Dynamic model for nutrient uptake by tomato plant in hydroponics

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Abstract
Aquaponics, combining fish production (aquaculture) and soilless plant cultivation (hydroponics), is an innovative food production system for future agriculture. Ideally, through recirculating the water and nutrients between the two coupled systems, the total requirements for fertilizer and water could be largely reduced, while achieving high efficiency of the system. In order to maximize the production potential of aquaponics, it is necessary to apply proper management on both water and nutrients usage. Particularly, the nutrient management of hydroponics is important considering the high chance of nutrient imbalance and the following negative effect on crop production.

The growth of greenhouse tomato plant and the related dynamic nutrient uptake were modelled in order to get insight in nutrient consumption as well as better management of nutrients in hydroponics. State space modelling approach with zero and first differential equations are used for modelling tomato growth. For describing nutrient uptake, a simple yet effective Michaelis-Menten shape equation is implemented which correlate the uptake with solar irradiance. The major inputs of the model are temperature, irradiance, $CO_2$ concentration and electrical conductivity of hydroponics considering their crucial effects on nutrient uptake. The validation of the model was performed on growth and nutrient uptake of tomato plant. Particularly, parameter estimation was performed on calibrating nutrient uptake model. In total, six nutrients were studied including nitrogen, phosphorus, potassium, sulfur, calcium and magnesium.

The validation results showed good prediction accuracy of tomato growth model (normalized root mean square error< 16%). The results of parameter estimation suggest that the nutrient uptake model could have a set of feasible parameter vectors instead of one optimal parameter vector. For validation of nutrient uptake model, it was found that current model is suitable for predicting nutrients which were absorbed actively. For nutrients which were absorbed both passively and actively, a more comprehensive model is required for accurate simulation.
1. Introduction

1.1 Background

Hydroponics, also referred to as soilless cultivation, is a method of cultivating crops in growing medium or in pure water culture where the nutrients are added to the irrigation water (Raviv 2007). Different from conventional soil-based production systems, the water and nutrients will be conserved in the hydroponic systems, preventing the excessive usage of water and fertilizers. Besides, for crops grown in hydroponic systems, the maximum yields can be achieved by maintaining optimal concentration of nutrients in the root zone. This would make the system economically feasible in expensive land areas e.g. peri-urban (Jones Jr 2004).

In open (non-recirculating) hydroponic systems, the average utilisation of main nutrients varies between 30 to 80%, and up to 1000 kg N ha\(^{-1}\) can be lost through drainage of waste solution (Van Noordwijk 1990, Rácz 2007). If not re-used, the discharge of the drained nutrient solution would cause serious water pollution. To alleviate environmental degradation caused by open hydroponic systems, closed (recirculating) hydroponic systems are developed, where the drainage solution is captured and recirculated (Sonneveld and Voogt 2009). Through the re-use of irrigation water, the use efficiency of water and nutrients are improved. However, the management of closed hydroponic systems is more difficult compared with open systems, as the nutrient concentrations of the system are largely influenced by the re-used drainage solution. Moreover, the plant nutrients uptake efficiency would vary depending on growth stage and environmental factors.

To cope with these difficulties, various techniques for real-time management of closed hydroponic systems are used. A standard method involves mixing of drainage and water by aiming at a pre-set electrical conductivity (EC) in the outgoing mixture (Kreij et al. 1999). By implementing this method, the EC of the supplied solution can be controlled at a desired point. Unfortunately, only the EC of the nutrient solution can be controlled, not the ratios between individual nutrients.

To deal with the risk of nutrients imbalance, nutrient uptake models for crops are used to support the nutrients management of hydroponic systems. By precisely predicting the nutrient requirements during the whole life cycle of the crops, nutrient uptake models can prevent the imbalance of nutrients and improve the efficiency of fertilizer usage.

The traditional nutrient uptake models consider the roots as a single cylinder surrounded by an infinite extent of soil, the nutrient diffuses through the soil water (via the pore water) driven by the difference in concentration. However this kind of models are used for field grown systems, thus are not adequate for hydroponic systems where nutrient concentrations are not limited. Regarding the hydroponic systems, many models were developed using Michaelis-Menten kinetics to describe the nutrient uptake by crop. The plant nutrient uptake is thought to occur through binding to certain specific ion-binding proteins in the root wall (Bowling 1976), in this way the mechanism is kinetically similar to the Michaelis-Menten reaction of enzyme kinetics (Murray 1993). Epstein et al. (1966) first noticed that the relation between potassium uptake and its concentration conform to a rectangular hyperbola which resembles the Michaelis-Menten enzyme kinetics. One important assumption for applying Michaelis-Menten kinetics on determining nutrient uptake is that the nutrients uptake is directly correlated to their concentration in the nutrient solution, that is:

\[
U_n = \frac{U_{\text{max}}(C - C_{\text{min}})}{K_m + (C - C_{\text{min}})}
\]  

(1.1.1.)

Where \( U \) is the nutrient uptake rate; \( U_{\text{max}} \) is the maximum rate of nutrient uptake; \( C \) is the concentration of the nutrient in the solution; \( C_{\text{min}} \) is the concentration where influx equals efflux; \( K_m \) is the Michaelis-Menten constant at which the uptake rate is half of the maximum uptake rate. It was found that \( U_{\text{max}} \) and \( K_m \) are affected by internal as well as external conditions (Massa et al. 2008). This effect can be either incorporated using a sub-model describing the effect of environmental conditions or by measurements and calculations of \( U_{\text{max}} \) and \( K_m \) at different environmental conditions (Sago et al. 2011). Mattson et al. (2006) developed a nutrient uptake model which takes into account the dynamic nature (change in growth, nutrient demand, storage, and reallocation) for hydroponically grown roses. In this model, the nutrient uptake was correlated with tissue nutrient concentration and dry matter. However, the tissue nutrient concentration and dry matter were directly obtained from data, the influences of environmental factors were not considered. Silberbush et al. (2005) proposed a model for water and nutrient uptake by plants grown in hydroponic systems. The influence of salinity on nutrient uptake rate was incorporated in the model. In practice, the salinity of hydroponic systems is well controlled, weakening the application of salinity based models. Kempen (2015) developed a model for simulation of water and nutrient uptake rate of soilless grown tomatoes. The nutrient uptake rate was correlated with photosynthetic photon flux density (PPFD) using Michaelis-Menten type equation. Yet, the model can only
simulate the total nutrients uptake for a certain period, the dynamic behaviour of nutrient uptake by tomato plant is not well studied.

Among the studies regarding the nutrient uptake by tomato plant in hydroponics, none of them has focuses on the dynamic behaviour of nutrient uptake during the whole development of the tomato plant. Besides, most of the existing models only considered the influence of one environmental factor. A comprehensive model considering multiple factors is needed. Thus, the innovation carried out by this study consists in developing a dynamic nutrient uptake model with consideration of possible environmental factors and the following model calibration using dynamic profile of nutrient content of tomato plant.

1.2 Research objectives
The current research is aimed at developing a dynamic model for predicting nutrient uptake by tomato plants grown using nutrient film technology (NFT) under greenhouse conditions, meanwhile accounting for influence of environmental factors on plant development.

This nutrient uptake model will be used for nutrient management of tomato cultivation in aquaponics. The nutrients of interest are macro nutrients, including nitrogen (N), phosphorus (P), potassium (K), calcium (Ca) and magnesium (Mg).

1.3 Research questions
1. Is Michaelis-Menten kinetics suitable for describing the nutrient uptake by tomato plants in NFT systems under greenhouse conditions?
2. Which factors should be considered for modelling tomato plant dry matter production?

1.4 Thesis outline
The general approach of this thesis is presented in section 2. A detailed model for tomato plants nutrient uptake is described in section 2.1 and 2.2. Mineral composition data of tomato plants are collected and shown in Appendix 3. Parameter estimation of the nutrient uptake model using ordinary least square OLS method was presented in section 2.4.

Results and discussions are listed in chapter 3 and 4. Finally, the recommendations are in chapter 5.
2. Model description

Many models have been developed both for nutrient uptake and growth of tomato plants, therefore, the current research will develop a model by learning from previous studies. A Michaelis-Menten based model will be built to simulate the tomato plant nutrient uptake rate. In order to predict the nutrient consumption per tomato plant, the nutrient uptake rate has to be integrated to the whole plant. This can be achieved by correlating the nutrient uptake model with the tomato plant growth model (using a sub-model of root growth).

For model calibration, data on plant nutrient uptake are required. Experimental data of tomato plant nutrient uptake rate are barely found in the existing literature, however, mineral composition data are available in many studies. Through modelling the plant growth and nutrient uptake against data of mineral composition, it would be easy to obtain the dynamic profile of tomato plant nutrient uptake. Consequently, this profile can be used for calibration of the nutrient uptake model.

Calibration of the model will be done by an ordinary least squares method.

A general overview of the model development is presented in Figure 1:

![Figure 1. Overview of the model development](image)

2.1 Modelling nutrient uptake

2.1.1 Modelling uptake based on radiation

Generally, plant growth is the result of transforming \( \text{CO}_2 \), \( \text{H}_2\text{O} \) and mineral nutrients into plant materials. This process is mainly done by plant photosynthesis. Research has shown a strong correlation between nutrients uptake rates and incident irradiance levels. In order to describe the effect of irradiance on nutrient uptake, a Michaelis-Menten type equation with rectangular hyperbola shape was used (Mankin and Fynn 1996):

\[
U_n = \frac{U_{\text{max}} \cdot \text{PAR}}{K_m + \text{PAR}} \quad (2.1.1.1)
\]

Where \( U_n \) (mg m\(^{-2}\)[root h\(^{-1}\)]) is the plant nutrient uptake rate; \( U_{\text{max}} \) (mg m\(^{-2}\)[root h\(^{-1}\)]) is the maximum rate of nutrient uptake; \( \text{PAR} \) (\( \mu \text{mol} m^{-2}[gh] s^{-1} \)) is the photosynthetically active radiation; \( K_m \) (\( \mu \text{mol} m^{-2}[gh] s^{-1} \)) is Michaelis-Menten constant (the photosynthetic active radiation at half of \( U_{\text{max}} \)).

Michaelis-Menten type kinetics is suitable for describing nutrients which are absorbed actively including K, N, P, Ca and Mg. However, the uptake of Ca is active, but also passively driven by the transpiration
stream (Mengel and Kirkby 1987, Adams 1993). Suggested by Silberbush et al. (2005), the following equation was used to calculate the calcium uptake:

\[
U_{\text{Ca}}^{\text{Ca}} = \frac{U_{\text{max}}^{\text{Ca}} \cdot PAR}{K_{\text{Ca}}^{\text{Ca}} + PAR} + \beta \cdot T_r \cdot a \cdot LAI(t) \cdot C^{\text{Ca}}
\]  
(2.1.2.)

Where \(U_{\text{Ca}}^{\text{Ca}}\) (mg m\(^{-2}\)[RSA] h\(^{-1}\)) is the uptake rate of Ca, \(U_{\text{max}}^{\text{Ca}}\) (mg m\(^{-2}\)[RSA] h\(^{-1}\)) is the maximum rate of Ca uptake, \(K_{\text{Ca}}^{\text{Ca}}\) (m\(^{-2}\)[gh] h\(^{-1}\)) is the Michaelis-Menten constant of Ca uptake, \(\beta\) is the fraction of water influx active in Ca uptake (assumed a constant value of 0.25), \(T_r\) (mm day\(^{-1}\)) is the water losses due to transpiration, \(a\) (m\(^2\)) is the cell area (assumed a constant value of 0.2), \(LAI(t)\) (m\(^2\) [leaf] m\(^{-2}\) [gh]) is the leaf area index at time \(t\), \(RSA(t)\) (m\(^2\) [RSA]) is the root surface area at time \(t\) and \(C^{\text{Ca}}\) (mg m\(^{-3}\)) is the Ca concentration in hydroponics.

The \(T_r\) can be calculated using a relative transpiration function (Silberbush, Ben-Asher et al. 2005):

\[
T_{\text{rel}} = \begin{cases} 
\sin \left(2\pi(t - (\tau - 1))\right) & \text{when } > 0 \\
0 & \text{otherwise} 
\end{cases}
\]  
(2.1.3.)

\[
T_r = T_{\text{rel}} \cdot T_{\text{max}} \cdot LAI(t)
\]  
(2.1.4.)

Where \(T_{\text{rel}}\) is relative transpiration (starting at 06:00 with a value of 0, increases to 1 at 12:00, and decline to 0 at 18:00), \(\tau\) is an integer number of the current day, \(T_{\text{max}}\) (mm day\(^{-1}\)) is the maximum transpiration rate, which occurs at midday. This simplified method assumes a sine wave of hourly solar irradiance and would give reasonable predictions under ideal situation. However, in reality, the solar irradiance has large uncertainties (without implementing artificial lighting), weakening the applicability of Silberbush’s transpiration model. A more suitable approach would be simulating the plant transpiration based on the relevant factors.

According to the research of Jolliet and Bailey (1992), the transpiration of tomato plant is affected by various climate factors, e.g. air speed, vapour pressure deficit, solar radiation and \(C_0\) concentration. It was also found that transpiration rate increases linearly with solar radiation, air vapour pressure deficit and air speed. Based on these findings, a regression equation is used to simulate the transpiration rate of tomato plant:

\[
T_r = 0.141 \cdot R_{\text{in}} + 28.1 \cdot D_a \quad R^2 = 0.87
\]  
(2.1.5.)

Where \(T_r\) (mg m\(^{-2}\) [gh] s\(^{-1}\)) is the transpiration rate, \(R_{\text{in}}\) (J m\(^{-2}\) [gh] s\(^{-1}\)) is the incoming global solar radiation on the horizontal surface inside the greenhouse, \(D_a\) (kPa) is the air vapour pressure deficit. \(D_a\) is taken as constant (0.5 kPa) in the simulation assuming the humidity is well controlled.

By replacing the transpiration \(T_r\) with the equation (2.1.5.), the modified equation for predicting the calcium uptake is:

\[
U_{\text{Ca}}^{\text{Ca}} = \frac{U_{\text{max}}^{\text{Ca}} \cdot PAR}{K_{\text{Ca}}^{\text{Ca}} + PAR} + \beta \cdot T_r \cdot a \cdot LAI(t) \cdot C^{\text{Ca}}
\]  
(2.1.6.)

For simplicity, \(C^{\text{Ca}}\) is assumed to be constant.

### 2.1.2 Parameters of uptake model

As mentioned before, the Michaelis-Menten parameters \(U_{\text{max}}\) (mg m\(^{-2}\)[RSA] h\(^{-1}\)) and \(K_m\) (\(\mu\)mol m\(^{-2}\)[gh] s\(^{-1}\)) are affected by both internal (e.g. development stage) and external (e.g. irradiance, electrical conductivity, temperature) factors. In order to reproduce these effects, the following equations are used to simulate \(U_{\text{max}}\) and \(K_m\) (Kempen 2015):

**Table 1. Michaelis-Menten parameters**

<table>
<thead>
<tr>
<th>Ion</th>
<th>(U_{\text{max}})</th>
<th>(K_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(NO_3^-)</td>
<td>(p^N_{O_3} \cdot (EC \cdot PTU) + p^N_{O_3})</td>
<td>(p^N_{O_3} \cdot (EC \cdot PTU) + p^N_{O_3})</td>
</tr>
<tr>
<td>(K^+)</td>
<td>(p^K_{+/} \cdot (EC \cdot PTU) + p^K_{+/})</td>
<td>(p^K_{+/} \cdot (EC \cdot PTU) + p^K_{+/})</td>
</tr>
<tr>
<td>(H_2PO_4^-)</td>
<td>(p^H_{2PO_4} \cdot (EC \cdot PTU) + p^H_{2PO_4})</td>
<td>(p^H_{2PO_4} \cdot (EC \cdot PTU) + p^H_{2PO_4})</td>
</tr>
<tr>
<td>(Mg^{2+})</td>
<td>(p^{Mg_{2+}} \cdot (EC \cdot PTU) + p^{Mg_{2+}})</td>
<td>(p^{Mg_{2+}} \cdot (EC \cdot PTU) + p^{Mg_{2+}})</td>
</tr>
<tr>
<td>(SO_4^{2-})</td>
<td>(p^{SO_4_{2-}} \cdot (EC \cdot PTU) + p^{SO_4_{2-}})</td>
<td>(p^{SO_4_{2-}} \cdot (EC \cdot PTU) + p^{SO_4_{2-}})</td>
</tr>
<tr>
<td>(Ca^{2+})</td>
<td>(p^{Ca_{2+}} \cdot (EC \cdot PTU) + p^{Ca_{2+}})</td>
<td>(p^{Ca_{2+}} \cdot (EC \cdot PTU) + p^{Ca_{2+}})</td>
</tr>
</tbody>
</table>
In order to adjust the time step of equation 2.1.7. from day to hour, following equation is used:

\[
PTU(t_d) = GDD(t_d) \cdot R(t_d)
\]  
(2.1.7.)

\[
GDD(t_d) = \sum_{i=d_{rad}}^{t_d} \left( T_{\text{max}}(i) + T_{\text{min}}(i) \right) / 2 - T_{\text{base}}
\]  
(2.1.8.)

Where \( t_d \) is days after transplanting of tomato plant into the hydroponics, \( T_{\text{max}} \) and \( T_{\text{min}} \) are the daily maximum and minimum temperature, respectively.

In order to adjust the time step of equation 2.1.7. from day to hour, following equation is used:

\[
PTU(t) = \frac{GDD(t_d)}{24} \cdot R_h(t)
\]  
(2.1.9.)

Where \( R_h \) (\( MJ \) \( m^{-2} \) \( [gh] h^{-1} \)) is the hourly global solar radiation.

### 2.1.3 Coupling uptake with plant growth

The simulated nutrient uptake rate by tomato plant has a unit of \( mg \) \( m^{-2} \) \( [RSA] h^{-1} \). Thus, in order to get the nutrient uptake per greenhouse area (\( mg \) \( m^{-2} \) \( [gh] h^{-1} \)) or per plant (\( mg \) \( plant^{-1} h^{-1} \)), the uptake rate should be coupled dynamically with tomato plant root surface area \( RSA \) (see equation 2.1.2.).

Typically, \( RSA \) is calculated by assuming the roots to be smooth cylinders with a constant mean radius \( rad \) (m) (Silberbush et al. 2005):

\[
RSA(t) = 2\pi \cdot rad \cdot RL(t)
\]  
(2.1.10.)

Where \( RL \) (m) is the root length of tomato plant. According to Barber (1995), the \( RL(t) \) can be simulated using a logistic function:

\[
RL(t) = \frac{RL_{\text{max}}}{1 + r_{g_1} \cdot \exp\left(-r_{g_2} \cdot t\right)}
\]  
(2.1.11.)

Where \( RL_{\text{max}} \) (m) is the maximal length of root; \( r_{g_1} \) and \( r_{g_2} \) are coefficients.

However, as \( RSA \) serves as a key coupler between the nutrient uptake model and the plant growth model, the result of plant growth model should be reflected in \( RSA \) for consistency. Kim et al. (2012) found a linear relation between root dry mass and root surface area in roses grown hydroponically. By assuming a similar root growth between tomato plant and rose, the \( RSA \) of tomato plant can be calculated as follow:

\[
RSA(t) = SSA \cdot W_{\text{root}}(t)
\]  
(2.1.12.)

Where \( SSA \) (\( m^2 [\text{root}] m^{-2} \text{[gh]} g^{-1} \)) is the specific root surface area which converts the root dry weights \( W_{\text{root}} \) (\( g \text{[DM]} m^{-2} \text{[gh]} \)) to root surface area \( RSA \) (\( m^2 [\text{root}] \)).

Wolterbeek (1987) studied the methods for determine the surface area of fine tomato roots. The results from the microscopic experiments gave a mean value of 0.011 for \( SSA \) (\( m^2 [\text{root}] g^{-1} \text{[fresh weight]} \)) (standard deviation = 5%, \( n = 102 \)). Considering an 88.5% of water content for tomato root system (Zhou and Luo 2009), the mean value for root surface area per root dry weight is 0.096 (\( m^2 [\text{root}] g^{-1} \text{[dry weight]} \)).

Using equation 2.1.1., 2.1.2 and 2.1.12., the nutrients uptake by tomato plant \( U(t) \) (\( mg \) \( m^{-2} \) \( [gh] h^{-1} \)) can be expressed as:

\[
U(t) = U_d(t) + SSA \cdot W_{\text{root}}(t)
\]  
(2.1.13.)

Based on the cumulative nutrients uptake \( U_{\text{cumul}} \) (\( mg \) \( m^{-2} \) \( [gh] \)) and plant dry weight \( W_{\text{plant}} \) (\( g \text{[DM]} m^{-2} \text{[gh]} \)), dynamic content of nutrients in tomato plants \( C_{\text{content}} \) (%) can be computed using:

\[
C_{\text{content}}(t) = \frac{U_{\text{cumul}}(t)}{W_{\text{plant}}(t)}
\]  
(2.1.14.)
\[ U_{\text{cumul}}(t) = \sum_{i}^{t} U(\tau) \]  

(2.1.15.)

### 2.2 Modelling tomato growth

In order to construct the dynamic profile of nutrients content in tomato plant, a growth model is needed including the dry matter partitioning to different organs.

Many models have been developed for simulation of growth of tomato plant. The growth of tomato plant is affected by various environmental factors including temperature, solar irradiance, CO\textsubscript{2} concentration, humidity and nutrient availability. For greenhouse tomato plant cultivation, factors like humidity have relatively small influence on the growth of tomato plant, thus their influences on growth can be neglected. Solar irradiance, CO\textsubscript{2} concentration and temperature are the three main factors which have large influence on tomato growth. For the purpose of building a system-level model, the solar irradiance, CO\textsubscript{2} concentration and temperature are taken into account as the environmental factors in model development.

#### 2.2.1 Photosynthetically active radiation inside the greenhouse

Photosynthesis of tomato plant is based on the flux of photosynthetically active radiation \( \text{PAR} \, (\text{Wm}^{-2}) \) which arrives at the top of the canopy. \( \text{PAR} \) designates the spectral range of solar radiation from 400 to 700 nanometres. According to the measurements performed by Anonymous (1981), it was calculated that daily radiation in the 400 to 700 nm range was on average 47% in daily global radiation. This percentage can vary slightly depending on weather conditions (clear or cloudy) and seasonal changes of solar irradiance. Since these factors are not in the scope of this research, a percentage \( \text{PAR} \) of 47% is used.

Due to the transmission of solar radiation by greenhouse, the \( \text{PAR} \) inside and outside of a greenhouse could vary largely. Bot (1983) developed a model for predicting the transmissivity based on solar position, greenhouse roof angle, dimensions of the roof construction parts, transmissivity of the glass panes and orientation of the greenhouse. According to the model validation performed by Heuvelinka and Batta (1995), average transmissivity for Venlo-type greenhouse is 62%. This value for greenhouse transmissivity is adopted. The \( \text{PAR} \) inside the greenhouse is calculated by multiplication of the \( \text{PAR} \) outside the greenhouse by the transmissivity.

#### 2.2.2 Canopy photosynthesis

The dry matter production of tomato plant is simulated based on ASKAM model developed by Gijzen (1992) and TOMSIM model developed by Heuvelink and Bertin (1994). In general, the leaf gross photosynthetic rate \( P_g \, (\text{gCO}_2 \text{m}^{-2} \text{leaf} \text{h}^{-1}) \) is calculated from the negative exponential light response curve (Spitters, Toussaint et al. 1986):

\[ P_g = P_{\text{max}} \times \left(1 - \exp \left(\frac{-\varepsilon \times \text{PAR}_{\text{abs}}}{P_{\text{max}}} \right)\right) \]  

(2.2.1.)

Where \( P_g \, (\text{gCO}_2 \text{m}^{-2} \text{leaf} \text{h}^{-1}) \) is leaf gross photosynthetic rate, \( P_{\text{max}} \, (\text{gCO}_2 \text{m}^{-2} \text{leaf} \text{h}^{-1}) \) is leaf maximum photosynthetic rate, \( \varepsilon \, (\text{gCO}_2 \text{m}^{-2} \text{J}^{-1}) \) is the leaf initial light use efficiency by photorespiration and \( \text{PAR}_{\text{abs}} \, (\text{Jm}^{-2} \text{leaf} \text{h}^{-1}) \), is absorbed photosynthetically active radiation.

According to Van Straten et al. (2010), the absorbed photosynthetically active radiation \( \text{PAR}_{\text{abs}} \) depends on the position of a leaf in the canopy. It is determined by the gradual extinction of radiation with canopy depth as a whole. Therefore, the canopy photosynthetic rate is calculated through a three-point Gaussian integration over the crop depth (if leaf area index \( \text{LAI} \, (\text{m}^2 \text{leaf} \text{m}^{-2} \text{gh}) \) is higher than 3, a five-point Gaussian integration should be used for accuracy). The Gaussian integration determines the canopy photosynthetic rate from the average photosynthetic rate for the three layers in the canopy. Based on the research of Goudriaan and Van Laar (2012) and Spitters (1996), the gross canopy assimilation rate \( P_{gc} \, (\text{gCO}_2 \text{m}^{-2} \text{gh} \text{h}^{-1}) \) can be computed using the following equations:

\[ L_{\text{GUSS}}(t) = RD_t \times \text{LAI}(t) \quad (i = 1,2,\ldots) \]  

(2.2.2.)

\[ L_t(t) = \text{PAR}(t) \times k_{\text{ext}} \times \exp(-k_{\text{ext}} \times L_{\text{GUSS}}(t)) \]  

(2.2.3.)

\[ P_{gc}(t) = P_{\text{max}} \times \left(1 - \exp \left(\frac{-\varepsilon \times L_t(t)}{P_{\text{max}}} \right)\right) \]  

(2.2.4.)
The relative depth \( RD_1 \) of the canopy and the weight factor \( WT_i \) for the three-point Gaussian integration are:

\[
RD_1 = (0.5 - \sqrt{0.15}, \ 0.5, \ 0.5 + \sqrt{0.15})
\]

\[
WT_i = (0.1127, \ 0.5, \ 0.8873)
\]

The leaf maximum photosynthetic rate \( P_{\text{max}} \) \( (g[CO_2] \text{m}^{-2}\text{[leaf]} \text{h}^{-1}) \) is limited by \( CO_2 \) concentration, \( C_a (\mu l[CO_2] \text{[air]}) \) and the stomatal resistance \( \left(\text{mol CO}_2 \text{ mol}^{-2} \text{ photon} \right) \) is 0.084 (mol CO\text{2} mol^{-2} photon). 

\[
\Gamma_1 = 42.7 + 1.68 \times (T_i - 25) + 0.012 \times (T_i - 25)^2
\]

This equation holds at \( 0 \) concentration 210 \( ml \) \text{ l}^{-1} \text{ (21%).} 

The leaf maximum photosynthetic rate \( P_{\text{max}} \) \( (g[CO_2] \text{m}^{-2}\text{[leaf]} \text{h}^{-1}) \) is limited by \( CO_2 \) concentration \( C_a (\mu l[CO_2] \text{[air]}) \). \( P_{\text{max}} \) depends on the \( CO_2 \) gradient between ambient air and chloroplast, and three resistances to \( CO_2 \) transport (Goudriaan et al. 1985):

\[
P_{\text{max}} = \frac{6.48 \times (C_a - \Gamma_1)}{r_m + 1.36 + r_b + 1.6 \times r_s}
\]

Where constant 6.48 converts \( \mu l[CO_2] \text{[air]} \) to \( g[CO_2] \text{m}^{-2} \text{[leaf]} \text{h}^{-1} \), \( r_m (s \text{ m}^{-1}) \) and \( r_s (s \text{ m}^{-1}) \) are the boundary layer resistance and the stomatal resistance for water vapour diffusion respectively. The ratio between \( CO_2 \) diffusion and water vapour diffusion are 1.36 and 1.6 (Von Caemmer and Darquhar 1981) for the boundary layer resistance and the stomatal resistance, respectively.
The dry matter production of tomato plant is calculated by converting the net photosynthesis to the dry matter production.

\[
DMP = \frac{W_{plant} \cdot T_{25}^{25}}{Q_{10}}
\]

Where \( W_{plant} \) is the dry weight of the plant, \( T_{25} \) is the temperature in °C, and \( Q_{10} \) is the temperature factor. The calculation of \( DMP \) is used to model the influence of temperature on maintenance respiration of tomato plant, with \( Q_{10} \) taken as 2 for maintenance respiration (Heuvelink and Bertin 1994).

In the model proposed by Gijzen (1992), the maintenance coefficient is assumed to be constant as the tomato plant is considered as a whole. However, in reality, the maintenance coefficients vary between different plant organs (leaves, stems, roots, fruits). With the development of tomato plant, the dry weights ratio of different organs to whole plants would change dynamically, resulting dynamic changes in maintenance coefficients. Thus, the tomato plant should be treated separately as different organs for calculation of maintenance respiration to improve accurate predictions. Based on the four types of organs (leaves, stems, roots, fruits), \( R_m(T) \) is calculated as follows:

\[
R_m(T) = \left( R_{stem}^{25} \cdot W_{stem}(t) + R_{root}^{25} \cdot W_{root}(t) + R_{fruit}^{25} \cdot W_{fruit}(t) \right) \cdot Q_{10}^{\frac{T-25}{10}}
\]

Where \( R_{stem}^{25} \) is the maintenance respiration coefficient at 25°C, \( W( \text{g[DM]} \) m\(^{-2}\)\{gh\}) is the organ dry weight per unit of greenhouse area with the subscripts referring to leaf, stem, root and fruit, respectively.

Amthor (2012) pointed that the crops maintenance respiration depends on the metabolic activity of the crop. When the crop dry weight is high and/or the irradiance is small, equation (2.2.12.) tends to over-estimate the maintenance respiration. Suggested by Heuvelink (1995), relative growth rate \( RGR \) (h\(^{-1}\)) was used as a measurement for metabolic activity:

\[
R_m(T) = R_{in}^{25} \cdot \left(1 - \exp(-f_{SGR} \cdot RGR)\right)
\]

Where \( f_{SGR} \) is the maximum maintenance rate of tomato plant, \( f_{SGR} \) is a regression parameter. The calculation of \( R_m(T) \) is the same with \( R_m(T) \) in equation (2.3.12.).

Relative growth rate \( RGR \) is the growth rate relative to the dry weights of the tomato plants \( W_{plant} \) (g[DM] m\(^{-2}\)\{gh\}):

\[
RGR(t) = \frac{1}{W_{plant}(t)} \cdot \frac{dW_{plant}(t)}{dt}
\]

To simplify the calculation, a simple regression equation is used to compute the \( RGR \) (Appendix 2):

\[
RGR(W_{plant}) = -0.0008 \cdot \ln(W_{plant}) + 0.0055 \quad R^2 = 0.5588
\]

Growth respiration of tomato plants refers to the \( CO_2 \) release during the synthesis, breakdown processes and energy delivery for uptake of nutrients and transport of material within the plant (Gijzen 1992).

The \( CO_2 \) release by growth respiration can be calculated using following equation (Spitters et al. 1989):

\[
C_p(t) = C_{pf} \cdot \frac{dW_{plant}(t)}{dt}
\]

Where \( C_p \) is the \( CO_2 \) release by growth respiration, \( C_{pf} \) is the \( CO_2 \) production factor of the tomato plant.

2.2.4 Net photosynthesis and dry matter production

To calculate the net photosynthesis rate \( P_{nc} \) (g[CO\(_2\]) m\(^{-2}\)\{gh\} h\(^{-1}\)), the \( CO_2 \) release by the maintenance respiration and growth respiration should be subtracted from the gross photosynthesis \( P_{gc} \) (equation 2.2.5.):

\[
P_{nc}(t) = P_{gc}(t) - R_m(t) \cdot C_{ac} - C_p(t)
\]

Where \( C_{ac} \) is the conversion factor of assimilates (\( CH_2O \)) to \( CO_2 \) (ratio of molar weights between \( CO_2 \) and \( CH_2O \), which is 44/30).

The dry matter production of tomato plant is calculated by converting the net photosynthesis to the dry matter production:
Dry matter partitioning

Dry matter partitioning is the process of allocation of produced biomass to different plant organs including leaves, roots, stems and storage organs (fruits). For crops which grow indeterminately, e.g. cucumber, sweet pepper and tomato, the distribution of dry matter may change dynamically (Hall 1977, De Koning 1988, Marcelis 1994). To account for this dynamic nature of partition, sink strength, defined as the potential requirement for assimilates of individual organs (Wolsinkwel 1985), is widely used in the area of modelling crop dry matter partitioning (Heuvelink and Marcelis 1989, Jones et al. 1991, Dayan et al. 1993, de Koning 1994).

The distribution of assimilates among different plant organs are mainly regulated by the sinks themselves. The available assimilates are partitioned among N sinks according to their sink strength $S_i$ ($g d^{-1}$) relative to the total sink strength. The fraction of dry matter distributed to an individual organ $f_i$ is calculated using:

$$f_i = S_i / \sum_{i=1}^{N} S_i$$

The growth rate of each organ can be calculated by multiplying $f_i$ by the total amount of dry matter available for plant growth. Each organ will grow at its potential rate when the available dry matter equals or exceeds the total sink strength. The abundant assimilates will be stored and utilized added to the newly formed assimilates (Heuvelink 1996).

Heuvelink (1996) performed several experiments to determine the factors which influence the dry matter partitioning of tomato plant. The partitioning was separated into two main parts: vegetative (leaves, stems and roots) and generative (fruits) plant parts. It was concluded that the generative sink strength was proportional to the number of fruits, besides, there exists a constant ratio (2.96) between the average sink strength of a vegetative unit (three leaves and stem internodes between two trusses) and the average sink strength of one fruit. The temperature experiments revealed no important direct influence of temperature on the ratio between generative and vegetative sink strength, which was in accordance with studies of De Koning (1994).

Based on these findings, the dry matter partitioning of TOMSIM model was adjusted by Heuvelink (1995), the major elements in the model are the individual fruit trusses, whereas the vegetative plant parts are lumped together as one sink. A truss is composed of a known number of identical fruits. For simplicity, the individual fruits within trusses are not simulated. As for the partition ratios between leaf, stem and roots, Jones et al. (1991) and Heuvelink and Bertin (1994) reported a constant distribution between leaf and stem growth. These authors reported values of 2.3 and 2.2, respectively. Pointed by Heuvelink, there is a tendency that the distribution ratio between leaves and stem will decrease with plant age, as stem becomes thicker and longer to support the head of the plant which is further and further away from the root system. However this effect is probably unimportant for tomato plants with short growth periods.

In general, the approach used in TOMSIM model gives reasonable prediction on partitioning between vegetative and generative parts of tomato plant. However, the partitioning model contains a large number of parameters thus may not be suitable for this research as the goal is to develop a simple model. An alternative modelling approach uses the partitioning coefficient or index theory (Singels and Bezuidenhout 2002, Ni, Luo et al. 2006, Yuan, Luo et al. 2006, Tang, Zhu et al. 2007). Ni et al. (2006) used the cumulative product of thermal effectiveness and PAR (TEP) to simulate the partitioning index of different organs in tomato plant. For tomato plant cultivation in greenhouse, the temperature is
controlled in the optimal range for tomato growth. Thus, instead of $\text{TEP}$ the cumulative photosynthetic active radiation $\text{CPAR (MJ m}^{-2}[\text{gh}]\text{)}$ is used for simulating partitioning index in the current model.

The growth rates of different organs are dependent on their individual partitioning index. According to Heuvelink (1996), it is often assumed that the dry matter is partitioned first to roots and shoots, and then to stems, leaves and fruits based on the amount of dry matter which is already partitioned to the shoots. Thus, the growth of tomato shoots and roots can be calculated using the following equations:

$$\frac{dW_{\text{shoot}}(t)}{dt} = \frac{dW_{\text{plant}}(t)}{dt} \times PIS$$  \hspace{1cm} (2.2.21.)

$$\frac{dW_{\text{root}}(t)}{dt} = \frac{dW_{\text{plant}}(t)}{dt} \times PIR$$  \hspace{1cm} (2.2.22.)

Where $W_{\text{shoot}} (g[\text{DM}] m^{-2}[\text{gh}])$ and $W_{\text{root}} (g[\text{DM}] m^{-2}[\text{gh}])$ are the dry weights of tomato shoots and roots, respectively. $PIS$ is the partitioning index of tomato shoots, $PIR$ is the partitioning index of tomato roots, and $\frac{dW_{\text{plant}}(t)}{dt}$ is described in equation 2.2.19.

The growth rate of tomato leaves, stems and fruits can be calculated based on the growth rate of shoots:

$$\frac{dW_{\text{leaf}}(t)}{dt} = \frac{dW_{\text{shoot}}(t)}{dt} \times PIL$$  \hspace{1cm} (2.2.23.)

$$\frac{dW_{\text{stem}}(t)}{dt} = \frac{dW_{\text{shoot}}(t)}{dt} \times PIST$$  \hspace{1cm} (2.2.24.)

$$\frac{dW_{\text{fruit}}(t)}{dt} = \frac{dW_{\text{shoot}}(t)}{dt} \times PIF$$  \hspace{1cm} (2.2.25.)

Where $W_{\text{leaf}} (g[\text{DM}] m^{-2}[\text{gh}]), W_{\text{stem}} (g[\text{DM}] m^{-2}[\text{gh}])$ and $W_{\text{fruit}} (g[\text{DM}] m^{-2}[\text{gh}])$ are the dry weights of tomato leaves, stems and fruits; $PIL$, $PIST$ and $PIF$ are the partitioning index of tomato leaves, stems and fruits, respectively.

The partitioning index can be computed based on a correlation with cumulative photosynthetic active radiation $\text{CPAR}$ (Ni et al., 2006):

$$PIS(t) = 1 - 0.12 \times \exp\left(-\frac{\text{CPAR}(t)}{100}\right) \hspace{1cm} R^2 = 0.96 \hspace{0.5cm} \text{RMSE} = 0.008$$  \hspace{1cm} (2.2.26.)

$$PIR(t) = 1 - PIS(t)$$  \hspace{1cm} (2.2.27.)

$$PIL(t) = 0.23 + 0.59 \times \exp\left(-\frac{\text{CPAR}(t)}{110}\right) \hspace{1cm} R^2 = 0.89 \hspace{0.5cm} \text{RMSE} = 0.04$$  \hspace{1cm} (2.2.28.)

$$PIST(t) = \begin{cases} 
0.02 \times \text{CPAR}(t) & (\text{CPAR} \leq 21) \\
0.2 + 0.3 \times \exp\left(-\frac{\text{CPAR}(t)}{108}\right) & (21 < \text{CPAR} \leq 515)
\end{cases} \hspace{1cm} R^2 = 0.8 \hspace{0.5cm} \text{RMSE} = 0.03$$  \hspace{1cm} (2.2.29.)

$$PIF(t) = 1 - PIST(t) - PIL(t)$$  \hspace{1cm} (2.2.30.)

2.2.6 Simulation of leaf area index

Greenhouse vegetable crops have a vertical structure in the greenhouse, so light filters down through layers of leaves before a smaller percentage actually reaches the floor. Leaf area index $\text{LAI (m}^2[\text{leaf}] m^{-2}[\text{gh}])$ is widely used to indicate the ratio of the area of leaves over the area of ground which the leaves cover (Salisbury and Ross 1978). $\text{LAI}$ can reaches up to 8 for many mature crop communities, depending on species and plant density $\rho (\text{plant m}^{-2}[\text{gh}])$.

Leaf area index $\text{LAI}$ is simulated based on the partitioning of leaf dry matter and specific leaf area $\text{SLA (cm}^2[\text{leaf}] g^{-1}[\text{dm}])$. Specific leaf area $\text{SLA}$ is defined as the ratio of leaf area to leaf dry mass.
Heuvelink mentioned that specific leaf area \( SLA \) can be influenced by environmental factors including light intensity (Bruggink 1992), temperature (Harssema 1977), \( CO_2 \) concentration (Madsen 1973) and sink-source ratio (Heuvelink and Buiskool 1995). However, due to the scarcity of related studies a good explanatory model is not available at present. A forcing function (Heuvelink 1995) which only considers the seasonal effects (mainly radiation) is applied:

\[
SLA(t_{\text{day}}) = 266 + 88 \times \sin\left(\frac{2\pi(t_{\text{day}} + 68)}{365}\right)
\]  

(2.2.31.)

Where \( t_{\text{day}} \) (day) is the day of the year (day 1 = 1 January).

By multiplying the \( SLA \) with the leaf growth rate \( \frac{dW_{\text{leaf}}}{dt} \), the growth rate of \( LAI \) is obtained:

\[
\frac{dLAI(t)}{dt} = \begin{cases} \frac{dW_{\text{leaf}}(t)}{dt} \times SLA & 0 \leq LAI \leq 3 \\ 0 & \text{LAI} > 3 \end{cases}
\]  

(2.2.32.)

Here, \( LAI \) is modelled to have a maximum value of 3, as it is assumed that for greenhouse tomato plant cultivation, the old leaves are removed (naturally/manually) to keep \( LAI \) constant when the crop is fully grown. This is mainly due to the inhibition of fruit production when the \( LAI \) is high. Since there exist a strong correlation between \( LAI \) and \( \rho \), it is assumed that the \( \rho \) is 2.1 considering a maximum value of 3 for \( LAI \).

### 2.2.7 Model implementation

From equation 2.2.5., 2.2.19., 2.2.21., 2.2.23. and 2.2.32., it can be noticed that the simulation of dry matter production uses \( LAI \) as input while the growth of \( LAI \) is modelled based on the dry matter production. This interdependence between the two state variables, \( LAI \) and \( W_{\text{plant}} \), makes it impossible for simulating both variables simultaneously.

To solve this problem, an alternative approach is used: using numerical value of \( LAI \) from previous time step as input for simulation of dry matter production (in equation 2.2.2. and 2.2.5.). Given an initial value for \( LAI \), the model could simulate dry matter production and \( LAI \) growth stepwise. The model is implemented in Python 3.5.

### 2.3 Composition data

Dynamic mineral composition data of tomato plants are collected from available literature. For most of the studies, the mineral content is reported separately for different organs, e.g. leaves, fruits. Assuming that the mineral content of tomato vegetative part \( C_{\text{vegetate}} \) (including leaves, roots and stems) equals the mineral content of tomato leaves \( C_{\text{leaf}} \) (\%) makes it possible to obtain the mineral content for whole tomato plant \( C_{\text{plant}} \) (\%):

\[
C_{\text{plant}}(t) = C_{\text{vegetate}}(t) \times W_{\text{vegetate}}(t) + C_{\text{fruit}}(t) \times W_{\text{fruit}}(t)
\]  

(2.3.1.)

\[
W_{\text{vegetate}}(t) = W_{\text{leaf}}(t) + W_{\text{root}}(t) + W_{\text{stem}}(t)
\]  

(2.3.2.)

Where \( C_{\text{fruit}} \) (\%) is the mineral content of tomato fruits, \( W_{\text{vegetate}}(g[DM] m^{-2}[gh]) \) is the dry weight of vegetative part of tomato plant.

For \( C_{\text{fruit}} \), no dynamic data is available as \( C_{\text{fruit}} \) is usually reported at harvest stage. Currently, the \( C_{\text{fruit}} \) is assumed to be constant during the whole development of tomato fruit.

### 2.4 Parameter estimation

It is assumed that all tomato growth parameters are fixed and known. The unknown Michaelis-Menten parameters \( U_{\text{max}} \) and \( K_m \) can be calculated by using regression equations (Table 1) (Kempen 2015). The general form for computing \( U_{\text{max}} \) and \( K_m \) is based on \( EC \) and \( PTU \):

\[
U_{\text{max}} = p_1 \times (EC \times PTU) + p_2
\]  

(2.4.1.)

\[
K_m = p_3 \times (EC \times PTU) + p_4
\]  

(2.4.2.)

and thus \( p_1, p_2, p_3, p_4 \) are parameters that need to be estimated.
Generally, the parameter estimation utilizing least square method can be defined as a minimization problem:

\[ V(p) = \sum_{i} (OBS_i - SIM_i(p))^2 \]  

(2.4.3.)

\[ \hat{p} = \arg \min(V(p)) \]  

(2.4.4.)

Where \( p \) is the parameter vector that needs to be estimated, \( OBS \) and \( SIM \) are observed and simulated results, respectively, \( V(p) \) is the sum of squared residuals, \( \hat{p} \) is the parameter vector which gives the minimum value for \( V(p) \).

The covariance matrix of the parameter estimates \( Cov \hat{p} \) can be calculated by (Keesman 2011):

\[ Cov \hat{p} = \frac{1}{N - Np} \sum_{i=1}^{N} e^2(i) \cdot [\phi^T \phi]^{-1} \]  

(2.4.5.)

Where \( N \) is the number of observed outputs, \( Np \) is the number of parameters, \( e \) is the prediction error and \( \phi \) is the regressor matrix. With the covariance matrix, the standard deviation of the parameters can be easily computed by:

\[ \sigma = \sqrt{\text{diag}(Cov \hat{p})} \]  

(2.4.6.)

By using the data of dynamic mineral composition in tomato plant, parameter estimation is performed to estimate \( p_1, p_2, p_3, p_4 \) in equation (2.4.1.) and equation (2.4.2.).

The parameter estimation is implemented using the 'least_squares' function (scipy optimization package) in Python 3.5. The method used for minimization is the Trust Region Reflective algorithm.
3. Results

3.1 Simulation of tomato plant growth

Simulation of the tomato growth model was performed using growth data measured by Heuvelink (1995). Since only the plot of dry matter production is available, numerical data is collected using PlotDigitizer. In total 12 growth experiments using Lycopersicon esculentum ‘Counter’ were carried out in heated greenhouses without CO₂ enrichment. Basic information on the 12 greenhouse experiments is presented in Table 2.

Table 2. Basic information on the 12 greenhouse experiments. Dates expressed as day of year (day 1 = 1 January), n is the number of plants in each destructive measurement (Heuvelink 1995).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Year</th>
<th>Sowing date</th>
<th>Planting date</th>
<th>Ending date</th>
<th>Duration (d)</th>
<th>n</th>
<th>Global radiation (MJ m⁻² d⁻¹)</th>
<th>Temperature (°C)</th>
<th>CO₂ (µmol mol⁻¹ /ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1992</td>
<td>6</td>
<td>61</td>
<td>166</td>
<td>105</td>
<td>4</td>
<td>13.7</td>
<td>20.4</td>
<td>355</td>
</tr>
<tr>
<td>3</td>
<td>1989</td>
<td>2</td>
<td>66</td>
<td>166</td>
<td>100</td>
<td>4</td>
<td>15.3</td>
<td>20.0</td>
<td>331</td>
</tr>
<tr>
<td>4</td>
<td>1988</td>
<td>87</td>
<td>138</td>
<td>237</td>
<td>99</td>
<td>6</td>
<td>14.9</td>
<td>21.3</td>
<td>301</td>
</tr>
<tr>
<td>5</td>
<td>1988</td>
<td>98</td>
<td>151</td>
<td>251</td>
<td>100</td>
<td>6</td>
<td>14.3</td>
<td>20.9</td>
<td>297</td>
</tr>
<tr>
<td>6</td>
<td>1991</td>
<td>119</td>
<td>158</td>
<td>262</td>
<td>104</td>
<td>4</td>
<td>15.3</td>
<td>23.2</td>
<td>297</td>
</tr>
<tr>
<td>7</td>
<td>1988</td>
<td>136</td>
<td>186</td>
<td>286</td>
<td>100</td>
<td>6</td>
<td>11.7</td>
<td>21.1</td>
<td>294</td>
</tr>
<tr>
<td>8</td>
<td>1990</td>
<td>142</td>
<td>186</td>
<td>281</td>
<td>95</td>
<td>8</td>
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<td>332</td>
</tr>
<tr>
<td>9</td>
<td>1989</td>
<td>155</td>
<td>200</td>
<td>306</td>
<td>106</td>
<td>4</td>
<td>11.5</td>
<td>20.8</td>
<td>337</td>
</tr>
<tr>
<td>10</td>
<td>1988</td>
<td>164</td>
<td>215</td>
<td>327</td>
<td>112</td>
<td>6</td>
<td>8.2</td>
<td>19.2</td>
<td>317</td>
</tr>
<tr>
<td>11</td>
<td>1990/1991</td>
<td>232</td>
<td>268</td>
<td>43</td>
<td>140</td>
<td>8</td>
<td>3.7</td>
<td>18.4</td>
<td>375</td>
</tr>
<tr>
<td>12</td>
<td>1988/1989</td>
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<td>67</td>
<td>120</td>
<td>3</td>
<td>2.9</td>
<td>18.2</td>
<td>392</td>
</tr>
</tbody>
</table>

Temperature set-point was 18°C day and night, independent of radiation level and ventilation temperature was 19°C day and night. The daily global radiation was measured outside the greenhouse.

Since the original hourly input data for greenhouse radiation, temperature and CO₂ concentration were not available, average values were adopted for temperature and CO₂ concentration. As for solar radiation, using an average value is clearly not suitable for accurate simulation. Thus, the hourly greenhouse PAR data reported by Thomas (Thomas 2015) was used. In Thomas's study, the greenhouse PAR is computed based on the hourly climate data obtained from the Royal Dutch Meteorological Institute (KNMI) measured between January 2008 and March 2009 in de Bilt, the Netherlands. Thus, the average global radiation is different between the 12 experiments conducted by Heuvelink (between 1987 and 1991 in Wageningen, the Netherlands) and Thomas's study. In order to reproduce the solar radiation level accurately, the average daily global radiation of the greenhouse PAR data computed by Thomas was set to the same value as the measurement of each experiment.

3.2 Validation of the dry matter production model

The simulated $W_{plant}$ (equation 2.2.18.) and measured dry matter production of tomato plant for experiments 1-12 are presented in Figure 2.
Figure 2. Simulated and measured dry matter production of tomato plant

Model precision was verified using root mean square errors $RMSE$, normalized root mean square error $NRMSE$ and coefficient of determination $R^2$ of the results. Smaller $NRMSE$ and larger $R^2$ indicate smaller deviation between simulated values and measured values, and simulation results of the model were more accurate and reliable (Chen et al. 2011).

$RMSE$, $NRMSE$ and $R^2$ are defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (OBS_i - SIM_i)^2}{n}}$$  \hspace{1cm} (3.2.1.)

$$NRMSE = \frac{RMSE}{OBS_{max} - OBS_{min}}$$  \hspace{1cm} (3.2.2.)

$$SS_{res} = \sum_{i=1}^{N} (OBS_i - SIM_i)^2$$  \hspace{1cm} (3.2.3.)

$$SS_{tot} = \sum_{i=1}^{N} (OBS_i - \bar{OBS})^2$$  \hspace{1cm} (3.2.4.)

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$  \hspace{1cm} (3.2.5.)

Where $n$ is the number of measurements, $OBS$ is the observed (measured) values, $SIM$ is the simulated values, $OBS_{max}$ and $OBS_{min}$ are the maximum value and minimum value of the measurements, respectively, $SS_{tot}$ is the total sum of squares of the measured data, $\bar{OBS}$ is the average value of the
measured, $SS_{res}$ is the sum of squares of residuals. The $RMSE$, $NRMSE$ and $R^2$ of the 12 experiments are presented in Table 3.

**Table 3.** Root mean square error $RMSE$, normalized root mean square error $NRMSE$ and coefficient of determination $R^2$ for dry matter production

<table>
<thead>
<tr>
<th>Experiments</th>
<th>RMSE (g DM m$^{-2}$ d$^{-1}$)</th>
<th>NRMSE (%)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75.23</td>
<td>6.8</td>
<td>0.9520</td>
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<tr>
<td>2</td>
<td>77.95</td>
<td>7.87</td>
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</tr>
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<td>90.44</td>
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</tr>
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<td>0.9196</td>
</tr>
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<td>15.81</td>
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<td>14.96</td>
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<td>12</td>
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<td>6.93</td>
<td>0.9560</td>
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</tbody>
</table>

As can be observed in Table 3, the $NRMSE$ for all experiments are in a low range (< 16%) and $R^2$ are relatively large (> 0.81). This indicates that in general the current model is capable of giving reasonable predictions on the dry matter production of tomato plant. Even for experiments 11 and 12 where the daily global radiation is relatively low (3.7 and 2.9 MJ m$^{-2}$ d$^{-1}$, respectively), the model still gives accurate predictions which shows good response to solar radiation. For experiments 8 and 10, the model shows relatively poor predictions (overestimation of total dry matter production), these might be caused by the difference between the current input data (solar radiation, greenhouse temperature and $CO_2$ concentration) and the original input data. After a close examination of the tendency of the simulated dry matter production in Figure 2, it can be found that the total dry matter production at the final stage (2000 hours after transplanting) were all underestimated in experiment 3-7 and experiment 9. One possible explanation could be the underestimation of LAI: in the current model, the LAI is modelled to have a maximum value of 3, while in the original experiments, the final value for LAI are all larger than 3 for experiment 1-9. From equation 2.2.1 to 2.2.5., it can be deduced that the dry matter production would increase with a larger value of LAI. Therefore, the underestimation of LAI at final stage could be the main reason for underestimation of dry matter production. However, such phenomenon is not observed in experiment 1, 2 and 8. This can be caused by the underestimation of maintenance or the overestimation of canopy photosynthesis. Both maintenance and canopy photosynthesis are depending on greenhouse temperature and $CO_2$ concentration. Thus, to better interpret the validity of the current model, hourly input data (greenhouse radiation, temperature and $CO_2$ concentration) of the original experiments would have been needed.

Due to the lack of data on tomato plant dry matter partitioning, the validation of the model is performed only on the dry matter production. Since the tomato plant maintenance respiration is modelled based on the partitioned dry weight of different plant organs (see equation 2.2.12.), the partitioning could influence the total dry matter production in turn. Hence, the underestimation/overestimation of the total dry matter production could also be the result of inaccurate predictions on dry matter partitioning into different organs. Nevertheless, reasonable predictions were obtained.

### 3.3 Nutrient uptake (estimation and simulation)

The parameter estimation results for nutrient uptake are listed in Table 4.

**Table 4.** Results of parameter estimation (with standard deviations)

<table>
<thead>
<tr>
<th>Ion</th>
<th>Initial guess (p1, p2, p3, p4)</th>
<th>$U_{max}$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>Values of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NO_3^-$</td>
<td>1, 1, 1, 1</td>
<td>2.18e-53+1.69e1</td>
<td>8.25e5+2.81e10</td>
<td>1.45e3+4.91e7</td>
<td>3.40e6+1.16e11</td>
<td>4.1449</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.18e-53, 8.25e5, 1.45e3, 3.40e6</td>
<td>3.23e-23+3.38e1</td>
<td>1.65e8+1.10e11</td>
<td>2.89e3+1.93e8</td>
<td>6.81e6+4.55e11</td>
<td>4.1449</td>
<td></td>
</tr>
<tr>
<td>$K^+$</td>
<td>1, 1, 1, 1</td>
<td>3.85e2+3.34e7</td>
<td>2.44e6+2.12e11</td>
<td>1.19e4+1.03e9</td>
<td>2.49e6+2.16e11</td>
<td>8.9916</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.85e2, 2.44e6, 1.19e4, 2.49e6</td>
<td>1.60e4+1.28e10</td>
<td>1.01e8+8.11e13</td>
<td>4.92e5+3.94e11</td>
<td>1.03e8+8.25e13</td>
<td>8.9916</td>
<td></td>
</tr>
<tr>
<td>$H_2PO_4$</td>
<td>1, 1, 1, 1</td>
<td>2.50e-42+6.57e-4</td>
<td>2.77e1+1.43e2</td>
<td>1.15e-1+7.47e-1</td>
<td>6.30e2+4.59e3</td>
<td>0.0962</td>
<td></td>
</tr>
</tbody>
</table>
In Table 4, it can be observed that the estimation differs with different initial guesses. This might be explained by the strong correlation between Michaelis-Menten parameters: $U_{max}$ and $K_m$. To visualize this effect, the estimated plant nutrient uptake rate $U_n (mg m^{-2}[root] h)$ is pairwisely plotted against $PAR (J m^{-2}[gh])$ and product of $EC$ and $PTU$ for each nutrient. (Note that the units of $PAR$ is different from equation 2.1.1., for detailed units conversion, see Appendix 2)
In Figure 3, it can be observed that regardless of the differences in estimated parameters, the uptake rate of each nutrient, except for phosphorus, is more or less the same. For phosphorus, the maximum uptake rate varied largely between two estimates: with the values of 25 and 80 (mg m$^{-2}$[root] h) for estimation 1 and 2, respectively. However, a close look at the variation tendency of the uptake rate reveals that these large differences between two estimates only appear when the PAR is relatively large (PAR > 1e6 J m$^{-2}$[gh] h) and product of EC and PTU is relatively small EC * PTU < 1.5e4). Thus, considering the fact that the PAR inside the greenhouse is seldomly larger than 1e6 J m$^{-2}$[gh] h, the differences caused by these different estimates can be neglected. This was proved by the close values of objective functions between different estimates: the returned value for objective function is 0.0962 and 0.0974 for estimation 1 and 2, respectively. These results suggest that the nutrient uptake model could have a set of feasible parameter vectors instead of one optimal parameter vector. Additionally, this might explain the unexpected values for standard deviations of estimated parameters which are larger than parameters itself (normally, estimation with standard deviation larger than estimates would be deemed as unreliable estimation).

The simulated and measured (from literature) nutrient contents for six macronutrients (N, P, S, K, Ca, Mg) are presented in Figure 4.
Figure 4. Simulated and measured nutrient content for six mineral nutrients (nitrogen, phosphorus, sulfur, potassium, calcium, magnesium). For each nutrient, both two
estimates are plotted (as simulate 1 and simulate 2). For calcium, passive uptake is plotted separately.

The RMSE for the different nutrient uptakes are listed in Table 5.

Table 5. RMSE for nutrient uptake

<table>
<thead>
<tr>
<th>Nutrients</th>
<th>RMSE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>3.84</td>
</tr>
<tr>
<td>P</td>
<td>0.82</td>
</tr>
<tr>
<td>S</td>
<td>0.83</td>
</tr>
<tr>
<td>K</td>
<td>4.95</td>
</tr>
<tr>
<td>Ca</td>
<td>2.00</td>
</tr>
<tr>
<td>Mg</td>
<td>0.47</td>
</tr>
</tbody>
</table>

From Figure 4, it is clear that the current nutrient model yields decent predictions on dynamic nutrient content of magnesium, phosphorus, nitrogen, sulfur and potassium. However, the dynamic mineral content of calcium is poorly predicted. This may suggest that the current model is limited for predicting U shape of calcium content, as the Michaelis-Menten parameters $U_{max}$ and $K_m$ are assumed to be linearly related to the product of $EC$ and $PTU$. This assumption may be true for nutrients which do not have much variation in uptake rate throughout the plant growth. However, the mechanism of calcium uptake is more complicated compared with other nutrients (driven both passively and actively). Apart from $EC$ and $PTU$, the calcium uptake could also be influenced by other factors e.g. the potential growth rates of different tissues.

As mentioned before, the calcium uptake is driven both passively and actively. The passive uptake of calcium is directly correlated with calcium concentration of the hydroponic solutions $C_{Ca}$ (equation 2.1.2.). For simplicity, $C_{Ca}$ was assumed to be constant in simulation. However, for literature where the calcium content data is collected, the corresponding dynamic $C_{Ca}$ is barely reported. Without knowing the exact dynamic $C_{Ca}$, the validity of the parameter estimation and simulation would be weakened.

Another possible reason is the potential discrepancies between the data from literature. Generally, the calcium is believed to move in the plant with the xylem sap flow. Consistent with this, the direction and rate of calcium flow should be depending on water potential gradients in response to transpiration and growth rates of different tissues (de Freitas, McElrone et al. 2014). In that study, the partitioning of calcium uptake by roots towards leaves and fruits would depend on the calcium concentration of xylem sap, as well as transpiration and growth rates of leaf and fruit. In this study, the calcium content of tomato fruits is assumed to be constant for the whole development. While, the variation of ratio between transpiration and growth rates of fruit and leaf would lead to dynamic change in calcium content of different tissues. Studies showed that reducing leaf transpiration can potentially decrease calcium movement into leaves, and increase its movement into fruit (Guichard, Gary et al. 2005, de Freitas, Shackel et al. 2011). These findings undermine the correctness of the computed calcium content data for whole tomato plant, as the leaf and fruit data is collected from different literature (Armstrong and Kirkby 1979, Cerda, Martinez et al. 1984, Zekki, Gauthier et al. 1996, SANTOS, ESMEL et al. 2007, Borgognone, Colla et al. 2013, Roosta and Hamidpour 2013) where no further information on transpiration and humidity was provided.

For sulfur content, the model gives relative high values for the early development stage. This phenomenon is mainly caused by the lack of data in the early stage. In order to improve the accuracy of the prediction, more scattered data points of the whole life cycle of tomato plant are needed.

4. Discussions

A model for predicting dry matter production, dry matter partitioning and nutrient uptake of hydroponically grown tomato plant was developed and implemented in Python 3.5. The dry matter production part is based on the ASKAM (Gijzen 1992) and TOMSIM (Heuvelink and Bertin 1994) model which describe the crop photosynthesis as a function of light intensity, $CO_2$ concentration and temperature. The partitioning part is based on an empirical partitioning index model (Ni, Luo et al. 2006) expressed as a function of cumulative light intensity. The nutrient uptake part is modelled using Michaelis-Menten type function based on light intensity.

For calibrating the nutrient uptake model, the dynamic profile of nutrient content of tomato plant was constructed based on literature. The following parameter estimation was performed using ordinary least square method. The estimation results indicate that for a Michaelis-Menten type model, there could exist a set of feasible parameter vectors instead of one optimal parameter vector due to the strong correlation
between the Michaelis-Menten parameters. And such nonuniqueness of solution could be the explanation for the large deviation of estimates (larger than estimates).

Since in this study no greenhouse tomato experiment was performed, the model was validated using dynamic data from literature. The validation result showed good prediction accuracy for simulation of dry matter production (with $\text{NRMSE} < 16\%$ for twelve experiments). The same goes for the uptake of six macronutrients, except for calcium. For uptake of calcium, the current model could not simulate the actual behaviour of calcium content in the tomato plant (which exhibits an U shape during the whole development stage of the plant). It was found that this could be caused by several factors:

1. The linear dependency of the Michaelis-Menten parameters $U_{\text{max}}$ and $K_m$ on product of $EC$ and $PTU$ is not suitable for describing calcium uptake mechanism, as the uptake of calcium could also be influenced by other factors.
2. The assumption that a constant value for $C_{Ca}^{\text{sd}}$ during whole plant development might not be true for experimental data used for validation.
3. There may exist discrepancy between the mineral composition data used for validation. As the data is collected from different literature, while the environmental factors (input for simulation) are barely reported. Thus, those complied data may not suitable for model validation.

Still, the model has several other limitations that may reduce its application range:

1. The model is designed for greenhouse tomato plant cultivation where the environmental factors are more or less controlled.
2. The plant density $\rho$ is not taken into account in the current model (assumed as constant 2.1). While in tomato plant cultivation, the plant density may vary depending on cultivation methods and varieties. A different plant density may leads to overestimation/underestimation of the dry matter production and nutrient uptake.

5. **Recommendations**

In order to perform a thorough validation of the model, more accurate data is needed for dry matter production, partitioning and mineral composition together with the corresponding environmental conditions. To improve the prediction accuracy and certainty of the model, it is suggested to use/measure more scattered data during the whole development of plant. If possible, it is also suggested to measure the mineral composition of the whole tomato plant instead of separate organs during the cultivation. By doing this, the potential error cause by inaccurate prediction by partitioning model could be eliminated.

If possible, it is recommended to use measurement values rather than simulated values for $LAI$. As there exist a strong correlation between $LAI$ and $\rho$, using measurements of $LAI$ would extend the application range of the model to various $\rho$ in an indirect way.

For calcium uptake model, it is recommended to include intrinsic factors e.g. plant potential growth rate, plant development stage in the model development. It is also recommended to include factors like plant density, humidity for model development as these factors are crucial inputs which would affect the plant growth and nutrients uptake as well.

As mentioned before, the nonuniqueness of estimation results might explain the large deviation of estimates. However, to verify this assumption, further research regarding the parameter estimation is recommended.
6. References


Appendix 1. List of parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.2</td>
<td>$m^2$</td>
<td>Cell area</td>
</tr>
<tr>
<td>$c$</td>
<td>3e8</td>
<td>$m \times s^{-1}$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$C$</td>
<td>$mg m^{-3}$</td>
<td></td>
<td>Nutrient concentration</td>
</tr>
<tr>
<td>$C_a$</td>
<td></td>
<td></td>
<td>$CO_2$ concentration in air ppm</td>
</tr>
<tr>
<td>$C_{ac}$</td>
<td>44/30</td>
<td>$g[CO_2] g^{-1}[CH_2O]$</td>
<td>Conversion factor of assimilates to $CO_2$</td>
</tr>
<tr>
<td>$C_{ad}$</td>
<td>0.7</td>
<td>$g[DM] g^{-1}[CH_2O]$</td>
<td>Conversion factor of assimilates to dry matter</td>
</tr>
<tr>
<td>$C_{ca}$</td>
<td>30/44</td>
<td>$g(CH_2O) g^{-1}[CO_2]$</td>
<td>Conversion factor of $CO_2$ to assimilates</td>
</tr>
<tr>
<td>$C_{content}$</td>
<td></td>
<td></td>
<td>Nutrients content of tomato plant</td>
</tr>
<tr>
<td>$C_{fruit}$</td>
<td></td>
<td></td>
<td>Mineral content of tomato fruits (obtained from literature)</td>
</tr>
<tr>
<td>$C_{leaf}$</td>
<td></td>
<td></td>
<td>Mineral content of tomato leaf (obtained from literature)</td>
</tr>
<tr>
<td>$C_{min}$</td>
<td></td>
<td></td>
<td>Concentration where influx equals efflux</td>
</tr>
<tr>
<td>$C_p$</td>
<td></td>
<td></td>
<td>$CO_2$ releasement by growth respiration</td>
</tr>
<tr>
<td>$C_{pf}$</td>
<td>0.4</td>
<td>$g[CO_2] g^{-1}[DM]$</td>
<td>$CO_2$ production factor of tomato plant</td>
</tr>
<tr>
<td>$C^{int}_{plant}$</td>
<td></td>
<td></td>
<td>Mineral content of tomato plant (obtained from literature)</td>
</tr>
<tr>
<td>$C_{vege}$</td>
<td></td>
<td></td>
<td>Mineral content of vegetative part of tomato plant (obtained from literature)</td>
</tr>
<tr>
<td>$CPAR$</td>
<td></td>
<td></td>
<td>Cumulative photosynthetic active radiation</td>
</tr>
<tr>
<td>$C^{ca}$</td>
<td>1e5</td>
<td>$mg m^{-3}$</td>
<td>Calcium concentration in hydroponics</td>
</tr>
<tr>
<td>$D_a$</td>
<td>0.5</td>
<td>kPa</td>
<td>Air vapour pressure deficit</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
<td></td>
<td>particulate photon energy</td>
</tr>
<tr>
<td>$E_1$</td>
<td></td>
<td></td>
<td>Energy of one molar photons</td>
</tr>
<tr>
<td>$EC$</td>
<td>1.5</td>
<td>$mS cm^{-1}$</td>
<td>Electrical conductivity of nutrient solution</td>
</tr>
<tr>
<td>$f$</td>
<td></td>
<td></td>
<td>The fraction of dry matter distributed to an individual organ</td>
</tr>
<tr>
<td>$f_{SCR}$</td>
<td>792</td>
<td>h</td>
<td>Coefficient in computing relative growth rate</td>
</tr>
<tr>
<td>$GDD$</td>
<td></td>
<td></td>
<td>Growing degree days</td>
</tr>
<tr>
<td>$h$</td>
<td>6.626e-34</td>
<td>$J s$</td>
<td>Planck constant</td>
</tr>
<tr>
<td>$k_{ext}$</td>
<td>0.8</td>
<td></td>
<td>Extinction coefficient of canopy</td>
</tr>
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<td>$K_m$</td>
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<td></td>
<td>Michaelis-Menten constant</td>
</tr>
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<td>$K^{ca}_m$</td>
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<td></td>
<td>Michaelis-Menten constant for calcium uptake</td>
</tr>
<tr>
<td>$L$</td>
<td></td>
<td></td>
<td>Quantity of PAR arriving</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAI</td>
<td>Leaf area index at a certain layer of canopy</td>
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<td></td>
</tr>
<tr>
<td>LGUSS</td>
<td>Leaf area index of a certain layer of canopy</td>
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</tr>
<tr>
<td>(N_a)</td>
<td>Avogadro constant</td>
<td></td>
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<td>NRMSE</td>
<td>Normalized root mean square error</td>
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<tr>
<td>OBS</td>
<td>Observed/Measured values</td>
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<tr>
<td>OBS\text{max}</td>
<td>Maximum observed/measured value</td>
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<td>OBS\text{min}</td>
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<tr>
<td>(p_1), (p_2)</td>
<td>Parameters for calculating (U\text{max})</td>
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</tr>
<tr>
<td>(p_3), (p_4)</td>
<td>Parameters for calculating (K\text{m})</td>
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<td>(\beta)</td>
<td>A set of parameters which gives the best fit between data and output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_g)</td>
<td>(g[CO_2] m^{-2}[leaf] h^{-1}) Leaf gross synthetic rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_{gc})</td>
<td>(g[CO_2] m^{-2}[g/h] h^{-1}) Gross canopy assimilation rate</td>
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<tr>
<td>(P\text{max})</td>
<td>(g[CO_2] m^{-2}[leaf] h^{-1}) Leaf maximum photosynthetic rate</td>
<td></td>
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</tr>
<tr>
<td>(P_{gc})</td>
<td>(g[CO_2] m^{-2}[g/h] h^{-1}) Net photosynthesis rate</td>
<td></td>
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</tr>
<tr>
<td>PAR</td>
<td>(J m^{-2}[g/h] h^{-1}) Photosynthetically active radiation</td>
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<td></td>
</tr>
<tr>
<td>PAR\text{abs}</td>
<td>(J m^{-2}[leaf] h^{-1}) Absorbed photosynthetically active radiation</td>
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<tr>
<td>PIF</td>
<td>Partitioning index of tomato fruits</td>
<td></td>
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<td>PIR</td>
<td>Partitioning index of tomato roots</td>
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<td>PIST</td>
<td>Partitioning index of tomato stems</td>
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<td>PTU</td>
<td>°C MJ m^{-2}[gh] Photo-thermal units</td>
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<td>(Q_{10})</td>
<td>2.0 (Q_{10}) -value for temperature effect on maintenance respiration</td>
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<tr>
<td>(rad)</td>
<td>(m) Mean radius of tomato roots</td>
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<td>(rg_{1},) (rg_{2})</td>
<td>Coefficients for calculating root length</td>
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<tr>
<td>(r_b)</td>
<td>100 (s m^{-1}) Boundary layer resistance for water vapour diffusion</td>
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<td>(r_m)</td>
<td>250 (s m^{-1}) Mesophyll resistance to (CO_2) transport</td>
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<td>(r_s)</td>
<td>50 (s m^{-1}) Stomatal resistance for water vapour diffusion</td>
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<td></td>
</tr>
<tr>
<td>(R)</td>
<td>(MJ m^{-2}[gh]) Daily solar radiation</td>
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</table>
| \(R^2\) | Coefficient of
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$R_{in}$</td>
<td>$f m^{-2}[gh] s^{-1}$</td>
</tr>
<tr>
<td>$R_{m}(T)$</td>
<td>$g[CH_{2}O] m^{-2}[gh] h^{-1}$</td>
</tr>
<tr>
<td>$R_{m}^{25}$</td>
<td>$g[CH_{2}O] g^{-1}[DM] h^{-1}$</td>
</tr>
<tr>
<td>$R_{m}^{25}$</td>
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<td>$R_{stem}^{25}$</td>
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<td>$R_{root}^{25}$</td>
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<td>$R_{fruit}^{25}$</td>
<td>0.00042</td>
</tr>
<tr>
<td>$RD$</td>
<td>0.1127, 0.5, 0.8873 for $1^{st}$, $2^{nd}$, $3^{rd}$ layer respectively</td>
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<td>$h^{-1}$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$m$</td>
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<td>$g d^{-1}$</td>
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<td>$SS_{res}$</td>
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<tr>
<td>$t$</td>
<td>hour</td>
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<td>$\degree C$</td>
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<td>$T_{l}$</td>
<td>$\degree C$</td>
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</tr>
<tr>
<td>$T_{min}$</td>
<td>18 $\degree C$</td>
</tr>
<tr>
<td>$T_{min}$</td>
<td>4 $mm day^{-1}$</td>
</tr>
<tr>
<td>$T_{r}$</td>
<td>$mg m^{-2}[leaf] s^{-1}$</td>
</tr>
<tr>
<td>$T_{rel}$</td>
<td></td>
</tr>
<tr>
<td>$T_{w}$</td>
<td>$mm day^{-1}$</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>$mg m^{-2}[RSA] h^{-1}$</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>$mg m^{-2}[RSA] h^{-1}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>$U_n$</td>
<td>$mg \cdot m^{-2} [RSA] \cdot h^{-1}$</td>
</tr>
<tr>
<td>$U_n^a$</td>
<td>$mg \cdot m^{-2} [RSA] \cdot h^{-1}$</td>
</tr>
<tr>
<td>$V(p)$</td>
<td></td>
</tr>
<tr>
<td>$W_{plant}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$W_{leaf}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$W_{stem}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$W_{root}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$W_{fruit}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$W_{vege}$</td>
<td>$g [DM] \cdot m^{-2} [gh]$</td>
</tr>
<tr>
<td>$WT$</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_0$</td>
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</tr>
<tr>
<td>$\Gamma_n$</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.1</td>
</tr>
<tr>
<td>$\rho$</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 2. Additional calculation

Relative growth rate

In order to relate relative growth rate $RGR$ with plant dry weight, a regression analysis was performed using experimental data measured by Heuvelink (1995) in Microsoft Excel.

Assuming the values of two consecutive measurements of dry weight are $M_1$ and $M_2$, and corresponding time of measurements are $t_1$ and $t_2$, respectively. According to the definition of $RGR$, following equation can be used for computing $RGR$:

$$RGR = \frac{M_2 - M_1}{t_2 - t_1} / \left(\frac{M_1 + M_2}{2}\right)$$

The results of regression analysis are presented in Figure 5.

![Figure 5. Regression of $RGR$ on dry weight.](image)

A logarithm type regression equation is used as the $R^2$ is the largest compared with linear type ($R^2 = 0.3975$) and polynomial type ($R^2 = 0.5484$).

$$RGR(W_{plant}) = -0.0008 \times \ln(W_{plant}) + 0.0055 \quad R^2 = 0.5588$$

Units conversion

Conversion of $mol^{-1} photon$ to $J^{-1}$

$CO_2$ concentration is usually reported in ppm, assuming the molar mass of $CO_2$ equals 44 $g/mol$, 1 ppm $CO_2$ equals 1.94 mg/m$^3$ $CO_2$. Leaf initial light use efficiency $\epsilon_0$ is reported as 0.084 $mol CO_2 mol^{-1} photon$. Assuming the average wavelength of incoming solar radiation is 550 nm, according to Planck-Einstein relation, the particulate photon energy $E$ ($J$) can be calculated using the following equation:

$$E = \frac{h \times c}{\lambda}$$

Where $h$ is the Planck constant ($6.626 \times 10^{-34} J s$), $c$ is the speed of light ($3 \times 10^8 m s^{-1}$), $\lambda$ is the wavelength of the particle. Thus, the energy for one photon is $3.9756 \times 10^{-19} J$. The energy of one molar of photons can be calculated using:

$$E_1 = E \times N_A$$

Where $N_A$ is the Avogadro constant and has the value of $6.022 \times 10^{23} mol^{-1}$.

The energy of one molar of photons assuming a wavelength of 550 nm is $217.6 kJ mol^{-1}$. 

36
Appendix 3. Literature data on mineral composition

Table 6. Nitrogen content (% DM)

<table>
<thead>
<tr>
<th>Week</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaves</td>
<td>3.456</td>
<td>4.06</td>
<td>4.65</td>
<td>2.78</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>5.075</td>
<td>5.51</td>
<td>4.77</td>
<td>2.96</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.14</td>
<td>5.03</td>
<td>4.47</td>
<td>2.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.27</td>
<td>5.3</td>
<td>4.9</td>
<td>2.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.98</td>
<td>3.94</td>
<td>4.48</td>
<td>3.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td></td>
<td>4.56</td>
<td></td>
<td>2.31</td>
</tr>
<tr>
<td>Fruits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.42</td>
</tr>
</tbody>
</table>

(Zekki, Gauthier et al. 1996, Borgognone, Colla et al. 2013)

Table 7. Potassium content (% DM)

<table>
<thead>
<tr>
<th>Time</th>
<th>Day 5</th>
<th>Day 10</th>
<th>Day 14</th>
<th>Day 18</th>
<th>Day 20</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 8</th>
<th>Week 12</th>
<th>Week 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaves</td>
<td>5.55</td>
<td>6.25</td>
<td>6.85</td>
<td>6.25</td>
<td>6.75</td>
<td>4.91</td>
<td>6.9</td>
<td>3.68</td>
<td>5.11</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td>4.69</td>
<td>3.96</td>
<td>5.53</td>
<td>5</td>
<td>4.78</td>
<td>4.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.53</td>
<td>3.91</td>
<td>5.53</td>
<td>5</td>
<td>4.78</td>
<td>4.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.72</td>
<td>4.6</td>
<td>5.51</td>
<td>5</td>
<td>4.78</td>
<td>4.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.87</td>
<td>4.76</td>
<td>4.2</td>
<td>5</td>
<td>4.78</td>
<td>4.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fruits</td>
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<td></td>
<td>3.77</td>
<td></td>
<td></td>
<td>3.75</td>
<td></td>
</tr>
</tbody>
</table>


Table 8. Calcium content (% DM)

<table>
<thead>
<tr>
<th>Time</th>
<th>Day 5</th>
<th>Day 10</th>
<th>Day 14</th>
<th>Day 18</th>
<th>Day 20</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 8</th>
<th>Week 12</th>
<th>Week 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaves</td>
<td>2.45</td>
<td>2.95</td>
<td>3.4</td>
<td>3.3</td>
<td>2.55</td>
<td>2.55</td>
<td>2.75</td>
<td>1.71</td>
<td>1.49</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>2.49</td>
<td>1.85</td>
<td>1.17</td>
<td>1.12</td>
<td>4.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.69</td>
<td>1.83</td>
<td>1.78</td>
<td>1.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.14</td>
<td>1.47</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>1.87</td>
<td>1.16</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.7</td>
<td>1.84</td>
<td>1.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Fruits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.06</td>
<td></td>
<td></td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>


Table 9. Phosphorus content (% DM)

<table>
<thead>
<tr>
<th>Week</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaves</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
<td>0.59</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>0.68</td>
<td>0.95</td>
<td>0.95</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.48</td>
<td>0.9</td>
<td>0.9</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.82</td>
<td>0.82</td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>0.87</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fruits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.92</td>
</tr>
</tbody>
</table>

(Zekki, Gauthier et al. 1996, Borgognone, Colla et al. 2013)

Table 10. Magnesium content (% DM)

<table>
<thead>
<tr>
<th>Time</th>
<th>Day 5</th>
<th>Day 10</th>
<th>Day 14</th>
<th>Day 18</th>
<th>Day 20</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 8</th>
<th>Week 12</th>
<th>Week 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaves</td>
<td>0.55</td>
<td>0.65</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.513</td>
<td>0.6</td>
<td>0.4</td>
<td>0.38</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.528</td>
<td>0.5</td>
<td>0.34</td>
<td>0.17</td>
<td>1.27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Zekki, Gauthier et al. 1996, Borgognone, Colla et al. 2013)
### Table 11. Sulfur content (% DM)

<table>
<thead>
<tr>
<th>Time</th>
<th>Day 30</th>
<th>Day 45</th>
<th>Day 92</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composition in leaves</td>
<td>0.86</td>
<td>0.6</td>
<td>1.12</td>
</tr>
<tr>
<td>Composition in fruits</td>
<td></td>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

Appendix 4. Python code of the model

Dry matter production
# Calculate the photosynthesis rate of a certain layer of the canopy
def layer_photo(eps, P_gm, DIS_1, DIS_2, DIS_3, WT_1, WT_2, WT_3, LAI, PAR, k_ext):
    # DIS_i is the distance coefficient of gauss integral (i is the layer number
    # of the canopy layers)
    # LAI is leaf area index
    # PAR is photosynthesital active radiation
    # k_ext is the extinction coefficient of canopy
    # P_gm is the maximum photosynthesis rate
    LGUSS_1 = DIS_1*LAI
    LGUSS_2 = DIS_2*LAI
    LGUSS_3 = DIS_3*LAI
    # LGUSS_i is the canopy depth of gauss layer
    L_1 = PAR*k_ext*np.exp(-k_ext*LGUSS_1)
    L_2 = PAR*k_ext*np.exp(-k_ext*LGUSS_2)
    L_3 = PAR*k_ext*np.exp(-k_ext*LGUSS_3)
    # L_i is the quantity of PAR arriving at the ith layer of the canopy
    P_g1 = P_gm*(1-np.exp(-eps*L_1/P_gm))
    P_g2 = P_gm*(1-np.exp(-eps*L_2/P_gm))
    P_g3 = P_gm*(1-np.exp(-eps*L_3/P_gm))
    P_g = (P_g1*WT_1+P_g2*WT_2+P_g3*WT_3)*LAI
    return P_g

# Simulate the dry matter production of tomato plant
def dry_matter_production(W_p0, LAI0, leaf0, root0, stem0, fruit0, t, t_start, T_average,C_ppm, DIS_1,
                           DIS_2, DIS_3, PAR, TEP, k_ext, WT_1, WT_2, WT_3, C_f, f):
    # W_p0, LAI0, leaf0, root0, stem0, fruit0 are the intial value for each state variables
    # W_p is the dry weight of whole plant
    # LAI is leaf area index
# leaf is the dry weight of leaf
# root is the dry weight of root
# stem is the dry weight of stem
# fruit is the dry weight of fruit
# t is the time step
# t_start is the planting data (t_start_th day of the year)
# C_ppm is the CO2 concentration
# T_average is the average temperature inside greenhouse

def diff(y, t_):
    # define state variables
    # TEP is the cumulative PAR
    # C_f is the conversion factor from assimilates to dry matter
    # f is a regression parameter
    W_p = y[0]  # [g DM]
    LAI = y[1]
    leaf = y[2]  # [g DM]
    root = y[3]  # [g DM]
    stem = y[4]  # [g DM]
    fruit = y[5]  # [g DM]
    # calculate the eps and P_gm based on temperature and CO2 concentration
    C_a = C_ppm
    ghe = 42.7+1.68*(T_average-25)+0.012*(T_average-25)**2
    eps = 1.544*10**-5*(C_a-ghe)/(C_a+2*ghe)
    P_gm = 0.013*(C_a-ghe)
    # calculate the plant photosynthetic rate [g CO₂ m⁻² h⁻¹]
    P_g = layer_photo(eps, P_gm, DIS_1, DIS_2, DIS_3, WT_1, WT_2, WT_3, LAI, PAR, k_ext)
    # simulate growth rate (dry matter production rate) of plant [g DM h⁻¹]
    # maintenance rate R_m is calculated based on the four type of organs of tomato plant
    R_m = (0.03*leaf+0.015*stem+0.01*root+0.01*fruit)/24*2**(5*(T_average-25)/10)
    # CO2 released by growth respiration is calculated as a factor times dWdt, for implementation
    # , the equation is divided by 1.191
\[
dWdt = C_f * \left( \frac{P_g * 30/44 - R_m * (1 - np.exp(-0.0008*np.log(W_p) + 0.0055)))}{1.191} \right)
\]

# RGR is computed by equation -0.0008*np.log(W_p)+0.0055 obtained using linear regression analysis (Appendix 2)
# 30/44 converts from CO2 to assimilates [g CH2O g**-1 CO2]

# tomato dry matter partition (first partition to root and shoot)
# PIS is the partition index of shoot
# PIR is the partition index of root
PIS = 1 - 0.12*np.exp(-TEP/100)
PIR = 1 - PIS

# simulate root growth
d_rootdt = dWdt*PIR

# partitioning of leaf, stem and fruit is based on the dry matter partitioned to the shoot
# PIL is the partition index of leaf
# PIST is the partition index of stem
# PIF is the partition index of fruit
PIL = PIS*(0.23+0.59*np.exp(-TEP/110))

# simulate leaf growth rate
d_leafdt = dWdt*PIL

PIST = np.piecewise(TEP, [0<=TEP<=21, 21<TEP], [0.02*TEP, 0.2+0.3*np.exp(-TEP/108)])*PIS

# simulate stem growth rate
d_stemdt = dWdt*PIST

PIF = PIS*(1-PIL-PIST)
PIF_p = np.piecewise(PIF, [PIF<0, 0<=PIF], [0, PIF])

# simulate fruit growth rate
d_fruitdt = dWdt*PIF_p

# simulate specific leaf area based on t_start
SLA = (266+88*np.sin(2*np.pi*(t_/24+68+t_start)/365))*1*10**-4
# simulate leaf area index using leaf dry matter production

d_laidt = np.piecewise(LAI, [0<=LAI<=3, LAI>3], [SLA*d_leafdt, 0])

val = [dWdt,
       d_laidt,
       d_leafdt,
       d_rootdt,
       d_stemdt,
       d_fruitdt]

return val

# set initial state for the differential equation
y0 = np.asarray([W_p0, LAI0, leaf0, root0, stem0, fruit0])

# solve the differential equation using 'odeint' (integrate a system of ordinary differential equations)
rez = odeint(diff, y0, t, full_output=0)

return rez

Nutrient uptake (take Mg as example)
# simulate magnesium uptake by tomato plant

def Mg_uptake(t, RSA, PAR, EC,a,b,c,d,DSR):
    # simulate magnesium uptake using Michaelis-Menten active uptake model
    # t is the hour of the simulation
    # RSA is the root surface area of tomato plant [m^2]
    # PAR is photosynthetically active radiation [J m^-2 h^-1]
    # EC is the electrical conductivity of nutrient solution
    # U_max is the maximum rate of magnesium uptake [mg m^-2 h]
    # K_m is the Michaelis-Menten constant (the photosynthetic photon flux density at 1/2 J_max)
    # DSR is daily solar radiation [MJ m^-2]
    # a,b,c,d are regression parameters for calculating U_max and K_m
    # assuming the maximum and minimum temperature of greenhouse equals 26 and 18 degrees
    # the growing degree days is simulated using t
    T_max = 26
    T_min = 18
    T_base = 8
    GDD = ((T_max+T_min)/2-T_base)*t/24

    # simulate photo thermal unit
PTU = GDD*DSR

# compute Michaelis-Menten parameters
U_max = a*EC*PTU+b
K_m = c*EC*PTU+d

# converting [umol m^-2 s] to [J m^-2 h^-1] assuming an average wavelength of 550 nm
wl = 550*1*10**-9  # [m]
c = 3*10**8         # [m/s]
h = 6.626*10**-34
NA = 6.022*10**23

# unit conversion
K_m = c/wl*h*NA*1*10**-6*3600*K_m

# calculate uptake based on Michaelis-Menten type model
uptake = RSA*PAR*U_max/(K_m+PAR)
return uptake

Parameter estimation (take K as example)

def residuals(p, data_K , t, root_dm,total_dm,PAR,DSR):
    # calculate the difference between the simulated and measured data (percentage of composition [DM/DM])
    # p is the vector of parameters that need to be estimated
    # data_K is the literature data of potassium content
    # root_dm is the dynamic dry weight of root obtained from simulation (vector)
    # total_dm is the dynamic dry weight of whole tomato plant (vector)
    # PAR is photosynthesis active radiation
    # DSR is daily solar radiation
    # root_dm, total_dm are obtained from simulation of dry matter production model

    k_rsa = 0.096   # coefficient which convert root dry matter to root surface area
    # a,b,c,d are parameters that need to be estimated
    a = p[0]
b = p[1]
c = p[2]
d = p[3]

    # compute the dynamic root surface area (vector)
    RSA = root_dm*k_rsa
# create time vector for simulation
tc = np.linspace(0, t[-1], t[-1]+1)

# parameters for uptake (irrelevant)
EC = 1.5  # electrical conductivity
# calculate uptake of potassium
uptake_K = np.zeros_like(tc)
for j,k in enumerate(DSR):
    uptake_K[j] = K_uptake(j, RSA[j], PAR[j], EC, a, b, c, d, DSR[j])
# calculate cumulative uptake of potassium
cuml_K = np.cumsum(uptake_K)
# calculate potassium content
percentage_K = cuml_K/1000/total_dm*100
# extract data point from simulated
sim_K = np.zeros_like(data_K)
for i,l in enumerate(t):
    sim_K[i] = percentage_K[l]
# delete the last element (artificial data for the last time step to ensure the size of the vector are correct
sim_K = np.delete(sim_K, 29)
data_K = np.delete(data_K, 29)
# compute difference between data and simulated result (residuals)
e_K = abs(data_K-sim_K)
self = e_K
return self

if __name__ == '__main__':
    # load the ratio between tomato fruit and whole plant (DM) for computing the mineral content of whole plant based on mineral content of leaves and fruits
    r_f = np.load('r_f.npy')
    # construct dynamic content data based on literature
data_K = (np.array([5.55, 6.75, 6.85, 6.75, 6.75, 6.75, 4.91, 4.69, 4.53, 3.68, 3.96, 3.91, 4.02, 3.72, 3.87,]...
\begin{verbatim}
5.11*(1-r_f[24*56])+3.76*r_f[24*56], 5.51*(1-r_f[24*56]), 5.25*(1-r_f[24*56])+3.76*r_f[24*56],
4.6*(1-r_f[24*56])+3.76*r_f[24*56], 4.76*(1-r_f[24*84])+3.76*r_f[24*84], 5*(1-r_f[24*84])+3.76*r_f[24*84], 4.75*(1-r_f[24*84])+3.76*r_f[24*84],
5.51*(1-r_f[24*84])+3.76*r_f[24*84], 4.72*(1-r_f[24*112])+3.76*r_f[24*112], 4.07*(1-r_f[24*112])+3.76*r_f[24*112],
4.07*(1-r_f[2879])+3.76*r_f[2879])/5.5

# time vector of the data point
t = np.array([5*24,10*24,14*24,18*24,20*24,22*24,21*24,21*24,
28*24,28*24,28*24,28*24,28*24,28*24,
56*24,56*24,56*24,56*24,56*24,56*24,
84*24,84*24,84*24,84*24,84*24,84*24,
112*24,112*24,2879])

# initial guess
p0 = [0.0007,241.491,0.006,2198.3]

# -- least_squares --
print('--- least_squares function results ---
')
# The known parameters can be passed as args
lsresult = least_squares(residuals, p0, args=(data_K,t,root_dm,total_dm,PAR,DSR),
method='trf', bounds=([0,0,0,0],[np.inf,np.inf,np.inf,np.inf]))

print('Estimated parameters
', lsresult['x'],'
')
# The function returns jac, which seems to be X matrix
lsX = lsresult['jac']
lscovx = inv(np.dot(lsX.T,lsX))
print('Calculated inv(X.T * X) with X=returned jac =
',lscovx,'\n')
# The residuals are returned as fun in lsresult
lsres = lsresult['fun']

# The calculated variance of residuals is
lsvarres = 1/(lsX.shape[0]-lsX.shape[1]) * np.dot(lsres.T,lsres)
# The covariance matrix of parameters is
lscovp = lsvarres*lscovx
\end{verbatim}
print('Covariance matrix of parameters 
', lscovp, '
')

# The standard deviations of the parameters are:
lssd = np.sqrt(np.diag(lscovp))
print('Standard deviations of parameters 
', lssd, '
')
print('Objective function value 
', lsresult['cost'])
print('-'*25, '
')