A 'Big Leaf, Big Fruit, Big Substrate' Model for Experiments on Receding Horizon Optimal Control of Nutrient Supply to Greenhouse Tomato

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Abstract

A dynamic model was set up to describe the mineral content, fruit dry matter content, and biomass of greenhouse tomato, for use in an experiment aiming at controlling the fertigation so as to reach the best compromise between disinfection costs of the recycled water and income from fruit fresh weight production, while ensuring that the dry matter of the fruits is above a given quality standard. The model describes the effect of mineral shortage on growth, and has a simple mechanism to relate dry matter content to the total ion concentration in the substrate.

Optimal control requires a quantitative model in state-space form. In order to reduce on-line computation time, the number of states was kept within reasonable limits by lumping leaves, fruits and substrate into three compartments. Therefore the model is labelled 'big leaf, big fruit, big substrate' or 3Bigs model. Using parameters from literature and some calibration on previous experiments, quite acceptable fits were obtained for biomass, and dry matter and K, N, and Ca contents of the fruits.

The model was subsequently used in a receding horizon optimal control experiment. Ion-concentrations in the drain as measured by ion-specific electrodes are fed back to correct the state of the model before the next values of the required drain flow and the fertilizer fluid aliquots are computed.

Although it was not possible to operate the controller for more than six weeks, it could be observed that the controller behaviour was consistent with expectations in view of the given model and goal function. The fresh weight yield and the dry matter constraint dominate the behaviour. The results also suggest that currently used ion concentrations are higher than necessary for uninhibited growth.

INTRODUCTION

The development of sensors to measure individual ions in drain water offers the perspective of more advanced control of nutrient supply. Such systems would help prevention of yield loss due to occasional nutrient shortage, lead to reduction of fertilizer use and loss to the environment, and eventually will contribute to improve crop quality. Optimal control enables full exploitation of these sensors, and is a powerful methodology to achieve explicit user goals in the best possible way. The methodology requires a systems model in state-space form.

The first half of this paper describes the development of a greenhouse tomato biomass and nutrient model in state-space form. In the second half the model is applied in an experiment in which a receding horizon optimal controller calculates on line the required drain flow and the amount of fertilizer fluids needed to provide the irrigation flow and composition that provides the best trade off between cost of disinfection of recirculation water and income from fruit fresh weight production. This work was part of a project called Hydrion-line III, in which university, research institutes and industry cooperated.

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LUMPED COMPARTMENT MODEL: THE 3BIGS

One of the project partners had a comprehensive model to describe the evolution in time of a tomato or cucumber crop (Marcelis et al., 1998). For the purpose of rigorous optimal control, this model could not be used directly, as it was not in a suitable form. Moreover, the industrial partners in Hydrion-line III wanted a simpler and more generic model. Hence, in the new model, it was decided to incorporate the production of assimilates by photosynthesis, and its conversion by growth to fruit and leaf biomass, which is universal for almost all plants, and to refrain from detailed modelling of node and fruit numbers. These ideas were already used in the so-called big leaf – big fruit model of Tap (2000), but the current model is a complete rebuild. We felt strengthened in this approach by the success of reducing a complex model like TOMGRO to a few states (Jones et al., 1999) or even a single state model (Seginer and Ioslovich, 1998).

The model uses the principle of mass conservation. Since the model has to be used for control of nutrient supply, the model has not only states for the carbon compounds, but also for several minerals. The model was extended with nutrient uptake, and the effect of nutrient shortage on growth. Nutrients are supplied via the substrate, so the substrate has been modelled as well. As in the current study leaves, fruits and substrate are lumped, the model is coined the Big fruit – Big leaf – Big substrate model, or, shortly, the ‘3Bigs’.

Description of the Model

Fig. 1 summarizes the various model compartments and the flows of carbon and nutrient ions. Table 1 lists the symbols used. The molar mass states *M* are subscripted to denote the compartment (*n*, *l*, *f*, *r*, *s*) and the relevant element (C or *i*, where *i* is either N, K, or Ca). They are expressed in moles per m^2 ground surface. Because of page limitations we only present the principal state equations, and explain the special features of the model. The full model description is given in Vanthoor (2005).

Carbon States

The carbon states of the model are modelled in a similar way as with the so called NICOLET model for lettuce (Seginer et al., 1999):

$$\dot{M}_{nC} = h_{an\_la}\, h_{an\_nC}\, r_{anC}{}^{pot} - (1+\theta)\left(r_{nlC} + r_{nfC} + r_{nrC}\right) - h_{n\sigma C}\left(r_{laC} + r_{faC} + r_{raC}\right) \tag{1}$$

$$\dot{M}_{lC} = r_{nlC} - (1-h_{n\sigma C})r_{laC} - r_{lhC} \tag{2}$$

$$\dot{M}_{fC} = r_{nfC} - (1-h_{n\sigma C})r_{faC} - r_{fhC} \tag{3}$$

$$\dot{M}_{rC} = r_{nrC} - (1-h_{n\sigma C})r_{raC} \tag{4}$$

The three terms in the non-structural C balance (Eqn. 1) represent the photosynthesis, growth and maintenance respiration, respectively. The notation expresses the direction of flow, e.g. r_{anC} is the flux of C from the ambient air *a* to the non-structural compartment *n* (i.e. photosynthesis), and r_{nlC} the flux of C from the non-structural compartment *n* to the leaves *l*, r_{fhC} is fruit harvest, and so on. A general principle applied throughout the model is that rate limiting factors are introduced as smooth sigmoid inhibition functions *h* that take values between 0 and 1. The function $h_{n\sigma C}\{G_{nl}\}$, for instance, is 1 as long as there are sufficient assimilates in the non-structural pool, represented by the ratio of non-structural C to structural C, i.e. $G_{nl} = M_{nC} / M_{lC}$. If the pool happens to be almost empty, the function approaches 0, and maintenance will go at the expense of leaf, fruits and roots (Eqns. 2, 3, 4). Similarly, potential photosynthesis, which is a function of the radiation intensity and the CO_2-concentration inside the greenhouse (not shown), is limited by the leaf area limit function $\mathrm{h}_{\mathrm{an\_la}}\{M_{lC}\}$, which is less than 1 if the canopy has not yet reached full closure. It is also assumed in the model that photosynthesis comes to a halt when the assimilate buffer gets full, in which case $h_{an\_nC}\{G_{nl}\} \rightarrow 0$.

The growth rates of leaves, fruits and roots all have the form

$$r_{njC} = h_{njC}\, h_{n\sigma C}\, h_{nj,i}^{\min}\, k_{gj0} Q_{gj}^{c_g(T_l - T_{l0})} M_{jC} \qquad j = \{l, f, r\} \tag{5a-c}$$

Thus, growth rate is proportional to biomass, with a temperature dependent rate coefficient. Inhibitions are introduced to limit growth when there are not sufficient assimilates available ($h_{n\sigma C}$ where σ stands for the sum of all structural compartments), and when the internal mineral content is not sufficient to support further unlimited growth ($h_{nj,i}^{\min}$), as discussed below. The function $h_{njC}\{M_{jC}\}$ with $j = \{l, f, r\}$ equals 1 when there are few organs (for a 'young' plant), and decreases when the crop develops. This is introduced because otherwise the ratio between vegetative and generative mass would be determined almost entirely by the initial states, which is unrealistic. In Seginer et al. (1999) this problem is solved by reducing the respiration with a mass dependent term to express aging. In the current model, respiration is a simple temperature dependent first order process of the biomass of each compartment:

$$r_{jaC} = k_{mj0} Q_{mj}^{c_m(T_l - T_{l0})} M_{jC} \qquad j = \{l, f, r\} \tag{6}$$

The Mineral Content States

The mineral contents of leaves and fruits, expressed in moles per square meter greenhouse, normally follow from the growth of leaves (r_{nlC}) and fruits (r_{flC}), using nominal stoichiometric ratios $g_{i/C,l}^{dem}$ and $g_{i/C,f}^{dem}$, respectively. However, when the associated nutrient flux cannot be supported by the transport in the substrate, the intake is less than demanded, and the mineral mass concentration will drop. This is modelled as follows

$$\dot{M}_{li} = \alpha_i\, r_{sli}^{dem} = \alpha_i\, g_{i/C,l}^{dem} \left(r_{nlC} - r_{lhC}\right) \tag{7}$$

$$\dot{M}_{fi} = \alpha_i\, r_{sfi}^{dem} = \alpha_i\, g_{i/C,f}^{dem} \left(r_{nfC} - r_{fhC}\right) \tag{8}$$

where $\alpha_i, i \in \{K, N, Ca\}$ is the ratio of the flux that can be supported by the substrate and the demanded flux:

$$\alpha_i = \min\left\{1, r_{s\sigma i}^{\sup} \Big/ \left(r_{sli}^{dem} + r_{sfi}^{dem}\right)\right\} \tag{9}$$

When the mineral mass concentration decreases, due to shortage in supply, initially growth remains unimpaired (Marcelis et al., 2003). Only when the mineral composition $g_{i/C,l}^{act} = M_{li} / M_{lC}$ decreases below a certain level growth is reduced. This is expressed by the smooth s-shaped inhibition factor

$$h_{nj,i} = \left(1 + \left(b_{ji} g_{i/C,j}^{dem} \Big/ g_{i/C,j}^{act}\right)^{s_{ji}}\right)^{-1} \qquad j = \{f, l\} \tag{10}$$

where b and s are parameters determining the shape of the s-curve. Which nutrient is the actual limiting factor in Eqns. 5a-c is expressed by

$$h_{nj,i}^{\min} = \min\left\{h_{nj\_N}, h_{nj\_K}, h_{nj\_Ca}\right\} \qquad j = \{f, l\} \tag{11}$$

Supportable Nutrient Flux from the Substrate, and Substrate States

The description of the nutrient flux supported by mass flow and diffusion to the roots is taken from an analysis by van Straten and Gieling (2004). Basically, the supportable uptake depends upon the specific root length, which in the model is a function of the root biomass, and the water uptake rate, which is modelled by a Penman-Monteith like formula. Furthermore, the substrate is treated as a simple continuous stirred tank reactor for all nutrients, whereas it is assumed that the water volume is kept constant due to the implemented feed-back drain flow controller according to Gieling et al. (2000). The set-point of this controller is one of the control inputs of the optimal control.

Other States: Development Stage and Fruit Dry Matter

There are two exceptions on the mass conservation principle in the model. First, there is a development state, which is, in fact, a temperature integral. The development stage is used solely to determine the onset of fruit harvest.

Secondly, in lack of a true water status model, an empirical positive relationship between dry matter content in the fruits and EC in the substrate is used. The function is made dynamic via a simple first order equation, on the basis of experiments reported by Li et al. (2002). The EC is computed from the substrate concentrations of the modelled ions NO_3^-, and K^+ by an empirical formula.

Summary of States, Inputs, and Outputs

The states of the model are the 4 carbon contents M_{nC}, M_{lC}, M_{fC}, M_{rC}, the 2 x 3 mineral contents in the plant M_{li}, M_{fi}, the 3 nutrient contents in the substrate, the development stage, and the fruit dry weight, altogether 15 states. In this application the inputs solar radiation S, the CO_2 concentration in the greenhouse C_{aC}, and the temperature inside the greenhouse T_{gh} are given. Controllable inputs are the drain flow controller set-point, and the aliquots of the fertilizer liquids used to make up the irrigation water. A matrix equation based on the ion-balance is used to link these controls to the concentrations of the ions in the irrigation water.

TRANSPARENCY OF THE STATE SPACE MODEL

A Single Time Frame

All processes are modelled as instantaneous continuous processes. This means that assimilate production by photosynthesis and consumption by growth are described as continuous processes, in contrast to first integrating the photosynthesis on a daily basis, and then distributing the assimilates once a day over crop organs. Such mixed continuous – discrete time models complicate the interpretation of parameters obtained from continuous experiments, and the use of two time scales is cumbersome in optimal control.

No Need for Iteration or Asynchronous Nutrient Demand Compensation

As all processes are instantaneous, their effect can be taken into account immediately. This is significant in particular in the exchange between substrate and crop. In the continuous – discrete approach there is a need to calculate by the end of the day whether the demanded nutrients can, in fact, be delivered by the substrate. If not, the crop model must be recalculated with nutrient limitation. Iteration can be avoided by assigning a time pattern to the demand, but then the calculation is asynchronous and may lead to over- or under-estimations at the end of the day. In the 3Bigs set-up, this effect is automatically taken into account, demand and supply occur simultaneously, and no iteration is needed.

Smooth Inhibition Functions

There are no if-then-else constructs, and there is no need to distinguish between potential and actual rates. This feature ensures that the model has smooth transitions from inhibited to non-inhibited mode, thus enabling the use of several analysis tools such as local sensitivity analysis. It also improves identifiability of the model, and it facilitates the computation of optimal controls.

CALIBRATION OF THE MODEL

Most parameters were obtained from literature data and previous experiments. A preliminary calibration was performed on data from Hydrionline II (Elings et al., 2004) for cucumber. Literature parameters already gave reasonable results, and further improvements were obtained by introducing the mass (age) dependent growth inhibition (Eqn. 5) and discontinuous harvest. Model N, and K contents were within 20% of nutrient data from Hydrion II for leaves and 25% for fruits. A cross-validation on the Hydrion-line III data, which refer to tomato, showed that it was necessary to increase the photosynthetic rate by 40%, and to assume a two-fold faster leaf growth rate. Also changes in nominal N/C, K/C and Ca/C composition ($g_{i/C,l}^{dem}$, $g_{i/C,f}^{dem}$) were required. An impression of the kind of results that is obtained is given in Fig. 2.

APPLICATION: OPTIMAL CONTROL OF FERTIGATION

Experimental Set-up

The Hydrion-line experiment was carried out in a greenhouse with tomato *Lycopersicon esculutum* L. 'Cedrico', grown on Grodan Expert substrate, at a plant density of 2.67 plants m^{-2}. Drain water was collected from a HortiMax ProDrain measurement gully with 16 plants, and samples were analysed with a time varying sample interval for NO_3^-, K^+ and Ca^{2+} ions with a Hydrion analyser. The fertigation solution was produced with a ModiFeed fertigation unit with 8 fertilizer solutions, on the basis of a recipe provided by a Synopta operation system.

Cost Function

The following economic cost criterion is used in the experiment:

$$J = \int_{t_c}^{t_c+t_{horizon}} \left(p_{desinf} F_{dW} - p_{fruit} \eta_{DM/C} \frac{dM_{fC}}{dt} G_f^{-1} \right) dt \qquad (12)$$

where t_c is the current time, $t_{horizon}$ the horizon length, J (€.m^{-2}[gh]) are the cumulative running costs of disinfection of recirculation water minus the value of cumulative fruit fresh weight, F_{dw} (m^3[water].m^{-2}[gh].s^{-1}) drain water flow rate, M_{fC} (mol[C].m^{-2}[gh]) molar mass of structural C in fruit, G_f fruit dry matter content (kg[dw] kg^{-1}[fw]), p_{desinf} the disinfection costs of drain water (0.054 €.m^{-3}[water]), p_{fruit} price for fresh tomatoes (1.0 €.kg^{-1}[fw]), and $\eta_{DM/C}$ the conversion from carbon mass to dry weight (kg[dw].mol[C]). The increment of fruit carbon matter is evaluated as $r_{nfC} - (1 - h_{n\sigma C})\, r_{fam}$, i.e. it is assumed that all net fruit production will eventually be harvested.

There are chemical and technical constraints to the liquid fertilizer aliquots and the nutrient concentrations in the irrigation water. Upper and lower constraints are set to drain flow, pH and EC, to prevent the model from leaving its validity range. The fruit dry matter is constrained at a lower boundary to ensure a minimum fruit quality.

Receding Horizon Control

Every hour the receding horizon controller computes control patterns over the coming 48 hours that minimise the cost function (Eqn. 12). The state is updated using a degenerated Kalman filter, i.e. that states are updated by the measurements, when available. Sequential Programming with control parameterisation by Chebyshev polynomials is used (Vlassenbroek, 1988). The control computed for the coming sampling interval is implemented, after which the optimal control computation is repeated over the new horizon.

RESULTS AND DISCUSSION

Fig. 3 shows that the resulting drain flow set point, and the N, K and Ca concentrations of the irrigation water, computed by the controller and applied to the system, are bang-bang, i.e. on either bound. During the day the controller tries to realise the maximum allowable drain flow rate, whereas the nutrients are chosen as low as possible.

This behaviour can be explained as follows. It appears that the disinfection costs in Eqn. 12 are negligible. So, the controller tries to realise a maximum fresh weight. Fresh weight increase occurs during the day, due to dry matter increase. However, fresh weight can be increased further by increasing the water content. A high water content, i.e. a low dry matter content, is favoured by reducing the EC, according to the positive correlation between EC and dry matter. During the night the opposite holds, in order to minimise the loss by maintenance respiration. The latter is possibly an artefact of the model, in view of the lack of a realistic water status description. The low EC chosen during the day must still be high enough to prevent growth limitation due to nutrient shortage, and to ensure that the dry matter content does not drop below the given minimum standard. In the current experiment these effects are hidden because the lowest permissible value was set according to (non-limiting) common practice. The controller behaviour suggests that such conservative limit is unnecessary when applying optimal control.

CONCLUSION

The state space approach made it possible to implement a receding horizon scheme in a consistent and relatively fast way. It offers transparency, which is valuable to the scientific quest for ever better models, and for the dissemination of advanced control.

As to the optimal control experiment it became clear that the dry matter constraint associated to the current goal function has a dominant influence on the result. Unfortunately, this part of the model is the most empirical, and is therefore the first candidate for improvement. When more confidence is gained in the quality aspects of the model, a trade-off of growth in favour of fruit quality can be a real option.

It has been demonstrated that optimal control with feed-back of measured ion-concentrations is technically feasible. The proof-of-principle paves the way to more advanced fertigation control in practice, and for active steering of crop quality in the future.

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Tables

Table 1. Notation of the 3Bigs Model.

Physical quantities			Subscripts	
G	content	mole[C].mole[C]$^{-1}$	Compartments	
C	concentration	mole.m^{-3}	n	non-structural biomass
M	mass	mole.m[gh]$^{-2}$	l	leaves and stem
T	temperature	°C	f	fruits
I	PAR inside	mole[phot].m^{-2}s^{-1}	r	roots
S	global radiation outside	W.m[gh]$^{-2}$	σ	leaf+fruit+root
F	flow rate	m^{3}.m[gh]$^{-2}$s^{-1}	s	substrate
D	development stage	-	a	air
V	volume	m^{3}.m[gh]$^{-2}$	i	irrigation
r	rate (molar flux)	mole.m[gh]$^{-2}$.s^{-1}	d	drain
Generic parameters			h	harvest environment
k	rate coefficient	s^{-1}	Elements/chemical substances	
c, Q	temperature coeff.	°C^{-1}, -	i	i = {N,K,Ca}
θ	growth resp. fraction	-	C, W	carbon, water
g	mineral content	mole[.].mole[C]$^{-1}$	Superscripts	
η	ratios / unit conversions (in output only)		pot	potential
Functions			dem	demanded
f{.}	function (of argument {.})		sup	supported
h	inhibition function		act	actual
other abbreviations: gh (greenhouse), dw (dry weight), fw (fresh weight), m (maintenance) subscript use: r_{anC} = C from air to non-structural biomass, i.e. rate of photosynthesis				

Figures

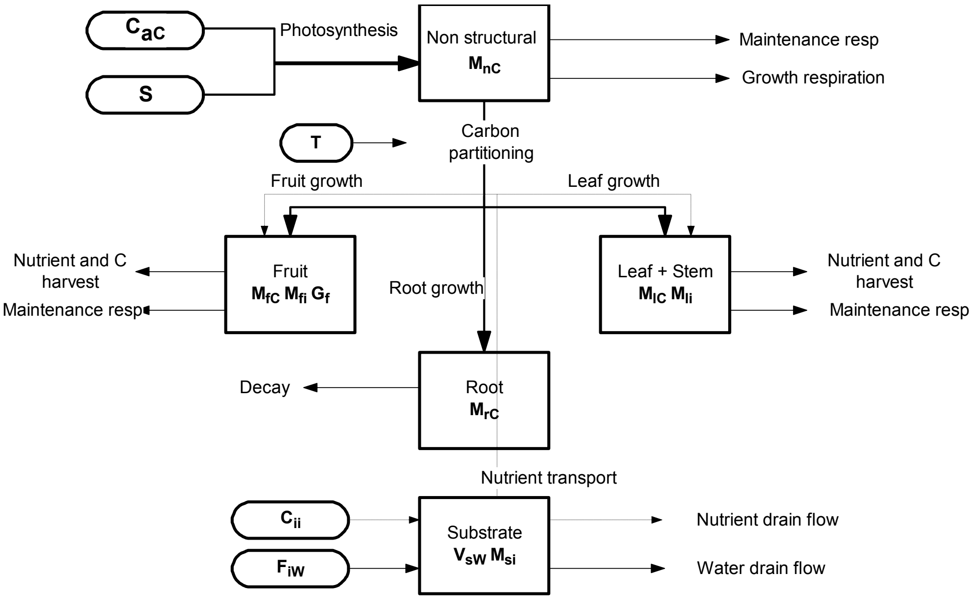


Fig. 1. Structure of the 3Bigs Model. Symbols see Table 1.

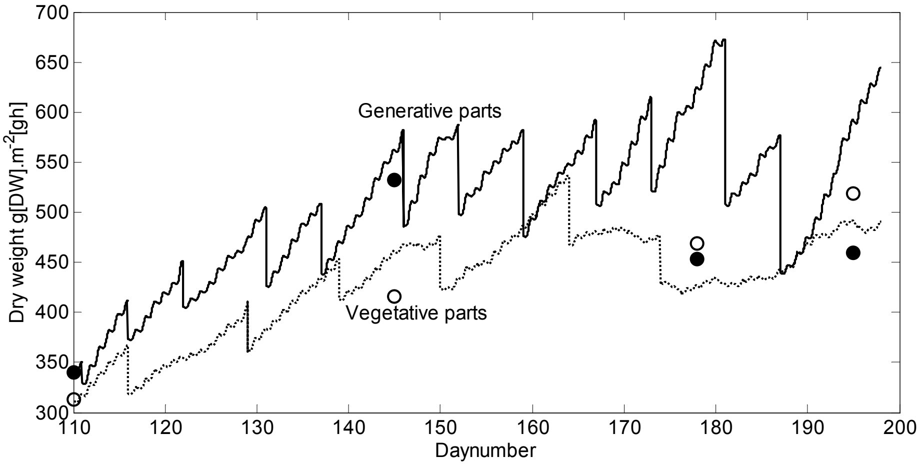
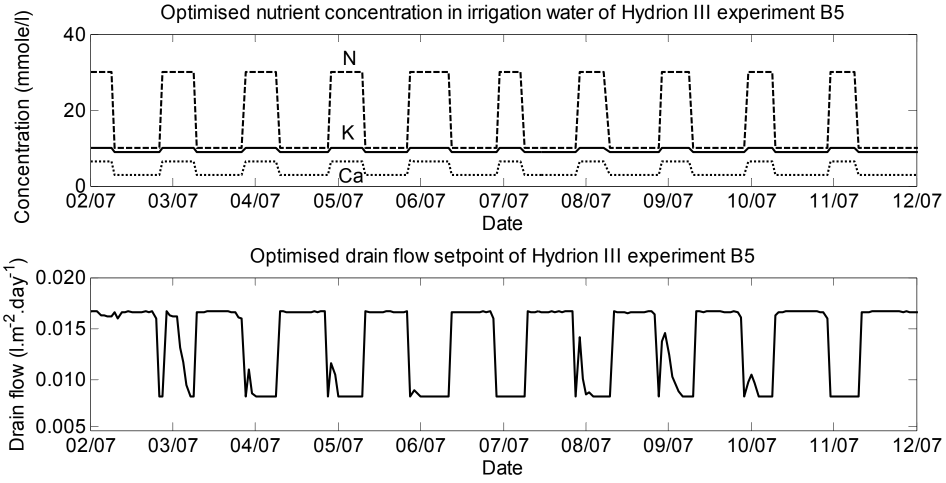


Fig. 2. Dry weight model fit on data of Hydrionline III. Solid line and closed circles: generative parts, dashed line and open circles: vegetative parts.



Optimised nutrient concentration in irrigation water of Hydrion III experiment B5

Optimised drain flow setpoint of Hydrion III experiment B5

Fig. 3. Experimental optimal control patterns; N, K, and Ca concentrations in irrigation water (top), and drainflow set-point (bottom).