_b} \right)^{-1/b} \right),
\]
which replaces equation (3) in the stochastic framework. The parameters \( s_{\text{lim}} \) and \( s_{c}\) represent the soil saturation point where the soil shifts from drainage behavior to capillary uptake behavior and is therefore equal to the hydrostatic point. This value of \( s \) at the hydrostatic equilibrium is a shifting “field capacity” soil saturation for which the magnitude depends on the depth of the groundwater and the soil hydraulic parameters.

The suggested form of the function which predicts the capillary and drainage fluxes \( q_{\text{total}} \) as a function of the soil saturation \( s \), which has a similar shape to the \( q(s, z) \) curves, is

\[
q_{\text{total}}(s) = \begin{cases} 
-m_2 & s_{c} < s \leq s^* \\
-m_1 \left[ 1 - e^{-b(s - s_{c})} \right] & s^* < s \leq s_{\text{lim}} \end{cases}
\]

where \( m_2 \) and \( m_1 \) are two constants: \( m_2 \) represents the maximum capillary flux for a given groundwater depth and hydraulic properties (encapsulated in \( G \)), while \( m_1 \) is equal to \( m_2 \) normalized for the reduction in capillary flux with increased saturation.

The dimensionless parameter \( G \) is a function that describes the relationship of the capillary flux with the groundwater depth, the bubbling pressure \( h_b \), and the hydraulic shape parameters \( a \) and \( 2 + 3/b \) [Eagleson, 1978] and is suggested to have the following functional form [Eagleson, 1978]:

\[
G = \alpha_e \left( \frac{h_b}{Z - Z_c} \right)^{2+3/b}.
\]

In equation (5), below \( s^* \) the actual capillary flux will be driven by the ET demand and can be lower than the potential capillary flux, while above \( s^* \) the capillary fluxes slowly decline with increased saturation. Basically, the impact of the capillary flux is that at some value of \( s \) the total loss \( p_{\text{new}} = \text{ET} + q_{\text{total}} \) actually equals zero (Figure 2). The soil will never dry out below this level of soil saturation because at this point (and below), the potential capillary flux is either equal to or greater than the actual evaporation losses and thus all evaporation demand can be supplied by the capillary flux. We will call this saturation point \( s \) “critical” \((s_{c})\). In reality, this means that below \( s_{c} \) the potential flux will be reduced until the capillary flow matches the actual ET. This also implies that \( s_{c} \) is the minimum soil saturation level the soil will reach for that particular groundwater level, ET demand curve, and soil type. This means \( s_{\text{lim}} \) and \( s_{c} \) are two important points on the saturation scale, both of which are dependent on the depth of groundwater.
groundwater level and the hydraulic properties of the soil. They both represent boundary values at which point the behavior of the loss function changes.

[16] For deep groundwater tables \( s_{cr} \) will be equal to \( s_w \), and for shallow groundwater tables it will be equal to \( s^* \). In the function including groundwater uptake, we need to use \( s_{lim} \) rather than \( s_{cr} \) as this point on the saturation curve becomes a variable rather than a fixed parameter and, as mentioned earlier, is defined as

\[
s_{lim} = \left( \frac{Z - Z_r}{h_b} \right)^{-1/b}.
\] (7)

In the rest of this paper we will use \( s_{lim} \) in the new piecewise functions.

This means the new loss function can be defined as

\[
\rho_{new} = \begin{cases} 
\left( \eta - m_2 \right) \left( \frac{s - s_{cr}}{s^* - s_{cr}} \right)^{\eta - \eta_s} e^{-\eta_s} & s_{cr} < s \leq s^* \\
\eta - m_1 \left[ 1 - e^{\left( s^* - s_{lim} \right)} \right] & s^* < s \leq s_{lim} \\
\eta + m \left[ e^{\left( s^* - s_{lim} \right)} - 1 \right] & s_{lim} < s \leq 1,
\end{cases}
\] (9)

\[
m_2 = \frac{K_s G}{\phi Z_r} \frac{1}{n} \frac{\left( s^* - s_{lim} \right)}{e^{\left( s^* - s_{lim} \right)} - 1}.
\]

All other parameters have been defined earlier. After integration, the probability density function, \( p(s) \), becomes

\[
p(s) = \begin{cases} 
\frac{C}{\left( \eta - m_2 \right)} \left( \frac{s - s_{cr}}{s^* - s_{cr}} \right)^{\eta - \eta_s} e^{-\eta_s} & s_{cr} < s \leq s^* \\
\frac{C}{\eta} \frac{\eta - m_1 \left( 1 - e^{\left( s^* - s_{lim} \right)} \right)}{\left( \eta - \eta_s \right)} e^{-\eta_s} \frac{\left( s^* - s_{lim} \right)}{e^{\left( s^* - s_{lim} \right)} - 1} & s^* < s \leq s_{lim} \\
\frac{C}{\eta} \left( \frac{1}{\eta} + m \left( e^{\left( s^* - s_{lim} \right)} - 1 \right) \right) \frac{\eta - m_1 \left( 1 - e^{\left( s^* - s_{lim} \right)} \right)}{\left( \eta - \eta_s \right)} e^{-\eta_s} \frac{\left( s^* - s_{lim} \right)}{e^{\left( s^* - s_{lim} \right)} - 1} & s_{lim} < s \leq 1.
\end{cases}
\] (10)

The parameter \( C \) in equation (10) is an integration constant. This constant can be derived analytically for the RI model [Laio et al., 2001], but we have approximated it numerically by using the fact that the area under a probability density function (pdf) should equal unity [Laio et al., 2001].

[18] We will now consider the situation \( m_1 \geq \eta \) and \( m_2 \geq \eta \), which means the capillary fluxes are too small to maintain evapotranspiration at maximum capacity. From the above discussion and the definition of \( s_{cr} \), it also follows that the total loss of soil moisture below \( s_{cr} \) is equal to zero. This implies that \( s_{cr} \) can, in this case, be defined by equating \( \eta \left( s - s_w \right) \left( s^* - s_w \right) \) with \( m_2 \):

\[
s_{cr} = \frac{m_2}{\eta} \left( s^* - s_w \right) + s_w.
\] (8)

\[
\rho_{new} = \begin{cases} 
\left( \eta - m_2 \right) \left( \frac{s - s_{cr}}{s^* - s_{cr}} \right)^{\eta - \eta_s} e^{-\eta_s} & s_{cr} < s \leq s^* \\
\eta - m_1 \left[ 1 - e^{\left( s^* - s_{lim} \right)} \right] & s^* < s \leq s_{lim} \\
\eta + m \left[ e^{\left( s^* - s_{lim} \right)} - 1 \right] & s_{lim} < s \leq 1,
\end{cases}
\] (11)

\[
m_2 = \frac{K_s G}{\phi Z_r} \frac{1}{n} \frac{1}{e^{\left( s^* - s_{lim} \right)} - 1}.
\]

\[
m_1 = \frac{K_s G}{\phi Z_r} \frac{1}{1 - e^{\left( s^* - s_{lim} \right)}}.
\]

\[
m = \frac{K_s}{\phi Z_r} \left( e^{1 - s_{lim}} - 1 \right).
\]
The critical groundwater depth \( Z_{cr} \) at which this will happen can be calculated by setting \( m_1 \) equal to \( \eta \) and including the definition of \( G \) (equation (7)) into the definition of \( m_1 \) (equation (9)).

\[
Z_{cr} = Z_b + \frac{h_b}{\left[ \frac{\delta n \left( 1 - e^\left( x - s_{cr} \right) \right)}{k_c \omega_c} \right]^{1/\lambda}}.
\]

As a result, the probability density function \( p(s) \) for this situation can be rewritten as

\[
p_{new}(s) = \begin{cases} 
C \frac{\exp(-\beta(s - s_{cr}) - \gamma s)}{\eta} 
\cdot \exp\left[ \frac{\gamma}{\eta} \left( 1 - \exp(-\beta(s - s_{lim})) \right) \right] 
& \text{s}_{cr} < s \leq \text{s}_{lim} \\
C \left[ 1 + \frac{m}{\eta} \left( e^{\beta(s - s_{cr})} - 1 \right) \right] \frac{\eta}{\exp(s_{lim})}^{1/\gamma} 
& \text{s}_{lim} < s < 1.
\end{cases}
\]

The overall loss function for this model becomes

\[
\rho_{new}(s) = \begin{cases} 
(\eta - m_2) \frac{\left( s - s_{cr} \right)}{s_{cr} - s_{cr}} & \text{s}_{cr} < s \leq \text{s}_{w} \\
(\eta - m_2) + (\eta - \eta_1) \frac{\left( s - s_{cr} \right)}{s_{cr} - s_{cr}} & \text{s}_{w} < s \leq s^* \\
\eta - m_1 \left[ e^{\beta(s - s_{cr})} - 1 \right] & s^* < s \leq \text{s}_{lim} \\
\eta + m \left[ e^{\beta(s - s_{cr})} - 1 \right] & \text{s}_{lim} < s \leq 1,
\end{cases}
\]

After integration, for the resulting \( p(s) \) we obtain

\[
p_{new}(s) = \begin{cases} 
C \frac{\left( s - s_{cr} \right)}{\eta - s_{cr}} \frac{\left( \eta - s_{cr} \right)}{\eta - \eta_1} \frac{\eta}{\exp(s_{cr})}^{1/\gamma} & \text{s}_{cr} < s \leq \text{s}_{w} \\
\frac{C}{\eta - m_2} \left[ 1 + \frac{\eta - \eta_1}{\eta - m_2} \right] \frac{\eta}{\exp(s_{cr})}^{1/\gamma} & \text{s}_{w} < s \leq s^* \\
\frac{C}{\eta} \left[ 1 + \frac{\eta - \eta_1}{\eta - m_2} \right] \frac{\eta}{\exp(s_{lim})}^{1/\gamma} & \text{s}^* < s \leq \text{s}_{lim} \\
\frac{C}{\eta} \left[ 1 + \frac{\eta - \eta_1}{\eta - m_2} \right] \frac{\eta}{\exp(s_{lim})}^{1/\gamma} & \text{s}_{lim} < s \leq 1.
\end{cases}
\]

The replacement of equation (3) in the stochastic framework now consists of three cases: equations (9), (11), and (15). Basically, the value of the potential capillary flux \( K_c G \) needs to be compared with the maximum evapotranspiration rate, after which the different forms of the probability density function \( p(s) \) can be calculated using equations (10), (13), or (16).

2.3. Calculations

To compare how well the suggested new model is able to represent changes in the soil saturation under a varying climatic input and for different soils, water balance...
calculations over 10,000 days were performed on the basis of equations (3), (9), and (15). For the water balance calculations, rainfall was generated on the basis of a Poisson distribution of the storm frequency and an exponential distribution of the rainfall amounts [Rodriguez-Iturbe et al., 1984]. The first 365 days of the water balance calculations were deleted to create a 1 year warm-up period for the derivation of means and variances.

Further, soil and vegetation parameters were derived from the literature [e.g., Porporato et al., 2001] and databases of Australian soils (Tables 1 and 2). The porosity parameter \( \phi \) was set equal to \( \theta_i \) as estimated with the van Genuchten pedotransfer functions in Neurotheta [Minasny and McBratney, 2002], while for \( s_{EC} \) the value at \( h = -100 \) cm was chosen from the same pedotransfer functions.

Some representative climate parameters (Table 3) were calculated from long-term rainfall data for several locations in Australia [Rodriguez-Iturbe et al., 1984], and this defined the range of possible values for \( \alpha \) and \( \lambda \).

Maximum evaporation data \( (E_{\text{max}}) \) was based on the average data for the listed weather stations in Table 3 (see http://www.bom.gov.au/climate/averages).

### 3. Results and Discussion

The proposed new function (equation (9)) fits the optimized \( q \) values quite well (Figure 3). This suggests that equation (9) is similar in behavior to the solution of equation (2) and can be used to simulate the capillary and drainage fluxes for the system in Figure 1 in the stochastic framework. There is only a slight overestimation of the capillary flux close to \( s_{\text{lim}} \) (Figure 3). Figure 3 clearly indicates how the point where drainage and capillary fluxes are both zero (i.e., the hydrostatic point or \( s_{\text{lim}} \)) shifts depending on the soil type and the groundwater depth. It also demonstrates that the capillary fluxes rapidly become very small at deeper groundwater levels (i.e., deeper than 150 cm below the bottom of the root zone for most soils).

A comparison of the water balance results of the two versions of the suggested piecewise linear models (equations (9) and (15)) indicates that there is only a very small penalty for ignoring the section below \( s_w \) (Table 4). The model with four limits is possibly more accurate for drier climates and deeper groundwater levels, as excursions below \( s_w \) are more frequent; however, the difference in the variance and the means is small (Table 4). This means using equation (9) is appropriate for most situations.

In comparison with the original model [Rodriguez-Iturbe and Porporato, 2004], the new model predicts lower soil saturations under wetter climates (Figure 4). This is because in wetter climates \( (\alpha \lambda = 0.7) \), the drainage process will dominate over the capillary processes, which means the value of the hydrostatic point \( s_{\text{lim}} \) becomes important (which is the point where fluxes are zero). Equations (9) and (15) include a variable \( s_{\text{lim}} \) which is a function of the groundwater depth to reflect the hydrostatic point. This results in a continuation of the drainage process compared to using a fixed \( s_{EC} \) in the original RI model, particularly at deeper groundwater levels. However, this is a more accurate representation of the real process. A constant \( s_{EC} \) value is valid for simulating drainage above deep groundwater tables but will probably overestimate the hydrostatic point for most soils (Table 1 and Figure 3). This would result in an underestimation of the actual drainage and thus relatively higher soil saturation values.

Overall, the new model clearly demonstrates the effect of groundwater on the soil water balance. Shallower groundwater tables tend to increase the amount of water in the soil, and this effect is larger for drier climates than for wetter climates (Figure 4). In addition, the effect dissipates rapidly with increasing depth of the groundwater table. For the sandy clay loam used in Figure 4, the impact of the capillary fluxes on the soil water balance is minor for a groundwater table at 2 m below the root zone, even under the driest climate \( (\alpha \lambda = 0.18) \). For soils with lower \( K_v \) values, the influence of groundwater will decrease even earlier.

The models presented here all assume that the majority of the root water uptake is concentrated in the root zone. This is not always the case. Groundwater-dependent vegetation could also have dimorphic root systems or varying root hydraulics which means that the majority of the root water uptake takes place from only a small fraction of the roots close to the groundwater table [Dawson and Pate, 1996]. This is not considered in this study but could be included in extensions on this work.

In this study, we have also assumed that the groundwater level is not directly affected by the daily atmospheric inputs or the evaporation from the vegetation. This assumption is only valid if the lateral transmissivity of the aquifer is much larger than the vertical transmissivity or if the aquifer system storage is very large compared to the capillary fluxes. Incorporation of the impact of evaporation and

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**Table 2. Vegetation Properties Used in the Simulations Following Porporato et al. [2001]**

<table>
<thead>
<tr>
<th>Property</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_i ) (cm)</td>
<td>100</td>
</tr>
<tr>
<td>( \Delta ) (cm)</td>
<td>0.2</td>
</tr>
<tr>
<td>( E_{\text{max}} ) (cm/d)</td>
<td>0.5</td>
</tr>
<tr>
<td>( E_{\text{w}} ) (cm/d)</td>
<td>0.01</td>
</tr>
<tr>
<td>( \psi_{s,\sigma} ) (MPa)</td>
<td>-0.12</td>
</tr>
<tr>
<td>( \psi_{s,p} ) (MPa)</td>
<td>-5.0</td>
</tr>
<tr>
<td>Leaf area index ( \xi )</td>
<td>2.5a</td>
</tr>
</tbody>
</table>

*From Whitehead and Beadle*
atmospheric inputs is a further step, and a recent paper has made some progress in that area [Ridolfi et al., 2008]. In another approach the groundwater table could be varied exogeneously (e.g., using a seasonal time series model in relation to the annual rainfall [Salas and Obeysekera, 1992]) where this variation is then incorporated into the model in this paper. In this case the function $G$ would become related to $\lambda$ and $\alpha$ through $Z$, and this is part of our ongoing research in this area.

3.1. Probabilistic Representation of $s$

[29] The probability density functions ($p(s)$) also indicate a clear difference between deep to shallow groundwater (Figure 5). The probability density functions spread out with shallower water table depths until $Z_{cr}$ is approached. The bounding at $s_{cr}$ for the lower end of $p(s)$ is also demonstrated (Figure 5). The value of $s_{cr}$ will shift toward $s_{lim}$ with shallower groundwater depths as higher capillary fluxes and thus higher evaporative demand are supported. Heavier soils (such as the light medium clay) will show a smaller shift as a function of groundwater depth because of lower hydraulic conductivities. In addition, the effect of groundwater on $p(s)$ decreases more rapidly than for more highly conductive (large $K_s$) soils, such as the sandy clay loam (Figure 6).

[30] With increasingly shallow groundwater depth, both $s_{cr}$ and $s_{lim}$ change. However, initially far from $Z_{cr}$, because of the nonlinearity of equation (12), $s_{lim}$ will increase much faster than $s_{cr}$, as equation (8) is approximately linear. This will cause the probability density function to spread (Figure 6). For groundwater levels closer to $Z_{cr}$, $s_{lim}$ increases less quickly, causing the probability density function to narrow. Groundwater levels shallower than $Z_{cr}$ finally generate a steeper and narrower $p(s)$ curve (equation (13)). This steepness of $p(s)$ under shallow groundwater tables is partly due to the fact that $p(s)$ is bounded on the upper end because of the sharp increase in losses from drainage above $s_{lim}$. Under wetter climate conditions the overall functions broaden because of an increase in excursions above $s^*$ for deeper groundwater depths and an increase in the variance relative to the dry climate (see Table 4). The same narrow probability density functions occur at shallow groundwater levels, as $p(s)$ is pushed up against the upper limit.

[31] Means and variances of $s$ were calculated numerically by integrating the probability density functions:

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**Figure 3.** Comparison of the optimized $q$ capillary values from the integral in equation (2) (solid black lines) with the new loss function including groundwater interaction ("new model" equation (9) dashed lines) for different groundwater depths ($Z$) below the root zone. Data have been plotted against relative saturation, $Se = (s - s_h)/(1 - s_h)$, to improve comparison between the soil types.


Table 4. Means and Standard Deviations of Soil Saturation $s^r$

<table>
<thead>
<tr>
<th>Climate (s)</th>
<th>0.18</th>
<th>0.33</th>
<th>0.5</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of $s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
New model with three limits | 0.48 | 0.57 | 0.66 | 0.67 |
New model with four limits | 0.47 | 0.56 | 0.66 | 0.66 |
| Variance of $s$ ($\times 10^{-3}$) | 
New model with three limits | 2.20 | 4.15 | 3.40 | 2.25 |
New model with four limits | 2.10 | 4.38 | 3.67 | 2.35 |

The top compares the two new models (with three limits, equation (9), and with four limits, equation (14)) over 10,000 days for different climates (different combinations of $s$) on a sandy clay loam. The bottom compares the new piecewise model with three limits (equation (9)) and four limits (equation (14)) to the original model without groundwater (equation (3)) for the same soil at two groundwater depths (175 and 300 cm below the surface) for the same climate combinations and soil.

The mean of $s$ increases with increasingly shallow groundwater levels (Figure 7) and decreases to the mean as predicted without accounting for groundwater at deeper groundwater depths (RI model). While there is some increase in the mean with wetter climates, the increase is small relative to the increase in the mean as a result of shallower groundwater tables. Again, the influence of groundwater on the mean soil saturation decreases rapidly as soon as groundwater levels are lower than 1–2 m below the root zone (depending on the soil type), indicating that capillary fluxes only affect the soil saturation over only a small range of groundwater depths.

In contrast, the variances of $s$ of the new model deviate quite considerably from the variance obtained for the RI model. Overall, the variance increases for wetter climates compared to drier climates, but this is true for both models (Table 4 and Figure 8). However, the variances of the new model reveal a maximum with respect to groundwater depth, which depends, among other things, on climatic forcing (Figure 8). With increasingly shallow groundwater depths, the variances of the new model first increase until close to $Z_{cr}$, after which the variances steeply decrease to a constant value (i.e., the variance of equation (13)). For deeper groundwater levels the variance of equation (10) also deviates from the original RI model. This is due to the changes in the distance between $s_{cr}$ and $s_{lim}$ compared to the RI model with a fixed $s_{cr}$. A fixed value of $s_{cr}$ might be close to the steady state soil saturation value, which is higher than the hydrostatic point for deep water tables [Salvucci and Entekhabi, 1994]. Assuming that soils in steady state equilibrium at $s_{cr}$, this also indicates that most soils will have drainage fluxes greater than zero under steady state conditions above deep water tables.

3.2. Water Stress Calculations

Water stress is a useful summary statistic for the growing season, as it incorporates both the variation and mean of the stress. Basically, two types of stress can be identified: the mean static stress, which represents the average stress vegetation experiences during a season, and the dynamic water stress, which takes into account the duration as well as the frequency of the stress [Porporato et al., 2001]. The water stress calculations and all additional parameters for those calculations exactly followed Porporato et al. [2001], and the reader is referred to that paper for further details. In terms of parameters, following Porporato et al., the sensitivity parameter $k$ was set to 0.5, the degree of nonlinearity (exponent) was set to 3, and the growing season $T_{seas}$ was assumed to be 250 days.

The mean static water stress (Figure 9, top) increases steadily with increasingly deeper groundwater depth and converges on the no-groundwater case for the same climate. The difference in the curves between the different soils is small for the mean static water stress. However, for the dynamic water stress (Figure 9, bottom), including the duration of the excursion below $s^*$ (denoted by $T_{s^*}$) and the frequency of the water stress ($n_{s^*}$ means that there are greater differences between the different soils. This is because $T_{s^*}$ and $n_{s^*}$ have greater values for the sandy clay loam than for the light medium clay for several reasons. The values of $T_{s^*}$ and $n_{s^*}$ are strongly driven by the size of soil storage given by $\phi Z_r$ [Porporato et al., 2001] and thus by the porosity differences between the two soils and by the distance between $s_{lim}$ and $s^*$. The sandy clay loam has a lower porosity and a narrower soil water characteristic (distance between $s_{lim}$ and $s^*$) which results in a higher frequency of crossings ($n_{s^*}$) and a greater duration of the excursions ($T_{s^*}$). This is also reflected in the fact that the sandy clay loam has a much larger variance than the light medium clay (Figure 8). Overall, these results suggest the model might be used to understand the effect of either increasing or lowering groundwater tables on groundwater-dependent ecosystems [Groom et al., 2000].

The water stress calculations also demonstrate that groundwater uptake through capillary fluxes does have an effect beyond the 1–2 m below the root zone suggested earlier. However, in terms of supplying sufficient water needed for the survival of trees in a semiarid environment, the groundwater levels still need to be relatively close to the root zone. If this is not the case, then the process of groundwater uptake is most probably through single roots accessing the groundwater directly. The implication is that the developed model can help in understanding the mechanism of groundwater uptake by roots and can lead to a better description of groundwater dependency in models.

4. Conclusions

Capillary groundwater fluxes influence the soil moisture balance but only in a limited range of groundwater levels. Groundwater levels deeper than 2 m below the
Figure 4. Comparison of the water balance for the original model (RI model 0, equation (3), dotted line) without groundwater uptake and the new model with three limits (equation (9)) for two different groundwater depths ($Z = 175$ cm, solid line, and $Z = 300$ cm, dashed line). The vegetation is trees (Table 2), and the soil is a sandy clay loam (Table 1). The different plots show different combinations of $\alpha \lambda$ (climate).
Figure 5. Probability density functions ($p(s)$) for two soils (sandy clay loam and light medium clay) and different groundwater depths. The $p(s)$ curves shift to the wet end of the spectrum under the influence of shallower water tables. This shift occurs with shallower water tables for the soil with the lower $K_s$ (light medium clay). Curves were calculated for a climate with $\lambda = 0.33$ and $\alpha = 1.0$. 

Figure 6. Probability density functions ($p(s)$) for four different soils (loamy sand, sandy clay loam, light medium clay, and heavy clay), different climates (dry $\lambda = 0.21$, wet $\lambda = 0.45$, and $\alpha = 1$ for both climates), and different water tables ($Z = 150$, 175, 250, and 350 cm) for the new model including groundwater interactions (equations (10), (13), and (16)).
Figure 7. Mean of the probability density functions for three different soils, different climates, and different groundwater levels, indicating an increase in the mean soil saturation at shallower groundwater tables for the new model including groundwater interaction and a sharp decrease in the effect of groundwater on the soil saturation for groundwater tables 2–3 m below the soil surface.
Figure 8. Variance of $s$ for three different soils (loamy sand, sandy clay loam, and light medium clay), different climates, and different groundwater levels, indicating a general decrease in the variance with groundwater levels from $Z_{cr}$. The variances above $Z_{cr}$, in contrast, are much lower. The deviation of the variance for the new model including groundwater interactions compared to the RI model at deep groundwater depths is due to the difference between a constant $s_{fc}$ and a variable $s_{lim}$. 
bottom of the root zone are unlikely to influence the soil moisture balance even for soils with high values of $K_s$.

The new proposed model, which is based on the RI model but additionally accounts for capillary fluxes from the groundwater, reflects the effect of changing groundwater depths on the soil water storage. The model consists of three different cases depending on the balance between the potential capillary flux and the actual evaporative demand. The probability density functions ($p(s)$) are bounded on the lower end at $s_{cr}$, which represents the point where the evaporative demand equals the capillary flux. Ignoring the $s$ range below the wilting point has little effect on the results, as $s_{cr}$ is generally above $s_w$ if there is any noticeable effect on the soil moisture balance. One key difference between the older model and our model is the inclusion of a variable $s_{lim}$, which represents the hydrostatic point, and this variable replaces the boundary at $s_f$ in the RI model. This extension results in drainage continuing at lower values of $s$ in the new model (equations (9), (11), and (15)) compared to the original model (equation (3)), particularly under wetter climates.

The mean of $s$ shifts predictably to a higher value with shallower groundwater tables; however, the variance shift is less predictable because of the simultaneous changes of $s_{cr}$ and $s_{lim}$ with changes of groundwater depth. The new model correctly predicts a decrease in the water stress for vegetation over a shallow groundwater table and provides opportunities to better understand groundwater dependency and the mechanisms involved in groundwater uptake by vegetation.

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