SOME CALCULATIONS CONCERNING THE LIGHT TRANSMISSION OF GLASS WITH A SHADE COATING AND CLEAN GLASS, WHEN DRY AND WHEN WETTED

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1 INTRODUCTION

Many growers shade their glasshouses in spring and summer with various kinds of whitewash. To overcome the disadvantage that during dark periods there will not be enough light available in these glasshouses, some firms developed coatings that show a remarkable increase in transparency when they become wetted.

At the Glasshouse Crops Research and Experiment Station at Naaldwijk and the Institute of Agricultural Engineering (IMAG) at Wageningen, some experiments were carried out in 1975 to analyse these shade coatings with respect to their property of becoming more transparent when wet. The investigations were conducted with pieces of glass of 20 x 20 cm. Some other aspects were examined as well. A report about this work by G.P.A. VAN HOLSTEIJN and D.A. LIEFTINK is in preparation. A full description of materials and methods will be given in that report.

The need arose during this work to find an equation that could describe the observed differences in transparency between the dry and wetted samples, as this might perhaps provide a means of comparing the values obtained for the various brands.

Some general calculations regarding clean glass are also included in this report. Some very high values for the transparency in wetted conditions were found, and it was convenient to have some theoretical values.

2 TRANSPARENCY OF GLASS WITH SHADE COATING WHEN DRY AND WHEN WET

The relationship between the transmission of light in a dry and wet condition was examined on the basis of 28 samples, chosen from the glass panels with a certain brand of shade coating. Some of these glass panels, had been sprayed once with a mixture of water and the coating material, some several times, so that there were different degrees of transparency. If a mathematical equation, expressing the relationship between the transparency in a wet and in a dry condition could be found, then it should be simple to examine further and define the differences between the various makes of shade coating on a programmable calculator.

The measurements were carried out with light falling perpendicularly on to the glass. The transmission of light in air, i.e. without glass between the light source and sensor, was taken to be 100%. In Fig. 1 the results of the 28 measurements are plotted in a graph. The horizontal axis shows the transmission of light in a dry condition, the vertical axis that of the wetted glass.

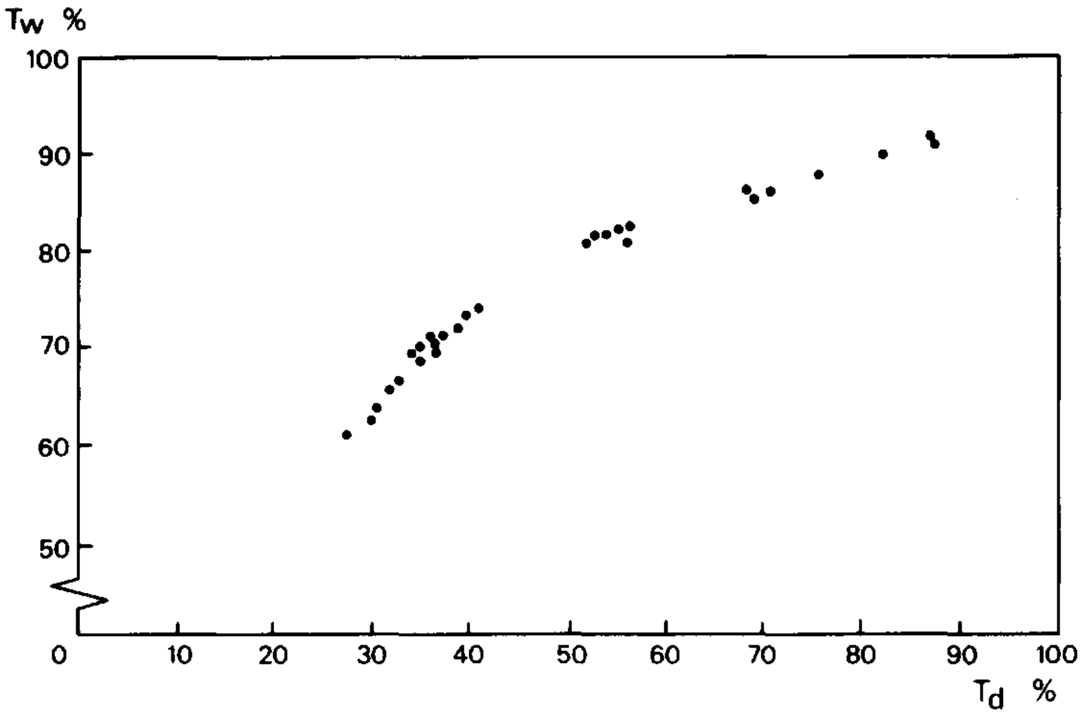

Figure 1 Transmission in a wet condition (T_w) plotted against the transmission when dry (T_d).

It will be seen that there is a definite mathematical relation between the transmission when dry (henceforward called T_d) and the transmission when wet (henceforward called T_w). But if one wishes to make further calculations, one needs to know whether the relation is logarithmic, quadratic or of some other kind. After some calculation and trial it appeared possible to proceed by taking the relative increases in transmission and to plot these against T_d.

The relative increase (z) is defined here as:

$$z = \frac{T_w - T_d}{T_d} \times 100\% \qquad (2.1)$$

So, if T_d = 40% and T_w = 60%, the relative increase in transmission is

$$z = \frac{60 - 40}{40} \times 100\% = 50\%.$$

Plotted against T_d, this z appeared to give a quadratic relation. By taking the square root of z and plotting it against T_d, we get a batch of points, that can be well described by a straight line.

The transmission of the samples in a dry and in a wet condition, the relative increase in transmission and the square root from the latter are given in Table I. In Fig. 2 the square root of the relative increase in transmission is plotted against T_d.

Nr	T_d	T_w	z	$\sqrt{z}$	Nr	T_d	T_w	z	$\sqrt{z}$
1	55.9	81.4	45.6	6.75	15	75.4	87.4	15.9	3.99
2	34.0	69.3	103.8	10.19	16	69.0	84.7	22.8	4.77
3	34.3	68.6	100.0	10.00	17	51.8	80.9	56.2	7.50
4	27.4	60.9	122.3	11.06	18	53.8	81.9	52.2	7.23
5	56.1	81.0	44.4	6.66	19	52.5	81.0	54.3	7.37
6	35.6	69.9	96.3	9.82	20	55.9	82.6	47.8	6.91
7	36.0	70.2	95.0	9.75	21	38.8	71.5	84.3	9.18
8	30.2	63.4	109.9	10.48	22	39.2	72.8	85.7	9.26
9	54.9	80.9	47.4	6.88	23	39.2	72.8	85.7	9.26
10	35.4	69.2	95.5	9.77	24	40.4	73.8	82.7	9.09
11	35.8	69.8	95.0	9.75	25	31.8	65.8	106.9	10.34
12	30.1	62.9	109.0	10.44	26	32.5	66.2	103.7	10.18
13	86.9	91.6	5.4	2.33	27	68.4	84.9	24.1	4.91
14	70.6	86.2	22.1	4.70	28	82.6	89.9	8.8	2.97

Table I Light transmission of the specimens when dry (T_d) and when wetted (T_w), relative increase in transmission (z) and the square root from z.

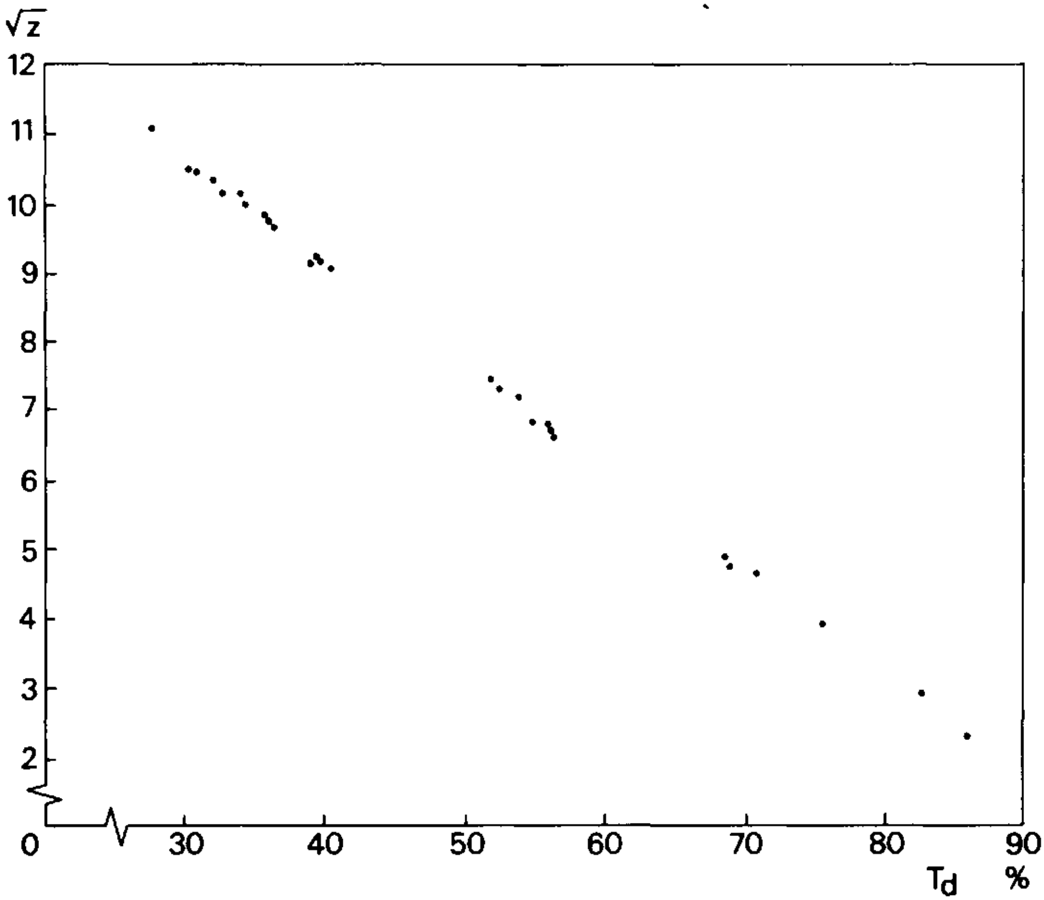

Figure 2 Square root of the relative increase in transmission by wetting plotted against the transmission in a dry condition.

The equation describing the straight line, having in general the shape y = ax + b, is in this case

$$\sqrt{z} = aT_d + b \tag{2.2}$$

The relation between T_w and T_d follows from the equations (2.1) and (2.2) and is described by

$$T_w = 0.01a^2T_d^{\ 3} + 0.02abT_d^{\ 2} + (1 + 0.01b^2)T_d \tag{2.3}$$

Using a regression technique, the coefficients a and b can be calculated, so that the transmission when wetted can be calculated from the transmission when dry.

The correlation (r) is obtained from the equation

$$r = \frac{a\sigma(x)}{\sigma(y)} \text{ or, in this case: } r = \frac{a\sigma(T_d)}{\sigma(\sqrt{z})} \tag{2.4}$$

In the example discussed here we get:

$a = - = -0.1459$
$b = \ = 14.9690$
$\sigma(T_d) = 17.1940$
$\sigma(V_z) = 2.5096$
$r = -0.9996$

The correlation appears to be very high, so it can be assumed that in the domain in which the trial is performed, the phenomenon can be described by the above equation. As follows from the 28 specimens, the equation for the brand of coating examined, is:

$$T_w = 0.01\cdot(-0.146)^2T_d^{\ 3} + 0.02\cdot(-0.146)\cdot 14.969T_d^{\ 2} +$$
$$(1 + 0.01\cdot 14.969^2)T_d$$

$$\text{or } T_w = 2.132\cdot 10^{-4}T_d^{\ 3} - 4.371\cdot 10^{-2}T_d^{\ 2} + 3.241T_d \tag{2.5}$$

From the same collection of specimens, some that were not included in the calculation above (i.e. kept out of the 28 used), were taken for a comparison of measured and calculated values of T_w. The results are given in Table II. The calculated values agree very well with the measured data.

T_d (%)	T_w measured (%)	Tw calculated (%)	Difference (%)
32.2	66.2	66.2	0
37.4	71.0	71.2	0.2
37.6	71.6	71.4	-0.2
53.5	81.5	80.9	-0.6
54.1	81.4	81.2	-0.2
72.5	85.9	86.5	0.6
84.0	90.1	90.2	0.1

Table II Measured and calculated values of T_w

Similar test pieces were taken for some other makes of shade coating, both from the series at Wageningen and at Naaldwijk, keeping to the same number of 28 specimens. The general mathematical relation appeared to apply to all series of specimens. The correlation coefficients varied from -0.98 to -1.0.

In the above-mentioned report the series from Wageningen and Naaldwijk were combined for each brand of lime. This gave slighty different values, but the relationship obtained was not affected.

3 INCREASE IN TRANSMISSION BY WETTING

Once an equation had been found, the question arose, for which transmission in a dry condition the increase in transmission by wetting the shade coating on the glass would be the largest. This question can be answered by taking the difference between the transmission when dry and when wetted and differentiating it with respect to the transmission in a dry condition.
Reverting to equation (2.3):

$T_w = 0.01a^2T_d^{\,3} + 0.02abT_d^{\,2} + (1 + 0.01b^2)T_d$ we get

$$T_w - T_d = 0.01a^2T_d^{\,3} + 0.02abT_d^{\,2} + 0.01b^2T_d \tag{3.1}$$

$$\text{and } \frac{d(T_w - T_d)}{dT_d} = 0.03a^2T_d^{\,2} + 0.04abT_d + 0.01b^2 \tag{3.2}$$

The extremes are obtained by writing:

$$0.03a^2T_d^{\,2} + 0.04abT_d + 0.01b^2 = 0 \tag{3.3}$$

$$\text{Then } T_d = \frac{-4ab \pm \sqrt{4^2a^2b^2 - 4\cdot 3a^2b^2}}{6a^2} = \frac{-2b \pm b}{3a} \tag{3.4}$$

$$\text{and } T_d = \frac{-b}{a}, \text{ or } T_d = \frac{-b}{3a} \qquad (3.5),\ (3.6)$$

The second derivative with respect to T_d is

$$\frac{d^2(T_w - T_d)}{dT_d^{\,2}} = 0.06a^2T_d + 0.04ab \tag{3.7}$$

$T_d = \frac{-b}{a}$ substituted in (3.7) gives: $0.06a^2(\frac{-b}{a}) + 0.04ab = -0.02ab$

As a and b are of opposite sign in our case, this value is positive. The function is thus at a minimum at this point.

$T_d = \frac{-b}{3a}$ in (3.7) gives: $0.06a^2(\frac{-b}{3a}) + 0.04 = 0.02ab$

This value for the second derivative is negative, if a and b are of opposite sign. This means that for $T_d = -b/3a$ the function, described by (3.1), reaches a maximum here.

The relative increase (z) at this point can be obtained using equation (2.2): $\sqrt{z} = aT_d + b$.

Substitution of $T_d = \frac{-b}{3a}$ in (2.2) gives:

$$\sqrt{z} = a\left(\frac{-b}{3a}\right) + b = \frac{2}{3}b$$

$$z = \frac{4}{9}b^2$$

For the example discussed in Section 2, where we found $a = -0.1459$ and $b = 14.9690$, we can similarly calculate the value for T_d at which the largest increase in transmission by wetting is obtained:

$$T_d = \frac{-b}{3a} = \frac{-14.969}{3\cdot(-0.1459)} = 34.2\%.$$

The corresponding value of z is $\frac{4}{9}\,b^2 = \frac{4}{9}\cdot 14.969^2 = 99.6\%$

Note that for $b = 15$, $z = (4/9) \times 15^2 = 100\%$, and that for $b = 15$ and $a = -0.01b$ for the point of maximum increase $T_d = T_w - T_d = 33.3\%$.
In the experiments the values for b ranged from 14.31 to 17.15 and the value of a from $-0.0101b$ to -0.0093.

4 TRANSMISSION OF DRY AND WETTED CLEAN GLASS, NORMAL INCIDENCE

The reflection of light by a boundary plane between two media depends on the difference in the indices of refraction of the two media involved. Let n_1 and n_2 be the refractive indices of the two media, then the ratio of the reflected to the incident light (R) in the case of normal incidence, i.e. perpendicular to the boundary-plane, is given by the equation:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad (4.1)$$

The indices of refraction of the different kinds of glass (n_g) vary. For the following calculation a value of $n_g = 1.52$ is assumed, this being a value frequently used in calculations. As is well known, the index of refraction of air can be approximated by $n_a = 1$; that of water is $n_w = 1.33$.

The calculation of dry glass is then as follows (see Fig. 3):

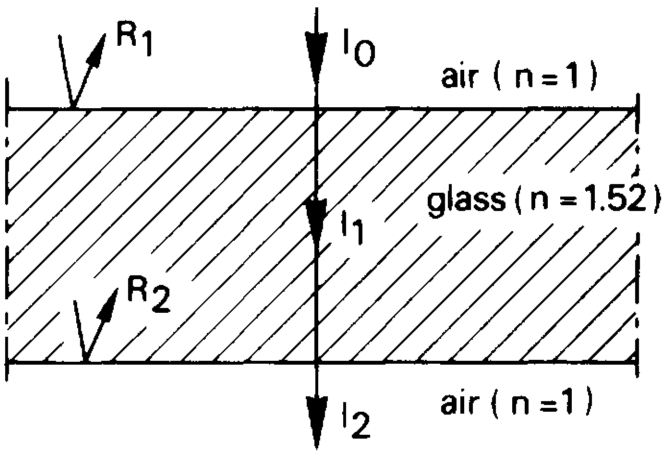

Figure 3 Dry glass, incidence perpendicular.

$$I_1 = (1 - R_1)I_0 \qquad (4.2)$$

$$I_2 = (1 - R_2)I_1 = (1 - R_1)(1 - R_2)I_0$$

$$R_1 = \left(\frac{1 - 1.52}{1 + 1.52}\right)^2 = 0.0426$$

$$R_2 = \left(\frac{1.52 - 1}{1.52 + 1}\right)^2 = 0.0426$$

$$I_2 = (1 - 0.0426)^2 \cdot I_0 = 0.9166\ I_0$$

This means that in a dry condition the transmission of light is 91.66%.

The calculation for wetted glass, i.e. glass with a film of water on one side, is similar to the preceeding calculation (see Fig. 4):

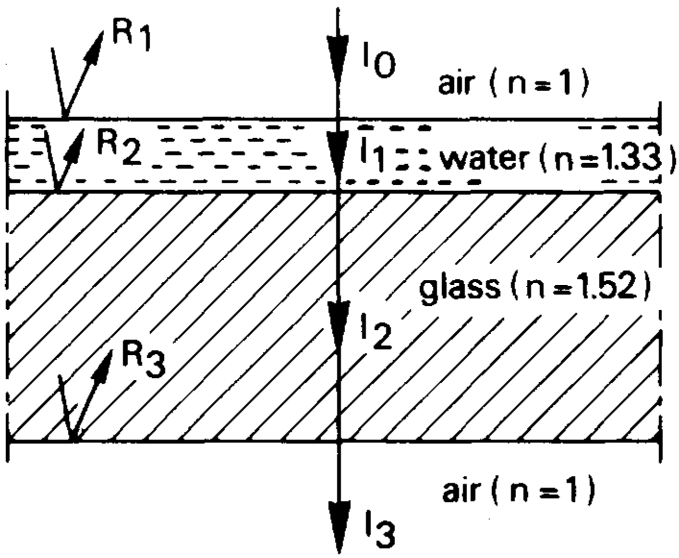

Figure 4 Wet glass, incidence perpendicular.

$$I_3 = (1 - R_1)(1 - R_2)(1 - R_3)I_0 \quad (4.3)$$

$$R_1 = \left(\frac{1 - 1.33}{1 + 1.33}\right)^2 = 0.0204$$

$$R_2 = \left(\frac{1.33 - 1.52}{1.33 + 1.52}\right)^2 = 0.0044$$

$$R_3 = \left(\frac{1.52 - 1}{1.52 + 1}\right)^2 = 0.0426$$

$$I_3 = 0.9796 \cdot 0.9956 \cdot 0.9574 \cdot I_0 = 0.9377\ I_0$$

Thus the transmission with a film of water on one side of the glass is 93.37%.

The relative increase in light transmission by wetting the glass is

$$\frac{93.37 - 91.66}{91.66} \cdot 100\% = 1.9\%.$$

Repeated reflections and transmissions between the surfaces and the light absorption in the medium glass or water are not taken into account in these calculations.

5 LIGHT TRANSMISSION OF DRY AND WET CLEAN GLASS FOR LIGHT FROM DIFFERENT DIRECTIONS

5.1 Starting points for the calculations

The angle between the light ray and the normal is called the angle of incidence; it is expressed here by i or i_i. The angle of refraction, r or i_{i+1}, is the angle between the broken light ray and the normal. The index of refraction (n) is equal to the ratio between the sines of the angle of incidence and the angle of refraction, or:

$$n = \frac{\sin i}{\sin r} = \frac{\sin i_i}{\sin i_{i+1}} \quad (5.1)$$

If the indices of refraction from glass to air ($n_{g.a}$) and from air to water ($n_{a.w}$) are known, the index of refraction from glass to water can be determined from

$$n_{g.w} = n_{g.a} \times n_{a.w} \quad (5.2)$$

The reflection taking place at every boundary plane can be calculated using Fresnel's equation:

$$R = \frac{1}{2} \left\{ \frac{\sin^2 (i - r)}{\sin^2 (i + r)} + \frac{\tan^2 (i - r)}{\tan^2 (i + r)} \right\} \quad (5.3)$$

For purposes of calculations with several angles of incidence the values for the indices mentioned below are used. As several values are given in the literature for the index of refraction of glass, the calculations are carried out with $n_g = 1.52$ and $n_g = 1.50$. The influence of small differences in the index of refraction can then be seen.

From air to glass:	n = 1.520	n = 1.500
From glass to air:	n = 0.658	n = 0.667
From air to water:	n = 1.333	n = 1.333
From water to air:	n = 0.750	n = 0.750
From glass to water:	n = 0.877	n = 0.889
From water to glass:	n = 1.140	n = 1.125

5.2 Example of calculation

As an example, the course of the calculations is shown for the situation that the angle of incidence is 45°, both for dry and for wetted glass, i.e. glass that has a water film on one side only. A value of 1.52 has been chosen in this example for n_g. The results obtained for other angles of incidence are given in paragraph 5.3.

Angle of incidence 45°, dry glass (see Fig. 5)

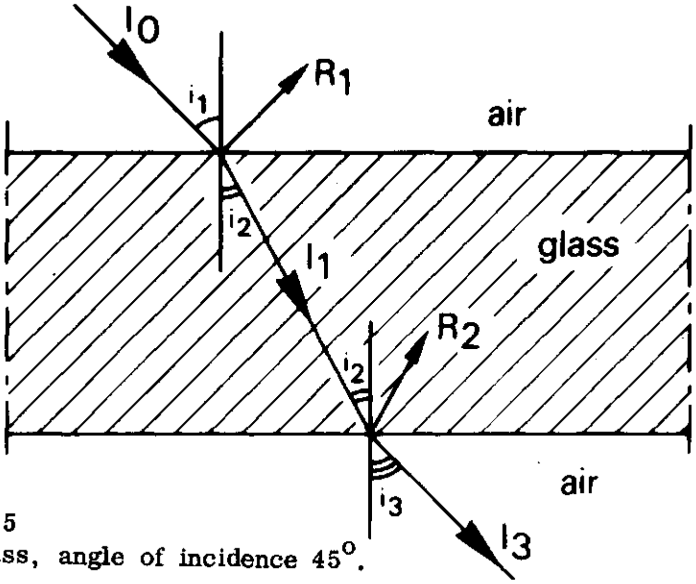

Figure 5
Dry glass, angle of incidence 45°.

$i_1 = 45^o$ $\qquad \sin i_2 = \frac{\sin 45^o}{1.52}$

$i_2 = 27.72^o$ $\qquad \sin i_3 = \frac{\sin 27.72^o}{0.658}$

$i_3 = 45^o$

$$R_1 = \frac{1}{2}\left\{\frac{\sin^2 17.28^o}{\sin^2 72.72^o} + \frac{\tan^2 (17.28^o)}{\tan^2 72.72^o}\right\} = 0.0531$$

$$R_2 = \frac{1}{2}\left\{\frac{\sin^2 (-17.28^o)}{\sin^2 72.72^o} + \frac{\tan^2 (-17.28^o)}{\tan^2 72.72^o}\right\} = 0.0531$$

Hence $I_2 = (1 - 0.0531)^2 \cdot I_o = 0.8966\ I_o$

Thus the transmission of light in the case of dry glass and an angle of incidence of 45° is 89.66%.

Angle of incidence 45°, glass wet on one side (see Fig. 6)

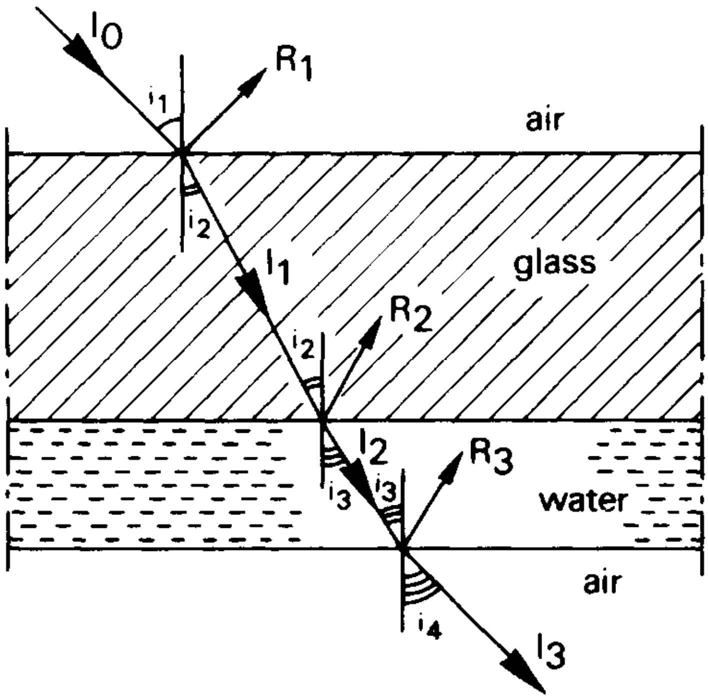

Figure 6 Wet glass, angle of incidence 45°.

$i_1 = 45^{o}$ $\qquad$ $\sin i_2 = \dfrac{\sin 45^{o}}{1.52}$

$i_2 = 27.72^{o}$ $\qquad$ $\sin i_3 = \dfrac{\sin 27.72^{o}}{0.877}$

$i_3 = 32.04^{o}$ $\qquad$ $\sin i_4 = \dfrac{\sin 32.04^{o}}{0.750}$

$i_4 = 45^{o}$

$$R_1 = \tfrac{1}{2}\left\{\frac{\sin^2 17.28^{o}}{\sin^2 72.72^{o}} + \frac{\tan^2 17.28^{o}}{\tan^2 72.72^{o}}\right\} = 0.0531$$

$$R_2 = \frac{1}{2}\left\{\frac{\sin^2(-4.32^o)}{\sin^2 59.76^o} + \frac{\tan^2(-4.32^o)}{\tan^2 59.76}\right\} = 0.0048$$

$$R_3 = \frac{1}{2}\left\{\frac{\sin^2(-12.96^o)}{\sin^2 77.04^o} + \frac{\tan^2(-12.96^o)}{\tan^2 77.04^o}\right\} = 0.0279$$

Hence $I_3 = (1 - 0.0531)(1 - 0.0048)(1 - 0.0279) \cdot I_o = 0.9161\ I_o$

That means that with a film of water on one side of the glass and an angle of incidence of 45° the transmission of light is 91.61%.

5.3 Some calculated values

Some values obtained with the above method are given in Table III.

Angle of incidence	Transmission (%) $n_{glass} = 1.52$		Transmission (%) $n_{glass} = 1.50$	
	dry	wet	dry	wet
0°	91.66	93.37	92.16	93.71
30°	91.36	93.12	91.87	93.46
45°	89.66	91.61	90.21	91.97
60°	82.36	84.84	82.96	85.23
75°	55.25	58.10	55.80	58.47

Table III Transmission of light with different angles of incidence

No physical meaning should be given to the rounding off of the figures to two decimals. However, if they were rounded off to one decimal, then in the case of n = 1.52 and dry glass, nearly all values would get higher and in the case of wet glass nearly all values would get lower. This would introduce a bias to the difference between the two values;this would be even less real.

6 DISCUSSION

The relation between the light transmission through glass with some special brands of shade coating when dry and when wetted is analysed only mathematically. No physical analysis of the phenomenon is undertaken. The experiments and calculations show, that it can be worthwhile to shade the glasshouse with the makes of coating investigated and to wet the glass in dull weather to improve the light transmission into the glasshouse, at least from

a point of view of light transmission, which is the subject of this report.
It was thought useful to describe the calculations in some detail, because often more is remembered when it is explained.

The calculations in sections 4 and 5 offer no new theories or insights, but in our investigations it was convenient to have the results available. As these results seem not to be generally known, they are also included in this report.

It should be noted that repeated reflections and transmissions are neglected (as is usually done) and that losses by absorption of the light by the medium are not accounted for either. The attenuation by absorption of the light on its path through a medium is proportional to the attenuation coefficient k, so the attenuation dI over a path dx can be expressed as $\frac{dI}{dx} = -k.I_x$, I_x being the intensity after traversing a path x through the medium. Solving this as a differential eqation and putting the light intensity at the beginning of the path through the medium I_o, gives the better known form of Beer's law: $I_x = I_o.e^{-kx}$.

In this report, however, only thin layers of clear glass and pure water are considered, so this absorption can be neglected.

A thin film of water on one side of the glass gives some percent gain in light transmission, compared to dry glass. Especially with light rays having a large angle of incidence, the gain is evident. In cases, not included in the above calculations, where a water film is present on both sides of the glass, the proportional gain is higher. This situation arises in glasshouses if during rain a condensate film is formed on the inside surfaces of the glass.

The condensate film on the inside of the glasshouse should then be regarded as a flat film of water and not as a collection of water droplets. The latter situation often occurs in glasshouses, but is not as well suited for calculations. The distribution of the droplets can differ, the geometry of the droplets is not constant, the calculation of the paths of the light rays coming from different directions and passing through several points of a sphere etc., is very complex, so that the calculations are much less simple than those shown here. Measurements at the IMAG at Wageningen by STOFFERS have shown, however, that also in the case of condensation in the form of droplets, there is a slight increase in light transmission.

7 SUMMARY

In experiments with special brands of shade coating made for glasshouses the need arose to know the mathematical relationship between the transmission of light through glass with this coating when dry and when wet.

The calculations described show the relationship between the transparency in a dry and in a wet condition obtained. A calculation is also made for non-coated glass, with the light falling perpendicularly on to the glass, to check some very high values obtained for the transparency of wetted, clean glass.

Some calculations of the difference in transparency between dry glass and glass with a condensed water film on the inside of the glass are also included for several angles of incidence. The latter case is often encountered in practice in glasshouses.