A Mixed Sampling Approach for Temporal Trends of Spatial Means

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Outline

Motivation and aim

Four statistical approaches in space–time

$\mathrm{D_S M_T}$ approach

Estimation of Trend and its Variance

Case study

Conclusions

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Monitoring in Europe

- Water Framework Directive
- European Soil Strategy
- Habitats Directive
- Kyoto-agreement

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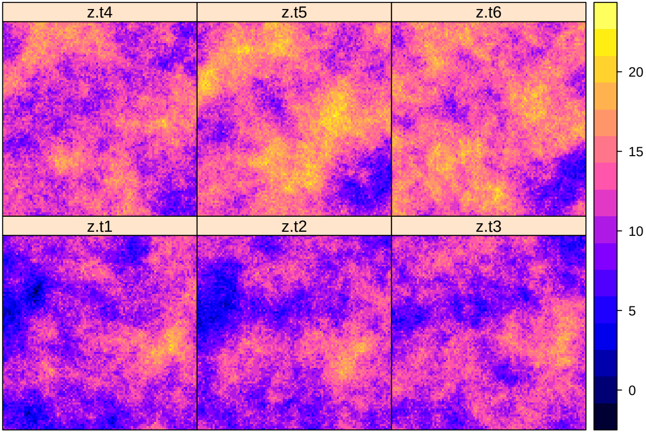
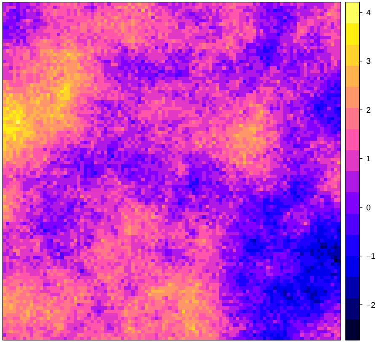
Simulated Space–Time Field

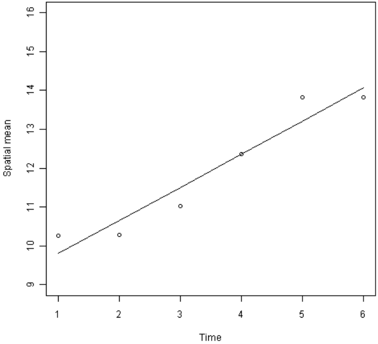


Map of Trend versus Trend of Spatial Means

Map of Trend at Points



Trend of Spatial Means



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Design-based and model-based approach

Definition of design-based and model-based approach

Type of approach	Sampling unit selection	Statistical inference
Design-based	Probability sampling	Design-based
Model-based	No requirement (purposive)	Model-based

Four statistical approaches in space–time

		Space	
	Statistical approach	Design-based	Model-based
Time	Design-based	$D_S D_T$	$M_S D_T$
	Model-based	$D_S M_T$	$M_S M_T$

Remarks on four approaches

- Fully design-based approach, e.g. compliance monitoring of space–time mean (Brus and Knotters (2008), Water Resources Research **44**)
- Fully model-based approach. Ter Braak *et al* (2008) JABES **13** used geostatistical space–time model to compare space-time patterns for predicting the temporal trend of spatial means

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$D_S M_T$ approach

- Design-based approach in space followed by model-based approach in time
- Probability sampling in space at all sampling rounds; design-based estimation of spatial means and of sampling variances
- No requirements on selection of sampling times
- For estimating temporal trend, purposive selection of sampling times best option
- Constant interval, first round at the start, last round at the end of monitoring period

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$\mathrm{D_S M_T}$ approach

- In estimating the trend a stochastic time-series model for the spatial means is used, i.e. model-based estimation
- Space–time field is a realisation of a stochastic space–time process
- Space–time process is only partly described by a model of the temporal variation of the spatial mean
- Uncertainty about trend accounts for
  - random selection of sampling locations
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Time–Series Model for Spatial Means

- $$\bar{y}(t) = \sum_{j=1}^{q} \beta_j x_j(t) + \eta(t)$$

  with $\eta(t)$ the model error, mean 0 and covariance matrix $\mathbf{C}_\xi$
- $\bar{y}(t)$ unknown, must be estimated:

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- If we take $x_1(t) = 1$ and $x_2(t) = t$, then

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- $$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{C}_{\xi p}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\mathbf{C}_{\xi p}^{-1}\hat{\hat{\mathbf{y}}})$$

  with

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Monitoring Soil Acidification in Forest Soils

- Four sampling rounds, interval one year
- Twenty locations per round
- Simple random sampling of 20 locations per round
- Rotational pattern, matching proportion 0.5
- Three sampling depths (depth depends on soil horizons)
- Four soil properties were measured: pH, NO_3 (mg kg^{-1}), NH_4 (mg kg^{-1}), NO_3 (mg l^{-1})

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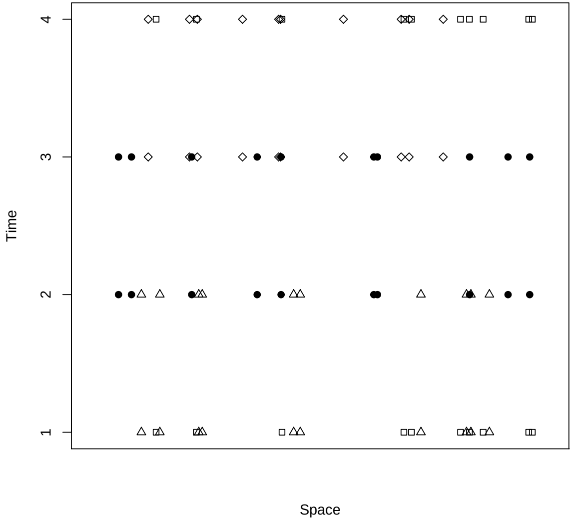
Rotational pattern



Estimation of variance-covariance matrix

- Estimation of sampling covariance

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- We assumed model-independence of spatial means
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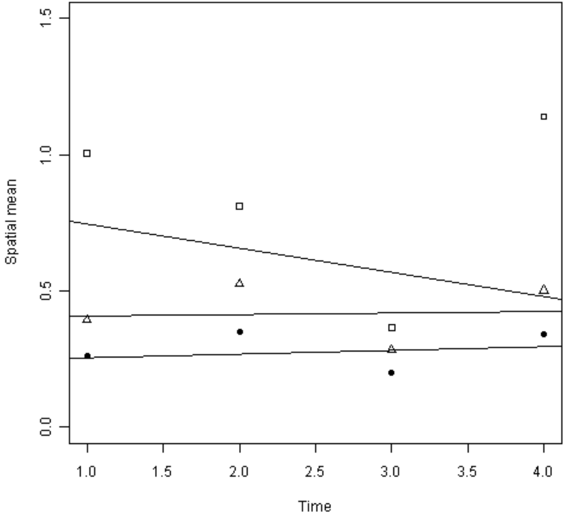
Fitted trend for NO_3 (mg N kg^{-1})



Results

Depth	trend	se	se$_p$	se$_\xi$
top	-0.089	0.19	0.077	0.18
mid	0.0054	0.071	0.047	0.052
sub	0.014	0.043	0.030	0.030

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Mixed, design-based model-based sampling approach can be an attractive alternative to fully model-based approach when data are sparse and interest is in global space–time quantities such as the temporal trend of the spatial mean:

- More simple
- Better validity
- More robust

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