EVALUATION OF PREDICTIVE MODELS DESCRIBING THE GROWTH OF LISTERIA MONOCYTOGENES
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Abstract

Various mathematical models have been developed to predict the growth of micro-organisms in food and to estimate shelf-life and safety of food products. Before they can be used in practice, predictive models must be shown to predict accurately the behaviour of micro-organisms in foods. Validation of models should involve comparison of predicted responses to observations in product, independent of those used to generate the model. Typically, growth rates or generation times predicted by the model are compared to those observed for the same organism in foods.

In this study, we used general applicable models and specific growth models for Listeria to evaluate the ability to predict the growth of this pathogen on meat products. Literature values, obtained from a large number of publications, for growth rates were compared with predictions given by different models by graphical and mathematical analysis.

Overall, the gamma model performed best. However, only small differences between the various models were observed. Model predictions were accurate within a factor of about two. These predictions should therefore not be considered as absolute, but in many cases the accuracy will be sufficient to use these models as a tool in management decisions.

Introduction

In recent years, the interest in developing mathematical models to describe growth of micro-organisms as a function of controlling factors (e.g. water activity ($a_w$), pH, temperature and oxygen availability) has increased. Predictive growth models have been developed in model media for a range of pathogens, for example Staphylococcus aureus (Sutherland et al., 1994), Listeria monocytogenes (Farber et al., 1996; McClure et al., 1997), Bacillus cereus (Sutherland et al., 1996), Yersinia enterocolitica (Adams et al., 1991; Sutherland and Bayliss, 1994) and Escherichia coli O157:H7 (Sutherland et al., 1997).

*L. monocytogenes* has been recognized as an important foodborne pathogen that causes listeriosis. Outbreaks of listeriosis have been associated with milk, cheese, vegetables and salads, and meat products. The organism is particularly problematic for the food industry because it is widespread in the environment. *L. monocytogenes* is able to grow over a wide range of temperatures (-1.5 to 45°C), pH values (4.39 to 9.4), and osmotic pressures (NaCl concentrations up to 10%). It is also facultatively anaerobic (ICMSF, 1996).

Several mathematical models to describe the combined effect of temperature, pH, $a_w$, NaNO$_2$, CO$_2$ concentrations and irradiation on growth of *L. monocytogenes* have been published (Grau and Vanderlinde, 1993; Patterson et al., 1993; Duffy et al., 1994 and Farber et al., 1996).
To determine whether predictions provide a good description of growth in foods, models should be validated to evaluate the predictive ability of a model. The accuracy of models can be assessed graphically by plotting the observed values against the corresponding predictions of a model. Furthermore, mean-square-error (MSE), $r^2$ values and the recently described indices, bias factor and accuracy factor (Ross, 1996), can also be used as an indication of the reliability of models when applied to foods.

The aim of this study was to determine the accuracy of general models (the Gamma concept (Zwietering et al., 1996) and Pathogen Modeling Program (Buchanan, 1993)) and specific models (applied to estimate the growth rate of *Listeria* on meat products) Grau and Vanderlinde, 1993; Patterson et al., 1993; Duffy et al., 1994 and Farber et al., 1996) to predict the growth of *L. monocytogenes* on meat products.

Therefore, growth rates on meat products reported in literature were compared with predictions given by the various growth models, by means of graphical and mathematical analysis.

### Materials and methods

#### Models

**General models**

**Gamma-concept (Zwietering et al., 1996):**

This model is based on the fact that the effect of various factors affecting the growth rate of micro-organisms can be combined by multiplying the separate effects. The effect of water activity is assumed to be linear, the effect of pH parabolic and the effect of temperature is supposed to follow the quadratic Ratkowsky equation:

$$
\mu = c(a_w - a_{w,\text{min}})(pH - pH_{\text{min}})(pH_{\text{max}} - pH)(T - T_{\text{min}})^2
$$

(1)

The equation can be extended to include models describing the influence of additional effects, e.g. preservatives, packaging conditions. The various hurdles can be quantified by separating the effects:

$$
\gamma = \frac{\mu}{\mu_{\text{opt}}} = \gamma(T) \cdot \gamma(pH) \cdot \gamma(a_w)
$$

(2)

The relative effect of one variable can be described by the gamma-factor of that variable:

$$
\gamma(T) = \left(\frac{T - T_{\text{min}}}{T_{\text{opt}} - T_{\text{min}}}\right)^2
$$

(3a)
\[
\gamma(pH) = \frac{(pH - pH_{\min})(pH_{\max} - pH)}{(pH_{\text{opt}} - pH_{\min})(pH_{\max} - pH_{\text{opt}})} \tag{3b}
\]

\[
\gamma(a_w) = \frac{(a_w - a_{w\min})}{(1 - a_{w\min})} \tag{3c}
\]

Symbols:

- \(T_{\min}, pH_{\min}, a_{w\min}\) = organism parameters; minimal growth temperature, pH and water activity of a micro-organism;
- \(T_{\text{opt}}, pH_{\text{opt}}\) = organism parameters; optimal growth temperature and pH;
- \(pH_{\max}\) = organism parameter; maximal pH at which growth just stops;
- \(T, pH\) and \(a_w\) = product- and process parameters; storage temperature, pH and water activity of the product under study.

Pathogen Modeling Program (PMP, version 5.0) (Buchanan, 1993):

Pathogen Modeling Program (PMP) was developed at the US Department of Agriculture. PMP is based on large experimental data sets. Polynomial models (like the example given in equation 4) are used to predict the growth of various pathogenic micro-organisms as function of controlling growth factors (e.g. temperature, pH, \(a_w\), availability of oxygen).

\[
\ln(\mu) = a + bT + cT^2 + dpH + epH^2 + fa_0 + ga_w^2 + hTpH + iTa_0 + jpH. \tag{4}
\]

Specific models

**Modified Arrhenius equation** (Grau and Vanderlinde, 1993); to describe the effect of temperature and pH on growth of \(L.\ monocyctogenes\) on beef:

\[
\ln(\text{gen} / h) = A_0 + A_1 + A_2 + A_3 / T + A_4 / T^2 + A_5 / pH + A_6 / pH^2 \tag{5}
\]

\(T = \) temperature (K); \(A_0, A_1, A_2, A_3, A_4, A_5, A_6\) = coefficients for the equation: \(A_0 = -232.64\); \(A_1 \times 10^{-5} = 1.4041\); \(A_2 \times 10^{-7} = -2.1908\); \(A_3 \times 10^{-5} = 1.1586\); \(A_4 \times 10^{-2} = -4.0952\)

**Quadratic equation** (Patterson et al., 1993); to estimate the effect of irradiation and temperature on the growth rate of \(L.\ monocyctogenes\) on poultry:

\[
(\text{gen} / \text{dag}) = A_0 + A_1T + A_2D + A_3T^2 + A_4D^2 + A_5TD + A_6T^2D + A_7TD^2 \tag{6}
\]

In this evaluation, irradiation was not taken into account \((D = 0)\); this leads to equation 7 in which only the effect of temperature on growth of \(L.\ monocyctogenes\) is described:
\[(gen / dag) = A_0 + A_1 T + A_2 T^2 \quad (7)\]

\[T = \text{temperature (°C)}; \quad D = \text{irradiation dose (kGy)}; \quad A_1 - A_2 = \text{coefficients for the quadratic equation}; \quad A_0 = -0.899; \quad A_1 = 0.252; \quad A_2 = -0.045\]

**Quadratic equation (Duffy et al., 1994)** to describe the effect of pH and \(a_w\) on growth of *L. monocytogenes* on meat at 5 °C:

\[(gen / h) = A_0 + A_1 \pH + A_2 a_w + A_3 \pH a_w + A_4 \pH^2 + A_5 a_w^2 \quad (8)\]

\[A_0 = -19.684; \quad A_1 = 0.5085; \quad A_2 = 36.254; \quad A_3 = -0.4970; \quad A_4 = 0.0046939 \quad (A_4 \text{ was left out since this term was shown to be not significant}); \quad A_5 = -16.581\]

**Quadratic equation (Farber et al., 1996)** to predict the influence of temperature, pH and \(C_0 \text{O}_2\) concentration on growth of *L. monocytogenes*:

\[
\ln(gen / dag) = A_0 + A_1 \pH + A_2 T + A_3 CO_2 + A_4 pH T + A_5 pH CO_2 + A_6 T CO_2 + A_7 \pH^2 + A_8 CO_2 \quad (9)
\]

In this case \(CO_2\) not taken into account \((CO_2 = 0)\). This leads to the following quadratic equation:

\[
\ln(gen / dag) = A_0 + A_1 \pH + A_2 T + A_3 pH T + A_4 T^2 \quad (10)
\]

\[T = \text{temperature (°C)}; \quad CO_2 = \text{percentage CO}_2 \text{ in the package}; \quad A_1 - A_8 = \text{coefficients for the quadratic equation}; \quad A_0 = 3.9651; \quad A_1 = -0.4823; \quad A_2 = -0.6517; \quad A_4 = 0.0395; \quad A_7 = 0.0153\]

**Validation**

Validation can be carried out on the basis of the same data as the model was set up with, i.e. internal validation. External validation uses new data, e.g. literature data, to assess the quality of the predictions of the model. The adequacy of a model to predict data can be assessed graphically or on the basis of mathematical and statistical parameters.

**Graphical comparison**

Literature values for growth rate in foods can be plotted against the corresponding predictions of a model. From this plot, predictions which would be unsafe in practice can be visualized readily, and the overall reliability of the model assessed.
Mathematical/statistical comparison

Several mathematical and statistical indices can be used to evaluate the performance of predictive growth models. These are described below.

1) percentage goodness of fit (Grau and Vanderlinde, 1993)

\[
100 \times \frac{\text{predicted } \mu - \text{observed } \mu}{\text{observed } \mu}
\]  
Equation (11)

The difference between the calculated and experimental growth rate (\(\mu \text{ (h}^{-1}\)) expressed as a percentage of the experimental value.

2) Mean square error (MSE) (Adair et al., 1989; Sutherland et al., 1994).

\[
MSE = \frac{\sum (\mu \text{ observed} - \mu \text{ predicted})^2}{n}
\]  
Equation (12)

The MSE, the residual sum of squares divided by the number of datum points, is a measure of variability remaining after fitting a model, that is not accounted for by deliberate changes in factors such as temperature, pH and \(a_w\). This error may come from several sources including natural variability, systematic errors and bias. The lower the MSE; the better the adequacy of the model to fit the data.

3) "multiple regression coefficient" or "coefficient of determination" (\(r^2\)) (Grau en Vanderlinde, 1993; Duffy et al., 1994; Sutherland et al., 1994).

The \(r^2\) statistic is often used as an overall measure of the fit attained. It measures the fraction of the variation about the mean that is explained by the fitted model. The higher the value (0 < \(r^2\) < 1), the better is the prediction by the model.

4) bias factor (Ross, 1996). The bias factor answers the question whether, on average, the observed values lie above or below the line of equivalence and, if so, by how much. It gives the structural deviations of a model.

\[
\text{bias factor} = 10^{(\sum \log(GT\text{predicted}/GT\text{observed})/n)}
\]  
Equation (13)

A bias factor < 1 indicates a 'fail safe' model, i.e. observed generation times were larger than predicted values, so that predicted values give a margin of safety.

5) accuracy or precision factor (Ross, 1996). The accuracy factor averages the distance between each point and the line of equivalence as a measure of how close, on average, predictions are to observations.
\[\text{accuracy factor} = 10^{\left(\sum \log\left(\frac{G_{\text{predicted}}}{G_{\text{observed}}}\right) / n\right)} \] (14)

The larger the value, the less accurate is the average estimate. An accuracy factor of 2 indicates that the prediction is, on average, a factor of 2 different from the observed value, i.e. either half as large or twice as large.

Results and discussion

The reliability of the models predicting growth of *Listeria* in meat products was tested by comparing predictions with observations for growth in meat(products) reported in literature. The literature validation exercise revealed marked deficiencies in the literature itself. In many publications information about the foods, experimental design and/or methods was incomplete or data were not suitable for curve-fitting and deriving kinetic parameters. A total of 83 data sets from 20 references were used.

In Figure 1 an example of the comparison of observed and predicted growth rates is given for the Gamma model.

![Figure 1 Comparison of published growth rates (h⁻¹) and those predicted using the Gamma model for Listeria in meat(products).](image)

In general, good agreement across the range of growth conditions was shown between observed and predicted values. The trend over a large range of decades (0.002 - 0.5) is predicted well. Most points fall close to the line of equivalence, indicating that the model predicts growth rates similar to those reported in published studies. Sometimes, there was poor agreement, this may be due experimental error, natural variability, model inaccuracy, additional relevant factors influencing growth (e.g. preservatives, modified atmosphere packaging) not (yet) implemented in the models or near-limiting growth conditions.
For the other models similar trends were observed, except for the model of Duffy et al., 1994. This model was developed to describe growth of *Listeria* on meat at 5 °C, therefore, it cannot be extrapolated for use at other temperatures. Only the effect of water activity and pH were taken into account in this model, while temperature is generally the most important controlling factor in these type of products.

The mathematical and statistical comparison of 6 models (both general and specific models) for prediction of growth of *Listeria* on meat(products) is given in Table 1.

Table 1 Evaluation of 6 models predicting growth of *Listeria* on meat(products) according to various mathematical/statistical characteristics (83 data sets)

<table>
<thead>
<tr>
<th>Model</th>
<th>Gamma</th>
<th>PMP</th>
<th>GvdL</th>
<th>Patterson</th>
<th>Duffy</th>
<th>Farber</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 70 %&lt;sup&gt;a&lt;/sup&gt;</td>
<td>25</td>
<td>33</td>
<td>30</td>
<td>27</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>MSE</td>
<td>0.0058</td>
<td>0.0089</td>
<td>0.0148</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0062</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.655</td>
<td>0.627</td>
<td>0.477</td>
<td>0.294</td>
<td>0.053</td>
<td>0.179</td>
</tr>
<tr>
<td>bias</td>
<td>0.918</td>
<td>0.778</td>
<td>0.719</td>
<td>0.705</td>
<td>1.080</td>
<td>0.828</td>
</tr>
<tr>
<td>accuracy</td>
<td>1.733</td>
<td>1.780</td>
<td>1.768</td>
<td>1.885</td>
<td>2.398</td>
<td>1.861</td>
</tr>
</tbody>
</table>

<sup>a</sup> = the number of observations for which the predicted growth rate differs more than 70 % from the reported growth rate

Comparing the various mathematical and statistical characteristics in Table 1, it can be observed that overall the Gamma model performs best. However, differences between the models were small. All models are 'fail safe' models (bias factor < 1), except the model of Duffy et al., 1994. As mentioned before this model should not be extrapolated, as it has only been developed to describe growth at 5 °C.

The agreement between the various models and the literature data shows that the concept of predictive microbiology can give appropriate predictions. Predictive models offer guidelines to the likelihood of bacterial growth under given conditions, assist in product development, eliminating some bacterial testing during product formulation studies. However, models can only be used to support decisions. The accuracy factor of about 2 implies that while the models can be used as tool for management decisions they should not be relied upon as the sole determinant of the products' safety.

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