CALCULATION METHODS OF POTENTIAL EVAPOTRANSPIRATION

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INTRODUCTION

A number of methods have been evolved to calculate the potential evapotranspiration. Some of these methods are based on an empirical correlation with monthly air temperature [THORNTHWAITE (1948), BLANEY (1951)]. MAKKINK (1955) shows that the curve of the monthly values calculated according to THORNTHWAITE agrees with the observed potential evapotranspiration only after application of a correction for a time lag and for the wind velocity. No method based on monthly temperature alone can be expected to give reliable results for different regions [VAN WIJK and DE VRIES (1954)].

PENMAN (1948) has evolved a formula on a basis of sound physical reasoning. Recently MAKKINK (1957b) published a correlation formula with incoming radiation and air temperature. TURC (1954b, 1955) has constructed a correlation formula for evapotranspiration in which he uses rainfall, temperature and radiation. HAUDE (1952, 1954) makes use of an empirical formula principally based on the saturation deficit at 14.00 p.m.

In this study six different methods will be used to calculate the potential evapotranspiration.

SOME GENERAL REMARKS CONCERNING THE DIFFERENT METHODS

a. The water balance method: Experiments based on this principle are performed at Wageningen with weighable monolith lysimeters covered with grass. Six of them were filled with a sandy soil and they had a constant ground water table at 50 cm. below the surface. This is a depth which on sand might be presumed to ensure that the evapotranspiration is not limited. The evapotranspiration can be calculated with the following hydrologic equation:

$$E = P + I - D \pm \Delta W$$

where E = evapotranspiration, P = precipitation, I = infiltration, D = drainage and ΔW = change in water content of the soil-block being considered.

The calculated data of the potential evapotranspiration have been corrected to a grass-length of two centimeters (MAKKINK 1957a).

b. Evapotranspiration from a water surface: The evaporation of water was measured with two evaporation pans (diameter 50 cm., depth 23 cm.). The water surface was kept 3.5 cm. below the rim. The rim of the pans was at the same level as the surface of the soil. They were surrounded by grass cut short. The data have been multiplied by PENMAN's reductionfactor [PENMAN (1948)] to get a value comparable with potential evapotranspiration.

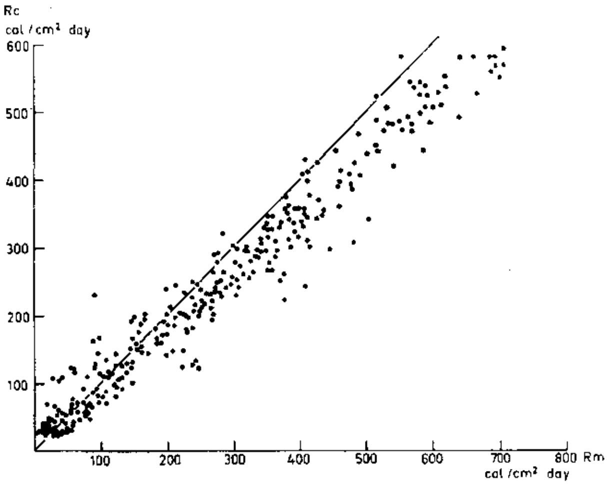

FIG. 1.
Relation between measured incoming radiation (Rm) and calculated incoming radiation (Rc) during 1957 at Wageningen

c. PENMAN's formula: For a full discussion of PENMAN's method of obtaining the potential evapotranspiration from meteorological observations, reference should be made to PENMAN's original papers (1948, 1956). He evolved the following equation to calculate the evaporation of free water[1]):

$$E_o = \frac{\Delta H_o + \gamma E_a}{\Delta + \gamma}$$

Multiplying E_o by an empirical reduction factor gives the potential evapotranspiration (E_p) for a soil covered with a crop. The values of this reduction factor were deduced from experiments at Rothamsted, [PENMAN (1948)].

The ratio $\frac{E_p}{E_o}$ has the following values:

0.6 from November to February
0.7 March, April, September, October
0.8 from May to August

MAKKINK (1957b) found that for some years the calculated incoming radiation (Appendix I) was smaller than the measured incoming radiation. In fig. 1, a comparison has been made between calculated and measured values of incoming radiation during 1957 at Wageningen. From this figure follows also that the calculated value is too small. DE VRIES (1955) suggests that on days with a partly clouded sky, an extra amount of radiation can reach the earth surface round the rim of the clouds. To get an idea of the influence of the discrepancy between the measured relative sunshine and the calculated „effective" relative sunshine a comparison was made between the calculated values of evapotranspiration and the measured evaporation from a pan. The figures 2 and 3 show the results of this. It is clear that the evapotranspiration calculated, with the measured incoming radiation and the calculated relative sunshine, gives the best agreement with the results of the evaporation pan.

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[1]) For the symbols see Appendix I.

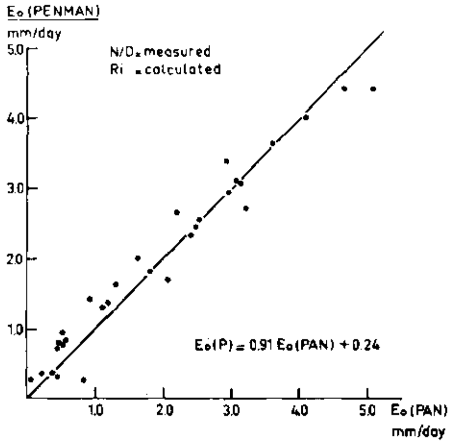

FIG. 2.
Relation between E (Pan) and E_o (PENMAN) calculated with measured relative sunshine (N/D) and calculated incoming radiation (Ri). Average values of 10-day periods

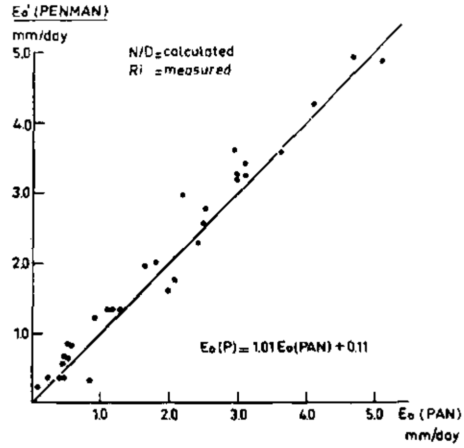

FIG. 3.
Relation between E (Pan) and E_o (PENMAN) calculated with measured incoming radiation (Ri) and calculated relative sunshine (N/D). Average values of 10-day periods

d. MAKKINK's formula: Recently MAKKINK (1957b) published a new formula for calculating the potential evapotranspiration. He based his formula on the measured incoming radiation and temperature. He evolved the equation[1]:

$$E_p = 0.61\, R_m \frac{\Delta}{\Delta + \gamma} - 0.12$$

MAKKINK evolved his formula for average monthly potential evapotranspiration values at Wageningen. In this study the formula was used over periods of 10 days.

e. TURC's formula: TURC (1954b, 1955) has evolved a correlation formula based on rainfall, temperature and radiation. He gives the following equation[2]:

$$E\,(\text{mm/10 days}) = \frac{P + a + V}{\sqrt{1 + \left(\frac{P + a}{L} + \frac{V}{2L}\right)^2}}$$

The small area of the lysimeters, presents difficulties in determining TURC's cropfactor V. He gives for a luxurious growing crop, without shortage of water at any time, a cropfactor V = 70, (1954a). This value has been used here. It must be expected that the calculated values with TURC's formula are somewhat higher than the results of the lysimeters, since these last values were corrected to a grass length of two centimeters.

f. HAUDE's formula: This method is principally based on the saturation deficit at 14.00 p.m. This value is multiplied by a reduction factor. HAUDE (1952) and UHLIG (1954) give the following values for the reduction factor:

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[1]) For the symbols see Appendix II.
[2]) See also Appendix III.

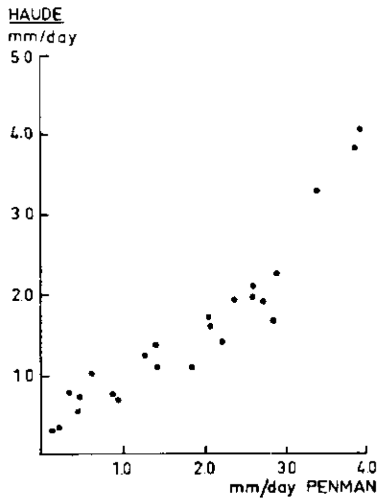

FIG. 4.
Relation between the calculated evapotranspiration with HAUDE's formula and PENMAN's formula at Wageningen. Average values of 10-day periods

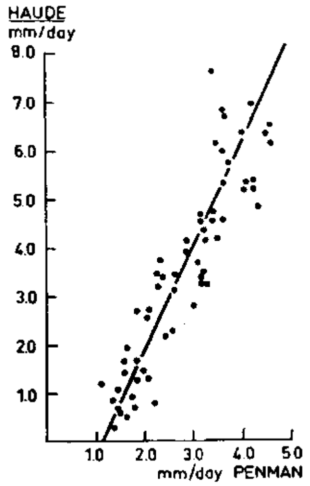

FIG. 5.
Relation between the calculated evapotranspiration with HAUDE's formula and PENMAN's formula after UHLIG (1954) at Bad Kissingen (day values)

0.3 from October to April
0.33 September
0.36 May and August
0.38 June and July

UHLIG (1954) gives the relation between the calculated evapotranspiration after PENMAN and after HAUDE in the equation: E (PENMAN) = 0.44 E (HAUDE) + 1.2. This relation holds true at Bad Kissingen (Germany). At Wageningen the calculation of the evapotranspiration for 10-day periods gives other results. The figures 4 and 5 represent the relation between both methods, respectively at Wageningen and Bad Kissingen. Obviously a great discrepancy exists between the results in both places. A comparison between the calculated values with HAUDE's formula and the results of the lysimeters gives the same discrepancy. Later, HAUDE (1955) corrected his reduction factor for wind-velocity, measured from 11.00 a.m. till 15.00 p.m. at 10 meters height.

A new reduction factor was calculated at Wageningen, consisting of a day-length factor and a wind-velocity factor. The reduction factor can be calculated at Wageningen with the formula:

$$f = \left(1.12\frac{D}{24} - 0.26\right)(0.32\,u_2 + 0.19)$$

where D = day-length in hours
u_2 = average wind-velocity in m/sec at 2 m. height

This equation has been tested for the neighbourhood of Wageningen only, so one should be careful in using it for other regions.

Discussion and conclusion

Potential evapotranspiration has been calculated with the six methods mentioned above. The evapotranspiration was calculated from meteorological data measured at Wageningen during 1957 for the mean values of 10-day periods expressed in mm/day. To get an idea of the similarity of the results each method has been plotted against the average of the six methods (fig. 6 to 11). Table 1 gives the correlation coefficients between the methods.

Table 1

Method	Pan × red. fact.	Penman	Makkink	Turc	Haude	Average of the six methods
Lysimeter	0.96	0.96	0.97	0.94	0.86	0.98
Pan × red. factor . .	–	0.99	0.97	0.96	0.96	0.99
Penman	–	–	0.98	0.98	0.95	0.99
Makkink	–	–	–	0.96	0.94	0.99
Turc	–	–	–	–	0.93	0.98
Haude	–	–	–	–	–	0.94

One may expect to get a fair impression of the error of each method by comparing the results with the average of all six, since the results of the six methods do not differ very much from each other. One must assume in this case that, though the results of the different methods have been correlated, there does not exist any correlation between the errors of the methods. It may therefore be expected that the average of the methods has a very small error relative to those of each method alone. Assuming that this average is without error, makes it possible to calculate the variance of each method (viz. table 2, column 2).

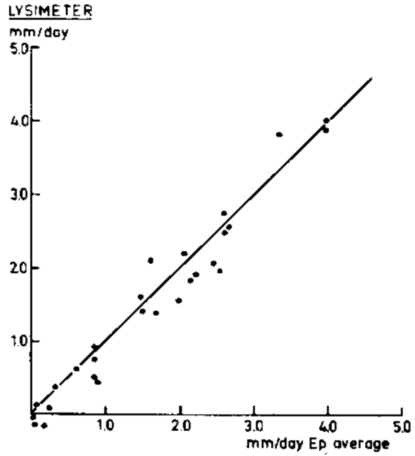

Fig. 6.

Relation between the average potential evapotranspiration of six methods and the evapotranspiration of the lysimeter. Average values of 10-day periods

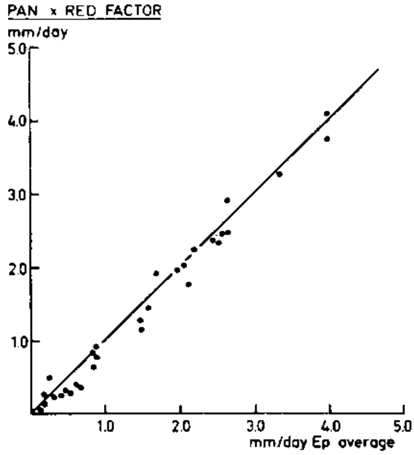

Fig. 7.

Relation between the average potential evapotranspiration of six methods and the measured pan evaporation multiplied by a reduction factor. Average values of 10-day periods

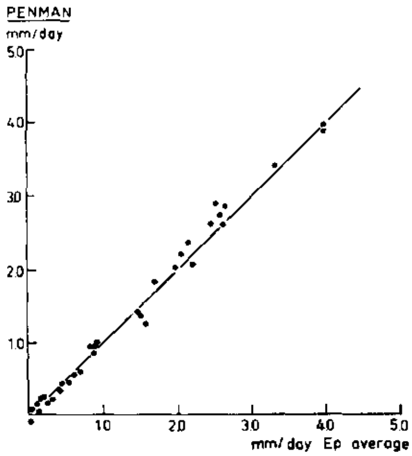

FIG. 8.

Relation between the average potential evapotranspiration of six methods and the calculated evapotranspiration with PENMAN's formula. Average values of 10-day periods

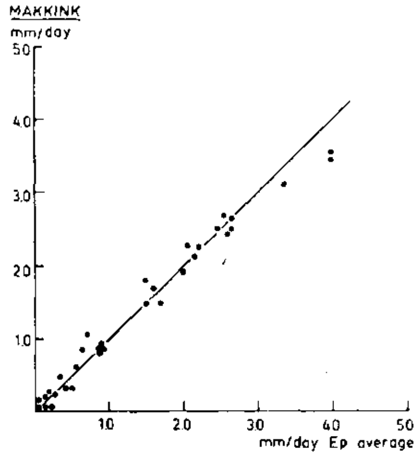

FIG. 9.

Relation between the average potential evapotranspiration of six methods and the calculated evapotranspiration with MAKKINK's formula. Average values of 10-day periods

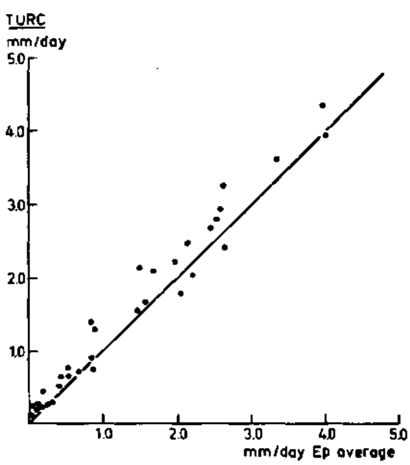

FIG. 10.

Relation between the average potential evapotranspiration of six methods and the calculated evapotranspiration with TURC's formula. Average values of 10-day periods

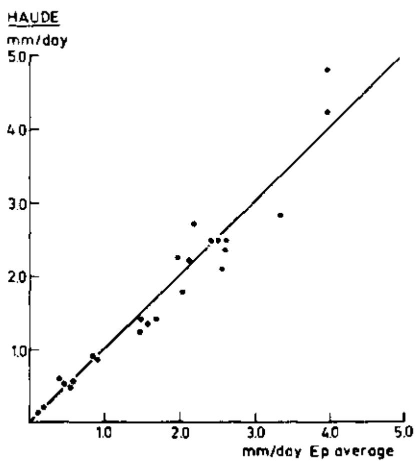

FIG. 11.

Relation between the average potential evapotranspiration of six methods and the calculated evapotranspiration with HAUDE's formula. Average values of 10-day periods

The data of the direct measurements of the lysimeters and the evaporation pan are independent of the data from the indirect methods. The data calculated with these formulae almost certainly will not show a mutual independence of the errors, since the same meteorological data such as radiation, temperature and relative humidity were used. It is possible therefore to combine three methods that have mutually independent errors by comparing the results of the lysimeters, the pan and one of the indirect methods. A combination of three independent methods offers a possibility to calculate the error without the help of repeats. VISSER (1958) gives a formula to calculate the error of results from a comparison of three methods (x, y and z) having errors with a zero mutual correlation. He evolved the equation

$$\sigma_x{}^2 = \overline{uu}\left(1 - \frac{r_{uv}\, r_{uw}}{r_{vw}}\right)$$

where σ_x = error in method x
u = $x - \bar{x}$; $v = y - \bar{y}$; $w = z - \bar{z}$
r_{uv} = correlation coefficient between x and y
r_{uw} = correlation coefficient between x and z
r_{vw} = correlation coefficient between y and z

The error of each method calculated with this formula is shown in table 2, collumn 3.

TABLE 2

Method	Error calculated with the average of the six methods in mm/10 days	Error calculated with the formula $\sigma_x{}^2 = \overline{uu}\left(1 - \frac{r_{uv} r_{uw}}{r_{vw}}\right)$ in mm/10 days
Lysimeter	2.7	2.8
Pan × red. fact. .	1.5	1.9
Penman	2.5	2.7
Makkink	2.0	2.1
Turc	2.2	3.2
Haude	2.9	4.2

It appears that the methods have about the same degree of accuracy. The error in the results of the calculations after HAUDE seems to be somewhat higher. A careful handling of HAUDE's formula is necessary, since the given formula of the reduction factor only holds true at Wageningen. In other regions a new reduction factor will have to be determined. But where the method of HAUDE is properly controlled, it will give a fair estimate of the evapotranspiration and it can be a valuable aid to hydraulic calculations.

It is possible to calculate the potential evapotranspiration from meteorological data after PENMAN, MAKKINK and TURC with the same accuracy as is obtainable with lysimeters and evaporation pans. The reduction factor of PENMAN holds true only for the potential evapotranspiration of a short crop. TURC's formula is the only one which gives some possibilities to calculate also the actual evapotranspiration.

Regarding the lack of knowledge concerning the reduction factor, the accuracy of the results for the potential evapotranspiration will be no restricting factor in this field of science. One may therefore assume, that further progress will not result from an increased accuracy of these modern evapotranspiration formulae but from a further study of the influence of environmental conditions on the reduction factor.

Summary

Values of potential evapotranspiration, calculated with the formulae of PENMAN, MAKKINK, TURC and HAUDE, have been compared with measured values from a pan and from lysimeters covered with grass.

In the calculations with PENMAN's formula, calculated values of the relative sunshine have been used. There is a good agreement for 10-day periods between E_o calculated after PENMAN and the evaporation of the pan.

HAUDE and UHLIG give coefficients, with which HAUDE's formula must be multiplied to get values of the potential evapotranspiration. These coefficients do not give correct results at Wageningen. A new reduction factor was calculated, consisting of a day-length factor and a wind-velocity factor. It is possible to calculate the potential evapotranspiration with the formulae of PENMAN, MAKKINK and TURC with the same degree of accuracy as is obtainable with lysimeters or evaporation pans. HAUDE's formula gives results that seem to be somewhat less accurate.

Résumé

Méthodes pour calculer l'évapotranspiration potentielle

On a comparé entr'eux l'évapotranspiration potentielle calculée au moyen des formules de PENMAN, MAKKINK, TURC et de HAUDE et des valeurs mesurées d'une cuve évaporimétrique et de lysimètres herbus. Pour le calcul effectué au moyen de la formule de PENMAN on s'est servi de valeurs calculées pour l'insolation relative.

Pour les moyennes de décades il existe un rapport assez étroit entre le E_0 calculé selon PENMAN et les valeurs mesurées de la cuve evaporimétrique.

Le facteur de réduction donné par HAUDE et UHLIG pour l'application de la formule de HAUDE donne pour Wageningen de fausses valeurs d'évapotranspiration. A Wageningen on a calculé un nouveau facteur de réduction se composant d'un facteur de longueur de journée et d'un facteur de la vitesse du vent.

Il est possible de calculer avec autant d'exactitude l'évapotranspiration en se servant des formules de PENMAN, MAKKINK et TURC, qu'au moyen de cuves évaporimétriques et de lysimètres. Il se peut que la formule de HAUDE ait des résultats quelque peu moins précis. Pour cette dernière méthode il y a en outre la difficulté que le facteur de réduction doit etre fixé dans chaque région.

Zusammenfassung

Berechnungsmethoden der potentiellen Evapotranspiration

Die potentielle Evapotranspiration wie diese berechnet wird mit den Formeln von PENMAN, MAKKINK, TURC und HAUDE wurde mit den Werte gemessen an einer Wasserschale und an Lysimetern mit Gras verglichen.

Bei der Berechnung mit der PENMANschen Formel sind berechnete Werte für den Sonnenscheindauer gebraucht. Es gibt eine gute Zusammenhang für Dekaden zwischen dem berechneten E_0 nach PENMAN und die gemessenen Werte an einer Wasserschale.

HAUDE und UHLIG geben Koeffizienten womit der HAUDEschen Formel multipliziert werden soll. Das Gebrauch dieser Koeffizienten zu Wageningen verursachte abweichende Verdunstungswerte. Deshalb sind zu Wageningen mit Hilfe der Tageslänge und Windgeschwindigkeit neue Koeffizienten berechnet worden.

Es ist möglich mit meteorologischen Angaben, unter Verwendung der Formeln von PENMAN, MAKKINK und TURC, die potentielle Evapotranspiration mit einem selben Grad von Genauigkeit zu berechnen wie diese gemessen wird an einer Wasserschale oder an Lysimetern. Die HAUDEsche Formel gibt möglich etwas weniger genaue Erfolge. Ausserdem sollen für diese Methode die Reduktionskoeffizienten in jedem Gebiete bestimmt werden.

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APPENDIX I: PENMAN's formula

$$E_o = \frac{\Delta H_o + \gamma E_a}{\Delta + \gamma}$$

where: E_o = evaporation from a water surface in mm/day
H_o = net gain in radiation-energy per unit of surface
Δ = slope of temperature – vapor-pressure curve
γ = psychrometer constant = 0.49 mm. Hg/degree centigrade
E_a = $0.35\,(0.5 + 0.54\,u_2)\,(e_a - e_d)$
u_2 = wind-velocity at 2 m. height in m/sec
e_a = saturation vapor-pressure at air temperature
e_d = saturation vapor-pressure at dew point

The incoming radiation can be calculated with the formula

$$R_i = \left(0.29 + 0.71\,\frac{n}{D}\right) Q$$

where: R_i = incoming radiation per unit of surface
$\frac{n}{D}$ = relative sunshine
Q = incoming radiation on totally clear days

DE VRIES (1955) gives day-values of Q at Wageningen.

APPENDIX II: MAKKINK's formula

$$E_p = 0.61\,R_m \frac{\Delta}{\Delta + \gamma} - 0.12$$

where: E_p = potential evapotranspiration
R_m = measured incoming radiation in mm/day
Δ = slope of temperature – vapor-pressure curve
γ = psychrometer constant = 0.49 mm. Hg/degree centigrade.

APPENDIX III: TURC's formula

$$E \text{ (mm/10 days)} = \frac{P + a + V}{\sqrt{1 + \left(\frac{P + a}{L} + \frac{V}{2L}\right)^2}}$$

where: P = precipitation in mm/10 days

a = soil factor. This factor can be calculated as follows: $a = 35 - \Delta$ with a maximum value $a = 10$ and a minimum value $a = 1$

Δ = deficit of soil moisture at the beginning of the ten-day period that must be calculated

L = $1/16\,(t + 2)\,\sqrt{R}$

t = average air temperature during the period

R = average incoming radiation in cal/cm² per day

TURC states that if $L < 10$ the crop factor $V = 0$

The crop factor V can be calculated as follows:

$$V = 25\sqrt{\frac{Mc}{Z}} \text{ or}$$

$$V = \left(\Delta_o + 30 + 1.5\,Mc\,\frac{z}{Z}\right) - \Delta$$

where: M = production of dry matter in 100 kg/ha

Z = length of growing season in periods of 10 days

z = number of the period considered

c = crop constant with the following values

$^4/_6$ corn and beets

$^5/_6$ potatoes

$^6/_6$ cereals, carrots, flax

$^7/_6$ beans, clover, other leguminous plants

$^8/_6$ lucerne and grass

Δ_o = deficit of soil moisture at the beginning of the period.

The smallest of the two calculated V-values should be used.

The growing season begins twenty days after the sowing date.

DISCUSSION

J. M. LYSHEDE:

It is quite certain that the formula of PENMAN is too simple, but I feel sure that a universal formula must be found by physical and climatological considerations sooner than by means of statistics.

Answer: (W. C. VISSER)

Physics has to present new functions, statistics will have to prove and eventually reject them and will be able to hint to better approaches. The cooperation between the two fields of science will not be difficult. I quite agree that the qualitative accuracy – taking into account all factors concerned – will be of more importance than an increase in the quantitative accuracy. As regards statistics, we will have to select with preference those methods that are free of a presumed functional relation.