Boundary-Layer Meteorology (2005) 116:385–392
DOI 10.1007/s10546-004-1986-2 © Springer 2005

Research Note

VARIANCE METHOD TO DETERMINE TURBULENT FLUXES OF MOMENTUM AND SENSIBLE HEAT IN THE STABLE ATMOSPHERIC SURFACE LAYER

H. A. R. DE BRUIN* and O. K. HARTOGENSIS
Wageningen University, Meteorology and Air Quality Group, Duivendaal 2, 6701 AP, Wageningen, The Netherlands

(Received in final form 25 May 2004)

Abstract. Evidence is presented that in the stable atmospheric surface layer turbulent fluxes of heat and momentum can be determined from the standard deviations of longitudinal wind velocity and temperature, σ_{u} and σ_{T} respectively, measured at a single level. An attractive aspect of this method is that it yields fluxes from measurements that can be obtained with two-dimensional sonic anemometers. These instruments are increasingly being used at official weather stations, where they replace the standard cup anemometer–wind vane system. With methods such as the one described in this note, a widespread, good quality, flux network can be established, which would greatly benefit the modelling community. It is shown that a 'variance' dimensionless height (ζ_σ) defined from σ_u and σ_T is highly related to the 'conventional' dimensionless stability parameter $\zeta = z/L$, where z is height and L is the Obukhov length. Empirical functions for ζ_σ are proposed that allow direct calculation of heat and momentum fluxes from σ_u and σ_T. The method performs fairly well also during a night of intermittent turbulence.

Keywords: CASES-99, Sonic anemometers, Stable surface layer, Variance method.

1. Introduction

Joost Businger has always encouraged exploring new methods to obtain turbulent fluxes from simple, single-level sensors. A good example can be found in Tillman (1972) who showed that in the unstable surface layer the sensible heat flux and the momentum flux can be determined from the standard deviation, σ_T, and skewness of the temperature (see also Businger, 1973). This method is often referred to as the *variance method*. Others elaborated on this idea and showed that daytime heat and momentum fluxes can be derived from σ_T and the horizontal wind speed (Weaver, 1990; Lloyd et al., 1991; Vugts et al., 1993; De Bruin et al., 1993; Katul et al., 1995). The

* E-mail: henk.debruin@wur.nl

variance method also appears to be a good indicator as to whether Monin–Obukhov similarity is valid under special conditions (De Bruin et al., 1991, 1993, 1999). So far, flux estimation using the variance method in the stable surface layer has not received much attention, mainly because fluxes are then small and the results often reveal much scatter.

In this note we will explore the variance method to determine the sensible heat flux and the friction velocity for stable conditions. In an accompanying paper, we evaluate these quantities from measurements of ε and C_T^2 (Hartogensis and De Bruin, 2005).

An attractive aspect of the variance method is that variances of temperature and horizontal wind speed can be determined with a relatively simple instrument, the two-dimensional (2-D) sonic anemometer. A recent development at standard weather stations is the replacement of the cup anemometer and wind vane for 2-D sonic anemometers. Apart from mean wind speed and direction, these also provide mean temperature, and the variances of, and covariances between these variables. From the raw data, also the dissipation rate of turbulence kinetic energy, ε, and the structure temperature parameter, C_T^2, can be derived. From these parameters more accurate fluxes could be obtained for a wide network of stations. Meteorologists dealing with weather forecast and climate models need actual flux measurements for validation and 'nudging' of their models, so that the ability to obtain surface fluxes from routine weather station data would be a great step forward in this respect (Holtslag and Van Ulden, 1983).

2. Theory and Approach

The variance method is based on the following formulations that follow from Monin–Obukhov similarity theory (MOST):

$$\frac{\sigma_T}{\theta_*} = \phi_T(\zeta) \tag{1}$$

and

$$\frac{\sigma_u}{u_*} = \phi_u(\zeta), \tag{2}$$

in which σ_T is the standard deviation of temperature, T, σ_u is the standard deviation of the longitudinal wind velocity, u, θ_* is a temperature scale, u_* is the friction velocity, and ϕ_T and ϕ_u are universal functions of the dimensionless height, $\zeta = z/L$, with z the height above a zero-plane displacement and L the Obukhov length. The definition of θ_* is $\theta_* \equiv -\frac{\overline{w'T'}}{u_*}$, where $\overline{w'T'} = \frac{H}{\rho c_p}$, in which H is the sensible heat flux density ρ, the air density and c_p, the specific heat of air at constant pressure. The definition of ζ is $\zeta = -\left(\frac{\kappa g z}{T}\right)\frac{\overline{w'T'}}{u_*^3}$. Hereafter, we refer to $\overline{w'T'}$ in any discussion of the sensible heat flux.

Several authors found that ϕ_T and ϕ_u have a constant value, c_T and c_u respectively, for a fairly wide range of ζ; see, for instance, Tillman (1972), De Bruin et al. (1993) and Pahlow et al. (2001) for ϕ_T, and Nieuwstadt (1984), Smedman (1988), De Bruin et al. (1993), Chu et al. (1996), Hsieh and Katul (1997) for ϕ_u. It is noted that at near-neutral conditions ϕ_T is a ratio of two quantities, σ_T and θ_*, that are both close to zero, and consequently its experimental accuracy is very low. Furthermore, most authors find that ϕ_u increases strongly with ζ in very stable conditions, where the experimental values of ϕ_u might be affected by spurious correlations, which are inevitable when applying MOST (see e.g., Pahlow et al., 2001).

The findings of the above cited authors suggest that over a fairly wide stability range the quantities $u_{*_\sigma} \equiv \frac{\sigma_u}{c_u}$ and $\overline{w'T'}_\sigma \equiv -u_{*_\sigma}\frac{\sigma_T}{c_T}$ are expected to be close to the actual u_* and $\overline{w'T'}$. If that is true, then $\zeta_\sigma = -\left(\frac{\kappa g z}{T}\right)\frac{\overline{w'T'_\sigma}}{u^3_{*_\sigma}}$ will be close to the actual ζ.

With this in mind, we now propose an alternative MOST variance approach by introducing a dimensionless friction velocity defined as $\frac{u_*}{u_{*_\sigma}}$ and a dimensionless heat flux defined as $\frac{\overline{w'T}}{\overline{w'T_\sigma}}$, which are both expected to be a function of ζ_σ. In this way we arrive at:

$$\frac{u_*}{u_{*_\sigma}} = F_u(\zeta_\sigma) \tag{3}$$

and

$$\frac{\overline{w'T'}}{\overline{w'T'_\sigma}} = F_{wT}(\zeta_\sigma), \tag{4}$$

in which F_u and F_{wT} are functions of ζ_σ. Our next step will be to explore this new approach with experimental data.

3. Experimental

The data presented were collected as a contribution to the CASES-99 field experiment that took place during October 1999 at a grassland site in Kansas, U.S.A (Poulos et al., 2002). We used a CSAT3 sonic anemometer from Campbell Scientific Inc., Logan, U.S.A. installed at a height of 2.65 m. Raw 20 Hz data were stored and processed with the *EC-pack* flux-software package developed by Wageningen University (http://www.met.wau.nl/projects/jep/index.html). We calculated 5-min values of σ_u, σ_T, and u_* and $\overline{w'T'}$ and averaged these to 10-min values. Several corrections were applied in calculating the fluxes: (1) the planar fit method proposed by Wilczak et al. (2001) with rotation angles determined for every 24 h, (2) linear detrending of the data, (3) humidity corrections for the sonic temperature according to Schotanus et al. (1983), and (4) frequency response corrections following Moore

(1986). The same dataset is analysed by Hartogensis and De Bruin (2005). There, we explain a special filtering that we applied, i.e., we excluded all 10-min data points for which the u spectrum did not show a clear inertial range.

4. Results

To evaluate the quantities $u_{*\sigma}$, $\overline{w'T'_\sigma}$ and ζ_σ, we required numerical values for constants c_u and c_T (see definitions of $u_{*\sigma}$ and $\overline{w'T'_\sigma}$). We used $c_T = 2.3$ and $c_u = 2.5$, which fall within the range of values reported in the literature (see e.g., Pahlow et al., 2001). To show that ζ_σ is a suitable scaling parameter we present Figure 1, which depicts ζ_σ versus ζ. Although the scatter is large for $\zeta > 1$, the agreement is accurate enough for our purpose.

Next, we fitted the functions F_u and F_{wT} to our data and found that

$$F_{wT} = F_u^2 = (1 - 1.5\zeta_\sigma + 1.8\zeta_\sigma^2)^{-1/2} \tag{5}$$

are suitable working formulae, which is demonstrated in Figures 2 and 3. There u_* and $\overline{w'T'}$, estimated from σ_u and σ_T using Equations (3)–(5), are compared with the measured values. It is seen that, except for a few outliers, the agreement is very good. Two eddy correlation systems installed close together can give considerably more scatter in stable conditions (see e.g., De Bruin et al., 2002).

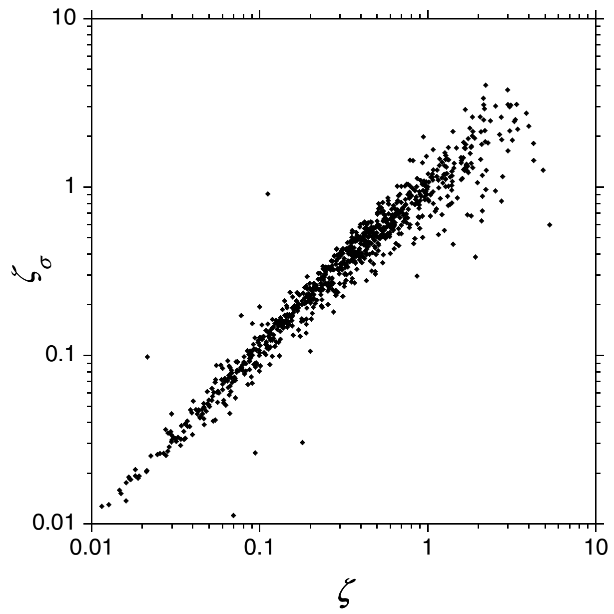
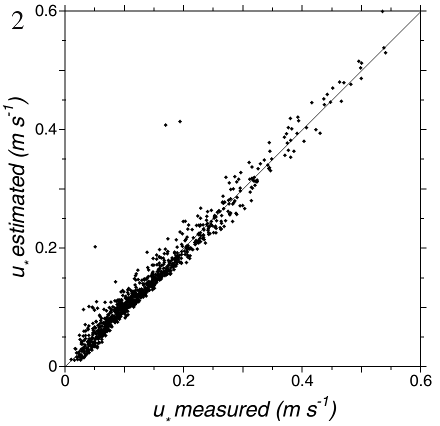


Figure 1. Dimensionless height, ζ_σ, evaluated from σ_u and σ_T versus the measured 'conventional' dimensionless height $\zeta = z/L$, where z is the height above the surface and L is the Obukhov length.



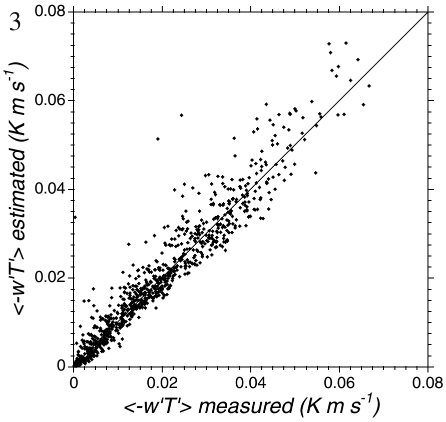


Figure 2. Friction velocity, u_*, estimated with the alternative variance method defined in Equations (3) and (5) versus the measured u_* using eddy correlation.
Figure 3. Heat flux, $-\overline{w'T}$, estimated with the alternative variance method defined in Equations (3)–(5) versus the measured $-\overline{w'T}$ using eddy correlation.

To illustrate the performance of our approach under conditions where turbulence is not continuously present, we show Figures 4 and 5, where we compared the estimated and measured values u_* and $\overline{w'T'}$ for the night of 4–5 October with so-called intermittent turbulence. It is seen that also for these conditions the results are satisfactory. Hartogensis et al. (2002) and Van de Wiel et al. (2003) showed this intermittent case also.

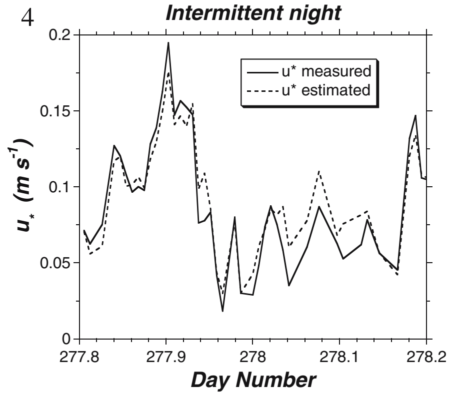
5. Discussion and Concluding Remarks

It is shown that for stable conditions, u_* and $\overline{w'T'}$ can be derived fairly accurately from the standard deviations of longitudinal wind velocity and temperature over a ζ range between 0 and about 10 using Equations (3) and (4). The method when compared with eddy correlation fluxes gives less scatter than a comparison between two eddy correlation systems installed close together. Furthermore, it is found that the performance of the variance approach is acceptable during a night with low wind speeds where turbulence was intermittent.

We realize that we used empirical expressions for the functions $F_u(\zeta_\sigma)$ and $F_{wT}(\zeta_\sigma)$ that likely need to be re-calibrated for different surface types and conditions, as was found by e.g., Weaver (1990) and Katul et al. (1995) for the temperature variance method under unstable conditions.

The dataset we used was prepared for an accompanying paper (Hartogenis and De Bruin, 2005), which deals with the ε–C_T^2 MOST relationships for 10-min time intervals (in order to be used later in scintillometry). It is

4



5

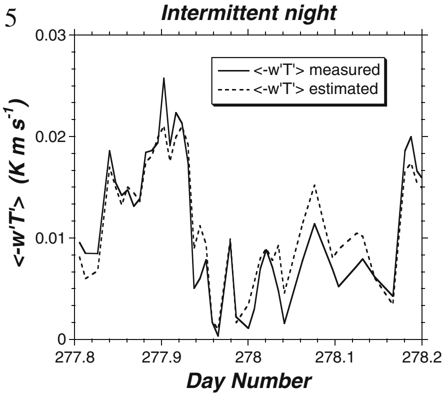


Figure 4. Estimated and measured friction velocity, u_*, as a function of time for the 'intermittent' night of 4–5 October. Estimated u_* is determined with the alternative variance method defined in Equations (3) and (5). Measured u_* is determined using eddy correlation.
Figure 5. Estimated and measured heat flux, $-\overline{w'T}$, as a function of time for the 'intermittent' night of 4–5 October. Estimated $-\overline{w'T}$ is determined with the alternative variance method defined in Equations (3)–(5). Measured $-\overline{w'T}$ is determined using eddy correlation.

important to note that we treated our eddy-correlation fluxes differently to others in the past. Especially, the use of the planar fit method and the filtering based on inertial range behaviour in the longitudinal wind speed spectrum component are different. Moreover, we calculated variances for 5-min intervals, which we subsuequently averaged to 10-min values.

During the CASES-99 experiment that lasted one month there was little or no rain, and nighttime evaporation was therefore small, with clear sky conditions dominating. It might be that stable conditions determined by different physical processes yield different results to those found herein. For instance, stable conditions can develop due to strong evaporation from irrigated crops in dry environments, where the surface layer is stably stratified even during daytime and evaporation (in energy units) can exceed net radiation. Under these conditions the correlation coefficient between temperature and specific humidity, r_{Tq}, is often close to -1 (see e.g., De Bruin et al., 1999), and the correlation between u and T, r_{uT}, a quantity that can be measured with 2-D sonic anemometers, is strongly related to r_{Tq} (De Bruin et al., 2004).

Our results are promising. However, more research is needed to show that our variance method is suitable for operational practice. To start with, the performance of 2-D sonic anemometers at weather stations in determining the standard mean horizontal wind speed and direction and more advanced turbulence parameters has to be investigated. Attention should be paid to moisture corrections of the 2-D sonic temperatures (see e.g., Schotanus et al., 1983) and the effects of using different time intervals or statistical treatment of the data (other trend corrections or different data filtering; in this note

10-min values when the u spectrum did not show a clear inertial range were excluded). An interesting aspect is the height dependency of the alternative MOST relationships we introduced. It is known that under free convection conditions variances are not constant with height. In the near-neutral regime Högström (1990) reported a height dependency for the vertical wind speed variance, whereas Thomas and Foken (2002) found a dependency on the Coriolis parameter f. Furthermore, the variance method, or the $\varepsilon - C_T^2$ approach explored in Hartogensis and De Bruin (2005), has to be validated over a wide range of circumstances and, if necessary, calibrated locally.

Acknowledgements

The authors thank Job Kramer who assisted with the CASES-99 fieldwork. The help of Drs. Steve Oncley and Gordon Maclean of N.C.A.R. during this field experiment is acknowledged also. The second author received a grant from NWO (grant nr 810.33.005).

References

Businger, J. A.: 1973, 'Turbulent Transfer in the Atmospheric Surface Layer', in D. A. Haugen (ed.), *Workshop on Micrometeorology*, Amer. Meteorol. Soc., Vol. 45 Beacon Street, Boston, MA., pp. 67–100.

Chu, C. R., Parlange, M. B., Katul, G. G., and Albertson, J. D.: 1996, 'Probability Density Functions of Turbulent Velocity and Temperature in the Atmospheric Surface Layer', *Water Resour. Res.* **32**, 1681–1688.

De Bruin, H. A. R., Bink, N.J., and Kroon L. J. M.: 1991, 'Fluxes in the Surface Layer Under Advective Conditions', in T. J. Schmugge and J. C. André (eds) *Land Surface Evaporation*, Springer Verlag, New York, pp. 157–171.

De Bruin, H. A. R., Kohsiek, W., and van den Hurk B. J. J. M.: 1993, 'A Verification of Some Methods to Determine the Fluxes of Momentum, Sensible Heat and Water Vapour Using Standard Deviation and Structure Parameter of Scalar Meteorological Quantities', *Boundary-Layer Meteorol.* **63**, 231–257.

De Bruin, H. A. R., Van Den Hurk, B. J. J. M., and Kroon, L.J.M.: 1999, 'On the Temperature-Humidity Correlation and Similarity', *Boundary-Layer Meteorol.* **93**, 453–468.

De Bruin, H. A. R., Meijninger, W. M. L., Smedman, A.-S., and Magnusson, M.: 2002, 'Displaced-Beam Small Aperture Scintillometer Test Part I: The WINTEX Data-Set', *Boundary-Layer Meteorol.* **105,** 129–148.

Hartogensis, O. K., De Bruin, H. A. R., and Van De Wiel, B. J. H.: 2002, 'Displaced-Beam Small Aperture Scintillometer Test. Part II: CASES-99 Stable Boundary Layer Experiment', *Boundary-Layer Meteorol.* **105**, 148–176.

Hartogensis, O. K. and De Bruin, H. A. R.: 2005, 'Monin–Obukhov Similarity Functions of the Structure Parameter of Temperature and TKE Dissipation Rate in the Stable Boundary Layer', *Boundary-Layer Meteorol.*, this issue.

Högström, U.: 1990, 'Analysis of Turbulent Structures in the Surface Layer with a Modified Similarity Formulation for Near Neutral Conditions', *J. Atmos. Sci.* **47**, 1949–1972.

Holtslag, A. A. M. and Van Ulden, A. P.: 1983, 'A Simple Scheme for Daytime Estimates of the Surface Fluxes from Routine Weather Data', *J. Clim. Appl. Meteorol.* **22**, 517–529.

Hsieh, C. I. and Katul, G. G.: 1997, 'Dissipation Methods, Taylor's Hypothesis, and Stability Correction Functions in the Atmospheric Surface Layer', *J. Geophys. Res.* **102**, 16391–16405.

Katul, G. G., Golz, S. M., Hsieh, C. I., Cheng, Y., Mowry, F., and Signon, J.: 1995, 'Estimation of Surface Heat and Momentum Fluxes Using the Flux-Variance Method Above Uniform and Non-uniform Terrain', *Boundary-Layer Meteorol.* **74**, 237–260.

Lloyd, C. R., Culf, A. D., Dolman, A. J. and Gash, J. H. C.: 1991, 'Estimates of Sensible Heat Flux from Observations of Temperature Fluctuations', *Boundary-Layer Meteorol.* **57**, 311–322.

Moore, C.: 1986, 'Frequency Response Corrections for Eddy Correlation Systems', *Boundary-Layer Meteorol.* **37**, 17–35.

Nieuwstadt, F. T. M.: 1984, 'The Turbulent Structure of the Stable Boundary Layer', *J. Atmos. Sci.* **41**, 2202–2216.

Pahlow, M., Parlange, M. B., and Porté-Agel, F.: 2001, 'On Monin–Obukhov Similarity in the Stable Atmospheric Boundary Layer', *Boundary-Layer Meteorol.* **99**, 225–248.

Poulos, G. S., Blumen, W., Fritts, D. C., Lundquist, J. K., Sun, J., Burns, S. P., Nappo, C., Banta, R., Newsom, R., Cuxart, J., Terradellas, E., Balsley, B., and Jensen, M.: 2002, 'CASES-99: A Comprehensive Investigation of the Stable Nocturnal Boundary Layer', *Bull. Amer. Meteorol. Soc.* **83**, 555–581.

Schotanus, P., Nieuwstadt, F., and De Bruin, H.: 1983, 'Temperature Measurement with a Sonic Anemometer and Its Application to Heat and Moisture Fluxes', *Boundary-Layer Meteorol.* **26**, 81–93.

Smedman, A.-S.: 1988, ' Observations of a Multi Level Turbulence Structure in a Very Stable Boundary Layer', *Boundary-Layer Meteorol.* **44**, 231–235.

Thomas, C. and Foken, Th.: 2002, 'Re-Evaluation of Integral Turbulence Characteristics and Their Parameterisations', in *Proc. 15th Symp. on Boundary Layers and Turbulence*, 15–22 July, Wageningen, Amer. Meteorol. Soc., Vol. 45 Beacon Street, MA., pp. 129–132.

Tillman, J. E.: 1972, 'The Indirect Determination of Stability, Heat and Momentum Fluxes in the Atmospheric Boundary Layer from Simple Scalar Variables during Dry Unstable Conditions', *J. Appl. Meteorol.* **11**, 783–792.

Van De Wiel, B. J. H., Moene, A. F., Hartogensis, O. K., De Bruin, H. A. R., and Holtslag, A. A. M.: 2003, 'Intermittent Turbulence and Oscillations in the Stable Boundary Layer over Land. Part III: A Classification for Observations During CASES99', *J. Atmos. Sci.* **60**, 2509–2522.

Vugts, H. F., Waterloo, M. J., Beekman, F. J., Fruhmau, K. F., and Bruijnzeel, L. A.: 1993, ' The Temperature Variance Method, a powerful Tool in the Estimation of Actual Evapotranspiration Rates', *IAHS Publ.* **216**, 251–260.

Weaver, H. L.: 1990, 'Temperature and Humidity Flux-Variance Relations Determined by One-Dimensional Eddy Correlations', *Boundary-Layer Meteorol.* **53**, 77–91.

Wilczak, J. M., Oncley, S. P., and Stage, S. A.: 2001, 'Sonic Anemometer Tilt Correction algorithms', *Boundary-Layer Meteorol.* **99**, 127–150.