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Innovative techniques in modelling large-scale river morphology

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ABSTRACT
The Rhine River is considered the backbone of the Northwest European waterways network. Due to the morphological behaviour of the river, navigation is subject to hindrance in some locations and frequent maintenance is being carried out. It is of a primary interest for the river manager to be able to evaluate different maintenance strategies to ensure a sustainable navigation channel. Accordingly, a detailed quasi 3-dimensional morphological model that covers the Rhine branches in the Netherlands was created. In this paper we present some innovative techniques which have been recently developed to undertake this task, e.g. domain decomposition, simulation management tool, and a novel dredging and dumping functionality. Domain decomposition is a technique in which a model domain is subdivided into several smaller model domains. Computations can be carried out separately on these domains. The simulation management tool allows for efficiently carrying out long-term computation with variable discharge; it helps reduce the computational time significantly. Dredging and dumping operations that allow carrying out actual river maintenance strategy. We briefly presents a case study that evaluates the effect of enlarging the navigation channel on the yearly dredging volume by applying a simple dredging scenario in the Waal River.

Keywords: River morphology, numerical modelling, large-scale, dredging.

1 INTRODUCTION

The Rhine River is considered the backbone of the Northwest European waterways network. All efforts are made to ensure the navigability of the river all year round. Due to the morphological behaviour of the river, navigation is subject to hindrance in some locations and frequent maintenance is being carried out. Plans in the Netherlands are being evaluated to expand the navigation channel. The aim of the navigation channel expansion is to secure a channel with a 170 m wide and 2.8 m deep along the route Rijnmond-Duisburg. The initial plan included the construction of bottom vanes in some selected locations in addition to additional dredging of the navigation channel (Rijkswaterstaat, 1993).

However, because of uncertainty of the effectiveness of the bottom vanes, a new plan has been proposed to achieve a sustainable navigation channel (Smedes, 2005). The plan is called "*Duurzame Vaardiepte Rijndelta*; in English: *Sustainable Navigation Channel Rhine delta."* (acronym: DVR).

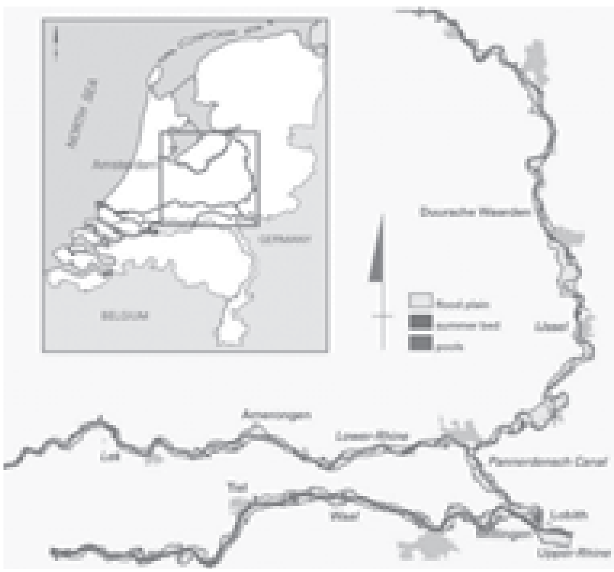

Figure 1: Rhine branches in the Netherlands.

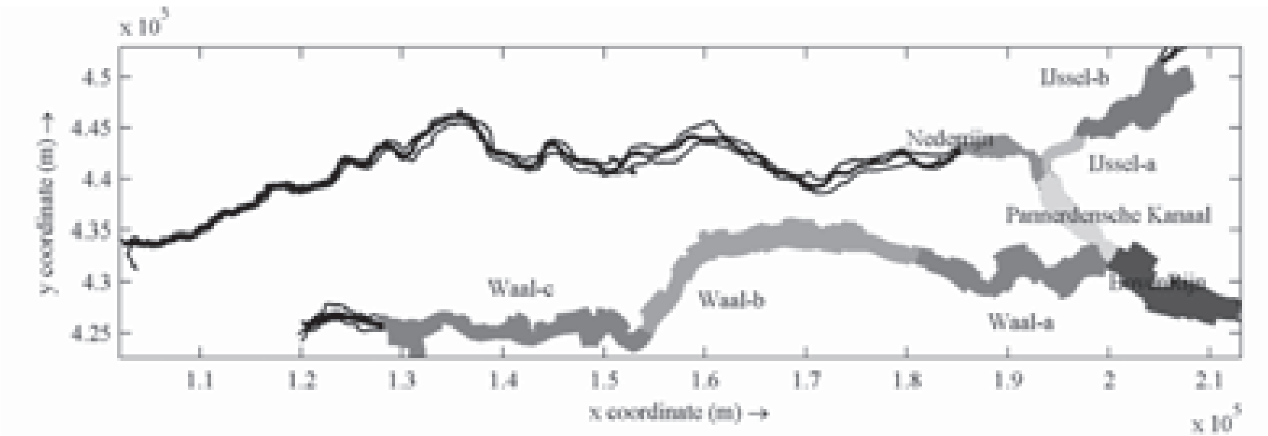

Figure 2: Overview of the model grid with indication to the different model domains.

To carry on with this plan, there is a need for a tool that is capable of predicting large-scale morphological response to a multitude of interventions, diagnosis of historical trends in navigation channel dimensions and forecasting of future trends. Preliminary studies have been carried out by Sieben et al (2005), Barneveld & Vermeer (2005) and Mosselman et al (2005).

Accordingly, a detailed quasi three dimensional morphological model that covers the Rhine branches in the Netherlands was created; the model is called the DVR-model. Both the intermediate and large-scales morphological developments can be evaluated using this model. The intermediate-scale concerns the behaviour of relatively short reach, few kilometres (*spatial distinction*), for a relatively short period of time, weeks to months (*temporal distinction*). In this way we provide a tool that enables the river manager to take decisions concerning local effects as well as short term possible dredging activities. The large-scale concerns the behaviour of the entire river (*spatial distinction*), that takes years to decades to take place (*temporal distinction*). In this way we are able to evaluate the response of the river to several interventions that aims to maintain navigability.

Until recently, such a problem would have been evaluated using two different types of models, the first would comprise several, local, 2-D or 3-D models which would be dedicated to evaluate the intermediate-scale; the second would be a 1-D or analytical model that aggregates the local models to evaluate the large-scale. However, recent developments in morphological modelling enable us to combine the two scales in one model. A crucial element of this model is the ability to carry out large-sale/long-term simulations with high accuracy in a relatively short computational time.

2 THE MODEL

2.1 Model construction

A detailed numerical quasi-3D morphological model was setup using the numerical software package Delft3D. The model covers, nearly, the entire Rhine system in the Netherlands, including the five main branches, viz. Upper Rhine (Emmerich to the Pannerdensche Kop km 853-867); the Waal (Pannerdensche Kop to Werkendam, km 867-953); the Pannerdensche Kanaal (km 876-879), the IJssel (IJsselkop to Doesburg, km 879-912), and the Nederrijn (IJsselkop to Driel, km 879-889).

The model was initially divided into 5 domains, one domain for each branch; with domain boundaries at bifurcations. Later on, for

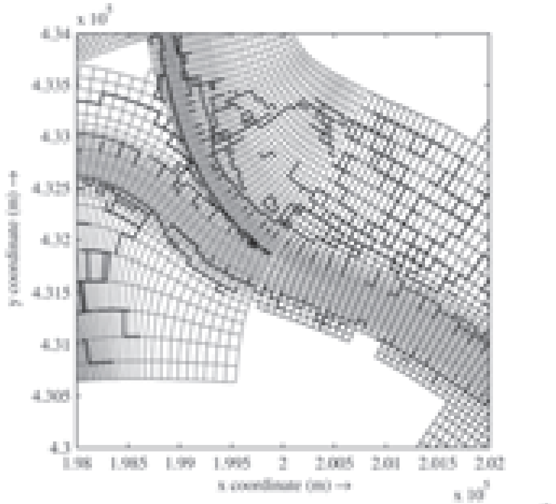

Figure 3: Detailed overview of the model grids at the 1st bifurcation point including three domains – thick lines represent groynes and thin dams.

equal computational load over multiple processors, the longer domains were split; the Waal was divided into 3 domains and the IJssel was divided into 2 domains.

Thus the final model consisted of 8 computational domains as shown in Figure 2. Figure 3 shows the domain boundary between the Bovenrijn, the Waal, and Pannerdensche Kanaal.

For reliable hydrodynamic computations the grid should fulfil the requirements of smoothness, orthogonality, and should have an aspect ratio close to unity (see Thompson et al., 1985; Wijbenga, 1985; and Mosselman, 1991). The smoothness requirement implies that difference between successive grid cell dimensions should be less than 10% in the main channel and 25% in the floodplains. The orthogonality requirement boils down to the fact that a grid cell corner point should not deviate more than 5^o from an angle of 90^o. The aspect ratio between the size of the grid in longitudinal and transverse directions should be kept close to unity, as the flow predominantly along stream-wise direction, an aspect ratio less than 6 is considered acceptable. The applied grid satisfied the above mentioned criteria.

After the construction of the grid, and with the help of GIS, all relevant hydraulic and morphological features were projected on the grid. These include: bed topography, bed composition (including fixed layers and spatial distribution of median sediment diameter D_{50}, see Sloff et al. 2006), hydraulic roughness (in the floodplains it was based on vegetation coverage), hydraulic structures (including groynes, summer dikes and longitudinal dams). The boundary conditions of the model were deduced from long term measurements. The hydraulic boundary conditions were as follows, a time-dependent discharge at the upstream boundary, water levels at the end of the Waal and the IJssel, and discharge withdrawal at the end of the Nederrijn. One morphological boundary condition of gradual bed degradation as described at the upstream boundary. For a complete overview of the model construction procedures we refer to Van Vuren, (2006) and Van Vuren et al. (2006).

2.2 *Morphological calibration*

The morphological calibration was carried out in two steps. The first focuses on the 1-D morphological behaviour and the second concerns the 2-D behaviour.

The 1-D-calibration is focused on the following cross-section-averaged quantities: annual sediment transport volumes/rates, celerity of bed disturbances, annual bed level changes, and time-averaged bed level gradients. An example of the calibration result representing a comparison between the calculated bed form celerity and that based on measurements is given in Figure 4; note that the large variations in measurements-based celerity is due to inaccuracies in field data.

The 2-D morphological patterns in the Rhine branches in the Netherlands are forced by the curvature of the channel and channel width variations. In principle, the 2-D patterns do not migrate through the river system. The 2-D calibration focuses on a correct reproduction of these patterns. Two important features define the 2-D bar-pool pattern of the river, the amplitude and location; they can be evaluated by the transverse slopes in bends, and the locations of crossing between two opposite bends. An example showing the calculated and measured bar-pool patterns along the left and right banks of the upper Waal is given in Figure 5.

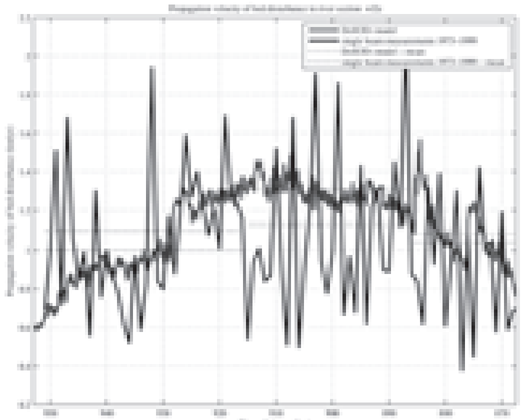

Figure 4: Comparison between calculated bed form celerity and that deduced from measurements.

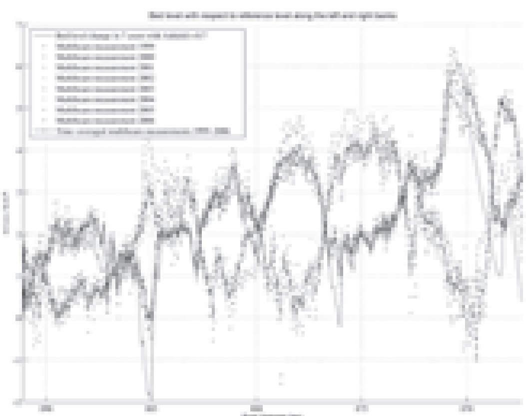

Figure 5: Comparison of calculated and measured bed levels along the left and right banks of the upper Waal.

3 INNOVATIVE ASPECTS

3.1 *Domain decomposition*

A crucial element in this model is the application of the technique of domain decomposition with grid refinement (Hummel and de Goede, 2000) see for example Figure 3. Domain decomposition is a technique in which a model domain is subdivided into several smaller model domains, which are called sub-domains. Computations can be carried out separately on these sub-domains; if computations are carried out simultaneously, we speak of parallel computing. The communication between the sub-domains takes place online along internal open boundaries; no additional boundary conditions are required. Besides the reduction of computational time, domain decomposition is particularly important when schematising river bifurcations or intake channels. De Heer and Mosselman, (2004) emphasise the important of the grid orientation on the model results at bifurcation. Additional possibilities that domain decomposition offers are, the ability to use different transport formulations for the different domains and the possibility to use finer grid when more resolution is required.

Important aspects that should be checked when using domain decomposition are the continuity of the solution at the domain boundaries and that there are no morphological disturbances generated at the boundaries. Figure 6 shows velocity fields of a single domain and multiple domain simulations; we can see that the results are nearly identical. Figure 7 gives the result of a test simulation that evaluates the evolution of a trench across a domain boundary; it gives identical result to the single domain model.

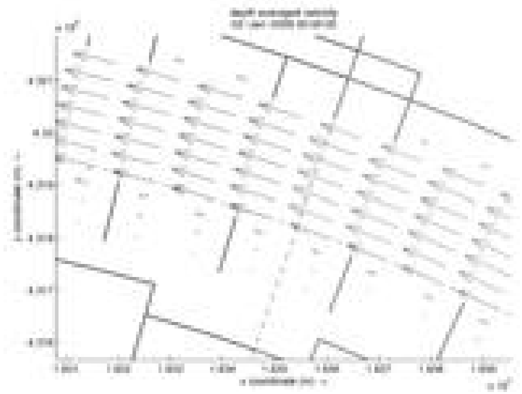

Figure 6: Depth averaged velocity using a single domain and multiple domains (2, nearly identical, sets of arrows). The thick lines indicate groynes. The dash-dot line is the domain boundary.

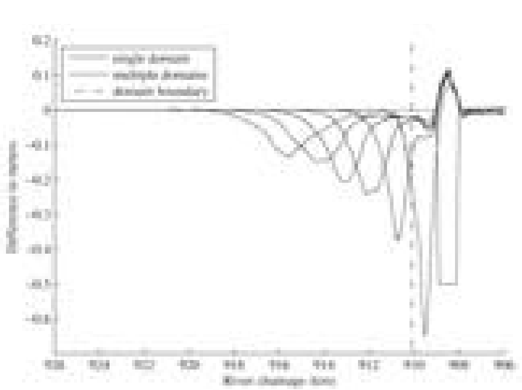

Figure 7: The evolution of a trench in the Waal after a period of 2 years for a simulation with a single domain and multiple domains (note that lines of the two cases are identical). The dash-dot line is the domain boundary.

3.2 *Simulation management tool*

Carrying out a long-term morphological simulation is an essential requirement of this model. To optimise the performance of these long-term computations, a well tested methodology is to schematise the variable discharge into a number of representative discharges (see Figure 8); this methodology is often referred to as a quasi-steady approach. One problem associated with this approach is the sharp transition between discharges that may cause model instabilities. In systems that use off-line coupling between hydrodynamic and morphodynamic modules, the hydrodynamic computation is given enough time to adapt to the new discharge. Subsequently, sediment transport and morphology are executed. Thus, the impact of the sharp transitions in discharge was avoided. Currently, Delft3D uses an online coupling approach between flow, sediment transport, and morphology, such that the full morphodynamic cycle is done in one time step (see Lesser et al., 2004; and Roelvink, 2006). However, the application of different discharges in one simulation is not trivial. Accordingly, we developed a simulation management tool that enables the use of quasi-steady discharge approach.

The simulation management tool is composed of two main components; the first handles reading and writing to database(s); and the second handles the adaptation of the input and boundary conditions files in accordance to the requirement of every discharge in the hydrograph used.

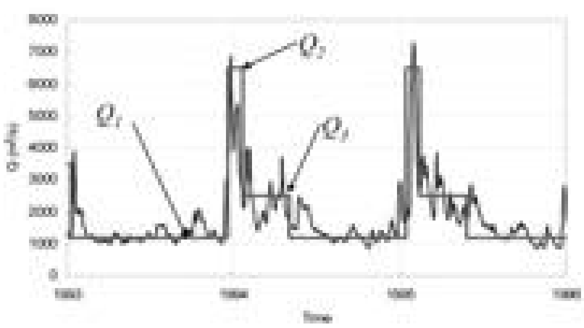

Figure 8: Comparison between schematised discharge and actual discharge time-series.

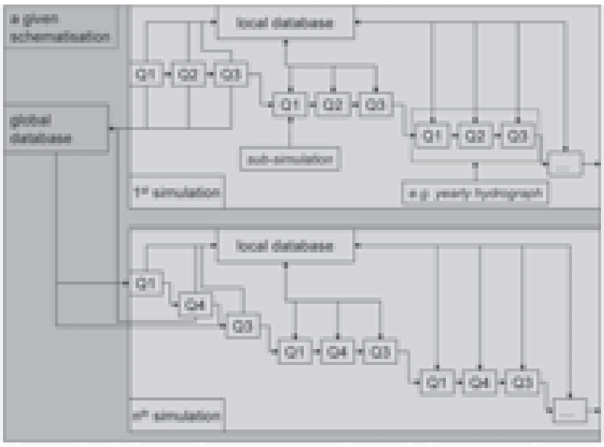

Figure 9: Schematic representation of the execution order of simulation with multiple discharges and the way communication is carried out with the databases.

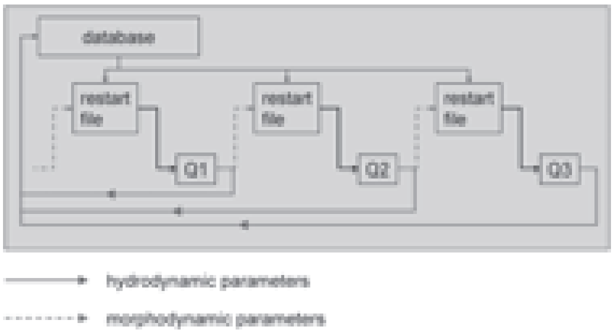

Figure 10: Detailed representation of the communication procedure between sub-simulations.

Running a multiple-discharge simulation takes the schematic form presented in Figure 9, where we can identify two different databases. The first is a global one which is meant to be used across several simulations that are carried out for a given model schematisation. The global database is written-to at the end of any sub-simulation corresponding to a discharge that appears for the first time; in subsequent sub-simulations for the same discharge, the global database is not updated. The second is the local database which is used frequently during a single simulation. The local discharge is updated at the end of every sub-simulation. Both databases are used to store only hydrodynamic results.

Running a simulation for the 1st time starts from the initial morphology with empty databases. When a discharge appears for the 1st time both databases are filled with the hydrodynamic parameters at the end of the sub-simulation. The end morphological condition of this sub-simulation is used as a starting condition for the subsequent one. Any subsequent simulation starts from a restart file that is created offline. The restart file is created by combining the end morphological condition of the previous sub-simulation and the hydrodynamic conditions coming from the relevant database (Figure 10). The hydrodynamic condition used to fill the restart file corresponds to the discharge that is going to be simulated. In case the discharge appears for the first time, the hydrodynamic condition of the closest (lower) discharge is used as initial condition. In the case of multiple domains, the same procedures hold for every domain. To avoid disturbance due to sharp transition between discharges, any sub-simulation is given enough spin-up time to adapt the hydrodynamic conditions coming from the database to the more recent morphology coming from the previous sub-simulation.

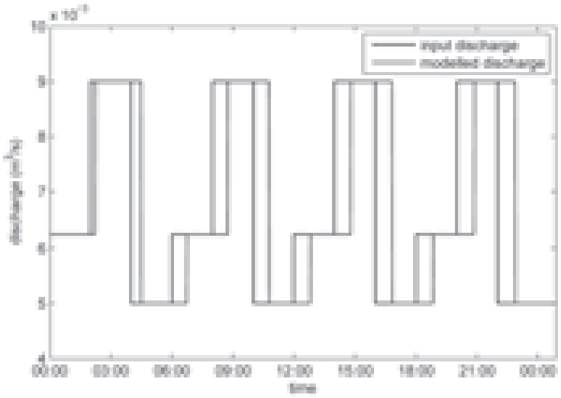

Figure 11: Comparison between input and modelled discharge time series.

After the end of a simulation, the results from all sub-simulation are combined in one file. This file contains the results of the whole simulation and can be used for post-processing. The functionality of the simulation management tool has been tested for both single domain and multiple domain simulations. The tests were carried out using a simpler model. An example that shows the difference between the input discharge time series and the modelled one is given in Figure 11 where we can spot a time shift in the modelled discharge. This time shift is attributed to spin-up times of individual sub-simulations.

3.3 *Dredging and dumping*

Due to the continuous maintenance of the navigation channel, the Rhine branches in the Netherlands experience several dredging and dumping operations. These operations are triggered by the formation of 'local' shallow areas that don't meet the minimum navigation requirements. In turn, these dredging operations trigger morphological response that affects their vicinity. Due to the large extent of the dredging operations over both the temporal and spatial scales, the small-scale local effect of these activities extends to affect the large-scale morphology of the river. Hence, a proper modelling of these dredging operations is important for any attempt to model the Rhine branches. Moreover, modelling the dredging and dumping activities will be utilised to analyse different dredging strategies. To cope with this, additional options to carry out flexible dredging and dumping operations have been implemented in Delft3D.

The dredging and dumping feature allows the use of dredging and dumping areas (see Figure 12). Within each dredging area the bed levels are lowered to a user-specified depth.

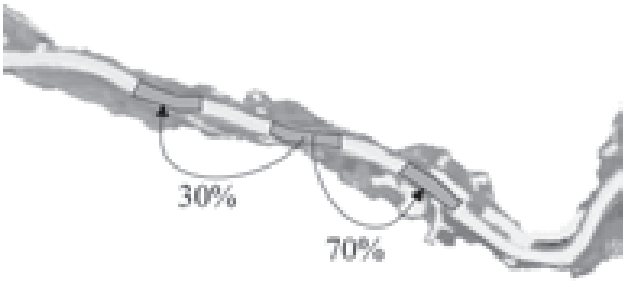

Figure 12: An example of dredging from an area and dumping into two different areas.

The dredged sediment can be dumped outside the model or in another area(s). It can be carried out in any number of dredging and dumping areas including overlapping areas; it is performed as follows: for each grid cell in a dredge area: if the bottom is higher than the dredge depth, then the bottom is set to (dredge depth – a clearance); then the volume of dredged material is summed over all cells in a dredge area and distributed over the dump area(s) using user-specified link percentages as shown in Figure 12.

The following dredging options are possible:

- Use a maximum dredging rate, by which, it is possible to define the maximum dredging rate that simulates the capacity of available dredgers.
- A possibility to dredge using different dredging rules in different areas.
- Dredge considering dune heights: It is now possible to consider the influence of the bedform height when undertaking a dredging action. This option can only be used when running a simulation which uses a dune-height predictor.
- Dredging and nourishment intervals: with this option it is possible to activate dredging actions only in certain intervals; e.g. only during low-discharge periods.
- Nourishment of material from outside the model that can be dumped in the model.
- Sequential dumping in a series of dumping blocks; coupled with the ability to evaluate the possibility of dumping in a given dump location. In a series of dump locations; if dumping capacity is exceed in the first listed dump in the first dump location will be automatically diverted to the subsequent dump location. This process is repeated tell reaching the last dump location then material which cannot be dumped is tracked using an outlet option. In this manner sequential dumping with preference to the first dump location is possible. This functionality allows complex dredge and dump scenarios similar to the situation in the Waal River.

Dredging criteria are designed to specify the conditions that trigger a dredging operation. The following dredging criteria are possible:

- Bed level above a threshold (Figure 13a), that could be:

○ prescribed threshold level;
[redacted]

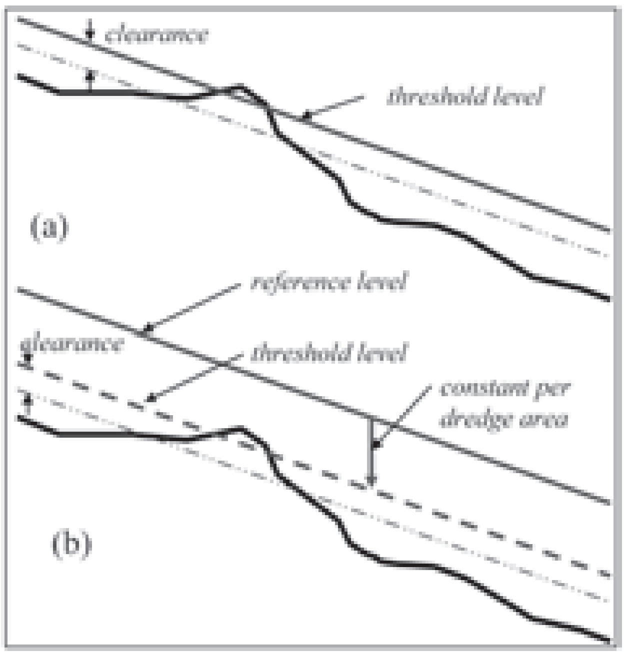

Figure 13: Dredging criteria; (a) prescribed threshold level, (b) depth below a specified reference level.

• input file that contains spatially varying threshold level or

- Constant depth below a specified reference level (Figure 13b); suitable for navigation maintenance.
- A maximum dredge rate: mining activates can be modelled utilising this criterion disregarding the bed level.

The following dredging methods are available:

- dredge top places first (Figure 14a),
- dredge proportional to availability of dredge material (Figure 14b), or
- dredge uniformly over the whole area (Figure 14c).

The following dumping methods are available:

- dump deepest parts first (Figure 15a), or
- dump uniformly over the whole area (Figure 15b).

Figure 14: Dredging method; (a) dredge top first, (b) dredge proportional, or (c) dredge uniform.

Figure 15: Dumping method; (a) dump deepest first, (b) dump uniform.

For bookkeeping, the following quantities, per area, for every sediment fraction, are stored during a simulation:

- total dredged material from an area,
- total dumped material to an area, and
- portion of the dredged material from an area that has been dumped to another area.

4 CASE STUDY – EFFECT OF ENLARGING THE NAVIGATION CHANNEL ON THE YEARLY DREDGING VOLUME

4.1 Setup of the case study

Simulations in this case study were carried out utilising the model that has been presented earlier. The recently developed Delft3D with additional functionality for dredging and dumping was used. In addition to a reference case without dredging operations, the following three cases were considered. Case B150: current navigation channel requirement of 2.50 m deep and 150 m wide channel. Case B170: future navigation channel requirement of 2.80 m deep and 170 m wide channel. Case B152: a navigation channel requirement of 2.80 m deep and 150 m wide channel.

For all cases the following settings were applied:

1. Schematised hydrograph (see Figure 16).
2. The least water level of 2002 was used as a reference level. This level was not updated during the simulation.
3. Simulation period: the simulations lasted for 5 years as given in Figure 16.
4. Dredging and dumping locations:
   a) Dredging: The *shape files* of the navigation channel of 150 m and 170 m width as well were utilised to define the dredging locations. These shape files were converted into dredging blocks such that every dredging block is 1.0 km long and has a the width of the navigation channel, viz. 150 m, 170 m, or 150 m consecutively. Dredging takes place in the reach from KM 868 to KM 952 with a total number of 85 dredging blocks. The dredging blocks were named *KM_i*, with *i* as the km number e.g. *KM_952*.
   b) Dumping: The shape file of the main channel was utilised to create dumping blocks for the full channel width. The dumping blocks were named *dump_KM_i* e.g. *dump_KM_952*; an additional block was created outside the river to dump excess material that will be removed from the river.

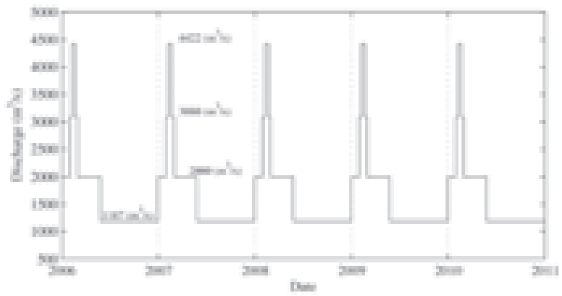

Figure 16: Schematised Hydrograph for the Waal River that was used in this case study.

5. Strategy:
   a) Locations: A dredging and dumping locations strategy was created such that the dredged material will be dumped in the upstream side of the dredging location as follows: for every dredging block *KM_i*, dumping will take place in the reach $dump\_KM\_i\text{-}x_{min}$ to $dump\_KM\_i\text{-}x_{max}$, with x_{min} and x_{max}, as the minimum and maximum distances from the dredging block, with a priority to dump the full amount in the nearest dumping block $dump\_KM\_i\text{-}x_{min}$ and to move 1.0 km at a time, i.e. all material will be dumped in $dump\_KM\_i\text{-}x_{min}$ and when it

is full dump in *dump_KM_i*-(x_{min}-1) and so on to reach *dump_KM_i*-x_{max}, then *outside* if all dump blocks are full. In this case study x_{min} = 0 km and x_{max} = 5 km where chosen (Figure 17), special attention was paid in the most upstream part of the model such that dumping will take place outside the model.

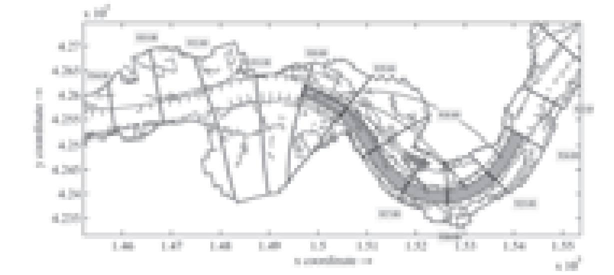

Figure 17: An example showing the dredging and dumping strategy; dredging from KM_929 and sequential dumping in dump_KM_929 (distance = 0.0 km) to dump_KM_924 (distance = 5.0 km).

b) Times: dredging time was specified such that dredging will take place during the low discharge period of 1187 m³/s. Due to the way that simulations are carried out, i.e. using the simulation management tool.

6. For a quantitative comparison between dredging scenarios, a simple cost function is used to calculate the cost of the dredging operation. The function takes the form:

$$\text{Cost} = A \cdot 1.4V\ ,$$

and

$$A = \begin{cases} 2 + 0.6\,L & L \leq 30\,\text{km} \\ 20 & L > 30\,\text{km} \end{cases}$$

with: V = Volume of dredged material [m^3]; the constant value of 1.4 to account for the overall [redacted], A = dredging unit price [EURO/m^3], and L = distance between dredging and dumping locations [km].

4.2 Results

Figure 18 shows that the river reaches a state of dynamic equilibrium after the first year (see the behaviour of the point without dredging). Dredging is triggered on reaching the threshold level and the shallow part within the dredging block is then removed.

In Case B150 that evaluates the current dredging. The result of this strategy is summarised in Figure 19 where we can see that, during a period of 5 years, dredging takes place in two locations with a total volume of 42,960 m³. Using the cost function given earlier, the dredging cost is evaluated to amount 120 KEuro/5 years. Figure 22 shows the temporal behaviour of dredging in the two locations. It is clear that dredging in KM 870 stops after 3 years, whereas, in KM 885 continues over the entire period.

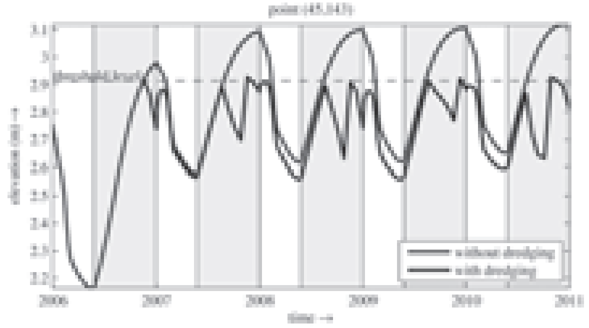

Figure 18: Comparison between the time-dependent behaviour of a point with dredging activities (Case B150) and reference case; shadows indicate dredging periods.

Case B170 evaluates the future dredging strategy of maintaining the navigation channel at a depth of 2.8 m and a width of 170 m. The result of this strategy is summarised in Figure 20 where we can see that dredging takes place in 17 locations with a total volume of 700,456 m³ during a period of 5 years with a total cost that amounts to 1,960 KEuro/5 years. Figure 23 shows the temporal behaviour of all dredging locations. It indicates that dredging stops in several locations after some time, whereas for several other locations it behaves in the same manner as in KM 885 in Case B150 by carrying out successive dredging. Dredging in this case extends over several kilometres; it nearly covers the whole reach from Km 870 to Km 890.

Case B152 evaluates a dredging strategy of maintaining the navigation channel at a depth of 2.8 m and a width of 150 m. The result of this strategy is summarised in Figure 21 where we can see that dredging takes place in 13 locations with a total volume of 293,538 m³ during a period of 5 years; the cost of dredging amounts to 820 KEuro/5 years.

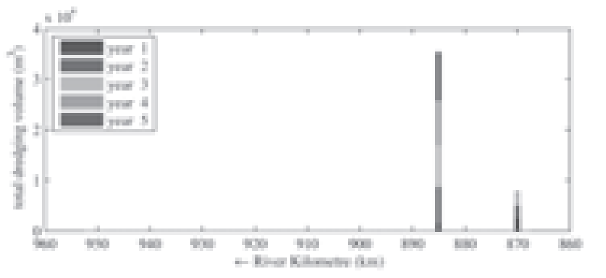

Figure 19: Spatial distribution of the total dredging volumes, [redacted]

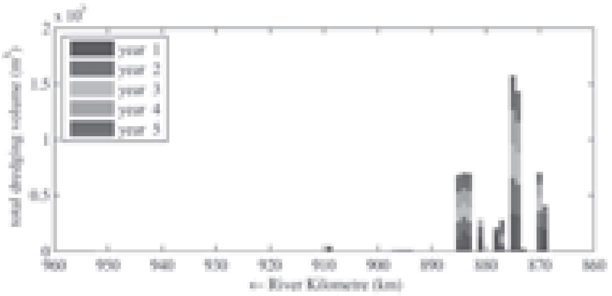

Figure 20: Spatial distribution of the total dredging volumes, Case B170.

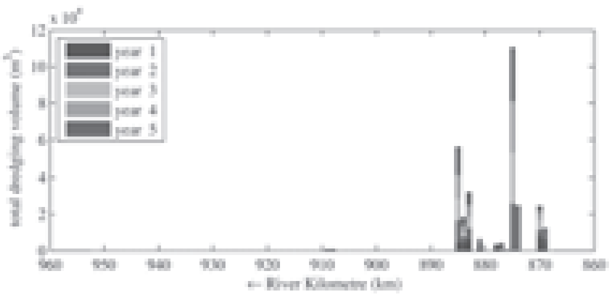

Figure 21: Spatial distribution of the total dredging volumes, Case B152.

Figure 24 shows the temporal behaviour of all dredging locations. It has a very similar pattern as observed in Case 170. Dredging in this case extends over several kilometres; it covers great part of the reach from km 870 to km 890. This case may be considered as an intermediate stage between Cases 150 and 170, where only the dredging depth is increased from 2.5 m to 2.8 m. However, the resulting dredging locations are closer to Case 170. Note that the dredging locations in Case 152 are comparable to Case 170.

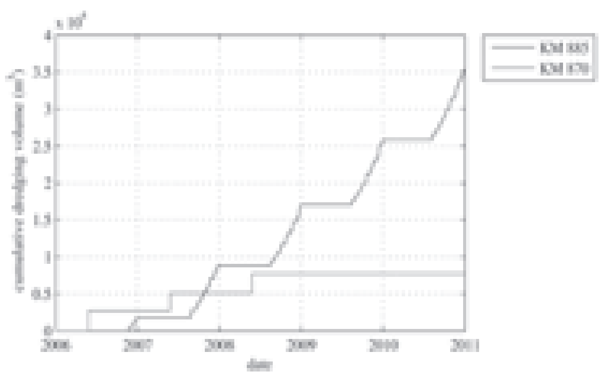

Figure 22: Time-series of cumulative dredging volume for all activated dredging operations in Case B150.

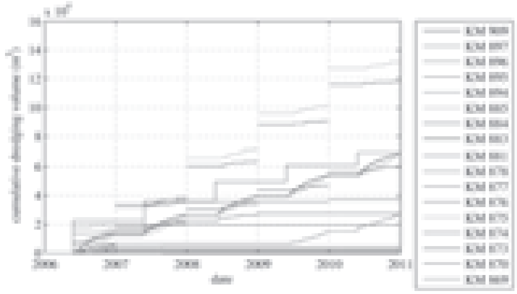

Figure 23: Time-series of cumulative dredging volume for all activated dredging operations in Case B170.

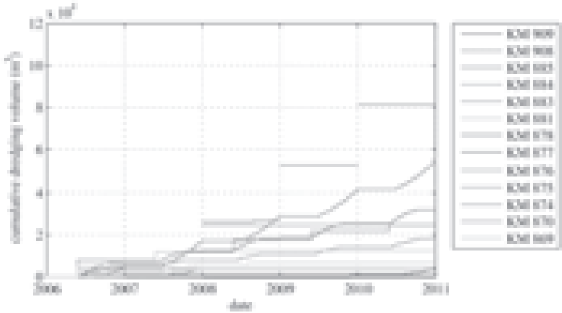

Figure 24: Time-series of cumulative dredging volume for all activated dredging operations in Case B152.

4.3 Conclusion

A comparison between the current situation given in Case 150 and the future navigation channel requirement given in Case 170 indicate that the cost of dredging would increase from 120 KEuro/5 years in Case B150 to 1,960 KEuro/5 years in Case B170; this is some 16 times higher.

Considering that Case 150 with a dredging volume of 42,960 m^3 as a reference case, we see that the dredging volume increases by increasing the minimum navigation depth as indicated in Case 152 to reach 293,538 m^3 (7 times more), and reaches up to 700,456 m^3 if we also increase the width of the navigation channel as given in Case 170 (16 time more). This indicates that increasing the width of the navigation channel nearly doubles the dredging volume when compared to only increasing the navigation depth.

By increasing the depth of the navigation channel nearly all dredging blocks, which are active by increasing the width, are activated.

5 CONCLUSION

In this paper we have presented a methodology to model large-scale river morphology for operational purposes. The methodology was presented using the case of the Rhine branches in the Netherlands where a detailed quasi 3-dimensional morphological model was created. Several innovative techniques were applied in this model; including domain decomposition, simulation management tool and flexible dredging and dumping functionality. These techniques improve the performance of the model in terms of flexibility and reduction of computational time. They are applicable for morphological modelling of large rivers.

We demonstrated the ability of the model by presenting a case study for dredging activities in the Waal River. The results demonstrate the strength of the model in offering answers to river mangers concerning planning activities. The model can be further extended to evaluate other aspects of importance such as water quality and ecology.

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