A Microphysical Interpretation of Radar Reflectivity–Rain Rate Relationships

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ABSTRACT

The microphysical aspects of the relationship between radar reflectivity *Z* and rainfall rate *R* are examined. Various concepts discussed in the literature are integrated into a coherent analytical framework and discussed with a focus on the interpretability of *Z–R* relations from a microphysical point of view. The forward problem of analytically characterizing the *Z–R* relationship based on exponential, gamma, and monodisperse raindrop size distributions is highlighted as well as the inverse problem of a microphysical interpretation of empirically obtained *Z–R* relation coefficients. Three special modes that a *Z–R* relationship may attain are revealed, depending on whether the variability of the raindrop size distribution is governed by variations of drop number density, drop size, or a coordinated combination thereof with constant ratio of mean drop size and number density. A rain parameter diagram is presented that assists in diagnosing these microphysical modes. The number-controlled case results in linear *Z–R* relations that have been observed for steady and statistically homogeneous or equilibrium rainfall conditions. Most rainfall situations, however, exhibit a variability of drop spectra that is facilitated by a mix of variations of drop size and number density, which results in the well-known power-law *Z–R* relationships. Significant uncertainties are found to be associated with the retrieval of microphysical information from the *Z–R* relation coefficients, but even more so with shortcomings of the measurement of rainfall information and the subsequent processing of that data to obtain a *Z–R* relation. Given a proper consideration of the uncertainties, however, valuable microphysical information may be obtained, particularly as a result of long-term monitoring of rainfall for fixed observational settings but also through comparisons among different climatic rainfall regimes.

1. Introduction

The purpose of this study is to elaborate on the potential for a microphysical interpretation of the coefficients of the relationship between radar reflectivity factor Z and rain rate R, often expressed as

$$Z = \alpha R^{\beta} \tag{1}$$

(Marshall 1969; Battan 1973; Wilson and Brandes 1979; Austin 1987; Rinehart 1997; Uijlenhoet 2001). Early attempts to establish a link between cloud microphysical processes and the coefficients of the *Z–R* relationship, for example, include Fujiwara (1965) who highlighted characteristic patterns of the multiplicative factor α and exponent β with the type of rainfall. Stout and Mueller (1968) and Cataneo and Stout (1968) investigated the potential advantages of a stratification of the *Z–R* relationship coefficients by type of rainfall, synoptic situation, and thermodynamic instability. Most recent efforts are documented by Rosenfeld and Ulbrich (2003), who discuss differences in *Z–R* relations between maritime and continental, convective, transition and stratiform, and orographic precipitation. The present study aims at providing a basis for a microphysical interpretation of the *Z–R* relation coefficients by integrating various concepts discussed in the literature into a coherent analytical framework. A graphical diagnostic tool is presented that aids in distinguishing special microphysical modes. In addition, the uncertainties associated with a microphysical interpretation of the *Z–R* relation coefficients are assessed.

The problem of describing the *Z–R* relationship coefficients in terms of characteristics of the raindrop size distribution is straightforward. However, the inverse problem—that is, a microphysical interpretation of these coefficients—is more challenging. Observations indicate a significant variability of raindrop size distributions within storms and from storm to storm (Fujiwara 1965; Stout and Mueller 1968; Cataneo and Stout 1968; Battan 1973; Waldvogel 1974; Carbone and Nelson 1978; Austin 1987; Tokay and Short 1996; Doelling et

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al. 1998; Atlas et al. 1999; Cifelli et al. 2000; Steiner and Smith 2000; Maki et al. 2001; Rao et al. 2001; Bringi et al. 2003; Uijlenhoet et al. 2003b,c), implying highly variable *Z–R* relationship coefficients. Moreover, empirically derived relationships between radar reflectivity and rain rate are burdened by a range of sampling issues and the methodology applied (Cornford 1967; Joss and Waldvogel 1969; Gertzman and Atlas 1977; Joss and Gori 1978; Wong and Chidambaram 1985; Chandrasekar and Bringi 1987; Krajewski and Smith 1991; Smith et al. 1993; Ulbrich 1994; Sheppard and Joe 1994; Ciach and Krajewski 1999; Kostinski and Jameson 1999; Campos and Zawadzki 2000; Ciach et al. 2000; Steiner and Smith 2000; Jameson and Kostinski 2001; Tokay et al. 2001; Meagher and Haddad 2002; Uijlenhoet et al. 2003a, manuscript submitted to *J. Atmos. Sci.*) that question the extent to which *Z–R* relations may be interpreted in microphysical terms.

The paper is organized as follows: the underlying theory and assumptions that lead to power-law formulations of the relationship between radar reflectivity and rain rate, as expressed in Eq. (1), are reviewed in section 2. That section is also concerned with specific aspects of *Z–R* relations based on exponential, gamma, and monodisperse raindrop spectra. Section 3 highlights microphysical conditions that lead to special *Z–R* relations and introduces a diagnostic graphical tool to identify these conditions. Moreover, this section includes a microphysical excursion into the realm of drastic changes in drop size distribution characteristics, and how they are related to precipitation type. The microphysical interpretation of the *Z–R* relationship coefficients—that is, the inverse problem to section 2—and its limitations are discussed in section 4. Evaluations based on raindrop spectra observations are included in sections 3 and 4 to complement the analytical analyses. The conclusions from this study are summarized in section 5.

2. Mathematical framework

a. Basic theory

In order to relate the multiplicative factor α and exponent β of the *Z–R* relationship (1) to microphysical parameters, we assume a gamma raindrop size distribution described by

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D), \tag{2}$$

where D (mm) is the drop diameter, N_0 [m^{-3} mm$^{-(1+\mu)}$] is the intercept,[1] Λ (mm^{-1}) the slope coefficient, and μ the distribution shape factor (Clark 1974; Ulbrich 1983; Willis 1984; Chandrasekar and Bringi 1987; Russchenberg 1993; Testud et al. 2001; Illingworth and Blackman 2002; Bringi et al. 2003). The exponential raindrop size distribution (Marshall and Palmer 1948; Ohtake 1970; Sekhon and Srivastava 1971; Waldvogel 1974; Joss and Gori 1978; Uijlenhoet and Stricker 1999) is obtained for the special case of $\mu = 0$. Other analytical parameterizations that have been proposed to characterize raindrop spectra are the generalized gamma distribution (Best 1950b) with the Weibull distribution (Weibull 1951; Jiang et al. 1996) as a special case, and the lognormal distribution (Bradley and Stow 1974; Markowitz 1976; Feingold and Levin 1986; Smith and Krajewski 1993; Sauvageot and Lacaux 1995). Furthermore, Sempere Torres et al. (1994, 1998) and Uijlenhoet et al. (2003b) provide a formulation of the raindrop size distribution in terms of a scaling law, and Smith (1993) characterizes drop spectra in terms of a drop arrival rate and properties of raindrop size (mean and standard deviation). Meagher and Haddad (2002) propose a principal-component-based description of the raindrop size distribution.

The radar reflectivity factor Z (mm^6 m^{-3}) at radar wavelengths at which the Rayleigh approximation is valid and the rain rate R (mm h^{-1}) are defined as

$$Z = \int_0^\infty N(D) D^6 \, dD \quad \text{and} \tag{3}$$

$$R = \frac{6\pi}{10^4} \int_0^\infty N(D) D^3 v(D) \, dD, \tag{4}$$

respectively. For simplicity, the integration over drop diameter is carried out from zero to infinity. Effects of truncation of the drop size distribution, which may occur as an instrumental artifact or as a natural phenomenon, are discussed in Ulbrich (1985, 1992, 1994), Feingold and Levin (1986), Ulbrich and Atlas (1998), and Illingworth and Blackman (2002). Truncation effects appear to be entirely contained in the multiplicative factor α of *Z–R* relations and do not affect the exponent β. The advantage of using gamma drop size distributions is that with increasing μ the distribution becomes narrower and thus effectively truncated. In the limiting case of $\mu \to \infty$ the gamma distribution approaches a Dirac δ function, that is, a monodisperse drop spectrum.

The relation between drop diameter D and its fall velocity v (m s^{-1}) shall be approximated by a power-law expression of the form

$$v(D) = v_0 D^p, \tag{5}$$

with the coefficients $v_0 = 3.778$ (m s^{-1} mm^{-p}) and $p = 0.67$, as proposed by Atlas and Ulbrich (1977), which provides a close fit to the data of Gunn and Kinzer (1949) for the diameter interval $0.5 < D < 5.0$ mm that matters most for rainfall. More sophisticated raindrop terminal fall speed parameterizations have been advocated in the literature (Best 1950a; Atlas et al. 1973; Berry and Pranger 1974; Beard 1976; Brazier-Smith 1992; Gossard et al. 1992), but the power-law (5) is the

[1] Strictly speaking N_0 is a concentration scaling parameter that becomes the intercept parameter only for the case of exponential raindrop spectra ($\mu = 0$). However, for simplicity we refer to N_0 as intercept throughout this paper.

only functional form consistent with power-law relationships between rainfall integral parameters such as Z and R (e.g., Uijlenhoet 2001). More recent measurements reveal raindrop fall speeds that differ from the Gunn and Kinzer (1949) values (e.g., Hauser et al. 1984; Hosking and Stow 1991; Löffler-Mang and Joss 2000; Salles and Creutin 2003); however, a detailed discussion of these effects is beyond the scope of this study.

Using (2) and the substitution $x = \Lambda D$, Eq. (3) can be restated as

$$Z = \int_0^\infty N_0 D^\mu \exp(-\Lambda D) D^6\, dD = \frac{N_0}{\Lambda^{(7+\mu)}} \int_0^\infty e^{-x} x^{(6+\mu)}\, dx = \frac{N_0}{\Lambda^{(7+\mu)}} \Gamma(7 + \mu), \quad (6)$$

where use was made of the gamma function, $\int_0^\infty e^{-x} x^n\, dx = \Gamma(n + 1)$. Similarly, using Eqs. (2) and (5), Eq. (4) can be rearranged as

$$R = \frac{6\pi}{10^4} \int_0^\infty N_0 D^\mu \exp(-\Lambda D) D^3 \upsilon_0 D^p\, dD = \frac{6\pi\upsilon_0}{10^4} \frac{N_0}{\Lambda^{(4+p+\mu)}} \int_0^\infty e^{-x} x^{(3+p+\mu)}\, dx = \frac{6\pi\upsilon_0}{10^4} \frac{N_0}{\Lambda^{(4+p+\mu)}} \Gamma(4 + p + \mu). \quad (7)$$

To ease interpretation of subsequent results in microphysical terms, the coefficients N_0 and Λ will be replaced by the raindrop density, that is, the total number of drops N_T (m^{-3}) per unit volume of air,

$$N_T = \int_0^\infty N(D)\, dD = \frac{N_0}{\Lambda^{(1+\mu)}} \Gamma(1 + \mu), \quad (8)$$

and the mass-weighted mean drop diameter $\overline{D}_m$ (mm),

$$\overline{D}_m = \frac{\int_0^\infty N(D) D^4\, dD}{\int_0^\infty N(D) D^3\, dD} = \frac{4 + \mu}{\Lambda}, \quad (9)$$

respectively.[2] The mass-weighted mean diameter $\overline{D}_m$ is used because of its reduced sensitivity to sampling limitations for small drop sizes, a problem of many instruments when measuring drop size distributions. The drop number density N_T is affected by the same problem; however, use of an equivalent concentration or normalized intercept parameter as suggested by Sekhon and Srivastava (1971), Willis (1984), Chandrasekar and Bringi (1987), Testud et al. (2001) or Illingworth and Blackman (2002) renders the microphysical interpretation less intuitive. The median drop diameter D_0 (i.e., the diameter dividing the distribution's liquid water content in half), which is frequently used as a characteristic drop diameter, is related to the mass-weighted mean diameter by

$$D_0 \approx \overline{D}_m \frac{3.67 + \mu}{4 + \mu} \quad (10)$$

(e.g., Atlas 1953; Ulbrich 1983; Tokay and Short 1996; Illingworth and Blackman 2002; Bringi et al. 2003). The mass-weighted standard deviation σ_D of drop diameters is defined as

$$\sigma_D = \left[\frac{\int_0^\infty N(D) D^3 (D - \overline{D}_m)^2\, dD}{\int_0^\infty N(D) D^3\, dD}\right]^{0.5} = \frac{(4 + \mu)^{0.5}}{\Lambda} = \frac{\overline{D}_m}{(4 + \mu)^{0.5}}. \quad (11)$$

Equation (11) relates the mean drop size and standard deviation of drop sizes to the distribution shape factor μ. A coefficient of drop variation may be defined as $\mathrm{CV}_D = \sigma_D/\overline{D}_m = (4 + \mu)^{-0.5}$. The raindrop number density N_T, mean drop size $\overline{D}_m$, and either one of the standard deviation of drop sizes σ_D, coefficient of drop variation CV_D, or distribution shape factor μ determine the drop size distribution given by (2) based on microphysically meaningful terms.[3] Note that for $\mu \rightarrow \infty$ the standard deviation of drop sizes goes to zero, which results in a monodisperse distribution as the limiting case.

Making use of Eqs. (8) and (9), the radar reflectivity factor (6) can be expressed as

$$Z = N_T \frac{\Gamma(7 + \mu)}{\Gamma(1 + \mu)} \left(\frac{\overline{D}_m}{4 + \mu}\right)^6 \quad (12)$$

and, similarly, the rain rate (7) as

$$R = N_T \frac{6\pi\upsilon_0}{10^4} \frac{\Gamma(4 + p + \mu)}{\Gamma(1 + \mu)} \left(\frac{\overline{D}_m}{4 + \mu}\right)^{(3+p)}. \quad (13)$$

The radar reflectivity factor Z and rain rate R are thus directly proportional to the raindrop number density N_T and proportional to the mass-weighted mean drop size $\overline{D}_m$ to the power of 6 and $3 + p$, respectively. The drop size distribution shape factor μ affects the numerical coefficient of relationships (12) and (13) only.

[2] Throughout this paper we may refer to the mass-weighted mean diameter (and the mass-weighted standard deviation of drop diameters) also simply as mean diameter (and standard deviation).

[3] The parameters suggested here are likely not statistically independent, but easily understood in terms of a concentration, mean drop size, and drop size dispersion. See Haddad et al. (1996) for a formulation of the raindrop size distribution in terms of statistically independent yet less intuitive parameters.

b. Z–R relationships

A power-law relation between the radar reflectivity factor Z and rain rate R, such as given in (1), can be obtained in different ways. For example, if (13) is solved for N_T and the resulting expression substituted into (12), we obtain

$$Z = \frac{10^4}{6\pi v_0}\frac{\Gamma(7+\mu)}{\Gamma(4+p+\mu)}\left(\frac{\overline{D}_m}{4+\mu}\right)^{(3-p)} R. \quad (14)$$

For a linear Z–R relationship like (14) to be physically meaningful (i.e., deterministic), the mean drop size $\overline{D}_m$ and distribution shape factor μ have to remain constant and the variability of the raindrop size distribution has to be facilitated by variations in drop number density. On the other hand, if (13) is solved for $\overline{D}_m$ and the resulting expression substituted into (12), this yields

$$Z = \frac{\Gamma(7+\mu)}{\Gamma(1+\mu)}\left[\frac{10^4}{6\pi v_0}\frac{\Gamma(1+\mu)}{\Gamma(4+p+\mu)}\right]^{[6/(3+p)]} \times \left(\frac{1}{N_T}\right)^{[(3-p)/(3+p)]} R^{[6/(3+p)]}. \quad (15)$$

Power-law Z–R relations (15) with exponent $\beta = 1.63$ [based on using $p = 0.67$, as suggested by Atlas and Ulbrich (1977)] are the consequence of a constant drop number density N_T and μ, while the variability of the drop spectrum may be accommodated through variations in mean drop size. Alternatively, Eq. (7) could be solved for either the intercept coefficient N_0 or slope coefficient Λ, and the respective result substituted into (6). The former approach, and using (9) to replace Λ by the mean drop size $\overline{D}_m$, results in (14) as well. The latter approach, however, combined with Eqs. (8) and (9) to replace N_0 by the raindrop number density N_T and mean drop size $\overline{D}_m$, produces

$$Z = \Gamma(7+\mu)\left[\frac{10^4}{6\pi v_0}\frac{1}{\Gamma(4+p+\mu)}\right]^{[(7+\mu)/(4+p+\mu)]} \times \left[\frac{\Gamma(1+\mu)}{(4+\mu)^{(1+\mu)}}\frac{(\overline{D}_m)^{(1+\mu)}}{N_T}\right]^{[(3-p)/(4+p+\mu)]} R^{[(7+\mu)/(4+p+\mu)]}, \quad (16)$$

which represents the case of a power-law Z–R relation, where the exponent β depends on the drop size distribution shape factor μ. Here, the mean drop size $\overline{D}_m$ and drop number concentration N_T may both vary, albeit in a coordinated manner such that the ratio $(\overline{D}_m)^{(1+\mu)}/N_T$ remains constant, which implies that the intercept N_0 has to be constant, as we will see. Specific aspects of this behavior will be further investigated in section 3. More generally, the power-law relation between the radar reflectivity factor Z and rain rate R may be written as (A. R. Jameson 2003, personal communications)

$$Z = \left(\frac{10^4}{6\pi v_0}\right)^{\beta}\frac{\Gamma(7+\mu)}{\Gamma(1+\mu)}\left[\frac{\Gamma(1+\mu)}{\Gamma(4+p+\mu)}\right]^{\beta} \times \left(\frac{\overline{D}_m}{4+\mu}\right)^{[6-(3+p)\beta]}\left(\frac{1}{N_T}\right)^{(\beta-1)} R^{\beta}. \quad (17)$$

Equations (14)–(16) are obtained from (17) as a result of selecting $\beta = 1$ (i.e., eliminating the dependence on N_T), $\beta = 6/(3+p)$ (eliminating the $\overline{D}_m$ term), and $\beta = (7+\mu)/(4+p+\mu)$, respectively.

The exponent β of Z–R relation (16) depends on the distribution shape factor μ. This relationship between β and μ is visualized in Fig. 1. The exponent β asymptotically reaches a value of 1 for large μ (positive or negative), but increases/decreases dramatically as μ approaches a value of -4.67 (vertical dotted line), where β is no longer defined. The two special cases of number- ($\beta = 1$) and size-controlled ($\beta = 1.63$) variability of the raindrop size distribution are indicated by the thin horizontal lines. The exponential case ($\mu = 0$, $\beta = 1.50$) is shown by the bold dot. The invalid or highly unlikely parts of the relationship between β and μ are shown by the bold dashed lines.

Exponents $\beta < 1$ are not admissible from a microphysical perspective; thus, values of $\mu \leq -4.67$ are not valid. Typical values of μ found in the literature are $-3 < \mu < 15$ (e.g., Willis and Tattelman 1989; Kozu and Nakamura 1991; Tokay and Short 1996; Wilson et al. 1997; Illingworth and Blackman 2002; Bringi et al. 2003). Smith and Krajewski (1993) show that $1 \leq \beta \leq 3.125$ (shaded in Fig. 1),[4] which would place the minimum possible value for $\mu \sim -3.6$. A slightly higher minimum of $\mu \sim -1.8$ is suggested by Doelling et al. (1998) and Steiner and Smith (2000), who find $1 \leq \beta < 1.8$ on the basis of raindrop spectra observations using Joss–Waldvogel (Joss and Waldvogel 1967) disdrometers. This upper value for the exponent of the Z–R relation is consistent with Smith and Krajewski's (1993) size-controlled case ($\beta = 1.79$) based on lognormal raindrop size distributions. From an analytical perspective, values of $\mu \leq -1$ are void in order to have all nonnegative moments of the raindrop spectrum (i.e., the physically relevant bulk rainfall variables such as N_T, R, Z, etc.) defined. Rainfall characterized by a size-controlled variability of the drop size distribution, therefore, provides a limiting case of the valid range for the distribution shape factor μ.[5] On the other hand, analytically there is no upper limit that the value μ may attain. However, for μ going toward infinity the mass-

[4] The upper bound of $\beta \leq 3.125$ is a consequence of the least squares estimation technique applied by Smith and Krajewski (1993), who used lognormal instead of gamma drop size distributions for their analysis.

[5] The exact value of β depends on the analytical form of the raindrop size distribution, for example, $\beta = 1.63$ for exponential and gamma raindrop size distributions, but $\beta = 1.79$ for lognormal raindrop spectra.

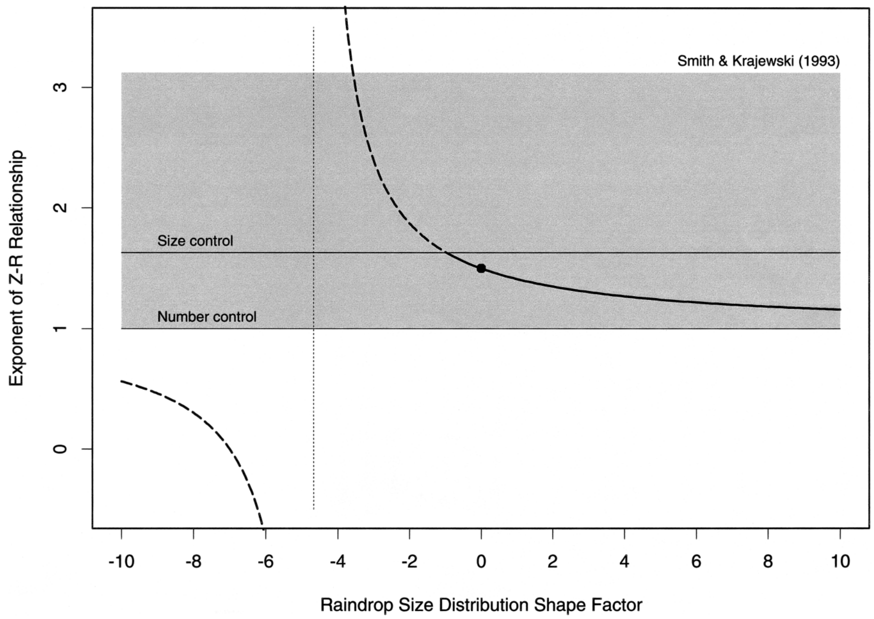

FIG. 1. Dependence of the exponent β of the *Z–R* relationship (16) as a function of the raindrop size distribution shape factor μ using the fall velocity exponent $p = 0.67$. The relationship between the exponent β and the drop size distribution shape factor μ is shown by the bold solid line for the microphysically valid range and by the bold dashed lines for the invalid range. The vertical dotted line indicates the asymptotic behavior of the relationship for $\mu = -(4 + p)$. The potential range of $1 \leq \beta \leq 3.125$ according to Smith and Krajewski (1993) is shaded in gray. The exponents for the two limiting cases of number-controlled and size-controlled drop spectra are indicated by solid lines. The bold dot marks the case of exponential raindrop size distributions ($\mu = 0$, $\beta = 1.5$).

weighted standard deviation of drop sizes, as defined by Eq. (11), approaches zero and the drop spectra become increasingly monodisperse, ultimately reaching a number-controlled condition. Consequently, the number-controlled ($\beta \sim 1$) case provides another limiting case. *The variability of the raindrop size distribution is thus bounded by either size- or number-controlled conditions, with conditions of a coordinated mixed control embedded in between those extremes.* This appears to be true independent of the analytical form of the raindrop size distribution. One might argue, therefore, that the deviation of β from 1 could be a measure of how far the underlying drop size distributions are from equilibrium or steady state (Sempere Torres et al. 1994; Uijlenhoet et al. 2003b), as discussed in section 3. The sensitivity of the exponent β to variations of the distribution shape factor μ decreases with increasing value of μ.

The relation between the multiplicative factor α and exponent β of the *Z–R* relationship (16) is visualized in Fig. 2 for various characteristic mean drop sizes $\overline{D}_m$ and raindrop size distribution shape factors $\mu \geq 0$.[6] The number- ($\beta = 1$) and size-controlled ($\beta = 1.63$) cases are not shown. Clearly, the *Z–R* relationship coefficients are highly sensitive to changes in characteristic raindrop size $\overline{D}_m$, number concentration N_T, and distribution shape factor μ. The infinitesimal possibilities of obtaining a *Z–R* relation for combinations of mean drop size and number concentration easily explain the multitude of such relationships reported in the literature (e.g., Twomey 1953; Stout and Mueller 1968; Battan 1973).

For the special case of monodisperse raindrop spectra, containing N_s ($=N_T$) drops of size D_s ($=\overline{D}_m$), the resulting *Z–R* relationship for the number-controlled case is

$$Z = \frac{10^4}{6\pi v_0}(D_s)^{(3-p)}R. \tag{18}$$

This equation may be derived either analytically or obtained from (14) for the limiting case of $\mu \rightarrow \infty$. Note that monodisperse raindrop spectra do not exhibit a mixed-control case, such as is seen for exponential or gamma raindrop size distributions. In addition, the size-controlled case—although analytically feasible—is not meaningful, because any significant changes in drop size would invariably lead to drop spectra deviating from

[6] Note that the value $\mu = 100$ used in several of the figures (i.e., Figs. 2, 3, 9, and 10) is unrealistically large; however, it is used to visualize the asymptotic behavior of the respective relationships.

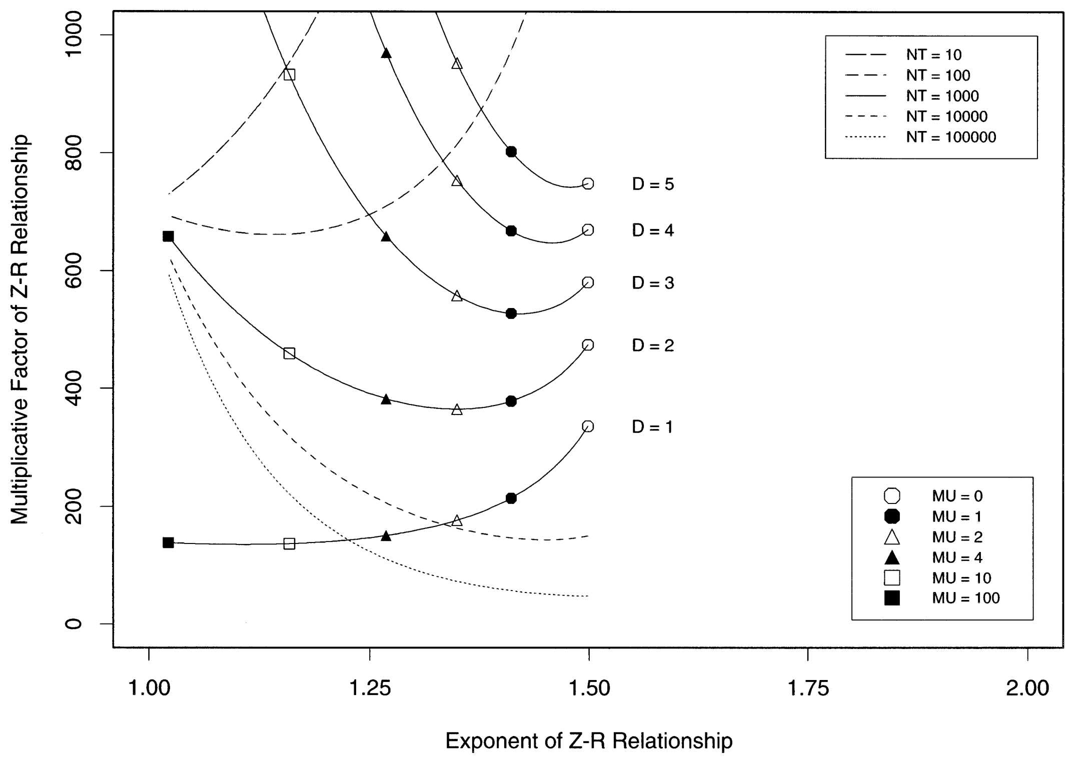

FIG. 2. Relationship between multiplicative factor α and exponent β of the *Z–R* relation (16) for various characteristic mean drop sizes $\overline{D}_m$ and raindrop size distribution shape factors μ. Results are shown for $\mu \geq 0$ and a total number of drops per unit volume of air $N_T = 1000$ (solid lines with symbols). In addition, for $\overline{D}_m = 2$ mm, the dependence of the *Z–R* relation coefficients on the total number of drops N_T is highlighted (broken lines).

being monodisperse.[7] Applying the power-law fall velocity coefficients suggested by Atlas and Ulbrich (1977) to (18) yields

$$Z = 140.42(D_s)^{2.33}R, \quad (19)$$

which is consistent with Waldvogel's (1975) result.

Note that all of the above *Z–R* relationships are derived for sea level pressure. These results, however, can easily be adjusted to different altitudes by multiplying υ_0 with a term $(\rho_0/\rho)^\chi$, where ρ_0 and ρ are the air densities at sea level and the desired altitude, respectively, and the exponent χ ranges from 0.4 to 0.5 depending on drop size (Foote and duToit 1969; Beard 1985). Moreover, Sekhon and Srivastava (1971), Battan (1976), Austin (1987), Dotzek and Beheng (2001), and Dotzek and Fehr (2003) discuss the effect of vertical air motions on the *Z–R* relation. Atlas and Chmela (1957), Austin (1987), and Li and Srivastava (2001) elaborate on the effect of evaporation of rain below cloud base.

[7] Slow processes such as evaporation of drops below cloud base or growth by accretion of cloud droplets exemplify size-controlled conditions; however, the corresponding time scale is on the order of hours.

3. Microphysical considerations

a. Special microphysical conditions

For any given shape (μ) of the raindrop size distribution, Eqs. (14)–(16) represent three special microphysical conditions, where (i) all variability of the raindrop size distribution is controlled by variations in number concentration (*number controlled*), (ii) all variability of the raindrop size distribution is controlled by variations in characteristic raindrop size (*size controlled*), and (iii) the variability of the raindrop size distribution is controlled by a *coordinated mix* of variations in number concentration and characteristic size.

This result is consistent with the one obtained by Uijlenhoet et al. (2003b), which is based on a scaling-law formalism that does not make any assumptions about the raindrop size distribution. Smith and Krajewski (1993) arrive at a similar result for lognormal raindrop size distributions and a statistical rainfall model (Smith 1993) based on the arrival rate and characteristic properties of raindrops. Thus, the above three microphysical modes exist independent of the form of the raindrop size distribution. Principally, one could imagine a case also where the variability of the drop spectrum may be governed by variations of the drop size distribution shape factor μ [analogous to the coefficient of drop size variation control in Smith and Krajewski (1993)]; however, analytically this is not straightforward based on gamma raindrop size distributions, as can be seen from Eq. (17), and will thus not be discussed.

Raindrop spectra governed by a number-controlled condition neither change their shape (constant μ) nor characteristic drop size (constant $\overline{D}_m$) while undergoing fluctuations in number density (variable N_T). This condition leads to all moments of the raindrop size distribution being linearly related to each other, as exemplified in Eq. (14). Linear Z–R relationships ($\beta = 1$) have been found previously for so-called equilibrium conditions in rainfall, where all variability of the raindrop size distribution is controlled by fluctuations in drop density (Pasqualucci 1982; Hodson 1986; List et al. 1987; List 1988; Hu and Srivastava 1995; Uijlenhoet et al. 2003b). Jameson and Kostinski (2001) find similar conditions in statistically homogeneous rain, where the Z–R relation has a clear physical meaning, as opposed to statistically inhomogeneous rain, where the relation between Z and R is of a purely statistical nature. Equilibrium or statistically homogeneous rainfall conditions reflect a balance between drop collisions, coalescence, and breakup (e.g., Srivastava 1971; Valdez and Young 1985; List et al. 1987; Brown and Whittlesey 1992; Hu and Srivastava 1995). If such conditions truly occur in nature, one might find them in the efficient warm rain process dominated growth phase of intense tropical rainfall (e.g., hurricanes), severe and long-lasting midlatitude storm systems (e.g., supercells), or persistent heavy orographic rainfall. A comment by Carbone (1985) on the paper by Willis (1984) and Willis' reply (1985) shed additional light on the existence of equilibrium drop spectra. Under such conditions, the characteristic mean drop size may remain constant for a given storm phase, although this drop size might vary among different situations. Therefore, even under this ideal (albeit extreme) condition there exists no unique Z–R relation: the multiplicative factor α depends on the mean drop size $\overline{D}_m$, as shown by (14). Ulbrich and Atlas (1998) show empirically that linear Z–R relations are obtained when drop spectra are stratified by mean drop size.[8] Moreover, conditions of approximate constant drop size have been observed for high rain rates by Hudson (1963), Blanchard and Spencer (1970), Srivastava (1971), Pasqualucci (1982), Hodson (1986), List (1988), Zawadzki and de Agostinho Antonio (1988), Atlas and Ulbrich (2000), Atlas and Williams (2003), and Uijlenhoet et al. (2003b).

In contrast, size-controlled raindrop size distributions exhibit variations in characteristic drop size (variable $\overline{D}_m$) while keeping the number density and distribution shape constant. This condition is described by (15), with a multiplicative factor that depends only on the raindrop number density N_T and an exponent $\beta = 1.63$. Rogers et al. (1991) indicate that such conditions may be applicable to steady stratiform-like drizzle rain ($R < 1$ mm h^{-1}, $Z < 20$ dBZ) falling from Hawaiian warm-based orographic clouds, where drops are neither created (no coalescence) nor destroyed (no breakup)—that is, the drop number density N_T remains approximately constant—and grow by accretion of cloud droplets. Carbone and Nelson (1978) provide evidence suggesting that size-controlled conditions may occur in dissipating convective cells, while Gunn and Marshall (1955) and Atlas and Chmela (1957) point to size sorting due to wind shear and/or turbulence.

The third mode represents variations of raindrop spectra conditioned on a constant ratio of characteristic drop size and number density while maintaining a constant shape of the distribution. For exponential raindrop spectra ($\mu = 0$), this microphysical condition is associated with a power-law relationship between Z and R with exponent $\beta = 1.5$ and a multiplicative factor α that depends both on the raindrop concentration N_T and the mean drop size $\overline{D}_m$, where α is directly related to the mean drop size but inversely to the number concentration. The ratio $N_T/\overline{D}_m$ is proportional to the intercept parameter N_0, as can be seen from Eq. (20) below. This is essentially the kind of rainfall reported by Marshall and Palmer (1948), Waldvogel (1974), and many others since.

b. Rain parameter diagram

It is instructive to visualize the three special modes in a diagram spanned by the drop number density N_T (ordinate) and the mean drop size $\overline{D}_m$ (abscissa), as shown in Fig. 3a. These special conditions occur under the assumption of a constant shape factor μ of the raindrop size distribution, as discussed earlier. The curves of constant N_0 are obtained by combining Eqs. (8) and (9) to yield

$$N_0 = \frac{(4 + \mu)^{(1+\mu)}}{\Gamma(1 + \mu)} \frac{N_T}{(\overline{D}_m)^{(1+\mu)}}, \tag{20}$$

which relates the intercept coefficient N_0 in Eq. (2) with the number density of drops N_T in a volume of air and the mass-weighted mean drop size $\overline{D}_m$ as a function of the raindrop size distribution shape factor μ. Figure 3a visualizes the relationship between N_T and $\overline{D}_m$ as a function of R and N_0, defined by Eqs. (13) and (20), for exponential raindrop spectra.

The effect of the distribution shape factor μ on these relationships is demonstrated in Fig. 3b. For fixed rain rate R and intercept coefficient N_0, increasing μ is accompanied by a reduction in total number of drops N_T and a slight

[8] Note that Eq. (14) provides an explanation for this linearity after stratification by mean drop size: the radar reflectivity factor Z, divided by the mass-weighted mean drop size raised to the power $3 - p$, is proportional to the rain rate R; i.e., $Z/(\overline{D}_m)^{(3-p)} \sim R$. It can be demonstrated that this proportionality to the rain rate holds for any characteristic drop size and for any parameterization of the raindrop size distribution.

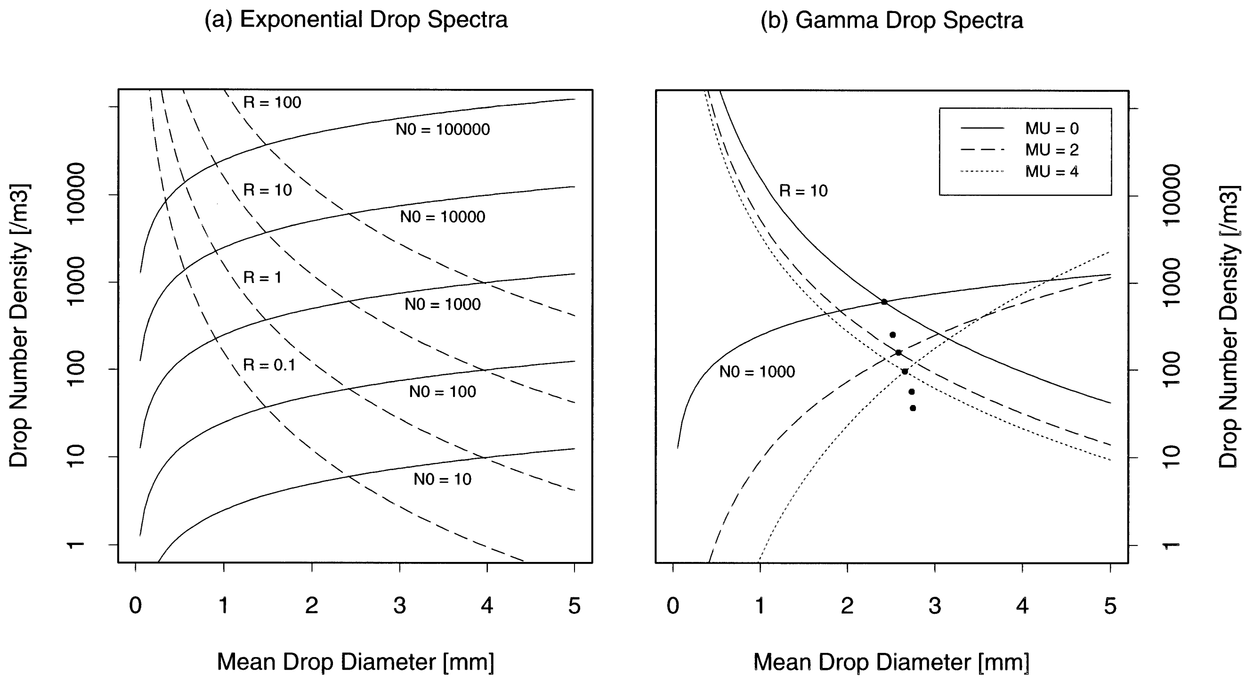

FIG. 3. (a) Relationship between the total number of drops per unit volume of air N_T and the mass-weighted mean drop diameter $\overline{D}_m$ as a function of the intercept coefficient N_0 and rain rate R as determined by Eqs. (13) and (20), respectively. The curves are based on exponential raindrop size distributions ($\mu = 0$). (b) Dependence of the relationship between N_T and $\overline{D}_m$ on the gamma distribution shape factor μ for fixed $N_0 = 1000$ m^{-3} mm$^{-(1+\mu)}$ and $R = 10$ mm h^{-1}. Points, where the lines for fixed N_0 and R are crossing, are shown for μ values of 0, 1, 2, 4, 10, and 100.

increase in mean drop size $\overline{D}_m$. The mean drop size as a function of N_0, R, and μ is described by

$$\overline{D}_m = (4 + \mu)\left[\frac{10^4}{6\pi v_0}\frac{R}{N_0}\frac{1}{\Gamma(4 + p + \mu)}\right]^{[1/(4+p+\mu)]}, \quad (21)$$

which is obtained by combining Eqs. (13) and (20). Examples of $\overline{D}_m$ for a wide range of shape factors μ are shown by the bold dots in Fig. 3b for fixed intercept coefficient $N_0 = 1000$ m^{-3} mm$^{-(1+\mu)}$ and rain rate $R = 10$ mm h^{-1}. Similarly, the total number of drops can be expressed as a function of N_0, R, and μ based on combination of Eqs. (13) and (20) to yield

$$N_T = \Gamma(1 + \mu)\left[\frac{10^4}{6\pi v_0}\frac{R}{\Gamma(4 + p + \mu)}\right]^{[(1+\mu)/(4+p+\mu)]} \times (N_0)^{[(3+p)/(4+p+\mu)]}. \quad (22)$$

Figure 3 can in fact be used as a *diagnostic tool* to evaluate the type of rainfall encountered and, therefore, which *Z–R* relationship may be applicable. For example, rainfall under number-controlled conditions (i.e., $\overline{D}_m$ constant) should fluctuate along a vertical line in Fig. 3, while rainfall under size-controlled conditions (N_T constant) should fall along a horizontal line. Rainfall, where all variability of the drop size distribution is controlled by coordinated variations of mean drop size and number density, such that the ratio of $(\overline{D}_m)^{(1+\mu)}/N_T$ remains constant, should follow lines of constant N_0 in Fig. 3. This diagram provides thus means to relate raindrop microphysics to *Z–R* relationships, analogous to the rain parameter diagram introduced by Atlas and Chmela (1957) and extensively discussed by Ulbrich and Atlas (1978).

Uijlenhoet et al. (2003b) discuss a storm with maximum rain rates peaking around 300 mm h^{-1} that was observed with the Illinois State Water Survey raindrop camera (Mueller 1962; Jones 1992) on 21 June 1958 near Miami, Florida. The raindrop size distributions of that storm displayed quasi-equilibrium behavior resulting in a *Z–R* relationship that was approximately linear (see Figs. 9 and 10 of Uijlenhoet et al. 2003b). The close to number-controlled nature of this rainfall is evidenced in Fig. 4. The median drop size[9] remained particularly constant during the initial burst of rainfall (1050–1100 UTC), while after that it hovered for 20 min around $2.0 < D_0 < 2.4$ mm and the corresponding drop number density fluctuated between $3000 < N_T < 9000$ m^{-3}. A bit later (1130–1140 UTC), the drop number density was significantly lower, yet the mean drop size was not much reduced ($1.4 < D_0 < 2.1$ mm). The outlier observation at 1102 UTC is likely attributable to a measurement problem (Uijlenhoet et al. 2003b).

[9] The difference between the median and the mass-weighted mean drop size was between 5% and 10% for this storm.

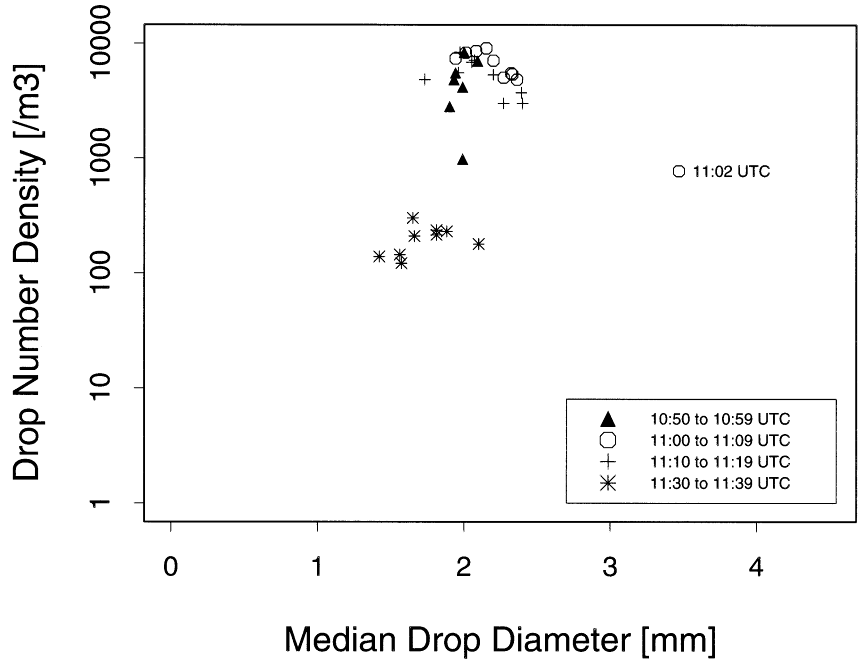

FIG. 4. Variability of the raindrop size distribution of the 21 Jun 1958 storm near Miami, FL, as reflected by the median drop size D_0 and drop number density N_T. Four 10-min periods are distinguished.

c. Microphysical discontinuities in rainfall—N_0 jumps

It is instructive to dwell on Eq. (20) for a moment, thinking in terms of possible microphysical causes for the so-called N_0 jumps. An observed characteristic of N_0 jumps, as originally pointed out by Waldvogel (1974), is that they occur for approximately constant rain rate. Order of magnitude changes in N_0, therefore, take place along the dashed lines in Fig. 3a, highlighting that such changes are the result of modifications of both *mean drop size and number concentration.*[10] Similar findings have been noted by Ulbrich (1983), Kostinski and Jameson (1999), and Jameson and Kostinski (2001). Jumps to larger N_0 values are accompanied by a decreasing $\overline{D}_m$ and increasing N_T. Microphysical processes associated with jumps to larger N_0 are increased breakup of raindrops due to enhanced number of collisions or onset of riming, which tends to suppress aggregation and thus inhibit the formation of larger snowflakes that would melt into bigger raindrops (Wacker 1995; Steiner and Smith 1998). Significant decreases in N_0, on the other hand, are related to an increase in mean drop size and a decrease in number concentration, which may be the result of increased coalescence and thus rapid growth of raindrops and/or increased aggregation of snowflakes (e.g., Waldvogel 1974). Equations (21) and (22) may be used to quantitatively assess changes in mean drop size and drop concentration for N_0 jumps.

[10] This is a direct consequence of the implied mass preservation, as can be seen from Eq. (13).

Not every drastic change in N_0, however, qualifies as an N_0 jump in the ‘‘classical’’ sense of Waldvogel (1974), where the rain rate remains approximately constant. According to (20), fluctuations in N_0 may be the result of any significant change in drop number density N_T, mean drop size $\overline{D}_m$, or a combination thereof, and likely cause fluctuations in rain rate as well. Significant changes within a given storm system are related to differences in microphysical growth processes, such as occurring between stratiform and convective rainfall (e.g., Waldvogel 1974; Tokay and Short 1996; Houze 1997; Steiner and Smith 1998; Cifelli et al. 2000; Maki et al. 2001; Rao et al. 2001; Uijlenhoet et al. 2003c). A continental squall line that passed over Goodwin Creek, a small research watershed (Alonso and Bingner 2000) in northern Mississippi, is used to exemplify significant differences in raindrop size distribution associated with the various phases of that type of storm. The 27 May 1997 storm system has been discussed in detail by Steiner et al. (1999) and Uijlenhoet et al. (2003c). Figure 5 shows the traces of reflectivity, rain rate, raindrop number density, mean drop size, and the standard deviation of drop sizes as derived from a Joss–Waldvogel (1967) disdrometer centered on the watershed. The thin vertical dotted lines delineate the three typical phases of squall lines: a leading convective line (C), trailing stratiform rainfall (S), and a transition zone (T) in between these two. Additional rain (L) fell later on this day (after 1230 UTC), though it was not directly associated with the squall line. The leading convective line exhibited rain

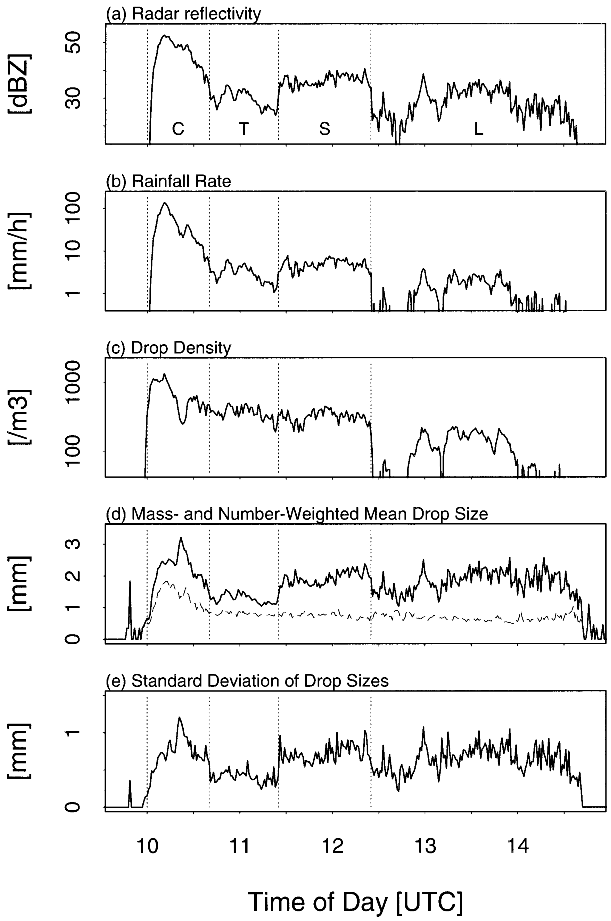

FIG. 5. Time series of (a) radar reflectivity factor [dBZ = 10 $\log_{10}$ (Z), where Z is given in mm^6 m^{-3}], (b) rainfall rate, (c) drop number density, (d) mass-weighted mean drop size (solid line), and (e) mass-weighted standard deviation of drop sizes based on the raindrop spectra recorded by a Joss–Waldvogel (1967) disdrometer during the passage of the 27 May 1997 squall line over the Goodwin Creek research watershed in northern MS. The number-weighted (or arithmetic) mean drop size is also shown for comparison by the dashed line in (d).

rates peaking around 140 mm h^{-1}, with raindrop spectra composed of several hundred drops per cubic meter, mass-weighted mean drop sizes ranging between 2 and 3 mm, and a standard deviation of drop sizes in excess of 0.5 mm. The transition zone following the convection was characterized by low intensity, a reduced drop concentration, and relatively narrow distributed smaller raindrops. The subsequent stratiform rainfall again displayed slightly increased intensity based on larger drops and larger spread of drop sizes, although the drop number density remained approximately the same as in the transition zone. Note the highly correlated fluctuations of the mass-weighted mean drop size $\overline{D}_m$ and standard deviation σ_D of drop sizes throughout the storm, clearly mirroring the three storm phases (Figs. 5d and 5e): the correlation between $\overline{D}_m$ and σ_D is approximately 0.92 for the convective leading line, 0.73 for the transition phase, and 0.78 for the stratiform rainfall.[11] The number-weighted (or arithmetic) mean drop size, in contrast, appears to distinguish only the convective leading line from the rest of the storm. Mean values for each storm phase, derived from the time-averaged raindrop spectra, are compiled in Table 1. The biggest raindrops occurred during the convective phase of the storm, which also exhibited the largest drop concentration. The transition zone marked a phase of decay, while the subsequent stratiform rainfall displayed a renewed growth of raindrops, albeit by a different mechanism as in the convective phase (e.g., Houze 1997). These significant changes in drop size distribution characteristics clearly show up in Z–R scatterplots (Fig. 6). Furthermore, they may be captured by sophisticated radars capable of determining the ratio of radar reflectivity measured under horizontal and vertical polarization (i.e., differential reflectivity), as pioneered by Seliga and Bringi (1976). Figure 7 highlights those changes in raindrop spectra within the parameter space spanned by the radar reflectivity factor and the differential reflectivity. Interestingly, the coefficient of drop variation remained remarkably constant during the entire storm (not shown). The average CV_D was approximately 34% for the convective and transition phases of the squall line, despite large differences in mean drop size (Table 1), and was only marginally larger (~40%) in stratiform rainfall. The drop size distributions of the squall line's stratiform rainfall and the rain that fell later on that day differed

[11] According to Eq. (11), this implies a rather constant distribution shape factor μ.

TABLE 1. Characteristic values of average raindrop spectra based parameters for the three distinct phases (C, T, and S) of the 27 May 1997 squall line system passing over Goodwin Creek and the subsequent rainfall (L).

	Distinct phases of rainfall			
Drop spectra–based parameters	C	T	S	L
Reflectivity (dBZ)	47.71	30.45	36.71	30.42
Rain rate (mm h^{-1})	41.6	3.1	5.0	1.1
Liquid water content (mm^3 m^{-3})	1679	181	228	52
Differential reflectivity (dB)	1.71	0.47	1.31	1.30
Raindrop number density (m^{-3})	686	375	329	97
Mass-weighted mean drop size (mm)	2.37	1.33	1.93	1.95
Standard deviation of drop sizes (mm)	0.82	0.45	0.76	0.78
Coefficient of drop variation (%)	34.6	33.8	39.4	40.0

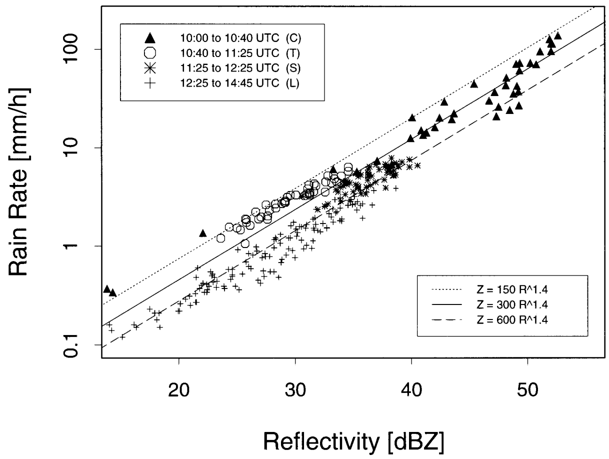

FIG. 6. Variability of the *Z–R* relationship between distinct phases of the 27 May 1997 squall line storm (Fig. 5), as reflected by the raindrop spectra data collected using a Joss–Waldvogel (1967) disdrometer in Goodwin Creek, northern MS. [Adapted from Steiner et al. (1999).]

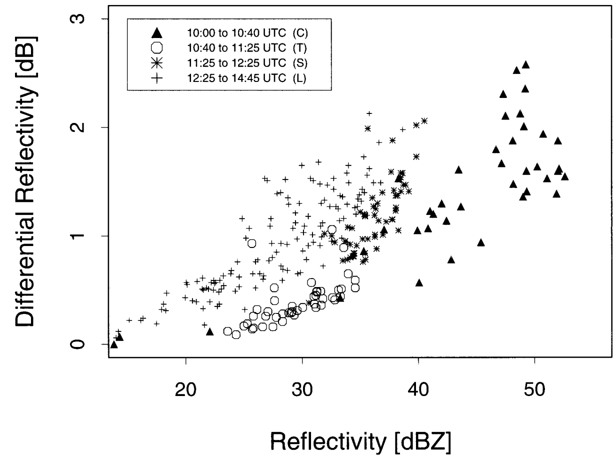

FIG. 7. Variability of the raindrop size distribution for the distinct phases of the 27 May 1997 squall line storm (Fig. 5), as reflected by the radar reflectivity factor Z and differential reflectivity Z_{DR}.

only in the raindrop number density, indicating similar microphysical growth processes, albeit likely under different conditions of moisture supply and mesoscale uplifting.

Rainfall that exhibits ''classical'' N_0 jumps cannot be described by a single physically meaningful *Z–R* relationship, as formulated in section 2b. Such a relationship between Z and R has to be of a statistical nature instead and should thus be expressed as $\langle Z\rangle = a\langle R\rangle^b$, where $\langle\ \rangle$ indicates expected values.[12] The N_0 jumps under an approximately constant rain rate are associated with changes in both mean drop size and number concentration, and these changes occur in opposite directions, as discussed earlier (Fig. 3). The variability of raindrop size distributions under N_0-jump conditions, therefore, is neither controlled purely by variations in drop size nor number concentration. Moreover, the ratio of $(\overline{D}_m)^{(1+\mu)}/N_T$ cannot remain constant or there would be no N_0 jump, as demonstrated by Eq. (20). In conclusion, N_0 jumps mark *discontinuities in rainfall* that are caused by a significant change in the microphysical precipitation production mechanisms.

Figure 8 shows how rainfall behaved during the different phases of the 27 May 1997 squall line in the parameter space spanned by $\overline{D}_m$ and N_T. The convective phase may have exhibited phases that were controlled by variations in drop size. Similarly, the transition phase may have been influenced by variations in drop size also. The stratiform rainfall, in contrast, behaved clearly differently, showing variations in both drop size and number density more like a mixed control.

Although a storm that exhibits N_0 jumps may not be characterized by a single, physically meaningful *Z–R* relationship, the rainfall in between such jumps may well exhibit a physical relation between Z and R. For example, rainfall may switch from one type of raindrop spectra fluctuation to another, as seen in Fig. 8. For simplicity assuming that both rainfall types may be characterized by *Z–R* relationship (16), albeit with a different constant ratio of $(\overline{D}_m)^{(1+\mu)}/N_T$, we are able to assess the effect of drastic changes in drop size distribution of the form $\tilde{N}_0 = \gamma N_0$ on the coefficients of the *Z–R* relationship. The exponent β is not affected; however, based on using Eqs. (16), (21), and (22), it can be shown that the multiplicative factor α undergoes significant modifications according to

$$\tilde{\alpha} = \alpha\left(\frac{1}{\gamma}\right)^{[(3-p)/(4+p+\mu)]}. \tag{23}$$

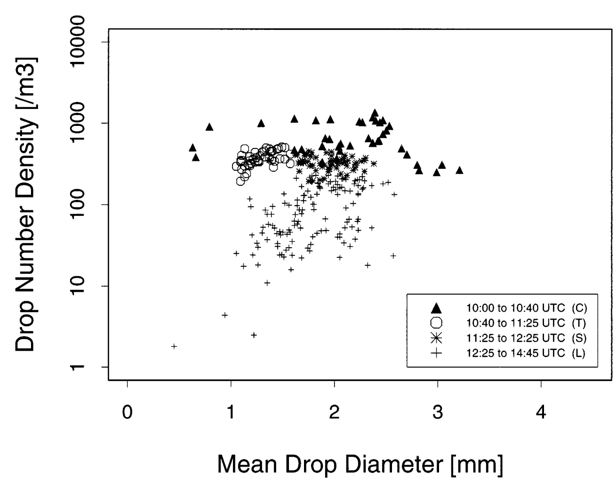

FIG. 8. Variability of the raindrop size distribution for the distinct phases of the 27 May 1997 squall line storm (Fig. 5), as reflected by the mass-weighted mean drop size $\overline{D}_m$ and drop number density N_T.

[12] Different symbols are used here for the multiplicative factor and exponent to distinguish them from the ones used in Eq. (1) and subsequent microphysical derivations.

For exponential drop size distributions ($\mu = 0$), a doubling of the intercept coefficient N_0 under the assumption that the rain rate remains constant, causes a reduction of the multiplicative factor α of the *Z–R* relation to 70% of its original value. And vice versa, a reduction of N_0 to half its original value will increase α by a factor of 1.4. These effects are somewhat smaller for gamma drop spectra with $\mu > 0$. For example, changes of approximately $\pm$15%–20% result for drop spectra with $\mu = 5$.

Besides significant changes in raindrop spectra within storms, there are also systematic differences among climatic precipitation regimes. For example, a recent study of Bringi et al. (2003) highlights that continental convective rainfall tends toward raindrop spectra with lower concentrations of larger-sized drops as compared to maritime convective rainfall. Also, Rao et al. (2001) point out significant differences in drop spectra, and thus *Z–R* relations, between northeastern (October–December; $\alpha = 155$ and $\beta = 1.39$) and southwestern (June–September; $\alpha = 407$ and $\beta = 1.32$) monsoon season rainfall observed at Gadanki in southeastern India.

4. Interpretation of *Z–R* relation coefficients

A myriad of *Z–R* relationships has been reported upon in the literature (e.g., Stout and Mueller 1968; Battan 1973; see Rosenfeld and Ulbrich 2003 for a recent review). One may wonder to what extent the coefficients of these relationships can be interpreted in microphysical terms. Answering this question involves an assessment of the type of rainfall and the effect of shortcomings of the underlying measurements or the technique of obtaining the relationship. Considering the associated uncertainties is key to a meaningful interpretation of the obtained results. The analyses presented here are making use of uncertainty values reported by Doelling et al. (1998), Steiner and Smith (2000), and Hagen and Yuter (2003). These studies document that the multiplicative factor α of the *Z–R* relation may be typically estimated with an uncertainty of approximately $\pm$30% and the raindrop size distribution shape factor μ with an uncertainty of $\pm$2. The latter is consistent with an uncertainty of the *Z–R* relation exponent β of $\pm$0.1. The raindrop number density N_T may be estimated within a factor of 2.

The subsequent evaluations are based on the assumptions that the appropriate *Z–R* relationship shall be known (more on that in section 4c), a power-law relationship between drop size and fall velocity according to Atlas and Ulbrich (1977) is valid, and typical values for the multiplicative factor $\alpha = 400$, number density of drops $N_T = 1000$, and distribution shape factor $\mu = 5$. This latter value is becoming more widely accepted (e.g., Wilson et al. 1997; Illingworth and Blackman 2002). We discuss both linear and power-law *Z–R* relations.

a. Linear Z–R relations

Rainfall based on monodisperse raindrop spectra exhibits a linear *Z–R* relation as represented by (18). The relationship between the multiplicative factor α and drop size D_s is visualized in Fig. 9a. The drop size monotonically increases with an increasing multiplicative factor of the *Z–R* relationship. For a multiplicative factor $\alpha = 400$, we estimate a drop size of $D_s = 1.57$ mm. Assuming an uncertainty of 30% for α (i.e., $280 < \alpha < 520$), the drop size estimate may be $1.34 < D_s < 1.75$ mm, representing an uncertainty of approximately 13%. This uncertainty of the estimated drop size will be somewhat larger for small values of α (Fig. 9a).

Steady and statistically homogeneous or equilibrium rainfall (number controlled) may be described by gamma drop spectra with an associated linear *Z–R* relation according to (14). The relationship between the mass-weighted mean drop size $\overline{D}_m$ and the multiplicative factor α is shown in Fig. 9b for various values of the raindrop distribution shape factor μ. Assuming gamma raindrop spectra with $\mu = 5$, we estimate a mean drop size of $\overline{D}_m = 1.37$ mm based on a multiplicative factor $\alpha = 400$. An uncertainty of 30% for α results in a mean drop size estimate of $1.17 < \overline{D}_m < 1.53$ mm—that is, an uncertainty of approximately 13%. Using Eq. (11), we estimate a corresponding mass-weighted standard deviation of drop diameters of $0.39 < \sigma_D < 0.51$ mm. However, not knowing the drop size distribution factor μ a priori will contribute additional uncertainty to the mean drop size estimate. Assuming $\mu \pm 2$, this additional uncertainty in mean drop size raises the overall drop size uncertainty to roughly 16%. The estimates of $\overline{D}_m = 1.18$ mm and $\sigma_D = 0.59$ mm based on exponential drop spectra ($\mu = 0$) fall within the above range of values derived for gamma raindrop size distributions.

List (1988) reported a linear $Z = 742R$ relationship for steady tropical rain with maximum rain rates of approximately 50 mm h^{-1} collected in Malaysia. Assuming a gamma drop size distribution with $\mu = 5$ (this is only a rough approximation to the data shown in List's Fig. 3), we estimate a mass-weighted mean drop size of $\overline{D}_m = 1.78$ mm ($\pm$16%), which appears close to the observed value (List's Fig. 10). The corresponding mass-weighted standard deviation of drop diameters, according to Eq. (11), is $\sigma_D = 0.59$ mm ($\pm$16%).

Uijlenhoet et al. (2003b) discuss two extreme events of quasi-equilibrium rainfall observed near Miami, Florida. Using a scaling-law formalism, they obtained $Z = 1540R^{1.06}$ for the 13 May 1958 storm (maximum rain rates reaching 500 mm h^{-1}) and $Z = 853R^{1.13}$ for the 21 June 1958 storm (maximum rain rates peaking around 300 mm h^{-1}), respectively. Based on gamma drop size distributions with $\mu = 2.63$ (13 May) and $\mu = 0.91$ (21 June), and ignoring the slight nonlinearity of these *Z–R* relationships for simplicity reasons, we estimate mean drop sizes of $\overline{D}_m = 2.33$ mm ($\pm$16%)

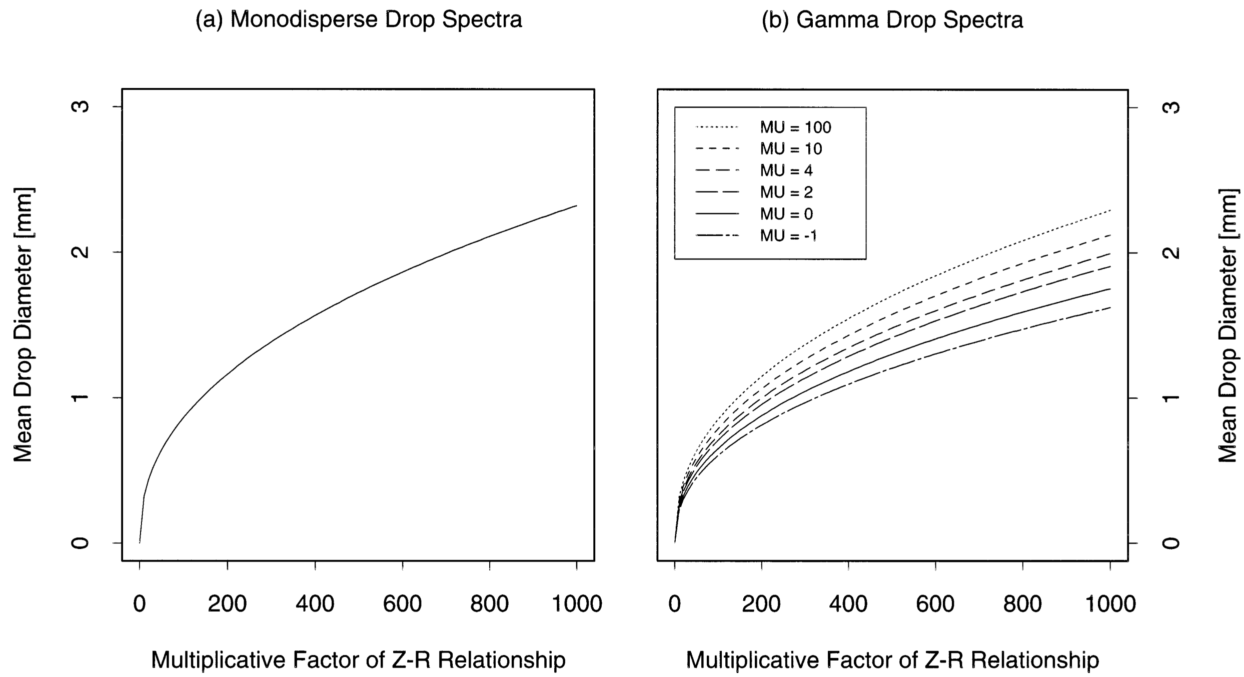

FIG. 9. Relationship between mass-weighted mean drop size $\overline{D}_m$ (D_s for monodisperse drop spectra) and multiplicative factor α for linear *Z–R* relations. The power-law relation between fall velocity and drop size according to Atlas and Ulbrich (1977) is used. (a) Case of monodisperse raindrop spectra based on Eq. (18). (b) Case of gamma raindrop spectra based on Eq. (14) for various values of the distribution shape factor μ. The exponential case is shown by the solid line, while the dashed lines indicate cases with $\mu \neq 0$. The dotted line representing the case of $\mu = 100$ closely mirrors the monodisperse relationship shown in (a).

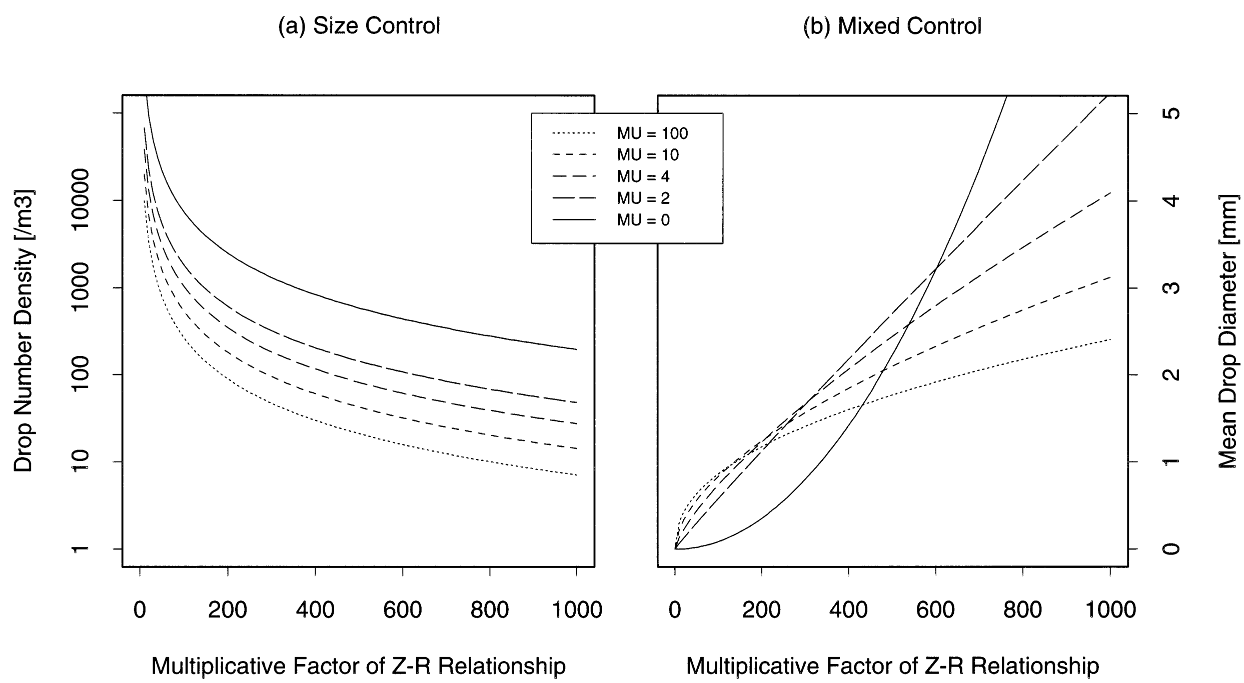

FIG. 10. (a) Relationship between total number of drops per unit volume of air N_T and multiplicative factor α for size-controlled power-law *Z–R* relations. (b) Relationship between mass-weighted mean drop size $\overline{D}_m$ and multiplicative factor α for mixed-controlled power-law *Z–R* relations. Results are shown for gamma drop spectra with various distribution shape factors $\mu \geq 0$ and a power-law relation between fall velocity and drop size according to Atlas and Ulbrich (1977). In addition, (b) is based on the assumption of a total number of drops per unit volume of air $N_T = 1000$.

TABLE 2. Estimated mass-weighted mean drop size $\overline{D}_m$ (mm) and standard deviation of drop size σ_D (mm) based on the multiplicative coefficient α, drop size distribution shape factor μ, and total number of drops N_T according to (16) and (11), respectively.

		$\overline{D}_m$			σ_D		
N_T	α	$\mu = 3$ ($\beta = 1.18$)	$\mu = 5$ ($\beta = 1.24$)	$\mu = 7$ ($\beta = 1.30$)	$\mu = 3$ ($\beta = 1.18$)	$\mu = 5$ ($\beta = 1.24$)	$\mu = 7$ ($\beta = 1.30$)
500	280	1.34	1.40	1.42	0.50	0.47	0.43
	400	1.79	1.79	1.77	0.68	0.60	0.53
	520	2.22	2.15	2.09	0.84	0.72	0.63
1000	280	1.59	1.57	1.54	0.60	0.52	0.47
	400	2.13	2.01	1.93	0.81	0.67	0.58
	520	2.64	2.41	2.27	1.00	0.80	0.69
2000	280	1.89	1.77	1.68	0.71	0.59	0.51
	400	2.53	2.26	2.10	0.96	0.75	0.63
	520	3.14	2.71	2.48	1.19	0.90	0.75

and $\overline{D}_m = 1.71$ mm ($\pm16\%$), respectively, and corresponding drop size standard deviations $\sigma_D = 0.90$ mm ($\pm16\%$) and $\sigma_D = 0.77$ mm ($\pm16\%$), as observed by Uijlenhoet et al. (2003b).

Atlas and Williams (2003) elaborate on equilibrium conditions observed in the rainfall of a continental tropical convective storm (with maximum rain rate of about 100 mm h^{-1}) in the Rondonia region of Brazil, finding a near-linear $Z = 1260R^{1.04}$ relation applicable to the storm's growth phase, where warm rain processes are dominant. Assuming gamma drop spectra with $\mu = 5$ yields a mass-weighted mean drop size of $\overline{D}_m = 2.24$ mm ($\pm16\%$) and standard deviation $\sigma_D = 0.75$ mm ($\pm16\%$), which is close to their observations based on polarization radar and ground-based drop spectra data. Assuming exponential ($\mu = 0$) or monodisperse ($\mu = \infty$) drop spectra instead, we obtain mean drop size estimates of 1.94 and 2.53 mm, respectively, falling again within the uncertainty range of the previous result.

b. *Power-law Z–R relations*

For rainfall situations, where all variability in intensity is controlled by variations in drop size, Fig. 10a displays how the number density of drops N_T in a volume of air may be estimated based on the multiplicative coefficient α of a power-law *Z–R* relation as given by (15). Based on a gamma drop size distribution with a typical value of $\mu = 5$, a drop concentration $N_T = 98$ m^{-3} is obtained for a coefficient $\alpha = 400$. An uncertainty of $\alpha \pm 30\%$ results in $65 < N_T < 172$ m^{-3} and considering an additional uncertainty of the distribution shape factor of $\mu \pm 2$ yields $51 < N_T < 258$ m^{-3}, which represents an overall uncertainty of at least a factor of 2.

A more typical case of a power-law *Z–R* relation is given by (16), describing rainfall where the variability in intensity is the result of variations in drop size and number concentration, and maybe the drop size distribution shape as well. Figure 10b shows how the mean drop size $\overline{D}_m$ may be estimated from the coefficient α, although the underlying relationship depends both on the distribution shape factor μ and the total number of drops N_T. The exponent β provides a measure of μ (Fig. 1), which in turn may be used for retrieval of N_0 from α. A mean drop size may be obtained only based on further assumptions. For example, assuming again a gamma drop size distribution with a typical value of $\mu = 5$ ($\beta = 1.24$), a power-law relationship between drop size and fall velocity as before, and a total number of drops $N_T = 1000$ m^{-3}, we estimate a mean drop size $\overline{D}_m = 2.01$ mm based on $\alpha = 400$. The corresponding standard deviation of drop sizes is $\sigma_D = 0.67$ mm according to Eq. (11). A $\pm5\%$ uncertainty of the exponent β (i.e., $1.18 < \beta < 1.30$) translates into an uncertainty of $\mu \pm 2$ for the distribution shape factor. Moreover, assuming that we know the multiplicative coefficient $\alpha \pm 30\%$, and the total number of drops N_T to within a factor of 2, yields a mean drop size in the range of $1.34 < \overline{D}_m < 3.14$ mm and drop size standard deviation $0.43 < \sigma_D < 1.19$ mm (Table 2). Thus, reasonable assumptions about the uncertainty of the various parameters result in approximately 45% uncertainty for the estimated mean drop size and 60% uncertainty for the standard deviation of drop sizes.

c. *Discussion*

A microphysical interpretation of the *Z–R* relationship coefficients is difficult, because typically one seeks to *estimate more parameters than independent observations available*. In addition, the available observations are likely burdened by uncertainties associated with the sampling of information as well as how this information gets processed. The discussions in section 4 so far have essentially ignored measurement uncertainties. Despite that, substantial uncertainties have been found for the estimation of microphysical parameters from the *Z–R* relationship coefficients, particularly for the case where the variability of the raindrop size distribution is governed by a mix of variations in drop size and number concentration. Moreover, assessing the proper microphysical condition based on the available rainfall information is nontrivial, which leads to a significant additional uncertainty in the choice of the *Z–R* relationship, as discussed next.

Assuming that relevant information is available, the ''steadiness'' of rain may be evaluated, which will provide guidance with regard to the anticipated type of rainfall and thus picking an appropriate type of *Z–R* relationship. According to Jameson and Kostinski (2002b), steadiness implies statistical stationarity and lack of correlation between raindrops in neighboring volumes. This is the same as saying that the drops have to be distributed according to a Poisson process at all scales. Jameson and Kostinski (2002b) argue that one may simply compare the variance of the observed rain rate *R* with that anticipated based on Poissonian rain: whenever the variance is much larger than the expected mean, the rain is not likely to be steady. In addition, if raindrop spectra information is available (either measured directly or retrieved from radar observations), the temporal variation of rainfall may be evaluated within the parameter space spanned by the mean drop size and drop number density, such as shown in Figs. 3, 4, and 8.

Polarimetric radar provides excellent means to assess statistically homogenous or equilibrium rainfall conditions. In particular, combined evaluation of the radar reflectivity factor and the differential reflectivity fluctuations within the parameter space spanned by these two parameters is illuminating: for statistically homogenous or equilibrium rainfall, the mean raindrop size should remain constant, while intensity variations are accommodated through fluctuations in drop number density. The differential reflectivity, which is a measure of the mean raindrop size (e.g., Seliga and Bringi 1976), therefore, has to remain approximately constant independent of variations of the radar reflectivity factor.

Another, more qualitative approach might be to evaluate radar reflectivity and Doppler spectrum data to assess the horizontal and vertical structure of a precipitation system and its variability with time (e.g., Atlas and Williams 2003). This may provide a rough classification into shallow layered clouds (likely exhibiting narrow to monodisperse drop spectra), intense yet very persistent clouds (number-controlled drop spectra), and others (mixed- or size-controlled drop spectra).

A major concern for a microphysical interpretation of the *Z–R* relationship coefficients is based on the practical limitations of rainfall sampling by raindrop spectrometers, rain gauges, and radar, or any combination thereof. More often than not, the obtained *Z–R* relationships are thus of a statistical rather than a physical nature (Jameson and Kostinski 2001). In addition, instrumental sampling limitations in space and time may lead to biased relationships (Cornford 1967; Joss and Waldvogel 1969; Gertzman and Atlas 1977; Joss and Gori 1978; Wong and Chidambaram 1985; Chandrasekar and Bringi 1987; Krajewski and Smith 1991; Smith et al. 1993; Ulbrich 1994; Sheppard and Joe 1994; Ciach and Krajewski 1999; Campos and Zawadzki 2000; Ciach et al. 2000; Steiner and Smith 2000; Jameson and Kostinski 2001, 2002a) and a mixture of different rainfall types will result in averaged *Z–R* relation coefficients that no longer reveal a meaningful microphysical picture (e.g., Waldvogel 1975; Jameson and Kostinski 2002a). Fitted power-law relationships between rainfall and radar parameters, introduced by Marshall et al. (1947) and used heavily thereafter, fall in this latter category. Measurement uncertainties, such as sensor accuracy and sensitivity or inhomogeneous filling of the sampling volume including effects of brightband and hail contamination (e.g., Wilson and Brandes 1979; Austin 1987; Steiner et al. 1999), and a sensitivity to the way the observed information is processed (e.g., Doelling et al. 1998; Ciach and Krajewski 1999; Campos and Zawadzki 2000; Ciach et al. 2000; Steiner and Smith 2000; Tokay et al. 2001; Meagher and Haddad 2002; Salles and Creutin 2003; Hagen and Yuter 2003), significantly worsen the achievable microphysical retrieval accuracy.

Based on the above discussions, it is clear that we may not easily be able to obtain an accurate depiction of the microphysical properties of rainfall. Nonetheless, for a given observational setting, it may still be very insightful to *monitor relative changes* of the *Z–R* relationship coefficients, which will provide feedback regarding significant changes in the microphysical and dynamic conditions. For example, systematic seasonal changes of the coefficients may be detected (e.g., Rao et al. 2001; Tokay et al. 2002), which in turn could lead to improved operational rainfall estimation procedures.

5. Conclusions

The relationship between the radar reflectivity *Z* and rainfall rate *R* was illuminated from a microphysical perspective. The *Z–R* relationship was examined using exponential, gamma, and monodisperse raindrop size distributions. Three special modes that a power-law *Z–R* relation of the form $Z = \alpha R^{\beta}$ may attain were identified. These three distinct modes are associated with conditions where the variability of the raindrop size distribution is controlled by either variations in number concentration N_T, mean drop size $\overline{D}_m$, or a combination thereof in which the ratio $(\overline{D}_m)^{(1+\mu)}/N_T$ is constant. For exponential and gamma drop spectra this ratio is proportional to the intercept coefficient N_0^{-1}. The variability of the raindrop size distribution is bounded by either size-controlled or number-controlled conditions, with conditions of a mixed control embedded in between those extremes.

The inverse problem of a microphysical interpretation of obtained *Z–R* relationship coefficients was evaluated. Significant uncertainties were found that are related to the fact that typically more (microphysical) parameters have to be estimated than independent observations available. These uncertainties may be of the order of 15%–20% for estimation of the mass-weighted mean drop size based on the coefficients of linear *Z–R* relations ($\beta = 1$) associated with steady and statistically homogeneous or equilibrium rainfall (i.e., number controlled), but likely at least twice as much for more typical rainfall situations characterized by power-law *Z–R* relationships with an exponent $\beta > 1$ (size or mixed controlled).

Uncertainties associated with the measurement of rainfall information and the data processing may substantially burden the retrieval accuracy of the microphysical properties of rainfall based on empirically derived *Z–R* relationships. In addition, identification of the appropriate *Z–R* relation mode may be difficult based on the accessible rainfall information, or potentially be masked beyond recognition due to practical constraints of the observations or measurement inaccuracies. Information provided by polarimetric observations (e.g., Ryzhkov et al. 1997; Illingworth et al. 2000; Bringi et al. 2003) may go a long way in reducing some of this uncertainty. However, given a proper consideration of the associated uncertainties and limitations, valuable microphysical insight may still be obtained, particularly as a result of long-term monitoring of rainfall for fixed observational settings.

Provided that there are three special modes for the *Z–R* relationship—their existence in nature remains to be thoroughly evaluated—one wonders about the microphysical and dynamic conditions that may produce these modes, and how the transition from one mode to another might be facilitated. Such transitions are likely not smooth and may be marked by discontinuities in the raindrop size distributions (N_0 jumps), reflecting significant changes in the underlying microphysical growth processes. Can the number-controlled case be linked to efficient growth of precipitation particles in tropical convective clouds, the size-controlled case to the onset and dissipating stages of convection, and the rest to a mixed-control case? Obviously, answering such questions requires additional research that is beyond the scope of this paper.

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REFERENCES

Alonso, C. V., and R. L. Bingner, 2000: Goodwin Creek experimental watershed: A unique field laboratory. *ASCE J. Hydraul. Eng.,* **126,** 174–177.

Atlas, D., 1953: Optical extinction by rainfall. *J. Meteor.,* **10,** 486–488.

——, and A. C. Chmela, 1957: Physical-synoptic variations of raindrop size parameters. *Proc. Sixth Weather Radar Conf.,* Cambridge, MA, Amer. Meteor. Soc., 21–29.

——, and C. W. Ulbrich, 1977: Path- and area-integrated rainfall measurements by microwave attenuation in the 1–3-cm band. *J. Appl. Meteor.,* **16,** 1322–1331.

——, and ——, 2000: An observationally based conceptual model of warm oceanic convective rain in the Tropics. *J. Appl. Meteor.,* **39,** 2165–2181.

——, and C. R. Williams, 2003: The anatomy of a continental tropical convective storm. *J. Atmos. Sci.,* **60,** 3–15.

——, R. C. Srivastava, and R. S. Sekhon, 1973: Doppler radar characteristics of precipitation at vertical incidence. *Rev. Geophys. Space Phys.,* **11,** 1–35.

——, C. W. Ulbrich, F. D. Marks, E. Amitai, and C. R. Williams, 1999: Systematic variation of drop size and radar–rainfall relations. *J. Geophys. Res.,* **104D,** 6155–6169.

Austin, P. M., 1987: Relation between measured radar reflectivity and surface rainfall. *Mon. Wea. Rev.,* **115,** 1053–1070.

Battan, L. J., 1973: *Radar Observations of the Atmosphere.* University of Chicago Press, 324 pp.

——, 1976: Vertical air motions and the *Z–R* relation. *J. Appl. Meteor.,* **15,** 1120–1121.

Beard, K. V., 1976: Terminal velocity of cloud and precipitation drops aloft. *J. Atmos. Sci.,* **33,** 851–864.

——, 1985: Simple altitude adjustments to raindrop velocities for Doppler radar analysis. *J. Atmos. Oceanic Technol.,* **2,** 468–471.

Berry, E. X., and M. R. Pranger, 1974: Equations for calculating the terminal velocities of water drops. *J. Appl. Meteor.,* **13,** 108–113.

Best, A. C., 1950a: Empirical formulae for the terminal velocity of waterdrops falling through the atmosphere. *Quart. J. Roy. Meteor. Soc.,* **76,** 302–311.

——, 1950b: The size distribution of raindrops. *Quart. J. Roy. Meteor. Soc.,* **76,** 16–36.

Blanchard, D. C., and A. T. Spencer, 1970: Experiments on the generation of raindrop size distributions by drop breakup. *J. Atmos. Sci.,* **27,** 101–108.

Bradley, S. G., and C. D. Stow, 1974: The measurement of charge and size of raindrops. Part II: Results and analysis at ground level. *J. Appl. Meteor.,* **13,** 131–147.

Brazier-Smith, P. R., 1992: On the shape and fall velocities of raindrops. *Quart. J. Roy. Meteor. Soc.,* **118,** 749–766.

Bringi, V. N., V. Chandrasekar, J. Hubbert, E. Gorgucci, W. L. Randeu, and M. Schoenhuber, 2003: Raindrop size distribution in different climatic regimes from disdrometer and dual-polarized radar analysis. *J. Atmos. Sci.,* **60,** 354–365.

Brown, P. S., and S. N. Whittlesey, 1992: Multiple equilibrium solutions in Bleck-type models of drop coalescence and breakup. *J. Atmos. Sci.,* **49,** 2319–2324.

Campos, E., and I. Zawadzki, 2000: Instrumental uncertainties in the *Z–R* relations. *J. Appl. Meteor.,* **39,** 1088–1102.

Carbone, R. E., 1985: Comments on ''Functional fits to some observed drop size distributions and parameterizations of rain.'' *J. Atmos. Sci.,* **42,** 1346–1348.

——, and L. D. Nelson, 1978: The evolution of raindrop spectra in warm-based convective storms as observed and numerically modeled. *J. Atmos. Sci.,* **35,** 2302–2314.

Cataneo, R., and G. E. Stout, 1968: Raindrop-size distributions in humid continental climates, and associated rainfall rate–radar reflectivity relationships. *J. Appl. Meteor.,* **7,** 901–907.

Chandrasekar, V., and V. N. Bringi, 1987: Simulation of radar reflectivity and surface measurements of rainfall. *J. Atmos. Oceanic Technol.,* **4,** 464–478.

Ciach, G. J., and W. F. Krajewski, 1999: Radar–rain gauge comparisons under observational uncertainties. *J. Appl. Meteor.,* **38,** 1519–1525.

——, M. L. Morrissey, and W. F. Krajewski, 2000: Conditional bias in radar rainfall estimation. *J. Appl. Meteor.,* **39,** 1941–1946.

Cifelli, R., C. R. Williams, D. K. Rajopadhyaya, S. K. Avery, K. S. Gage, and P. T. May, 2000: Drop-size distribution characteristics in tropical mesoscale convective systems. *J. Appl. Meteor.,* **39,** 760–777.

Clark, T. L., 1974: A study in cloud phase parameterization using the gamma distribution. *J. Atmos. Sci.,* **31,** 142–155.

Cornford, G. S., 1967: Sampling errors in measurements of raindrop and cloud-droplet concentration. *Meteor. Mag.,* **96,** 271–282.

Doelling, I. G., J. Joss, and J. Riedl, 1998: Systematic variations of *Z–R* relationships from drop size distributions measured in northern Germany during seven years. *Atmos. Res.,* **47–48,** 635–649.

Dotzek, N., and K. D. Beheng, 2001: The influence of deep convective motions on the variability of *Z–R* relations. *Atmos. Res.,* **59–60,** 15–39.

——, and T. Fehr, 2003: Relationship between precipitation rates at the ground and aloft—A modeling study. *J. Appl. Meteor.,* **42,** 1285–1301.

Feingold, G., and Z. Levin, 1986: The lognormal fit to raindrop spectra from frontal convective clouds in Israel. *J. Climate Appl. Meteor.,* **25,** 1346–1363.

Foote, G. B., and P. S. duToit, 1969: Terminal velocity of raindrops aloft. *J. Appl. Meteor.,* **8,** 249–253.

Fujiwara, M., 1965: Raindrop-size distribution from individual storms. *J. Atmos. Sci.,* **22,** 585–591.

Gertzman, H. S., and D. Atlas, 1977: Sampling errors in the measurement of rain and hail parameters. *J. Geophys. Res.,* **82,** 4955–4966.

Gossard, E. E., R. G. Strauch, D. C. Welsh, and S. Y. Matrosov, 1992: Cloud layers, particle identification, and rain-rate profiles from ZRV_f measurements by clear-air Doppler radars. *J. Atmos. Oceanic Technol.,* **9,** 108–119.

Gunn, K. L. S., and J. S. Marshall, 1955: The effect of wind shear on falling precipitation. *J. Meteor.,* **12,** 339–349.

Gunn, R., and G. D. Kinzer, 1949: The terminal velocity of fall for water droplets in stagnant air. *J. Meteor.,* **6,** 243–248.

Haddad, Z. S., S. L. Durden, and E. Im, 1996: Parameterizing the raindrop size distribution. *J. Appl. Meteor.,* **35,** 3–13.

Hagen, M., and S. E. Yuter, 2003: Relations between radar reflectivity, liquid-water content, and rainfall rate during the MAP SOP. *Quart. J. Roy. Meteor. Soc.,* **129,** 477–493.

Hauser, D., P. Amayenc, B. Nutten, and P. Waldteufel, 1984: A new optical instrument for simultaneous measurement of raindrop diameter and fall speed distributions. *J. Atmos. Oceanic Technol.,* **1,** 256–269.

Hodson, M. C., 1986: Raindrop size distribution. *J. Climate Appl. Meteor.,* **25,** 1070–1074.

Hosking, J. G., and C. D. Stow, 1991: Ground-based measurements of raindrop fallspeeds. *J. Atmos. Oceanic Technol.,* **8,** 137–147.

Houze, R. A., Jr., 1997: Stratiform precipitation in regions of convection: A meteorological paradox? *Bull. Amer. Meteor. Soc.,* **78,** 2179–2196.

Hu, Z., and R. C. Srivastava, 1995: Evolution of raindrop size distribution by coalescence, breakup, and evaporation: Theory and observations. *J. Atmos. Sci.,* **52,** 1761–1783.

Hudson, N. W., 1963: Raindrop size distribution in high intensity storms. *Rhod. J. Agric. Res.,* **1,** 6–11.

Illingworth, A. J., and T. M. Blackman, 2002: The need to represent raindrop size spectra as normalized gamma distributions for the interpretation of polarization radar observations. *J. Appl. Meteor.,* **41,** 286–297.

——, ——, and J. W. F. Goddard, 2000: Improved rainfall estimates in convective storms using polarization diversity radar. *Hydrol. Earth Syst. Sci.,* **4,** 555–563.

Jameson, A. R., and A. B. Kostinski, 2001: Reconsideration of the physical and empirical origins of *Z–R* relations in radar meteorology. *Quart. J. Roy. Meteor. Soc.,* **127,** 517–538.

——, and ——, 2002a: Spurious power-law relations among rainfall and radar parameters. *Quart. J. Roy. Meteor. Soc.,* **128,** 2045–2058.

——, and ——, 2002b: When is rain steady? *J. Appl. Meteor.,* **41,** 83–90.

Jiang, H., M. Sano, and M. Sekine, 1996: Radar reflectivity and rainfall rate relation from Weibull raindrop-size distribution. *IEICE Trans. Commun.,* **E79-B,** 797–800.

Jones, D. M. A., 1992: Raindrop spectra at the ground. *J. Appl. Meteor.,* **31,** 1219–1225.

Joss, J., and A. Waldvogel, 1967: Ein Spektrograph für Niederschlagstropfen mit automatischer Auswertung. *Pure Appl. Geophys.,* **68,** 240–246.

——, and ——, 1969: Raindrop size distribution and sampling size errors. *J. Atmos. Sci.,* **26,** 566–569.

——, and E. G. Gori, 1978: Shapes of raindrop size distributions. *J. Appl. Meteor.,* **17,** 1054–1061.

Kostinski, A. B., and A. R. Jameson, 1999: Fluctuation properties of precipitation. Part III: On the ubiquity and emergence of the exponential drop size spectra. *J. Atmos. Sci.,* **56,** 111–121.

Kozu, T., and K. Nakamura, 1991: Rainfall parameter estimation from dual-radar measurements combining reflectivity profile and path-integrated attenuation. *J. Atmos. Oceanic Technol.,* **8,** 259–271.

Krajewski, W. F., and J. A. Smith, 1991: On the estimation of climatological *Z–R* relationships. *J. Appl. Meteor.,* **30,** 1436–1445.

Li, X., and R. C. Srivastava, 2001: An analytical solution for raindrop evaporation and its application to radar rainfall measurements. *J. Appl. Meteor.,* **40,** 1607–1616.

List, R., 1988: A linear radar reflectivity–rainrate relationship for steady tropical rain. *J. Atmos. Sci.,* **45,** 3564–3572.

——, N. R. Donaldson, and R. E. Stewart, 1987: Temporal evolution of drop spectra to collisional equilibrium in steady and pulsating rain. *J. Atmos. Sci.,* **44,** 362–372.

Löffler-Mang, M., and J. Joss, 2000: An optical disdrometer for measuring size and velocity of hydrometeors. *J. Atmos. Oceanic Technol.,* **17,** 130–139.

Maki, M., T. D. Keenan, Y. Sasaki, and K. Nakamura, 2001: Characteristics of the raindrop size distribution in tropical continental squall lines observed in Darwin, Australia. *J. Appl. Meteor.,* **40,** 1393–1412.

Markowitz, A. H., 1976: Raindrop size distribution expressions. *J. Appl. Meteor.,* **15,** 1029–1031.

Marshall, J. S., 1969: Power-law relations in radar meteorology. *J. Appl. Meteor.,* **8,** 171–172.

——, and W. McK. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.,* **5,** 165–166.

——, R. C. Langille, and W. McK. Palmer, 1947: Measurement of rainfall by radar. *J. Meteor.,* **4,** 186–192.

Meagher, J. P., and Z. S. Haddad, 2002: Principal-component analysis for raindrops and its application to the remote sensing of rain. *Quart. J. Roy. Meteor. Soc.,* **128,** 559–571.

Mueller, E. A., 1962: Raindrop size distributions at Miami, Florida. Research Rep. 9B, Illinois State Water Survey Meteorological Laboratory, University of Illinois, 281 pp.

Ohtake, T., 1970: Factors affecting the size distribution of raindrops and snowflakes. *J. Atmos. Sci.,* **27,** 804–813.

Pasqualucci, F., 1982: The variation of drop-size distribution in convective storms: A comparison between theory and radar measurements. *Geophys. Res. Lett.,* **9,** 839–841.

Rao, T. N., D. N. Rao, K. Mohan, and S. Raghavan, 2001: Classification of tropical precipitating systems and associated *Z–R* relationships. *J. Geophys. Res.,* **106D,** 17 699–17 711.

Rinehart, R. E., 1997: *Radar for Meteorologists.* 3d ed. Rinehart Publications, 428 pp.

Rogers, R. R., I. I. Zawadzki, and E. E. Gossard, 1991: Variation with altitude of the drop-size distribution in steady light rain. *Quart. J. Roy. Meteor. Soc.,* **117,** 1341–1369.

Rosenfeld, D., and C. W. Ulbrich, 2003: Cloud microphysical properties, processes, and rainfall estimation opportunities. *Radar and Atmospheric Science: A Collection of Essays in Honor of David Atlas, Meteor. Monogr.*, No. 52, Amer. Meteor. Soc., 237–258.

Russchenberg, H. W. J., 1993: Doppler polarimetric radar measurements of the gamma drop size distribution of rain. *J. Appl. Meteor.,* **32,** 1815–1825.

Ryzhkov, A., D. Zrnić, and D. Atlas, 1997: Polarimetrically tuned $R(Z)$ relations and comparison of radar rainfall methods. *J. Appl. Meteor.,* **36,** 340–349.

Salles, C., and J.-D. Creutin, 2003: Instrumental uncertainties in Z–R relationships and raindrop fall velocities. *J. Appl. Meteor.,* **42,** 279–290.

Sauvageot, H., and J.-P. Lacaux, 1995: The shape of averaged drop size distributions. *J. Atmos. Sci.,* **52,** 1070–1083.

Sekhon, R. S., and R. C. Srivastava, 1971: Doppler radar observations of drop-size distributions in a thunderstorm. *J. Atmos. Sci.,* **28,** 983–994.

Seliga, T. A., and V. N. Bringi, 1976: Potential use of radar differential reflectivity measurements at orthogonal polarizations for measuring precipitation. *J. Appl. Meteor.,* **15,** 69–76.

Sempere Torres, D., J. M. Porrà, and J.-D. Creutin, 1994: A general formulation for raindrop size distribution. *J. Appl. Meteor.,* **33,** 1494–1502.

——, ——, and ——, 1998: Experimental evidence of a general description for raindrop size distribution properties. *J. Geophys. Res.,* **103D,** 1785–1797.

Sheppard, B. E., and P. I. Joe, 1994: Comparison of raindrop size distribution measurements by a Joss–Waldvogel disdrometer, a PMS 2DG spectrometer, and a POSS Doppler radar. *J. Atmos. Oceanic Technol.,* **11,** 874–887.

Smith, J. A., 1993: Marked point process models of raindrop-size distributions. *J. Appl. Meteor.,* **32,** 284–296.

——, and W. F. Krajewski, 1993: A modeling study of rainfall rate–reflectivity relationships. *Water Resour. Res.,* **29,** 2505–2514.

Smith, P. L., Z. Liu, and J. Joss, 1993: A study of sampling-variability effects in raindrop size observations. *J. Appl. Meteor.,* **32,** 1259–1269.

Srivastava, R. C., 1971: Size distribution of raindrops generated by their breakup and coalescence. *J. Atmos. Sci.,* **28,** 410–415.

Steiner, M., and J. A. Smith, 1998: Convective versus stratiform rainfall: An ice-microphysical and kinematic conceptual model. *Atmos. Res.,* **47–48,** 317–326.

——, and ——, 2000: Reflectivity, rain rate, and kinetic energy flux relationships based on raindrop spectra. *J. Appl. Meteor.,* **39,** 1923–1940.

——, ——, S. J. Burges, C. V. Alonso, and R. W. Darden, 1999: Effect of bias adjustment and rain gauge data quality control on radar rainfall estimation. *Water Resour. Res.,* **35,** 2487–2503.

Stout, G. E., and E. A. Mueller, 1968: Survey of relationships between rainfall rate and radar reflectivity in the measurement of precipitation. *J. Appl. Meteor.,* **7,** 465–474.

Testud, J., S. Oury, R. A. Black, P. Amayenc, and X. Dou, 2001: The concept of ''normalized'' distributions to describe raindrop spectra: A tool for cloud physics and cloud remote sensing. *J. Appl. Meteor.,* **40,** 1118–1140.

Tokay, A., and D. A. Short, 1996: Evidence from tropical raindrop spectra of the origin of rain from stratiform versus convective clouds. *J. Appl. Meteor.,* **35,** 355–371.

——, A. Kruger, and W. F. Krajewski, 2001: Comparison of drop size distribution measurements by impact and optical disdrometers. *J. Appl. Meteor.,* **40,** 2083–2097.

——, ——, ——, P. A. Kucera, and A. J. Pereira Filho, 2002: Measurements of drop size distribution in the southwestern Amazon basin. *J. Geophys. Res.,* **107D,** 8052, doi:10.1029/2001JD000355.

Twomey, S., 1953: On the measurement of precipitation intensity by radar. *J. Meteor.,* **10,** 66–67.

Uijlenhoet, R., 2001: Raindrop size distributions and radar reflectivity–rain rate relationships for radar hydrology. *Hydrol. Earth Syst. Sci.,* **5,** 615–627.

——, and J. N. M. Stricker, 1999: A consistent rainfall parameterization based on the exponential raindrop size distribution. *J. Hydrol.,* **218,** 101–127.

——, J. A. Smith, and M. Steiner, 2003b: The microphysical structure of extreme precipitation as inferred from ground-based raindrop spectra. *J. Atmos. Sci.,* **60,** 1220–1238.

——, M. Steiner, and J. A. Smith, 2003c: Variability of raindrop size distributions in a squall line and implications for radar rainfall estimation. *J. Hydrometeor.,* **4,** 43–61.

Ulbrich, C. W., 1983: Natural variations in the analytical form of the raindrop size distribution. *J. Climate Appl. Meteor.,* **22,** 1764–1775.

——, 1985: The effects of drop size distribution truncation on rainfall integral parameters and empirical relations. *J. Climate Appl. Meteor.,* **24,** 580–590.

——, 1992: Effects of drop-size-distribution truncation on computer-simulations of dual-measurement radar methods. *J. Appl. Meteor.,* **31,** 689–699.

——, 1994: Corrections to empirical relations derived from rainfall disdrometer data for effects due to drop size distribution truncation. *Atmos. Res.,* **34,** 207–215.

——, and D. Atlas, 1978: The rain parameter diagram: Methods and applications. *J. Geophys. Res.,* **83C,** 1319–1325.

——, and ——, 1998: Rainfall microphysics and radar properties: Analysis methods for drop size spectra. *J. Appl. Meteor.,* **37,** 912–923.

Valdez, M. P., and K. C. Young, 1985: Number fluxes in equilibrium raindrop populations—A Markov-chain analysis. *J. Atmos. Sci.,* **42,** 1024–1036.

Wacker, U., 1995: Competition of precipitation particles in a model with parameterized cloud microphysics. *J. Atmos. Sci.,* **52,** 2577–2589.

Waldvogel, A., 1974: The N_0 jump in raindrop spectra. *J. Atmos. Sci.,* **31,** 1067–1078.

——, 1975: Tropfenspektren, Niederschlagstyp and Z–R Beziehungen. *Meteor. Rundsch.,* **28,** 33–36.

Weibull, W., 1951: A statistical distribution function of wide applicability. *J. Appl. Mech.—Trans. ASME,* **18,** 293–297.

Willis, P. T., 1984: Functional fits to some observed drop size distributions and parameterization of rain. *J. Atmos. Sci.,* **41,** 1648–1661.

——, 1985: Reply. *J. Atmos. Sci.,* **42,** 1349–1350.

——, and P. Tattleman, 1989: Drop-size distributions associated with intense rainfall. *J. Appl. Meteor.,* **28,** 3–15.

Wilson, D. R., A. J. Illingworth, and T. M. Blackman, 1997: Differential Doppler velocity: A radar parameter for characterizing hydrometeor size distributions. *J. Appl. Meteor.,* **36,** 649–663.

Wilson, J. W., and E. A. Brandes, 1979: Radar measurement of rainfall—A summary. *Bull. Amer. Meteor. Soc.,* **60,** 1048–1058.

Wong, R. K. W., and N. Chidambaram, 1985: Gamma size distribution and stochastic sampling errors. *J. Climate Appl. Meteor.,* **24,** 568–579.

Zawadzki, I., and M. de Agostinho Antonio, 1988: Equilibrium raindrop size distributions in tropical rain. *J. Atmos. Sci.,* **45,** 3452–3459.