GEOPHYSICAL RESEARCH LETTERS, VOL. 34, L07403, doi:10.1029/2007GL029409, 2007

Path-averaged rainfall estimation using microwave links: Uncertainty due to spatial rainfall variability

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Received 19 January 2007; revised 2 March 2007; accepted 9 March 2007; published 11 April 2007.

[1] Microwave links can be used to estimate the path-averaged rain rate along the link when precipitation occurs. They take advantage of the near proportionality between the specific attenuation affecting the link signal and the rain rate. This paper deals with the influence of the spatial variability of rainfall along the link on the accuracy of the rainfall estimates. We focus on single-polarization single-frequency links operating at frequencies ranging from 5 to 50 GHz and with path lengths ranging from 500 m to 30 km. A stochastic simulation framework is used to investigate the frequency at which the linearity occurs (found to be about 30 GHz) for intense Mediterranean precipitation. In addition, the error associated with path-averaged rain rate estimates is being quantified. For instance, the bias is found to be in the order of −15% with an uncertainty of about 10% for microwave links described in the recent literature.

Citation: Berne, A., and R. Uijlenhoet (2007), Path-averaged rainfall estimation using microwave links: Uncertainty due to spatial rainfall variability, *Geophys. Res. Lett.*, *34*, L07403, doi:10.1029/2007GL029409.

1. Introduction

[2] Microwave links offer the opportunity to alternatively estimate line-averaged evaporation [*Leijnse et al.*, 2007a] or precipitation [*Atlas and Ulbrich*, 1977; *Jameson*, 1991; *Leijnse et al.*, 2007b] along their paths. A link typically covers a range of a few kilometers, with a transmitter on one end and a receiver at the other, and at a height of a few tens of meters above the surface. Practical applications concerning rain rate estimation using radio links from commercial cellular phone networks have been demonstrated [*Messer et al.*, 2006; *Leijnse et al.*, 2007c], and offer great potential for instance for hydrological purposes over poorly gauged or ungauged basins (see http://pub.iwmi.org). For such quantitative applications, the accuracy and the uncertainty associated with the path-averaged rain rate estimated using microwave links still need to be analyzed. This paper presents an approach suited to investigate some aspects related to this issue, concerning in particular the influence of the frequency and the length of the link as well as the spatial variability of rainfall along the path.

[3] During rainy periods, the microwave link signal is attenuated by the raindrops along its path. Microwave links take advantage of the fact that over a certain range of frequencies, the specific attenuation k [dB km^{-1}] is almost linearly related to the rainfall intensity R [mm h^{-1}]. Consequently, the path-averaged rainfall intensity $\langle R \rangle$ is nearly proportional to the path-averaged specific attenuation $\langle k \rangle$ [*Atlas and Ulbrich*, 1977; *Jameson*, 1991]. The path-averaged specific attenuation itself is proportional to the path-integrated attenuation (PIA) which is measured by the link. In this way, it is possible to estimate the path-averaged rain rate. *Messer et al.* [2006] and *Leijnse et al.* [2007c] have shown the feasibility of rain rate estimation using microwave links from commercial telecommunication networks. Another possibility to estimate the path-averaged rain rate is to use the difference in attenuation at two distinct frequencies [e.g., *Rahimi et al.*, 2003], or at two orthogonal polarizations [*Ruf et al.*, 1996]. However, the associated uncertainties remain to be quantified for both approaches.

[4] As the propagation of the link signal depends on the scattering from every individual raindrop along its path, we expect the proportionality factor between $\langle R \rangle$ and $\langle k \rangle$ to depend on the frequency and the path length of the link as well as on the rain drop size distribution (DSD), which is a statistical means to describe the population of raindrops [*Jameson and Kostinski*, 2001]. Because of the variety of microphysical processes involved in the formation and evolution of raindrops, the DSD is highly variable in space and time [e.g., *Uijlenhoet et al.*, 2003]. Therefore it is important to investigate the influence of the variability of the DSD along the path on the rain-rate estimates using microwave links. This variability will depend on the type of rainfall and the climatic setting.

[5] The present paper focuses on the understanding of the influence of (1) the frequency and the length of the link and (2) the spatial variability of the DSD along the path on the accuracy of rainfall estimation using a single-frequency microwave link, through the $\langle R \rangle - \langle k \rangle$ relation. To investigate these issues, we use a stochastic simulator to generate range profiles of DSDs, from which all the microwave link variables can be derived. A Monte Carlo approach enables to quantify the impact of the DSD variability on the accuracy of the precipitation estimates as performed by a microwave link for a range of frequencies and path lengths.

2. Rainfall Estimation Using Single-Frequency Microwave Links

[6] In the frequency range of microwave links (from about 5 to 50 GHz), electromagnetic waves interact with raindrops when propagating through rainfall, because their wavelengths are of the order of magnitude of the raindrop sizes. The (one-way) specific attenuation k [dB km^{-1}]

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0094-8276/07/2007GL029409$05.00

affecting the link signal (at a wavelength λ) can be expressed as a function of the DSD according to

$$k = \frac{1}{\ln 10} \int_0^\infty \sigma_E(\lambda, D) N(D) dD \ , \qquad (1)$$

where D denotes the equivolumetric spherical drop diameter [mm] and $\sigma_E(D)$ denotes the extinction cross-section [cm^2] due to a drop of diameter D, and N denotes the DSD: $N(D)dD$ [m^{-3}] is the number of drops, in the diameter interval [D, $D + dD$], per unit volume. Similarly, the rainfall intensity R [mm h^{-1}] can be expressed as a function of the DSD according to

$$R = 6\pi \, 10^{-4} \int_0^\infty D^3 v(D) N(D) dD \ , \qquad (2)$$

where $v(D)$ [m s^{-1}] denotes the terminal fall velocity of a raindrop of diameter D. It is worth noting that R does not depend on the wavelength of the link because it is a flux of water, totally independent of the interaction between microwaves and hydrometeors.

[7] Because the integrands of (1) and (2) are similar at frequencies used for microwave links, R and k are almost linearly related [*Olsen et al.*, 1978]:

$$R = \alpha k^\beta, \qquad (3)$$

where α depends on the link frequency and on the DSD and $\beta \approx 1$. Therefore, the path-averaged rain rate $\langle R \rangle$ reads

$$\langle R \rangle \ \approx \ \alpha \, \langle k \rangle \qquad (4)$$

where $\langle k \rangle$ [dB km^{-1}] denotes the one-way path-averaged specific attenuation, which can be derived from microwave link measurements. Knowing the value of α, it is hence possible to estimate the path-averaged rain rate $\langle R \rangle$ using a single-frequency microwave link. However, the linearity of the $\langle R \rangle - \langle k \rangle$ relation and the uncertainty associated with its coefficients depend on the frequency of the link as well as on the variability of the DSD along the link. The objective of this paper is to investigate this issue, using a simulation framework based on a stochastic model of range profiles of DSDs.

3. Stochastic Rainfall Profile Simulator

[8] A microwave link can be thought of as a range profile of DSDs, through which an electromagnetic wave propagates. Because rainfall is highly variable at all scales [e.g., *Jameson and Kostinski*, 2001], we adopt a stochastic approach. Range profiles of DSDs are generated using the stochastic simulator proposed by *Berne and Uijlenhoet* [2005a]. It is based on an exponential DSD model and a first order vector auto-regressive process. This stochastic simulator is able to produce realistic range profiles of DSDs of equivolumetric spherical drops [*Berne and Uijlenhoet*, 2005b].

[9] The parameter values are derived from DSD time series measurements from an optical spectropluviometer, collected during the HIRE'98 experiment in Marseille, France [*Uijlenhoet et al.*, 1999]. In particular, we focus on a strongly convective period of 45 min associated with intense rainfall (about 30 mm of rain). The time series can be converted into a spatial profile assuming Taylor's hypothesis with a constant wind velocity (of about 12.5 m s^{-1} as estimated from radar images). To get a spatial resolution of 50 m, the DSD measurements used to parameterize the simulator are considered at a 4-s time step.

[10] One thousand 30-km long profiles of DSD parameters have been generated. From these DSD profiles, and for a range of frequencies between 5 and 50 GHz and a range of path lengths between 500 m and 30 km, the corresponding profiles of specific attenuation k and rain rate R have been computed, using the Mie theory for the scattering cross-sections [*van de Hulst*, 1981] and Beard's velocity model for the drop terminal fall velocities [*Beard*, 1976]. The mean rain rate and its standard deviation at the local (R) and link ($\langle R \rangle$) scales for the one thousand 30-km long profiles are 29.4, 39.8 and 12.1 mm h^{-1}, respectively. This controlled experiment framework allows to adopt a Monte Carlo approach to quantitatively investigate the influence of the frequency and the length of the link, as well as the spatial variability of the DSD on the ability of the microwave link to retrieve path-averaged rain rates.

4. Influence of the Frequency and the Length of the Link

[11] As highlighted in Section 2, the linearity of the $\langle R \rangle - \langle k \rangle$ relation depends on both the frequency and the DSD. To quantify this dependence, we use the set of 1000 profiles of DSDs and the corresponding R and k profiles. Each individual profile provides one pair of path-averaged values of R and k, i.e., $\langle R \rangle$ and $\langle k \rangle$. A power law is fitted (using a non-linear least-squares regression method with $\langle k \rangle$ as independent variable) on the 1000 $\langle R \rangle$ and $\langle k \rangle$ values, for frequencies ranging between 5 and 50 GHz and lengths ranging between 500 m and 30 km (see Figure 1).

[12] From Figure 1, it is clear that the length of the link has a limited influence on the prefactor and the exponent values, except at very short ranges (smaller than 2 km). This can be explained by (1) the near linearity of the relations, (2) the high spatial resolution (50 m) of the simulated profiles, and therefore the large number of points used to fit the power laws, and (3) the relatively small characteristic scale of the studied type of rainfall (about 4 km). The frequency, on the contrary, has obviously a significant influence as shown by the variability of the prefactor and the exponent values. It must be noted that these values also depend on the DSD parameterization. Hence the frequency at which the exponent is found to be equal to 1 with the intense rainfall parameterization used here (30 GHz) will be different for another type of rainfall. For instance, a similar set of simulated profiles for a moderate rainfall parameterization (not shown) indicates linearity at about 37 GHz.

[13] So far, we have estimated at which frequency a microwave link should be operated to have proportionality between $\langle R \rangle$ and $\langle k \rangle$, for intense Mediterranean rainfall. In the following section, the mean relative error and the

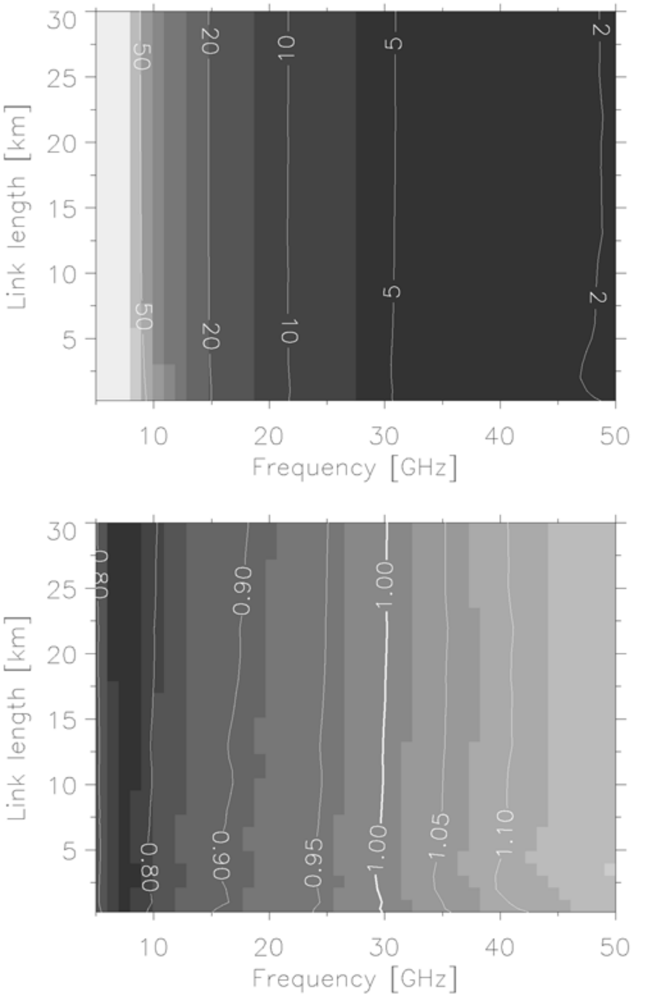

Figure 1. (top) Prefactor α and (bottom) exponent β of the relation $\langle R\rangle = \alpha\ \langle k\rangle^{\beta}$ as a function of the frequency and the length of the link, for intense Mediterranean rainfall.

uncertainty in the retrieved path-averaged rain rate $\langle R\rangle$ using the mean specific attenuation $\langle k\rangle$ are investigated.

5. Uncertainty of Rain Rate Retrieval Using a Single-Frequency Microwave Link

[14] The simulation framework enables us to quantify and to derive robust statistics about the error in the retrieved path-averaged rain rate using a microwave link. This error results from two main sources: (1) the error due to the use of a deterministic power law to describe the relation between the two stochastic variables $\langle R\rangle$ and $\langle k\rangle$; and (2) the discrepancy between the actually employed and the optimal power laws.

5.1. Error in the Retrieved Path-Averaged Rain Rate Due to the Use of $\langle R\rangle - \langle k\rangle$ Power Laws

[15] For each frequency and each link length, we have a set of 1000 values of $\langle R\rangle$ and $\langle k\rangle$. The retrieved path-averaged rain rate $\langle R\rangle' = \alpha\ \langle\ k\rangle^{\beta}$ is obtained by applying the fitted power law to $\langle k\rangle$. The mean relative error (MRE, defined as the mean of $\langle R\rangle'/\langle R\rangle - 1$) is used to quantify the error resulting from the use of this deterministic power law to relate $\langle R\rangle$ and $\langle k\rangle$. The uncertainty associated with $\langle R\rangle'$ is quantified as the difference between the 90% and 10% quantiles of the relative error. In this way, we obtain "error bars" which contain 80% of the values. We use the relative error in order to give an equal weight to small and large $\langle R\rangle$ values. Figure 2 presents the MRE and the associated uncertainty values as a function of the frequency and the length of the link. The "+" signs correspond to real microwave links from the literature, listed in Table 1.

[16] The MRE due to the use of a power law to describe the relation between $\langle R\rangle$ and $\langle k\rangle$ is found to be nearly negligible, between -2 and 0% for a large range of frequencies and link lengths. Its structure mainly depends on the length of the link because of the sampling effects on the coefficients of the fitted power laws. The path-averaged rain rate estimates can be slightly biased, as indicated by non-zero MRE values, because of the least squares fitting.

[17] The minimum uncertainty is found to be about 4% at about 30 GHz, the frequency at which the exponent is equal

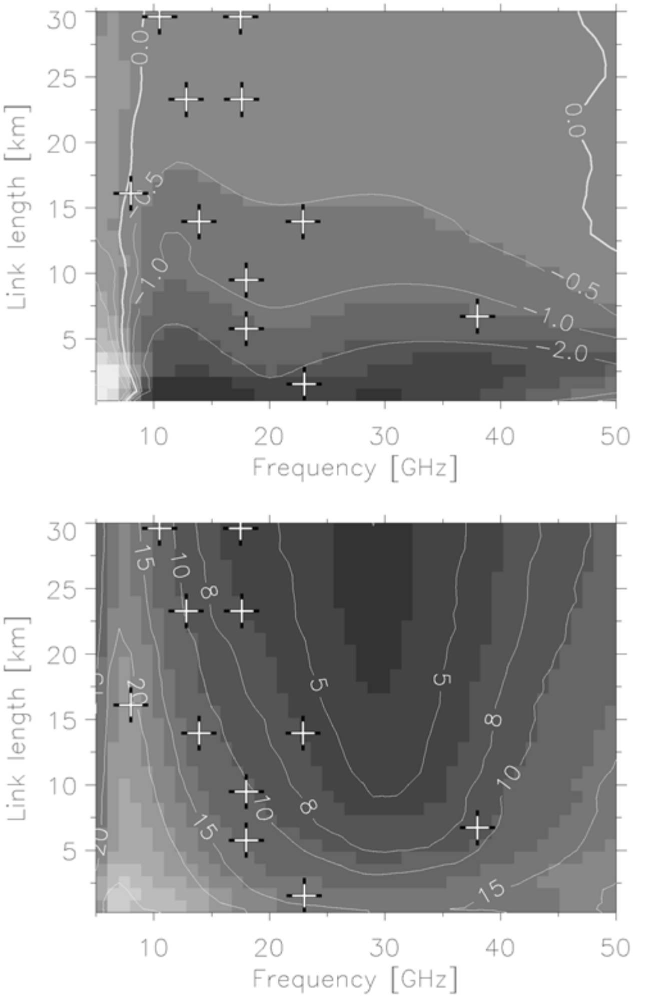

Figure 2. (top) Mean relative error (in %) and (bottom) associated uncertainty (in %) between the estimated ($\langle R\rangle'$) and the true ($\langle R\rangle$) path-averaged rain rate, using the fitted $\langle R\rangle - \langle k\rangle$ relations. The plus signs correspond to the real microwave links listed in Table 1.

Table 1. Frequency and Length of Microwave Links Operated in Commercial Communication Networks (Links 1–5) and for Research Purposes (Links 6–8)

Link	Length, km	Frequency, GHz	Reference
1	1.53	23	*Messer et al.* [2006]
2	5.77	18	*Messer et al.* [2006]
3	9.48	18	*Messer et al.* [2006]
4	16.11	8	*Messer et al.* [2006]
5	6.72	38	*Leijnse et al.* [2007c]
6	13.95	13.9–22.9	*Holt et al.* [2003]
7	23.29	12.8–17.6	*Holt et al.* [2003]
8	29.6	10.5–17.5	*Holt et al.* [2003]

to 1, and is symmetrical around this value. The uncertainty decreases when the link length increases, because the number of points to fit the power law is larger for longer lengths. Therefore the fitted coefficients are less sensitive to sampling effects. The bottom panel of Figure 2 gives an order of magnitude of the minimum uncertainty affecting path-averaged rain rates retrieved using microwave links, that cannot be corrected for.

5.2. Additional Error in the Retrieved Path-Averaged Rain Rate Due to the Use of Point-Scale Climatological Power Laws

[18] Because we employ a controlled experiment framework, we were able in the previous section to use the exact $\langle R \rangle$ to obtain the best fitted power-law relations. In reality however, the exact $\langle R \rangle$ is not available, and the problem of estimating the coefficients of the $\langle R \rangle - \langle k \rangle$ relation arises. One possible approach to tackle this issue would be to use the ITU Recommendations (for horizontal polarization state) [*International Telecommunication Union*, 2005] to derive the coefficients of the power law, which corresponds to a point-scale climatological $R - k$ relation. In this manner, it is possible to quantify the additional error in the retrieved path-averaged rain rate due to the use of such a climatological (point scale) power law. Similarly to Figure 2, Figure 3 presents the MRE and the associated uncertainty values for $\langle R \rangle' = \alpha_c \langle k \rangle^{\beta_c}$, where (α_c, β_c) denote the coefficients of the climatological point-scale $R - k$ relation.

[19] In comparison with Figure 2, there is a dramatic change in the structure of the MRE. It is one order of magnitude larger (between 0 and −20%), and the structure is mainly influenced by the frequency. This can be related to the strong effect of the inaccuracy in the coefficients of the $\langle R \rangle - \langle k \rangle$ relation, due to the use of a climatological point-scale $R - k$ power law. Concerning the associated uncertainty, the change with respect to Figure 2 is less sharp, but the minimum has nearly doubled (about 8%) and is located between 15 and 20 GHz. The uncertainty is systematically larger compared to the values obtained using the optimal link scale $\langle R \rangle - \langle k \rangle$ power laws (bottom panel of Figure 2), except for frequencies between 10 and 15 GHz (because there is a large underestimation in this frequency range, as indicated by the MRE). It is interesting to note that the path-averaged rain rates retrieved from real microwave links exhibit a limited uncertainty (below 15% in general) but are significantly underestimated (MRE below −10%).

[20] If no reliable statistics on the local climatology of the DSD are available, the use of the ITU-R recommendations may result in significant errors, and there is no clear optimal frequency for rain rate retrieval using microwave links. In this case, a trade-off has to be achieved between a limited bias and a limited uncertainty. Moreover, it must be noted that even if the MRE is about 0, individual profiles may exhibit a much larger relative error on the path-averaged rain rate, i.e., in the order of tens of percent.

6. Conclusions

[21] In this paper, we have investigated the influence of both the frequency and the length of microwave links, as well as the spatial variability of the DSD within the path of the link, on the retrieved path-averaged rain rates. A stochastic simulator was used to generate 1000 range profiles of DSDs from which the rain rate and the specific attenuation were obtained at frequencies ranging from 5 to 50 GHz and for path lengths between 500 m and 30 km. From disdrometric measurements of Mediterranean rainfall,

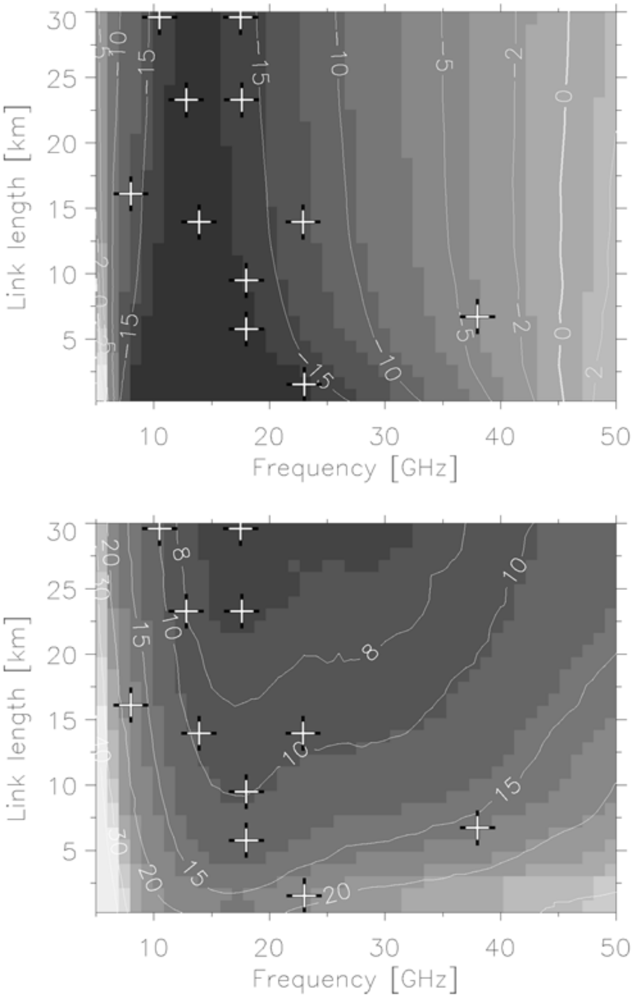

Figure 3. (top) Mean relative error (in %) and (bottom) associated uncertainty (in %) between the estimated ($\langle R \rangle'$) and the true ($\langle R \rangle$) path-averaged rain rate, using a climatological point-scale R–k power law from *International Telecommunication Union* [2005]. The plus signs correspond to the real microwave links listed in Table 1.

an intense rainfall parameterization for the DSD simulator was implemented.

[22] The retrieval of path-averaged rain rates using a single-frequency microwave link is based on the near linearity of the relation between the path-averaged specific attenuation $\langle k \rangle$ and the path-averaged rain rate $\langle R \rangle$. The length of the link is found to have no significant influence on the prefactor and exponent of the fitted $\langle R \rangle - \langle k \rangle$ power laws. For this relation to be nearly linear, the frequency must be about 30 GHz for intense Mediterranean rainfall.

[23] For practical applications, it is very difficult to obtain the appropriate $\langle R \rangle - \langle k \rangle$ relation. One possible approach is to use a climatological $R - k$ relation (at the point scale) derived from the ITU-R recommendations. Our simulation framework enables to quantify the relative error and the uncertainty due to the spatial variability of the DSD for real microwave links, which turns out to be significant when using an inadequate $\langle R \rangle - \langle k \rangle$ relation.

[24] Future developments of this work will concern the comparison with simulated path-averaged rain rates estimated from a line configuration of rain gauges, as well as the analysis of other sources of uncertainty like calibration issues and the accuracy of the measured power in an operational context. In addition, we envisage to use this simulation framework to investigate the accuracy of dual-frequency and dual-polarization microwave links for rainfall estimation.

[25] **Acknowledgments.** The second author is supported by the Netherlands Organization for Scientific Research (NWO, grant 016.021.003, *Vernieuwingsimpuls*).

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