Evaluation of Factors in the Soil-Loss Equation

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Recent research provides new basic information for evaluating factors comprising the equation for estimating soil loss

SOIL-LOSS estimation by means of empirical equations is a basic step in the now widely accepted slope-practice method of determining conservation practices to be applied to specific fields. This paper presents recent developments which add to the research information currently available to serve as a basis for factors comprising the soil-loss estimating equation. Several of the major factors are examined in the light of these developments.

Consolidation of Runoff and Soil-Loss Data

A national program to assemble all available runoff and soil-loss data at a single location for standardization, summarization, and further analysis was initiated in the Soil and Water Conservation Research Division, Agricultural Research Service, U.S. Department of Agriculture, in 1954. Basic data collected in the past three decades at 35 federal-state cooperative research projects have been assembled at the central runoff and soil loss statistical laboratory at Lafayette, Ind., and have been transferred to punched cards(13)*. The laboratory is operated in cooperation with the Purdue Agricultural Experiment Station and 20 other state agricultural experiment stations.

The Slope-Practice Equation for Estimating Field Soil Loss

The slope-practice equation for estimating the rate of erosion from a specific field area expresses the average annual soil loss in tons per acre as some base amount times a series of factors (2, 6, 8, 9, 10, 11, 12). These factors adjust the base loss for effects of differences in slope length, percent slope, soil erodibility, conservation measures, cropping and management. The factors are evaluated from actual measurements of soil and water loss on small experimental field plots.

Research and operations personnel in a regional workshop at Purdue University in July 1956 selected corn, spring grain, one-year meadow, with up-and-down-slope operation and only moderate soil treatments, as the base rotation for the slope-practice equation, to be adjusted to other rotations by means of a table of rotation indices (11). The slope-length, degree of slope, and soil erodibility factors were assigned unity values for the predominant plot length of 72.6 ft, a 9 percent slope, and Marshall soil, respectively.

The base soil loss thus defined averaged 8 tons per acre for 198 plot-years measured under this rotation in the north central states. The range of locations and years included in the study were assumed to provide a fairly representative sample with respect to timing of storms of different sizes relative to seedbed preparation and cover protection.

The Effects of Differences in Slope

Length of Slope. Zingg(15), in his study of the effect of length of slope on soil loss, concluded that soil loss per unit area varies as the 0.6 power of slope length. A group study in 1945 under the leadership of Musgrave(7) proposed 0.35 as the average value for the slope-length exponent for soil loss per unit area. The data from 15 studies in 9 states show a wide variation in values of the exponent. In a slope-practice group study at Purdue University in July, 1956, which included the authors, a slope-length exponent of 0.5±0.1 was recommended for field use in the north central states.

A total of 532 plot years of data with simultaneous measurements on two or three lengths of slope under natural rainfall are available for statistical analysis of the relationship. Maximum lengths in two of the 15 studies were 270 ft and 630 ft, respectively. In the other studies the maximum was 145 ft. Due to physical limitations, studies utilizing simulated rainfall have been restricted to very short slopes.

Over-all analyses indicate that runoff is significantly affected by interaction of slope length with soil type. At the majority of locations, runoff did not differ significantly with plot length. However, in 15 years at Temple, Tex., and 17 years at Hays, Kan., a decrease in runoff with increased slope length was significant at the 5 percent level. For these two studies, the length-exponent for the soil-loss equation is zero. In 25 years at Guthrie, Okla., and in a 9-year study at Bethany, Mo., on virgin Shelby soil, runoff showed a significant increase with increased slope length. For these two studies, the length-exponent for soil loss estimation averages 0.74.

Soil-loss data from three replications of two plot lengths on each of four degrees of slope on Fayette soil at LaCrosse, Wis., show a significant interaction of length and degree of slope. However, because of the confounding of variables and lack of replications, it has not been possible to determine whether or not a similar interaction effect exists in the over-all data.

The relationship of soil loss to slope length often varied more from year to year on the same plot than it varied

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*Numbers in parentheses refer to the appended references.

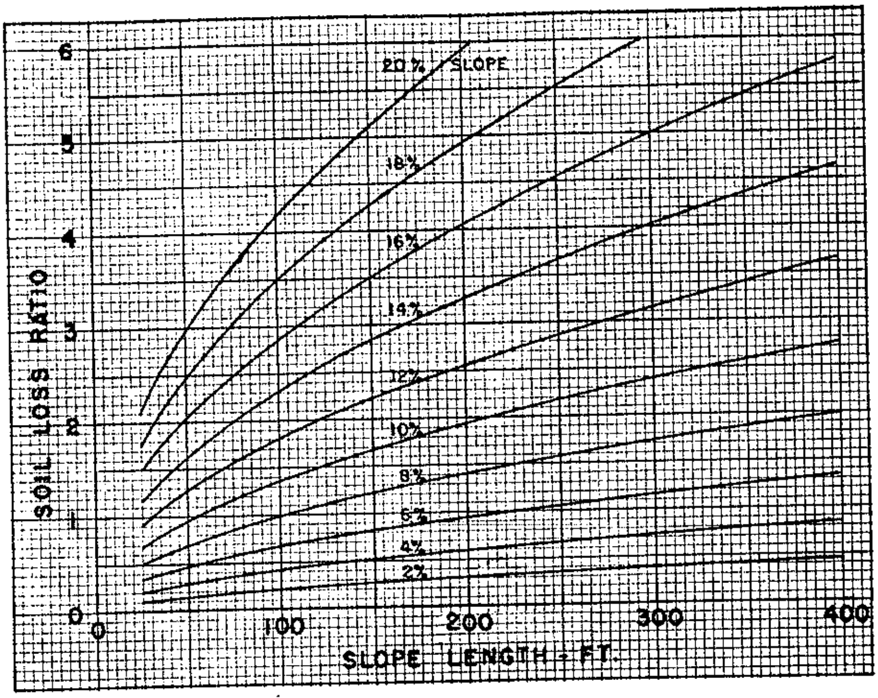

Fig. 1 Chart for adjusting plot soil loss to length and degree of slope

among locations. General trends were sometimes reversed in severe storms. The magnitude of the exponent appears to be influenced by storm size and soil characteristics, but extensive analyses on individual storm basis have not provided altogether satisfactory explanations of the storm-to-storm differences in the slope length-soil loss relationship.

Although slope-length exponents computed from each of the 15 sets of data vary rather widely, the differences in the average exponents derived from each of ten studies in the north central and northeastern states are not significant at the 10 percent level. The arithmetic mean of the ten exponents weighted by years is 0.46. For the studies in which field operations were up-and-down-slope, the average is 0.51; for across-slope farming, it is 0.42. However, the unexplained variance in these studies is large, and the difference between the two values is not significant. These analyses support the previously stated conclusion of the group meeting at Purdue University in 1956.

Percent Slope. Evaluation of the effect of percent slope on soil loss is complicated by three major weaknesses in the data: *(a)* Available data are too limited, *(b)* the slope effect is frequently completely confounded with the effectiveness of contouring which is itself believed to be a function of degree of slope, and *(c)* with few exceptions the range of slopes included in an experiment was too small to give a good indication of the type of curve that would best describe the relationship.

Available data lie within the 3 to 22 percent slope range. These data are somewhat more accurately fitted by a parabolic curve than by the exponential type. A recent manuscript by the authors(10) presented the equation

$$A=0.43+0.30S+0.04S^2$$

in which A is soil loss in tons per acre and S is percent of slope.

The family of curves in Fig. 1 indicates the appropriate factor for adjusting soil loss from "unity" plots to field-slope conditions. For example, the soil loss from an 8 percent slope averaging 290 ft in length is shown by the appropriate slope curve in the chart to be 1.7 times that from the 9 percent, 72.6-ft slope previously defined as unity.

Suppose an average annual loss of Y tons per acre has been measured or computed for some specific combination of practices on a 4 percent slope averaging 310 ft in length, and that it is desired to estimate the rate of erosion to be expected from the same cropping and farming practices on a field with a 10 percent slope 120 ft long. The ratio of the latter to the former (1.49/0.73) may be written directly from chart readings for the two combinations. Expected loss from the steeper slope would be Y times this ratio, or $2Y$ tons.

A Two-Equation Approach to Soil Loss Estimation

The rate of soil erosion from a field is influenced to various degrees by the characteristics of each individual rainstorm. The same storm characteristics appear also to influence some of the factor values in the slope-practice equation. In a comparison of annual average losses over a relatively short period, results may be significantly biased by effects of abnormal rainfall. Bias in measurement of treatment effects by extraneous variables is usually minimized in good statistical designs by randomization. But in soil and water loss studies, effective randomization over some of the extraneous variables may not be possible because of physical and economic limitations. For example, for soil factor evaluation, it would be difficult to find a range of major soil types within an area compact enough to have identical rainfall.

If the erosion-producing characteristics of individual rainstorms can be identified and their effects evaluated, these characteristics may be included as additional variables in a soil-loss estimating equation based on individual-storm data. The rate of erosion from a field is influenced by a large number of factors, the effects of some of which are additive while others are of a multiplicative nature. Combination of the two types of effects into a single equation would seriously complicate the use of least-squares procedures.

. . . Soil-Loss Equation

This difficulty is overcome by the use of a two-equation approach to soil-loss estimation.

The first equation combines the additive effects of rainfall characteristics, antecedent moisture, compaction by prior rains, and the interactions of these factors to estimate the soil loss associated with a specific rain falling on a plot devoid of cover, soil treatment and management effects. In other words, equation [1] computes the soil loss to be expected from a specific rain falling on untreated fallow with tillage operations comparable to row crops.

The second equation is similar to the slope-practice equation previously described. However, it now expresses soil loss in tons per acre for one specific storm. The base value of 8 tons is replaced by the soil loss computed in equation [1], and a factor evaluating the effect of the cover and rotation on the plot at the time of the storm. With the base value computed from rainfall data in equation [1], and the values of some of the other factors known from other studies, additional factors may be evaluated one at a time by solution of equation [2]. Thus the effect of storm size on the value of the factor under study may also be investigated.

Predicting Soil Loss Under Fallow Conditions

Individual storm runoff and soil-loss data are available for extended periods of continuous fallow at many of the locations where soil and water management studies have been conducted. The fallow plots received tillage operations comparable to plots in continuous row crop. These data served as the basis for extensive exploratory multiple regression analyses designed to identify the rainfall characteristics which influence erosion, and to evaluate combinations of rainfall characteristics and antecedent conditions as estimators of soil loss from fallow. Special emphasis was directed to detection of significant interaction effects. More than 40 factors and interaction terms were investigated.

Use of an electronic computer permitted the study of a large number of variables in a single equation. Coefficients of determination obtained with various subsets of fewer variables were then compared with the highest coefficient obtained. Some of the highest R^2 values for small subsets of variables are tabulated in Table 1.

A New Rain Parameter. A single parameter to describe the relative erosion-producing capacity of individual rainstorms is highly desirable for storm classification in analyses. The variable found to be outstanding as such a single measure is the product of the rainfall energy and maximum 30-min intensity of the storm. This term measures the interaction effect of these two storm characteristics and will be referred to as the $E \times I$ variable. In each of seven sets of fallow-plot data analyzed, a simple linear regression on this one variable accounted for a greater portion of the total storm-to-storm variation in soil loss than did a multiple regression on rainfall amount and maximum 5, 15, and 30 min intensities.

The ability to predict soil loss is considerably improved by this new parameter, and the cost of computing multiple-regression equations is reduced by the fact that one variable affords more information than four or five variables which would otherwise need to be included in the equation.

TABLE 1. PERCENT OF TOTAL SOIL LOSS VARIATION EXPLAINED BY VARIOUS FACTORS AND COMBINATIONS (R^2)

Soil type: Location:	Shelby[1] Bethany, Missouri	Shelby	Shelby[2]	Marshall[2] Clarinda, Ia.	Fayette La Crosse, Wis.
Percent slope:	8	8	8	9	16
Years of data:	10	10	10	7	6
No. of observations:	138	207	207	131	144
Variable(s),	R^2	R^2	R^2	R^2	R^2
Precip. amount	73.0	68.3	64.6	38.7	42.2
Rainfall Energy	81.7	78.2	73.9	54.9	61.6
15-min max. intensity	43.4	40.9	25.7	50.4	65.5
30-min max. intensity	56.2	59.8	35.1	56.0	79.9
Precip. amt., 15-min intensity and 30-min intensity	78.6	73.8	67.6	66.2	82.6
Interaction of $E \times I$	89.2	81.7	75.6	70.7	88.0
Four Variables[3]	92.1	85.8	80.2	78.6	88.3

[1] Rough-plow winter period excluded.

[2] Seven inches of topsoil mechanically removed.

[3] Rainfall energy, energy times 30 min intensity, total rainfall energy since last tillage of the soil, and antecedent precipitation index.

The New Equation. Little is added to the accuracy by combining rainfall amount and maximum intensities with the $E \times I$ variable in multiple regressions. Accuracy of the prediction equation is improved, however, by combining with the $E \times I$ variable in a multiple regression, a measure of rainfall energy and indices of soil moisture and surface compaction by rainfall. Correlation coefficients ranging from 0.89 to 0.96 were obtained from the five groups of data of Table 1 in the model:

$$Y_c = b_0 + b_1 X_e + b_2 X_i + b_3 X_p + b_4 X_c$$

where Y_c = computed soil loss, T/A

X_e = energy of the rain in foot-tons per acre

X_i = X_e times maximum 30-min intensity in inches per hour

X_p = antecedent precipitation index. (Antecedent net precipitation with daily between-rain reductions varied according to season)

X_c = accumulated rainfall energy since last tillage of the soil.

Least-squares evaluations of the coefficients in this equation are as follows for the soil groups of Table 1, taken in sequence from left to right:

Group No.	b_0	b_1	b_2	b_3	b_4
1	−2.46	.00163	.00389	.439	.0622
2	−3.10	.00353	.00268	.792	.0452
3	−2.88	.00317	.00195	.320	.0407
4	−1.63	.00036	.00439	.106	.0518
5	−1.24	−.00074	.01040	.035	.0383

The results of this mathematical analysis are consistent with the basic principles of erosion(14). The $E \times I$ term appears to be a good measure of the combined effects of *(a)* the decreasing infiltration rate during a rain, *(b)* the exponentially increasing erosion effect of surface flow, and (*c*) the protection against raindrop splash which is afforded by the film of flowing water. The antecedent precipitation index in the equations affords a measure of the point on the time-infiltration rate curve at the beginning of the rain. Total rainfall energy since last cultivation provides a measure of the effects of surface compaction by prior rains, reduction in surface detention due to leveling of the surface by splash erosion, and sealing of the land surface.

Inaccuracies in wintertime indices of soil moisture and compaction and failure to measure the effect of freezing and thawing tend to lower the estimating efficiency of an equation based on year-around data.

Computing Rainfall Energy. The importance of rainfall energy in estimating soil losses, calls for a routine procedure for obtaining a measure of the total kinetic energy imparted by the countless raindrops comprising a storm. Published data and literature show that, in natural rainfall, drop-size distribution is highly correlated with intensity. An energy equation was derived by the authors(14) from published data on drop velocity by Gunn and Kinzer(3), and by Laws (4), and on drop-size distribution by Laws and Parsons(5). The equation, $KE = 916 + 331 \log_{10} I$, expressing kinetic energy in foot-tons per acre-inch as a function of rainfall intensity in inches per hour, is the basis for Table 2.

To compute the energy value of a rainstorm, Table 2 is used with the information available from a recording rain gage chart. A tabular record of intensities, with the amount of rain falling in each of the successive intensity increments, is obtained from the recorder chart. The table is entered with the midvalue of a specific intensity increment. The corresponding energy figure from the table multiplied by the inches of rain falling at this rate describes the energy value of the increment of the storm. These partial products are accumulated to obtain the total energy value for the storm. The rain gage chart must be carefully read so as not to average high intensities with low ones.

In many instances the amount, duration, and maximum 5, 15, 30, and 60-min intensities have already been tabulated in the station records. If a rain has a single high-intensity period, this information will suffice for a fair estimate of rainfall energy. The recorded information is used to compute amounts and intensities for 5, 10, 15, and 30-min periods which are mutually exclusive and a fifth intensity which is the average for the remainder of the storm. Analyses as a part of this study show that energies computed by this method will be slightly biased and should be multiplied by a factor of 1.04 to place them on an equal basis with evaluations from detailed readings of the rain gage chart.

TABLE 2. KINETIC ENERGY OF NATURAL RAINFALL
(Foot-tons per acre-inch)

Intensity, in/hr	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	—	254	354	412	453	485	512	534	553	570
.1	585	599	611	623	633	643	653	661	669	677
.2	685	692	698	705	711	717	722	728	733	738
.3	743	748	752	757	761	765	769	773	777	781
.4	784	788	791	795	798	801	804	807	810	814
.5	816	819	822	825	827	830	833	835	838	840
.6	843	845	847	850	852	854	856	858	861	863
.7	865	867	869	871	873	875	877	878	880	882
.8	884	886	887	889	891	893	894	896	898	899
.9	901	902	904	906	907	909	910	912	913	915
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	916	930	942	954	964	974	984	992	1000	1008
2.	1016	1023	1029	1036	1042	1048	1053	1059	1064	1069
3.	1074	1079	1083	1088	1092	1096	1100	1104	1108	1112
4.	1115	1119	1122	1126	1129	1132	1135	1138	1141	1144
5.	1147	1150	1153	1156	1158	1161	1164	1166	1169	1171
6.	1174	1176	1178	1181	1183	1185	1187	1189	1192	1194
7.	1196	1198	1200	1202	1204	1206	1208	1209	1211	1213
8.	1215	1217	1218	1220	1222	1224	1225	1227	1229	1230
9.	1232	1233	1235	1237	1238	1240	1241	1243	1244	1246

Applications for the New Equation

Cropping and Soil Management Practice Factors. Four methods commonly employed to handle the effects of extraneous variables in statistical designs are rigid control, randomization, classification, and analysis of covariance procedures. In the case of the effects of differences in rainfall pattern, the new rain parameter, $E \times I$, with its high runoff and soil-loss prediction value, provides a means for employment of either of the latter two methods. It will serve as a basis for classifying storms according to their erosion-producing potential and will serve as a concomitant variable in analysis of covariance.

The soil-loss estimating equation for fallow soil, derived on the basis of four readily measured variables, provides a theoretical common check plot which in some instances makes it possible to compare data otherwise incomparable because of differences in time periods or physical location.

Soil Erodibility Factor. Due to the physical and economic aspects involved in experiments to evaluate soil-erodibility factors, soil effects are usually automatically confounded with the effects of differences in rainfall occurring in the different geographic regions. The approach developed in the preceding sections appears to be quite valuable in segregating the two effects to compute independent numerical values for the soil factor and for the rainfall factor.

Detailed precipitation records and soil-loss data are available for fallow plots on various soil types located in different geographic regions. If all the fallow plots receive up-and-down slope tillage similar to that for row crops, and the data are adjusted to a common slope base, differences in measured rates of erosion are a measure of the combined effects of differences in soil and rainfall. Soil-loss estimating equations based on the four measured variables listed in footnote 3 of Table 1 may be derived for the various soil types. A single rainfall pattern may then, in effect, be superimposed upon all the plots by substituting the pertinent measured characteristics for each storm into the estimating equations which have been derived for the several soil types. The result is a good estimate, with calculable error, of the erosion which would have been measured at each location under the fallow conditions had this been the rainfall pattern over all the regions. Differences in these calculated erosion rates are then a measure of soil erodibility, independent of rainfall effects.

A test for interaction effects was made by applying to four soil types a set of hypothetical precipitation data which included all combinations of three levels of each of the four variables on which the prediction equation is based. The test detected no significant interaction of soil type and rainfall characteristics.

. . Soil-Loss Equation

The procedure applied to the actual data for the soil groups and years of record included in Table 1 yielded the following soil factors:

Marshall, desurfaced (Clarinda, Ia.)	1.00
Shelby, moderately eroded (Bethany, Mo.)	1.20
Shelby, desurfaced (Bethany, Mo.)	0.85
Fayette (La Crosse, Wis.)	1.07

These analyses indicate that the relatively high soil losses measured at La Crosse as compared with those at Clarinda and Bethany were the result of effects of slope and rainfall differences rather than differences in erodibility of the soils. The analysis will be extended to include other soil types for which data are available.

Rainfall Factor. Relative rainfall factors for the period of plot measurements may be evaluated by an extension of the procedure developed for determining soil erodibility. For example, it was found that the equation based on four variables estimated the true mean soil loss for a specific storm at Bethany within a very narrow confidence band. If now the values of these four variables as they were measured at La Crosse are substituted in the equation developed for Bethany, the cumulative result is a good estimate of total soil loss that would have occurred had the La Crosse rains fallen on the Bethany plot. When both soil loss totals are converted to annual averages, the difference between the two is a measure of the effect of rainfall differences. By this procedure, the following relative rainfall factors were derived for four locations, using the Clarinda, Iowa, station rainfall for 1933-39 as unity:

Location	Years of Data	Rainfall Factor
Clarinda, Ia.	1933-39	1.00
Bethany, Mo.	1931-40	1.35
La Crosse, Wis.	1933-38	1.54
Lafayette, Ind.	1944-53	1.74

When these rainfall factors are combined with the soil factors of the preceding section and with the applicable slope and contouring factors, the product of the factors for each location very closely approximates the ratio of the average annual soil loss measured from the fallow plot at that location to the average annual loss from the Clarinda plot.

The relative factors tabulated above apply only for the respective short-time periods indicated in the tabulation. They are helpful in analyzing differences in soil losses measured at the various locations, but are not the *normal* factors for these localities. Also, the value of a factor in the slope-practice equation which evaluates the relative *annual* rainfall erosion potential is limited to comparisons of erosion at locations with identical distribution of rainfall by seasons of the year. For general field application, rainfall factors will need to be evaluated on the basis of seasons corresponding to different stages in crop development.

Derivation of relative seasonal rainfall erosion index values for specific localities from long range Weather Bureau data is planned. Availability of normal rainfall factors and soil factors will enhance the accuracy of the slope-practice technique.

Summary

Recent developments add to the information available to serve as a basis for factors comprising the soil-loss estimating equation.

The exponent for length of slope in the soil-loss estimating equation was analyzed on the basis of 532 plot-years of data assembled at the central soil and water loss data statistical laboratory at Lafayette, Ind. The magnitude of the exponent varies considerably from year to year as well as between locations. It appears to be affected by an interaction effect of slope length and soil type on runoff. Differences in the slope-length exponents measured for contour and with-slope farming and differences measured between locations in the north central and northeastern states are not significant at the 10 percent level. For these regions, 0.5 appears to be a good conservative estimate of the over-all average value for the length exponent.

The percent slope-soil loss relationship should be studied in new field tests without contouring, on as many slopes as physically possible. Very limited available data indicate a parabolic relationship for slopes under 20 percent. However, it is quite possible that the shape of this curve was influenced by the fact that most of the data are from plots with cultural operations on the contour. The effectiveness of contouring is generally considered to be a function of percent slope.

A family of curves in Fig. 1 presents the appropriate factors for adjusting soil-loss data for differences in length and degree of slope.

A single-storm, two-equation approach to soil loss prediction is presented as a means of minimizing bias in soil loss data due to significant differences in the characteristics of individual rains. Extensive analyses of fallow-plot data to evaluate the effects of rainstorm characteristics on soil and water loss yielded a *new rain parameter* which appears to be quite efficient as a basis for classification of storms according to erosion-producing potential. The parameter is the product of the rainfall-energy and maximum 30-min intensity of a storm. A table of rainfall-energy values associated with rainfall intensities is presented to facilitate computation of the new parameter.

Soil erodibility and rainfall factors may be evaluated independently by employment of the new relationships provided to segregate soil and rainfall effects. This greatly enhances the utility of plot data in geographic regions other than where the field studies were conducted. The procedure is illustrated in a partial study in which erodibility factors for four soils and rainfall factors for four locations are numerically evaluated. The study will be extended to other soils and localities.

A similar approach is feasible in the analysis of runoff data.

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