# A STATISTICAL ANALYSIS OF THE <br> DIFFERENCES BETWEEN PRECIPITATION AND EVAPORATION IN THE NETHERLANDS 

Рн. Тн. Stol<br>(Instituut voor Cultuurtechniek en Waterhuishouding, Wageningen, Nederland)

## Introduction

In elaborating problems in water management, data on the frequency distribution of precipitation are often used. In that case only a part of the climatic factors, affecting problems in hydraulic engineering are taken into account however.

A more complete description of the climate in a water balance is obtained by evaluating the evaporation as well. In the same way as up to now has been done for the rainfall alone, the difference between rainfall and evaporation can be integrated in a frequency research. It will then be possible to determine with greater precision the frequency of the precipitation (respectively evaporation) surplus.

In the Netherlands such an inquiry has been set up for one of the southern provinces, in which use was made of the 10-days total of the measured precipitation $(\mathrm{R})$ and the evaporation $\left(\mathrm{E}_{0}\right)$, which was determined with Penman's formula as a starting-point. The difference between precipitation and evaporation was calculated according the formula ( $\mathrm{R}-\mathrm{xE}_{\mathrm{o}}$ ), in which x is the reduction factor for $E_{0}$.

As variable factors, the following were included in this inquiry:

1. the months of the year;
2. periods of $10,20,30,60,90,120$ and 180 consecutive days respectively, calculated from the first of each month;
3. the quantity ( $\mathrm{R}-\mathrm{xE}_{\mathrm{o}}$ ) in mm . from the years 1933 to 1953 inclusive, for the periods mentioned sub 2 with a value of $x$ of $0.25,0.40,0.55,0.70,0.85$ and 1.00 respectively;
4. the cumulated relative frequencies of occurrence of the quantities sub 3.

## Analysis of elaboration possibilities

The factors which were included in the inquiry gave rise to the making and compilation of a great number of diagrams, each of which elucidates a definite facet of the relation between the data. On the axes of a system of coordinates only two variables can be plotted, but in the plane of the coordinates different curves can be drawn for a third variable. The diagram obtained in this way is valid for a specific value of a fourth variable.

Working in this manner, the number of potential permutations for every reduction factor is $4!=24$. Since there will always be a preference for the factors on the ordinate or on the abscissa and since changing the axes yields


Fig. 1.
The monthly totals of $\mathrm{R}-0.7 \mathrm{E}$ for the month April over the years 1933 to 1953 are given as summated relative frequencies. For R -$-0.7 \mathrm{E}_{o}>0$ there is a precipitation surplus, for $R-0.7 E_{0}<0$ the absolute value gives the evaporation surplus
no essentially new aspect, half the number of permutations mentioned, will be sufficient however.
In the elaboration the elementary frequency-polygon of which an example is given in figure 1 has been taken as a starting-point. On a normal probability scale the number of mm . evaporation surplus ( $\mathrm{R}-0.70 \mathrm{E}$ ) has been plotted for a period of 30 days from April Ist.
For the month of February it is shown in figure 2, how the curves for the different periods may be integrated into one figure.
A certain facet of the four-dimensional relation may be inferred from this stereogram for example by application of horizontal sectioning. This has in figure 2 as a result that contour lines (polygons) arise for a probability with a constant level. These polygons may be projected in a graph, which then gives the relation between the number of mm . evaporation surplus and the period length.

It is possible to make vertical sections through the stereogram in the same way (figure 3), which gives polygons for a constant $\mathrm{R}-0.7 \mathrm{E}_{\mathrm{o}}$ In that case the variables on the axes are the probability and the length of the period.

Instead of the absolute magnitude of the evaporation surplus, the values expressed in mm. in 24 hours can be used in many cases as well. Even though by doing so no new factors are introduced in this investigation, this way of representation may be illuminating in some cases.

## Applications of the frequency curves

The broken lines of the frequency polygons (see figure 1) were straightened out by means of flowing curves for which the technique of the intersection-check was adopted. In the procedure followed, it was taken as being essential that local corrections on the original polygon would not result in irregularities in one of the sections. The complete four dimensional model is included in the intersection-check by means of this operation.

The graphs obtained with this method may serve to solve certain problems on the frequency of occurrence of an evaporation- and precipitation surplus. Before applying the graphs the magnitude of the reduction factor to be used in each special case has to be chosen. Furthermore the order of magnitude should


Fig. 2. Three horizontal cross-sections for a 5,50 and $95 \%$ surplus probability respectively, through a collection of frequency polygons on several periods from February 1st onward (Fig. 2 and fig. 3 are both models for February)


Fig. 3. Three vertical cross-sections for a 0,20 and 50 mm . rainfall surplus respectively, through the same collection of frequency polygons as shown in fig. 2


Probobility in \%
be known of the maximum moisture deficit which is admissible for the profile and the same applies for the moisture capacity. Besides this, it will be of importance to know what underground drainage- and supply possibilities can occur. The graphs now make it possible to establish a connection between these quantities with the frequencies of the climatic factors e.g., the precipitation and the evaporation. When making improvement plans on land and water management, an appreciation can be obtained about the combination of possibilities which from a technical point of view as well as from an economic one are best to be realized and which at the same time will approach the agricultural desiderata the closest.

In the following only examples of the treatment with the reduction factor $\mathrm{x}=0.7$ will be given.

## Application on the problem of the water supply

By introducing the moisture capacity of a soil profile, when for the latter is asked what probability there exists of desiccation after a certain amount of time, that series diagrams can be selected in which the probability and the period length are taken as coordinates and each curve in the graphs represents a constant amount of moisture.

Figure 4 gives an example of this manner of representation. When it is assumed that in a profile a quantity of moisture, which the crop is allowed to use, of 120 mm . is available as a supply on April 1st, the profile can compensate an evaporation surplus of the same size without water supply from sprinkling or infiltration. Now, in figure 4 the 120 mm .-curve can be followed and it appears that in 5 out of a hundred years, or once in 20 years, an evaporation surplus of more than these 120 mm . will occur in approximately 70 consecutive days. The longer a period chosen, the larger the probability of the appearance of an evaporation surplus greater than in the case mentioned. The critical position lies at 140 days or almost 5 months, namely the period of April 1st up to and including July with 20 days in August. So in somewhat more than 32 years out of a 100 years or once in 3 years, the profile will be uncapable of supplying the required volume of moisture. In that case more than 1 mm . per day will have evaporated.

Calculated from July 1st, the probabilities of desiccation of the profiles

Fig. 5.
As fig. 4, but now calculated from July 1st

mentioned are much larger. If it is checked how the condition is from July 1st, the quantity already evaporated should be deducted in the first place. There is no saying how large this quantity is, but it may be inferred from figure 4 that from April up to and including June, a median evaporation surplus of 65 mm . will appear. In the above mentioned case the profile chosen as an example will have still available $120-65=55 \mathrm{~mm}$. moisture on July 1st. In figure 5 the probability of an evaporation surplus larger than the latter volume of moisture is shown, the probability being largest after well over 50 days and then amounting to $30 \%$.

This example shows how the course of the evaporation can be followed with the aid of the required diagrams for the specific months.

In the above, the attention has been drawn to the probabilities with which moisture deficits occur. The question of the possibility to cope with these deficits now presents itself.

Without going into the technical aspect of this problem, the frequencies in which the occurring evaporation will be in excess of the volume of water supplied, will be given attention.

An easy way of rendering this problem is a diagram in which on the ordinate $\mathrm{R}-0.7 \mathrm{E}_{\mathrm{o}}$ is projected and on the abscissa the period-length. It is now possible to draw lines in such a diagram that represent a constant mean supply in mm. per 24 hours and to establish a relation between them and the frequencies of the evaporation surplus.

A similar representation is given in figure 6. It can be concluded from this graph that with a probability of $25 \%$, or once in 4 years, an evaporation surplus may be expected of more than 25 mm . for a period of 30 days, starting on April 1st; for 60 days this is more than 55 mm . and for 90 days approximately 100 mm .

The advantage of working on water supply problems with a graph as given in figure 6 can be seen from the following example. If there exists a possibility of supplying from April 1st an amount of water with a maximum of 1 mm . per 24 hours then, over 2 months such an amount of water will be provided, that merely once in 4 years ( $25 \%$ probability) an evaporation will appear that will

CURVES FOR EQUAL PROBABILITIES, GEMERT R-07E, APRIL


Fig. 6.
A relation is given for various surplus probabilities (the figures with each curve) between evaporation surpluses and lengths of periods. The dotted line agrees with a supply of 1 mm . per 24 hours
exceed this quantity. After 90 days however, a moisture deficit will be of more frequent occurrence. Once in 4 years this deficit will be at least 12 mm ., that can be determined by measuring in figure 6 the vertical distance between the dotted line and the $25 \%$ curve at 90 days.

The interpolating of the value of the probabilities between the lines in figure 6 can not always be done with adequate accurracy. Also, when taking into consideration the effects of irrigation, corresponding with for instance $\frac{1}{2}, 1$, $1 \frac{1}{2}, 2$, etc., mm. per 24 hours, two families of curves have to be compared with each other. This becomes easier if one, starting with figure 4, constructs a new diagram which will show curves of equal quantities, expressed in mm. per 24 hours. This new diagram (figure 7) gives an other kind of representation of the same information that can be obtained from figure 6.

If it is determined how the situation is when calculated from July 1st, one obtains figure 8.

## Fig. 7.

For evaporation surpluses in $\mathrm{mm} /$ 24 hours (the figures at the right) the surplus probability over periods of 10 to 180 days is given, calculated from April 1st


Fig. 8.
As fig. 7, but calculated from July 1st


An evaporation surplus with an average of 1 mm . per 24 hours for the month of July is exceeded in approximately $45 \%$ of the number of cases, but calculated for 2 months it is less than $30 \%$. In this diagram is expressed the fact, that in the Netherlands the months of August and September are among those which are richest in precipitation while the evaporation is already decreasing in this period. Calculated from July 1st, precipitation surplus occurs after 90 days with a probability of already $50 \%$.

## Applications on drainage problems

Following the diagram on July, the series of diagrams can be continued with the corresponding one of the month of October (figure 9). This graph, concerning the winter months, indicates that now mainly the problem of the precipitation surplus appears. For the month of October there is a probability of $35 \%$ of such a surplus of more than 1 mm . per 24 hours, but this probability will reach a maximum of $80 \%$ after somewhat more than 90 days. After this period the surplus lessens because now months with a stronger evaporation are involved in the summation, months in which at the same time the volume of precipitation is decreasing.
CURVES FOR EQUAL OUANTITIES, GEMERT R-O,7E,OCT.
mm PER 24 HOURS


Fig. 9.
As fig. 7, but for precipitation surpluses, calculated from October 1st

From the diagrams it can be determined how large the amounts of precipitation surplus to be expected are going to be and have to be discharged by artificial means in low lying polder districts. In order to be able to drain away in October, in 10 days time the precipitation surplus occurring in this period with a probability of $20 \%$ (consequently once in 5 years), a drainage discharge of 2 mm . per 24 hours would be necessary. That, as a rule, this rate of discharge is taken much higher (viz.: well over 11 mm . per 24 hours) ensues from three facts.

In the first place the surplus probability is frequently taken much smaller than $20 \%$; the occurrence of a condition that a drainage system is unable to cope with the quantities of precipitation surplus is, as a rule, considered permissable only with a much smaller frequency than once in 5 years. According the agricultural investment in a certain area, this may vary from once in 10 or 20 or even more years.

In the second place the demand to be able to discharge the precipitation surplus of 10 days during that period is not put strong enough. Very often there will be the desire of removing the precipitation surplus within already 2 or 3 days. The circumstance that one has to work with at least 10 -days periods when calculating the evaporation to obtain the required accuracy, is the cause that extreme cases of precipitation surplus over a very short period of time do not show to a sufficient advantage in the given diagrams.

In the third case it should be pointed out that in the months November and December, with an equal probability and for the same-short - periods, a larger surplus may occur.

## Summary

The differences between precipitation and evaporation determined per 10-day period over 20 years provided the basis for a statistical analysis of the occurrence of precipitation and evaporation surplus.
In the inquiry, the months of the year, different lengths of periods, the quantity ( $\mathrm{R}-\mathrm{xE}_{\mathrm{o}}$ ) and the cumulated relative frequencies were taken as variables.
It is possible to give the relation for each reduction factor $x$ between this number of variables in 12 different ways. In the diagrams obtained by means of this procedure each time light is cast on another aspect of this relation. It is possible to make suitable selection from the diagrams available both for problems relating to the water supply in periods with an evaporation surplus as well as for problems in which the precipitation surplus is studied. The problems presented can be elucidated and studied now in the most appropriate manner.

[^0]
## Zusammenfassung

Eine Statistische Analyse von Niederschlags- und Verdunstungsüberschüssen in den Niederlanden

Die Unterschiede zwischen Niederschlägen und Verdunstung, pro Dekade über zwanzig Jahre bestimmt, ergaben die Grundlage für eine statistische Analyse des Auftretens von Niederschlags- und Verdunstungsüberschüssen.

Die variabelen Einheiten bei der Untersuchung waren die Monate des Jahres, die Längen verschiedener Perioden, die Menge ( $\mathrm{R}-\mathrm{xE}_{\mathrm{o}}$ ) in Millimetern und die summierten relativen Frequenzen.
Es hat sich als möglich herausgestellt den Zusammenhang zwischen diesen variabelen Einheiten für jeden Reduktionsfaktor auf 12 verschiedene Weisen darzustellen. In den dabei erzielten Figuren wird immer ein anderer Aspekt dieses Zusammenhanges beleuchtet. Es ist möglich, sowohl für Probleme, die die Zufuhr von Wasser in Perioden mit einem Verdunstungsüberschuss betreffen als für Probleme, in denen der Niederschlagsüberschuss studiert wird, eine geeignete Wahl aus den vorhandenen Figuren zu treffen, wodurch das fragliche Problem auf möglichst einfache Weise erklärt und studiert werden kann.


[^0]:    Résumé
    Une analyse statistique de la différence entre les précipitations et l'évaporation aux Pays-Bas

    Les différences entre les précipitations et l'évaporation, déterminées par décade sur une période de vingt années, ont fourni les bases d'une analyse statistique de la fréquence d'exces de précipitations et d'évaporation.

    L'analyse porte sur les variables suivantes: mois de l'année, périodes de différente longueur, la quantité ( $\mathrm{R}-\mathrm{xE} \mathrm{E}_{\mathrm{o}}$ ) en mm et les fréquences relatives additionnées.
    Il s'est trouvé possible de reproduire le rapport entre ce nombre de variables de 12 manières différentes pour chaque facteur de réduction. Les figures obtenues de cette façon illustrent chacune un aspect différent de ce rapport. Il est possible de faire un choix approprié parmi ces figures tant pour les problèmes relatifs à l'amenée d'eau dans les périodes à excès d'évaporation que pour les problèmes relatifs aux excès de précipitations, de sorte que les problèmes à résoudre peuvent être exposés et étudiés le plus simplement possible.

