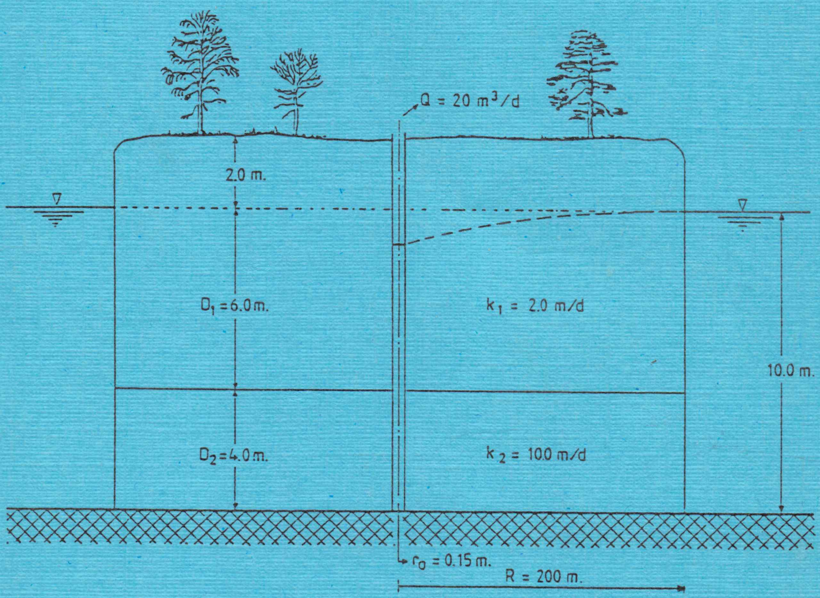
Department of Water Resources
Vakgroep Waterhuishouding

Introduction to Hydrology, exercises
Vraagstukken bij het college Inleiding Hydrologie

K150-001
K150-002



Landbouwuniversiteit Wageningen

06140108

VRAAGSTUKKEN BIJ HET COLLEGE
INLEIDING HYDROLOGIE

INTRODUCTION TO HYDROLOGY, EXERCISES

Department of Water Resources

Vakgroep Waterhuishouding

Printing history

First edition	*(1990)*	*300 copies*
Second edition	*(1992)*	*300 copies*
Reprint	*(1994)*	*100 copies*
Third edition	*(1995)*	*100 copies*
Reprint	*(1995)*	*70 copies*

CONTENTS

0 PREFACE

The exercises in this book should be regarded as an addition to the lecture notes on Hydrology. They constitute an important learning tool when preparing for the exam of course K150-001 or -002: *Introduction to Hydrology*.

The purpose of this collection is to give the student an opportunity to test his knowledge, as well as his ability to solve problems. Many questions are therefore provided with a complete solution. However they should not be consulted before trying a solution yourself.

I thank mr. B. Nijssen for his careful preparation of this text. If nonetheless any questions remain we hope they will be brought to our attention.

Wageningen, April 1990 R.W.R. Koopmans

In this new edition we have added a number of questions, while at the same time removing others which are no longer thought to be relevant as preparatory material for the exam.
Moreover at the end of each chapter you find a number of (Dutch) exam problems for which only the answers have been given.

Wageningen, December 1992 R.W.R. Koopmans

The symbols in this text have been brought in conformity with the new (1995) edition of the Dutch lecture notes.
Again some (Dutch) exam problems have been added.

Wageningen, February 1995 R.W.R. Koopmans

1 INTRODUCTION

Exercise 1.1

A polder (2000 ha) receives seepage flow (groundwater) from the surroundings at a net annual rate of 220 mm.
The average precipitation is 790 mm per year. Water is removed from the polder through a pumping station at an annual rate of 10 million m^3.
a. Compute the average evaporation in mm per year.

Exercise 1.2

Compute the average rainfall on earth, on the land surface and the on oceans, using the data given in figure 1.2-1 of the (Dutch) lecture notes and assuming that 70% of the surface of the earth is covered by oceans and seas.
Hint: consider the earth to be a sphere with a radius of 6400 km.

Answer to exercise 1.1

The annual evaporation follows directly from the waterbalance:

Precipitation + Seepageflow (in) = Discharge pump + Evaporation

We assume that the change in storage equals zero, when using average yearly values:

$$0.790 + 0.220 = \frac{10\ 10^6}{2.10^7} + E$$

$$E = 1.010 - 0.5 = 0.510\ m\ (= 510\ mm)$$

Answer to exercise 1.2

The total rainfall amounts to (107 + 398) 10^{12} m^3/year.

The surface of the earth is: $4\ \pi\ (64\ 10^5)^2 \approx 514.7\ 10^{12}\ m^2$.

Average yearly rainfall = 981 mm.

On the landsurface: $107/(0.3 \cdot 514.7) \approx 693$ mm.

In the oceans: $398/(0.7 \cdot 514.7) \approx 1105$ mm.

Examenvraagstukken bij hoofdstuk 1

1.1 (december 1991)

In een gebied met een warm, droog klimaat ligt een groot meer, waarin een rivier (afkomstig uit een natter gebied) uitmondt. Het meer heeft geen afvoer en de rivier voert jaarlijks gemiddeld 4.10^{10} m^3 water aan. Men wil 400 km bovenstrooms van het meer uit deze rivier één miljoen hectare bevloeien. Bij deze bevloeiïng wordt per jaar 1000 mm water per ha gebruikt (voornamelijk voor verdamping van de gewassen). In het droge klimaat waar zich het meer bevindt valt per jaar slechts 200 mm neerslag. De open water verdamping E_o kan men stellen op 1800 mm per jaar.

a. Bereken het huidige gemiddelde oppervlak van het meer in km^2.
b. Idem na voltooiing van de bevloeiingswerken.
c. Wat zal er gebeuren met het meerpeil na aanvang van de bevloeiing? Komt het meer geheel droog te staan?
d. Idem als het meer een diepe bak is met een constante oppervlakte bij ieder peil (gelijk aan uw antwoord van vraag a.) en 30 m diep? Komt het meer geheel droog te staan? Zo ja, na hoeveel jaar?

1.2 (augustus 1994)

Over de wereld-waterbalans worden de volgende gegevens verstrekt (in afgeronde getallen):

	oppervlakte in 10^6 km^2	neerslag in 10^3 km^3 per jaar
aarde	510	505
zeeën	375	398
humide streken (land opp.)	90	79
aride streken (land opp.)	45	28

waterdamptransport in 10^3 km^3 per jaar:	
van zee naar land	46
van land naar zee	10

a. Bereken, in eenheden van 10^3 km^3 per jaar, de gemiddelde verdamping van:

- de aarde als geheel
- de zeeën
- de humide streken (met afvoer)
- de aride streken (zonder afvoer)

b. Reken deze gegevens om tot mm waterschijf, gerekend over de betreffende oppervlakte. Geef aan of de getallen betrekking hebben op E_o, E_{pot} of E_{act}.

c. De zeeën bevatten 1400 10^{18} kg water. Bereken de gemiddelde verblijftijd van een waterdeeltje in zee (in jaren).

Examenvraagstukken bij hoofdstuk 2

2.1 (augustus 1992)

Geef een definitie van de volgende begrippen:

a. stroomgebied

b. afvoerverlooplijn

3 PRECIPITATION

Exercise 3.1

The department of Water Resources studies rainfall-runoff relationships and the waterbalance in a small catchment area (The Hupselse Beek catchment), which is situated in the Achterhoek, an area in the east of The Netherlands. In this and several other exercises data from this catchment are used. In the first exercise areal rainfall will be calculated.

To determine the average precipitation of an area the Thiessen polygons will be constructed for 7 stations (see figure 3.1.1). First the method for constructing the polygons will be explained.

The construction of so called Thiessen polygons is based on the assumption that the observations made at one particular point can be attributed to all points that are closer to the considered point of measurement (PM) than to any other PM where similar observations are or have been made simultaneously. Therefore we make use of the property that the perpendicular bisector (PB) of two points L and R is the locus of all points located at equal distance to both points. Hence all points left of the PB are closer to L and all points right of the PB are closer to R (see fig- ure 3.1.2.a). For three points we can divide the area in three parts (see figure 3.1.2.b and 3.1.2.c). Note that the PB's of three points always intersect in one point. In the case of more PM's we can construct an irregular network of so-called polygons delimiting the areas attributed to each PM.
(**Note**: Start with the polygon of one PM and proceed from there).

Now to determine the average precipitation of an area the measurements of each station are attributed a 'weight'. This weight or weighing factor is the ratio of the area of the polygon attributed to the PM under considera- tion and the total area:

$$W_i = \frac{A_i}{\Sigma A} \qquad (3.3.1)$$

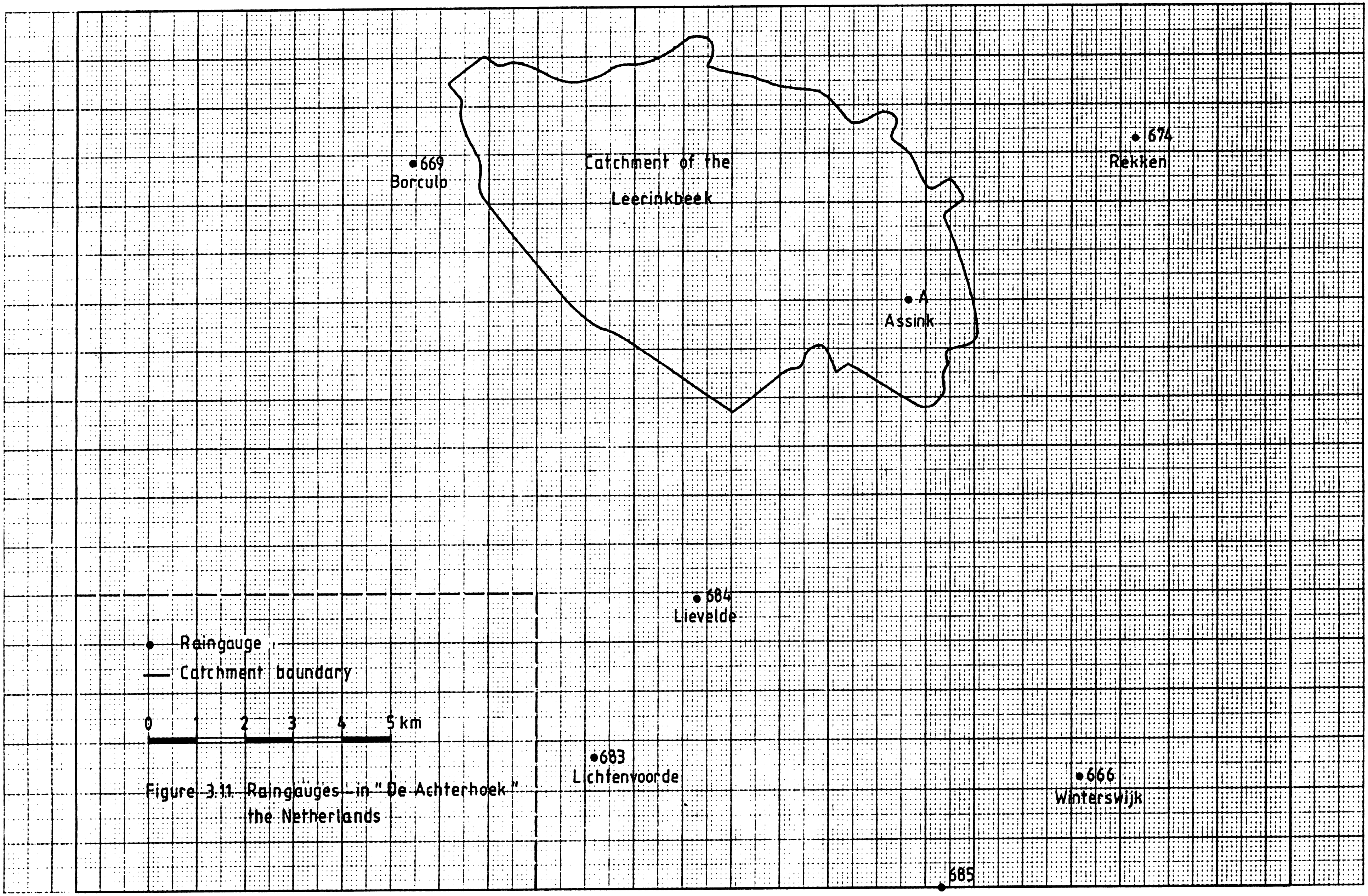


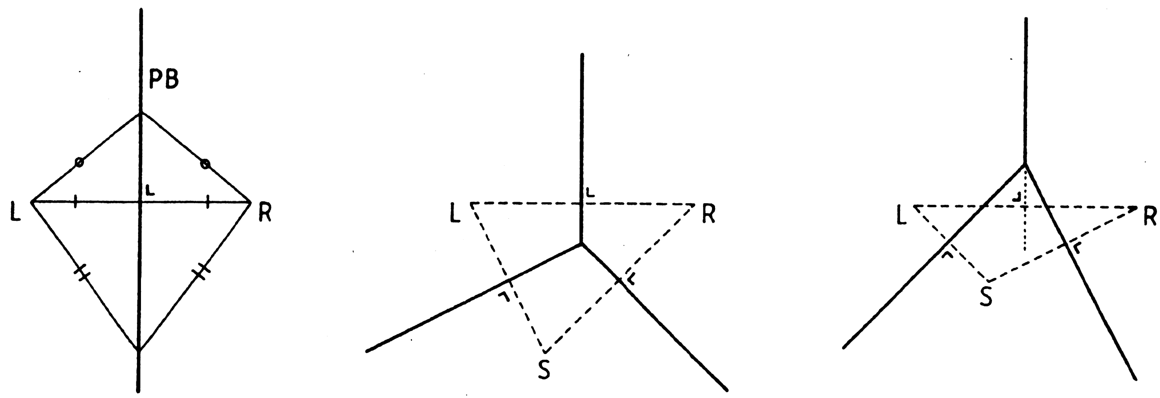
Figure 3.11 Raingauges in "De Achterhoek" the Netherlands

with:

W_i = weight of station i

A_i = area assigned to station i

ΣA = total area.



3.1.2.a 3.1.2.b 3.1.2.c

Figure 3.1.2 **Construction of a Thiessen network.**

For an area delimited by straight lines we can determine the surface area by calculation (divide each area into rectangles, triangles and/or trapezoids for which the surface area is easy to calculate) or by counting unit squares.

a. - Construct the Thiessen network for the rectangular area given in figure 3.1.1, using all stations.
  - Determine the surface areas and calculate the weight assigned to each rain gauge.
  - Determine the average (weighted) precipitation of the first decade of every month for the area, using the data in table 3.1.1.

In the 1960's hydrologic research was done in the Leerinkbeek area, to estimate the water deficit of agricultural crops. The Hupselse Beek is part of the Leerinkbeek catchment.
For an irregular area like the Leerinkbeek catchment (see figure 3.1.1) the surface area assigned to each PM can be determined:
- by counting unit squares (rather tedious for small squares);

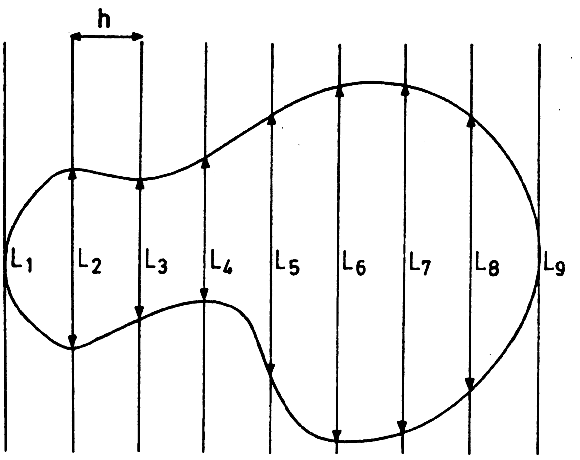
- by cutting out and weighing each part and their total (good quality paper, accurate balance, very easy and accurate but very destructive to your map);
- with the aid of a planimetric instrument (experience needed);
- with the aid of a computer and digitizing equipment (easy and accurate when one has the equipment);
- by the application of Simpson's rule (simple and accurate if used right).

Table 3.1.1 Precipitation in mm during the first decade of every month in 1982 for some stations in the Achterhoek.

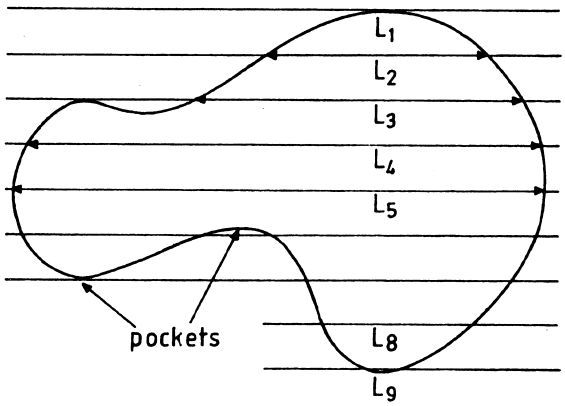
month station	J	F	M	A	M	J	J	A	S	O	N	D
Assink	41.6	6.5	47.6	13.8	36.1	10.8	5.7	8.9	3.3	21.3	5.2	32.7
666	33.1	5.6	32.8	10.7	32.6	1.1	6.6	7.0	9.4	22.7	5.7	24.0
669	31.1	7.1	37.1	16.4	39.9	5.0	34.3	17.4	12.7	28.3	5.8	28.6
674	36.2	5.5	32.0	11.3	37.6	5.7	5.8	8.3	4.5	23.4	5.2	25.0
683	35.1	5.5	32.7	13.2	33.2	36.2	4.5	22.5	5.1	28.4	5.4	30.3
684	36.0	5.5	29.6	10.4	31.6	7.4	6.2	10.6	7.5	24.5	5.4	29.1
685	35.1	6.9	34.9	12.7	32.7	4.5	11.2	16.3	6.2	24.5	6.7	30.6

(666 = Winterswijk, 669 = Borculo, 674 = Rekken, 683 = Lichtenvoorde, 684 = Lievelde, 685 = Corle)

(Note: The decades are defined as follows:
- first decade: day 1 to 10 of every month
- second decade: day 11 to 20 of every month
- third decade: remaining days of every month



3.1.3 a. Uneven number of lines ; even number of strips



3.1.3.b. Avoid pockets

Figure 3.1.3 Simpson's rule.

Simpson's rule can be used to determine the surface area of irregular surfaces. The total area is divided by an **uneven** number of evenly spaced, parallel lines into an **even** number of parallel strips (see figure 3.1.3.a). The first and last line must touch or coincide with opposite boundaries of the area under consideration. Choose the direction and number of lines in such a way that the development of 'pockets' is avoided (see figure 3.1.3.b). Now the area (A) in units squared can be calculated according to:

$$A = \frac{1}{3}\, h\ (\ell_1 + 4\ell_2 + 2\ell_3 + 4\ell_4 + 2\ell_5 + \ \ldots\ + 2\ell_{2n-1} + 4\ell_{2n} + \ell_{2n+1}) \qquad (3.1.2)$$

with:

2n+1 = number of lines

h = distance between lines

ℓm = length of line m (first and last are often zero)

n = 1, 2, 3, ...

b. Using Simpson's rule and the data from table 3.1.1 calculate the average (weighted) precipitation of the first decade of every month for the catchment of the Leerinkbeek.

c. Plot the results of exercise b and the amounts of precipitation for the other decades as given in table 3.1.2 in a graph depicting rainfall per decade for the Leerinkbeek catchment during 1982.

Table 3.1.2 Precipitation in mm per decade in the Leerinkbeek catchment in 1982.

month	dec.	P (mm)	month	dec.	P (mm)	month	dec.	P (mm)
January	2	0.1	May	2	6.2	September	2	2.2
	3	28.8		3	9.1		3	20.0
Februay	2	5.2	June	2	31.7	October	2	33.1
	3	1.5		3	41.9		3	14.8
March	2	25.4	July	2	2.7	November	2	49.6
	3	1.6		3	1.1		3	22.7
April	2	7.8	August	2	34.0	December	2	41.5
	3	5.1		3	14.7		3	10.8

Another way to determine the average areal rainfall is by means of isohyets, lines of equal rainfall. The lines are obtained by (linear) interpolation between different precipitation stations. At present there are computer programs available that do the job. Next the areas between two isohyets are determined and so the average rainfall on a surface can be calculated, again by using weights for the areas between the different isohyets.

d. Name two important disadvantages of the use of isohyets to determine the average precipitation in an area.
e. Not all Thiessen networks are as easy to draw as the one given in this example. As an additional exercise draw the Thiessen network for the stations given in figure 3.1.4.
(Note: Remember that the PB's of **three** points **always** intersect in one point).

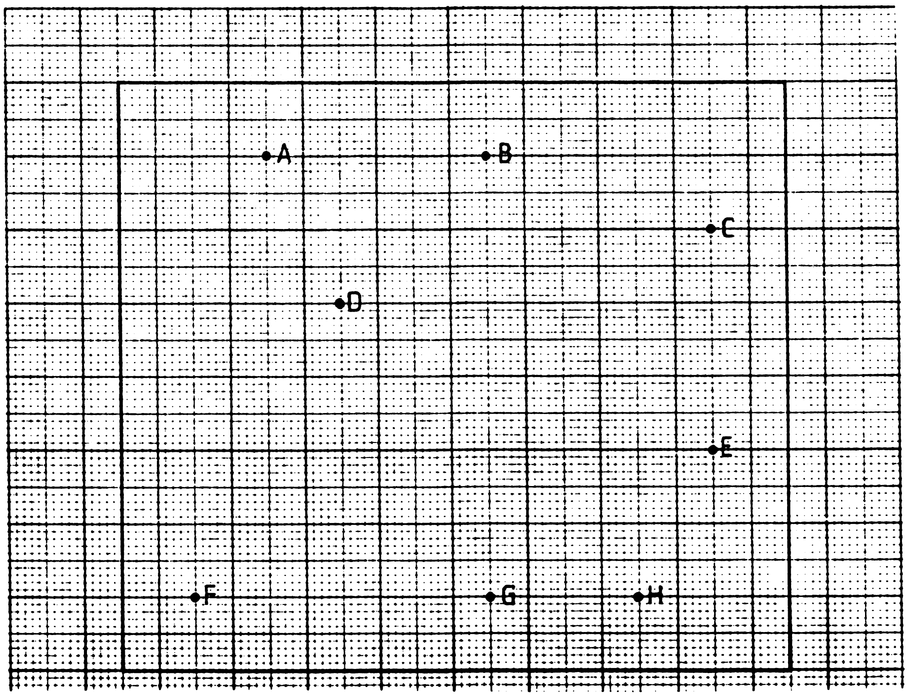


Figure 3.1.4 Rainstations.

Exercise 3.2

In this exercise extreme k-day precipitation amounts will be used to calculate return periods and to draw rainfall duration curves. First read the following pages about the use of Gumbel extremal probability paper.

Probability paper is a valuable and simple tool in testing whether observations are in accordance with theory and for the interpretation of results. In case of extremes (either maxima or minima) Gumbel extremal probability paper is used.
To explain and/or forecast extremes that may be expected to occur within a certain time and area, the following, essential conditions have to be met:
- that we deal with statistical variables;
- that the initial distribution and its parameters from which the extremes have been drawn, remain constant from one sample to the next;
- that the observed extremes are extremes of samples of independent data.

In addition each sample, from which the extreme value is taken, needs to be sufficiently large. For a more or less reliable forecast the number of samples (and therefore extremes) should be at least in the order of 10 to 20.
On Gumbel paper the observed extreme variable x is plotted along a linear scale on one axis (usually the vertical one or ordinate), while on the other axis (usually the horizontal one or abscissa) three scales are marked:
- a linear scale for the reduced variable y, where:

$$y = a(x_k - u) \qquad (3.2.1)$$

with:
y = reduced variable
x_k = observed extreme
a = the standard deviation of the Gumbel distribution
u = the modus of the Gumbel distribution.

- a non-linear scale, the cumulative frequency scale, which marks the theoretical probability P_n of the value x not exceeding a certain limit x_k. According to Gumbel's theory, extreme values obey the following distribution:

$$P_n(x \le x_k) = \exp(-\exp(-y)) \tag{3.2.2}$$

with:

P_n = probability of the value x not to exceed x_k

x_k = extreme value

x = variable

y = reduced variable.

Plotting x_k against $-\ln(-\ln(P_n))$ results in a straight line (because $-\ln(-\ln(P_n)) = y = a(x_k-u)$).

- a non-linear scale (on the upper side of the paper), marked T, the return period. It is linked to the cumulative frequency scale by:

$$T = \frac{1}{1-P_n} = \frac{1}{Q_n} \tag{3.2.3}$$

with:

T = return period

P_n = probability that $x \le x_k$

Q_n = probability that $x > x_k$

The procedure for using the Gumbel extremal probability paper will now be explained.

Procedure for Gumbel extremal probability paper

a. The observed extremes are arranged in increasing order for maxima and in decreasing order for minima.

b. Calculate the n fractions m/(n+1) where m = 1, 2, 3, ..., n and n is the number of observed extremes.

Choose and mark an appropriate (linear) scale for x on the ordinate. Plot m/(n+1) on the cumulative frequency scale against the corresponding extreme x_m.

c. If the plotted data do not show a tendency to lie on a straight line further steps are useless. In this case the data are not extremes, the samples are too small or are not drawn from the same population, the extremes do not obey the laws underlying this theory, etc..

d. As a first attempt a straight line can be drawn through the plotted points.

e. The regression line through the plotted points can also be calculated.
   - calculate the sum of the extremes Σx_m
   - calculate the square of the sum of the extremes $(\Sigma x_m)^2$
   - calculate the squares of the extremes $x_1^2, x_2^2, \ldots, x_n^2$
   - calculate the sum of squares $\Sigma(x_m)^2$
   - calculate the mean of the extremes $\Sigma x_m/n = \bar{x}$
   - calculate the variance $s_x^2 = (\Sigma(x_m)^2 - (\Sigma x_m)^2/n)/n$
   - calculate the standard deviation $s_x = \sqrt{(s_x^2)}$

     (Computation of the standard deviation can be carried out on most pocket calculators)
   - take $\bar{y}_n$ from table 3.2.1 and σ_n from table 3.2.2 (n is the number of extremes); $\bar{y}_n$ is the average of the reduced variable y and σ_n is the standard deviation of the reduced variable y
   - calculate the parameter $1/a = s_x/\sigma_n$
   - calculate the parameter $u = \bar{x} - \bar{y}_n/a$ (for maxima)

     $u = \bar{x} + \bar{y}_n/a$ (for minima)
   - now the regression line is given by:

     $x = u + y/a$ (for maxima)

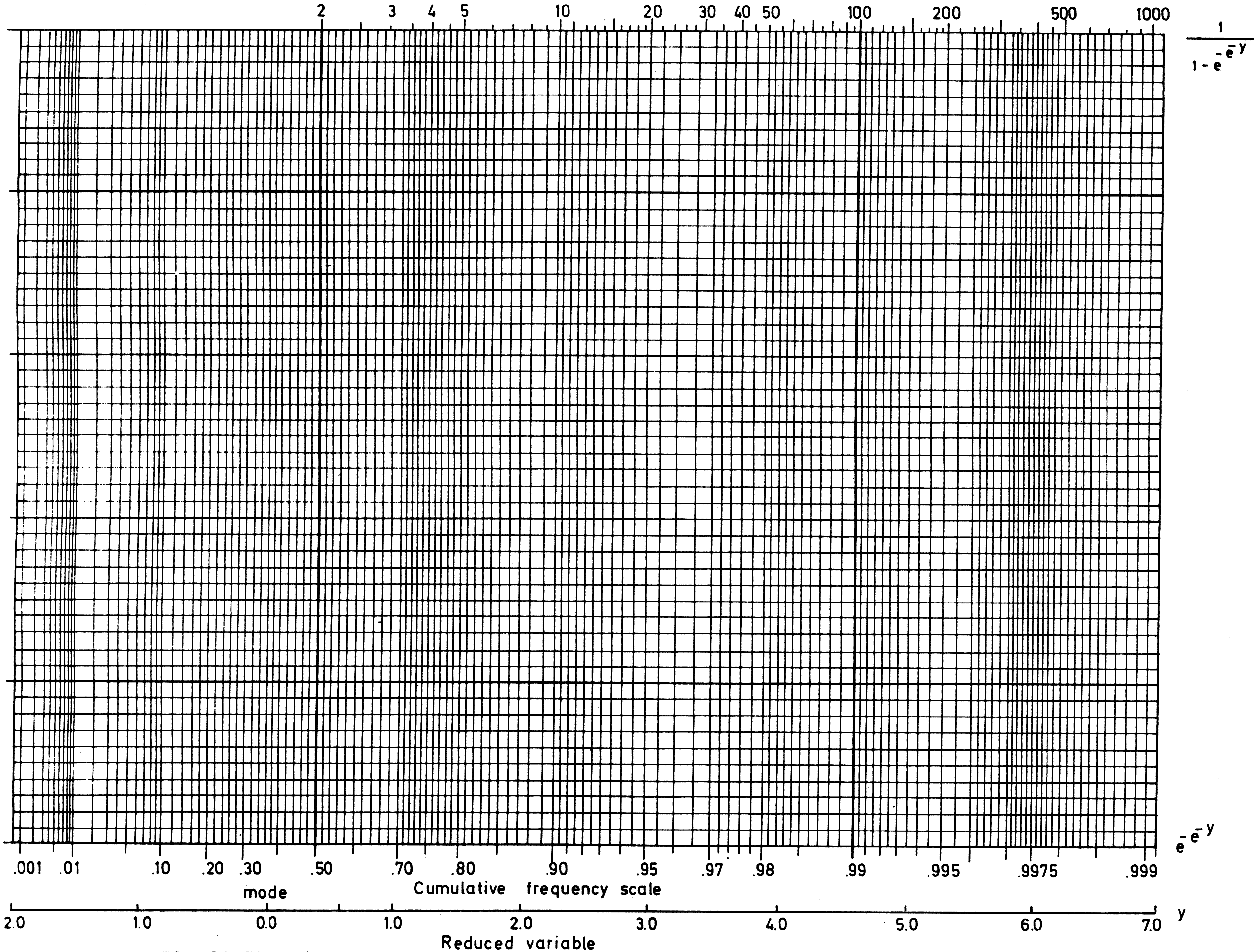
     $x = u - y/a$ (for minima)
   - For $y = 0.0$, x is equal to u. The position $y = 0.0$ is called the mode of the distribution and can be plotted. After choosing a second point on the (calculated) regression line (i.e. $y = 3.0$) the corresponding x may be calculated and plotted. Next the regression line may be drawn through these two points.

f. To find the expected extreme value x with a return period T, start at T on the scale along the top of the paper. Go down in a straight line until you cross the regression line. The corresponding x can then be found on the vertical scale.

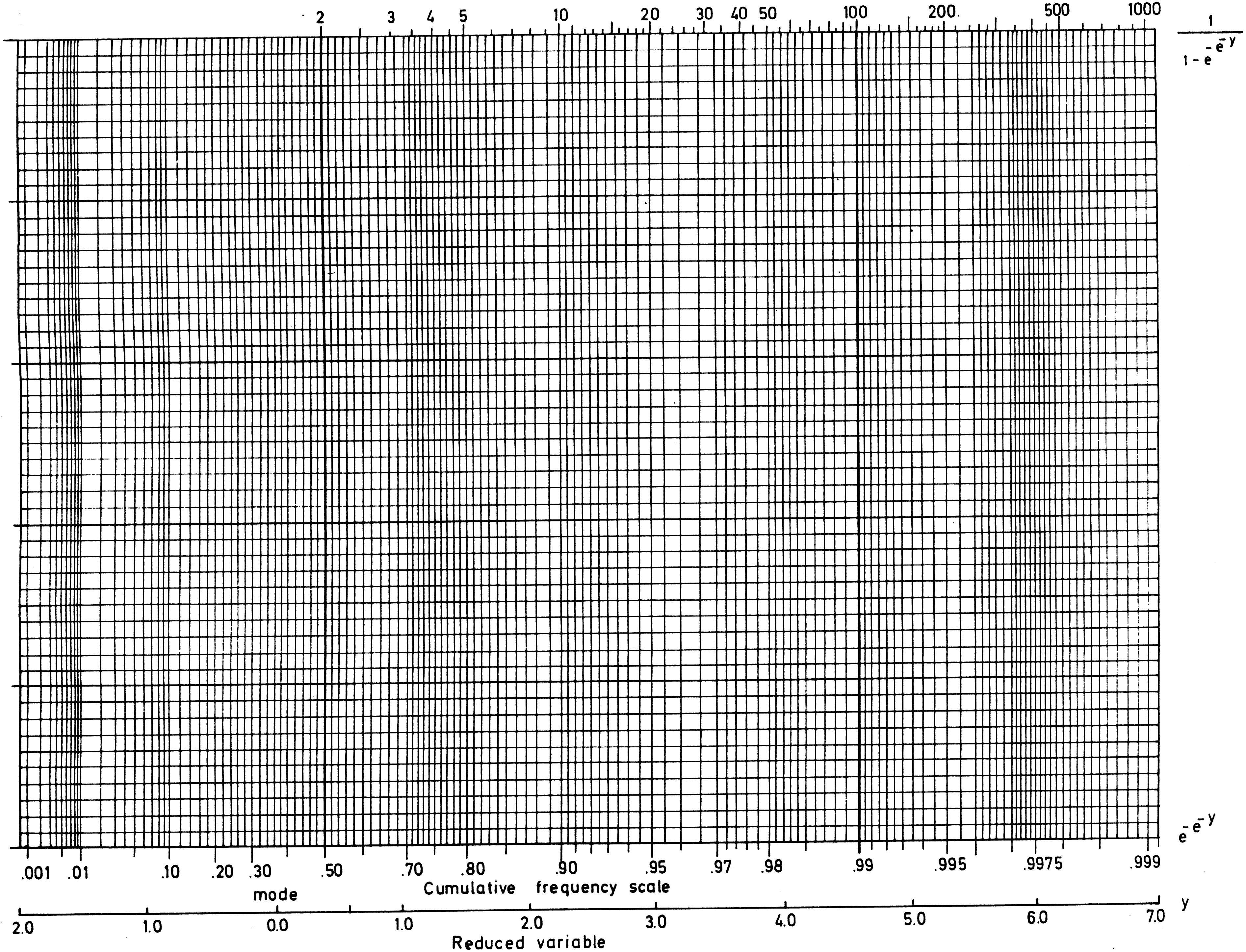
Note: For y (reduced variable) > 4.0 Gumbel paper does not differ appreciably from logarithmic paper. The distance between 0.99 and 0.999 is the same as between 0.999 and 0.9999. Thus the paper may be extended by taking the piece between T = 100 and T = 1000 from a second piece of paper and paste it onto the other piece in such a way that T_1= 100 coincides with T_1 = 1000. After correcting the y, P_n and T scales the assembled paper is extended to P_n = 0.9999 and T = 10000.

The above theory will be applied to daily rainfall records obtained with a precipitation gauge at the Assink Meteo Station. It has been assumed that the month of March is the critical month with regard to rainfall excess interfering with agricultural activities in the Leerinkbeek area (see exercise 3.1). However in general there are no serious drainage problems during the growing season, because the Leerinkbeek area consists mainly of well drained sandy soils.
In table 3.2.3 the daily rainfall records of March 1973 through 1987 are given.

a. Determine the maximum daily rainfall to be expected with a return period of respectively 2, 5, 10 and 50 years. To do this plot the extremes on Gumbel paper, calculate and draw the regression line and read the expected maximum 1-day rainfall with a return period of T years from the figure.
b. Do the same for the expected maximum 2-day rainfall.
c. Suppose that the expected maximum k-day rainfall has been calculated in the same way for k = 3, 4, 5, 7 and 10, with the results as in table 3.2.4.



GUMBEL PAPER



GUMBEL PAPER

Table 3.2.1 Expected mean $\bar{y}_n$ as function of the number of extremes n.

n	0	1	2	3	4	5	6	7	8	9
10	.4952	.4996	.5053	.5070	.5100	.5128	.5157	.5181	.5202	.5220
20	.5236	.5252	.5268	.5283	.5296	.5309	.5320	.5332	.5343	.5353
30	.5362	.5371	.5380	.5388	.5396	.5402	.5410	.5418	.5424	.5430
40	.5436	.5442	.5448	.5453	.5458	.5463	.5468	.5473	.5477	.5481
50	.5485	.5489	.5493	.5497	.5501	.5504	.5508	.5511	.5515	.5518
60	.5521	.5524	.5527	.5530	.5533	.5535	.5538	.5540	.5543	.5545
70	.5548	.5550	.5552	.5555	.5557	.5559	.5561	.5563	.5565	.5567
80	.5569	.5570	.5572	.5574	.5576	.5578	.5580	.5581	.5583	.5585
90	.5586	.5587	.5589	.5591	.5592	.5593	.5595	.5596	.5598	.5599
100	.5600									
150	.5646		300	.5699		750	.5738			
200	.5672		400	.5714		1000	.5745			
250	.5688		500	.5724						

Table 3.2.2 Expected standard deviation σ_n as function of the number of extremes n.

n	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0,9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0916	1.1004	1.1047	1.1089
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									
150	1.2253		300	1.2479		750	1.2651			
200	1.2360		400	1.2545		1000	1.2685			
250	1.2429		500	1.2588						

Table 3.2.3 Daily precipitation (mm day^{-1}) in March for a rain gauge at the Assink Meteo Station (1974-1987).

year/day	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
1	0.3	0.0	0.0	4.4	0.0	0.0	0.8	0.0	5.4	2.2	0.9	0.0	0.2	0.0	2.0
2	3.2	1.3	0.0	0.0	0.0	0.5	2.6	0.0	0.6	8.3	0.2	4.2	3.0	0.0	17.7
3	1.0	3.5	2.7	0.0	2.1	3.2	2.5	0.0	2.7	6.4	0.0	10.2	0.0	0.0	23.0
4	4.1	0.0	0.0	0.0	1.8	1.2	5.2	0.0	1.4	13.2	0.0	0.0	0.5	0.0	0.0
5	0.0	0.0	1.0	0.0	0.9	0.0	4.1	3.0	0.0	4.8	0.0	0.0	2.7	5.8	0.0
6	0.3	5.2	3.0	0.0	0.0	0.5	0.0	0.2	0.0	0.4	0.0	0.0	0.0	5.3	0.0
7	8.7	0.0	1.9	0.0	0.0	0.0	0.0	8.7	0.5	0.0	0.2	0.0	6.2	0.2	0.0
8	1.4	0.0	11.6	0.0	0.7	0.0	4.2	0.4	3.4	0.0	0.0	0.0	0.0	0.0	0.0
9	0.2	0.0	0.0	0.3	0.0	5.8	0.0	0.0	7.1	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.4	0.0	0.0	11.8	0.0	20.6	0.0	0.0	0.5	0.0	0.5	0.0
11	0.0	0.0	1.2	0.0	0.0	0.0	3.2	2.4	11.1	15.2	0.0	3.1	2.1	0.0	0.0
12	0.1	0.1	0.0	0.0	3.0	0.0	5.0	0.0	22.5	1.6	0.0	0.0	0.0	0.0	0.0
13	0.0	1.2	0.5	1.1	0.0	9.6	0.0	12.2	4.6	6.8	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.6	2.4	0.0	3.2	5.4	1.1	0.0	0.0	1.4	0.0	0.0
15	0.0	0.3	2.4	0.0	8.8	1.8	7.6	0.0	7.0	0.0	4.8	0.0	3.2	0.0	0.0
16	0.0	8.7	3.9	0.2	0.4	11.5	10.4	0.0	1.7	4.2	0.0	0.0	3.0	0.0	3.6
17	0.3	1.4	1.0	0.0	4.4	2.6	2.2	0.0	5.6	2.5	0.1	0.0	0.0	0.0	3.8
18	0.7	3.1	0.4	1.1	0.0	0.0	0.2	0.0	0.0	0.5	3.4	0.0	0.0	0.1	17.8
19	0.3	4.3	0.2	0.1	0.4	2.9	3.2	0.0	0.2	1.1	5.8	0.0	0.0	0.0	2.2
20	0.0	0.0	1.2	0.0	0.5	11.2	0.0	0.0	0.0	0.5	1.4	0.0	0.0	0.0	0.0
21	0.0	6.5	3.3	0.0	0.0	2.9	0.0	0.0	0.0	0.0	6.0	0.0	3.8	3.2	6.3
22	0.0	0.1	0.0	0.0	2.2	5.0	1.0	0.0	0.0	0.0	6.6	0.0	0.0	0.0	3.2
23	0.0	0.0	0.0	0.0	0.0	3.3	5.9	3.9	0.8	0.0	5.6	0.0	0.3	7.5	1.5
24	0.0	0.0	0.0	0.0	0.0	6.7	0.0	0.0	6.3	0.0	19.3	0.0	0.0	5.4	8.8
25	0.0	0.0	4.6	3.5	0.0	0.3	0.3	0.0	6.2	0.0	0.3	1.8	0.7	12.0	2.0
26	1.0	0.0	2.9	7.3	0.0	11.4	2.7	0.2	16.1	0.0	9.8	0.3	4.8	0.9	3.6
27	7.7	0.0	2.9	6.9	1.1	1.6	1.0	1.0	1.4	0.0	3.3	5.0	11.4	4.8	0.0
28	0.0	0.0	0.0	0.7	12.4	2.8	3.6	2.0	0.0	0.0	3.3	8.1	0.4	18.2	1.5
29	0.0	0.0	22.5	0.0	0.6	0.7	1.5	10.0	0.0	0.0	4.3	2.8	11.1	4.2	3.3
30	0.0	0.5	1.4	1.1	0.0	1.0	1.3	7.6	0.0	1.8	4.4	0.4	4.0	4.5	0.8
31	0.8	0.0	0.0	0.0	0.0	10.2	0.5	0.0	0.0	0.0	6.8	3.1	8.3	4.4	0.0
Total	30.1	36.2	68.6	27.1	39.9	99.1	80.8	54.8	130.6	70.6	87.6	39.5	67.1	77.0	101.1

Table 3.2.4 Expected k-day rainfall (in mm) in March with a return period of T years for the Assink Meteo Station.

length of period k (in days)	return period T (in years)			
	2	5	10	50
3	22.5	35.1	43.4	61.6
4	25.6	39.6	48.8	69.1
5	28.3	43.4	53.4	75.4
7	33.6	51.4	63.1	88.9
10	38.5	60.2	74.6	106.2

Now draw the rainfall duration curves for a return period of 2, 5, 10 and 50 years.

d. Assuming an available storage of respectively 15, 25 and 35 mm, make a first approximation of the required discharge capacity if flooding is allowed with a return period of 10 years.

Exercise 3.3

The extremely high discharge of the river Rhine of 16.500 m^3s^{-1} has a return period T = 1250 years. We assume that this estimate is correct.

a. What is the probability that this discharge will occur in the year 2000?
b. What is the probability that this discharge will not occur in the year 2000?
c. What is the probability that this discharge will occur within a 50 year period?
d. What is the probability that this discharge will not occur within a 50 year period?
e. What is the length of the period in which this discharge will occur with a probability of 0.5?

Answer to exercise 3.1

a. When the Thiessen polygons have been constructed in the right way the resulting network looks as depicted in figure 3.1.5.
The surface areas and the weights are given in table 3.1.3.

Table 3.1.3 Areas and weights for the different stations.

station	nr	area (km²)	weight
Assink		72.9	0.16
Winterswijk	666	56.8	0.13
Borculo	669	120.5	0.27
Rekken	674	50.5	0.11
Lichtenvoorde	683	85.0	0.19
Lievelde	684	53.2	0.12
Corle	685	14.2	0.03
total		453.1	1.01

The average decade sums are given in table 3.1.4. They are calculated according to:

$$P = \sum (W_i \; P_i)$$

with:

P = total precipitation [mm]
W_i = weight of station i [-]
P_i = precipitation recorded at station i [mm]

Table 3.1.4 Precipitation in mm per decade for the total area in 1982.

month	dec.	P (mm)	month	dec.	P (mm)	month	dec.	P (mm)
January	1	35.1	May	1	35.7	September	1	7.6
	2	0.0		2	6.9		2	0.6
	3	27.9		3	10.2		3	15.5
February	1	6.1	June	1	11.6	October	1	25.4
	2	5.4		2	33.7		2	33.4
	3	0.5		3	37.0		3	14.7
March	1	35.9	July	1	13.4	November	1	5.5
	2	31.0		2	3.0		2	47.5
	3	1.6		3	0.9		3	22.7
April	1	13.3	August	1	13.8	December	1	28.7
	2	6.1		2	30.6		2	40.6
	3	4.5		3	20.5		3	10.3

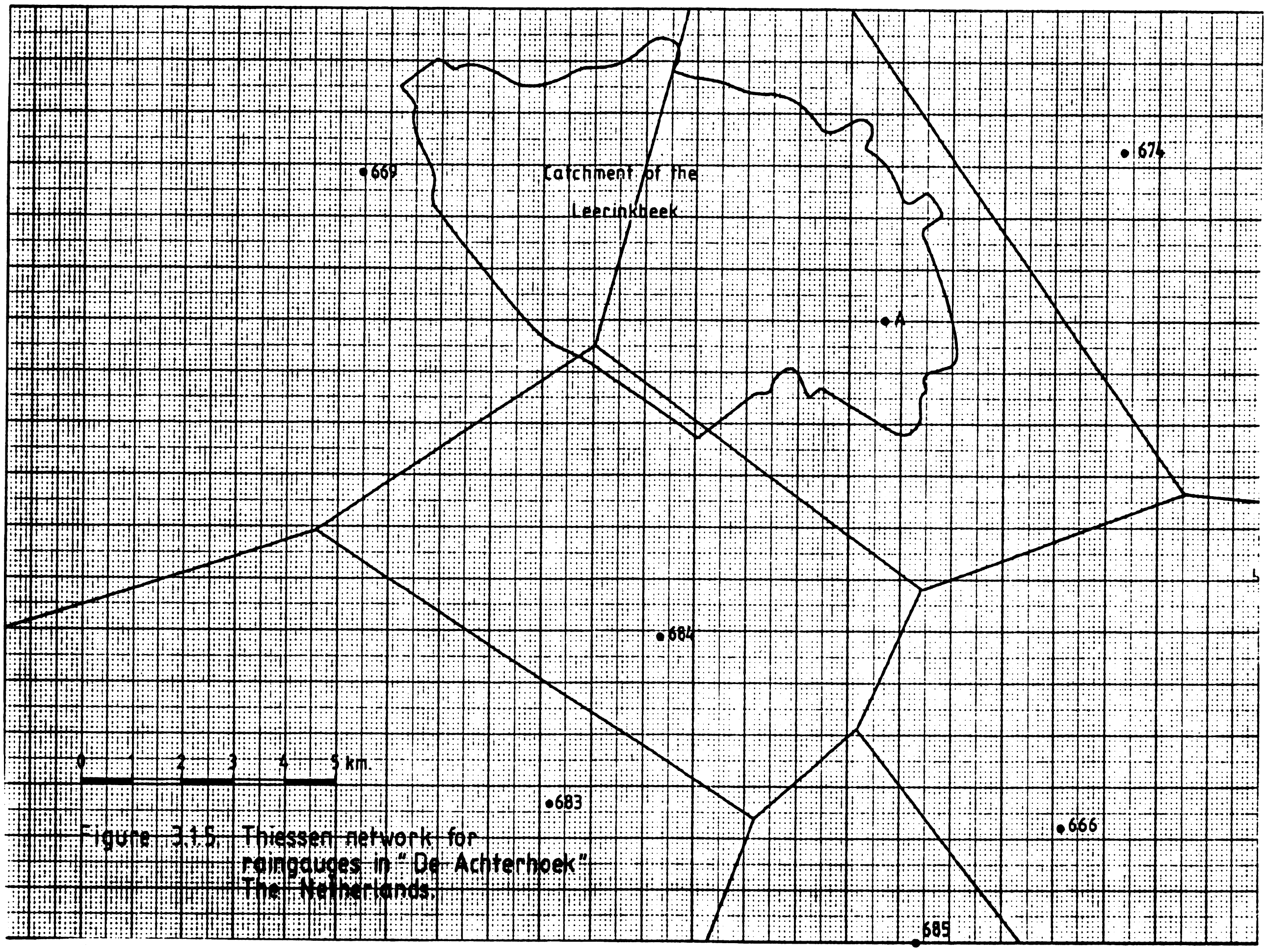


Figure 3.1.5. Thiessen network for raingauges in "De Achterhoek", The Netherlands.

b. The Leerinkbeek catchment is covered by the polygons of three stations, namely by 669 (Borculo), Assink and 684 (Lievelde). To determine the surface areas assigned to each station with Simpson's rule the areas are divided as shown in figure 3.1.6. In this case it proved advantageous to select three sections with different widths for the application of Simpson's rule. The resulting surface areas and weights are given in table 3.1.5. The average decade sums are given in table 3.1.6.

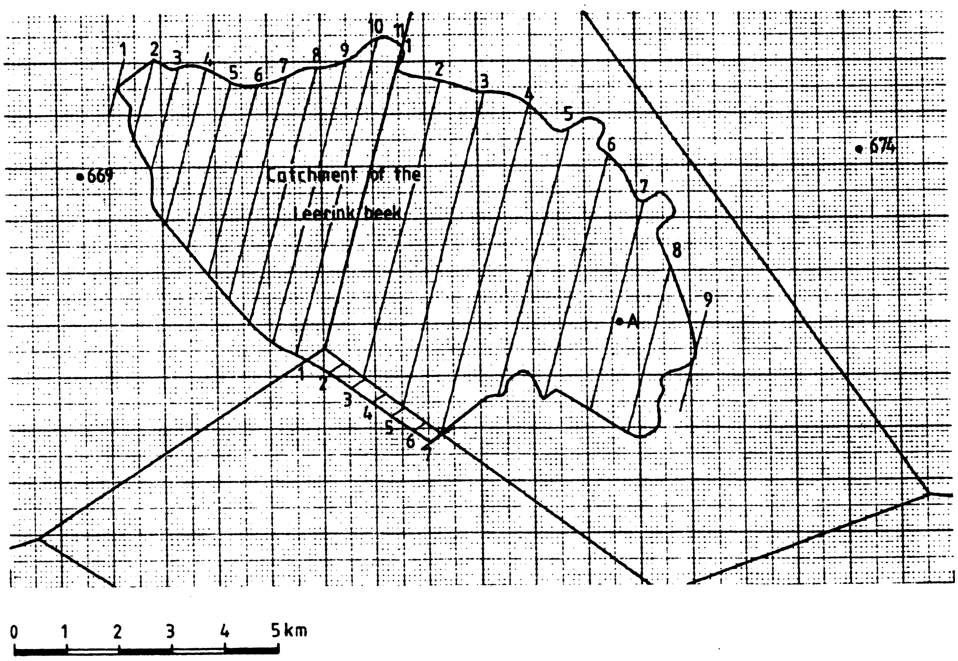


Figure 3.1.6 Appplication of Simpson's rule to the area of the Leerink beek catchment.

Table 3.1.5 Areas and weights for the three stations covering the Leerinkbeek catchment.

station	nr.	area (km²)	weight
Assink		33.0	0.63
Borculo	669	19.1	0.36
Lievelde	684	0.6	0.01
total		52.7	1.00

Table 3.1.6 Precipitation in mm per decade in the Leerinkbeek catchment in 1982 (first decade of every month).

month	dec.	P (mm)	month	dec.	P (mm)	month	dec.	P (mm)
January	1	37.7	May	1	37.4	September	1	6.8
February	1	6.7	June	1	8.7	October	1	23.9
March	1	43.6	July	1	16.1	November	1	5.4
April	1	14.7	August	1	12.0	December	1	31.2

c. See figure 3.1.7.

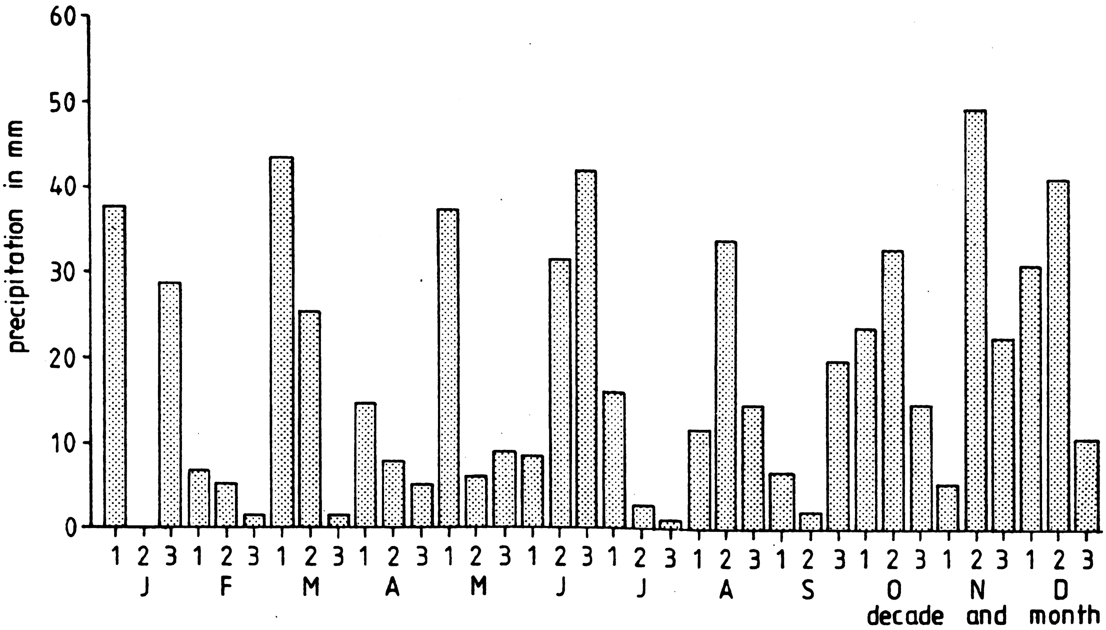


Figure 3.1.7 Precipitation in mm per decade in the Leerinkbeek catchment (1982).

d. Two important disadvantages of the use of isohyets are:

- the way the lines are drawn depends on the interpreter; this makes it more difficult to compare the results; this does not apply when using a computer program. However be careful to choose the appropriate program.
- the isohyets have to be drawn again for every new period; this also means that the weights have to be determined again and this is tedious and time-consuming.

e. The network will look as depicted in figure 3.1.8. In every node exactly three lines intersect.

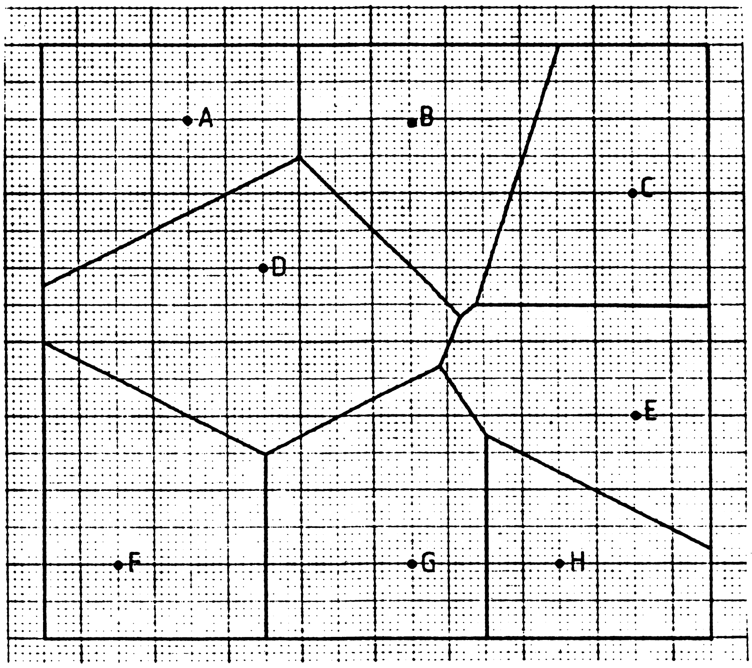


Figure 3.1.8 Constructed Thiessen network.

Answer to exercise 3.2

a. To plot the data on Gumbel paper, determine the maximum 1-day and 2-day values for each month and rearrange them in ascending order. To find the 2-day precipitation for each month first add the precipitation of every two consecutive days in a month. Next number the maxima, assigning 1 to the lowest and 15 (there are 15 years) to the highest maximum. These maximum values can be plotted against $y = i/(n+1)$, in which i is the number assigned to an observation and n is the number of observations. This results in table 3.2.5 and the dots in figure 3.2.1.

Table 3.2.5 Maximum 1- and 2-day precipitations in March (1973-1987) for the Assink Meteo Station.

number	1	2	3	4	5	6	7	8
y=i/(n+1) 1-day 2-day	1/16 7.3 10.1	2/16 8.7 10.1	3/16 8.7 13.5	4/16 10.2 14.1	5/16 11.4 14.2	6/16 11.5 14.4	7/16 11.8 16.2	8/16 12.2 17.6
number	9	10	11	12	13	14	15	
y=i/(n+1) 1-day 2-day	9/16 12.4 18.0	10/16 15.2 19.6	11/16 18.2 23.0	12/16 19.3 23.9	13/16 22.5 24.9	14/16 22.5 33.6	15/16 23.0 40.7	

To find the best fitting curve through these points the regression line is calculated. As a check on your calculations some intermediate results and the final results are given in tabel 3.2.6.

Table 3.2.6 Intermediate and final results when calculating the regression lines for the expected maxium 1- and 2-day precipitation.

	1-day	2-day
Σx_m	214.9	293.9
$(\Sigma x_m)^2$	46182	86377
Σx_m^2	3489	6780
$\bar{x}$	14.33	19.59
s_x^2	27.35	68.11
s_x	5.23	8.25

	1-day	2-day
y_n	0.5128	0.5128
σ_n	1.0206	1.0206
1/a	5.1	8.1
u	11.7	15.4
line	x=11.7 +5.1y	x=15.4 +8.1y

Now the regression line $x = \frac{1}{a} y + u$ can be drawn in figure 3.2.1. The expected maximum 1-day precipitation with a certain return priod T can be read from the figure in the following way. Start at the return period T on the scale along the top of the paper. Go down in a straight line until the regression line for k = 1 (maximum 1-day precipitation) is reached. Read the corresponding x-value on the vertical axis. (The x-value can also be calculated using the equation for the regression line). For T = 2, 5, 10 and 50 years, the results can be found in table 3.2.7.

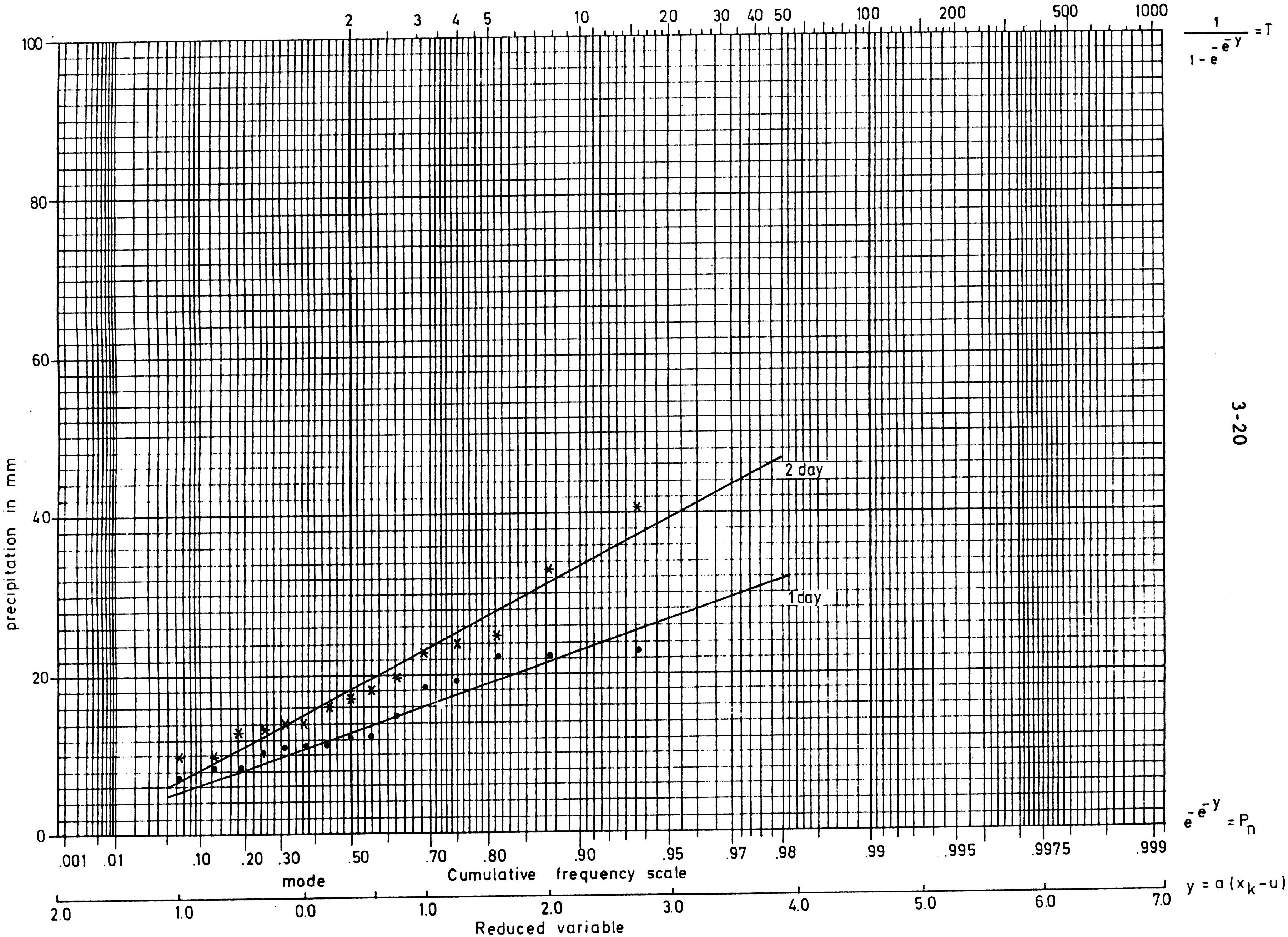


Figure 3.2.1. GUMBEL PAPER

Table 3.2.7 Expected maximum 1- and 2-day precipitation in March with return period T for the Assink Meteo Station.

	2	5	10	50
1-day	13.6	19.4	23.2	31.7
2-day	18.4	27.5	33.6	47.0

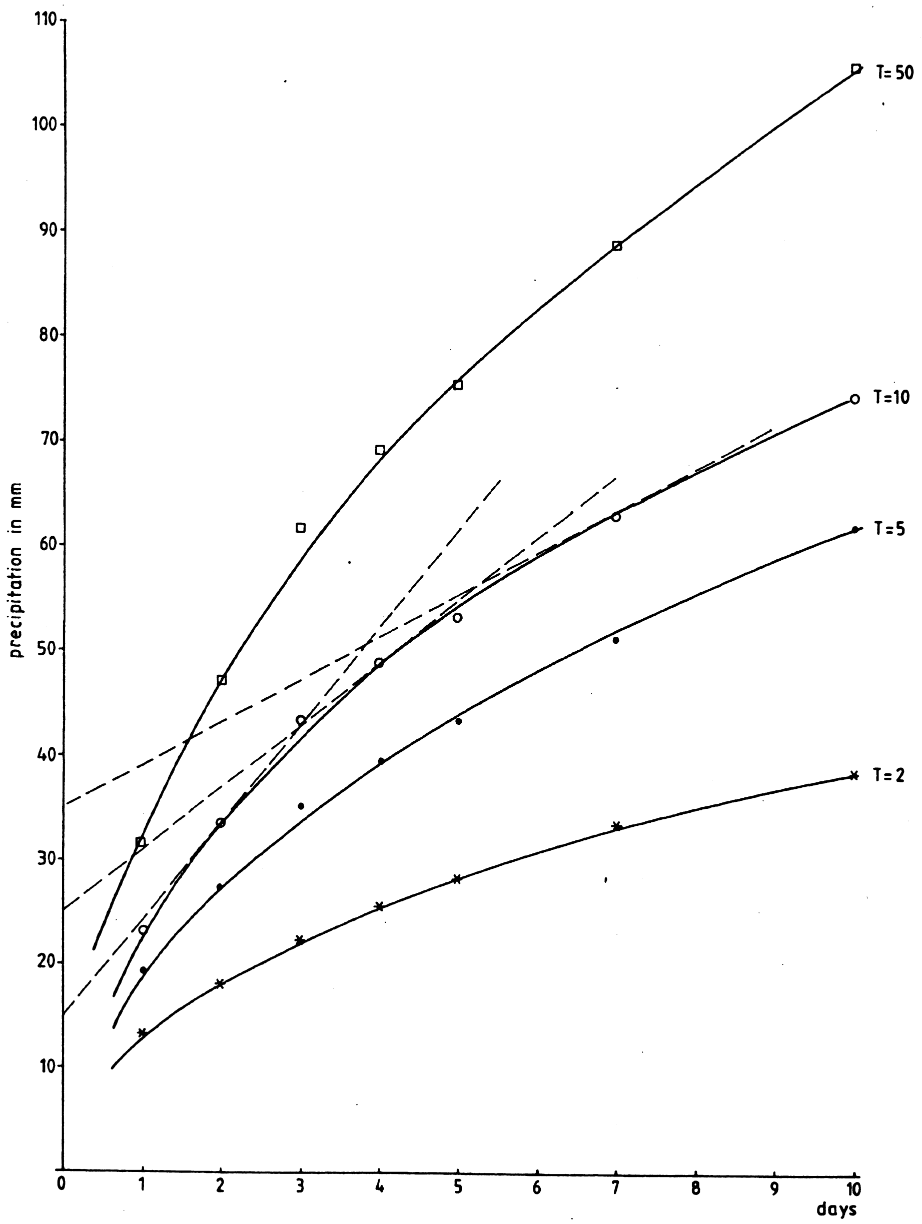


Figure 3.2.2 Rainfall duration curves for the Assink Meteo Station.

c. The rainfall duration curves are obtained by plotting the results of table 3.2.4 (page 3-13) and table 3.2.7 in one figure. Here the precipitation should be on the vertical axis and the number of days on the horizontal axis. The result is given in figure 3.2.2.d. To estimate the design discharge capacity with a return period of 10 years the following procedure can be followed. Plot the available storage on the precipitation axis of figure 3.2.2. Draw the tangent from this point to the rainfall duration curve for T=10. The slope of this line is a first estimate for the required discharge capacity in mm day^{-1}.

So the required discharge capacity with an available storage of:
15 mm is (40-15.0)/2.7 = 9.3 mm day^{-1}
25 mm is (50-25.0)/4.2 = 6.0 mm day^{-1}
35 mm is (50-35.0)/3.7 = 4.1 mm day^{-1}

Answer to exercise 3.3

a. The probability that this discharge will occur within any given year (and therefore also the year 2000) is 1/T = 1/1250.
b. The probability that this discharge will not occur within any given year (and therefore also the year 2000) is 1-1/T = 1249/1250 = 0.9992.
c. The probability (U) that the discharge will occur within a 50 year period is:

$$U = 1 - (1 - \frac{1}{T})^r$$

with:

U = probability that a certain value will be exceeded during a certain period
T = mean return period of that value [time]
r = length of period [time]

Here $T = 1250$ year and $r = 50$ year, so $U = 0.0392$. So the probability that the discharge of the river Rhine will exceed 16500 $m^3\ s^{-1}$ within a period of 50 years is 0.0392.

d. The probability that this discharge will not occur in a given 50 year period is $1 - U = 1 - 0.0392 = 0.9608$.

e. This means that $U = 0.5$ and r is unknown. So the question boils down to calculating r.

$U = 0.5 = 1 - (1 - 1/1250)^r$, so $(1 - 1/1250)^r = 0.5$ and $r = \log(0.5)/\log(1249/1250) = 866.1$ year.

Examenvraagstukken bij hoofdstuk 3

3.1 (december 1991)

In het stroomgebied van een rivier wordt de neerslag op een aantal punten waargenomen.

a. Welke methoden kent u om de gemiddelde neerslaghoeveelheid (voor een bepaalde periode) in het stroomgebied te berekenen?

b. Geef van elke methode (m.b.v. een eenvoudige vergelijking) aan hoe de berekening verloopt (niet met een getallen voorbeeld uitrekenen!).

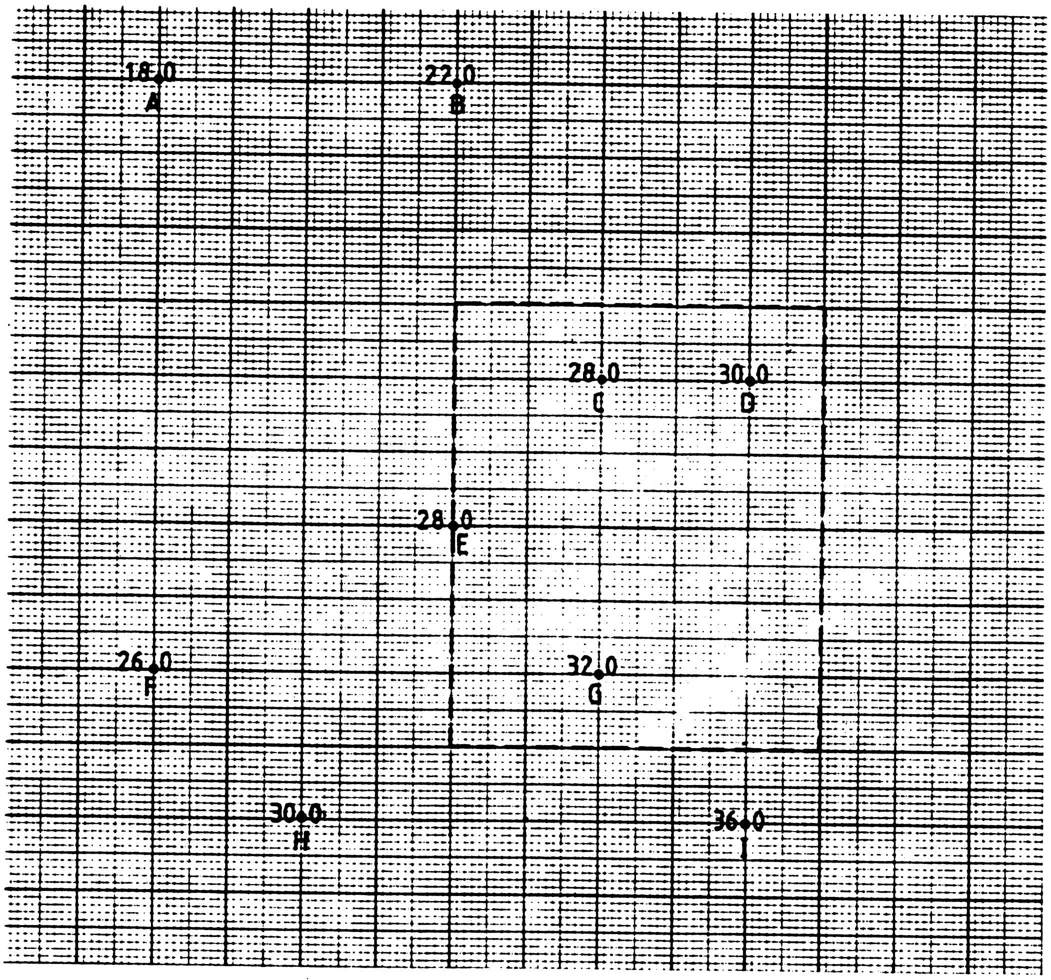
c. Geef een korte opsomming van de voor- en nadelen van elke methode.

3.2 (juni 1992)

In de onderstaande figuur zijn op de locaties van de punten A t/m I regenmeters geplaatst; de cijfers geven de neerslag (afgerond op hele mm's) voor een bepaalde periode. De met een streepjeslijn aangegeven rechthoek is een landbouwbedrijf.

a. Teken de polygonen van Thiessen (voor het landbouwbedrijf) in de figuur.

b. Bereken de gemiddelde neerslag voor het bedrijf met behulp van de Thiesen methode (in mm, 1 cijfer achter de komma).



Neerslagmetingen op locaties A t/m I.

Examenvraagstukken bij hoofdstuk 4

4.1 (juli 1994)

Infiltratie door het bodemoppervlak kan worden beschreven met de volgende vergelijking:

$$f = f_c + (f_0 - f_c)\, e^{-kt}$$

f is de infiltratie snelheid, f_0 en f_c respectievelijk voor $t = 0$ en $t \rightarrow \infty$.
Langs een beek in het oosten van Nederland ligt een leemgrond (tot 20.0 m uit de beek aan beide zijden onder een helling van 4%) waarvoor geldt:
$f_0 = 32.0$ mm uur^{-1}, $f_c = 5.0$ mm uur^{-1} en $k = 6.0$ uur^{-1}.
Tijdens een hevige regenbui worden de volgende regenhoeveelheden gemeten:
0 -15 min: 9.0 mm
15-30 min: 10.0 mm
30-45 min: 6.0 mm
45-60 min: 2.0 mm (totaal 27.0 mm in één uur)
a. Bereken f op t = 0, 15, 30, 45 en 60 min.
Maak een tekening waarin de regenbui en de infiltratie zijn uitgezet tegen de tijd.(eenheden van de schalen duidelijk aangeven !)
b. Bereken de hoeveelheid water(m^3) die per 100.0 m beek-lengte oppervlakkig afstroomt van de hellingen ter weerszijden, als gevolg van deze regenbui.

5 EVAPORATION

Exercise 5.1

The mean annual precipitation-surplus of the Veluwe amounts to 365 mm. This surplus is the difference between areally averaged precipitation and evapotransporation.

a. The evapotranspiration is determined by the evapotranspiration of deciduous forest, heather, sand and pine forest. Arrange these types of soil cover according to descending annual evapo(transpi)ration. Why in this sequence.
b. The annual precipitation is 800 mm with an average chloride concentration of 6 mg l^{-1}. The groundwater under the glacial ridges covered with forest has an average chloride concentration of 24 mg l^{-1}. Estimate the evapotranspiration of these forests in mm per year.

Exercise 5.2

From 1956 till 1 april 1987 the Royal Dutch Meteorological Institute (KNMI) used the Penman formula to calculate the evaporation of an open water surface (E_0). Modifications of the formula over time resulted in the use of several (slightly different) versions of the Penman formula by different people.
To end this confusion and for several other practical and theoretical reasons the KNMI started using the empirical Makkink formula to calculate the reference crop-evapotranspiration (E_r). This formula has been used since 1 April 1987 instead of the Penman formula.
The Makkink formula is given by:

$$\lambda E_r = 0.65 \frac{s}{s+\gamma} K\downarrow \qquad (5.2\text{-}1)$$

with:

λ	=	heat of vaporization	[J kg^{-1}]
λ	=	$\{2501 - 2,4(T - 273)\} \times 10^3$	
T	=	temperature	[K]
E_r	=	reference crop-evapotranspiration	[kg m^{-2} s^{-1}]

s = change of saturated vapour pressure with temperature [hPa K^{-1}]

γ = psychrometer constant ≈ 0.67 [hPa K^{-1}]

K↓ = incoming short wave radiation at the earth surface [W m^{-2}]

An important advantage of the Makkink formula over the Penman formula is that only the temperature and the incoming short wave radiation have to be measured in order to calculate the reference crop-evapotranspiration (E_r). These two entities are measured regularly at a large number of meteorological stations. To calculate E_0 according to the Penman formula it is necessary to measure the number of hours of sunshine, the temperature, the relative humidity of the air and the windspeed.

a. Calculate the reference crop-evapotranspiration for the first decade of every month at the Assink Meteo Station (East-Gelderland), using formula 5.2-1 and the table 5.5.2. The incoming short wave radiation at the earth surface (R_s) and the temperature (T) are given in table 5.5.1.

Table 5.5.1 Incoming short wave radiation at the earth surface (K) and the temperature (T) as measured at the Assink meteo station in 1982.

month decade	J 1	F 1	M 1	A 1	M 1	J 1	J 1	A 1	S 1	O 1	N 1	D 1
K↓ (W m^{-2}) T (°C)	31 0.7	51 3.7	77 4.5	139 8.8	158 7.5	259 20.0	216 17.5	167 20.2	150 15.9	74 12.1	37 10.1	23 3.4

The reference crop-evapotranspiration is calculated for an optimally growing grass surface (short) which transpires according to formula 5.1. To calculate the evaporation of real crops, crop factors (f) are used.

$$E_{crop} = f\, E_r \tag{5.2}$$

with:

E_{crop} = evapotranspiration of a real crop [kg m^{-2} s^{-1}]

f = crop factor [-]

E_r = reference crop-evapotranspiration [kg m^{-2} s^{-}]

b. Using the reference crop-evapotranspiration E_r (see Answer 5.2.a) and the crop factors in the Lecture notes, calculate the evapotranspiration per decade and the total evapotranspiration per growing season (April-Sept.) for grass and cereals.

Table 5.5.2 Change of saturated vapour pressure as a function of temperature (mbar $^{o}K^{-1}$ = hPa $^{o}K^{-1}$).

T	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-10	0.23	0.23	0.23	0.23	0.23	0.21	0.21	0.21	0.21	0.21
-9	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.23	0.23	0.23
-8	0.27	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.24
-7	0.28	0.28	0.28	0.28	0.27	0.27	0.27	0.27	0.27	0.27
-6	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.28	0.28	0.28
-5	0.32	0.32	0.32	0.32	0.31	0.31	0.31	0.31	0.31	0.31
-4	0.35	0.35	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.32
-3	0.36	0.36	0.36	0.36	0.36	0.36	0.35	0.35	0.35	0.35
-2	0.39	0.39	0.39	0.39	0.37	0.37	0.37	0.37	0.37	0.37
-1	0.41	0.41	0.41	0.41	0.40	0.40	0.40	0.40	0.40	0.40
-0	0.44	0.44	0.44	0.44	0.43	0.43	0.43	0.43	0.43	0.41
0	0.45	0.45	0.45	0.45	0.45	0.47	0.47	0.47	0.47	0.47
1	0.48	0.48	0.48	0.48	0.48	0.49	0.49	0.49	0.49	0.51
2	0.51	0.51	0.51	0.52	0.52	0.52	0.52	0.53	0.53	0.53
3	0.53	0.55	0.55	0.55	0.55	0.56	0.56	0.56	0.56	0.57
4	0.57	0.57	0.57	0.59	0.59	0.59	0.59	0.60	0.60	0.60
5	0.60	0.61	0.61	0.61	0.61	0.63	0.63	0.63	0.64	0.64
6	0.64	0.65	0.65	0.65	0.67	0.67	0.67	0.68	0.68	0.68
7	0.69	0.69	0.69	0.71	0.71	0.71	0.72	0.72	0.72	0.73
8	0.73	0.73	0.75	0.75	0.75	0.76	0.76	0.76	0.77	0.77
9	0.77	0.79	0.79	0.79	0.80	0.80	0.80	0.81	0.81	0.83
10	0.83	0.83	0.84	0.84	0.85	0.85	0.85	0.87	0.87	0.87
11	0.88	0.88	0.88	0.89	0.89	0.91	0.91	0.91	0.92	0.92
12	0.92	0.93	0.93	0.95	0.95	0.95	0.96	0.96	0.97	0.97
13	0.97	0.99	0.99	1.00	1.00	1.01	1.01	1.03	1.03	1.03
14	1.04	1.04	1.05	1.05	1.05	1.07	1.07	1.08	1.08	1.09
15	1.09	1.11	1.11	1.12	1.12	1.13	1.13	1.15	1.15	1.16
16	1.16	1.17	1.17	1.19	1.19	1.20	1.20	1.21	1.21	1.23
17	1.23	1.24	1.24	1.25	1.25	1.27	1.27	1.28	1.28	1.29
18	1.29	1.31	1.31	1.32	1.32	1.33	1.33	.135	1.36	1.36
19	1.37	1.39	1.39	1.40	1.40	1.41	1.41	1.43	1.43	1.44
20	1.44	1.45	1.45	1.47	1.47	1.48	1.49	1.49	1.51	1.52
21	1.52	1.53	1.55	1.55	1.56	1.57	1.57	1.59	1.60	1.60
22	1.61	1.63	1.63	1.64	1.65	1.65	1.67	1.68	1.68	1.69
23	1.71	1.71	1.72	1.73	1.73	1.75	1.76	1.76	1.77	1.79
24	1.79	1.80	1.81	1.81	1.83	1.84	1.84	1.85	1.87	1.87
25	1.88	1.89	1.89	1.91	1.92	1.93	1.95	1.96	1.97	1.99
26	1.99	2.00	2.01	2.03	2.04	2.04	2.05	2.07	2.08	2.09
27	2.09	2.11	2.12	2.13	2.13	2.15	2.16	2.17	2.17	2.19
28	2.20	2.21	2.23	2.24	2.25	2.27	2.28	2.29	2.29	2.31
29	2.32	2.33	2.35	2.36	2.37	2.39	2.39	2.40	2.41	2.41
30	2.43	2.44	2.45	2.47	2.48	2.49	2.51	2.52	2.53	2.55
31	2.56	2.57	2.59	2.60	2.61	2.63	2.64	2.65	2.67	2.68
32	2.69	2.71	2.72	2.73	2.75	2.76	2.77	2.79	2.80	2.81
33	2.83	2.84	2.85	2.87	2.88	2.89	2.91	2.92	2.93	2.95
34	2.96	2.97	2.99	3.01	3.03	3.04	3.05	3.07	3.08	3.11
35	3.12	3.13	3.15	3.16	3.17	3.19	3.20	3.21	3.23	3.25
36	3.27	3.28	3.29	3.31	3.32	3.35	3.36	3.37	3.39	3.41
37	2.43	3.44	3.45	3.47	3.48	3.51	3.52	3.53	3.55	3.57
38	3.59	3.60	3.61	3.63	3.64	3.67	3.68	3.71	3.72	3.75
39	3.76	3.77	3.79	3.80	3.81	3.84	3.85	3.87	3.88	3.91
40	3.92	3.93	3.96	3.97	3.99	4.00	4.01	4.03	4.05	4.07

Exercise 5.3

At the global scale the waterbalances of oceans and landsurfaces are mutually in balance. The landsurface amounts to 30% of the earthsurface. Above land the mean annual rainfall amounts ca. 800 mm and the evapotranspiration ca. 450 mm.
Does the mean annual rainfall above the oceans exceed or not exceed the mean annual evaporation above the oceans?
How many mm's/year is the mean difference above the oceans?

Exercise 5.4

We consider two identical crops, growing optimally due to sufficient moisture supply by sprinkling irrigation.
One crop receives each 4 days and the other crop each 8 days a watergift.
Is there any difference in evapotranspiration to be expected between the two crops? Explain your answer.

Exercise 5.5

Which term in the three surface balances do you consider to be the driving mechanism of all processes which determines the other terms? Explain your answer.

Exercise 5.6

In the empirical formula to calculate the net longwave radiation the value of ℓ amounts 0.0067 ($mbar^{-1/2}$).
Give the value of ℓ if expressed in mm mercury (Hg).

Exercise 5.7

The evapotranspiration is normally expressed in mm's water depth by hydrologists and in W/m^2 by meteorologists.
If the evapotranspiration amounts 4 mm day^{-1}, calculate the equivalent evaporative flux in $W\ m^{-2}$.

Exercise 5.8

Formulate the nightly radiation balance.
Is Q^* positive (+ to surface) or negative?

Exercise 5.9

We consider three different surfaces: water, pine forest and grass.
Give the sequence from max to min, for the three surfaces if we respectively consider $K\uparrow$, Q^* and heat flux to the medium (soil or water) below the surface.

Answer to exercise 5.1

a. In descending order according to annual evapo(transpi)ration: pine forest > deciduous forest > heather > sand.
Pine forest is an evergreen crop with relatively high interception-evaporation. Deciduous forest loses its leaves for a period of about 5 to 6 months a year. During that period evaporation is low due to the very limited amount of interception.
However in its green period deciduous forest evapo-transpires more than pine forest, but it does not compensate enough for exceeding the annual total of pine forest.
Heather is a drought-resistent crop, which consumes water very economically. The total annual evapotranspiration will be lower than the forests, but higher than the bare sand. Evaporation from bare soil will only be high during a short (hours) period after rainfall. As soon as there is a dry topsoil (≈ cm's) evaporation will be dramatically restricted.

b. $N = 800$ mm year^{-1} and $[Cl^-]_{rain} = 6$ mg l^{-1}.
The $[Cl^-]_{gr.w.} = 24$ mg l^{-1} and when we assume that all chloride comes from precipitation this means that only one quarter of the total annual precipitation reaches the groundwater. So the actual evapotranspiration of the forests on the glacial ridges is $3/4*800 = 600$ mm year^{-1} .

Answer to exercise 5.2

a. As an example the reference crop-evapotranspiration will be calculated for the first decade of January. The evapotranspiration for all decades during 1982 at the Assink Meteo Station is given in the table hereafter.
First decade of January:
$K = 31$ W m^{-2}, $T = 0.7$ °C.
If $T = 0.7$ °C then according to table 5.5.2: $s = 0.47$ hPa K^{-1}.
This results in $E_r = 3.32*10^{-6}$ kg m^{-2} s^{-1}.
Now 1 kg water per m² ≈ 1 l water per m² ≈ 1 mm water
Therefore $E_r = 3.32*10^{-6}$ kg m^{-2} s^{-1} ≈ 86400 * ($3.32*10^{-6}$) ≈ 0.3 mm day^{-1} = 3 mm decade^{-1}

Calculation of the reference evapotranspiration according to Makkink for the Assink Meteo Station (1982).

	January			February			March		
decade	1	2	3	1	2	3	1	2	3
K↓ ($W\ m^{-2}$)	31	49	19	51	49	72	77	97	123
T (°C)	0.7	-2.1	2.9	3.7	3.4	0.1	4.5	5.4	6.0
s (mbar °C-1)	0.47	0.39	0.53	0.55	0.55	0.45	0.59	0.61	0.64
λ ($10^5\ J\ kg^{-1}$)	25.0	28.3	24.9	24.9	24.9	25.0	24.9	24.8	24.8
E ($10^{-5}\ kg\ m^{-2}s^{-1}$)	3.32	4.14	2.19	5.77	5.77	7.52	9.43	12.12	15.75
E (mm day^{-1})	0.3	0.4	0.2	0.5	0.5	0.6	0.8	1.0	1.4
E (mm $decade^{-1}$)	3	4	2	5	5	5	8	10	15
	April			May			June		
decade	1	2	3	1	2	3	1	2	3
K↓ ($W\ m^{-2}$)	139	215	160	158	248	220	259	151	186
T (°C)	8.8	6.0	7.7	7.5	14.5	15.4	20.0	12.8	15.6
s (mbar °C-1)	0.77	0.64	0.72	0.71	1.07	1.12	1.44	0.97	1.13
λ ($10^5\ J\ kg^{-1}$)	24.7	24.8	24.7	24.8	24.6	24.6	24.5	24.7	24.6
E ($10^{-5}\ kg\ m^{-2}s^{-1}$)	19.56	27.53	21.81	21.35	40.30	40.30	46.90	23.50	30.85
E (mm day^{-1})	1.7	2.4	1.9	1.8	3.5	3.5	4.1	2.0	2.7
E (mm $decade^{-1}$)	17	24	19	18	35	35	41	20	27
	July			August			September		
decade	1	2	3	1	2	3	1	2	3
K↓ ($W\ m^{-2}$)	216	261	201	167	171	140	150	138	107
T (°C)	17.3	19.9	18.4	20.2	16.6	14.6	15.9	16.0	14.5
s (mbar °C-1)	1.27	1.44	1.32	1.45	1.20	1.07	1.16	1.16	1.07
λ ($10^5\ J\ kg^{-1}$)	24.6	24.5	24.5	24.5	24.6	24.6	24.6	24.6	24.6
E ($10^{-5}\ kg\ m^{-2}s^{-1}$)	37.44	47.26	35.37	30.30	28.99	22.75	25.12	23.11	17.39
E (mm day^{-1})	3.2	4.1	3.1	2.6	2.9	2.0	2.2	2.0	1.5
E (mm $decade^{-1}$)	32	41	34	26	29	22	22	20	15
	October			November			December		
decade	1	2	3	1	2	3	1	2	3
K↓ ($W\ m^{-2}$)	74	63	54	37	30	20	23	15	18
T (°C)	12.1	11.0	10.6	10.1	7.0	6.3	3.4	2.7	3.3
s (mbar °C-1)	0.92	0.88	0.85	0.83	0.69	0.65	0.55	0.53	0.55
λ ($10^5\ J\ kg^{-1}$)	24.7	24.7	24.7	24.7	24.8	24.8	24.9	24.9	24.9
E ($10^{-5}\ kg\ m^{-2}s^{-1}$)	11.27	9.41	7.95	5.39	3.99	2.58	2.71	1.73	2.12
E (mm day^{-1})	1.0	0.8	0.7	0.5	0.3	0.2	0.2	0.1	0.2
E (mm $decade^{-1}$)	10	8	8	5	3	2	2	1	2

E_{total} (mm $year^{-1}$) = 570

Potential evapotranspiration (E_p) of grass and cereals during the 1982 growing season.

	April			May			June		
decade	1	2	3	1	2	3	1	2	3
Er *mm decade^{-1})	17	24	19	18	35	35	41	20	27
f_c grass	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
E_p (mm decade^{-1})	17	24	19	18	35	35	41	20	27
f_c cereals	0.7	0.8	0.9	1.0	1.0	1.0	1.2	1.2	1.2
E_p (mm decade^{-1})	12	19	17	18	35	35	49	24	32
	July			Augustus			September		
decade	1	2	3	1	2	3	1	2	3
E_r (mm decade^{-1})	32	41	34	26	29	22	22	20	15
f_c grass	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9
E_p (mm decade^{-1})	32	41	34	26	29	20	20	18	14
f_c cereals	1.0	0.9	0.8	0.6					
E_p (mm decade^{-1})	32	37	27	16					

E_p total (mm)	
grass:	465
cereals:	354

Answer to exercise 5.3

800 mm (rainfall)

Land 30% $\longrightarrow$ Oceans (70%)

450 mm (evaporation) 350 mm (runoff)

Land: Rainfall - Evaporation - Discharge = Δ Storage (Mean: $\Delta S = 0$)

Ocean: idem (Mean: $\Delta S = 0$)

$\longrightarrow$ Ocean: Rainfall - Evap + 350 x 3/7 = 0

$\longrightarrow$ Rainfall - Evap = <u>-150 mm</u>

Evap > Rainfall

Answer to exercise 5.4

Evapotranspiration by each crop is determined by:

1) transpiration of the crop E_t (4 days irrigation) $\approx$ E_t (8 days irrigation)
2) evaporation of the soil E_s (4 days irrigation) > E_s (8 days irrigation)
3) interceptie-evaporation E_i (4 days irrigation) > E_i (8 days irrigation)

$\longrightarrow E_{act}$ (4d) > E_{act} (8d)

Answer to exercise 5.5

The incoming short wave radiation
K↓ is responsible for a positive net radiation (Q^*), which in turn drives the energy balance and the atmospheric watercycle (evapotranspiration → rainfall).

Answer to exercise 5.6

$$\ell = 0.0067 \frac{1}{mbar^{1/2}} \qquad 1\ mbar = 0,76\ mm\ mercury\ (Hg)$$

$$\rightarrow \sqrt{mbar} = \sqrt{0.76\ mmHg} = 0.872\ \sqrt{mmHg}$$

$$\rightarrow \ell = 0.0067\ x \frac{1}{0.872\ \sqrt{mmHg}} = 0.0077\ (mmHg)^{-1/2}$$

Answer to exercise 5.7

$$4\ mm/day \overset{?}{\rightarrow} \frac{W}{m^2} \qquad \frac{4\ mm}{day} = \frac{0.004\ m}{day} = \frac{0.004\ m^3}{day,\ m^2}$$

$$= \frac{4\ kg}{day, m^2} = \frac{4 \times 2450\ kJ}{day, m^2} = \frac{4 \times 2450\ kJ}{86400\ sec,\ m^2}$$

$$= 113,4 \frac{J}{sec,\ m^2} = 113,4 \frac{W}{m^2}$$

Answer to exercise 5.8

K↓ and K↑ are zero

→ L↓ + L↑ = Q^* L↓ < L↑ → Q^* is negative.

Answer to exercise 5.9

	K↑	Q^*	$H_{(medium)}$
water	min	max	max
grass	max	min	intermediate
pine forest	intermediate	intermediate	min

Examenvraagstukken bij hoofdstuk 5

5.1

Volgens figuur 5.4.4 (dictaat) bedraagt op 21 juni de $K_A\downarrow$-waarde in Nederland ca. 1000 cal $cm^{-2}dag^{-1}$. Indien op genoemde datum bij onbewolkte hemel de verdamping de typische waarde van 4,5 mm dag^{-1} bereikt, hoeveel % van de aangeboden energie aan de rand van de atmosfeer is dan omgezet in verdampingswarmte? Gegeven:

$$K\downarrow = K_A\left(0{,}2 + 0{,}48\frac{n}{N}\right)$$

5.2

We gaan nogmaals uit van 21 juni, een onbewolkte dag $\frac{n}{N} = 0{,}95$

met een gemiddelde etmaal temperatuur van 15° C. Van een uitgestrekt (ha's) grasoppervlak wordt die dag de actuele verdamping gemeten en blijkt, gesommeerd over de dag, 3,2 mm te bedragen.

Vragen: Welke methoden komen in aanmerking voor bepaling van het verloop van de actuele verdamping over de dag.

Is de actuele verdamping lager dan de referentie verdamping en hoeveel bedraagt de eventuele reductie in mm's.

5.3 (juni 1992)

a. Geef de grootte-volgorde van de energiebalans termen gesommeerd voor de overdagperiode van een zonnige dag.
   - van een goed van bodemvocht voorzien hoog maisgewas
   - van een woestijngebied

b. De volgende formule is gegeven

$$?(4) = \left(\frac{V_{act}}{V_{max}}\right)^{va} x\ \underbrace{f\ x\ \underbrace{0{,}65\ x\ \frac{s}{s+\gamma}\ \underbrace{K_A\left(a+b\frac{n}{N}\right)}_{?(1)}}_{?(2)}}_{?(3)}$$

- Benoem de diverse vraagtekens

- Benoem de afzonderlijke grootheden in de vergelijking

c. - Op welke soorten van oppervlakken is het evaporatieproces van toepassing?

- Omschrijf per oppervlak het verloop van het evaporatieproces in de tijd na een flinke regenbui in de vroege ochtend.

5.4 (augustus 1992)

a. Geef de drie vergelijkingen van de afzonderljke balansen aan het aardoppervlak.
b. Waarom verandert de nettostralingsintensiteit van positief overdag naar negatief 's nachts?
c. Noem twee belangrijke oorzaken waardoor voor landbouwgewassen de actuele verdamping achter kan blijven bij de referentiegewasverdamping.
d. Stel, we bekijken het tropisch regenbos en een semi-aride* gebied. Wat kun je zeggen over de mate van sterkte van de koppeling tussen waterbalans en energiebalans voor elk van deze gebieden.

*(droog seizoen en een (relatief kort) nat seizoen).

5.5 (augustus 1991)

Ten behoeve van de tuinbouw berekent men de verdamping in kassen met de gemeten netto straling RN.

a. Welke term in de basisformule van Penman mag nu worden verwaarloosd en waarom?
b. Bepaal bij benadering het verband tussen de netto straling en de verdamping, als Hbodem 0.
c. Bereken bij benadering de verdamping uit b. (in mm d^{-1}) als gegeven is:

$Q^* = 9.106$ J $m^{-2}d^{-1}$

$L = 2400.103$ J kg^{-1}

$\gamma = 0.66$ mb K^{-1}

$s = 1.34$ mb K^{-1}

$\rho = 1000$ kg m^{-3}

5.6 (mei 1991)

In het zuiden van Mozambique is in 1988 een stuwdam gereed gekomen in de (kleine) Umbeluzi rivier. Achter de dam is een stuwmeer ontstaan van 40 km^2.

Hoofddoelstelling is het waarborgen van een continue bron voor de (drink)-watervoorziening van de 20 km stroomafwaarts gelegen hoofdstad Maputo (aftappunt ligt benedenstrooms van de dam).

De belangrijkste nevendoelstelling is de ontwikkeling, door middel van irrigatie, van ca. 10000 ha landbouwgrond.

Gegevens:

- de gemiddelde neerslag (in het meer) bedraagt 730 mm j^{-1}.
- gemiddelde rivierafvoer (= aanvoer in het stuwmeer): 8,0 m^3 s^{-1}
- hoeveelheid benodigd voor de drinkwatervoorziening: 2,8 m^3 s^{-1}
- gemiddelde open water verdamping E_o = 4 mm d^{-1}
- minimale afvoer voorbij het inname punt van de watervoorziening (noodzakelijk voor de preventie van zout opdringing en t.b.v. ecologische eisen): 1 m^3 s^{-1}
- stel de netto irrigatiebehoefte gelijk aan de gewasverdamping E_{act} = 0,8 E_o
- de irrigatie verliezen bedragen 40% van de bruto aanvoer t.b.v. irrigatie.
- ondergrondse verliezen uit het stuwmeer zijn te verwaarlozen.
- verliezen tussen de stuwdam met het innamepunt van de drinkwatervoorziening, zijn gemiddeld over het jaar te verwaarlozen.

a. Onder welke balans-condities van het stuwmeer kan de nevendoelstelling worden gerealiseerd.
   Stel daarvoor de vergelijking op.

b. Hoeveel ha kan er maximaal geïrrigeerd worden?
   Wordt de nevendoelstelling wel of niet gehaald.

6 GROUNDWATER

Exercise 6.1

a. Write a definition for the hydraulic head (H).
b. Why is H not a proper measure for the potential in the case of multi-fluid problems?
c. Write the potential (Ψ), the pressure equivalent of the potential (P*) and the velocity potential (Φ) as a function of the hydraulic head (H).

Exercise 6.2

A permeable soil with a depth of 3.00 m lies upon an impervious layer. The groundwater is at rest and the phreatic level reaches 2.00 m above the impervious layer. In this soil two piezometers ($\varnothing$ 0.02 m) have been installed with a filter at respectively 0.50 and 1.50 m above the impermeable layer.

a. Sketch the height of the waterlevels inside the piezometers.
b. Calculate the magnitude of Ψ, H and P*.
   (choose the top of the impermeable layer as the reference level).

Exercise 6.3

The kinetic energy of flowing water can be expressed in the velocity head (h_v) and is calculated according to:

$$h_v = \frac{v^2}{2g} \tag{6.3.1}$$

Typical groundwater velocities are in the order of 1 m day^{-1}.
Calculate the velocity head for this case. Is it reasonable to neglect this velocity head?

Exercise 6.4

An important formula in groundwater flow is the formula given as:

$$v = -k\ \nabla H \qquad (6.4.1)$$

a. Which is the name of this formula?

b. What is the meaning of the minus sign?

c. What is the meaning of the symbols v, k, H and ∇H?

Exercise 6.5

Given a system as indicated in figure 6.5.1. A constant difference in water level of 0.20 m is maintained during the experiments; H_1 = 1.10 m, H_2= 0.90 m above the reference level.

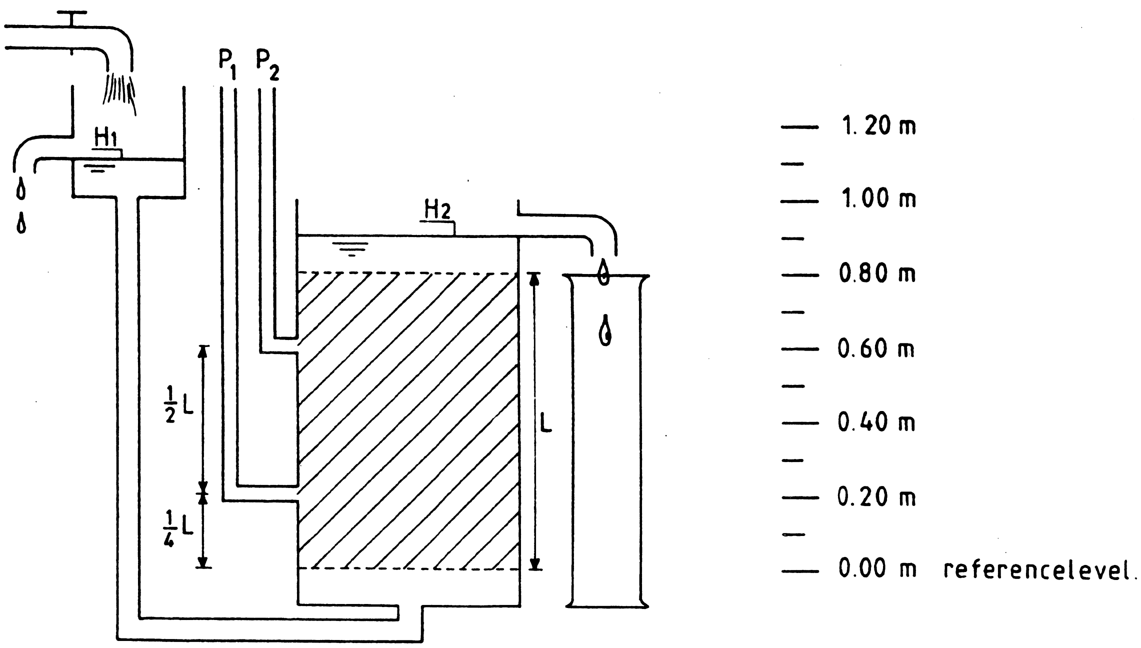


Figure 6.5.1

In the column with a cross-section $A = 0.02\ m^2$ a soil sample is placed with a length $L = 0.80$ m. The pore fraction of the soil sample $n = 0.15$. The discharge measured is $Q = 0.008\ m^3\ day^{-1}$.

a. Calculate the hydraulic conductivity (k) of this soil sample.
b. Calculate the heights (above the reference level) of the water in the tubes P_1 and P_2.
c. Calculate the flux density (v) and the effective or pore velocity (v_e) of the water in the soil column.

Exercise 6.6

Given a system as indicated in figure 6.6.1 in which a constant waterlevel difference is maintained.

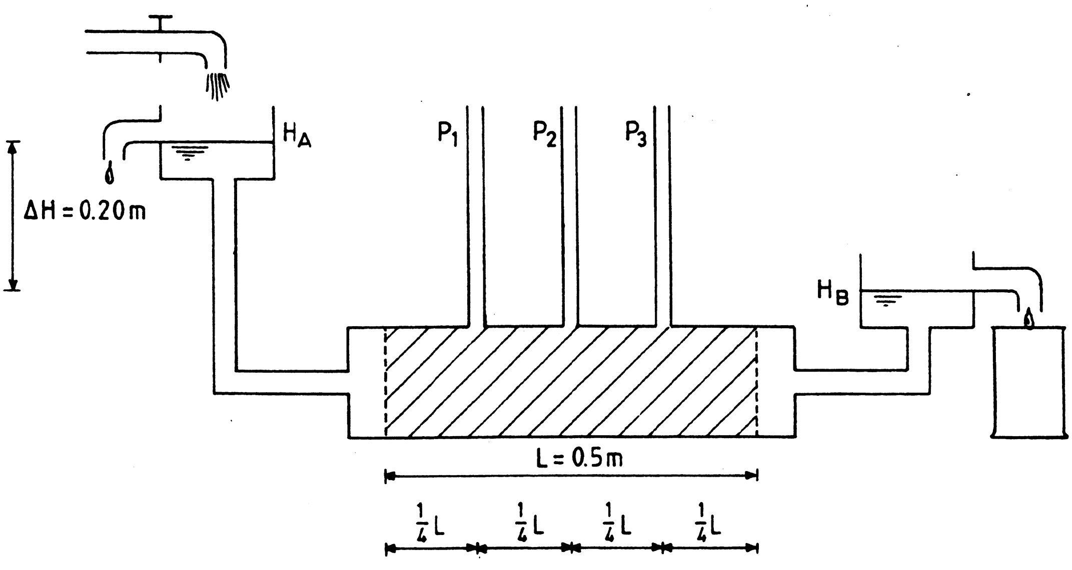


Figure 6.6.1

$H = 0.20$ m

$A = 0.04\ m^2$

$L = 0.50$ m

Three piezometers are installed at:

$P_1 = 0.25L = 0.125$ m

$P_2 = 0.5\ L = 0.25$ m

$P_3 = 0.75L = 0.375$ m.

(All resistances, except those occurring in the soil column, may be neglected).

a. The soil column has a hydraulic conductivity of $k = 0.5$ m day^{-1}.
   Calculate the discharge (Q), the percolation velocity (v) and the height of the waterlevels in the three piezometers.

b. Assume that the column is filled with two types of soil in such a way that the boundary between the layers is parallel to the length of the soil column. The hydraulic conductivities of the soil types are respectively $k_1 = 0.2$ m day^{-1} and $k_2 = 0.8$ m day^{-1}. ($A_1 = A_2 = 0.02$ m^2).
   Calculate Q and v of each layer and the height of the waterlevels in the three piezometers.

c. Assume that the same soil types are now brought into the column in such a way that the boundary between the soil types lies at the middle of the soil column and perpendicular to the length of the column; $L_1 = L_2 = 0.25$ m.
   Calculate Q and v of each layer and the height of the waterlevels in the three piezometers.

d. Now assume that the column is filled with three types of soil in the same way as under b. The hydraulic conductivities of the soil types are respectively $k_1 = 0.8$ m day^{-1}, $k_2 = 0.05$ m day^{-1} and $k_3 = 0.2$ m day^{-1}. Furthermore $A_1 = 0.02$ m^2, $A_2 = 0.004$ m^2 and $A_3 = 0.016$ m^2.
   Calculate Q and v of each layer and the height of the waterlevels in the three piezometers.

e. The column is now filled with the same three soil types as under d, but in the same way as under c. The lengths of the layers are respectively $L_1 = 0.24$ m, $L_2= 0.02$ m and $L_3 = 0.24$ m.
   Calculate Q and v of each layer and the height of the waterlevels in the three piezometers.

f. Compare the results of b, c, d and e.

g. Which value of ΔH must be chosen to obtain $Q = 8$ l day^{-1} in the situation under d. and in the situation under e?

Exercise 6.7 (MSc exam March 1988)

a. What is the Dupuit assumption in groundwater flow?
b. In the case of a phreatic aquifer what is the particular advantage of the Dupuit assumption?
c. Prove that in groundwaterflow under the Dupuit assumption the variation of the fluidpressure P as a function of z is the same as in a hydrostatic pressure distribution.

Exercise 6.8

Water is flowing in a steady state (in the saturated zone) between two parallel canals as shown in figure 6.8-1.

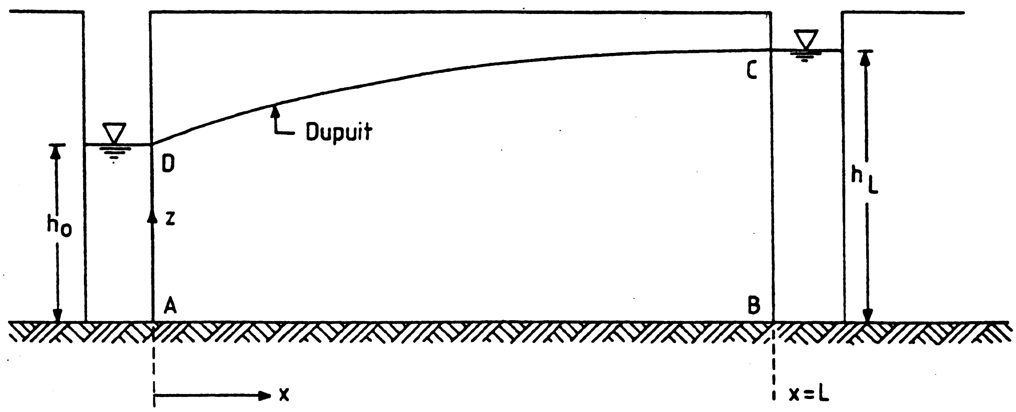


Figure 6.8-1 Groundwaterflow between parallel canals.

a. Use the Dupuit assumption and the data given below, to compute the amount of water (m^3) flowing into the lefthand canal per day and per meter length.
   Hint: find the equation of h as a function of x first.
b. What is the water level (above AB) at x = 50.0 m?
c. Estimate the traveltime for a particle from BC to AD.

data:
L = 100.0 m

k = 5.0 m d^{-1}
n_e = 0.25 (effective porosity)
h_0 = 4.0 m
h_L = 5.0 m.

Exercise 6.9

a. Derive the formula for steady flow to a well in phreatic water above an impermeable layer. The thickness of the water bearing layer is now equal to the hydraulic head (H) (reference level is chosen at the top of the impermeable layer).
b. Show that the formula is equivalent to the formula for flow towards a well in a confined layer when H is large in respect to the changes in H.

Exercise 6.10

A water supply company pumps each year $2*10^6$ m^3 water from a well in a confined aquifer with a thickness (D) of 30.0 m and a effective porosity (n_e) of 0.35. Calculate the time of residence (T) of a water particle at a distance of 1231 m from the well.

Exercise 6.11

A water supply company wants to start a new pumping station that will pump $1.5*10^6$ m^3 $year^{-1}$ from an unconfined aquifer. The phreatic water bearing layer has a thickness (D) of 40.0 m above an impermeable base and a hydraulic conductivity (k) of 30 m day^{-1}. The phreatic level in the existing situation is 39.60 m above the impermeable layer. In a steady state the lowering of the groundwater table will be 0.00 m at a distance of 2 km from the pumping station.
The pumping station is located close to a nature reserve and its caretaker is worried that the lowering of the groundwater table will have serious consequences for the marsh flora in the reserve.

a. Calculate the (steady) lowering of the groundwater table in the nature reserve if the pumping station will be 500 m away from the reserve. Should the caretaker be worried?
b. The possibility exists to transport surface water of acceptable quality into the reserve. This water can be infiltrated in the area around the pumping station by means of a network of ditches. What will be the amount of water in m^3 day^{-1} needed to limit the lowering of the groundwater table to 0.20 m?

Exercise 6.12

The phreatic level in a polder is NAP -4.20 m whereas the hydraulic head of the deeper groundwater is NAP -2.20 m.
Calculate the seepage in the polder when the resistance of the top boundary of the aquifer consisting of holocene peat- and claylayers is 1000 days.

Exercise 6.13 (Exam June 1992)

A circular island (R=200 m) is located in a fresh water lake, the soil surface is 2.0 m above the lake level. From a well (effective radius r_0 = 0.15 m) in the centre of the island water is withdrawn continuously at the rate of 20.0 m^3 per day. The soil profile of the island consists of two layers of which the thickness and permeability have been measured:
- layer 1 from soil surface to -8.0 m below soil surface, permeability 2.0 m/d
- layer 2 from -8.0 m to -12.0 m below soil surface, permeability 10.0 m/d

The bottom of the lake is the top of an impermeable layer, which is also found underneath the island. The well reaches down to this impermeable base.
a. Make a schematic drawing of the situation and compute the water level in the well.
b. Give a sketch of the hydraulic head in layer 1 and 2. Explain the shape of the function h(r). Are they different? If so why?

c. How many m^3 flow through layer 1 per day?

d. If the lake became seriously contaminated on July 1 1992, when would the contamination arrive in the well? The effective porosity of both layers is 0.33.

e. What would the travel time be, if layer 2 were 2.0 m thick?

Hint: Well flow (based on the Dupuit assumption) in a homogeneous aquifer of constant thickness D can be described by:

$$H_2 - H_1 = \frac{Q}{2\pi kD} \ln \frac{r_2}{r_1}$$

Exercise 6.14 (Exam December 1991)

Water is flowing (from east to west) in a confined aquifer, porosity = 0.30, permeability = 10.0 m/d. Two piezometers are placed 3000 m apart, one measures 8.5 m + MSL (reference level), the other 7.0 m + MSL.

a. Make a schematic drawing of the situation and compute the discharge (m^3/d) through 1 kilometer cross-section (perpendicular to the direction of flow), if the thickness of the aquifer = 50.0 m.

b. Compute the travel-time of a fluid particle between the two piezometers.

c. Suppose now that the thickness of the aquifer varies linearly from east to west, according to: D = 35.0 + 0.01 x', where x' = 0 m at the filter on the east side and x' = 3000 m at the filter on the west side. Compute the discharge (m^3/d) per km cross-section. You may assume Dupuit flow.

d. Compute the travel-time for the situation under c.

Exercise 6.15 (MSc exam May 1988)[1)]

The hydraulic conductivity of a soil sample is tested in the laboratory with a simple apparatus as shown in figure 6.15-1 a so called falling head permeameter. The amount of water above the sample is not replenished.

1) not for K150-001

During the experiment the height of the water above the sample is measured. The outflow at the bottom is free and at atmospheric pressure.

a. Is this a steady or a non-steady flow?
b. Derive the equation for the conductivity (k) as a function of h,L and time (t) (so k = f(h,L,t)).
c. Compute the hydraulic conductivity (k) in m day^{-1} on the basis of the following data:

$t = 0$ $\quad h_0 = 10.00$ cm $\quad L = 20.00$ cm

$t = 5$ min. $\quad h_5 = 8.19$ cm

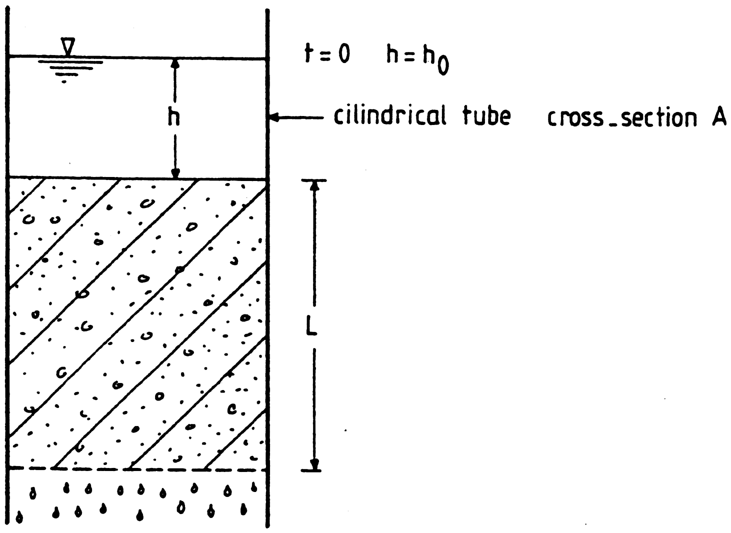


Figure 6.15-1

Exercise 6.16[2)]

Numerical methods have superseded most other methods in solving groundwater flow especially ,since the emergence of the computer. In this exercise an example will be given of a simple numerical method.

[2)] not for K150-001/002

The flow region for which the groundwater flow is to be calculated is divided into squares with side length a. For the stream function holds the Laplace equation $\nabla^2\Psi = 0$. In two dimensions this can be written as:

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} = 0 \tag{6.16.1}$$

As an approximation, $\delta^2\Psi/\delta x^2$ can be written as:

$$\frac{\partial^2\Psi}{\partial x^2} \approx \frac{(\Psi_1 - 2\Psi_0 + \Psi_3)}{a^2} \tag{6.16.2}$$

and in the same way:

$$\frac{\partial^2\Psi}{\partial y^2} \approx \frac{(\Psi_2 - 2\Psi_0 + \Psi_4)}{a^2} \tag{6.16.3}$$

Combining 6.16.2 and 6.16.3 results in:

$$\frac{(\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 - 4\Psi_0)}{a^2} = 0 \tag{6.16.4}$$

or:

$$\Psi_0 = \frac{\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}{4}$$

We will consider groundwater flow from two canals through a confined aquifer to a drain (see figure 6.16.1.a). If the canal wall AB is at sufficient distance from drain D the inflow along AB increases almost in a linear way with depth. We divide the area ABCD into 24 squares (see figure 6.16.1.b).

- AB is a streamline for which we assume $\Psi = 0\%$.
- BCD is a streamline for which we assume $\Psi = 100\%$.
- as inflow along AB is almost linear we may assume:
  for P: $\Psi = 25\%$
  for Q: $\Psi = 50\%$
  for R: $\Psi = 75\%$
- in other points the values for Ψ are estimated.

$$\begin{array}{ccc} & \circ\,\psi 2 & \\ \psi 1\,\circ & \circ\,\psi 0 & \circ\,\psi 3 \\ & \circ\,\psi 4 & \end{array} \qquad \psi 0 = \frac{\psi 1 + \psi 2 + \psi 3 + \psi 4}{4}$$

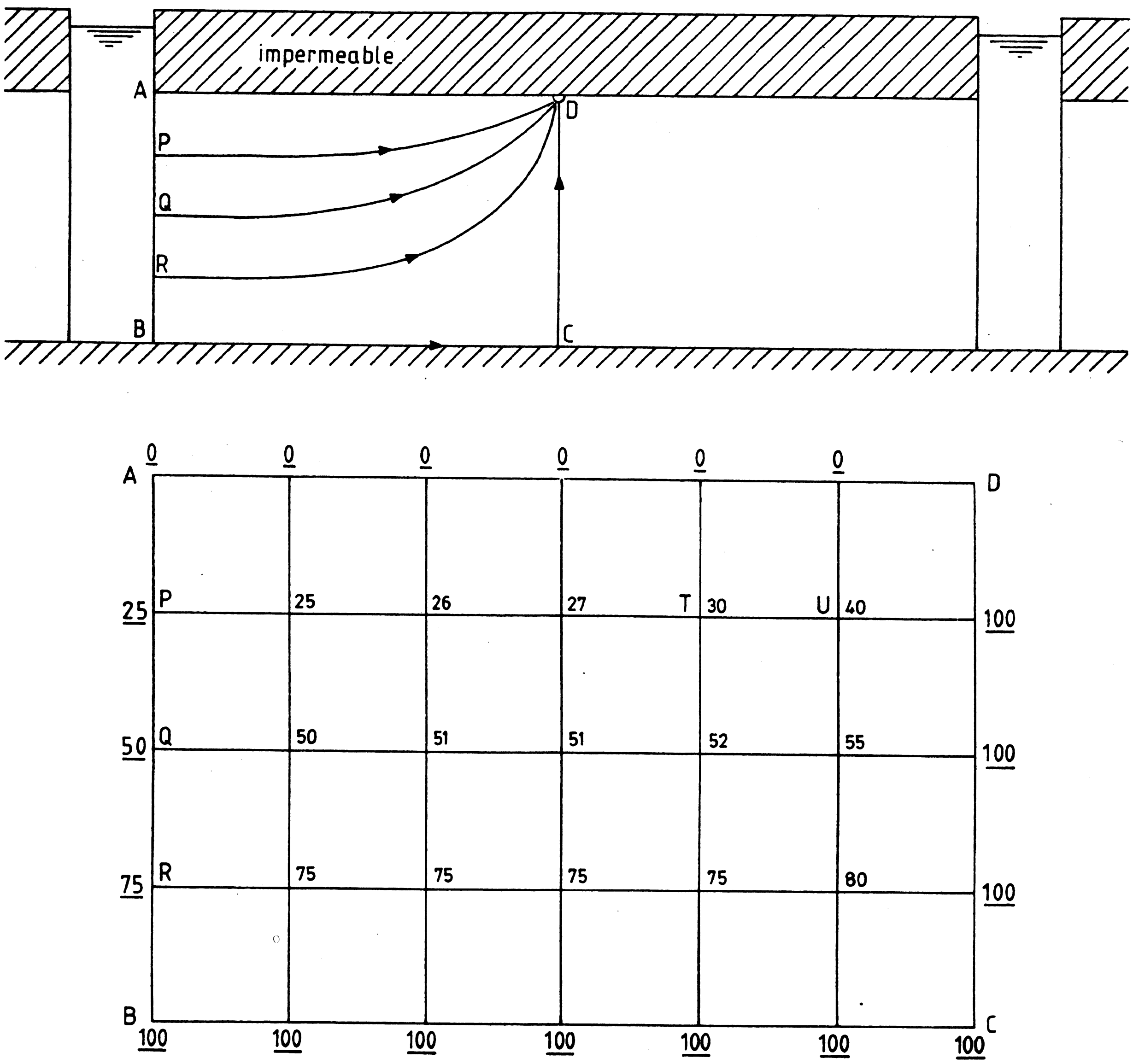


Figure 6.16.1 Numerical method for solving a flow problem.

a. Starting at point U we apply:

$$\Psi_u = \frac{\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}{4} = 46.25$$

Now for all points within the boundaries we apply this rule going left from U then one line down etc.. Then after the last point has been done start again at U. You can stop after 5 times. With a computer this simple procedure can be repeated until the required accuracy is achieved.

(Of course the boundaries are not recalculated because these are already known values).

b. Draw the streamlines for 25% 50% and 75%.

Answer to exercise 6.1

a. The hydraulic head (H) is a measure for the potential of the groundwater and is defined as energy per weight [J N^{-1} = m] in relation to a certain reference level.

$$H = z + \frac{P}{\rho g}$$

with:

H	= hydraulic head	[m]
z	= elevation head	[m]
P	= fluid pressure relative to P_{atm}	[Pa]
ρ	= density of the liquid	[kg m^{-3}]
g	= acceleration of gravity	[m s^{-2}]

b. The two assumptions underlying the use of H are:
- g = constant
- ρ = constant

In the case of multi-fluid problems the second condition is not met and therefore H may not be used.

c. $\Psi = gz + P/\rho = gH$ [J kg^{-1}]
$P^* = \rho gz + P = \rho gH$ [Pa]
$\Phi = kH$ [m^2 s^{-1}]

Answer to exercise 6.2

a. The groundwater reaches in both piezometers a height of 2 m above the impervious layer as drawn in figure 6.2.1.

b. $\Psi_A = gz_A + P_A/\rho = 0.50\ g + (1.50\ \rho g)/\rho = 2.00\ g = 20.0$ J kg^{-1}.
$H_A = z_A + P_A/(\rho g) = z_A + h_{pA} = 2.00$ m or $H_A = \Psi_A/g = 2.00$ m.
$P_A^* = \rho g\ z_A + P_v = \rho g\ (z_A + h_{pA}) = 20000$ Pa or
$P_A^* = \rho g\ H_A = 20000$ Pa
In the same way:

$\Psi_B = gz_B + P_B/\rho = 20.0 \text{ J kg}^{-1}$

$H_B = 2.00 \text{ m}$

$P_B{*} = 20000 \text{ Pa}$

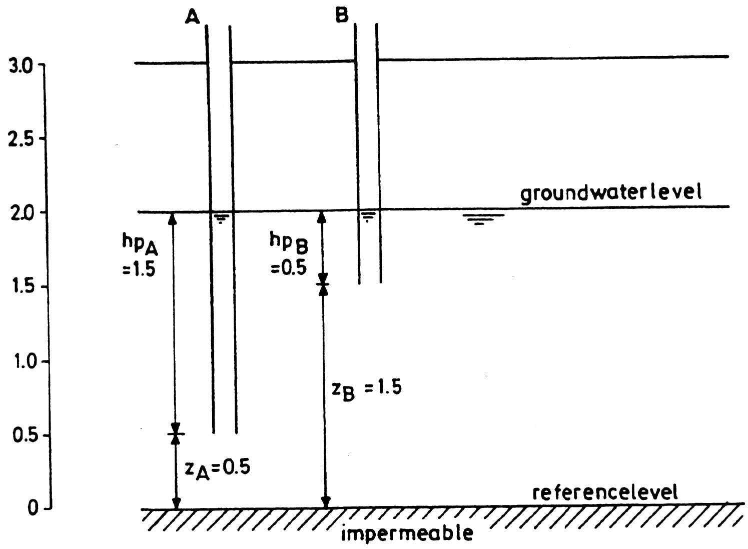


Figure 6.2.1

Answer to exercise 6.3

$v = 1 \text{ m day}^{-1} = 1/86400 \approx 1.16{*}10^{-5} \text{ m s}^{-1}$

so:

$h_v = (1.16{*}10^{-5})^2/(2{*}10) = 6.7{*}10^{-12} \text{ m}$

The gradient of the hydraulic head (H) is typically 10^{-3} to 10^{-4} m m^{-1} (for Dutch conditions). The velocity head (h_v) is so small compared to these gradients that it can be neglected.

Answer to exercise 6.4

a. Darcy's law written as a 'point' equation.

b. Water flows from a place where H is high to a place where H is low. The minus sign indicates that the direction of flow is opposite to the direction in which H increases so in order to obtain a positive quantity for v when it is directed in the positive x-direction a minus sign is needed.

c.

v	=	flux density	[m day^{-1}]
k	=	hydraulic conductivity	[m day^{-1}]
H	=	hydraulic head	[m]
∇H	=	gradient of the hydraulic head	[m m^{-1}]

Answer to exercise 6.5

a. Darcy's law $Q = kiA$ so $k = Q/(iA)$.
Here $i = \Delta H/L = 0.25$ m m^{-1} $Q = 0.008$ m^3 day^{-1} and $A = 0.02$ m^2 so $k = 1.6$ m day^{-1}.

b. Because i is constant along the soil column it follows that:
$H_{P1} = H_1 - 0.2i = 1.10 - 0.2i = 1.05$ m
$H_{p2} = H_1 - 0.6i = 0.95$ m.

c. The flux density (v) is:
$v = ki = Q/A = 0.4$ m day^{-1} .
The effective velocity (v_e) is larger than the flux density and is:
$v_e = v/n = 0.4/0.15 = 2.67$ m day^{-1}.

Answer to exercise 6.6

a. Darcy's law:

$Q = kiA$

Here:

$k = 0.5$ m day^{-1}

$i = \Delta H/L = 0.20/0.50 = 0.40$ m m^{-1}

$A = 0.04$ m^2

So:

$Q = 0.008$ m^3 day^{-1} and $v = ki = Q/A = 0.2$ m day^{-1}

$i = 0.40$ m m^{-1} is constant along the column so

$H_{P1} = H_A - 0.125i = 0.20 - 0.125*0.40 = 0.15$ m

$H_{P2} = H_A - 0.25\ i = 0.10$ m

$H_{P3} = H_A - 0.375i = 0.05$ m

b. $A_1 = A_2 = 0.02$ m^2

i = constant = $\Delta H/L = 0.40$ m m^{-1}

So:

$Q_1 = k_1 i A_1 = 0.0016$ m^3 day^{-1}

$Q_2 = k_2 i A_2 = 0.0064$ m^3 day^{-1}

$Q_{tot} = 0.0016 + 0.0064 = 0.0080$ m^3 day^{-1}

and:

$v_1 = k_1 i = Q_1/A_1 = 0.08$ m day^{-1}

$v_2 = 0.32$ m day^{-1}

$v_m = Q_{tot}/A_{tot} = 0.0080/0.04 = 0.2$ m day^{-1}

(v_m = mean flux density [m day^{-1}])

Because i is constant and has the same value as under a it follows that H_{P1}, H_{P2} and H_{P3} are also the same as under a.

c. $A_1 = A_2 = 0.04\ m^2$

$\Delta H_1 + \Delta H_2 = 0.20\ m$

$Q_1 = Q_2$ because what flows out of soil 1 must flow into soil 2 (there is no change of storage of water in the column).

So:

$Q_1 = k_1 i_1 A_1 = 0.008\ i_1\ m^3\ day^{-1}$

$Q_2 = k_2 i_2 A_2 = 0.032\ i_2\ m^3\ day^{-1}$

because $Q_1 = Q_2$ it follows that $i_1 = 4i_2$ and

because $L_1 = L_2$ also $\Delta H_1 = 4\ \Delta H_2$.

In addition we know that $\Delta H = \Delta H_1 + \Delta H_2 = 5\Delta H_2 = 0.20$ m resulting in $\Delta H_1 = 0.16$ m and $\Delta H_2 = 0.04$ m, the discharge and velocity can now be calculated easily.

$i_1 = 0.64\ m\ m^{-1}$, $i_2 = 0.16\ m\ m^{-1}$

$Q_1 = Q_2 = 0.00512\ m^3\ day^{-1}$

$v_1 = v_2 = 0.128\ m\ day^{-1}$

In this case i is not constant along the whole column but i_1 is constant along L_1 and i_2 is constant along L_2, so:

$H_{P1} = 0.20 - 0.125*0.64 = 0.12\ m$

$H_{P2} = 0.20 - 0.25*0.64 = 0.04\ m$

$H_{P3} = 0.20 - 0.25*0.64 - 0.125*0.16 = 0.02\ m$

d. For this case the calculations proceed in the same way as under b.

$i = \text{constant} = 0.40\ m\ m^{-1}$.

So:

$Q_1 = k_1 i_1 A_1 = 0.064\ m^3\ day^{-1}$

$Q_2 = 0.00008\ m^3\ day^{-1}$

$Q_3 = 0.00128\ m^3\ day^{-1}$

$Q_{tot} = 0.00776\ m^3\ day^{-1}$

and:

$v_1 = 0.32\ m\ day^{-1}$

$v_2 = 0.02\ m\ day^{-1}$

$v_3 = 0.08\ m\ day^{-1}$

$v_m = Q_{tot}/A_{tot} = 0.194\ m\ day^{-1}$

Because i is constant and has the same value as under a it follows that H_{P1}, H_{P2} and H_{P3} are also the same as under a.

e. For this case the calculations proceed in the same way as under c.

$A_1 = A_2 = A_3 = 0.04\ m^2$
$\Delta H_1 + \Delta H_2 + \Delta H_3 = 0.20\ m$
$Q_1 = Q_2 = Q_3$

So:
$Q_1 = k_1 i_1 A_1 = k_1 A_1 \Delta H_1 / L_1 = 0.1333\ \Delta H_1\ m^3\ day^{-1}$
$Q_2 = 0.1 \Delta H_2$
$Q_3 = 0.0333\ \Delta H_3$
and because $Q_1 = Q_2 = Q_3$ it follows that:
$\Delta H_1 = 0.75 \Delta H_2$
$\Delta H_3 = 3.0 \Delta H_2$
So: $4.75 \Delta H_2 = 0.20$ results in
$\Delta H_1 = 0.0316\ m$
$\Delta H_2 = 0.0421\ m$
$\Delta H_3 = 0.1263\ m$
$Q_1 = Q_2 = Q_3 = 0.00421\ m^3\ day^{-1}$

Here i is not constant along the whole column but i_1 is constant along L_1, i_2 is constant along L_2 and i_3 is constant along L_3, so:
$H_{P1} = 0.20 - 0.125*0.316/0.24 = 0.184\ m$
$H_{P2} = 0.20 - 0.316 - 0.01*0.0421/0.02 = 0.147\ m$
$H_{P3} = 0.20 - 0.316 - 0.0421 - 0.115*0.1263/0.24 = 0.066\ m$

f. Comparing the results from b, c, d and e it is evident that the flow of water is largely dependent on the configuration of the soil layers. A thin layer with a low conductivity, lying perpendicular to the main direction of flow, determines largely the amount of water flowing through a soil (compare d and e), whereas the same layer lying parallel to the main direction of flow has only a small impact (compare b and d).

g. Under d.

To obtain Q = 0.008 m^3 day^{-1}, we use the equation:

$$Q = k \frac{\Delta H}{L} A$$

Because k, L and A are constants (for this particular soil sample), Q/ΔH is also constant. Using the Q and ΔH under d, we find Q/ΔH = 0.00776/-0.20 = 0.0388 m^2 day^{-1}. So in order to obtain Q = 0.008 m^3 day^{-1}, ΔH should be ΔH = 0.008/0.0388 = 0.206 m.

Under e:

In this case Q/ΔH = 0.00421/0.20 = 0.02105 m^2 day^{-1} and therefore H = 0.008/0.02105 = 0.380 m.

Answer to exercise 6.7

a. The Dupuit assumption states that groundwater flow mainly takes place in a horizontal direction, so the vertical flow component can be neglected.

b. The advantage of the Dupuit assumption is that the flow problem is reduced with one dimension namely the vertical component.
For example flow in a cross-section becomes one dimensional and the two variables H (hydraulic head) and h (height of the phreatic level above the reference level) are reduced to one and the same variable (H = h).

c. In case of horizontal flow the vertical gradient of the hydraulic head (dH/dz) is 0. Now:

$$H = \frac{P}{\rho g} + z$$

with:

H	= hydraulic head	[m]
P	= fluid pressure	[Pa]
ρ	= density of liquid	[kg m^{-3}]

g = acceleration of gravity [$m^2\ s^{-1}$]

z = height above reference level [m]

So:

$$\frac{dH}{dz} = \frac{1}{\rho g}\frac{dP}{dz} + 1 = 0$$

which leads to:

$$\frac{dP}{dz} = -\rho g$$

and this is the hydrostatic pressure distribution.

Answer to exercise 6.8

a. The equation of h as a function of x can be found by applying equation (6.7-2) from the lecture notes.
The solution is:

$$h^2 = -2\frac{q}{k}x + constant$$

The boundary conditions are:
$x = 0$: $h = h_0 = 4.0$ m
$x = L$: $h = h_L = 5.0$ m

From the first b.c. it follows that the integration constant equals $h_0^2 = 16$.
Using the second b.c. we find:

$$q = \frac{-k(h_L^2 - h_0^2)}{2L} = -0.225\ m^3/d$$

And the equation reads:

$$h^2 = 0.09\ x + 16$$

b. $x = 50.0\ m \rightarrow h_{50} = 4.53\ m$

c. $$T = \int_{100}^{0} n_e \frac{dx}{v_x}$$

$$v_x = \frac{q_x}{h} = \frac{-\ 0.225}{h}$$

$$T = \int_0^{100} \frac{n_e\ h}{0.225}\ dx = \frac{0.25}{0.225} \int_0^{100} (0.09\ x + 16)^{½}\ dx$$

$$= \frac{0.25}{0.225} \left\{\frac{2}{3} \cdot (0.09\ x + 16)^{3/2} \cdot \frac{1}{0.09}\right\} \Big]_0^{100} =$$

$= 1028.8 - 526.7 = 502$ dagen

Now instead of using h insert the average thickness D ≈ 4.5 m.

$$\overline{v}_x = \frac{-0.225}{4.5} = -.05\ m/d$$

$$T = \frac{100}{0.05} = 500\ d$$

In view of the uncertainties with respect to k and n_e, one can therefore safely apply the second approach, using an average v_x.

Answer to exercise 6.9

a. According to Darcy's law the discharge at a distance r from the axis of the well is given by:

$$Q' = -k\ \frac{dH}{dr}\ 2\pi rD$$

with:

Q'	=	discharge at a distance r from the well	$[m^3\ s^{-1}]$
k	=	hydraulic conductivity	$[m\ s^{-1}]$
H	=	hydraulic head	[m]
r	=	distance from the well	[m]
D	=	thickness of the water bearing layer	[m]

In this case D = H and when we only pay attention to the amount of discharge (and not to the direction of the flow), we can write:

$$Q = k\,\frac{dH}{dr}\,2\pi r H$$

or:

$$HdH = \frac{Q}{2\pi k}\,\frac{dr}{r}$$

Because we consider steady flow Q = constant so integration between H_1, H_2 and r_1, r_2 yields:

$$H_2^2 - H_1^2 = \frac{Q}{\pi k}\,\ln\frac{r_2}{r_1}$$

also known as the <u>Thiem formula</u>.

b. $(H_2^2 - H_1^2)$ can also be written as $(H_2 + H_1)(H_2 - H_1)$. The average thickness (D) of the water bearing layer between r_1 and r_2 is $D = (H_2 + H_1)/2$. If this thickness is large in respect to the changes in H we may assume that D is (almost) constant in the whole flow region.
In that case $(H_2 + H_1) = 2D \approx$ constant so the Thiem formula converts into:

$$H_2 - H_1 = \frac{Q}{2\pi kD}\,\ln\frac{r_2}{r_1}$$

and is (approximately) equal to the flow towards a well in confined water.

Answer to exercise 6.10

The total volume of water (V) in the aquifer within a certain distance (r) from the well is:

$$V = \pi n_e r^2 D$$

with:

V	=	amount of water in the aquifer	[m^3]
n_e	=	effective porosity	[-]
r	=	distance from the well	[m]
D	=	thickness of the aquifer	[m]

So in this case V = $5.0*10^7$ m^3.

A water particle at a distance (r) from the well will reach the well when all the water closer to the well (V) has been removed by the water supply company. So the time of residence (T) is therefore:

$$T = \frac{V}{Q}$$

with:

T	=	time of residence	[years]
V	=	amount of water in the aquifer	[m^3]
Q	=	amount of water pumped	[m^3 $year^{-1}$]

So with Q = $2.0*10^6$ m^3 $year^{-1}$ the time of residence (T) of the water particle is 25 years.

Answer to exercise 6.11

a. Using the Thiem formula as derived in exercise 6.9:

$$H_2^2 - H_1^2 = \frac{Q}{\pi k} \ln \frac{r_2}{r_1}$$

the hydraulic head (H) at distance r_1 can be caluclated according to:

$$H_1 = \sqrt{H_2^2 - \frac{Q}{\pi k} \ln \frac{r_2}{r_1}}$$

So using the data:

H_2 = 39.6 m   k = 30.0 m day^{-1}

r_1 = 500 m   Q = 4109.6 m^3 day^{-1}

r_2 = 2000 m

The formula results in H_1 = 38.83 m and the lowering of the groundwater table is 0.77 m.

The caretaker has a good reason to be worried.

b. If the lowering of the groundwater table is limited to 0.20 m the net amount of water leaving the area is calculated according to:

$$Q = \frac{\pi k}{\ln\left(\frac{r_2}{r_1}\right)} (H_2^2 - H_1^2)$$

In this case with H_1 = 39.4 m, Q amounts to 1074 m^3 day^{-1}. Because the actual amount of water pumped is 4109 m^3 day^{-1}, the amount that has to be infiltrated is 4109-1074 = 3035 m^3 day^{-1}.

Answer to exercise 6.12

Darcy's law is:

$$q = -k \frac{\Delta H}{\Delta z}$$

with:

q =	flux density of groundwater	[m^3 m^{-2} d^{-1}]
k =	hydraulic conductivity	[m day^{-1}]
H =	hydraulic head	[m]
z =	vertical coordinate	[m]

Furthermore the resistance of the confining layer (c) is defined as:

$$c = \frac{D}{k}$$

with:

c = resistance of confining layer [days]

D = thickness of confining layer [m]

k = hydraulic conductivity [m day^{-1}]

So rewriting Darcy leads to:

$$q = -k\,\frac{\Delta H}{\Delta z} = -\,\frac{k}{\Delta z}\,\Delta H = -\,\frac{k}{D}\,\Delta H = -\,\frac{\Delta H}{c}$$

Here ΔH = -4.20 - (-2.20) = -2.00 m and c = 1000 days, resulting in an upward flow or seepage of 0.002 m day^{-1}.

Answer to exercise 6.13

a) If the flow takes place in the horizontal direction then: $kD = k_1D_1 + k_2D_2$. First we assume that for the upper layer D_1 = 6.0 m, then: $kD = 2\cdot 6 + 4\cdot 10 = 52$ m^2/d.
As a first approximation for the water level in the well, H_w, we find:

$$10 - H_w = \frac{20}{2\pi 52}\ln\frac{200}{0.15}$$

H_w = 10.0 - 0.440 = 9.56 m.

The average saturated thickness of layer 1 is therefore 5.78 m and k_1D_1 = 11.56 m^2/d hence kD = 51.56 m^2/d.

Once again:

$$10-H_w = \frac{20}{2\pi 51.56} \ln \frac{200}{0.15}$$

$H_w = 10.0 - 0.444 = 9.556 \rightarrow D_1 = 5.778.$

This is very close to the previously computed value (no adjustment necessary) and we choose $H_w = 9.56$ m and $D_1 = 5.78$ m.

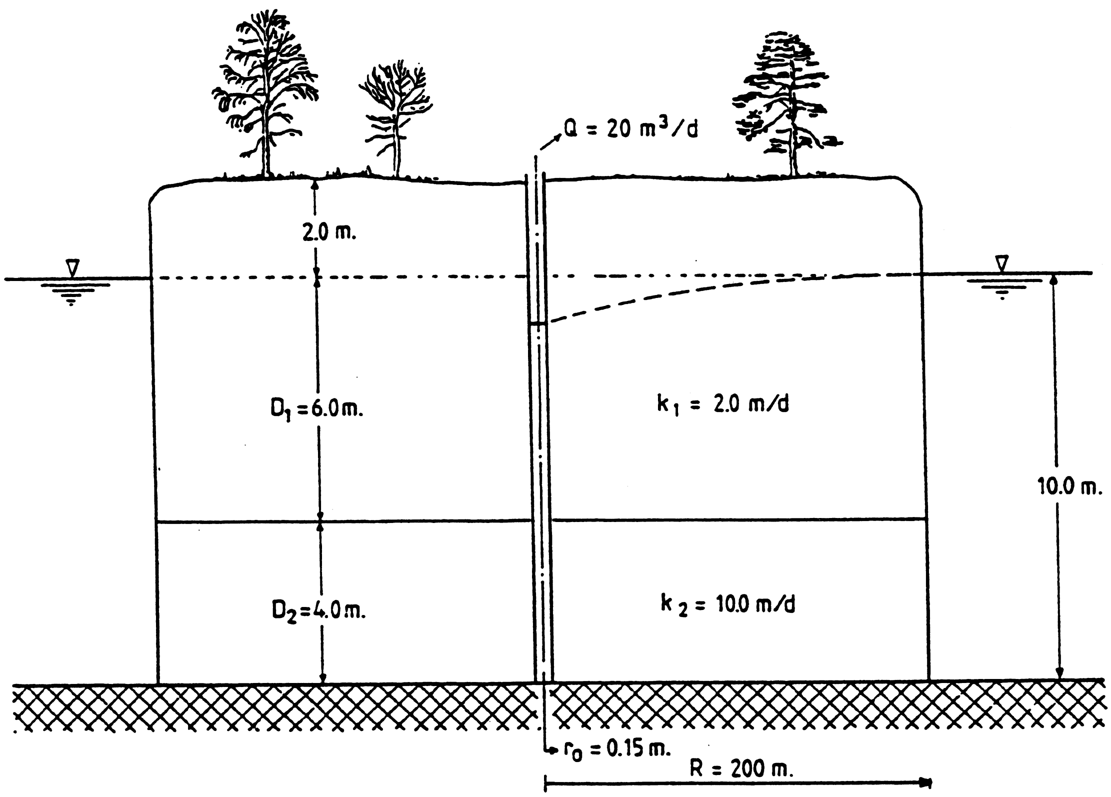


Figure 6.13-1

b) The slope of the curve is large for small r and small for large r.
This is because Q = Area $\cdot$ v_r = $2\pi rD$ $\cdot$ v_r.
The function h(r) is the same in both layers because the boundary conditions are the same:

$r = 0.15$: $H_w = 9.56$ m

$r = 200$ m: $H_R = 10.0$ m

and the Dupuit based equation applies to both layers.

c) $$Q_1 = \frac{2\pi k_1 D_1 (H_R - H_w)}{\ln r_2/r_1} \rightarrow Q_1 = 4.48\ m^3/d$$

or:

$Q_1 : Q = k_1 D_1 : kD$
$Q_1 : 20 = 11.56 : 51.56$
$Q_1 = 4.48\ m^3/d$
$Q_2 = 15.52\ m^3/d$

d) Water and solutes travel at higher velocities in layer 2 than in layer 1.

Therefore: $$T = \frac{\pi R^2 \cdot D_2 \cdot 0.33}{Q_2} = 10688\ d$$

e) The same.

Answer to exercise 6.14

a) The flux density equals:

$$v = -10.0 \cdot \frac{1.5}{3000} = 0.005\ m/d$$

v is constant because the aquifer is confined and the thickness is constant. The discharge through 1 km cross-section is:

$$Q = A \cdot v = 1000 \cdot 50 \cdot 5 \cdot 10^{-3} = 250\ m^3/d.$$

b) The travel time between the two piezometers may be computed from:

$$T = \frac{L}{v_e} = \frac{3000 \cdot 0.3}{5 \cdot 10^{-3}} = 18\ 10^4 d$$

(493 *years*)

or:

$$T = \int_{3000}^{0} \frac{-dx}{v_e} = 18.10^4 d$$

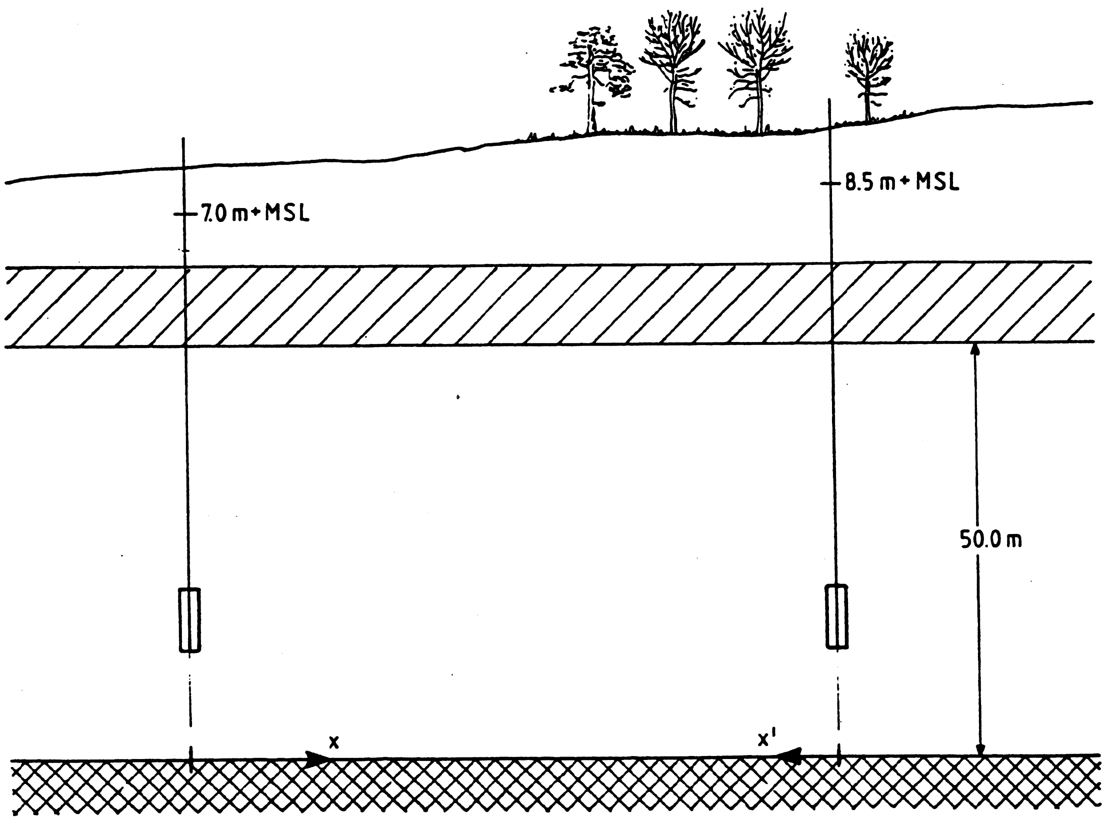


Figure 6.14-1

c) Since D is now a function of x and because Q remains constant (confined aquifer), we may write:

$$Q = (35+0,01\ x^1)1000(-10\ \frac{dH}{dx})$$

The x and x^1 coordinate are related as follows: x^1 = 3000-x. Hence:

$$Q = (65-0,01\ x)1000(-10\ \frac{dH}{dx})$$

Separation of the variables leads to:

$$\frac{Q}{10^4}\ \frac{dx}{(65-0.01x)} = -dH$$

solution:

$$-\frac{Q}{10^2}\ \ln\ (65-0.01x) = -\ H + constant$$

Boundary values are x = 0 H = 7.0 m

x = 3000 H = 8.5 m

$$\frac{Q}{100}\ \ln 65 = -7.0 + constant$$

$$\frac{Q}{100}\ \ln 35 = -8.5 + constant$$

Subtracting the second equation from the first equation it follows that:

$$\frac{Q}{100}\ \ln\frac{65}{35} = 1.5 \rightarrow Q = 242.3\ m^3$$

(*per km*)

d) In this case v_e is a function of x

$$T = \int_{3000}^{0} \frac{-dx}{v_e}$$

v_e can also be expressed as a function of x:

$$v_e = \frac{242.3}{(65-0.01x).300} = \frac{0.808}{(65-0.01x)}$$

Which makes solution possible (try for yourself).

However a simpler approach is:

$$T = \frac{\textit{volume of water}}{\textit{discharge}}$$

Volume of water:

$$0.3\left(\frac{65+35}{2}\right) \cdot 3000 \cdot 1000 = 4.5\ 10^7 m^3$$
$$\textit{(per km)}$$

$$T = \frac{4.5\ 10^7}{242.3} = 185720\ d$$

Answer to exercise 6.15

a. This is a non-steady flow, because the gradient of the hydraulic head (dH/dz) changes over time.

b. According to Darcy's law the flux density is:

$$v = -k\,\frac{dH}{dz} = -k\,\frac{h+L}{L}$$

with:

v = flux density	[m s^{-1}]
k = hydraulic conductivity	[m s^{-1}]
dH/dz = gradient of hydraulic head	[m m^{-1}]
h = height of water column on sample	[m]
L = length of soil sample	

But the flux density (v) is also equal to the change of h over time so:

$$v = \frac{dh}{dt} = -k\,\frac{h+L}{L}$$

which can be written as:

$$\frac{dh}{h+L} = -\frac{k}{L}\,dt$$

Assuming that $h=h_0$ at $t=0$ integration leads to:

$$\ln\left(\frac{h+L}{h_0+L}\right) = -\frac{k}{L}\,t$$

so:

$$k = -\frac{L}{t}\ln\left(\frac{h+L}{h_0+L}\right) = \frac{L}{t}\ln\left(\frac{h_0+L}{h+L}\right)$$

c. Using the data results in:

$$k = \frac{20}{5}\ln\left(\frac{30.0}{28.19}\right) = 0.249 \text{ cm min}^{-1}$$
$$= 3.58\ m\ day^{-1}$$

Answer to exercise 6.16

a. See figure 6.16.2.

b. The streamlines are determined by linear interpolation between the nodes in figure 6.16.2.
Close to the drain this interpolation is inaccurate, the network being too wide for that part of the flow region. The method can be improved by:
- finer mesh of the network (e.g. close to the ditches and the drains);
- introduction of more complicated boundary conditions (e.g. groundwater table seepage surface).

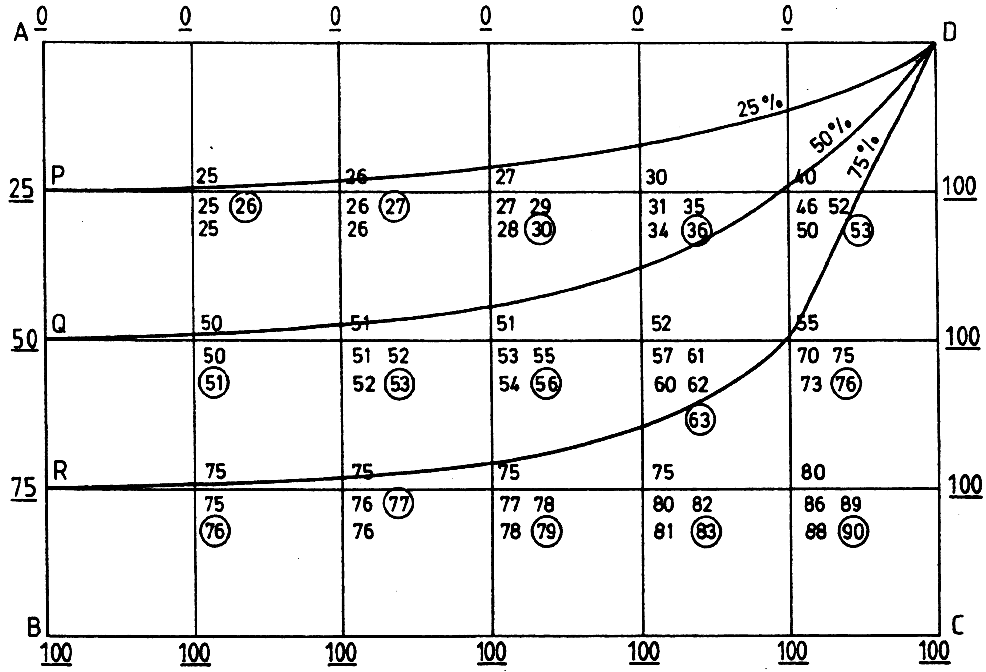


Figure 6.16.2 Numerical solution and streamlines.

Examenvraagstukken bij hoofdstuk 6

6.1 (augustus 1991)

In een gebied met een jaarlijkse neerslagsom van 833 mm ligt een afvoerloos meer van 86 ha. Uit zijn omgeving ontvangt dit meer per jaar 215000 m^3 water via enkele beekjes. Het meer is gemiddeld 2.00 m diep; de waterspiegel schommelt om het peil van 30.00 m boven gemiddeld zeepeil (GZP).
Het bodemprofiel is ter plaatse van het meer als volgt opgebouwd (hoogten t.o.v. GZP in meters):

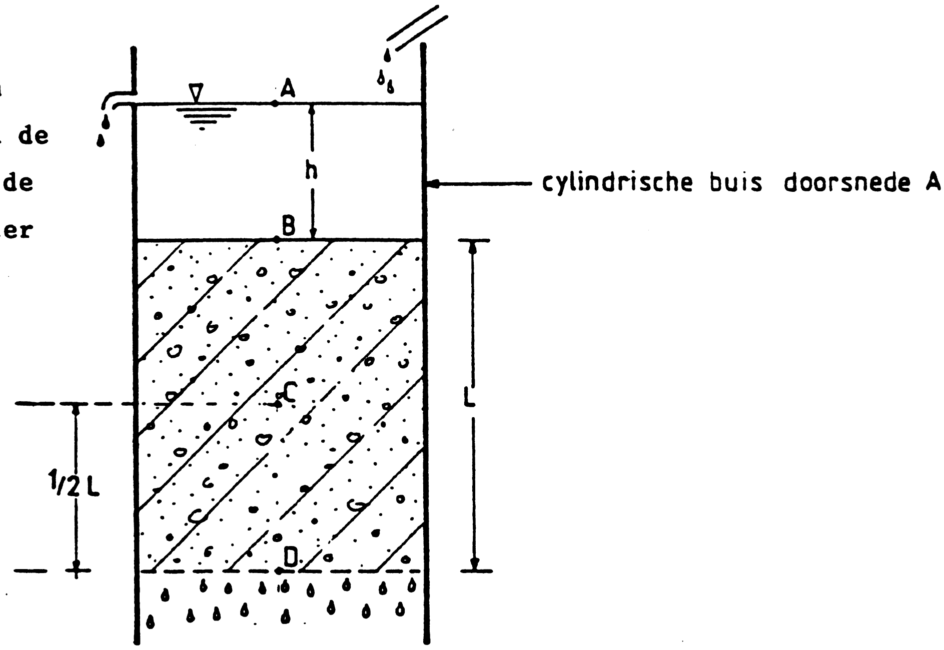
+ 30.00 tot + 28.00	water
+ 28.00 tot + 23.00	zand met k = 5.0 m d^{-1}
+ 23.00 tot + 8.00	leem met k = 1.0 m d^{-1}
+ 8.00 tot + 1.00	klei met k = 0.5 m d^{-1}
dieper dan GZP + 1.00	grof zand, k zeer groot

In het meer is een stijgbuis geplaatst waarvan het filter zich op NAP + 1.00 m bevindt, dus precies aan de bovenzijde van de grofzandige laag. De waterspiegel in deze stijgbuis blijkt 3.6 cm lager te liggen dan de waterspiegel van het meer.

- Gezien de uitgestrektheid van het meer (86 ha) mag worden aangenomen dat onder zijn bodem alleen vertikale stroming optreedt.
- Gevraagd wordt de jaarsom van E_o (mm j^{-1}) voor het meer. Geef een volledige berekening met duidelijke motivering van de achtereenvolgende handelingen en met vermelding van alle tussenresultaten.

6.2. (augustus 1992)

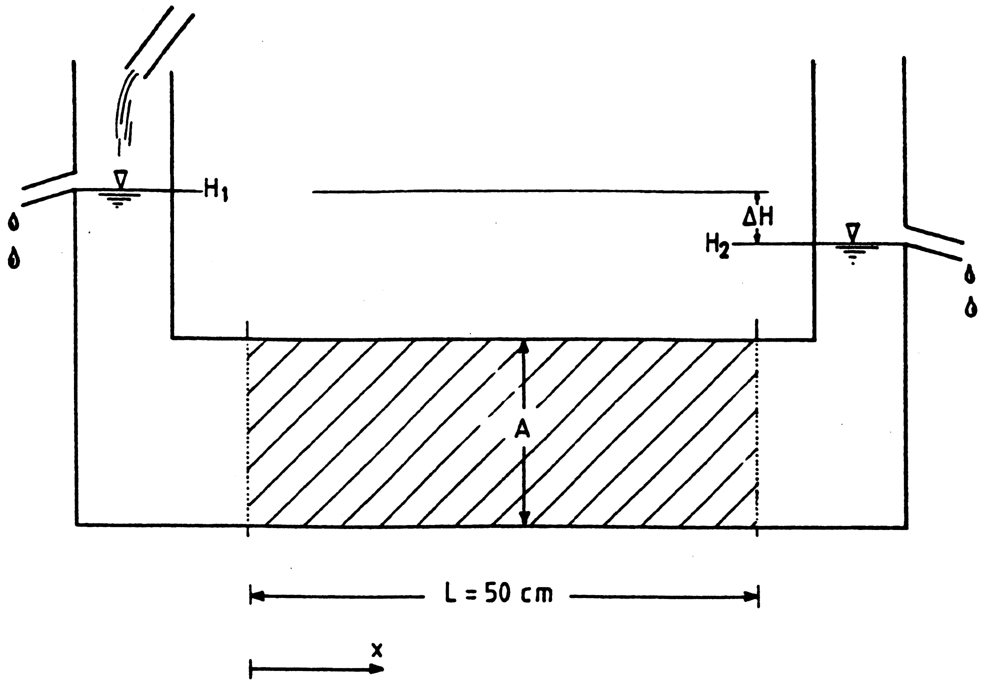
In een laboratorium opstelling stroomt water stationair door een kolom grond zoals aangegeven in de nevenstaande figuur. Aan de onderzijde treedt het water 'vrij' uit.
h = 0.25 m,
doorsnede A = 0.0707 m^2



a. Welke ontbrekende gegevens zijn nodig om met deze opstelling de doorlatendheid te meten?
b. Bereken de waarde van de stijghoogte in de punten A,B,C en D als L = 1.0 m (kies het horizontale vlak door D als referentievlak).
c. Wat is de vloeistofdruk (Pa) in de punten A,B,C en D?
   ρ = 1000 kg m^{-3}, g = 9.81 m s^{-2}.
d. Bereken de fluxdichtheid als k = 2.0 m d^{-1}.
e. Welk extra gegeven is nodig om de reistijd van een waterdruppel tussen B en D te kunnen berekenen?

6.3. (december 1990)

De onderstaande figuur is een schematische voorstelling van een laboratoriumproef om de doorlatendheid te meten.



Gegeven:
Oppervlakte dwarsdoorsnede (A) is 300 cm^2
Lengte van de grondkolom (L) is 50 cm
Stijghoogte verschil H_1-H_2 is 10 cm
Porositeit is 0.35

a. Als de kolom is gevuld met homogene grond en het debiet bedraagt 600 cm^3 per uur, bereken dan de waarde van de doorlatendheid k in m d^{-1}.
b. Vervolgens wordt de rechterhelft van de kolom (dus 25 cm) verwijderd en gevuld met een tweede grondsoort (de linker 25 cm blijft gevuld met dezelfde grond). Het debiet bedraagt in de nieuwe (evenwichts)situatie 300 cm^3 per uur. Wat is de doorlatendheid (m d^{-1}) van de tweede grondsoort?
c. Bereken de reistijd van een vloeistofdeeltje voor vraag a. en b.
d. Teken het verloop van de stijghoogte (als functie van x) voor beide gevallen a. en b. en geef een verklaring.

6.4. (augustus 1991)

a. Wat betekent de uitdrukking $\nabla\ v = 0$?
b. Geldt dit ook beneden de grondwaterstand in anisotroop doorlatende grond?
   Motiveer uw antwoord.
c. Dezelfde vraag als in b. maar dan voor de onverzadigde zone van de bodem? Zo ja onder welke voorwaarden.

6.5. (april 1991)

In de figuur van blz. 6-2 in het dictaat geeft de stippellijn de positie van de grondwaterspiegel aan.

a. Waarom stijgt de waterspiegel ten oosten van Bennekom tot grotere hoogte dan onder de Grebbeberg (bij Rhenen)? Verklaar uw antwoord m.b.v. een vergelijking (formule).
b. Aan de Bennekomse kant is k = 20. m d^{-1}, terwijl de grondwaterstand er in oostelijke richting 2.0 m per km stijgt. Bereken v (de fluxdichtheid) in m/d en in m $jaar^{-1}$.
c. Hoeveel water (m^3) stroomt jaarlijks per km lengte de Gelderse Vallei binnen door het bovenste pakket als de dikte ervan 30. m bedraagt?
d. Hoe groot is de werkelijke snelheid van het water in de poriën als de poriënfractie 0.35 bedraagt?

7 SOIL MOISTURE

Exercise 7.1

a. Give a definition of the pF concept.

b. What is the pF-value:
   - in the soil at the phreatic level
   - in the soil at field capacity
   - in the soil at wilting point
   - in a soil sample oven dried at 105°C?

Exercise 7.2

When we neglect osmotic effects, the potential of soil water can be written as:

$$\psi_t = \psi_g + \psi_p + \psi_m$$

with:

Ψ_t = total potential of soil water
Ψ_g = potential caused by gravity
Ψ_p = potential caused by pressure
Ψ_m = potential caused by matric forces

a. Give a definition of the potential and include its dimension.

b. What is the difference in gravity potential between two points at heights z_1 and z_2 ($z_2 > z_1$) in a water column in equilibrium?

c. An unsaturated soil sample is in equilibrium with a water column as depicted in figure 7.2.1. Calculate Ψ_m at the top side of the soil sample.

d. What is the pF-value in that point?

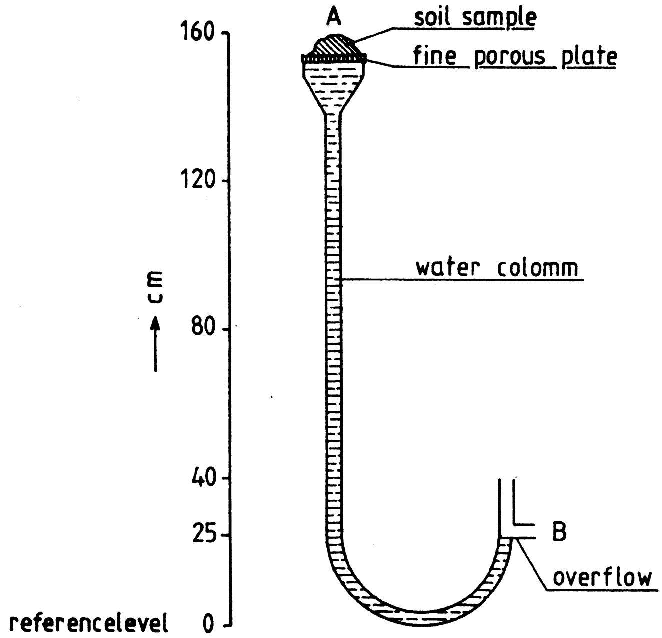


Figure 7.2.1 Unsaturated soil sample in equilibrium with a water column.

Exercise 7.3

a. What is the general differential equation for unsaturated flow?

b. A steady state situation for a period of one to several days is often assumed in calculating the vertical unsaturated flow between the phreatic level and the bottom side of the rootzone. How can the maximum height (z) above the phreatic level to which a given flux (v_z) can rise, be calculated in this case?

c. Which soil physical relation has to be known to calculate the answer under b?

Exercise 7.4

At the Staring Centre laboratory tests are done to establish pF-curves for different soils in the Netherlands. The data in this exercise and the next are taken from the Staring series.

a. The data series in table 7.4.1 are found for a very fine to fine sand (B1). Plot the pF-curve for this sand in figure 7.4.1.

Table 7.4.1 Matric heads (h_m) at different values for the moisture volume content (θ) for a very fine to fine sand (B1).

Δ ($cm^3\ cm^{-3}$)	h_m (cm)	θ ($cm^3\ cm^{-3}$)	h_m (cm)	θ ($cm^3\ cm^{-3}$)	h_m (cm)
0.371	0	0.250	-65	0.120	-301
0.370	-1	0.240	-70	0.110	-375
0.360	-14	0.230	-77	0.100	-476
0.350	-30	0.220	-84	0.090	-614
0.340	-33	0.210	-92	0.080	-821
0.330	-35	0.200	-101	0.070	-1161
0.320	-38	0.190	-112	0.060	-1819
0.310	-40	0.180	-125	0.050	-3147
0.300	-43	0.170	-141	0.040	-6112
0.290	-46	0.160	-160	0.030	-16500
0.280	-50	0.150	-184	0.020	-89800
0.270	-54	0.140	-214	0.010	-814000
0.260	-59	0.130	-250	0.000	$-1.0*10^8$

In figure 7.4.1 the pF-curves are given for three other types of soil as well:

- B3 is a very loamy, very fine to fine sand
- B12 is a very heavy clay
- B16 is sandy peat and peat.

b. Determine for each soil in figure 7.4.1 (including B1):
   - the pore fraction (n)
   - the moisture volume content (θ) at field capacity
   - the air volume content (θ^g) at field capacity
   - the moisture volume content (θ) at wilting point
   - the plant available moisture.

c. Discuss the differences between the four soils in regard to the properties determined under b.

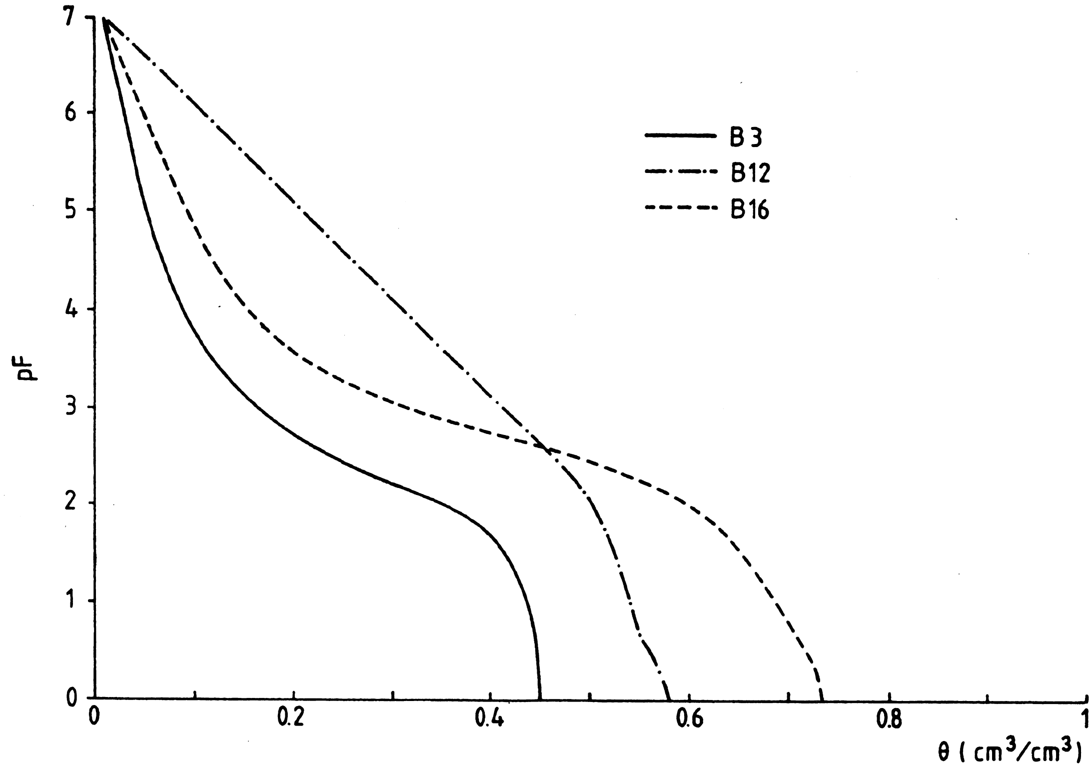


Figure 7.4.1 pF curves for several Dutch soils.

Exercise 7.5

To determine the volume fraction of water (θ) of a soil, a cylindrical soil sample is taken, stored in a plastic container to prevent evaporation losses and taken to the laboratory. There it is weighed, oven-dried (24 hours at 105°C) and weighed again. The density (ρ_s) of the solid fraction of this soil has been determined earlier and is known to be ρ_s = 2650 kg m^{-3}. .

The other data are:

- diameter of sample (D) = 0.05 m
- length of sample (L) = 0.10 m
- mass of sample before drying (M_w) = 306.38 gr
- mass of sample after drying (M_d) = 262.17 gr

Calculate the porosity (n) and the volume fraction of water (θ) of this soil.

Answer to exercise 7.1

a. The definition of pF is:

with:

S_m = soil moisture suction = $|h_p|$ [cm]

b. At the phreatic level: pF = $-\infty$ (S_m = 0 cm)
At field capacity: pF = 2.0 (S_m = 100 cm)
At wilting point: pF = 4.2 (S_m = 16000 cm)
Oven dried soil sample: pF = 7 (S_m = $1*10^7$ cm).

Answer to exercise 7.2

a. The potential at a point in a force field is the amount of work required to transfer a body of unit mass from a reference point to the point under consideration. It is expressed in energy per mass, so J kg^{-} .

b. The gravity potential in a point z is calculated according to:

$$\Psi_g = mgz/m = gz$$

with:

Ψ_g	=	gravity potential	[J kg^{-1}]
m	=	mass	[kg]
g	=	acceleration of gravity	[m s^{-2}]
z	=	height relative to reference level	[m]

So the difference in gravity potential ($\Delta\Psi_g$) is:

$$\Delta\Psi_g = g(z_2 - z_1)$$

c. The system is in equilibrium, so:

$$\Psi_{tA} = \Psi_{tB}$$

Now:

$$\Psi_{tA} = \Psi_{gA} + \Psi_{pA} + \Psi_{mA}$$

in which $\Psi_{pA} = 0$, because the pressure in A is equal to the atmospheric pressure which is the reference level.
In the same way:

$$\Psi_{tB} = \Psi_{gB} + \Psi_{pB} + \Psi_{mB}$$

in which $\Psi_{pB} = 0$ for the same reason as $\Psi_{pA} = 0$. But here also $\Psi_{mB} = 0$, because point B is not under the influence of matrix forces.
So:

$$\Psi_{gA} + \Psi_{mA} = \Psi_{gB} \text{ or } \Psi_{mA} = \Psi_{gB} - \Psi_{gA} = g(z_B - z_A) = -13.5\ m^2\ s^{-2}$$
$$= -13.5\ J\ kg^{-1}$$

d. According to the definition:

$$pF_A = {}^{10}\log S_{mA}$$

Now:

$$S_{mA} = -h_{mA} = -\Psi_{mA}/g = 1.35\ m$$
$$= 135\ cm$$

So:

$$pF_A = {}^{10}\log S_{mA} = {}^{10}\log 135 = 2.13$$

Answer to exercise 7.3

a. The general differential equation for unsaturated flow is:

$$\frac{\partial\theta}{\partial t} = - \nabla(k(h_m)\nabla H)$$

with:

θ = soil moisture content [cm^3 cm^{-3}]

t = time [days]

∇ = nabla or del operator

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

$k(h_m)$ = unsaturated hydraulic conductivity [m day^{-1}]

h_m = matric head of soil [m]

H = hydraulic head [m]

It is derived by combining the continuity equation and Darcy's law.

b. In case of steady state, vertical, unsaturated flow ($\frac{\partial\theta}{\partial t} = 0$) the equation reads:

$$\frac{d}{dz}\ \{k(h_m)\,(\frac{dh_m}{dz} + 1)\} = 0$$

or:

$$v_z = -k(h_m)\,(\frac{dh_m}{dz} + 1)$$

with:

v_z = vertical flux density [m day^{-1}]

This can also be written as:

$$dz = - \frac{dh_m}{v_z/k(h_m)+1}$$

Integration of this equation:

$$\int_0^z dz = \int_0^{h_p} \left(-\frac{dh_m}{v_z/k(h_m)+1}\right)$$

gives the maximum height (z) to which a steady state flux (v_z) can rise above the phreatic level.

c. To perform the integration it is necessary to know the relation between k and h_m.

Answer to exercise 7.4

a. Plotting the pF-curve for soil B1, is done by plotting pF against θ. The pF-value is calculated according to:

$$pF = {}^{10}\log|-h_m|$$

with:

h_m = matric head of soil moisture [cm]

So for $h_m = 0$, pF is $-\infty$ and cannot be plotted, but for the other values of h_m the procedure is clear.

b.
- The porosity (n) is found by extrapolation to pF $-\infty$. This procedure is demonstrated in figure 7.2-3 in your book.
- The moisture content ($\theta_{2.0}$) at field capacity is the moisture content at pF = 2.0.
- The air volume content (θ^g) at field capacity is found by calculating ($n - \theta_{2.0}$).
- The moisture content ($\theta_{4.2}$) at wilting point is the moisture content at pF = 4.2.
- The plant available moisture is found by evaluating ($\theta_{2.0} - \theta_{4.2}$).

The results for the four soils are given in table 7.4.2.

c.
- The pore fraction is highest in the peat and lowest in sand as could be expected.
- The moisture content at field capacity shows the same order as the porosity. The clay soil has only lost a small amount of its water between pF = 0.0 and pF = 2.0.
- The low air volume content at pF = 2.0 of the clay soil is most striking.
- At wilting point the fine sand has lost most of its water, whereas the heavy clay still contains almost 50% of the water present at saturation. The amount of water still stored at wilting point is not available to the plant.

Table 7.4.2 Properties of four different soils, derived from their water retention curves (pF curves).

	B1	B3	B12	B16
n	0.37	0.45	0.58	0.74
$\theta_{2.0}$ (cm^3 cm^{-3})	0.20	0.35	0.50	0.59
θ^g (cm^3 cm^{-3})	0.17	0.10	0.08	0.15
$\theta_{4.2}$ (cm^3 cm^{-3})	0.03	0.07	0.28	0.13
P.A.M. (cm^3 cm^{-3})	0.17	0.28	0.22	0.46
P.A.M. (mm)	170	280	220	460

- The peat has an extraordinary amount of water available for the plant, whereas clay with its high porosity has even less water available than soil B3.

Answer to exercise 7.5

The porosity of the soil is calculated according to:

$$n = \frac{V_v}{V}$$

with:

V_v = volume of pores [m^3]
V = volume of soil sample [m^3]

V_v can be determined using:

$$V_v = V - V_s$$

with:

V_s = volume of solids [m^3]

where V_s is calculated using:

$$V_s = \frac{M_d}{\rho_s}$$

with:

M_d = mass of sample after drying [kg]

ρ_s = density of solid fraction of sample [kg m^-]

So using the data leads to:

V_s = 0.26217/2650 = 9.89 $*10^{-5}m^3$ = 98.9 cm^3

V = 1/4 $\pi D^2 L$ = 1.964$*10^{-4}m^3$ = 196.4 cm^4

V_v = 196.4 - 98.9 = 97.5 cm^3

And so:

n = 97.5/196.4 = 0.496

The volume fraction of water (θ) is calculated according to:

$$\theta = \frac{V_w}{V}$$

with:

V_w = volume of water [m^3]

where V_w is calculated using:

$$V_w = \frac{(M_w - M_d)}{\rho}$$

with:

M_w = mass of soil sample before drying [kg]

ρ = density of soil water (≈1000 kg m^{-3}) [kg m^{-3}]

resulting in:

$V_w = (0.30638 - 0.26217)/1000 = 4.42*10^{-5}m^3 = 44.2$ cm

And so:

$\theta = 44.2/196.4 = 0.225$

Examenvraagstukken bij hoofdstuk 7

7.1 (april 1991)

Zestigjarig dennenbos ten oosten van Bennekom wortelt tot ongeveer 1.5 m diepte. De grondwaterspiegel bevindt zich ruim 20.0 m beneden maaiveld. De neerslag bedraagt gemiddeld 800 mm per jaar; de bodem is grof zand (stuwwal).

a. Schat de voeding van het grondwater onder het dennenbos. Kies daarbij tussen de volgende waarden: 600 mm, 400 mm of 200 mm.
b. Kan het bos in een droge zomer profiteren van het grondwater? Kies uit de volgende mogelijkheden:
   ja, nee, een beetje.
c. Als de k-θrelatie van het zand beschreven kan worden door de relatie:

$$\left(\frac{k_{onv}}{k_{verz}}\right) = \left(\frac{\theta}{\theta_{verz}}\right)^n$$

waarbij
k_{verz} = 10.0 m d^{-1}
θ_{verz} = 0.30
n = 5
Bereken het vochtgehalte op 15 m diepte beneden het maaiveld.

Aanwijzing: neem aan dat de stroming stationair is, daaruit volgt dat de gradiënt van de stijghoogte $\frac{dH}{dz}$ gelijk is aan 1.0 (zie Koorevaar et al, 1983) en bereken k_{onv}.

8 AGROHYDROLOGIE

Exercise 8.1 (Exam November '87)

The Thornthwaite & Mather model for computing the waterbalance of a rootzone, may be written as:

$$V = V_0 \exp[-E_{pot}\, t/V_o]$$

a. What is the meaning of the symbols?
b. Which are the (three) assumptions underlying the derivation of the formula?

Exercise 8.2

In this exercise the Thornthwaite & Mather model will be applied to several different situations. First the procedure for the calculation of the water balance according to Thornthwaite and Mather will be explained.
The Thornthwaite-Mather model will be used in the form:

$$ST = ST_0 \exp[-APWL/ST_0] \quad \text{[mm]} \qquad [1]$$

<u>Procedure for the calculation of the water balance</u>

1. Determine the monthly values of precipitation (P), potential evapotranspiration (PE) and (P-PE).
   Lowest mean month temperature below -1°C, then precipitation stays as snow on the surface (see 16).
   Lowest mean month temperature above -1°C (see 2).

2. Determine the year sums: ΣP, ΣPE and $\Sigma(P-PE)$.
   If $\Sigma(P-PE) > 0$, then a precipitation surplus (see 3).
   If $\Sigma(P-PE) < 0$, then a precipitation deficit (see 10).

3. *Precipitation surplus*
Most often there is a dry period ((P-PE) < 0) and a wet period ((P-PE) > 0), sometimes two dry and two wet periods (i.e. a long and a short rainperiod). At the end of the (long) rainperiod the soil is saturated and thus $ST = ST_o$.

4. As soon as (P-PE) gives negative values the subsequent values of (P-PE) are summarized. The accumulated precipitation deficit over N successive dry months is:

$$APWL = -\Sigma(P-PE) \quad \text{[mm]} \qquad [2]$$

As soon as rainfall exceeds potential evapotranspiration for a whole month ((P-PE) > 0) the series is stopped. When a dry month occurs later on, a new series is started according to 14.

5. For the different values of APWL the stored soil moisture ST is calculated using equation (1).

6. The change in storage ($\Delta ST = ST_2 - ST_1$) indicates whether the soil water is restored ($\Delta ST > 0$) or depleted ($\Delta ST < 0$).

7. In dry months ((P-PE) < 0) the actual evapotranspiration (AE) and the evaporation deficit (D) are calculated from:

$$AE = P - \Delta ST \quad (\Delta ST < 0) \quad \text{[mm]} \qquad [3]$$

and:

$$D = PE - AE \quad \text{[mm]} \qquad [4]$$

8. For wet months ((P-PE) > 0) it follows that AE = PE and ΔST = (P-PE) until the maximum soil moisture capacity ST_0 has been reached. A surplus (S) results when $ST = ST_0$ and this surplus will run off. The surplus is calculated from:

$$S = (P-PE) - \Delta ST \quad (S \geq 0) \quad [mm] \qquad [5]$$

9. As a check on your calculations can be used:

$\Sigma(\Delta ST) = 0$ [mm]
$\Sigma S = \Sigma P - \Sigma AE$ [mm]
$\Sigma D = \Sigma PE - \Sigma AE$ [mm]

10. *Precipitation deficit*
Summarize all negative values of P-PE : $\Sigma(P\text{-}PE)_{neg}$
Summarize all positive values of P-PE : $\Sigma(P\text{-}PE)_{pos}$.

11. If $\Sigma(P\text{-}PE)_{pos} < ST_0$ (see 12).
If $\Sigma(P\text{-}PE)_{pos} > ST_0$ (see 13).

12. If $\Sigma(P\text{-}PE)_{pos} < ST_0$ the soil will never be completely saturated. At the end of the wet period the moisture content of the soil attains its maximum, that is $ST_{max} < ST_0$. At the end of the dry period the moisture content of the soil reaches its minimum ST_{min}. During the following wet period the soil moisture content will increase again from ST_{min} to ST_{max}.
Now ST_{max} and ST_{min} can be calculated in the following way.
At the end of the wet period ST = ST_{max} and APWL = $APWL_{min}$. Using the formula, this can be written as:

$$ST_{max} = ST_0 \exp[-APWL_{min}/ST_0] \qquad [6]$$

In the same way it is clear that at the end of the dry period ST = ST_{min} and APWL = $APWL_{max}$. This can be written as:

$$ST_{min} = ST_0 \exp[-APWL_{max}/ST_0] \qquad [7]$$

Now we use:

$$(APWL_{max} - APWL_{min}) = \Sigma(P-PE)_{neg} \qquad [8]$$

$$(ST_{max} - ST_{min}) = \Sigma(P-PE)_{pos} \qquad [9]$$

Knowing ($APWL_{max}$ - $APWL_{min}$) and (ST_{max} - ST_{min}) the equations can be solved.
At the end of the wet period one uses the values $APWL_{min}$ and ST_{max} and continues as outlined under point 5 and following. As soon as a wet month appears (P-PE > 0), the series is stopped. If another dry period starts later then a new series is started according to 14.

13. If Σ(P-PE)pos > ST_0, then the soil will be saturated with water at the end of the wet period and one acts as outlined under point 4 and following.

14. Sometimes the dry period is interrupted by a few wet months (P-PE > 0). These positive values have to be added to ST. There are two possibilities:
    ST_0 is reached and a surplus occurs (see 4).
    ST_0 is not reached (see 15).

15. If in the short rainperiod ST_0 is not reached, but only the moisture content $ST_1 < ST_0$, then the corresponding $APWL_1$ can be calculated according to:

$$APWL_1 = ST_0 \ln[ST_0/ST_1] \qquad [10]$$

This $APWL_1$ is then used as the initial value of a new series.

16. *Snow cover*
    Precipitation will be stored in the form of snow (SN) when the mean monthly temperature is lower than -1°C. The total amount of stored moisture (SN + ST) maybe larger than ST_0.
    As soon as the mean monthly temperature rises over -1°C snowmelt takes place. As long as ST < ST_0 in the soil the snow may replenish the soil water content, when ST_0 is reached discharge takes place.
    In the original article by Thornthwaite and Mather some rules are given for this discharge but the general validity of these rules is to

be doubted.

THORNTHWAITE, C.W. and J.R. MATHER, 1955. *The waterbalance*. Publications in climatology, Drexel Institute of Technology, 8, no. 1.

THORNTHWAITE, C.W. and J.R. MATHER, 1955. *The water budget and its use in irrigation*. USDA yearbook of agriculture, pp. 346-358.

Calculate the waterbalance for the following cases:

a. The Netherlands, ST_0 = 150 mm.

	J	F	M	A	M	J	J	A	S	O	N	D
P	69	52	44	49	52	57	78	89	71	72	70	64
PE	6	16	37	71	99	114	110	92	59	28	10	3

b. Ciénaga de Zapata (Cuba), ST_0 = 300 mm.

	J	F	M	A	M	J	J	A	S	O	N	D
P	37	32	37	80	199	250	215	222	256	174	35	19
PE	72	73	96	117	141	155	170	164	140	124	86	76

c. Mountainside (Jamaica), ST_0 = 150 mm.

	J	F	M	A	M	J	J	A	S	O	N	D
P	80	78	68	130	161	79	69	148	123	145	142	44
PE	84	88	106	115	125	111	125	115	92	91	77	80

d. Cherfech (Tunisia), ST_0 = 200 mm.

	J	F	M	A	M	J	J	A	S	O	N	D
P	63	68	35	28	22	12	2	20	29	54	49	54
PE	19	21	28	48	86	116	156	147	106	77	40	23

Exercise 8.3 (MSc exam March '86)

A soil can hold 100 mm of water in storage. The potential evapotranspiration in a dry period is 8 mm/day; the initial storage is 100 mm.
We suppose that the theory of Thornthwaite & Mather is valid for this case.

a. How much will the actual evapotranspiration of a crop be when the storage is reduced to 25 mm?
b. How many days of dry weather are needed to reach this storage of 25 mm?

Exercise 8.4 (Exam October '86)

a. Calculate the waterbalance for Macia, Moçambique (ST_0 = 100 mm).

	J	F	M	A	M	J	J	A	S	O	N	D
P	124	191	125	79	42	44	27	24	30	55	78	111
PE	157	129	121	91	69	46	53	82	116	150	149	164

b. In case one would want to irrigate, how many mm of irrigation water would be needed in a year?

c. If the groundwatertable in the area of Macia is relatively high, would that increase or decrease the amount of water needed?

d. What problems can be expected, how can they be solved and how would that change your answer under c?

Exercise 8.5 (Exam August '88)

In case a crop suffers from moisture shortage, the actual evapotranspiration as well as the yield are reduced. According to Rijtema most crops behave according to:

$$y = ax + b$$

with:

y	: yield	[kg ha^{-1}]
x	: E_{act}/E_{pot}	[-]
E_{act}	: actual evapotranspiration	[mm growing $season^{-1}$]
E_{pot}	: potential evapotranspiration	[mm growing $season^{-1}$]
a,b	: constants	[kg ha^{-1}]

Doorenbos & Kassam proposed:

$$1-y/y_{max} = k\ (1 - E_{act}/E_{pot})$$

with:

y_{max}	: yield when E_{act}=E_{pot}	[kg ha^{-1}]
k	: constant	[-]

a. Show that both expressions are equivalent.

b. Express a and b in terms of y_{max} and k.

Exercise 8.6 (Exam December '87)

Give a description of the causes of indirect damage to crops in case of excess of water.

Exercise 8.7

Assume that a crop is grown on a soil consisting of 40 cm of loamy, very fine to fine sand (B2) on light clay (O11). The pF-curves for these soiltypes are given in figure 8.7.1.

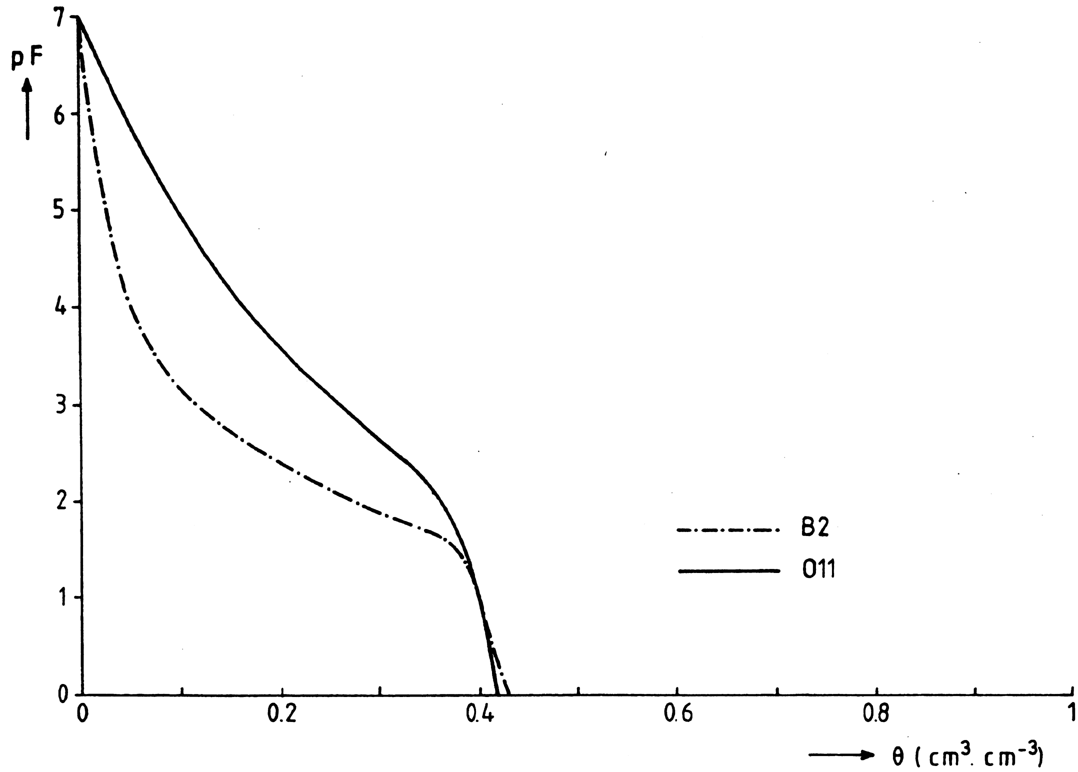


Figure 8.7.1 pF-curves for two Dutch soils.

At the beginning of the growing season the soil is at field capacity. Calculate the total available moisture and the easily available moisture according to Shockley (both in mm) for a crop that roots to a depth of 100 cm.

Answer to exercise 8.1

a. V = amount of available moisture in the soil [mm]

V_0 = amount of available moisture at the beginning of the dry period [mm]

E_{pot} = potential evapotranspiration [mm day^{-1}]

t = time [day]

$E_{pot}t$ = accumulated potential water loss [mm]

b.
- The actual evapotranspiration in dry periods is proportional to the amount of available moisture in the soil (E_{act} = c V).
- There is no recharge of the moisture in the rootzone by way of capillary rise.
- All precipitation occurring during a month is available for evapotranspiration.

Answer to exercise 8.2

a. The Netherlands, ST_0 = 150 mm.

	J	F	M	A	M	J	J	A	S	O	N	D	year
P	69	52	44	49	52	57	78	89	71	72	70	64	767
PE	6	16	37	71	99	114	110	92	59	28	10	3	645
P-PE	63	36	7	-22	-47	-57	-32	-3	12	44	60	61	122
wet					dry				wet				
APWL				22	69	126	158	161					
ST	150	150	150	130	95	65	52	51	63	107	150	150	
ΔST	0	0	0	-20	-35	-30	-13	-1	+12	+44	+43	0	0
AE	6	16	37	69	87	87	91	90	59	28	10	3	583
S	63	36	7						0	0	17	61	184
D				2	12	27	19	2					62

Notes:
- $\Sigma(P-PE)_{year}$ = 122 > 0 and for the rainy season Σ(P-PE) = 283, so ST = ST_0 = 150 at the end of the rainperiod. Because March is the last month with a precipitation surplus it follows that ST_{March} = 150.
- APWL is calculated (during the period with a precipitation deficit) according to APWL = $-\Sigma$(P-PE). So $APWL_{April}$ = 22, $APWL_{May}$ = $APWL_{April}$ + 47 = 69, etc.

- The storage during the dry months is calculated using equation (1), so $ST_{April} = ST_0 \exp(-APWL/ST_0) = 150 \exp(-22/150) = 130$, etc.

b. Ciénaga de Zapata (Cuba), $ST_0 = 300$ mm

	J	F	M	A	M	J	J	A	S	O	N	D	year
P	37	32	37	80	199	250	215	222	256	174	35	19	1556
PE	72	73	96	117	141	155	170	164	140	124	86	76	1414
P-PE	-35	-41	-59	-37	58	95	45	58	116	50	-51	-57	142
	dry				wet						dry		
APWL	143	184	243	280							51	108	
ST	186	162	133	118	176	271	300	300	300	300	253	209	
ΔST	-23	-24	-29	-15	+58	+95	+29	0	0	0	-47	-44	0
AE	60	56	66	95	141	155	170	164	140	124	82	63	1316
S					0	0	16	58	116	50			240
D	12	17	30	22							4	13	98

c. Mountainside (Jamaica), ST = 150 mm.

	J	F	M	A	M	J	J	A	S	O	N	D	year
P	80	78	68	130	161	79	69	148	123	145	142	44	1267
PE	84	88	106	115	125	111	125	115	92	91	77	80	1209
P-PE	-4	-10	-38	-15	36	-32	-56	33	31	54	65	-36	58
	dry			wet		dry		wet					
APWL	40	50	88		(17)	49	105					36	
ST	115	107	83	98	134	108	74	107	138	150	150	118	
ΔST	-3	-8	-24	+15	+36	-26	-34	+33	+31	+12	+12	-32	0
AE	83	86	92	115	125	105	103	115	92	91	91	76	1160
S				0	0			0	0	42	65		107
D	1	2	14			6	22					4	49

Notes:

- There are two dry periods, but $\Sigma(P\text{-}PE)_{year} = 58 > 0$ and $\Sigma(P\text{-}PE) = 183$ for the longest wet period, so $ST = ST_0 = 150$ at the end of that period. Therefore $ST_{November} = 150$.
- After calculating APWL and ST it becomes clear that the soil moisture is not totally restored during the short rainperiod. In other words, APWL > 0 at the end of the short rainperiod. To calculate $APWL_{May}$ we use equation [1] in a somewhat different form:

  $$APWL_{May} = ST_0 \ln(ST_0/ST_{May}) = 150 \ln(150/134) = 17$$

This value is written between brackets, because there is no real shortage of water during the month of May ((P-PE) > 0). Now $APWL_{June}$ can be calculated as $(APWL_{May} + 32) = 49$.

d. Cherfech (Tunisia), ST_0 = 200 mm.

	J	F	M	A	M	J	J	A	S	O	N	D	year
P	63	68	35	28	22	12	2	20	29	54	49	54	436
PE	19	21	28	48	86	116	156	47	106	77	40	23	867
P-PE	44	47	7	-20	-64	-104	-154	-127	-77	-23	9	31	-431
wet				dry							wet		
$\Sigma(P-PE)_{neg}$				(20)	(84)	(188)	(342)	(469)	(546)	(569)			
APWL			(62)	82	146	250	404	531	608	631			
ST	93	140	147	133	96	57	27	14	10	9	18	49	
ΔST	+44	+47	+7	-14	-37	-39	-30	-13	-4	-1	+9	+31	0
AE	19	21	28	42	59	51	32	33	33	55	40	23	436
S	0	0	0								0	0	0
D				6	27	65	124	114	73	22			431

Notes:

- $\Sigma(P-PE) = -431 < 0$, so the next step is to calculate $\Sigma(P-PE)_{pos}$ and $(P-PE)_{neg}$.
- $\Sigma(P-PE)_{pos} = 138 < ST_0$, so the soil will never be completely at field capacity. This means that at the end of the wet period the soil moisture content will be smaller than 200 mm. To be able to calculate the rest of the table it is necessary to find ST and APWL at the end of the wet and at the end of the dry period.
- At the end of the wet period:
  - $ST=ST_{max}$
  - $APWL=APWL_{min}$

  At the end of the dry period:
  - $ST=ST_{min}$
  - $APWL=APWL_{max}$

  Now we have the following system of equations:

  (a) $ST_{max} - ST_{min} = \Sigma(P-PE)pos = 138$

  (b) $APWL_{max} - APWL_{min} = \Sigma(P-PE)neg = 569$

  (c) $ST_{max} = ST_0 \exp(-APWL_{min}/ST_0) = 200 \exp(-APWL_{min}/200)$

  (d) $ST_{min} = ST_0 \exp(-APWL_{max}/ST_0) = 200 \exp(-APWL_{max}/200)$
- This can now be solved.

  (a) = (c) - (d) leads to:

  $138 = 200*(\exp(-APWL_{min}/200) - \exp(-APWL_{max}/200))$

  (b) can be rewritten as:

  $APWL_{max} = APWL_{min} + 569$

  Combination of these two equations results in:

$$138/200 = \exp(-APWL_{min}/200) - \exp(-(APWL_{min} + 569)/200)$$
$$= \exp(-APWL_{min}/200)*(1 - \exp(-569/200))$$

From this $APWL_{min}$ can be calculated:

$$APWL_{min} = -200*\ln\{\frac{138/200}{1-\exp(-569/200)}\} = 62$$

- So: $APWL_{min} = 62$, $APWL_{max} = 569+62 = 631$
  $ST_{max} = 200*\exp(-62/200) = 147$ and $ST_{min} = 147-139 = 9$
- With these values the rest of the table can be completed.

Answer to exercise 8.3

a. According to the theory of Thornthwaite & Mather the actual evapotranspiration during a dry period is proportional to the amount of water stored in the soil, $E_{act} = c\ V$.
   When $V=V_0$ at $t=0$ then $c = E_{pot}/V_0$
   So $c= 8/100 = 0.08$ and $E_{act} = 0.08*25 = 2$ mm/day.

b. The model developed by Thornthwaite & Mather can be written as:
   $V = V_0 \exp(-E_{pot}\ t/V_0)$
   So in this case $25 = 100 \exp(-8t/100) = 100 \exp(0.08\ t)$, which leads to $t = (-1/0.08) \ln(25/100) = 17.3$ days.
   So a storage of 25 mm is reached after 17 days of dry weather.

Answer to exercise 8.4

a.

	J	F	M	A	M	J	J	A	S	O	N	D	year
P	124	191	125	79	42	44	27	24	30	55	78	111	930
PE	157	129	121	91	69	46	53	82	116	150	149	164	1327
P-PE	-33	62	4	-12	-27	-2	-26	-58	-86	-95	-71	-53	-397
dry		wet		dry									
Σ_1		(62)	(66)										
Σ_2	(463)			(12)	(39)	(41)	(67)	(125)	(211)	(306)	(377)	(430)	
APWL	503		(40)	52	79	81	107	165	251	346	417	470	
ST	1	63	67	59	45	44	34	19	8	3	2	1	
ΔST	0	+62	+4	-8	-14	-1	-10	-15	-11	-5	-1	-1	0
AE	124	129	121	87	56	45	37	39	41	60	79	112	930
S		0	0										0
D	33			4	13	1	16	43	75	90	70	52	397

Note: $\Sigma_1 = \Sigma(P-PE)_{pos}$
$\Sigma_2 = \Sigma(P-PE)_{neg}$

Notes:

- $\Sigma(P-PE) < 0$ and $\Sigma(P-PE)_{pos} < ST_0$, so the soil moisture is not totally restored at the end of the wet season. Therefore $ST_{max} < ST_0$. The same procedure of calculation is followed as in exercise 2.d. Here only the results will be given.
  The equations are in this case:
  $ST_{max} - ST_{min} = \Sigma(P-PE)_{pos} = 66$
  $APWL_{max} - APWL_{min} = \quad (P-PE)_{neg} = 463$
  $ST_{max} = 100 \exp(-APWL_{min}/100)$
  $ST_{min} = 100 \exp(-APWL_{max}/100)$
  Solving this results in:
  $ST_{max} = 67$, $ST_{min} = 1$, $APWL_{max} = 503$, $APWL_{min} = 40$

b. The minimally needed amount of irrigation water is the amount needed by the crop to transpire optimally. So in this case the amount needed is $\Sigma(PE-P) = 397$ mm.

c. With a relatively high groundwatertable capillary rise can occur. This means replenishment of the soil moisture from the groundwater. Hence less than 397 mm of irrigation water is needed for the crop to transpire optimally.

d. In this climate the precipitation-surplus during the wet spell is too small to carry away the accumulated salts from the rootzone. Therefore one can expect salinization. This can be prevented by giving more irrigation water than under b. and at the same time establishing a good drainage system.

Answer to exercise 8.5

a. The expression proposed by Doorenbos & Kassam can be rewritten in the following way:

$$1-y/y_{max} = k(1 - E_{act}/E_{pot})$$
$$y = y_{max}(1 - k(1 - E_{act}/E_{pot}))$$
$$= y_{max} - ky_{max} + ky_{max}E_{act}/E_{pot}$$
$$= ky_{max}E_{act}/E_{pot} + (1 - k)y_{max}$$
$$= ky_{max}\cdot x + (1 - k)y_{max}$$

Because k and y_{max} are constant ky_{max} and $(1 - k)y_{max}$ are also constant. Therefore the two expressions are equivalent.

b. From the answer under a it is clear that $a = ky_{max}$ and $b = (1 - k)y_{max}$.

Answer to exercise 8.6

Indirect damage to crops in case of excess of water is caused by:
- deterioration of the soil structure, particularly the formation of crusts with a low permeability for water and air in light sandy clay soils;
- lack of nitrogen, because nitrogen mineralization and nitrification are hampered for considerable time;
- interruption of normal farm management practices. This is often the most serious cause for damage in modern farming. As a result of the excess of water, land is impassable and/or unworkable, spring activities start too late and harvesting is difficult.

Answer to exercise 8.7

The volume fractions of water at field capacity (pF = 2.0) and at wilting point (pF = 4.2) can be read from figure 8.7.1. The total available moisture is the moisture in the rootzone between field capacity and wilting point. According to Shockley the easily available moisture is calculated by dividing the rootzone in four layers of equal thickness and next assuming that:
- 100% of the available moisture in the first layer is easily available;
- 75% of the available moisture in the second layer is easily available;
- 50% of the available moisture in the third layer is easily available;

- 25% of the available moisture in the fourth layer is easily available.

The calculations and results are given in table 8.7.1.

Table 8.7.1 Total available and easily available moisture in the rootzone.

layer (cm)	θ at pF = 2.0 (cm^3 cm^{-3})	θ at pF = 4.2 (cm^3 cm^{-3})	T.A.M.[1] (mm)	E.A.M.[2] (mm)
0 - 25	0.276	0.045	57.75	57.75
25 - 40	0.276	0.045	34.65	25.99
40 - 50	0.365	0.150	21.50	16.13
50 - 75	0.365	0.150	53.75	26.88
75 - 100	0.365	0.150	53.75	13.44
total			221	140

[1] T.A.M.: Total Available Moisture

[2] E.A.M.: Easily Available Moisture according to Shockley.

T.A.M. (in mm) is calculated according to:

T.A.M. = (pF(2.0) - pF(4.2))*(thickness of layer in mm).

Examenvraagstukken bij hoofdstuk 8

8.1. (april 1992)

Voor een gebied in een Mediterraan klimaat zijn maandelijkse gegevens omtrent neerslag en potentiële verdamping bekend (zie bijlage). Bereken de gemiddelde waterbalans (volgens de methode van Thornthwaite-Mather) van een bewortelde zone voor de situatie waarin de grondwaterstand zich diep onder het maaiveld bevindt. De maximale hoeveelheid water die in de wortel-zone kan worden geborgen bedraagt 180 mm.

a. Completeer de tabel in de bijlage en laat zien hoe de berekening is uitgevoerd.

	J	F	M	A	M	J	J	A	S	O	N	D	jaar
P	64	75	40	32	22	15	10	25	36	51	50	52	
E_{pot}	18	23	30	51	86	110	140	132	100	71	35	19	
APWL													
ST													
Δ *ST*													
E_{act}													
Afvoer													
Verd. tekort													

9.0 Antwoorden van de Nederlandse examenvraagstukken

1.1.

a. 2.5 10^4 km^2

b. 1.875 10^4 km^2

c. zal dalen totdat het (nieuwe) oppervlak gelijk is aan 1.875 10^4 km^2; nee.

d. zal dalen totdat het meergeheel droog is (na 75 jaar).

1.2

a.

aarde als geheel	505	10^3 km^3
de zeeën	434	" "
de humide streken	43	" "
de aride streken	28	" "

b.

aarde	990 mm	(E_{act})
zeeën	1157 mm	(E_o)
humide streken	478 mm	(E_{act} - E_{pot})
aride streken	622 mm	(E_{act})

c. 3226 j

3.1.

a.
1. rekenkundig gemiddelde
2. Thiessen polygonen methode
3. isohyeten methode
4. hoogteklassen methode

$$\bar{P} = \frac{\sum_1^n P_i}{n}$$

b. 1.

n = aantal meetstations

2. $\bar{P} = w_1P_1 + w_1P_2 + \ldots\ldots + w_nP_n$

$w_1 \ldots\ldots w_n$ zijn gewichtsfactoren

$$w_i = \frac{A_i}{A_{tot}}$$

A_i = oppervlak Thiessen polygoon

A_{tot} = totale oppervlak van het stroomgebied.

3. Idem als 2. De gewichtsfactoren w_i ....w_n worden nu verkregen, door eerst de isohyeten te tekenen en de oppervlakte Ai tussen twee isolijnen te bepalen. P_1....P_n is de gemiddelde neerslag (tussen twee isolijnen)

4. Idem als 2. A_i is nu de oppervlakte die representatief is voor een bepaalde hoogteklasse met neerslag P_i.

3.2

30.3 mm

4.1

a.

t = 0 min f = 32,0 mm uur^{-1}

t = 15 min f = 11.0 mm uur^{-1}

t = 30 min f = 6,3 mm uur^{-1}

t = 45 min f = 5,3 mm uur^{-1}

t = 60 min f = 5,1 mm uur^{-1}

b.

70 m^3

5.1

26,3%

5.2

ca 1.4 mm/dag ofwel 30%

5.3

a. mais Q^*, λE, H, G.

woestijn Q^*, H, G, λE

b. kale grond, wateroppervlak, nat gewas

kale grond: gedurende korte periode (uren) zal de verdamping maximaal zijn, zolang de capillaire aanvoer van vocht door de bodem naar het oppervlak het verdampingsproces kan volgen.
Indien de aanvoer kleiner wordt dan de atmosferische vraag dan zal de toplaag snel uitdrogen en wordt de aanvoer nog sterker belemmerd. De verdamping valt dan snel scherp terug.
Wateroppervlak: In principe zal de verdamping gelijk zijn aan de atmosferische vraag (E_{pot})
Nat gewas: Als gevolg van interceptie zal de verdamping van het gewas boven de Epot liggen. Tijdens het opdrogen van het gewas zal de verdamping teruglopen naar E_{pot} indien het gewas voldoende vocht kan onttrekken aan de bodem. Indien dit niet het geval is zal de transpiratie na volledige opdroging van het gewas onder het nivo van E_{pot} komen te liggen.

5.4.

a. Zie dictaat

b. In het algemeen is de hoeveelheid uitgaande langgolvige straling tijdens het verloop over het etmaal groter dan de hoeveelheid inkomende langgolvige straling (zie figuur dictaat, straling). Aangezien 's nachts de kortgolvige stralingstermen nul zijn, wordt de nettostraling negatief.

c. - vochttekorten in de bodem
- onvolledige bedekkingsgraad van het gewas.

d. Zoals blijkt uit de balansen (zie fig. dictaat) zijn de energiebalans en waterbalans vooral gekoppeld via nettostraling, verdamping en neerslag.
Voor een tropisch regenwoud is de koppeling zeer sterk als gevolg van een direkt verband tussen de drie grootheden. Echter in een semi-aride klimaat zal de koppeling er slechts sterk zijn in het regenseizoen. In het droge seizoen zal Q^* zich verdelen over H_{lucht} en H_{bodem} met E_{act} naar de waarde nul. De koppeling tussen de energiebalans en de waterbalans vervalt nagenoeg.

5.5.

a. Ventilatieterm; u -> 0.

$$\lambda E = \frac{s}{\gamma + s} Q^*$$

b.

c. 2,5 mm/dag

5.6.

a. Wanneer de gemiddelde jaarlijkse toevoer naar het stuwmeer groter of gelijk is aan de totale afname, inclusief de afname voor 10000 ha geïrrigeerde landbouw.
Neerslag + toevoer naar stuwmeer = verdamping + afvoer t.b.v. drinkwater, zout en irrigatie.

b. 5337 ha
Nevendoelstelling kan ongeveer voor de helft worden gerealiseerd.

6.1. 645 mm j^{-1}.

6.2.

a. Q en L.

b. H_a = 1.25, H_b = 1.25 m, H_c = 0.625 m, H_d = 0.0 m.

c. P_a = 0Pa, P_b = 2452.5 Pa, P_c = 1226.3 Pa, P_d = 0 Pa.

d. 2.5 m d^{-1}.

e. n_e (effectieve porositeit).

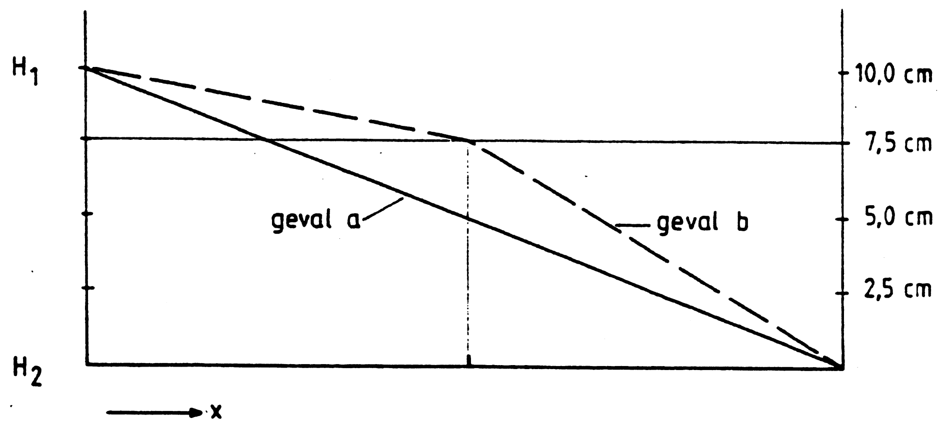
6.3.

a. 2.4 m d^{-1}

b. 0.8 m d

c. 8.75 respectievelijk 17.5 uur

d.



6.4.

a. divergentie van de fluxdichtheid is nul, betekent dat in een elementair volume evenveel instroomt als uitstroomt.

b. ja; in een verzadigde stroming is $\nabla . v = 0$ als
ρ = constant
anisotropie heeft hierop geen invloed.

c. ja, bij stationaire stroming.

6.5.

a. omdat de breedte van het infiltratiegebied bij Rhenen veel kleiner is dan op de Veluwe. Zie vergelijking 6.7 - 8 van het collegedictaat.

b. 0.04 m/d 14.6 m j^{-1}

c. 438000 m3 j^{-1}

d. 41.7 m j^{-1}

7.1

a. 200 mm $jaar^{-1}$

b. nee

c. gradient $\frac{dH}{dz} = 1.0$

$k_{onv} = 0.274\ 10^{-3}$ m d^{-1}

$\theta = 0.0367$

8.1.

APWL maart: 14 mm

APWL oktober: 513 mm

$\sum$ Eact: 472 mm

$\sum$ Afvoer: 0 mm

$\sum$ Verd. tekort: 343 mm

Naam : ______________________________

Reg.nr. : ______________________________

Adres : ______________________________

Plaats : ______________________________

Tel.nr. : ______________________________