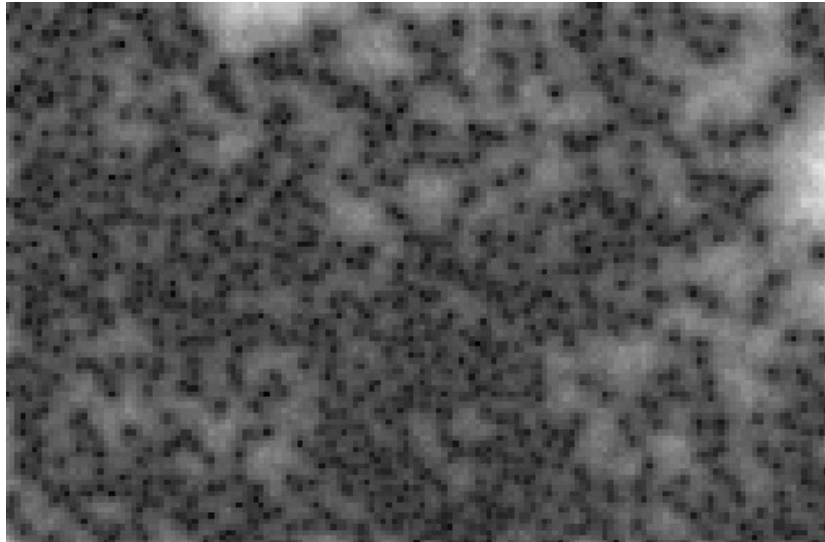


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Communicating spatial uncertainty to non-experts using R

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Abstract

Effective visualisation methods are important for the efficient use of uncertainty information for various groups of users. Uncertainty propagation analysis is often used with spatial environmental models to quantify the uncertainty within the information. A challenge arises when trying to effectively communicate the uncertainty information to non-experts (not statisticians) in a wide range of cases.

Due to the growing popularity and applicability of the open source programming language R, this project aimed to develop R functions to effectively communicate spatial uncertainty to non-experts. The spatial uncertainty information was generated using Monte Carlo algorithms, the output of which is represented by an ensemble of model outputs (i.e. a sample from a probability distribution). Three visualisation methods, adjacent maps, glyphs and an interactive application, were chosen for implementation. To provide the most universal visualisation tools for non-experts, a survey was conducted on a group of 12 university students. This survey assessed the effectiveness of the selected methods for visualising uncertainty in spatial variables such as elevation and land cover. The adjacent maps and glyphs were used for continuous variables. Both allow for displaying maps with information about the ensemble mean and standard deviation. Adjacent maps were also used for categorical data, displaying maps of the most probable class, as well as its associated probability. The interactive applications included a graphical user interface, which in addition to displaying the previously mentioned variables also allowed for comparison of joint uncertainties at multiple locations. The survey indicated that users could understand the basics of the uncertainty information displayed using the three methods, with all three having an approximately equal preference. The implementation of the visualisations was done via calls to the *ggplot2* package. This allowed the user to provide control over the content, legend, colours, axes and titles. The interactive methods were implemented using the *shiny* package allowing users to activate the visualisation of statistical descriptions of uncertainty through interaction with a plotted map.

This research brings uncertainty visualisation to a broader audience through the development of tools for visualising uncertainty using open source software.

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1. Introduction

1.1 Background

Geospatial information is rarely error-free and subsequently an understanding of spatial uncertainty is important. Errors are hard to quantify, as the measured value may not be identical to the 'true' value. Uncertainty relates to the level of confidence about knowledge of the 'true' value and therefore is a subjective measure of the deviation of a measured value from the 'true' value (Heuvelink et al., 2007). When conducting an analysis of geospatial information, it is integral to take the uncertainty of the data into account (Zhang and Goodchild, 2002).

When performing an operation using geodata, it needs to be taken into account that uncertainties in the input data will propagate through any operation (Heuvelink et al., 1989). Furthermore, models themselves can also have related uncertainties, leading to extra factors to account for when analysing the propagation of uncertainties in the output. A frequently used method for the analysis of uncertainty propagation is the Monte Carlo method. Based on a random sample from a probability distribution function (pdf) of the variable containing uncertainty, a series of possible realisations of a model output are created. These are used to assess the uncertainty of the model.

The uncertainty of outputs and data is beneficial for a range of different users including decision- and policy-makers, scientists and even the general public (Heuvelink et al., 2007). Communication of uncertainties is often done through visualisation of the data. However, there is no one-size-fits-all approach regarding the best way of communicating the uncertainty to the different users. Various studies (Senaratne et al., 2012, Kinkeldey et al., 2015, Spiegelhalter et al., 2011) have shown that when visualising the uncertainty, the preferred and most effective visualisation technique depends on the user group.

Digital visualisation of geodata has its roots in traditional cartography. As such, many symbols and symbology techniques developed over centuries of cartography have been easily transferred to digital visualisation (Wood, 1994). Bertin (1967) developed seven visual variables (location, size, shape, orientation, colour hue, colour value and texture) as a way of using symbols to represent information. These were extended to 12 variables by both Morrison (1974) and MacEachren (1992) with colour saturation, arrangement, crispness, resolution and transparency added. Since the 1990's the search for innovative and new techniques for visualising uncertainty has been ongoing. Many techniques have been developed through application of Bertin's visual variables as well as the extensions by Morrison and MacEachren. This

includes various types of static maps, yet with the help of modern technology, traditional map techniques have also been augmented using image sequences as well as graphical user interfaces. This could lead to more efficient methods of communication of uncertainty as well as wide-scale distribution of visualisation techniques.

1.2 Sources of uncertainty

When visualising uncertainty, it is useful to know where the uncertainty is coming from and why. Defining uncertainty is difficult, as there is no universal definition due to it being subjective, field dependent and can concern different aspects of our knowledge about phenomena (Longley et al., 2005). In the field of Geographic Information Science (GIS), uncertainty is often used 'to describe situations in which the digital representation is simply incomplete and as a measure of the general quality of the representation' (Longley et al., 2015). MacEachren et al. (2005) refer to uncertainty when inaccuracy is not known objectively, as otherwise it could be expressed as error. When trying to represent uncertainty, one commonly used approach is using a probability density function (pdf) (Heuvelink et al., 2007). This has a number of advantages, as a pdf allows for modelling correlation between uncertainties as well as applying methods for uncertainty propagation analyses (Heuvelink, 1998). Using a pdf also allows for the uncertainty to be quantifiable. Foody & Atkinson (2002) use the definition of uncertainty being a 'quantitative statement about the probability of error'. This definition of uncertainty will be used in this research, as in order to visualise spatial uncertainty, the uncertainty needs to be quantified in some sort of way.

The uncertainty pipeline is a concept used to identify three stages where uncertainty is introduced (Wittenbrink et al., 1995, Pang et al., 1997, Pang, 2008). These stages are the acquisition stage, the transformation stage and the visualisation stage and can be seen in Figure 1. The acquisition stage corresponds to uncertainties in the initial data and as such to the input uncertainty. This can come from a range of sources. Input data can come from observations and measurements, as well as numerical modelling. Uncertainty can be sourced from inaccuracies in the measuring instruments, as well as simplification of models. Many measurements or model runs can be done multiple times. Averaging these can eliminate the random error component and reduce the uncertainty in acquisition. The transformation stage corresponds to any transformations done on the input data. This means the data undergoes some sort of transformation and includes simple things such as unit conversions, as well as more complicated processes such as resampling or the use of algorithms to change data types (Pang, 2008). Monte Carlo analysis can be used

to assess the uncertainty after transformations have been done on the data. Uncertainty introduced in the visualisation stage can have its origins in the rendering process, interpolations used during visualisation as well as during animation processes. This research will not focus on the uncertainty produced during the visualisation stage. However, it is important to realise that miscommunication through visualisation can also lead to uncertainty. This research aims to visualise uncertainty from the output of Monte Carlo analyses in a way that there is no miscommunication in the visualisation.

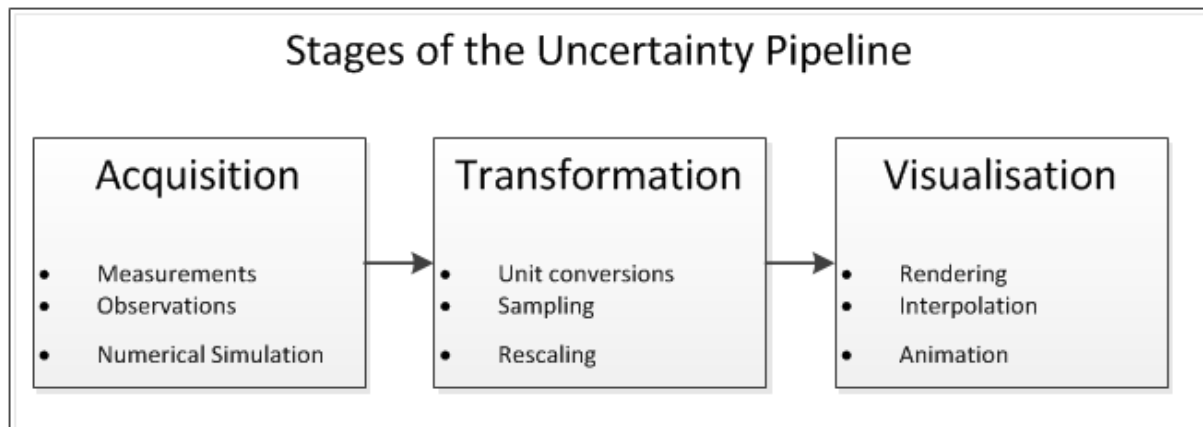


Figure 1: The uncertainty pipeline describing stages where uncertainty is introduced. After Wittenbrink et al. (1995); Pang et al. (1997); Pang (2008).

1.3 Spatial uncertainty

When evaluating spatial uncertainty, it is important to evaluate the types of spatial uncertainty. Burrough and McDonnell (1998) differentiate between two types of uncertainty: positional and attribute. Positional uncertainty refers to uncertainties in the position of an object, which as an example can be expressed as x- and y-coordinates. These uncertainties will often be produced in the acquisition stage, yet can also be a product of transformations. Attribute uncertainty refers to uncertainties in the attributes of an object. The attributes are generally the characteristics of the object and are mostly measured. This can for example be the height of a tree or the diameter of the tree measured at chest height. Once again, uncertainties in the attributes can originate from both the acquisition and transformation stage. In order to limit the scope of this research, only attribute uncertainty will be examined. As such, the measurement scales of attribute uncertainty are reviewed next.

When reviewing attribute uncertainty, Heuvelink et al. (2007) divide it into four categories depending on the measurement scales. The first two divide the numerical scale into two parts: continuous and discrete. Continuous attributes can take any value within a range, whereas discrete attributes can only take certain values within a

range. Attributes such as the depth of water in a flood area are continuous; the number of trees in a parcel of land is a discrete attribute. The third is the categorical scale. Categorical attributes belong to some sort of group or category, an example is land cover. The final measurement scale is narrative attributes. These attributes are a bit more abstract, as they involve the textual description of an attribute. When categorising uncertainty, MacEachren et al (2005) do not include the narrative measurement scale and Pang et al. (1997) only evaluated the numerical scales. In order to limit the scope, this research will only focus on continuous and categorical data, as these can be outputs of Monte Carlo analyses.

When evaluating attributes, one can also explore the concept that attributes can vary in both space and/or time. As such there are four categories of space-time variability. The first category contains attributes that are constant in both space and time. These include known constants as well as attributes that are assumed constant (Heuvelink et al., 2007). The next category includes attributes that vary in only space, yet not time. Alternatively, attributes can vary only in time, yet not in space. This means these attributes will only change over time without changing their position in space. Finally, attributes can also vary in both space and time. In order to limit the scope of this research, only attributes with spatial uncertainty will be considered. This means that at each location the attribute can take on a different value, which can in turn be uncertain. Furthermore, uncertainties in space can be correlated, which is elaborated on in Section 2.1. Temporal uncertainty and variability will not be taken into account as it adds an extra dimension of analysis, and as such falls out of the scope of this research.

1.4 Uncertainty representation

There are many different graphical variables that can be used to represent uncertainty. These variables include location, size, shape, orientation, colour hue, colour value, texture, colour saturation, arrangement, crispness, resolution and transparency. Using these variables, uncertainty can be added graphically to a map through encoding the uncertainty within the variable. There are various techniques used to represent uncertainty and this research splits these techniques into three categories: static techniques, dynamic techniques and interactive techniques. Static techniques involve one single map, on which the uncertainty is depicted using graphical variables. Dynamic techniques involve multiple maps, using a mechanism to shift from map to map. This shifting between maps highlights the areas of uncertainty. Interactive techniques involve the use of a graphical user interface where users can choose what information they want to have displayed, as well as a more detailed selection of areas.

While there are various visualisation methods in the three visualisation categories, three methods have been chosen for this research: adjacent maps, glyphs and an interactive application. The adjacent maps method presents two maps adjacent to each other. This allows for the first map to visualise the data, with the second map visualising the uncertainty on its own. Adjacent maps are one of the most popular methods for static representation of uncertainty. Gerharz and Pebesma (2009) and Senaratne et al. (2012) both found adjacent maps to be the most preferred method of the methods they tested. Glyphs are geometrically plotted specifiers that encode data values and information through their shape, colour, size, orientation and location (Pang et al., 1997, Potter, 2006). Using glyphs, the uncertainty of points can be placed on top of a map of the data. Senaratne et al. (2012) and Kinkeldey et al. (2014) both recognise glyphs as having high potential to display qualitative information. Interactive applications display uncertainty through an interactive environment where the user can in part decide what they want to see. Previous implementations have had mixed evaluations, with many having a large potential yet being tricky for the user to use intuitively (Gerharz and Pebesma, 2009, Sanyal et al., 2010, Senaratne et al., 2012).

1.5 Open source software

Open source software (OSS) is software that has its source code freely distributed (Mockus et al., 2002) and is often developed, maintained and distributed on a voluntary basis (Lakhani and von Hippel, 2003). OSS has grown tremendously over the last 20 years, with the total number of open source software projects growing exponentially (Deshpande and Riehle, 2008). OSS is also abundant within the GIS domain, with the website FreeGIS.org currently listing 356 open source GIS software programs with a growing trend (Steiniger and Bocher, 2009). One OSS is the R Project, which provides the R programming language and environment. R has advantages through its advanced statistical capabilities and high quality graphical output (Ripley, 2001). Furthermore, through the use of R packages, the software can be used for a variety of GIS analyses and visualisations. This research has chosen to visualise spatial uncertainty in R due to its strong graphical outputs and widespread use in the scientific community (Steiniger and Bocher, 2009).

1.6 Problem Definition

Although a range of uncertainty visualisation techniques are currently used, there is still a lack of a concrete framework for uncertainty visualisation. Johnson and Sanderson (2003) see this as a key research challenge and note “the realistic need to create a formal, theoretical error and uncertainty visualisation framework”. There

have been various proposed uncertainty visualisation frameworks over time including those by MacEachren (1992), Pang et al. (1997) and MacEachren et al. (2005). However, there is still no concrete knowledge about which visualisation techniques are most suitable for applications with spatial uncertainty.

Complicating the development of creating an uncertainty visualisation framework are the varied outcomes of surveys evaluating the visualisation techniques. Spiegelhalter et al. (2011) found that “there are few reproducible experimental findings for assessing best practice in visualising uncertainty”. There are conflicting reports as to whether the effectiveness of various techniques differs between different user groups (MacEachren et al., 2005). However, two studies (MacEachren et al., 2005, Kinkeldey et al., 2015) have shown that incorporation of uncertainty in a decision-making process depends on the user’s expertise. Furthermore, different types of users often have a discrepancy between their most preferred methods and the methods that were most effective (Senaratne et al., 2012, Kinkeldey et al., 2015, Spiegelhalter et al., 2011).

As such, there is a need to discover the best visualisation techniques for specific users and to offer tools for using these. Previous studies (MacEachren et al., 2005, Kinkeldey et al., 2015) investigating the effectiveness of uncertainty visualisation have often only involved expert users and students. This research proposes to investigate uncertainty visualisation focussing on non-experts as the user group. A non-expert is defined as a user without specific knowledge in the field of spatial uncertainty propagation. With the rise of the Internet globally as well as open source software, there is an enormous amount of potential for the distribution of new knowledge to this non-expert field. As such, uncertainty visualisation has the potential to help not only policy- and decision makers, but also the general public. This research will aim to bring uncertainty visualisation to a broader audience through the development of tools for visualising uncertainty using open source software.

1.7 Research objective and questions

This thesis is undertaken to supplement the Quantifying Uncertainty in Integrated Catchment Studies (QUICS) project. As part of this project, a spatial uncertainty propagation package is being developed in the statistical software R. This package consists in part of uncertainty propagation algorithms and in part of visualisation of these uncertainties. This thesis thus has the main objective:

“To implement and evaluate techniques for visualisation and communication of spatial uncertainty to a non-expert audience.”

In order to achieve this objective, two research questions have been defined:

RQ1: How can adjacent maps, glyphs and an interactive application be implemented to visualise spatial uncertainty in visualisation functions in an R package?

RQ2: How effective are the functions in communicating spatial uncertainty to a non-expert user?

2. Methodology

This chapter contains the methods used to implement the visualisation methods as functions in R and how they were evaluated. First, the statistical parameters for quantifying uncertainty are discussed. Thereafter, the way the chosen methods visualise uncertainty is briefly discussed before the implementation of the methods is discussed in more detail. The datasets used and targeted users are discussed before the evaluation process is finally discussed.

2.1 Statistical modelling of uncertainty

The data dealt within this research is continuous and categorical spatial data, as explained in Section 1.3. The spatial attributes in the datasets used are elevation for the Zlatibor DEM, inundation depth for the flooding case and land cover for the Soest case. These variables are uncertain, meaning their true value is unknown. In order to analyse the uncertainty, a series of possible realisations of the data was simulated using the Monte Carlo method. Each of these realisations has an equal probability of being the 'true' value.

When evaluating spatial uncertainty it is important that the uncertainty can be quantified. Sanyal et al. (2010) use the term uncertainty metrics to denote the most important parameters derivatives used to quantify uncertainty. These are the ensemble mean, standard deviation, interquartile range and prediction intervals. This project also uses the ensemble mean, standard deviation and interquartile range, yet also the relative error as well as the concept of spatial correlation. The following statistics apply to the values at all locations within the continuous datasets. The categorical dataset used the class probability to quantify the uncertainty of each class at all locations.

The ensemble mean of the realisations at each location is an important variable. Since every value in the realisations has an equal probability of being 'true', the average value is used to predict the value of the unknown 'true' value. In a group of n realisations, the sample values are denoted as X with $X = (x_1, x_2, x_3, \dots, x_n)$ for n realisations. Equation 1 shows how the mean, \bar{x} , is calculated.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (\text{Equation 1})$$

The sample standard deviation is another important variable, as it denotes the uncertainty. The standard deviation is used to represent the spread of the values in the realisations for each location. The standard deviation, s_n , of the samples in X was calculated using Equation 2.

$$s_n = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (\text{Equation 2})$$

In order to calculate the interquartile range, it is first important to know the quantiles. The quantiles are calculated using the empirical quantile function (Jones, 1992) shown as Equation 3. The probability, p , is between 0 and 1, with the function I being a function that returns 1 if the condition within the square brackets is true and 0 otherwise.

$$Q_n(p) = x_1 + \sum_{i=2}^n (x_i - x_{i-1}) I \left[\frac{i-1}{n} < p \right] \quad (\text{Equation 3})$$

Using Equation 3, the interquartile range is then calculated using Equation 4. This is the range between the 25th and 75th percentile and gives an indication of the spread of the realisations.

$$\text{Interquartile range} = Q_n(0.75) - Q_n(0.25) \quad (\text{Equation 4})$$

The relative error is the last metric and relates the size of the standard deviation relative to the mean. The relative error is calculated using Equation 5.

$$\text{Relative error} = \frac{s_n}{\bar{x}} \times 100 \quad (\text{Equation 5})$$

Correlation, ρ , measures the joint variability of two random variables, X and Y . It is used to observe linear relationships between two points. Correlation is a number between -1 and 1, with values of -1 or 1 representing high correlation and 0 representing no correlation. If there is positive correlation, high values of X will often appear with high values of Y . If the correlation is negative, high values of X will often appear with low values of Y . The correlation between X and Y is calculated using Equation 6.

$$\rho(X, Y) = \frac{\sum[(X - \bar{x}) * (Y - \bar{y})]}{\sqrt{\sum(X - \bar{x})^2 * \sum(Y - \bar{y})^2}} \quad (\text{Equation 6})$$

Spatial correlation refers to the correlation between two points in space. As the distance between the points increases, the correlation between the two points decreases. A correlogram shows the correlation as a function of distance. Using a correlogram one can find the range of high spatial correlation between two points. Figure 2 shows two correlograms and a scatterplot of two points at a certain distance apart. The left diagram shows two points with low spatial correlation, with the right diagram showing two points of high spatial correlation at the same distance apart.

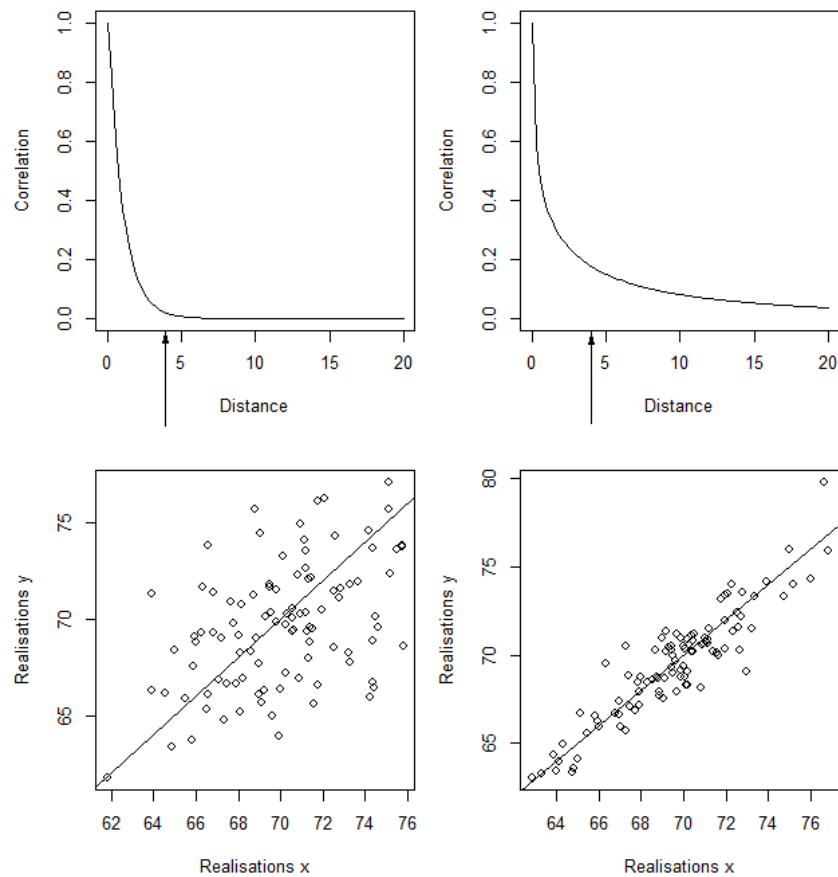


Figure 2: Scatterplots showing the correlation of two points the same distance apart but with different spatial correlation.

To quantify the uncertainty in the categorical dataset, the class probability of each class was calculated at all locations. At each location, the number of realisations the class was present in was denoted as x . The probability for each class is then calculated as in Equation 7, with n denoting the total number of realisations.

$$p(x) = \frac{x}{n} \quad (\text{Equation 7})$$

2.2 Selected visualisation methods

The first method chosen was the adjacent maps method, which is the most simple of the three methods. Two maps were placed next to each other, displaying the predicted value map on the left and the uncertainty on the right for continuous data. For categorical data, the left map displayed the most likely class and the right map the associated class probability. The colour scheme chosen varied for the different data types. Since one of the test cases was a DEM, the colour scheme chosen for the predicted values is similar to the terrain colours colour palette found in R. Low values received a dark green colour, with the gradient transitioning to white for the

highest values. The uncertainty was coloured from black (lowest uncertainty) to white (highest uncertainty). This was chosen as white is easy to recognise as being uncertain (Gerharz and Pebesma, 2009). For the categorical data the most likely classes received random colours to highlight different classes on the map. The class probability map was also coloured from black to white, however with black having the highest probability and white the lowest.

The second method was using glyphs to represent the uncertainty. Glyphs can encode data through various variables such as size, shape and colour (Pang et al., 1997, Potter, 2006) and this method was only used for continuous data. Glyphs have been used widely for displaying uncertainty, with a wide variety of shapes and colours used. Shapes can be arbitrary, though smart choices of the shape can also help the user understand the uncertainty. They have been effective in communicating uncertainty (Potter, 2006, Sanyal et al., 2009) yet can become visually overwhelming if too many are displayed on one map (Pang, 2001). The base map of predicted values used the same terrain colour scheme described in the previous paragraph. A circular, black glyph was chosen as the default, with the uncertainty encoded in the area of the circle. The colour black was chosen as it is not included in the base map, with the circle not being colour-filled in order to still be able to read the base map. In order to avoid displaying too many glyphs, the user can choose the density of glyphs.

The final method, the interactive application, displays the uncertainty in a single map, yet through interactivity more information about the uncertainties can be gained. Many interactive representations feature a clickable map where the user can display uncertainty information about a certain location or where the map is modified after clicking. Various implementations of interactive uncertainty visualisations (Pebesma et al., 2007, Potter et al., 2009, Sanyal et al., 2010) depended on a separate application, with a web tool created as part of the UncertWeb Project (Gerharz et al., 2012). Two interactive applications were created, one for continuous data and the other for categorical data. The colouring schemes for the maps were identical to the colouring schemes used in the adjacent maps. The maps were also clickable, displaying additional information about the realisations, spatial correlation and class probabilities. These plots used the default R colouring schemes.

2.3 Implementation of visualisation methods

The implementation of the visualisation methods was split into three parts corresponding to the three methods used. Furthermore, the visualisation functions had to be applicable for both continuous and categorical data for the adjacent maps and interactive application methods. The plotting of the data was based upon the *ggplot2* package. This package is an implementation of the grammar of graphics in R

(Wickham, 2006). Ggplot was chosen for its strengths in multi-layered plots as well as custom user layout.

Four functions were created: one to plot adjacent maps, one to plot glyph maps, one to create an interactive application for continuous data and finally one to create an interactive application for categorical data. The interactive application functions depend on the *shiny* package and the RStudio environment to run (RStudio, 2013). Pseudo-code of the functions can be found in Appendix 1 through to Appendix 4. Table 1 shows the compatible data types, inputs as well as the outputs these functions create.

Table 1: **Overview of data and inputs for the uncertainty visualisation functions**

Function	Adjacent maps	Glyphs	Interactive: continuous	Interactive: categorical
Data type	Continuous/categorical	Continuous	Continuous	Categorical
Required inputs	Simulations object	Simulations object Density	Simulations object	Simulations object
Optional inputs	Title, x-axis, y-axis, legend labels Grid and axis ticks Colour palettes for the two maps	Title, x-axis, y-axis, legend labels Grid and axis ticks Colour palettes for the two maps	None	None
Output	Two adjacent maps	One map with glyphs	Graphical user interface	Graphical user interface

A test class was created containing the expected characteristics of the data to be used with the functions. An overview of this test class is seen in Figure 3. An object of this class has slots for the realisations, mean, standard deviation, quantiles, most likely class, and class probability. The realisations slot was created to contain all the simulated outputs after a Monte Carlo simulation. For continuous variables, the mean and standard deviation slots contain the mean and standard deviation of the

realisations, with the quantiles slot containing the 5, 25, 50, 75 and 95 percentile of the realisations. For categorical variables, the most likely class slot contains the value of the class that is most likely in each cell, with the class probability slot containing the probability of the most likely class for each cell. All the objects in the slots were raster objects, taken from the *raster* package (Hijmans, 2012). The mean, standard deviation and most likely class slots were all of class *RasterLayer*. This means they were all single-layer raster layers. The remaining slots were all of class *RasterBrick*. *RasterBrick* objects allow for the storage of many single raster layers. This multi-layered approach allows for efficient storage of multiple raster layers. Furthermore, it simplifies the process of running calculations and statistics over a large dataset of similar rasters. The quantiles slot was a *RasterBrick* as it contains a raster layer for each quantile. The class probability slot had a layer for each class, containing the probabilities of each cell being the most likely cell from the realisations.

Simulations
- Realisations: <i>Rasterbrick</i>
- Mean: <i>RasterLayer</i>
- Standard deviation: <i>Rasterlayer</i>
- Quantiles: <i>RasterBrick</i>
- Most likely class: <i>RasterLayer</i>
- Class probabilities: <i>RasterLayer</i>
+ <code>plotadjacent()</code>
+ <code>plotglyphs()</code>
+ <code>interactivecontinuous()</code>
+ <code>interactivecategorical()</code>

Figure 3: Overview of the test class “Simulations”.

The adjacent map plotting function relies on the `ggmap` function from the *ggplot2* package. Its only required variable is an object of the test class. From this object it checks the data type and returns adjacent maps of the mean and standard deviation for continuous data and the most likely class and class probability for categorical data. Furthermore, the user is able to modify the title, x- and y-labels, the legend, presence of a grid, presence of axis ticks and labels and change the colouring of the plot.

The glyphs plotting function also plots a `ggmap` object, requiring both an object of the test class containing continuous data as well as the density of the glyphs. It returns a map of the mean, with the standard deviation placed on the map using circular glyphs in a grid. An increase in the density leads to a small grid, with more

information displayed, yet can show effects of clustering. Additionally, the user can modify additional information for the plot in the same way the adjacent map function does.

The interactive application combines the powerful plotting of the *ggplot2* package with the simple creation of user interfaces using the *shiny* package. The functionalities of the two functions created using *shiny* are explained in Table 2. The continuous data application allows the user to see either the mean or standard deviation map. The user can visualise areas with a relative error above a certain value using a slider. The user can also view areas with a prediction interval of 90% larger than a certain range using a slider. Additionally, single-clicking the map displays a histogram of the realisations at that point. Double-clicking the map adds a scatterplot displaying the realisations of both the single- and double-clicked points. This allows for visualisation of spatial correlation. The categorical data application is simpler, initially only displaying the map of the most likely class. Single-clicking on this map displays a pie chart of the probabilities of the various classes at that specific point.

Table 2: **Functionalities of the two interactive application functions**

Function	Interactive: categorical
Single-clicking on map	Displays pie chart of class probabilities of the realisations at that point
Function	Interactive: continuous
Selecting Mean	Displays map of predicted values
Selecting Standard Deviation	Displays map of standard deviation
Selecting Relative Error	Displays slider bar. Cells with relative error higher than the value on the slider bar turn grey on the map
Selecting Prediction Interval	Displays numeric input. Cells with an interquartile range larger than the prediction interval turn grey on the map
Single-clicking on map	Displays histogram of realisations at that point

Double-clicking on map	Displays scatterplot of realisations between single-clicked and double-clicked point
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2.4 Datasets

The three visualisation methods were developed and tested in conjunction with three datasets. Seeing as functions for both continuous and categorical spatial data were being developed, two continuous datasets and one categorical dataset were chosen. Furthermore, the targeted end users were kept in mind throughout the development of the functions. The locations of these datasets can be seen in Figure 4.

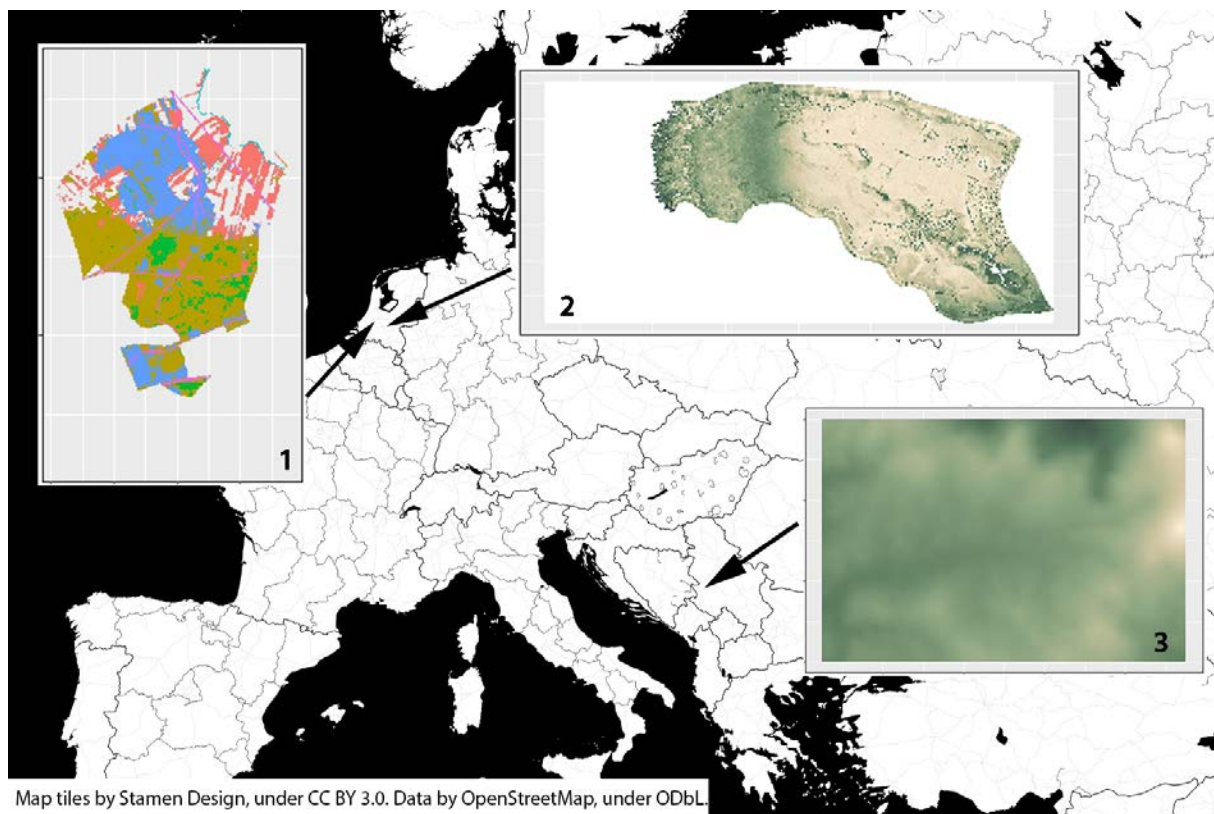


Figure 4: The locations of the three test datasets used

The first dataset was a categorical dataset, containing 50 realisations of multiple land cover classes generated by sequential indicator simulation (De Bruin et al., 2004). The land cover data contains the various land covers for the Dutch municipality of Soest. The various classes included agriculture, trees, nature, water, built-up areas and infrastructure. However, during the development of the functions the land cover class was not important and these land covers were only treated with unique values (1-6) representing the land cover classes. In this dataset the original files were all of an ESRI ArcInfo Coverage data format. The realisations were converted to an ASCII

raster format with a cell size of 25m x 25m. This way they were easily used in the R programming environment.

The second dataset used contained 100 realisations of a flooding case. These data contain the realisations of a flood event and was generated as an exercise in the Wageningen University course GRS-30306: Spatial Modelling and Statistics. It covers an area between the Maas and Waal rivers in the Netherlands. The model simulates the inundation depth of the floodplain six hours after a dike breach, with both the influx of water and the underlying DEM simulated and subsequently uncertain. The format of the data already was in an ASCII raster format with a cell size of 100m x 100m.

The last dataset used contained 50 realisations of a digital elevation model (DEM) in the Zlatibor region, Serbia (Hengl et al., 2010). The data contained a DEM as well as control points with measured elevation data. Using the control points and the DEM, the errors at the control points were determined. From these errors, 50 realisations of the DEM were simulated using simple kriging. The format of the realisations was again an ASCII raster with a cell size of 30m x 30m.

2.5 Users

The users of the visualisation functions are intended to be non-experts. This has been defined as a user without specific knowledge in the field of spatial uncertainty propagation. Keeping this in mind the visualisation functions were kept to be as simple as possible. However, the interactive application was created with some more advanced users in mind, combining both simple visualisations of the core uncertainty data, yet also allowing for a more in depth analysis of the data through relative errors and prediction intervals.

In order to find participants for the survey, fellow students were asked to complete the survey. Students were chosen as they fit the description of the non-expert target group. Furthermore, since the survey was hosted on one specific computer, it was convenient to have students studying in the same room complete the survey.

2.6 Evaluation of the implemented visualisation methods

The evaluation stage was intended to evaluate and receive feedback on the created functions. Moreover, it was intended to evaluate the level of understanding of the visualisations. The evaluation was done by use of an electronic survey, which non-experts were then asked to complete. The results of the survey aim to give insight into which visualisation methods work the most effectively.

The survey was created using Google forms and as such hosted online. The full survey can be found in Appendix 5. The survey was completed on one computer, as in addition to the visualisations in the survey the interactive application was also evaluated. For this the interactive application had to be run through RStudio on the same computer. The survey was structured into four parts. The first three parts each evaluated the various visualisation methods for a specific dataset. The datasets used here were the Zlatibor DEM simulation, the flooding case study and the land cover simulations for the municipality of Soest. The last part allowed for feedback on the various visualisation methods.

The part using the Zlatibor DEM first evaluated the understanding of both adjacent maps and glyphs through estimating values of the map for specific points. Furthermore, it asked the user for their preferences between the two types of visualisations. The questions using the interactive application aimed to evaluate the understanding of both the relative error, as well as the prediction interval functionalities of the application. Furthermore, it also evaluated the histogram and scatterplot functionalities of the application.

The second part evaluated the flooding case study in a similar fashion to the Zlatibor DEM. Here questions were again asked regarding the understanding of the adjacent and glyph maps through asking the user for values to be estimated from the map for certain points. The user was then again asked to state the preference between the two methods. The interactive application was evaluated through using the histogram to determine areas of higher uncertainty.

The third part evaluated the land cover simulations through the use of adjacent maps and the interactive application. The users were asked to estimate values from the maps as well as the probabilities of certain areas of the map using the interactive application.

Finally the users were asked to provide their general evaluations of the various visualisation methods through a ranking scale as well as providing their own feedback on these methods.

3. Results

The results are split into two parts. First the outputs of the visualisation functions are presented. The interactive application only is presented as a static screenshot of the application. Thereafter the results of the evaluation survey are presented.

3.1 Application of the visualisation functions

Figure 5 presents the output of the Adjacent Maps plotting function applied to the Zlatibor DEM dataset. It displays the predicted value, the mean, in the left plot and the standard deviation in the right plot. The right plot encodes the standard deviation, with areas of high values having a higher uncertainty compared to areas of low values.

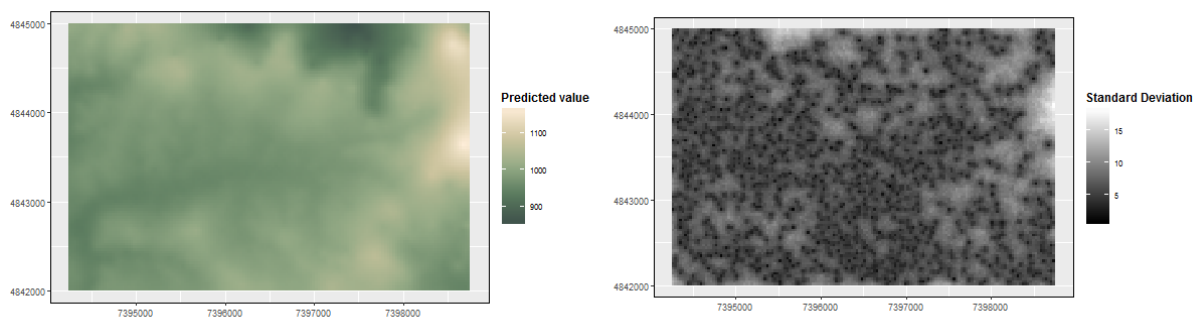


Figure 5: Zlatibor DEM predicted values and uncertainty using adjacent maps

Figure 6 presents the output of the Glyphs plotting function applied to the same dataset. This single plot displays both the predicted value as the background map, with the standard deviation represented with the circular glyphs overlaid on top. Larger glyphs encode a larger uncertainty.

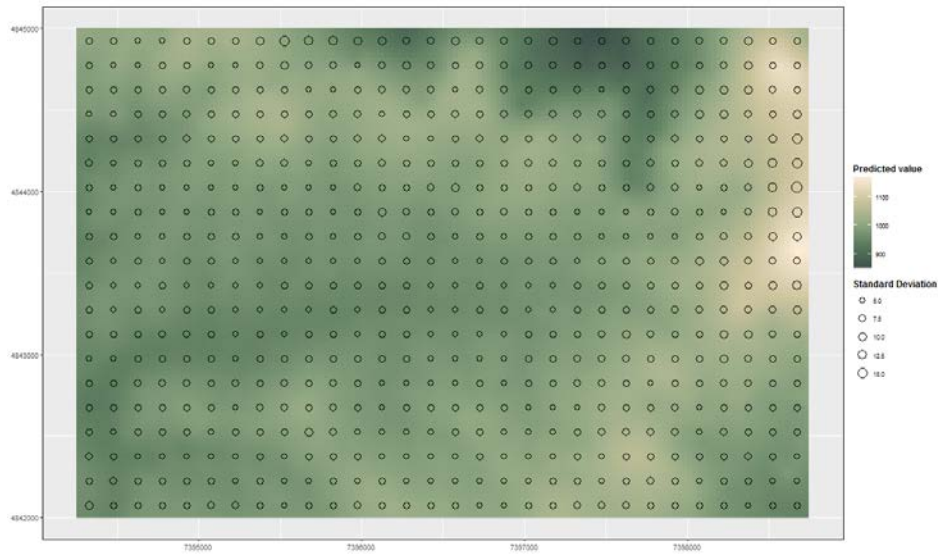


Figure 6: Zlatibor DEM predicted values and uncertainty using glyphs

Figure 7 displays the interactive application for continuous data again using the Zlatibor test case. It shows a map of predicted values, together with a histogram of the realisations of a point and a scatter plot between the realisations of two points. The extra features are only displayed as buttons here, with the output of them not shown.

Visualisation of Continuous Data

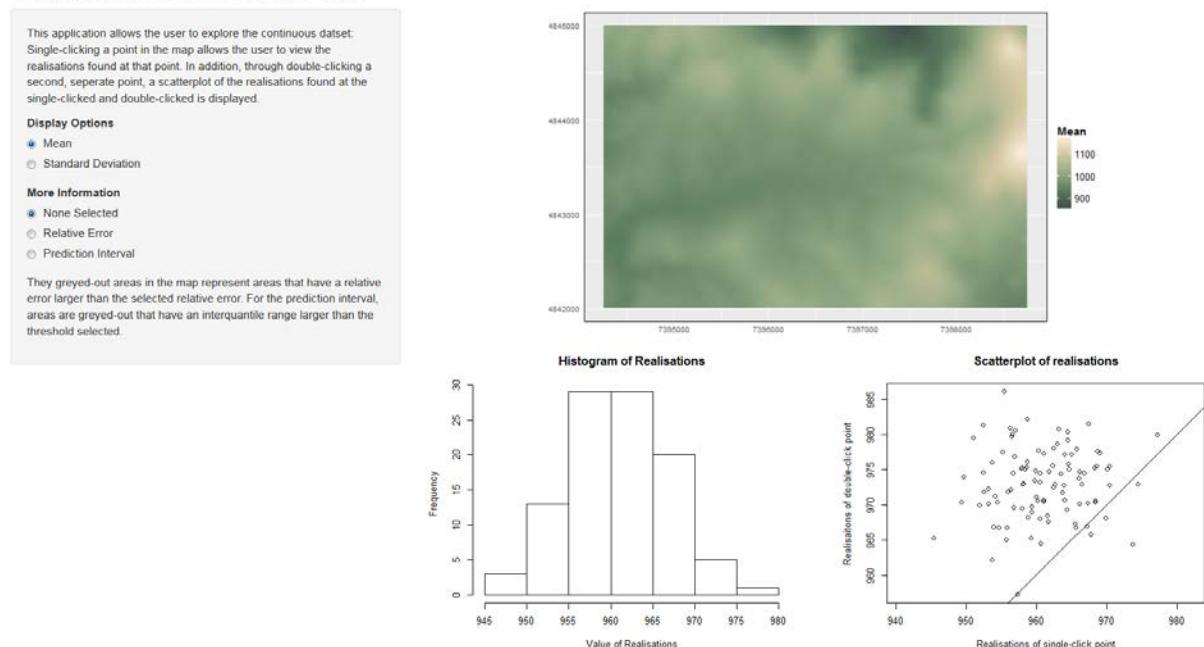


Figure 7: Zlatibor DEM predicted values shown in the interactive application

Figure 8 shows the interactive application for categorical data using the Soest land cover dataset. Using adjacent maps, the left map displays the most likely class for the region, with the right map displaying the class probabilities for a selected point.

Visualisation of Categorical Data

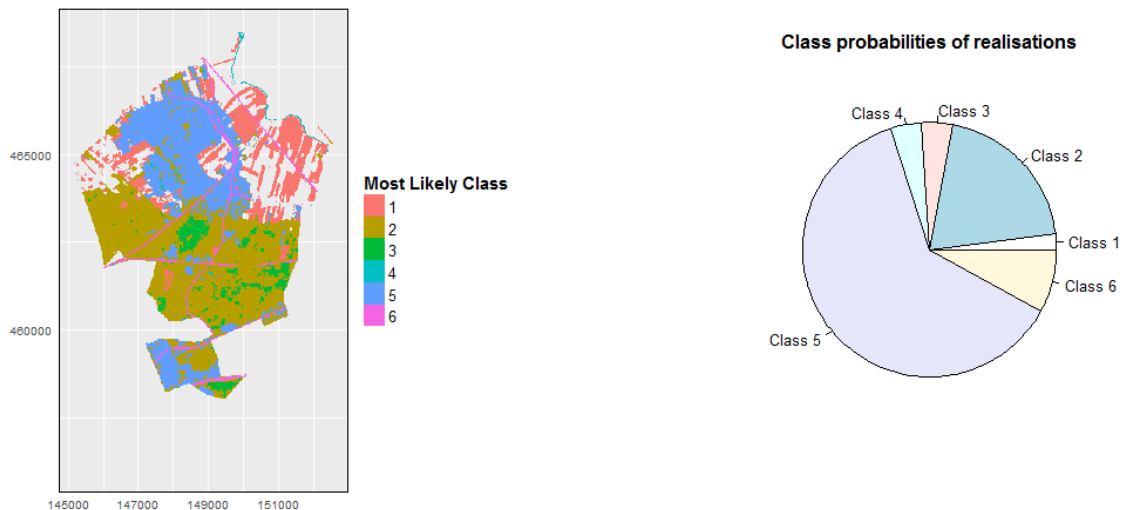


Figure 8: Most likely land cover classes for Soest shown in the interactive application

3.2 Evaluation survey results

Twelve students answered the survey; their responses have been split up into four categories. The first results are related to the adjacent maps method, the next results to the glyphs method. Thereafter, the results of the interactive applications are presented with the final results presenting the results about personal preferences of the test users group. The full data can be found in Appendix 6.

3.2.1. Adjacent maps

The results of using adjacent maps with the Zlatibor DEM dataset show a spread over the possible answers (Figure 9). To test the users' understanding, they were asked to estimate the standard deviation of three given points on the predicted value map. The correct answer of Point A was 0–5, with the highest number of respondents choosing this answer. However, the point was located in a local area of low uncertainty, surrounded by areas of high uncertainty, explaining the high number of responses in the 15+ category. For point B, the correct answer was 5-10. Half the respondents answered this correctly, with the point located next to areas of high and low uncertainty. Point C had a correct answer of 5-10, with the majority of responses also in this category.

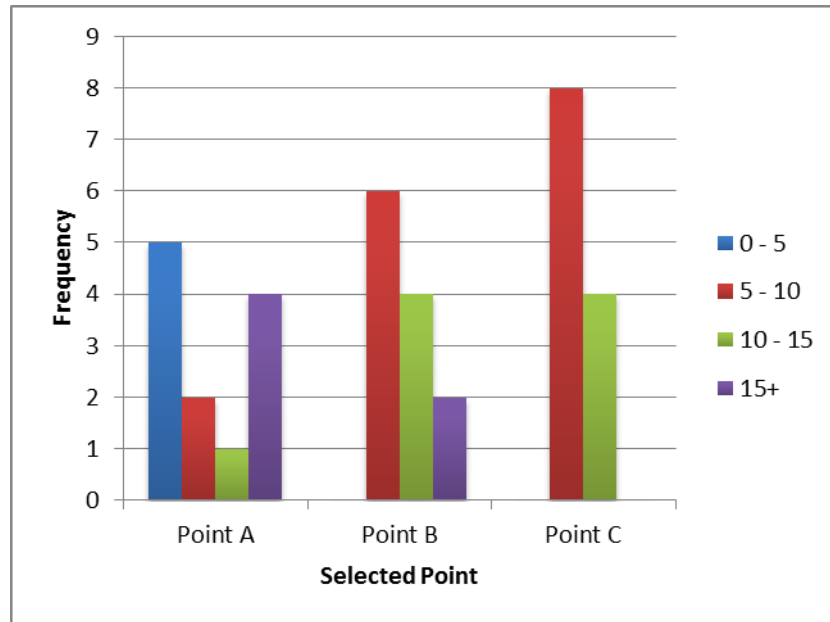


Figure 9: Frequency of selection of the standard deviation value for the indicated points in the Zlatibor DEM using adjacent maps

Figure 10 shows the results of estimating values from the adjacent maps using the flooding test case. Point A was located in an area with a lot of noise of medium to high uncertainty, with the results reflecting this noise. Eleven of the twelve students managed to read the standard deviation of point B from the map correctly. Point C was located in a region of medium uncertainty, with the majority of students identifying this point correctly.

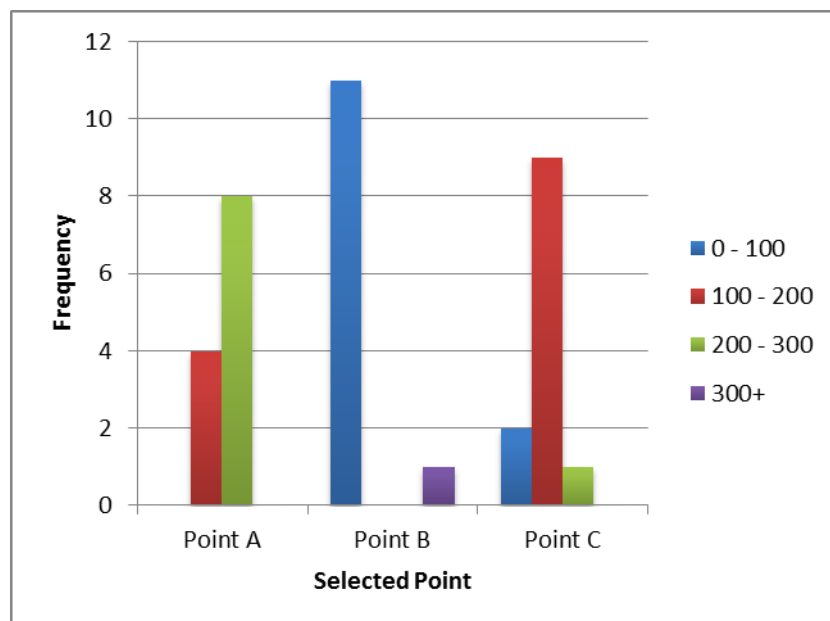


Figure 10: Frequency of selection of the standard deviation value for the indicated points in the flooding case using adjacent maps

The last case tested using adjacent maps was the land cover case of the municipality of Soest. The students were asked to rank the three points in descending uncertainty. The correct answer C-A-B was given by six students, with another three identifying point C being the most certain. Figure 11 also shows that three students identified either point A or B as being the most certain. Point A was located in a region of high uncertainty, yet with the exact probability hard to read. Thus the spread between 0.6-0.8 and 0.8-1.0 in Figure 12 was expected. Point B was in an area of low probability with a lot of noise. Here again the spread of responses was expected. Point C was clearly in an area of high uncertainty with 11 of the 12 students answering this question correctly.

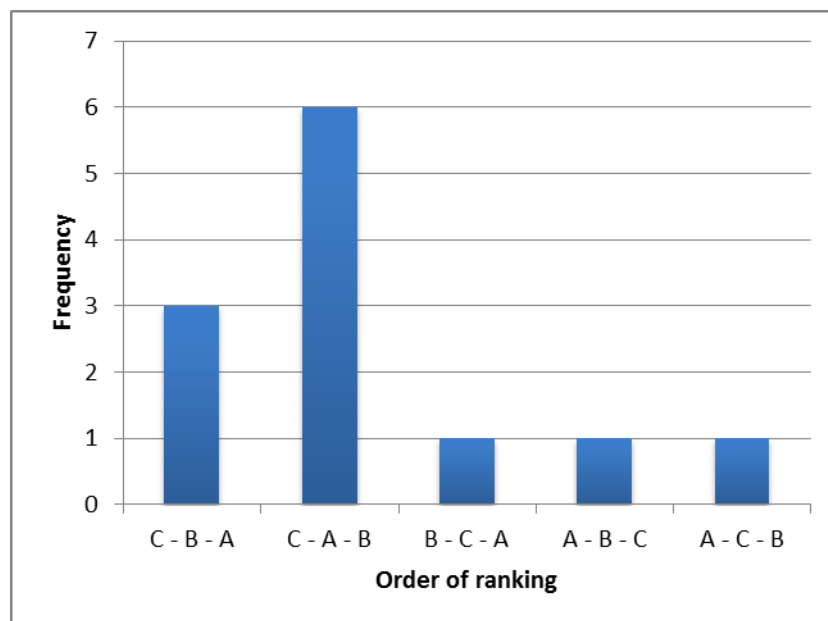


Figure 11: Frequency of ranking of probabilities for the indicated points in Soest using adjacent maps

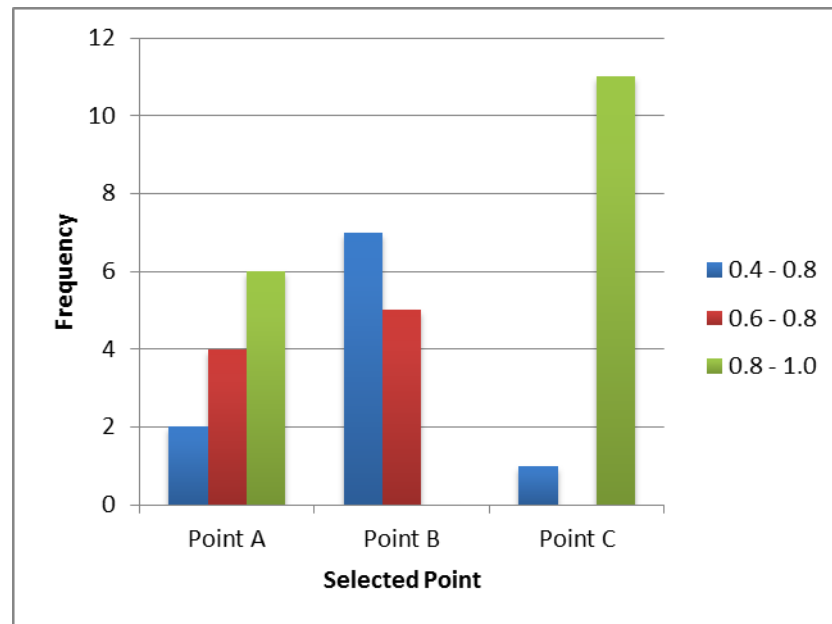


Figure 12: Frequency of selection of the probabilities for the indicated points in Soest using adjacent maps

3.2.2. Glyphs

Using glyphs with the Zlatibor DEM case study showed to also give a spread in answers when asked to estimate the standard deviation of three points on the glyph map (Figure 13). The correct answer for point A was a standard deviation of 7.5-10, which most of the students answered correctly. However, this relies on interpreting the size of the glyphs with the legend, so the other results were expected. Point B had a correct answer of 10-12.5, which was also answered correctly in large. Point C was clearly in an area with low uncertainty, thus the correct answer from all the students was not unexpected.

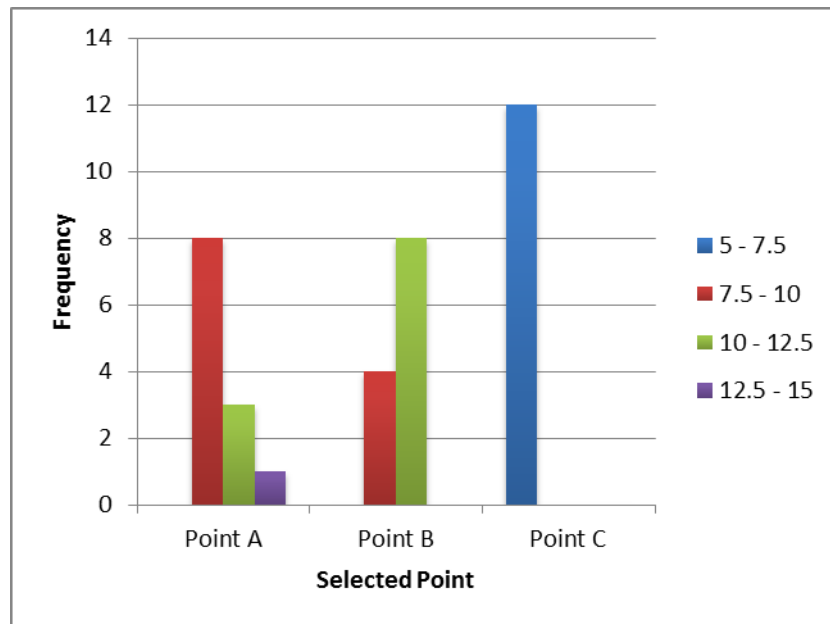


Figure 13: Frequency of selection of the standard deviation value for the indicated points in the Zlatibor DEM using glyphs

The glyphs were also used together with the flooding case study (Figure 14). This map had little local variations of uncertainty and subsequently the students answered the questions correctly to a large part. As seen in the bar plot in Figure 14, the standard deviation of Point A was correctly identified, with 11 out of 12 students also identifying the standard deviation of points B and C correctly.

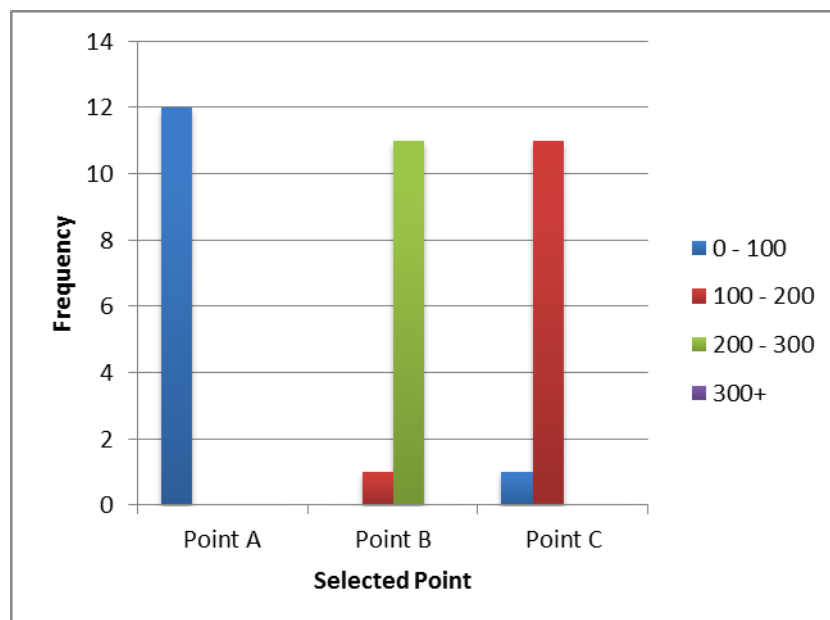


Figure 14: Frequency of selection of the standard deviation value for the indicated points in the flooding case using glyphs

3.2.3. Interactive application

Using the interactive application, the users were asked to identify the percentage of the map with a relative error higher than one percent. As seen in Figure 15, all students answered this question correctly. Furthermore, they were also asked to find the threshold of the interquartile range which half the map exceeds. All but one of the students were able to identify this threshold correctly.

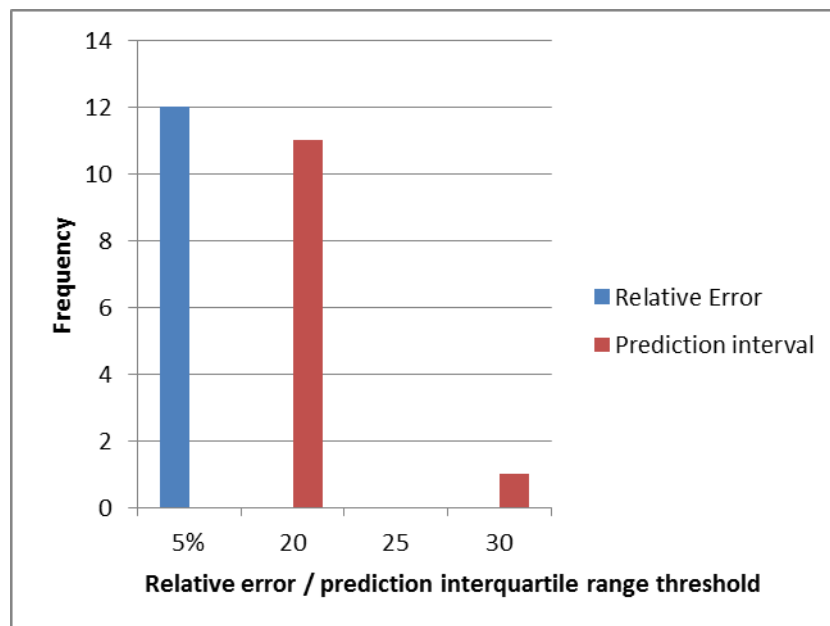


Figure 15: Frequency of relative error and prediction interval estimations of the Zlatibor DEM using the interactive application

Also using the Zlatibor DEM case study, the users were asked to identify the range of high spatial correlation between two selected points. As seen in Figure 16, the majority of students answered this question correctly with the spatial correlation decreasing rapidly at ranges larger than 5 cells between the points.

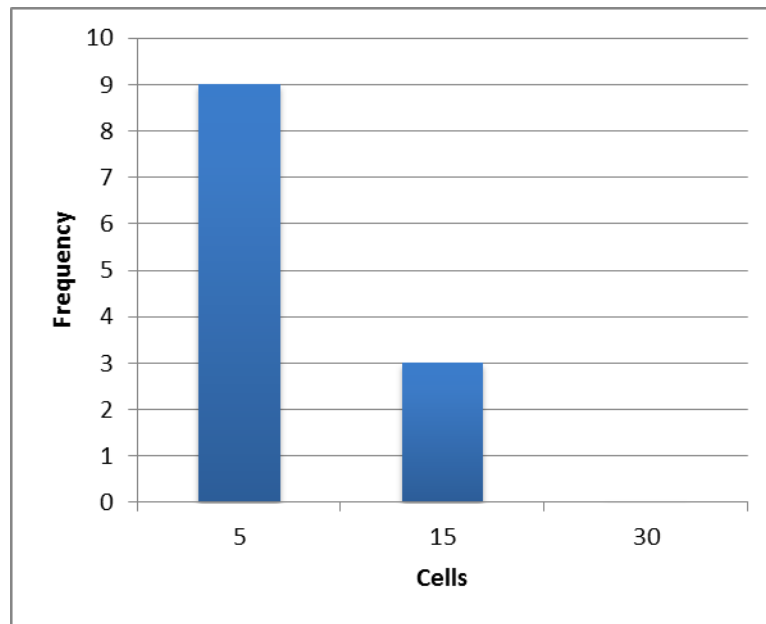


Figure 16: Frequency of the estimated range of spatial correlation between two points using the Zlatibor DEM study case

Using the interactive application, users were also asked to evaluate the probability of an area being assigned Class two. Figure 17 shows that half the students answered the question correctly, with another 5 choosing for a higher probability.

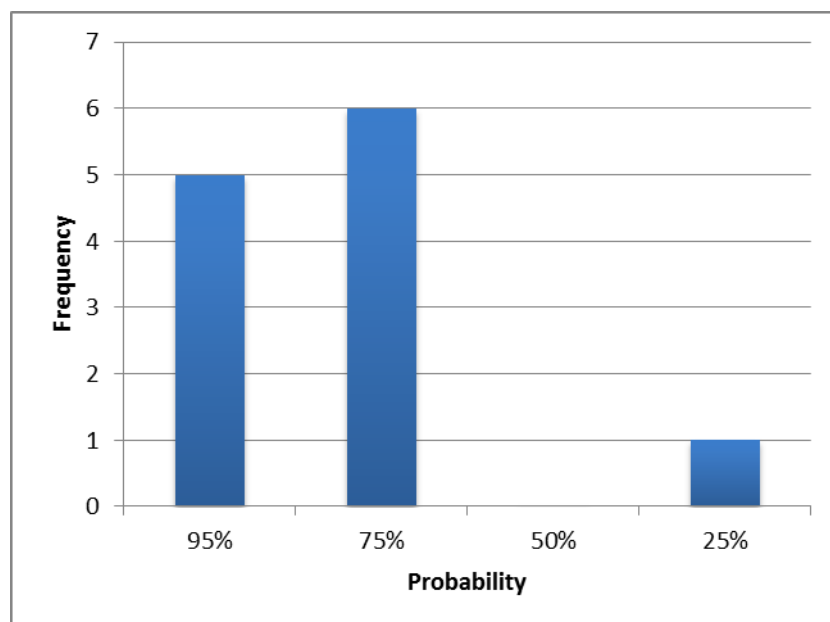


Figure 17: Frequency of the probability that an area was assigned Class two in the Soest study case

3.2.4. Personal preferences

The respondents of the survey were asked to state their preference for the size of the glyphs shown in the map. As Figure 18 shows, there was no clear preference for the larger or smaller glyph size. Furthermore, they were also asked to state whether the glyphs displayed on the map made it more difficult to read the actual values from the map of predicted values. Figure 19 shows that half the respondents did find it harder, though the other half did not find added difficulty created through the glyphs.

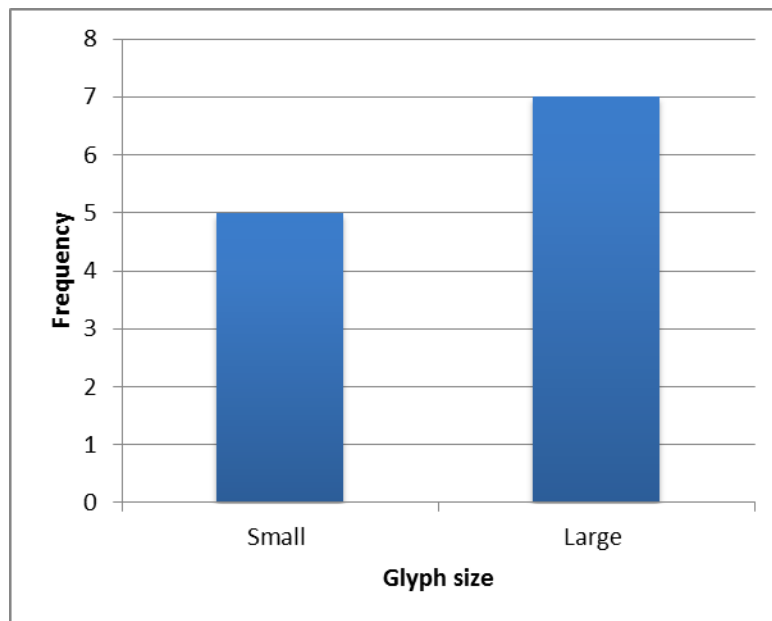


Figure 18: Frequency of glyph size preference for the Zlatibor DEM test case

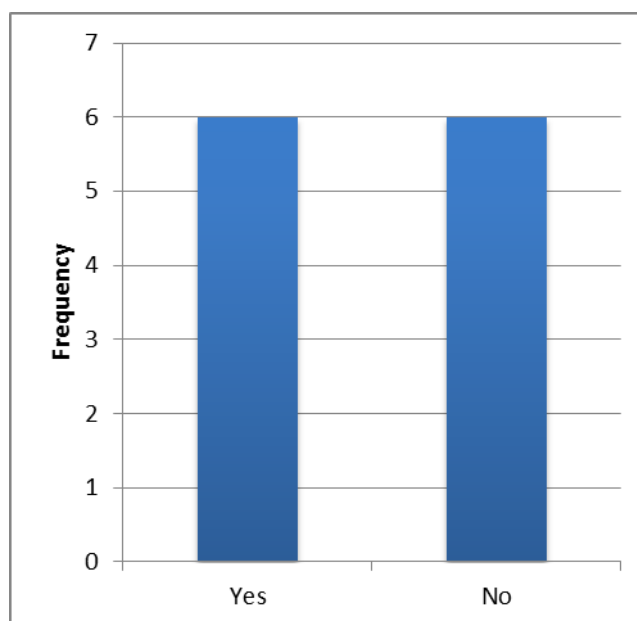


Figure 19: Frequency of users answers to whether adding glyphs complicates reading map values

There was not a clear preference for either of the two static methods evaluated (Figure 20). After using the two methods with the Zlatibor DEM dataset, the adjacent maps method had a slight preference over the glyphs method. However, as seen in Figure 20, after testing using the flooding dataset the glyphs method actually had a higher preference over the adjacent maps method. This suggests that both methods are applicable, though depending on the dataset one could be more appropriate.

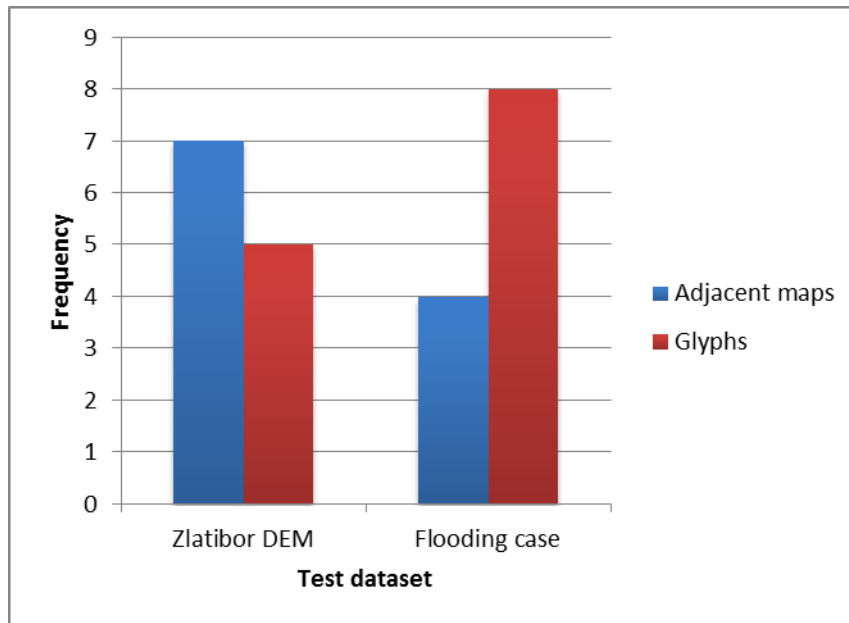


Figure 20: Preference of visualisation methods between adjacent maps and glyphs using two datasets

The overall ease of comprehension for the three methods evaluated varied depending on the method. The disagree category was only chosen once for the glyphs and interactive application methods and no times for the adjacent maps method. Figure 21 also shows that the glyphs method had two thirds of preferences in the agree or slightly agree categories, suggesting an overall ease of comprehension. The adjacent maps had a fairly even spread between agree and slightly disagree. Finally the interactive application also had a spread between agree and no preference, with two answers ranked below no preference. In general, there was no strong trend towards agree or disagree, suggesting the methods are comprehensible but could be improved.

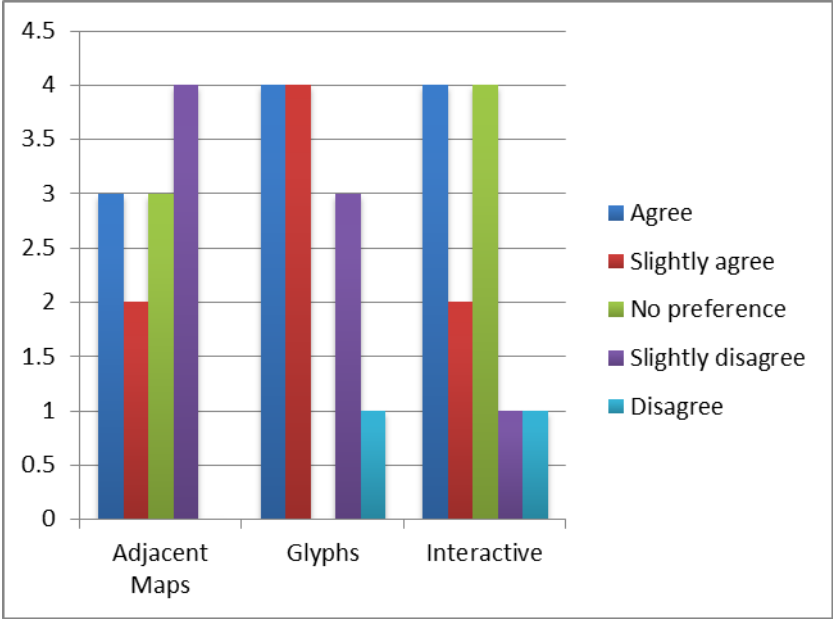


Figure 21: Perceived ease of comprehension of the three visualisation methods

4. Discussion

This research resulted in four functions written in R to visualise spatial uncertainty to non-experts. Three different visualisation methods were used, which were then also evaluated by a group of students. This chapter discusses implementation of the methods, the results discovered for each of the visualisation methods evaluated, as well as limitations and recommendations for future improvements.

4.1 Implementation of visualisation methods

The adjacent maps method was the first method to be implemented. The overall output of the adjacent maps function is satisfactory, yet it can still be improved. The output maps of the adjacent maps function are not identical in size. The amount of plotting space for the two maps is identical. However, since the standard deviation legend is three characters longer than the predicted value legend, the plot size of the standard deviation map is adjusted to fit the map and legend in the allocated plotting space. This was only realised after the evaluation of the three methods. This could also make it harder to visually match two points across the maps.

Using the adjacent maps function as a guideline, the glyph map function was then created. The sizing of the glyphs within the glyph map was readjusted multiple times before deciding to use the current layout. When adding the glyphs initially, the plotting function plots the glyphs with a large difference between the size of the largest and smallest glyph, regardless of the actual spread in values. This allows for quick visualisation of minimum and maximum values. However, it also can mislead the user into thinking that the largest uncertain value is many times bigger than the smallest value in cases where the spread of values is low. Keeping this in mind, it was chosen to relate the size of the glyph to the actual values. This does result in a minimal variety of sizes if the spread of values is small. However, it also returns a more true insight into the variation of the uncertainty. Furthermore, the user can adjust the density of the glyphs to their own liking. In cases of low spread of values, a higher density of glyphs could provide a more detailed insight of variation.

The interactive application functions built upon the knowledge gained from the previous two functions. Initially, only one function was to be built, which automatically recognised the data type and adjusted the user interface accordingly. However, due to the different needs regarding the user interface depending on the data types, it was decided to create two functions. This also had the benefit of keeping the code as succinct and efficient as possible. The interactive application functions do have a shortcoming that they are not very portable. The outputs of the adjacent maps and

glyphs functions can be printed and taken anywhere. However, the interactive application functions depend on an open RStudio environment. As such, a computer containing the data and RStudio is always required. The interactive applications could also be hosted on a website, yet this would also need an available server and website.

4.2 Evaluation of visualisation methods

Using adjacent maps was the first method to also be evaluated. Overall, the results of the adjacent map evaluation for the DEM case study, the flooding case study and the land cover case study were positive. They showed a general trend of understanding being able to read off the uncertainty from the adjacent maps. Values of points located in clear areas of a certain range of uncertainty were easily obtained. This was the case at both the upper and lower ends of the uncertainty scale. However, points with uncertainty values in a more central range of the scale were harder to distinguish correctly. This was seen across all three case studies. Gerharz and Pebesma (2009) found similar results, with adjacent maps easy to understand yet harder to perform in tasks where quantification was required. Furthermore, the probability map of the land cover case study had large areas where the probabilities were noisy. This makes it especially difficult to read off uncertainty values. The colouring chosen for the uncertainty visualisation may have influenced these added difficulties of values in the middle ranges. White was chosen to represent areas of high uncertainty, as it is easy to recognise and comprehend (Gerharz and Pebesma, 2009), with black representing areas of low uncertainty. However, it may be difficult to differentiate between different values of grey shades and values in the middle range of the scale. Hengl (2003) also found that the colours in the gradient to white are harder to distinguish. In order to improve this, a larger visual range for colour differentiation could be used.

The glyphs method was the next method evaluated and resulted in fairly similar results. In general, the results were positive, with the concept generally understood and easy to read off. The results also showed less spread between the answers compared to the adjacent maps. Once again, the points selected at the upper or lower end of the glyph scale were the easiest to distinguish. In both the DEM and flooding case studies, the point placed between the smallest glyph size returned correct results for all participants. Values in the middle of the scale again seemed harder to read off precisely. Kunz et al. (2011) found glyphs were hard to implement in datasets with great variation due to the scaling of the glyphs. This research agrees with this finding, as the scaling makes for difficult reading in the middle ranges. Furthermore, the DEM case study had four categories of glyph sizes in the legend,

with the flooding case study only having three categories of glyph sizes in the legend. The spread in the results for the two points placed in the middle ranges was larger for the DEM case study compared to the flooding case study. This suggests that more glyph sizes provided on the scale make it more difficult to correctly differentiate between the subtle differences in the glyph sizes. The different glyph densities were also briefly evaluated. The results showed that there was no preference for glyph density, suggesting that this could be manually customised for each use case.

The interactive application was evaluated by giving the users direct tasks using the relative error, prediction interval and cross-correlation for the continuous data. Class probabilities per point were used for the categorical data. In general, these statistics were well received, as shown by the positive results. All users answered the relative error question correctly and all but one answered the prediction interval question correctly. However, it cannot be said that the statistical concepts are understood, as the users were only assessed on reading the value for a statistic from the visualisation. In the evaluation, the questions were more of a step-by-step guide of the use of the functionality, so the outcomes were expected. The spatial correlation scatterplot yielded good results, with the deviances from the correct answer potentially coming from a user's perception of cell size and level of correlation still regarded as 'highly correlated'. Finally, the probability that an area belonged to a certain land cover class was also evaluated. Half the respondents answered this correctly, yet five also incorrectly estimated it too high. This may be due to the majority of points having high confidence and the user's interpretation of how to include the areas of lower confidence.

The evaluation survey also aimed to get an insight into user preferences between the different methods. The two static methods were ranked into first and second preference for both the DEM case study and the flooding case study. For the DEM case study, the adjacent maps were slightly preferred. However, for the flooding case study the glyphs were slightly preferred. From these results it is assumed that neither method has a strong preference over the other. The preference of the maps could change depending on the user as well as the data set used to create the outputs. Furthermore, when evaluating the ease of comprehension of all three visualisation methods used, none of the methods returned only negative results. All three methods displayed a spread of opinions across the range from agreeing that the method is easy to comprehend to disagreeing. This suggests that the tools are easy to comprehend to some users and subsequently the user could choose which method suits him or her best.

4.3 Limitations

This evaluation survey was not without its limitations. It only gathered 12 responses and due to this small sample size, only large contrasts would have turned out to be significant if significance testing were done. More importantly the sample used was also not a random sample from the population. The population was the group of non-experts, which covers anybody without specific knowledge in the field of spatial uncertainty propagation. This evaluation only used students, who are part of the target group but not random. Due to these reasons, no tests of significance were undertaken. Nevertheless, the results give valuable, general insights into what extent non-users can understand the visualisation methods.

The adjacent maps method relies highly on users being able to spatially correlate points and areas across two maps. The map of predicted values and the map of standard deviation were of slightly different sizes for both the DEM and flooding case studies. This may have remained unnoticed during the students' participation in the survey, though could have influenced their responses slightly. If the evaluation were to be done again, the map size would be adjusted to have equal size maps.

The survey only evaluated the output of the functions and did not include any use of the functions within the R environment. Calling the functions within an R script using provided data was not evaluated, though it is expected to be simple if the user has some basic knowledge using the R software. This is due to the functions conforming to other functions used within the R software.

Finally, it was also not evaluated if the users could use the functions in a context for their own use. The evaluation survey did not include any questions regarding decision making based on the outputs. This is an important step, as the outputs are intended as information to base decisions upon. Furthermore, it was also not evaluated if the statistical concepts such as relative error, prediction interval, cross correlation and class probabilities were understood. If understood, these additional concepts implemented in the interactive application have the potential to provide more information about the data and its uncertainty.

4.4 Recommendations

In order to statistically backup the results found in this study, the evaluation survey would have to be redistributed to a true random sample and a larger sample size. However, this has its challenges in both selecting a random sample and then receiving answers from the sample. As this is possibly not feasible, the results do still give a general indication of the potential of these functions.

The functions themselves also have large potential for expansion. The statistical information displayed in the interactive application (prediction interval, relative error) could also be added to the adjacent maps in the form of additional plots. The prediction interval could also be expanded to include the 90% and 95% prediction interval instead of only the interquartile range. In order to analyse the data sets to a larger degree, the Jensen-Shannon divergence could be included. This divergence measures the similarity between two probability distributions (Melville et al., 2005) and could be used in addition to the current scatter plot of correlation. The Jensen-Shannon divergence has previously been used by Tellenbach et al. (2009) and Xu et al. (2010). Topographic extensions such as base maps and contour lines could also be added to give the user a better sense of location. It is recommended that if extra functionalities are added that they do not make the functions more complicated to use. Rather, they provide additional optional functionalities, with the default options still kept as simple as possible.

The implementation of the test class was done using raster objects created from the *raster* package. This package depends in turn on the *sp* package (Hijmans, 2012). As such, it could be investigated to see if the test class could also be created using *sp* objects, eliminating an additional dependent library in the functions. Furthermore, the functions could also potentially be extended using the *ellipsis* parameter. This parameter allows for the input of any number of named or unnamed arguments. These are then passed onto functions later on. Since the visualisation functions use *ggplot2* to plot the output, the *ellipsis* could potentially be used for more personal customisation of the output plots. However, this would require the user to have detailed knowledge in the functionality of the *ggplot2* package.

5. Conclusions

This project was intended to provide non-experts a platform to communicate and understand spatial uncertainty. The three methods, adjacent maps, glyphs and an interactive application, were implemented as functions in R. All three provide a useful, easy to understand output.

The adjacent maps and glyphs methods were implemented in R as functions based upon the *ggplot2* package. They provide the user a ready-to-use output, with additional options to customise the labels, titles and legends. The interactive application was implemented as two separate functions using the *shiny* and *ggplot2* packages; one for continuous data and one for categorical data. These functions allow for basic visualisation similar to the adjacent maps, yet also allow for more detailed insights into the uncertainty of the data. This is done by using the additional statistics relative error, prediction interval and spatial correlation for continuous data and class probabilities for categorical data.

After evaluation of the functions using a survey two main conclusions about the effectiveness of the functions were found:

The three methods were generally well understood and effective, with the lower and upper extreme uncertainties easy to recognise. Uncertainties in the middle of the range were harder to identify. These were due to noise, difficulty in differentiating between greyscale colours and difficulty in differentiating between subtle differences in glyph sizes.

No single method had a strong preference over the other methods. The adjacent maps and glyphs methods received approximately equal preference from the test users. As such, the user can choose from the available visualisation methods and choose the method most suited to their dataset and liking.

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Appendices

Appendix 1: Adjacent map pseudo-code

```
load libraries

#' Visualise Spatial Uncertainty using Adjacent Maps
#'
#' Allows for visualising spatial uncertainty in both continuous and
#' categorical data using Adjacent Maps. For continuous data it provides
maps
#' of mean and standard deviation of realisations. For categorical data,
the
#' function provides maps of the most likely class and the associated
#' probability.
#'
#' @param simulations Object of class simulations
#' @param title,xlab,ylab,legend Character vector giving plot title, x
axis
#' label, y axis label and legend respectively. To be provided in
format
#' c("plot1", "plot2").
#' @param grid Logical. Add grid to background of graph
#' @param axis Logical. Add axis ticks and labels to graph
#' @param mean_col,std_col \code{\link[grDevices]{colorRampPalette}}.
Colours
#' to be used for plotting the mean and standard deviation of
continuous
#' data respectively.
#'
plotAdjacent <- function(simulations, title = NULL, xlab = NULL,
                        ylab = NULL, legend = NULL, grid = T, axis = T,
                        mean_col = NULL, std_col = NULL)

# Check if simulations is of correct class
if simulations object not of correct class
  stop

#Check if titles provided for both graphs
if titles not provided for both graphs
  warning titles should be provided

#Check if labels provided for both graphs
if labels not provided for both graphs
  warning labels should be provided

#Check if legends provided for both graphs
if legend not provided for both graphs
  warning legends should be provided

# Set background grid and axis ticks/labels
Initialise gridlines, axis ticks and text to be shown

#Define theme
Initialise theme for title, legend, background, axes

# Plot for continuous data
if of continuous data
```

```
Convert mean and standard deviation to data frames
Set default colour palette for mean and standard deviation
Set default legend labels for continuous data
Create mean and standard deviation plots
Arrange plots adjacently

# Plot for categorical data
if of categorical data
  Convert most likely class and class probabilities to data frames
  Set default colour palette for class probability and most likely
class
  Default legend labels for categorical data
  Create most likely class and class probability plots
  Arrange plots adjacently
```

Appendix 2: Glyph map pseudo-code

```
load libraries

#' Visualise Spatial Uncertainty using Glyphs.
#'
#' Allows for visualising spatial uncertainty for continuous data using
#' glyphs.
#'
#' @param simulations Object of class simulations containing continuous
data
#' @param density Numeric. Gives spacing of density of glyphs.
#' @param title,xlab,ylab Character vector giving plot title, x axis
label and
#' y axis label respectively.
#' @param grid Logical. Add grid to background of graph
#' @param axis Logical. Add axis ticks and labels to graph
#' @param mean_col \code{\link[grDevices]{colorRampPalette}}. Colours to
be
#' used for plotting the mean values of the realisations.
#' @param glyph_col Character giving the colour of the glyphs
representing the
#' standard deviation of the realisations.
#' @param legend Character giving the legend titles. To be provided in
format
#' c("legend1", "legend2").
#'
plotGlyphs <- function(simulations, density, title = NULL, xlab = NULL,
  ylab = NULL,grid = T, axis = T, mean_col = NULL,
  glyph_col = NULL, legend = NULL)

# Check if simulations is of correct class
if simulations argument not of correct class
  stop

# Check if simulations is of continuous data
if not continuous data
  stop
# Check if density is of correct class
if density argument not a number
  stop

#Check if legends provided for graph and glyphs
if legends not provided for both graphs
```

```
warning legends should be provided

Convert mean to data frame
Aggregate standard deviation and create data frame

# Set background grid and axis ticks/labels
Initialise gridlines, axis ticks and text to be shown

#Define theme
Initialise theme for title, legend, background, axes

Set colour palette for mean and colour for glyph
Set default legend labels

Create mean plot
Add standard deviation glyphs to mean plot
Return plot
```

Appendix 3: Interactive application for continuous data pseudo-code

```
#' Interactively Visualise Categorical Spatial Data
#'
#' Allows for visualising spatial uncertainty for categorical data in an
#' interactive interface. Allows for more knowledge about class
probabilities at
#' a certain location. If data contains NA values, consider changing these
to
#' NaN values for a more insightful visualisations.
#'
#' @param x Object of class simulations. Must contain continuous data.
#'
#' @return Interactive application to visualise the data.

interactiveContinuous <- function(x){
  require libraries

  # Check if x is of correct class
if object x not of class simulations
  stop

  # Check if x is of categorical data
if not categorical data
  stop

  Create user interface using shiny
  Add title
  Add sidebar with radiobuttons
  Add conditional panels that appear after clicking previous buttons
  Add three plot output panels

  Create server using shiny

  Convert mean, standard deviation to data frames
  Calculate relative error
  Initialise theme for title, legend, background, axes

if relative error radio button clicked
  create slider input panel

if prediction interval radio button clicked
```



```
    create numeric input panel

  if Mean radio button clicked
    if Relative error radio button clicked
      modify mean data frame to remove values that do not fulfil condition
    if prediction interval radio button clicked
      modify mean data frame to remove values that do not fulfil condition

    create first plot from mean data frame

  if Standard deviation radio button clicked
    if Relative error radio button clicked
      modify standard deviation data frame to remove values that do not
fulfil condition
    if prediction interval radio button clicked
      modify standard deviation data frame to remove values that do not
fulfil condition

    create first plot from mean data frame

  if first plot single-clicked on
    retrieve realisations of clicked point
    create histogram of realisations in second plot

  if first plot double-clicked on
    if first plot also single clicked on
      retrieve realisations of single and double-clicked points
      create scatterplot between realisations of both points
```

Appendix 4: Interactive application for categorical data pseudo-code

```
#' Interactively Visualise Continuous Spatial Data
#'
#' Allows for visualising spatial uncertainty for continuous data in an
#' interactive interface. Options include basic map of predicted values and
#' standard deviation, as well as observing relative error and prediction
#' intervals.
#'
#' @param x Object of class simulations. Must contain continuous data.
#'
#' @return Interactive application to visualise the data.

interactiveCategorical <- function(x)
  require libraries

  # Check if x is of correct class
  if object x not of class simulations
    stop

  # Check if x is of categorical data
  if not categorical data
    stop

  Create user interface using shiny
  Add title, sidebar and two plot output panels

  Create server using shiny

  Convert most likely class and class probabilities to data frames
```

Initialise theme **for** title, legend, background, axes

Create plot of most likely class

if plot is clicked on
 retrieve realisations of clicked point
 count occurrence of each class
 create pie chart of the occurrences of each class

Appendix 5: User evaluation survey

The online survey can be found at https://docs.google.com/forms/d/12NjQFBXzC2UM8lZJqwEcxMU_xmcpNjvhe_jp95Qvfho/formResponse

User Evaluation Survey of Spatial Uncertainty Visualisation

This survey aims to evaluate the spatial uncertainty visualisation functions developed as part of a master thesis project at Wageningen University (WU), The Netherlands. The research aims to effectively communicate information about spatial uncertainty through visualisation to a non-expert. As such, it is not required to have specific knowledge in the field of spatial uncertainty propagation.

Please read this survey carefully, it will take about 15-20 minutes.

The survey will evaluate three different visualisation methods using three data sets. The visualisation methods evaluated are adjacent maps (two maps next to each other), maps using glyphs (symbols on top of the map) and an interactive application.

We know that maps contain errors which implies that we do not know the true value of the environmental variable that is represented on the maps. Therefore, many different versions of the map were simulated, all having the same probability of representing the true situation. Jointly these outcomes express uncertainty about the true state of the environmental variable.

For maps of continuous variables such as elevation and flood depth, the mean (i.e., measure of centrality) and standard deviation (measure of spread) were computed from the simulated maps.

Often maps represent categories such as land cover types. For those cases the most frequently mapped category (most likely class) and the fraction of times that this class occurred in the entire series of simulated maps (probability of occurrence) were computed.

User Background

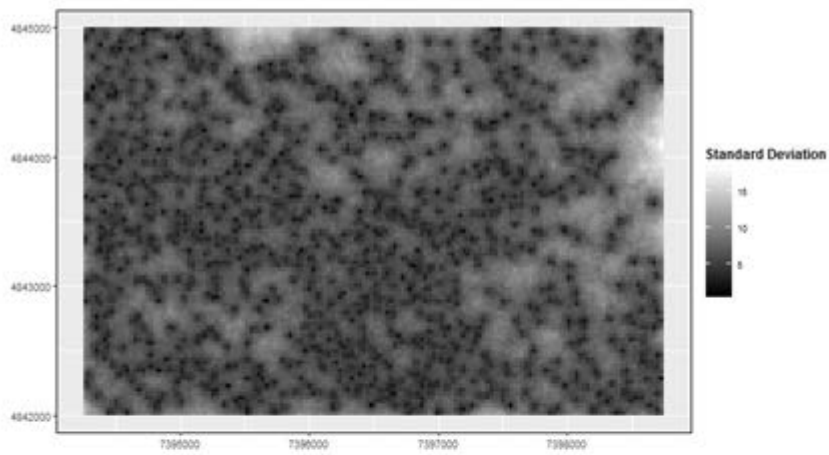
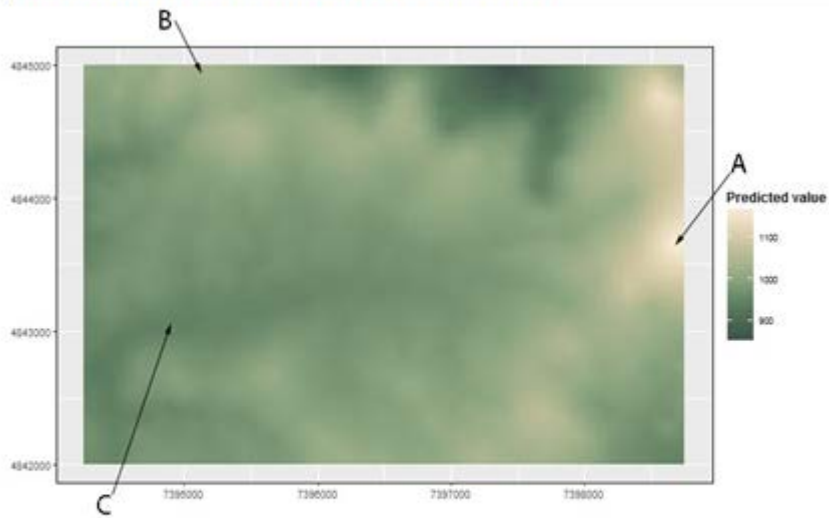
1. What is your field of expertise/study?

Mark only one oval.

- GIS
- Other: _____

This dataset contains a Digital Elevation Model (DEM) simulated from control points for the region of Zlatibor, Serbia. 100 realisations of the DEM were simulated. Of these 100 realisations the predicted value (mean) and uncertainty were taken (standard deviation).

DEM Uncertainty using Adjacent Maps



2. What is the approximate standard deviation at each point?

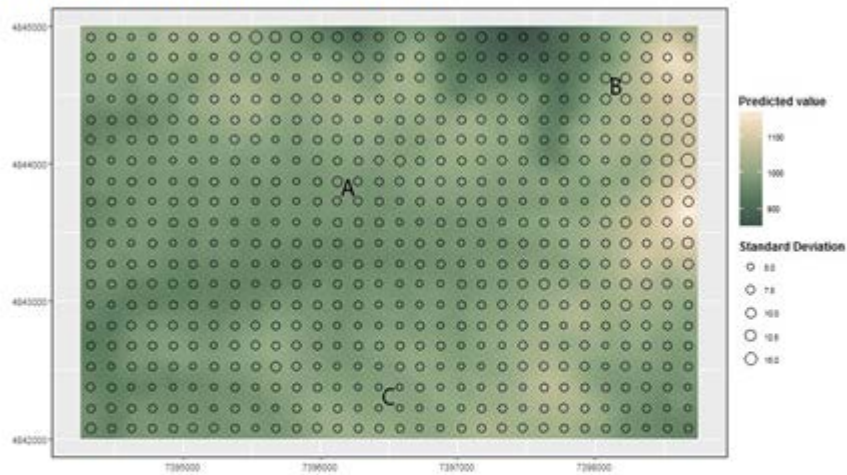
Please use the end of the arrowhead as the reference point.

Mark *only one oval per row*.

	0 - 5	5 - 10	10 - 15	15+
Point A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

I

DEM Uncertainty using Glyphs

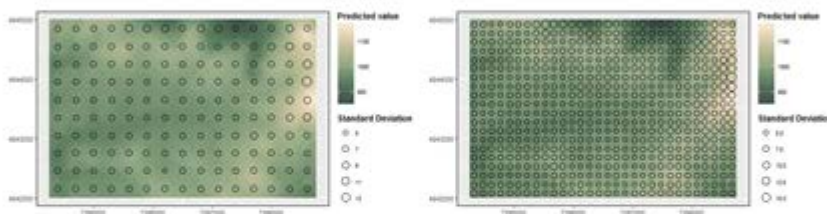


3. What is the approximate standard deviation at each point?

Please use the center of the letter as the reference point

Mark only one oval per row.

	5 - 7.5	7.5 - 10	10 - 12.5	12.5 - 15
Point A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>



From which map is easier to read off uncertainty values? Do you prefer a smaller or larger grid?

Mark only one oval.

Left map

Right map

4. Does the uncertainty visualisation make it harder to read the map?

Mark only one oval.

- Yes
 No

5. Please rank your preferred visualisation methods

1 denotes most preferred method. Each rank may only be chosen once.

Mark only one oval per row.

	1	2
Adjacent Maps	<input type="radio"/>	<input type="radio"/>
Glyphs	<input type="radio"/>	<input type="radio"/>

DEM Uncertainty using Interactive Application

The interactive application for this data set can be found in the second tab in this browser. Please explore the application, then answer the questions below.

6. What percentage of the map has a relative error higher than 1%?

The relative error is calculated as $(\text{standard deviation}/\text{predicted value}) * 100$ and denotes how relative the error is to the predicted value

Mark only one oval.

- 5%
 25%
 50%

7. Select the "Prediction Interval" button. The uncertainty of the data can also be characterised by the interval in which 90% of all cases will fall. This interval is called the interquartile range. Areas with an interquartile range larger than the threshold selected will be greyed-out in the map. At what value of the threshold of the interquartile range is the greyed-out area of the map larger than 50%?

Mark only one oval.

- 20
 25
 30

8. Do the histograms of a selected point give a better understanding of the distribution of the realisations compared to the standard deviation?

Mark only one oval.

- Yes
 No

9. **At what distance are the pixels in areas of high uncertainty still correlated?**

Start this question by displaying the standard deviation map, with no extra information selected. Find an area with a high standard deviation, then single- and double-click on the same point. The scatterplot shows the spatial correlation between the two points. Single-click points around the original point to find the distance at which the points are not strongly correlated anymore.

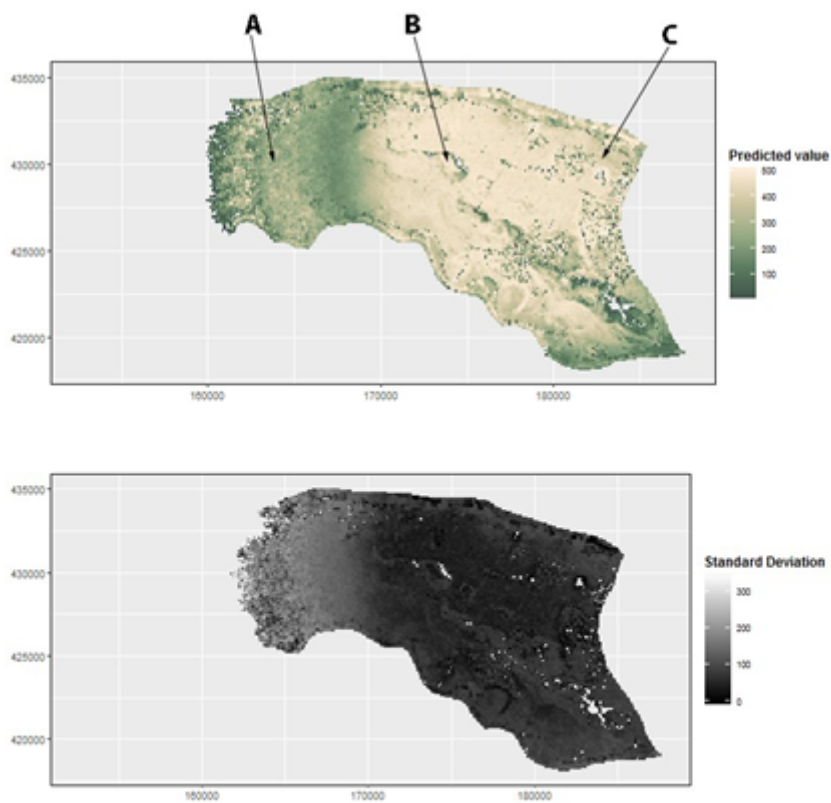
Mark only one oval.

- 5 pixels
- 15 pixels
- 30 pixels

Second Dataset: Flooding Case Maas & Waal

This dataset contains a simulation of a dike breach in an area between the Maas and Waal rivers in The Netherlands. 100 realisations of the flood depth were simulated by running a flood model with uncertain influx of water and an uncertain DEM. Of these realisations the predicted value (mean) and standard deviation were taken.

Flood Uncertainty using Adjacent Maps

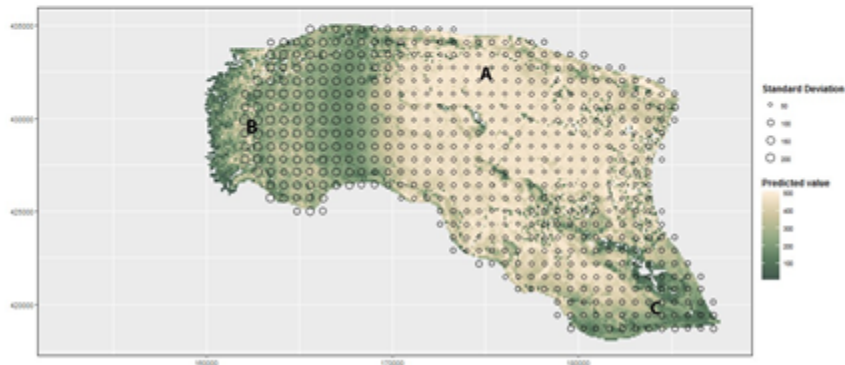


10. What is the approximate standard deviation at each point?

Please use the end of the arrowhead as the reference point
Mark *only one oval per row*.

	0 - 100	100 - 200	200 - 300	300+
Point A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Flood Uncertainty using Glyphs



11. What is the approximate standard deviation at each point?

Please use the center of the letter as the reference point
Mark *only one oval per row*.

	0 - 100	100 - 200	200 - 300	300+
Point A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

12. Please rank your preferred visualisation methods

1 denotes most preferred method. Each rank may only be chosen once.
Mark *only one oval per row*.

	1	2
Adjacent Maps	<input type="radio"/>	<input type="radio"/>
Glyphs	<input type="radio"/>	<input type="radio"/>

Flood Uncertainty using Interactive Application

The interactive application for this data set can be found in the third tab in this browser.
Please explore the application, then answer the questions below.
This dataset is fairly large, so please be patient when waiting for the data to load after moving a slider.

13. Using the single-click to explore the realisations, which areas of the map have the least realisations with values?

All cells have 100 realisations. However, as this was a flooding model, in some modelled floods the cells were not flooded and received an NA value. These are not seen in the histogram.

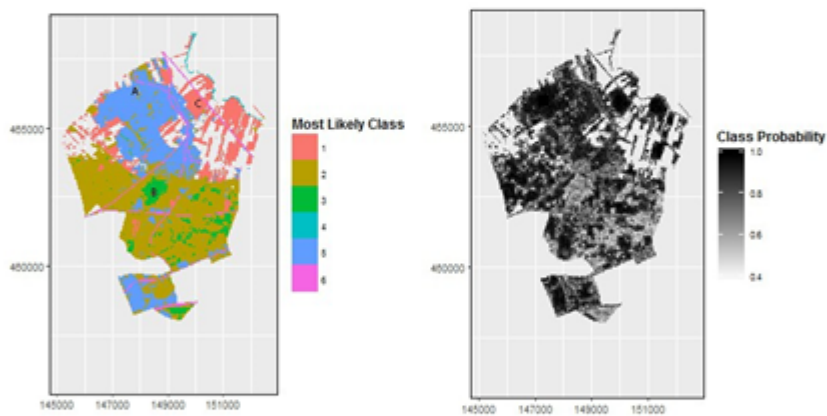
Mark only one oval.

- Left
- Center
- Bottom Right
- Top Right

Third Dataset: Landcover of Soest

This dataset contains simulated landcover for the Dutch municipality of Soest. Using thematic and reference data, 50 realisations were created using a model. Of these realisations the most likely class and its associated probability were taken.

Landcover Probabilities using Adjacent Maps



14. Please rank the letters in the image above from highest class probability to lowest class probability

Mark only one oval.

- A - B - C
- A - C - B
- B - A - C
- B - C - A
- C - A - B
- C - B - A

15. **What is the approximate class probability at each point?**

Mark only one oval per row.

	Low 0.4 - 0.6	Medium 0.6 - 0.8	High 0.8 - 1
Point A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Point C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Landcover Probabilities using Interactive Application

The interactive application for this data set can be found in the fourth tab in this browser. Please explore the application, then answer the questions below.

16. **How confident are you that the pixels assigned class two in the bottom area of the map are actually class two?**

The pixels in question are YELLOW and in the bottom part of the map.

Mark only one oval.

- 95%
- 75%
- 50%
- 25%

Evaluation of preferences

This section aims to evaluate the preferred methods of visualisation and to receive feedback.

17. **Using Adjacent Maps to communicate and visualise uncertainty is easy to comprehend.**

Mark only one oval.

	1	2	3	4	5	
Agree	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Do Not Agree

18. **Using Glyphs to communicate and visualise uncertainty is easy to comprehend.**

Mark only one oval.

	1	2	3	4	5	
Agree	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Do Not Agree

19. **Using an interactive application to communicate and visualise uncertainty is easy to comprehend.**

Mark only one oval.

	1	2	3	4	5	
Agree	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Do Not Agree

20. Do you have any comments/feedback on the adjacent maps method and its implementation?

21. Do you have any comments/feedback on the glyphs method and its implementation?

22. Do you have any comments/feedback on the interactive application method and its implementation?

23. Does an interactive application make understanding the uncertainty easier?
Mark only one oval.
- Yes
- No
- Equal to other representations

Appendix 6: User evaluation survey results

Number	What is the approximate standard deviation at each point? [Point A]	What is the approximate standard deviation at each point? [Point B]	What is the approximate standard deviation at each point? [Point C]	What is the approximate standard deviation at each point? [Point A]	What is the approximate standard deviation at each point? [Point B]	What is the approximate standard deviation at each point? [Point C]
1	0 - 5	5 - 10	10 - 15	7.5 - 10	10 - 12.5	5 - 7.5
2	15+	15+	5 - 10	12.5 - 15	10 - 12.5	5 - 7.5
3	15+	10 - 15	5 - 10	7.5 - 10	10 - 12.5	5 - 7.5
4	0 - 5	10 - 15	5 - 10	7.5 - 10	10 - 12.5	5 - 7.5
5	15+	5 - 10	10 - 15	7.5 - 10	10 - 12.5	5 - 7.5
6	10 - 15	5 - 10	10 - 15	10 - 12.5	10 - 12.5	5 - 7.5
7	15+	10 - 15	5 - 10	10 - 12.5	10 - 12.5	5 - 7.5
8	0 - 5	5 - 10	10 - 15	7.5 - 10	7.5 - 10	5 - 7.5
9	0 - 5	5 - 10	5 - 10	7.5 - 10	7.5 - 10	5 - 7.5
10	5 - 10	15+	5 - 10	7.5 - 10	10 - 12.5	5 - 7.5
11	0 - 5	10 - 15	5 - 10	10 - 12.5	7.5 - 10	5 - 7.5
12	5 - 10	5 - 10	5 - 10	7.5 - 10	7.5 - 10	5 - 7.5

Number	From which map is easier to read off uncertainty values?	Does the uncertainty visualisation make it harder to read the map?	Please rank your preferred visualisation methods [Adjacent Maps]	Please rank your preferred visualisation methods [Glyphs]	What percentage of the map has a relative error higher than 1%?	At what value of the threshold of the interquartile range is the greyed-out area of the map larger than 50%?
1	Right map	Yes	1	2	5%	20
2	Right map	Yes	1	2	5%	20
3	Right map	No	1	2	5%	30
4	Left map	No	2	1	5%	20
5	Right map	Yes	1	2	5%	20
6	Left map	No	2	1	5%	20
7	Left map	Yes	2	1	5%	20
8	Left map	No	2	1	5%	20
9	Right map	No	1	2	5%	20
10	Left map	No	2	1	5%	20
11	Left map	Yes	1	2	5%	20
12	Left map	Yes	1	2	5%	20

Number	Do the histograms of a selected point give a better understanding of the distribution of the realisations compared to	At what distance are the pixels in areas of high uncertainty still correlated?	What is the approximate standard deviation at each point? [Point A]	What is the approximate standard deviation at each point? [Point B]	What is the approximate standard deviation at each point? [Point C]	What is the approximate standard deviation at each point? [Point A]
1	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
2	No	15 pixels	100 - 200	0 - 100	100 - 200	0 - 100
3	Yes	5 pixels	200 - 300	300+	200 - 300	0 - 100
4	No	5 pixels	100 - 200	0 - 100	100 - 200	0 - 100
5	No	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
6	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
7	Yes	15 pixels	100 - 200	0 - 100	0 - 100	0 - 100
8	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
9	Yes	15 pixels	100 - 200	0 - 100	0 - 100	0 - 100
10	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
11	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100
12	Yes	5 pixels	200 - 300	0 - 100	100 - 200	0 - 100

Number	What is the approximate standard deviation at each point? [Point B]	What is the approximate standard deviation at each point? [Point C]	Please rank your preferred visualisation methods [Adjacent Maps]	Please rank your preferred visualisation methods [Glyphs]	Using the single-click to explore the realisations, which areas of the map have the least realisations with values?	Please rank the letters in the image above from highest class probability to lowest class probability
1	200 - 300	100 - 200		2	1 Left	C - A - B
2	200 - 300	100 - 200		2	1 Center	C - A - B
3	200 - 300	100 - 200		1	2 Left	B - C - A
4	200 - 300	100 - 200		2	1 Left	A - B - C
5	200 - 300	100 - 200		2	1 Center	C - B - A
6	100 - 200	0 - 100		2	1 Left	C - A - B
7	200 - 300	100 - 200		2	1 Left	A - C - B
8	200 - 300	100 - 200		2	1 Left	C - A - B
9	200 - 300	100 - 200		1	2 Left	C - A - B
10	200 - 300	100 - 200		2	1 Left	C - B - A
11	200 - 300	100 - 200		1	2 Left	C - B - A
12	200 - 300	100 - 200		1	2 Left	C - A - B

Number	What is the approximate class probability at each point? [Point A]	What is the approximate class probability at each point? [Point B]	What is the approximate class probability at each point? [Point C]	How confident are you that the pixels assigned class two in the bottom area of the map are actually class two?	Using Adjacent Maps to communicate and visualise uncertainty is easy to comprehend.	Using Glyphs to communicate and visualise uncertainty is easy to comprehend.
1	High 0.8 - 1	Medium 0.6 - 0.8	Low 0.4 - 0.6	95%	4	4
2	Medium 0.6 - 0.8	Low 0.4 - 0.6	High 0.8 - 1	75%	1	1
3	High 0.8 - 1	Low 0.4 - 0.6	High 0.8 - 1	75%	4	2
4	Low 0.4 - 0.6	Medium 0.6 - 0.8	High 0.8 - 1	95%	3	1
5	Low 0.4 - 0.6	Medium 0.6 - 0.8	High 0.8 - 1	75%	2	4
6	High 0.8 - 1	Low 0.4 - 0.6	High 0.8 - 1	75%	3	4
7	High 0.8 - 1	Medium 0.6 - 0.8	High 0.8 - 1	75%	2	1
8	Medium 0.6 - 0.8	Low 0.4 - 0.6	High 0.8 - 1	95%	4	2
9	High 0.8 - 1	Low 0.4 - 0.6	High 0.8 - 1	95%	1	2
10	High 0.8 - 1	Low 0.4 - 0.6	High 0.8 - 1	75%	4	1
11	Medium 0.6 - 0.8	Low 0.4 - 0.6	High 0.8 - 1	95%	1	2
12	Medium 0.6 - 0.8	Medium 0.6 - 0.8	High 0.8 - 1	25%	3	5

Number	Using an interactive application to communicate and visualise uncertainty is easy to comprehend.	Does an interactive application make understanding the uncertainty easier?
1		4 Yes
2		1 Yes
3		2 Yes
4		1 Yes
5		3 Equal to other representations
6		5 Yes
7		3 Equal to other representations
8		1 Yes
9		1 Yes
10		2 Yes
11		3 Yes
12		3 Equal to other representations