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# HEAT TRANSFER DURING THE COOLING PROCESS OF HEAT GENERATING PRODUCE

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## Summary

The cooling process of fresh agricultural produce is complicated by their heat generation due to the respirational activity. Especially large unit-loads can show the "heating" phenomenon, even during the cooling process.
A mathematical approach to this problem is presented, based on known relations.
Courses of center temperatures were calculated for different combinations of the characteristic numbers: Fo, Bi, Po. The results are shown in figures and compared with experimental results. Special interest is paid to the increase in cooling time and the "heating peak" in the beginning of the cooling process. Large values of $\frac{1}{Bi}$ and Po result in heating rather than cooling even if the body is cooled at the surface.
Practical limits are presented for the different parameters of the process, within which a certain cooling effect can be expected.

# HEAT TRANSFER DURING THE COOLING PROCESS OF HEAT GENERATING PRODUCE

## 1. Introduction

The cooling-down process of heat generating produce is not yet fully described in a mathematical model.
The work of Pflug and coworkers [1]) has explained a great deal of this process for the case of neglegible heat generation. The simple theory, however, does not cover the phenomena occurring during the cooling-down of heat generating produce, especially with great respirational activity.

## 2. Theory

The solution of the differential equation for the conduction of heat in an infinite slab with constant heat production:

$$\rho c \frac{\partial t}{\partial \tau} = \lambda \frac{\partial^2 t}{\partial x^2} + Q$$

with the initial and boundary conditions

$$t = t_o \qquad -X \leq x \leq +X; \quad \tau = 0$$

$$\frac{\partial t}{\partial x} = 0 \qquad x = 0$$

$$-\lambda \frac{\partial t}{\partial x} = \alpha(t - t_a) \qquad x = +X$$

is given by Luikov [2]) as:

$$\Theta = \frac{t-t_a}{t_o-t_a} = \frac{Po}{2}\left[1-\left(\frac{x}{X}\right)^2+\frac{2}{Bi}\right] + \sum_{n=1}^{\infty}\left(1-\frac{Po}{\mu_n{}^2}\right)A_n\cos\left(\mu_n\frac{x}{X}\right)\exp(-\mu_n{}^2 Fo) \tag{1}$$

with the dimensionless characteristic numbers
Pomerantsev:

$$Po = \frac{QX^2}{\lambda(t_o-t_a)}$$

giving the relation between generated heat and heat transfer by conduction,

Biot:

$$Bi = \frac{\alpha X}{\lambda} \quad \text{and}$$

Fourier:

$$Fo = \frac{\lambda}{c\rho}\,\frac{\tau}{X^2}$$

$A_n$ and $\mu_n$ are tabellized functions of Bi, given in the literature, i.e. Luikov [2]):

$$A_n = \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n}$$

and contains $\mu_n$ as the characteristic roots of the equation

$$\mu_n \sin \mu_n - Bi \cos \mu_n = 0$$

Equation 1 describes the development of the temperature field in time. The first term represents the steady state temperature distribution to which the vanishing time depending term is added.

The equation can be split into one describing external effects and one describing internal effects:

$$\Theta_{ext} = \sum_{n=1}^{\infty} A_n \cos(\mu_n \frac{x}{X}) \exp(-\mu_n{}^2 Fo) \qquad (2)$$

and

$$\Theta_{int} = \frac{Po}{2}\left[1-(\frac{x}{X})^2+\frac{2}{Bi}\right] - Po \sum_{n=1}^{\infty} \frac{1}{\mu_n{}^2} A_n \cos(\mu_n \frac{x}{X}) \exp(-\mu_n{}^2 Fo) \qquad (3)$$

indicating that the temperature field can be found by superposition of external and internal effects. Eq. 2 gives the well-known solution of the cooling problem without heat sources. Eq. 3 multiplied by the term $(t_o - t_a)$ gives the development of the temperature field in a heat producing medium in the case where $t_a = t_o$.

The center temperature (x=o), which as a maximum, is the most interesting with respect to quality, is given by:

$$\Theta_{center} = \frac{Po}{2}\left[1+\frac{2}{Bi}\right] + \sum_{n=1}^{\infty} (1-\frac{Po}{\mu_n{}^2}) A_n \exp(-\mu_n{}^2 Fo) \qquad (4)$$

This equation was solved numerically for several values of Po, Bi and Fo. The results are given in fig. 1 to 4.

The figures show the particular behaviour of the center temperature of heat generating produce: a heating peak and a general failure of the cooling process to reduce the temperature.

These effects are much more pronounced at high values of the Pomerantsev-number (fig.'s 1 and 2).

The effectiveness of the cooling-down process can be proved by a relation between Po and Bi.

Cooling does not take place if: $\Theta = \frac{Po}{2}\left(1+\frac{2}{Bi}\right) \geq 1$

which leads to: $Po \geq 2$

or if $Po < 2$ than $Bi \leq \frac{2Po}{2-Po}$

## 3. Experimental

During the cooling-down of a carton 0.28 x 0.48 x 1.20 $m^3$, filled with cut flowers, most of them roses, the centre temperature was recorded.

The thermal properties of the package were determined from previous experiments:

| | | | |
|---|---|---|---|
| half width of the package | $X$ | = | $0.14\ m$ |
| thermal conductivity | $\lambda$ | = | $0.12\ \frac{W}{m\ K}$ |
| specific heat | $c$ | = | $4000\ \frac{J}{kg\ K}$ |
| density in package | $\rho$ | = | $200\ \frac{kg}{m^3}$ |
| heat generation | $\frac{Q}{\rho}$ | = | $0.3\ \frac{W}{kg}$ |
| heat transmission coefficient | $k$ | = | $\left[\frac{1}{\frac{1}{\alpha_{air}}+\frac{d_{carton}}{\lambda_{carton}}}\right] = 8.7\frac{W}{m^2K}$ |
| heat transfer coefficient | $\alpha_{air}$ | = | $11\ \frac{W}{m^2K}$ |

carton thickness $d_{carton}$ = 0.003 m

thermal conductivity $\lambda_{carton}$ = $0.12 \frac{W}{m^2K}$

temperature difference $(t_o-t_a)$ = $15^oC$

The characteristic numbers are

$$Bi = \frac{kX}{\lambda} = 10$$

$$Po = \frac{QX^2}{\lambda(t_o-t_a)} = 0.66$$

Fig. 5 shows the measured center temperature; the calculated temperature according to eq. 4; and the results of eq. 2 and 3 for the center temperature, Bi = 10, Po = 0.66.

The cooling-down process can, within technical accuracy, in this case be treated as one-dimensional till a Fo-number of 0.5, corresponding to ca. 20 h.
Theoretical and experimental temperature are in good agreement during the starting period of the process
The heating-peak is very clearly developed in the range of $Fo \leq 0.15$. For $0.15 < Fo$ there is a growing positive deviation of the calculated temperature according to the theory of the slab, and the experimental temperature. This is caused by the condition of constant heat generation, which holds in practice only for small temperature ranges as the heat generation decreases with the temperature.

## 4. Conclusions

The theory of heat conduction with continuous heat sources in the analytical form as presented by Luikov [2]) can be applied to the cooling-down process of heat generating produce.

The one-dimensional approach holds for a certain period depending on the shape of the body.

The condition of constant heat generation restricts the application to small temperature ranges, but the heating peak is very clearly described.

A relation between the characteristic numbers Po and Bi can be given for the effectiveness of a cooling-down process if

$$Po \geqslant 2$$

or

$$Po < 2 \qquad \text{and} \qquad Bi \leqslant \frac{2Po}{2-Po}$$

cooling does not take place.

## 5. Acknowledgement

The authors are greatly indebted to Mr. A. J. Koster of the computercenter of the Agricultural University at Wageningen, for taking care of the execution of the Fortran IV program on the Control Data 3200 machine.

## 6. Literature

[1]). Pflug, I. J., and I. J. Kopelman, Correlating and predicting transient heat transfer rates in food products. Annex 1966-2, Bull. IIR, Paris, 89-100.

[2]). Luikov, A. V. (ed. V. P. Hartnett), Analytical Heat Diffusion Theory. Academic Press, New York and London 1968.

Symbols

| | | |
|---|---|---|
| $A_n$ | function of Bi (page 2) | |
| $\alpha$ | heat transfer coefficient | $\frac{W}{m^2 K}$ |
| c | specific heat | $\frac{J}{kg\ K}$ |
| k | heat transmission coefficient | $\frac{W}{m^2 K}$ |
| $\lambda$ | thermal conductivity | $\frac{W}{mK}$ |
| $\mu_n$ | function of Bi (page 2) | |
| Q | rate of heat generation | $\frac{W}{m^3}$ |
| $\rho$ | bulk density | $\frac{kg}{m^3}$ |
| $t(x,\tau)$ | temperature at x at time $\tau$ | $^oC$ |
| $t_o$ | uniform temperature of the body at the beginning of the process | $^oC$ |
| $t_a$ | ambient temperature | $^oC$ |
| $\Theta$ | dimensionless temperature difference | $\frac{t-t_a}{t_o-t_a}$ |
| $\tau$ | time | sec. |
| X | half thickness of slab | m |
| x | running coordinate | m |
| Po | Pomerantsev number: $\frac{QX^2}{\lambda(t_o-t_a)}$ | |
| Bi | Biot number: $\frac{\alpha . X}{\lambda}$; $\frac{k . X}{\lambda}$ | |
| Fo | Fourier number: $\frac{\lambda}{\rho c}\ \frac{\tau}{X^2}$ | |

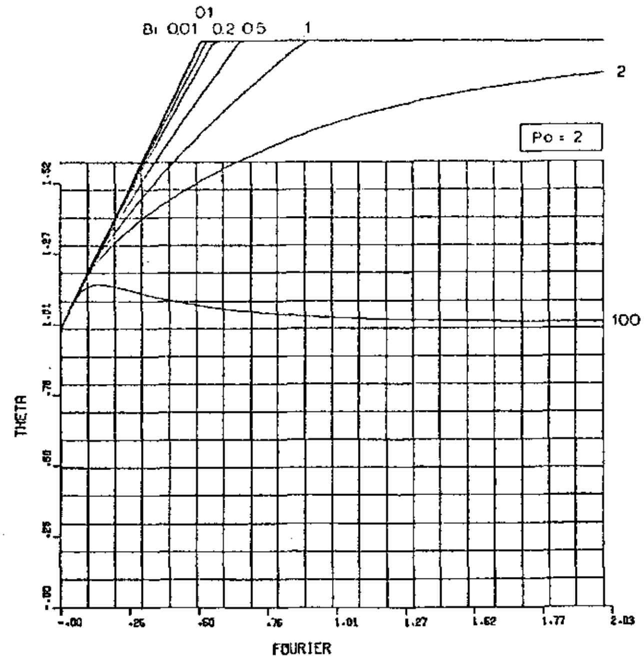


Fig. 1. Cooling curves of a heat generating slab, Θ vs. Fo; parameter: Bi; for Po = 2.

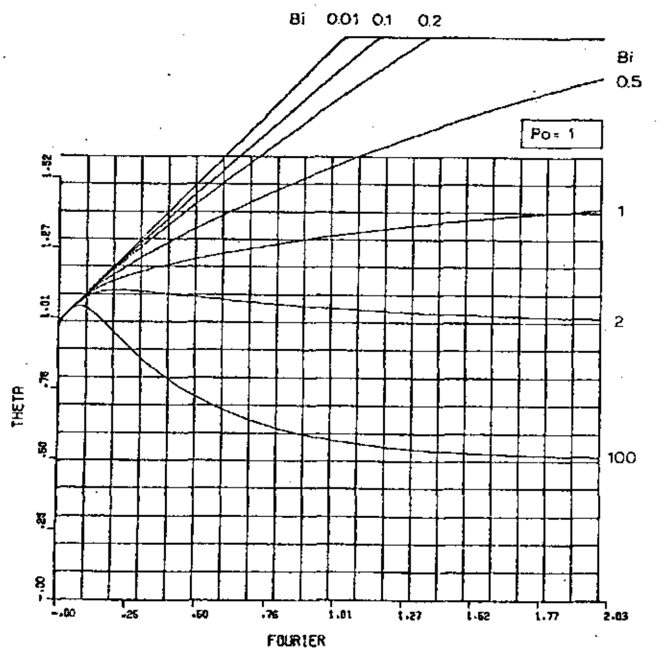


Fig. 2. Cooling curves of a heat generating slab, Θ vs. Fo, parameter: Bi; for Po = 1.

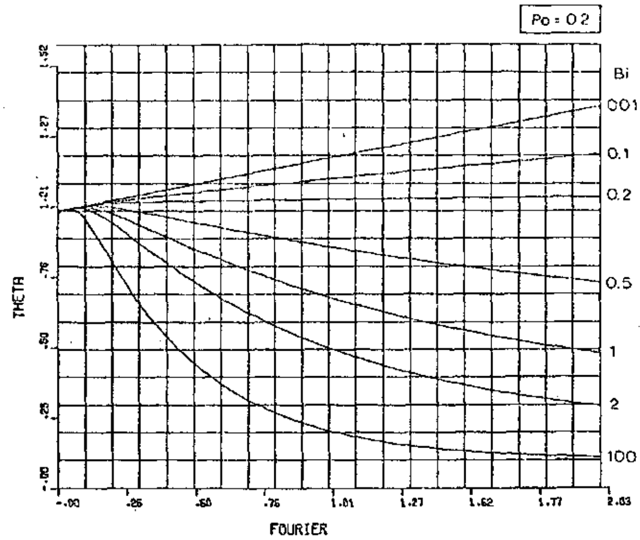


Fig. 3. Cooling curves of a heat generating slab,
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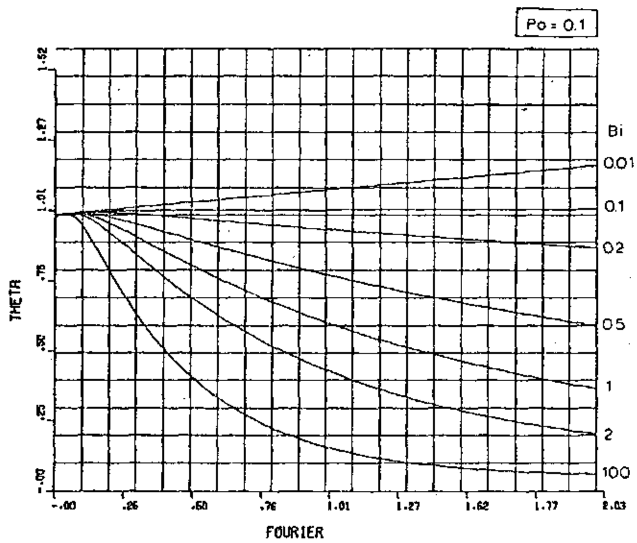


Fig. 4. Cooling curves of a heat generating slab,
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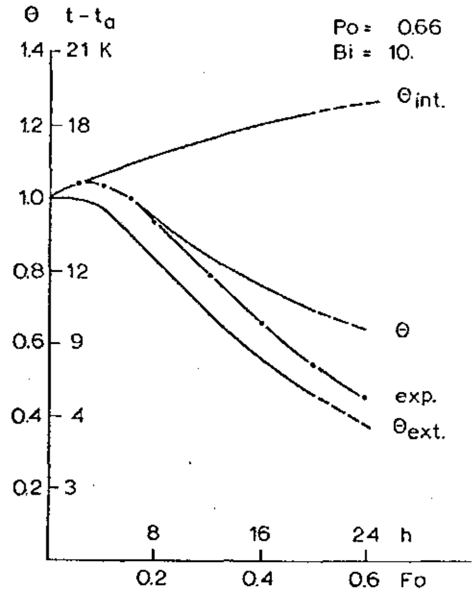
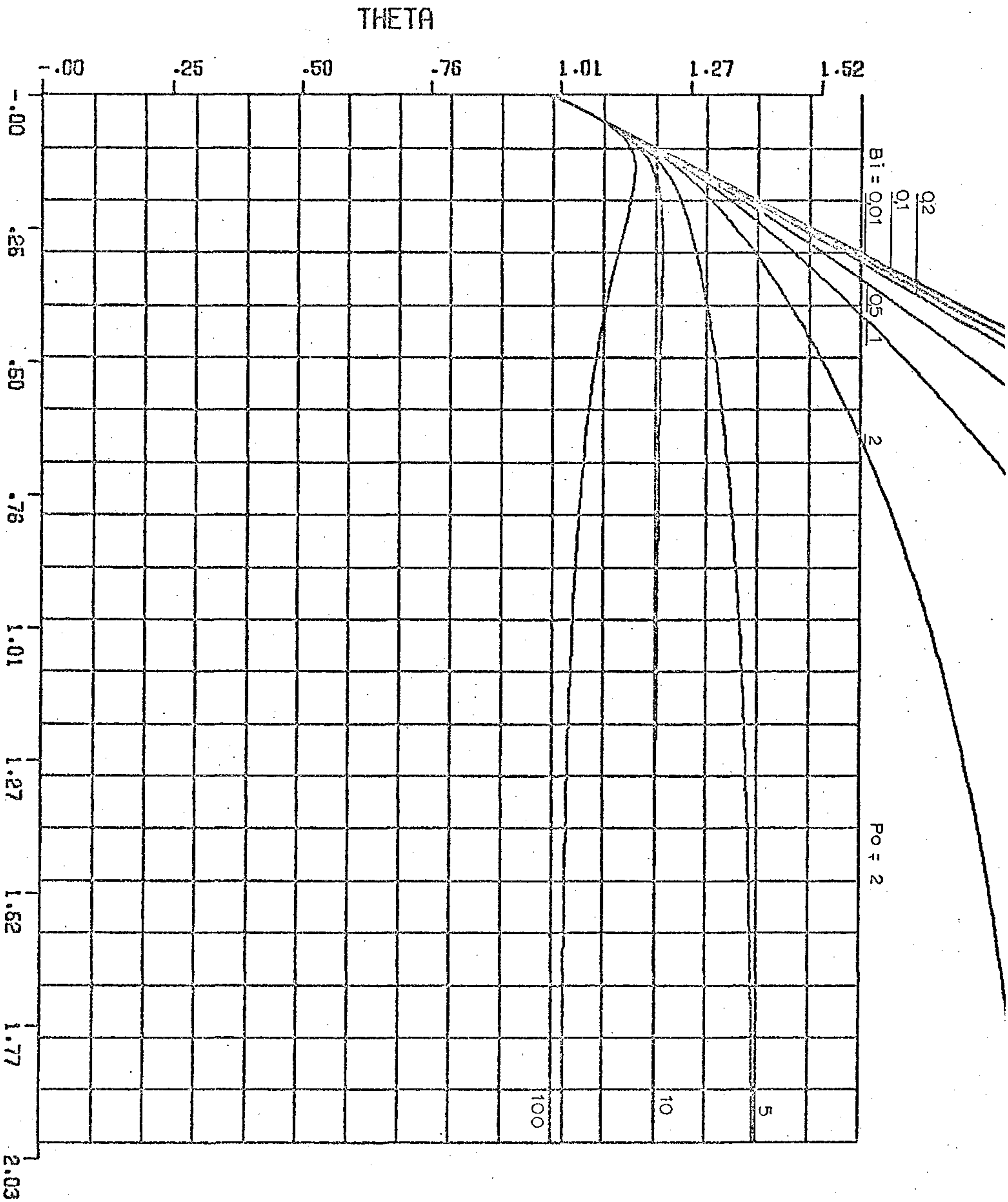



Fig. 5. Cooling curve of a box filled with flowers, t(Θ) vs. τ (Fo). See text page 4.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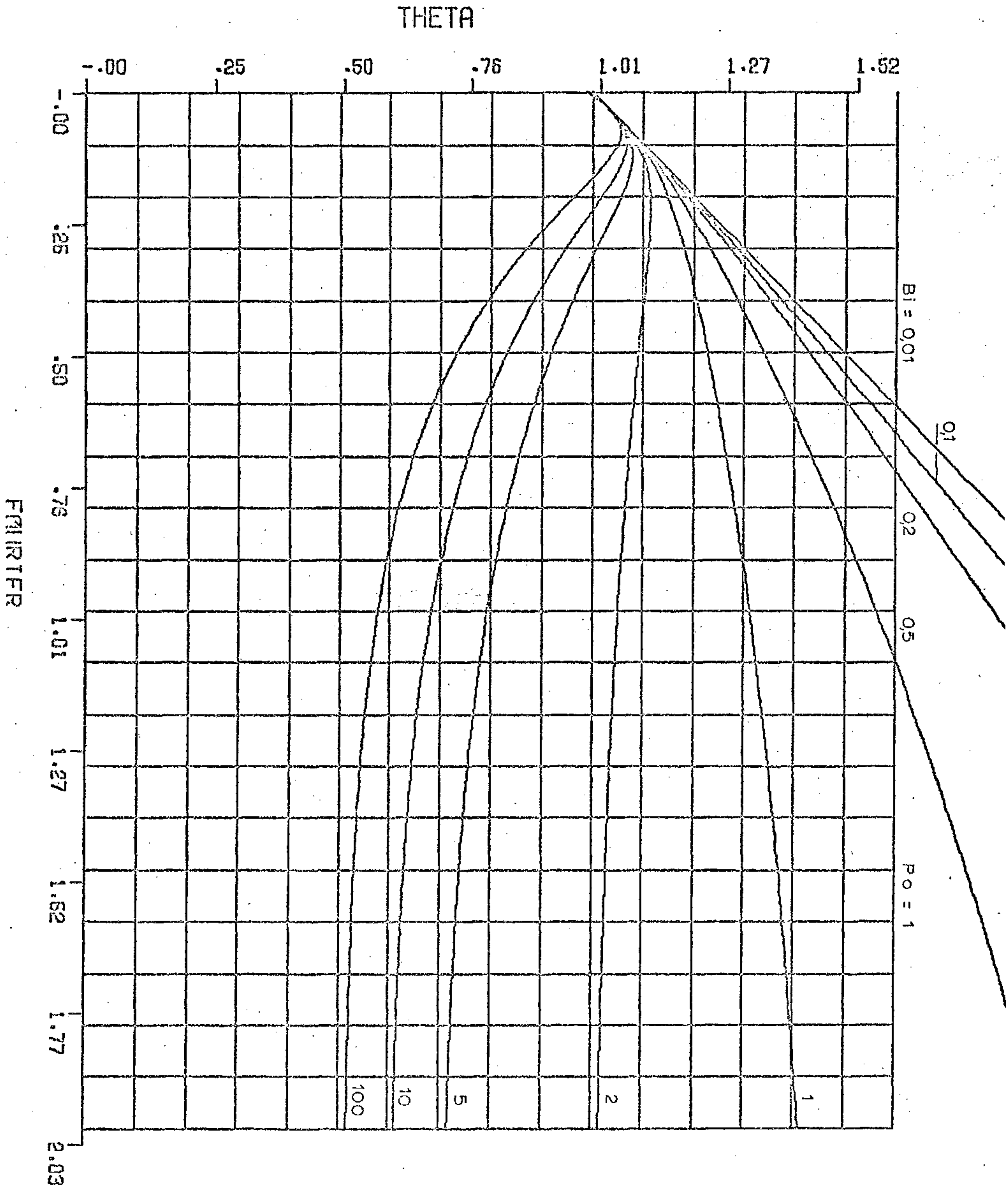
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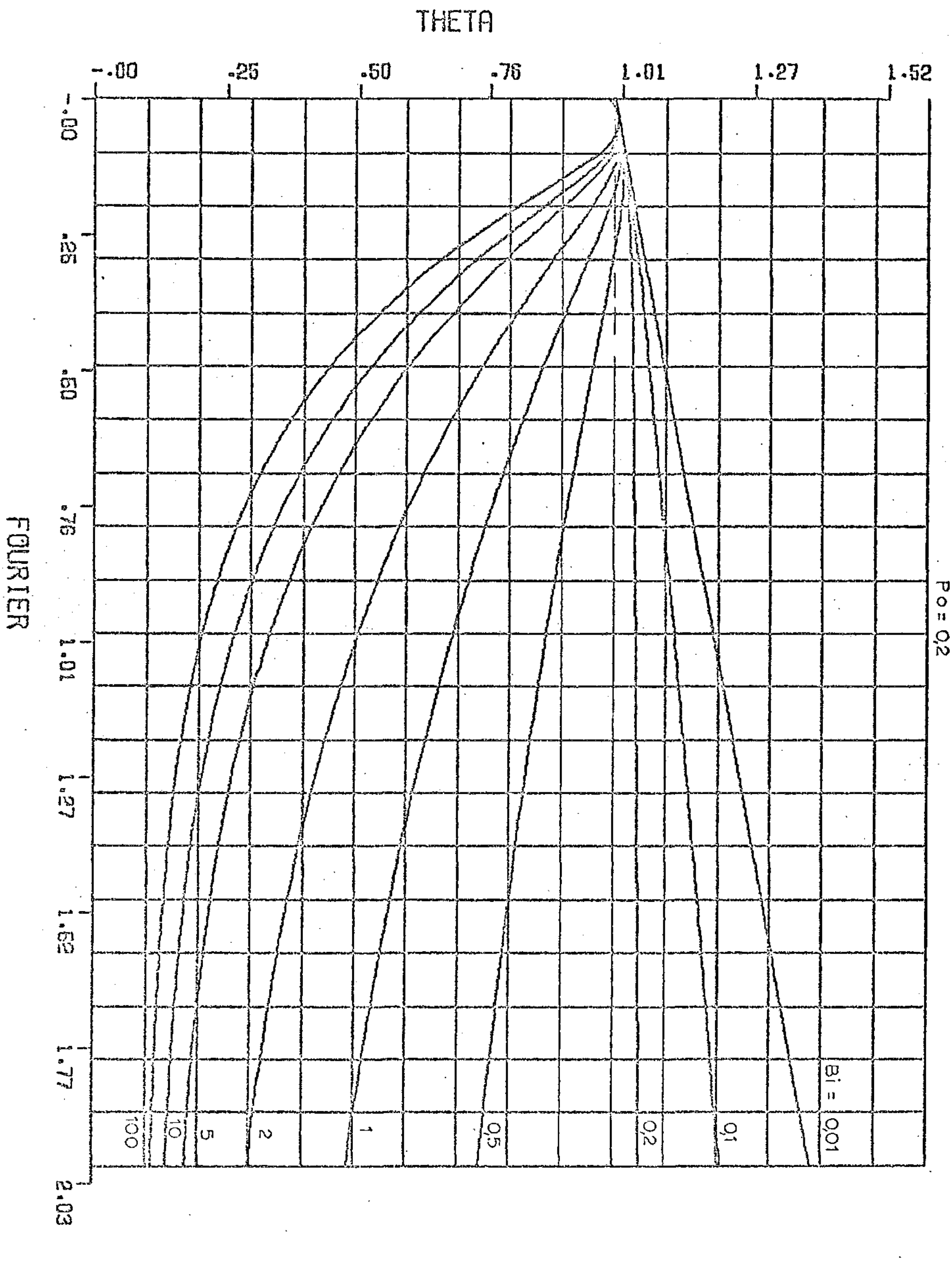
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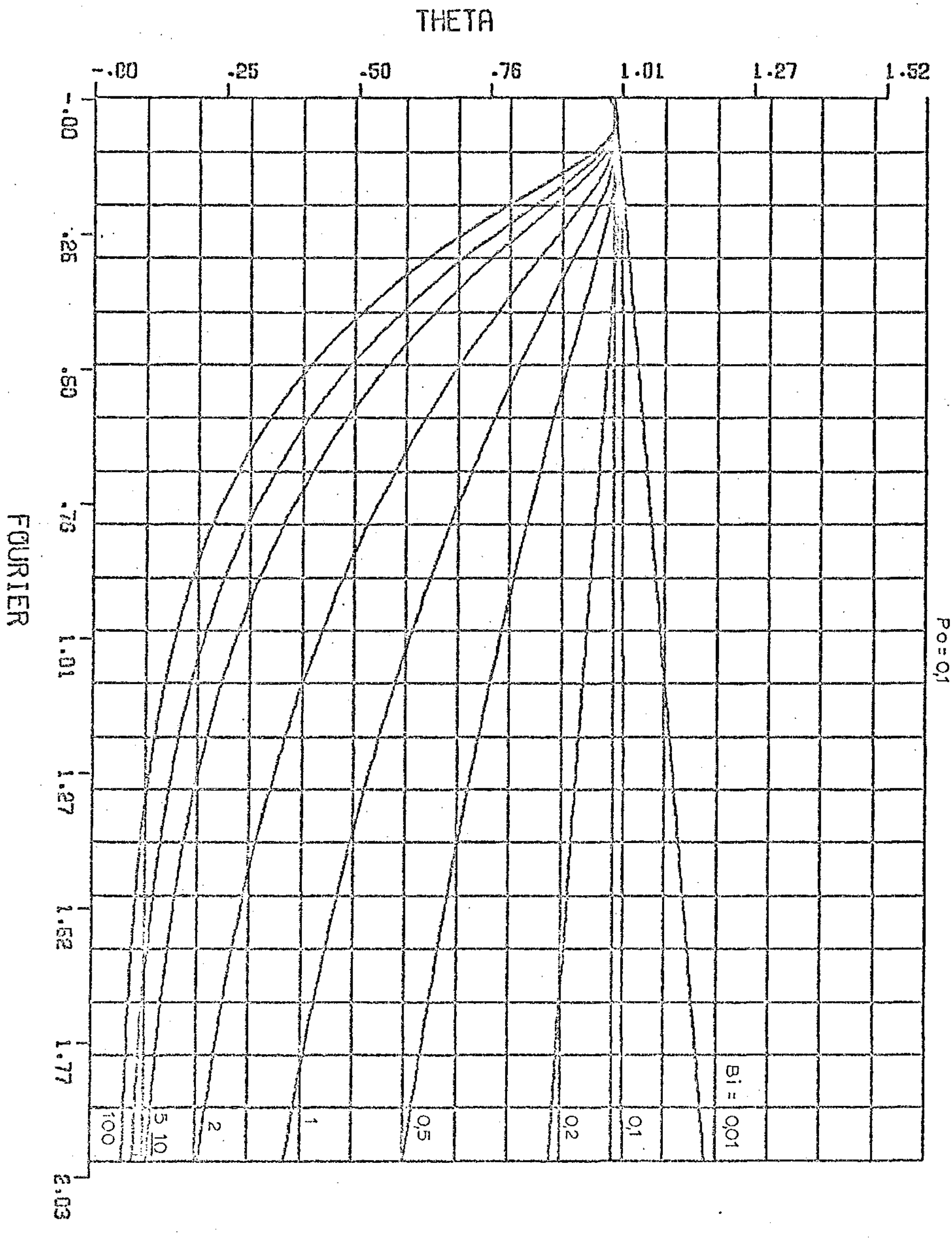
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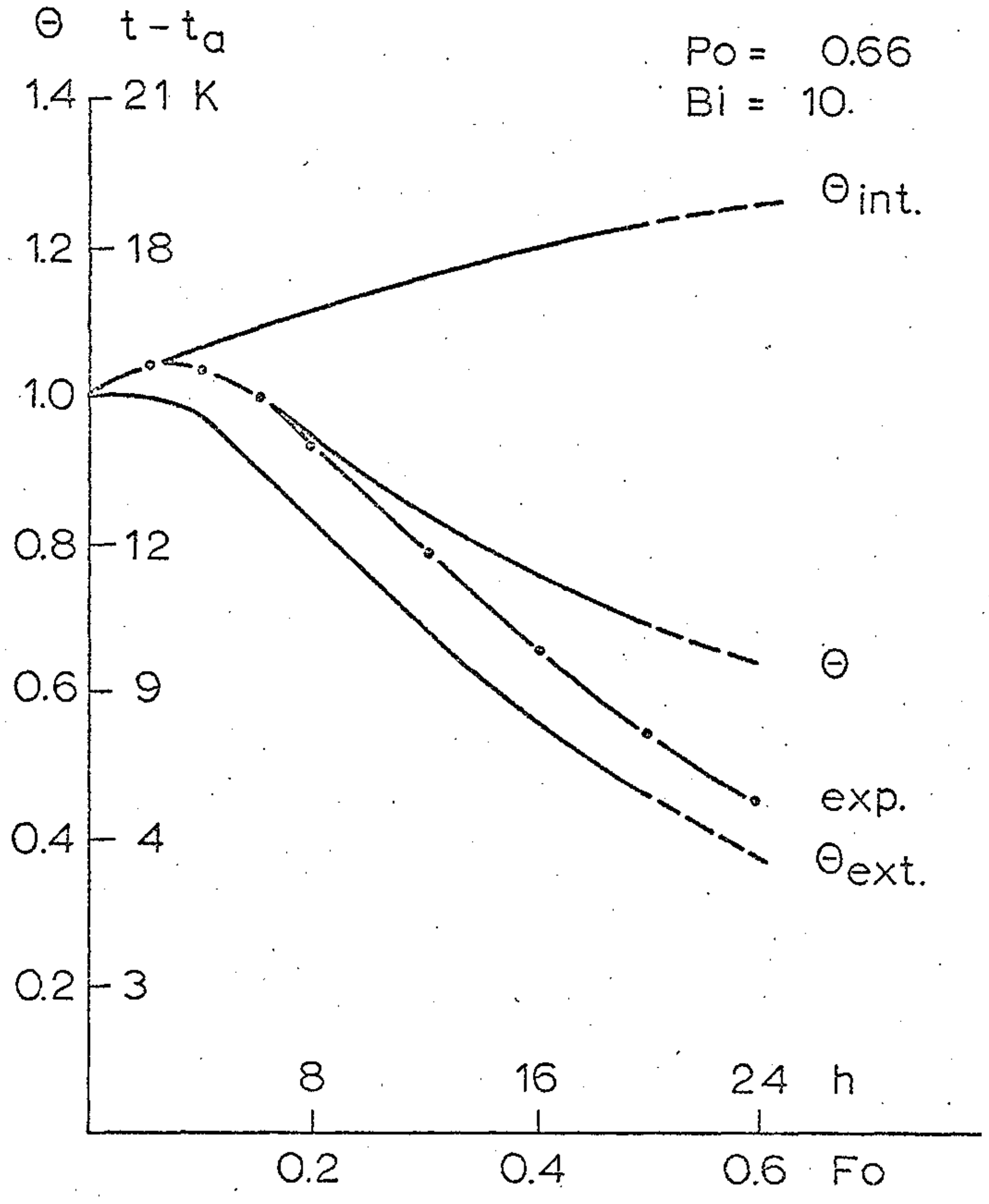
(5)



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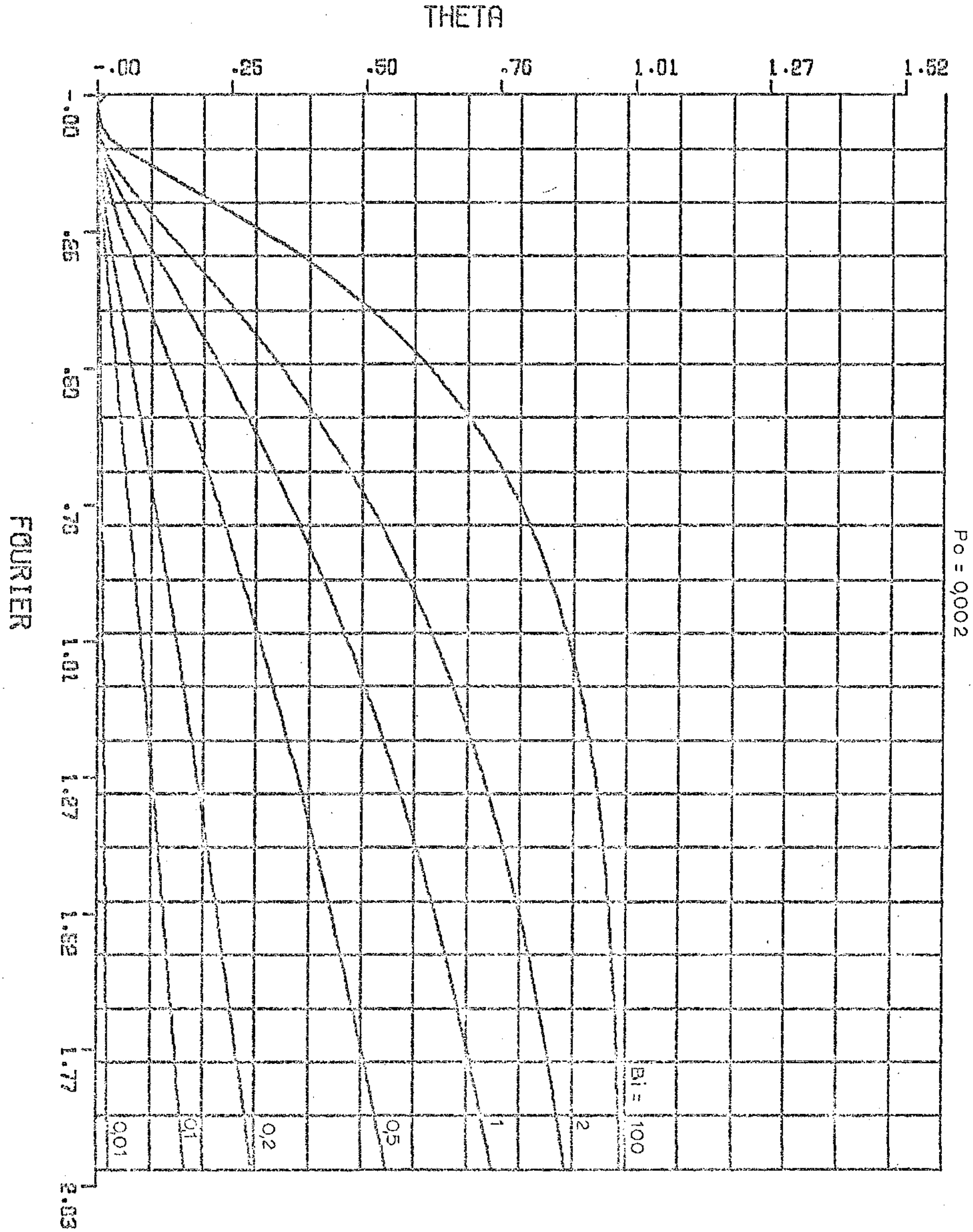
Fo

opwarmkrommen voor de centrum-
temperatuur van een plaat

Serie I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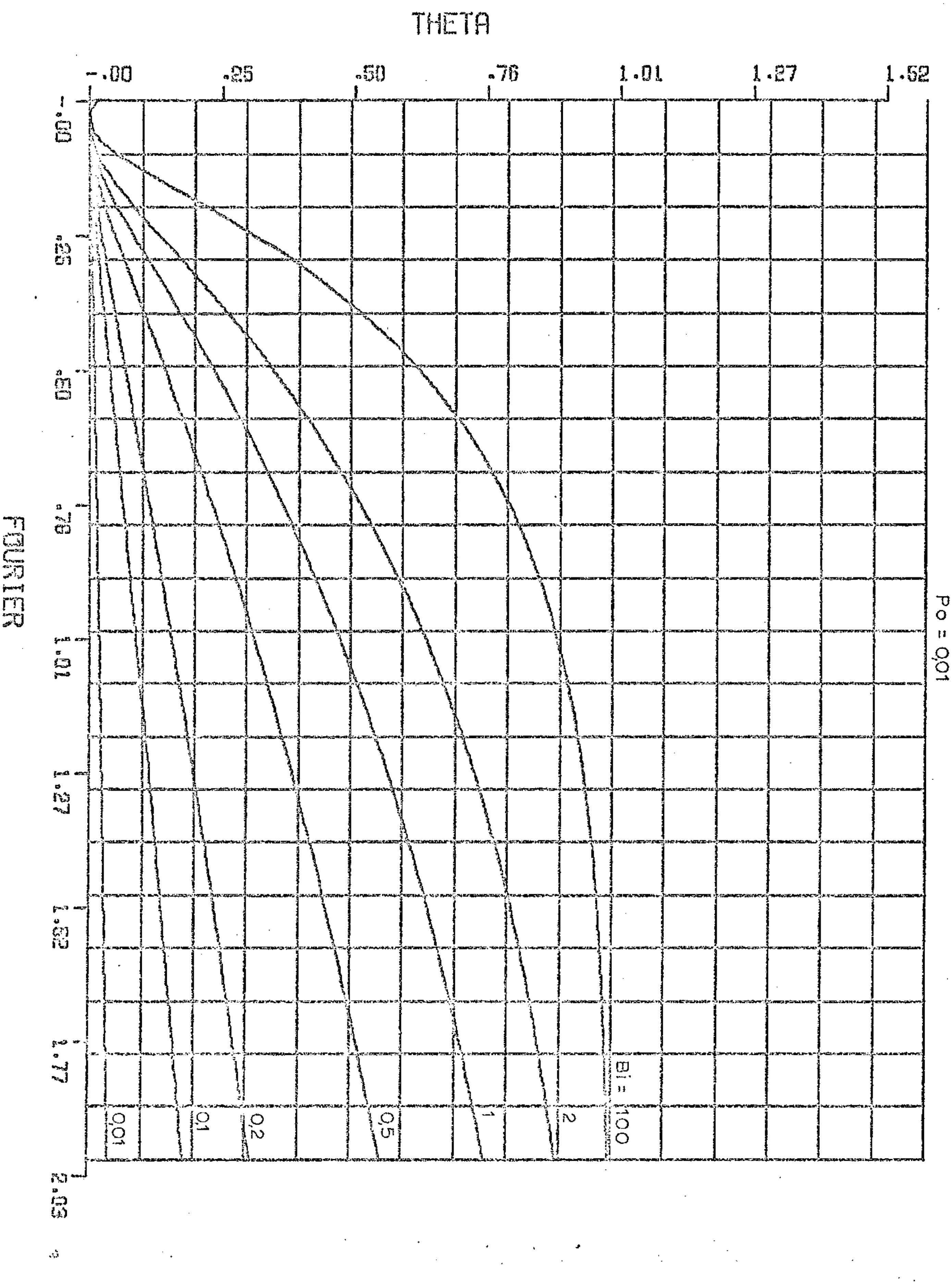
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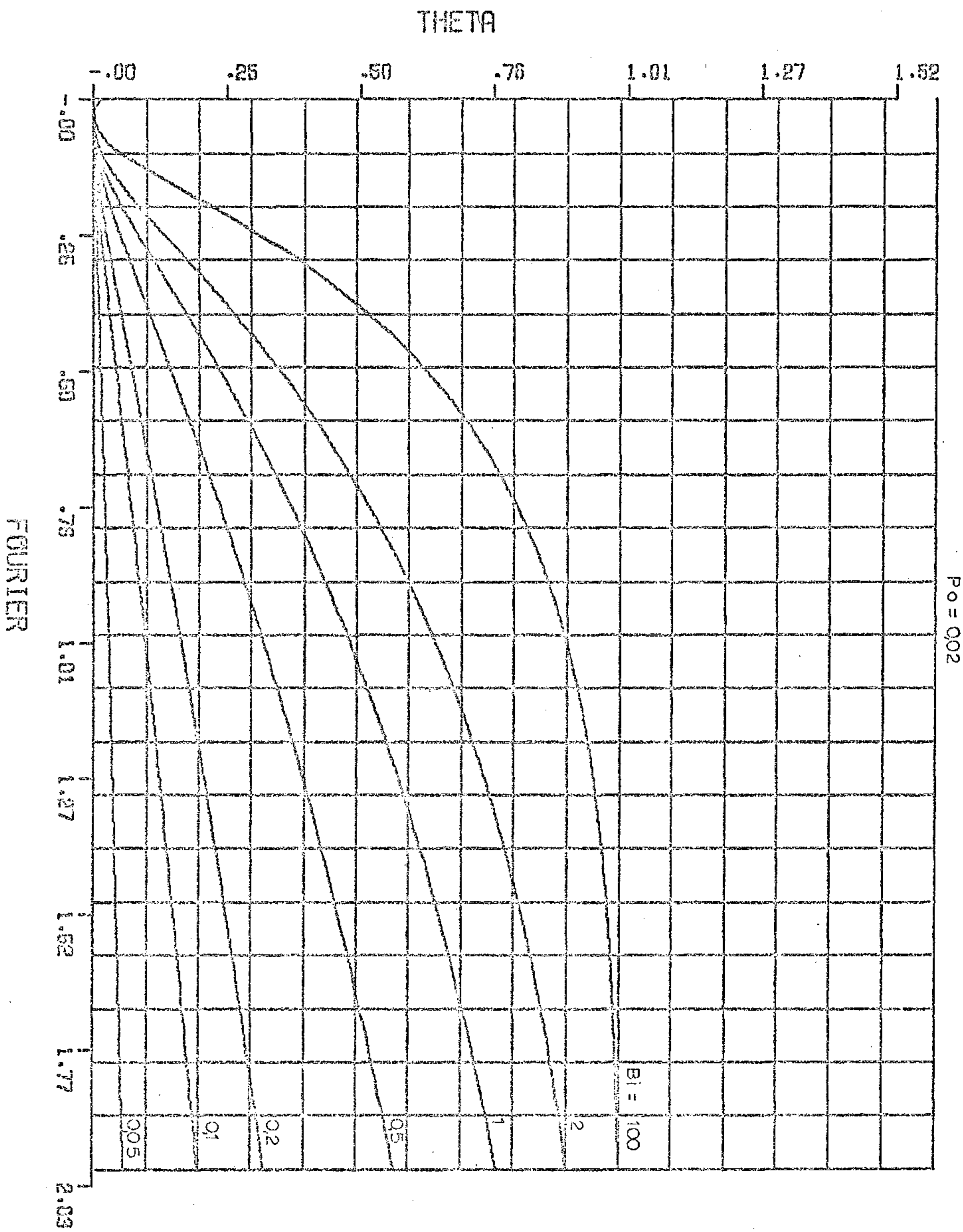

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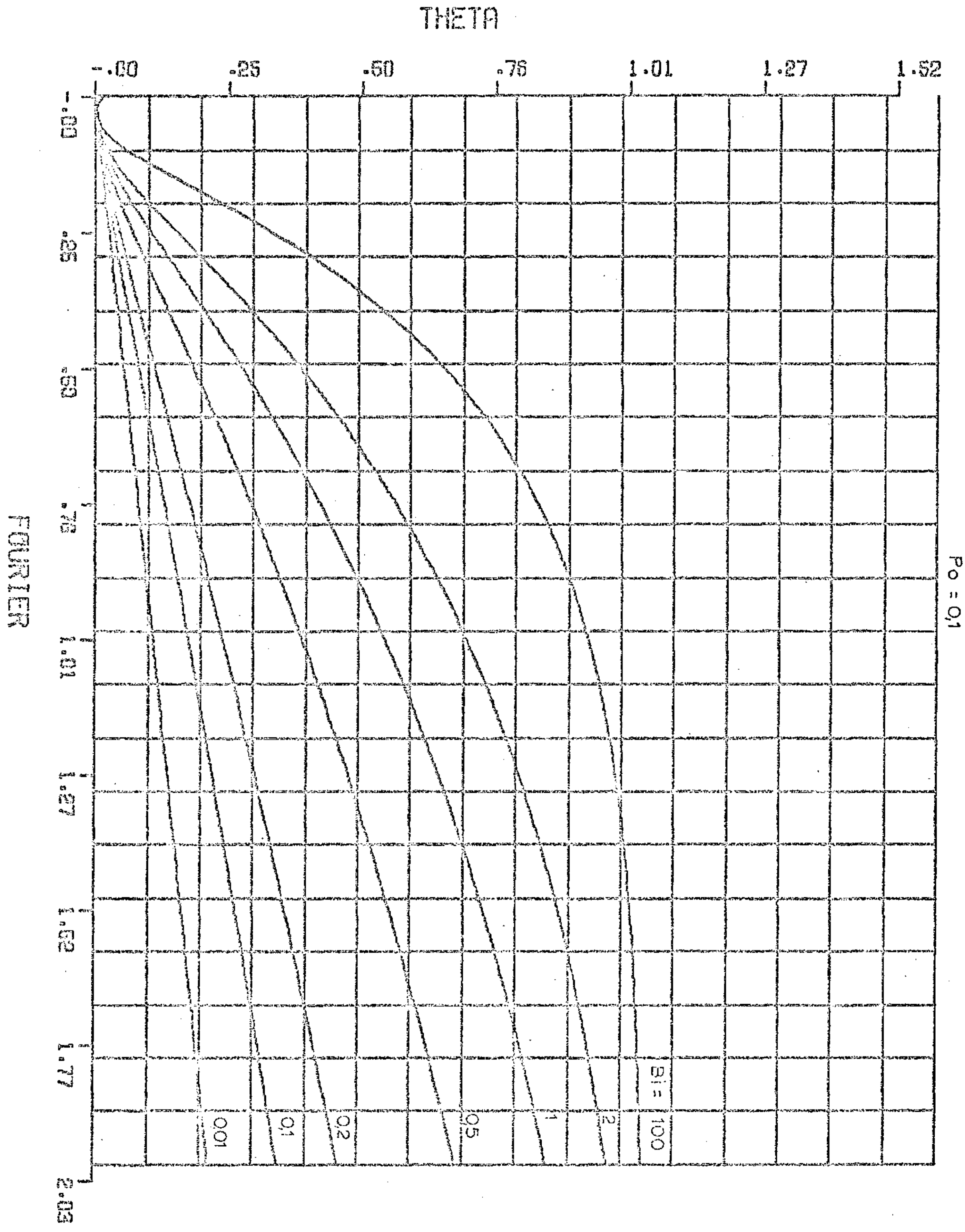
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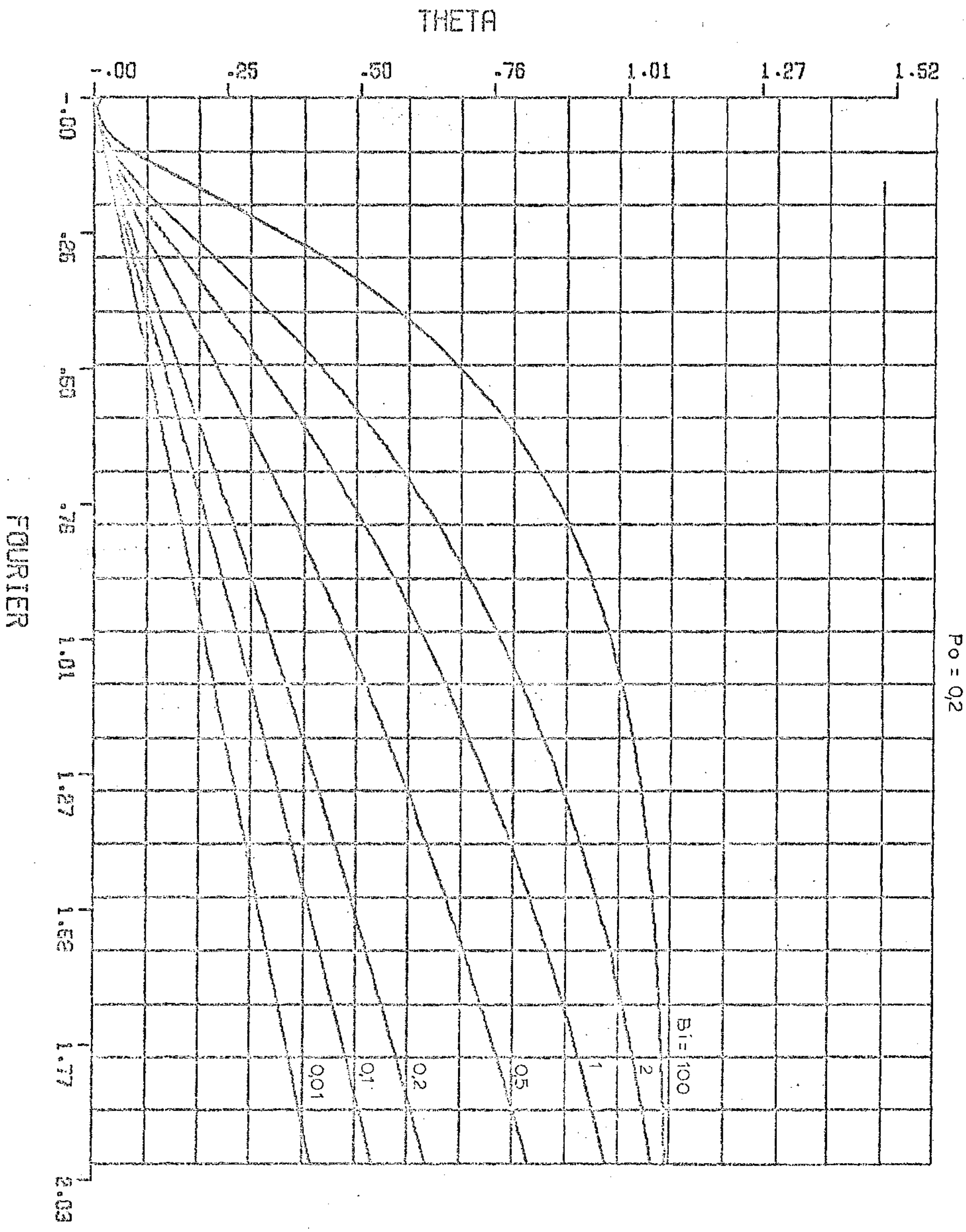
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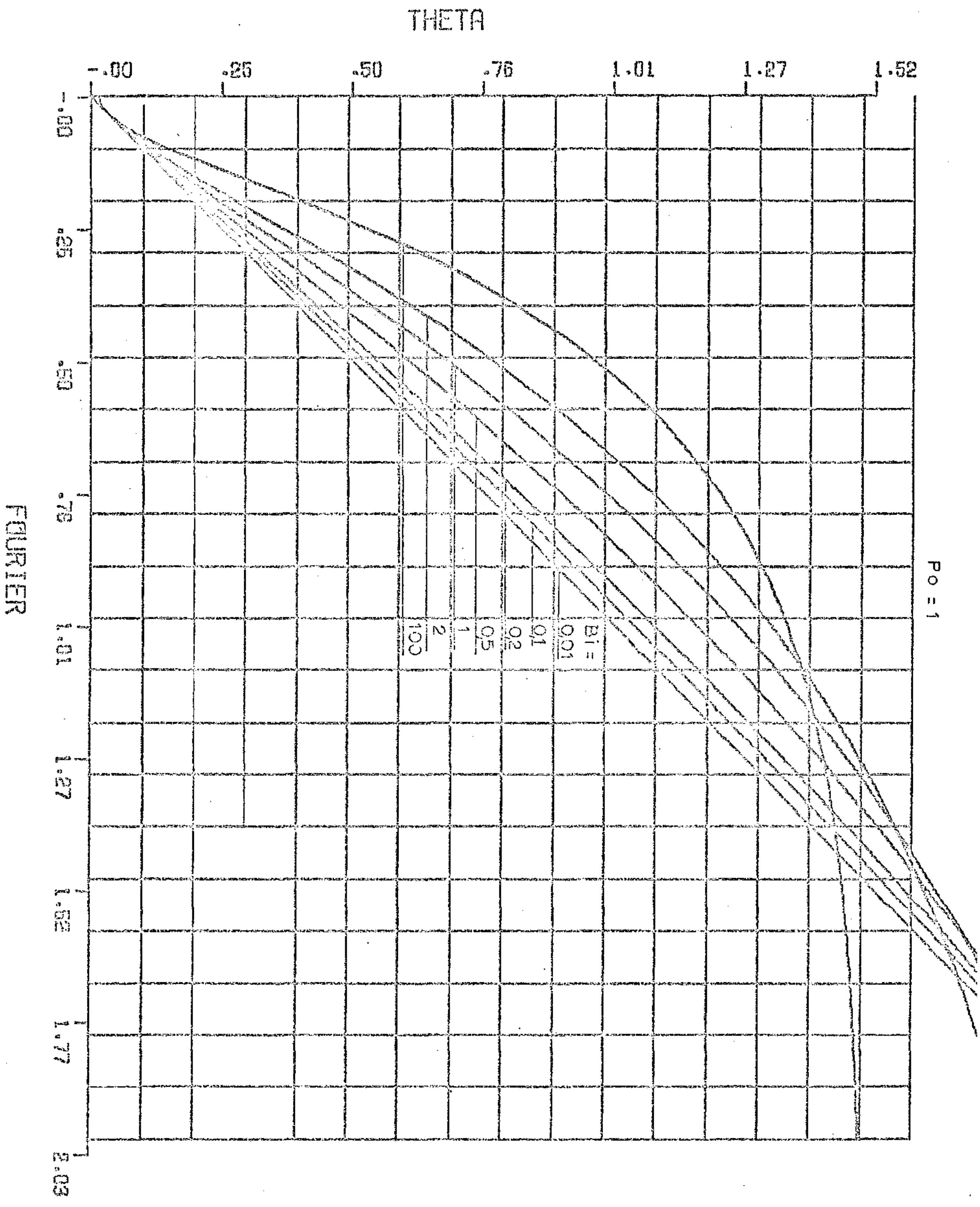

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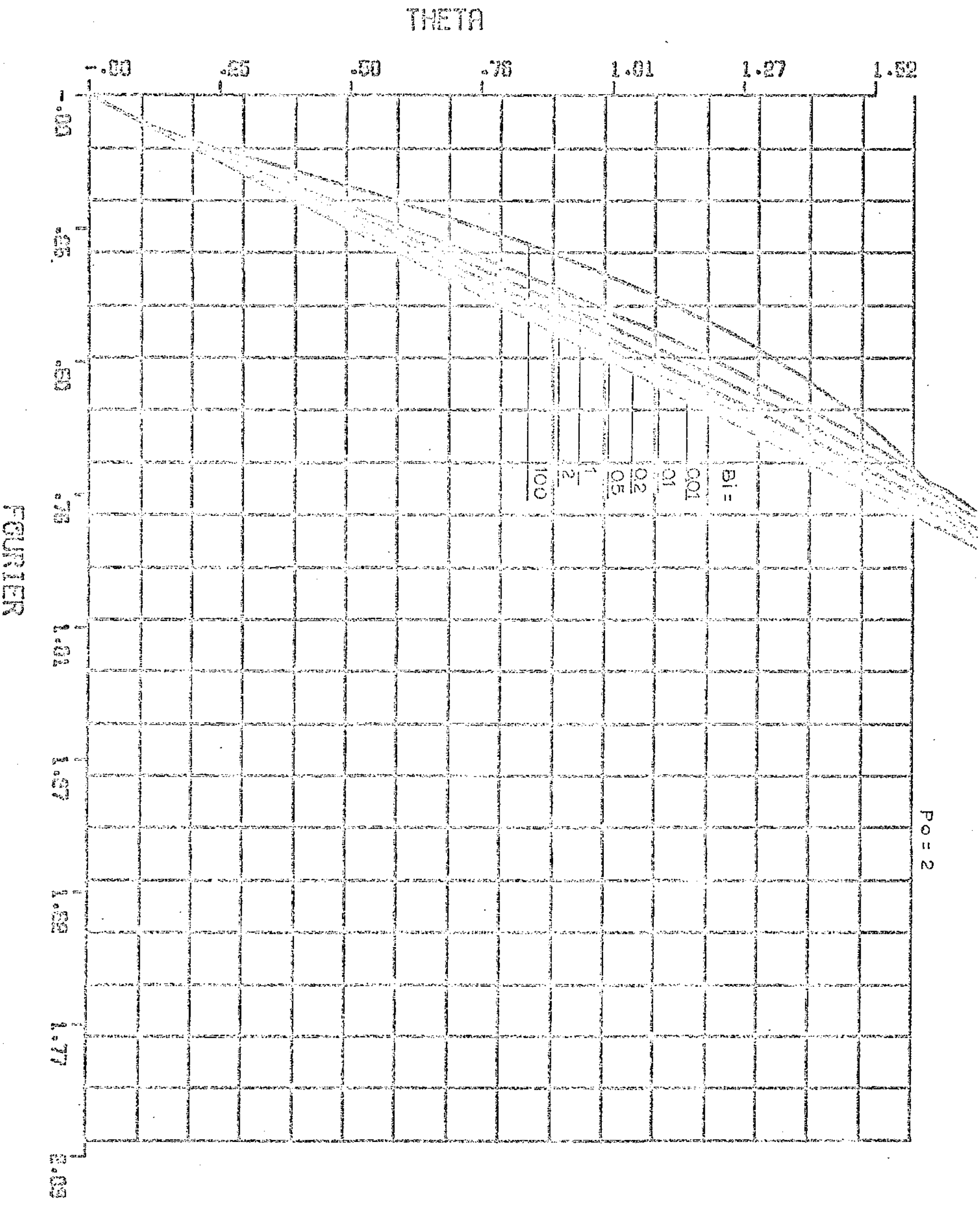
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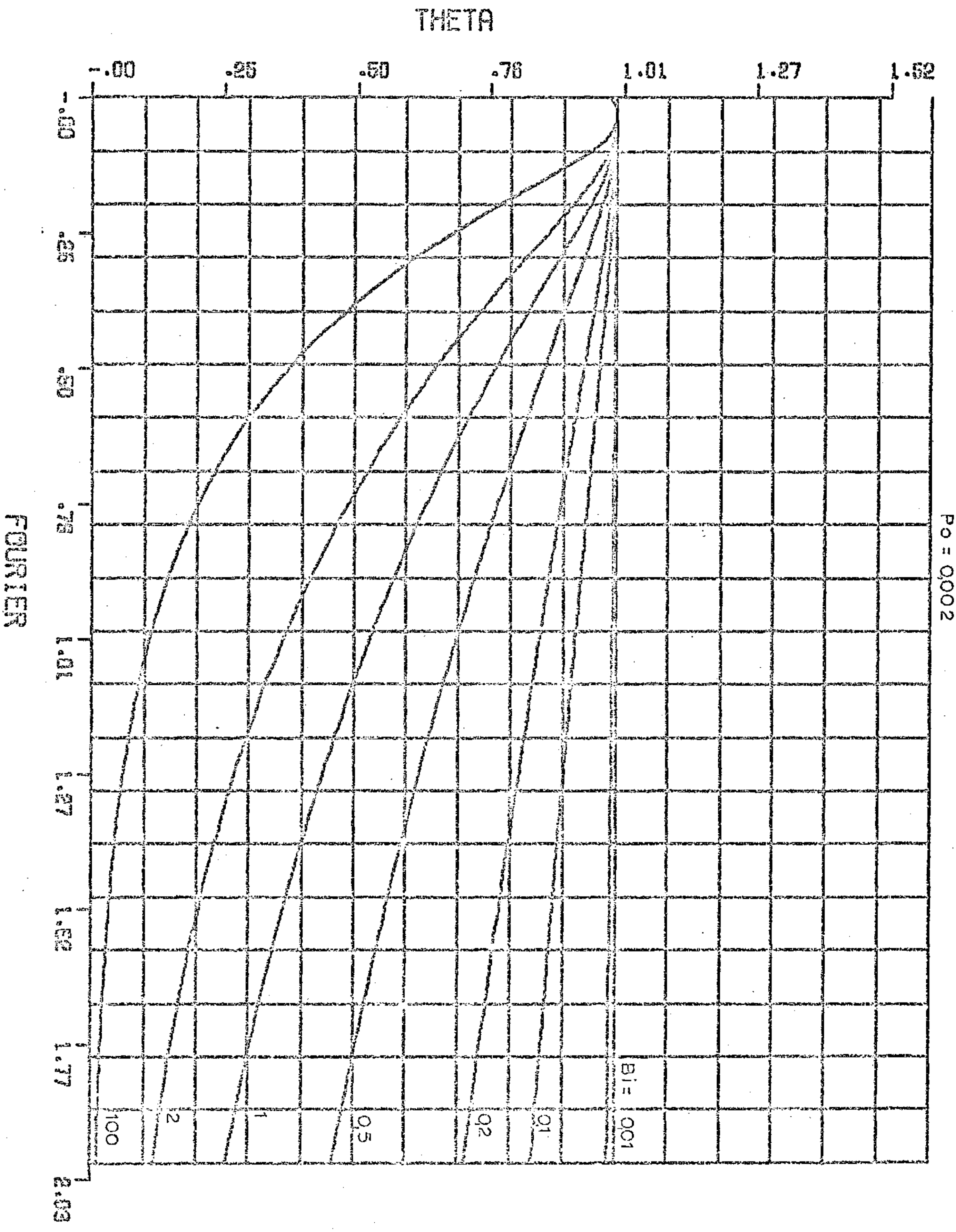
Afkoelkrommen voor de centrumtemperatuur van een plaat

Serie I



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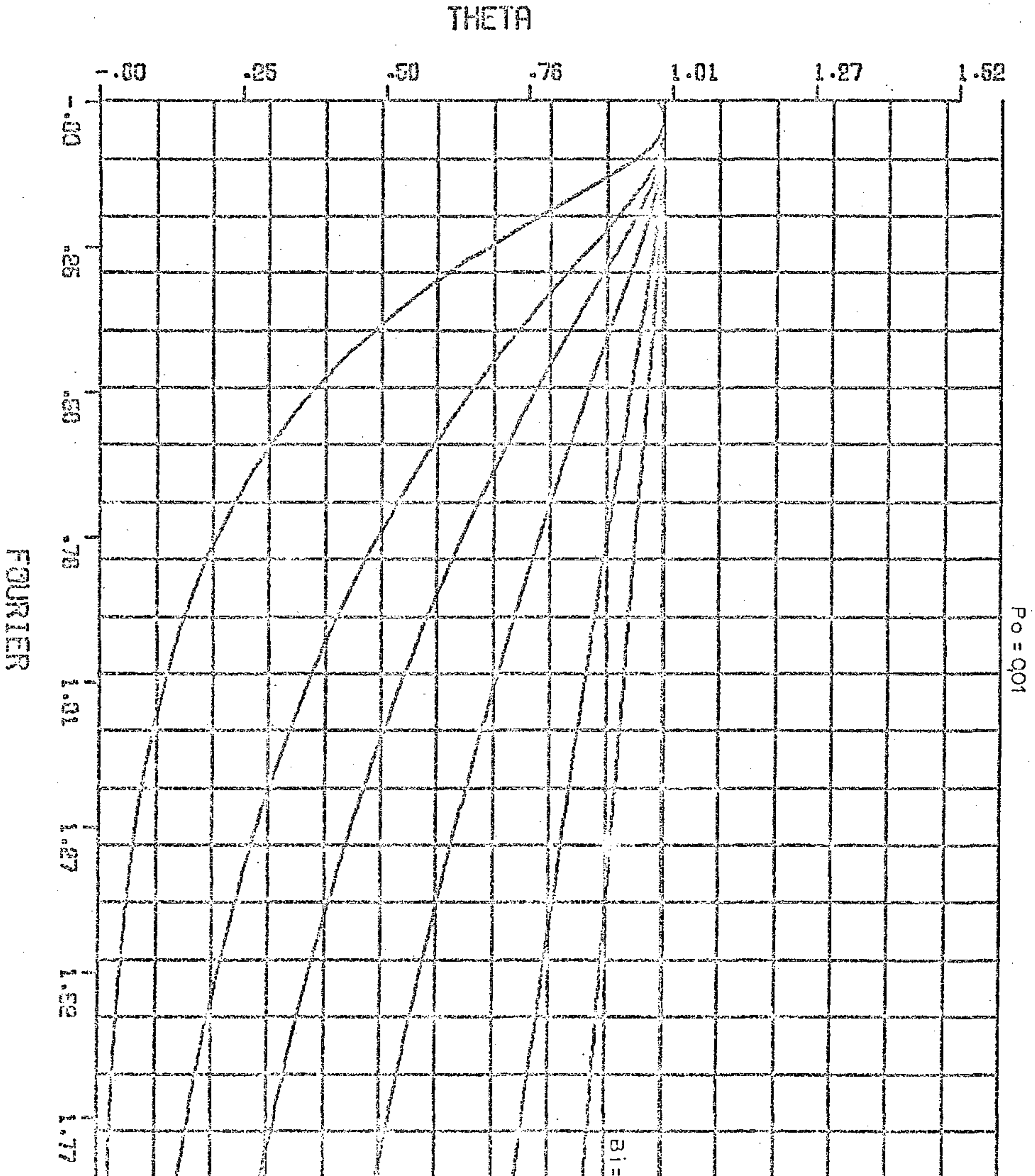
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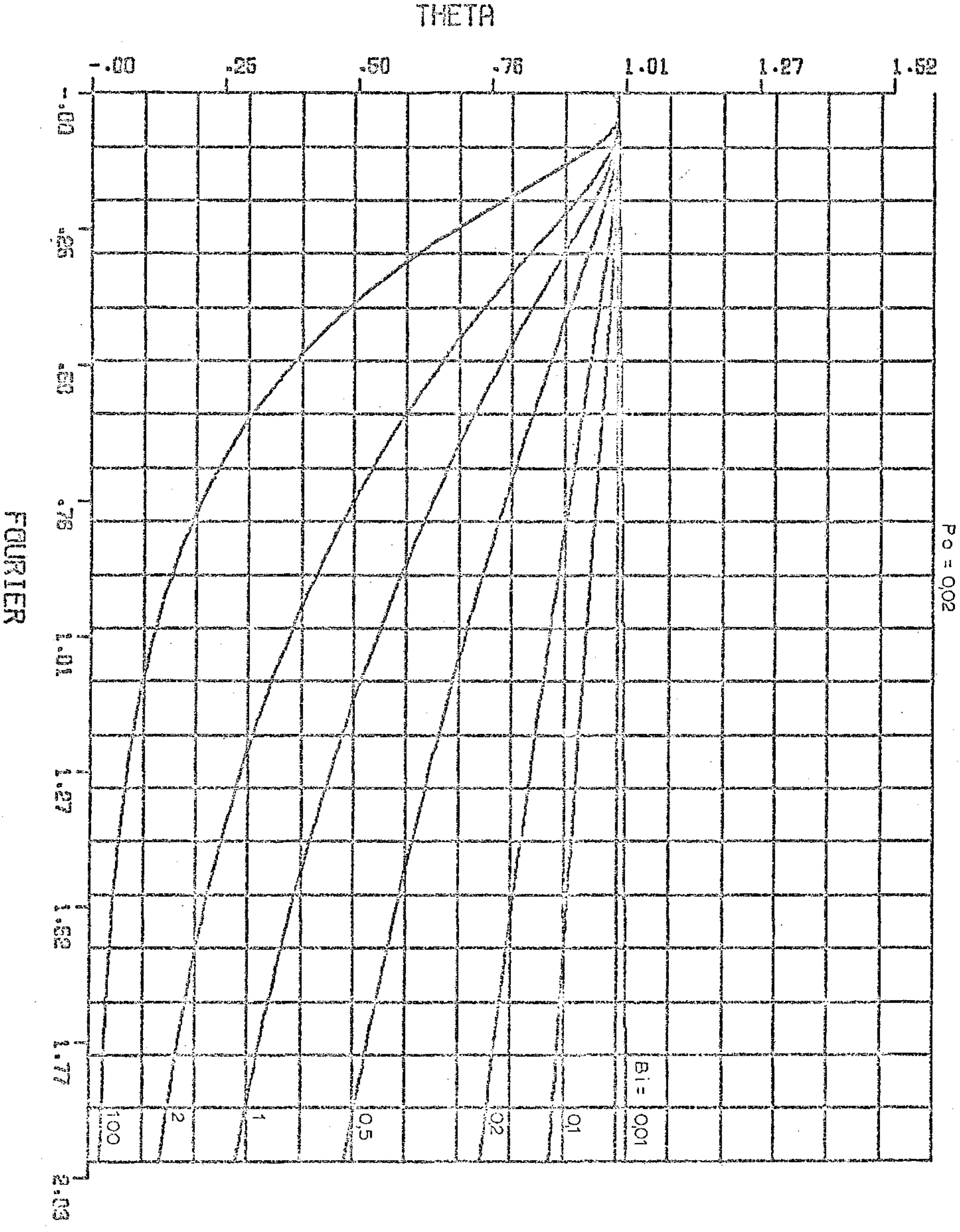
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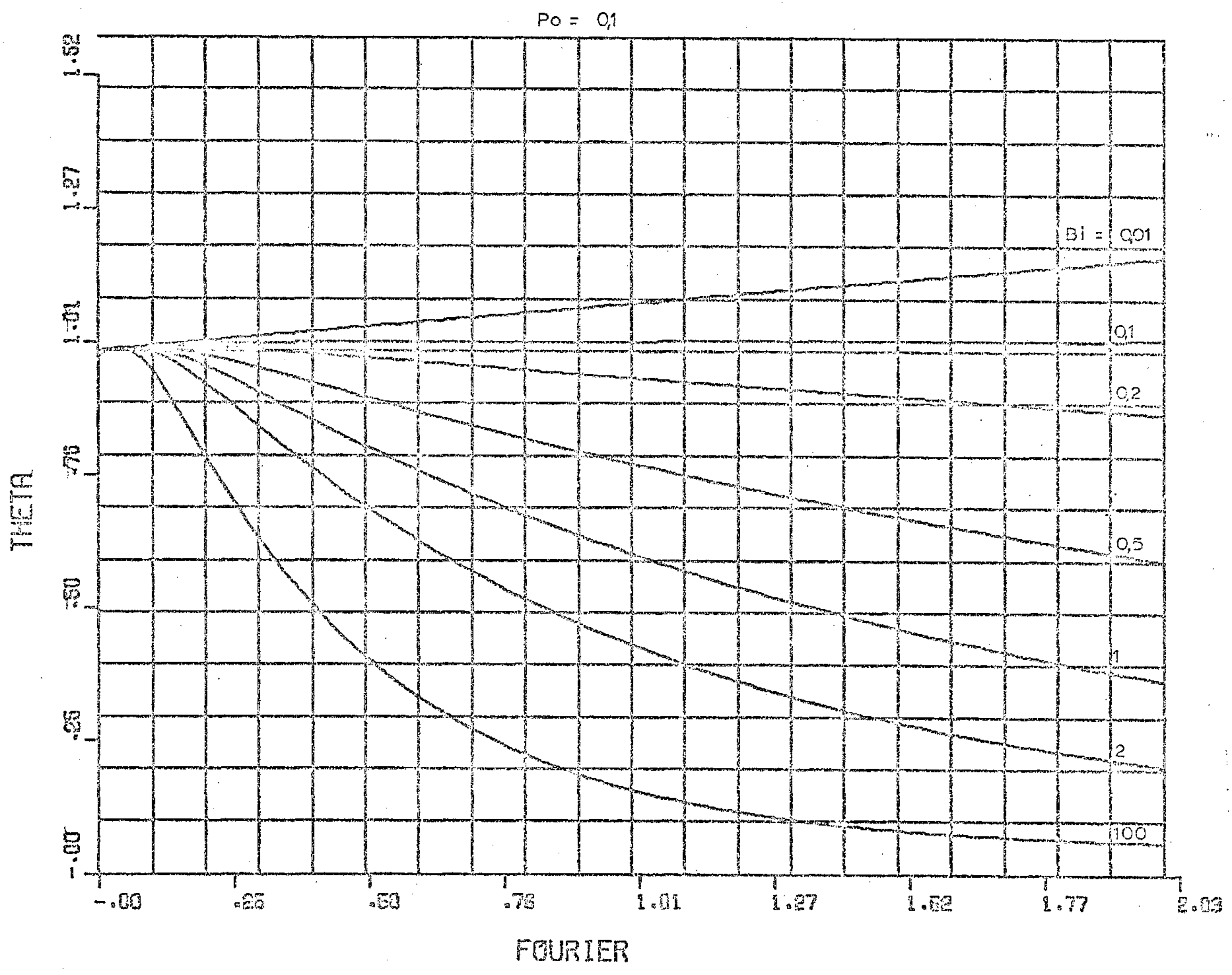
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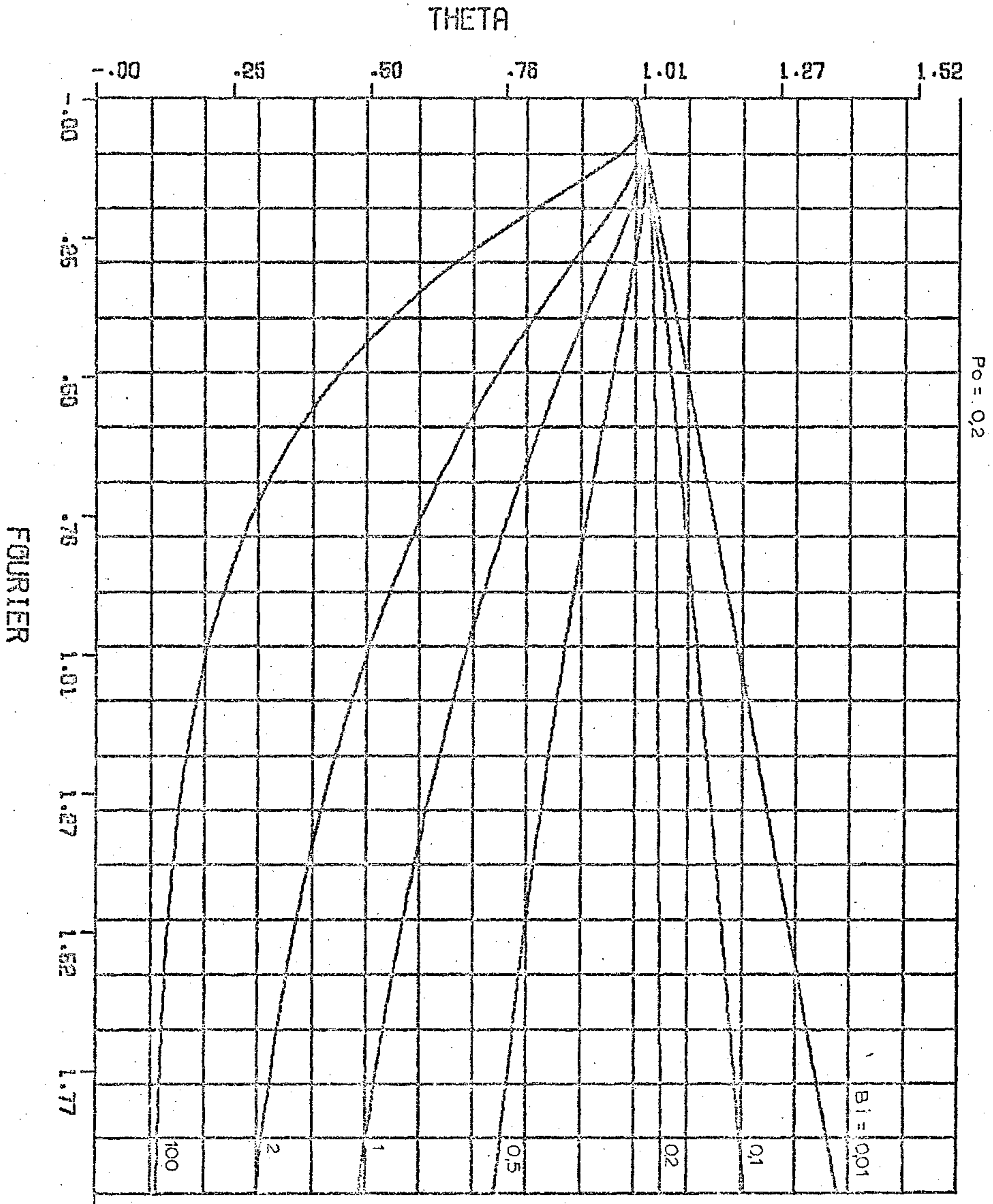

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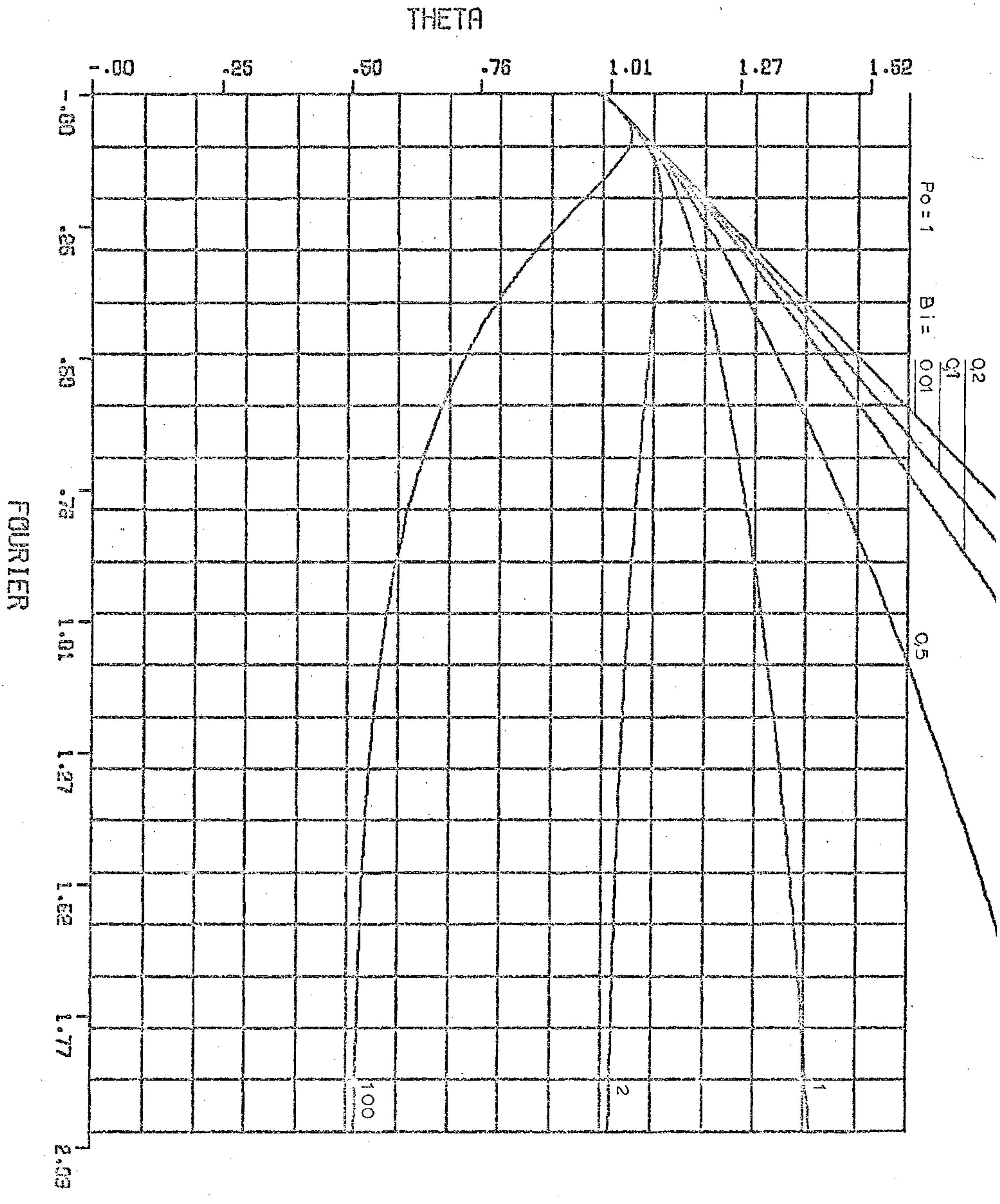
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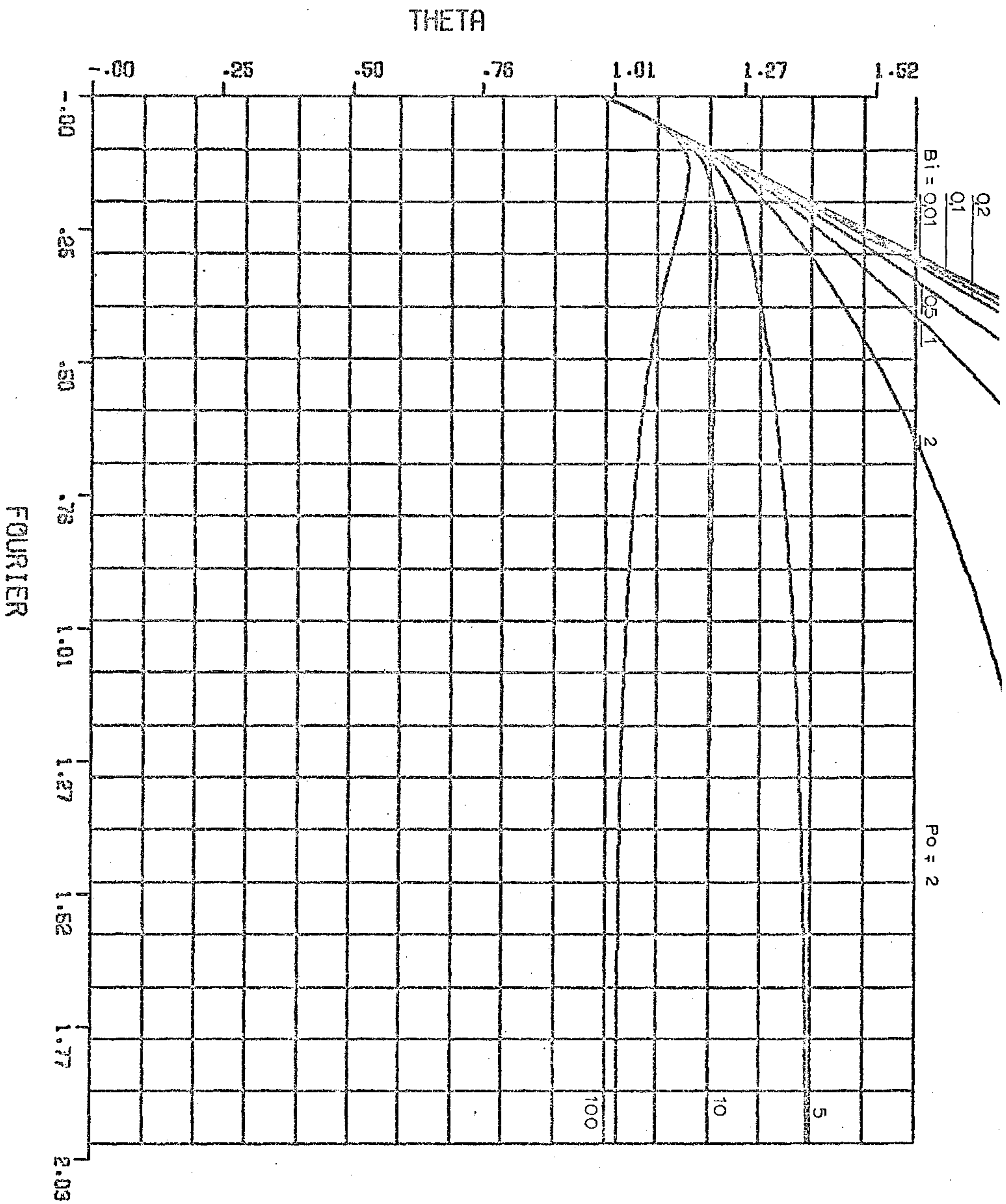
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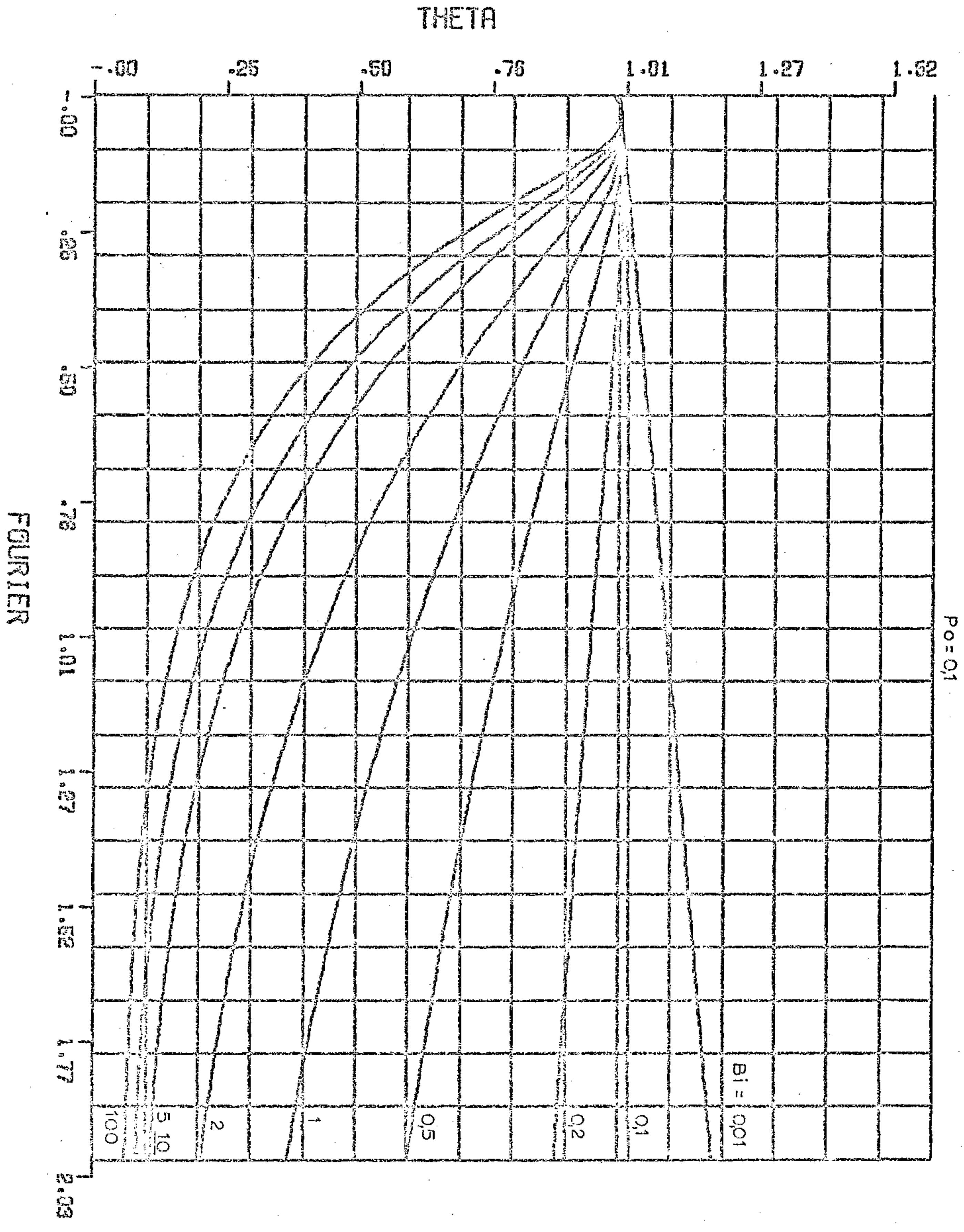
Afkoelkrommen voor de centrum-
temperatuur van een plaat

Serie II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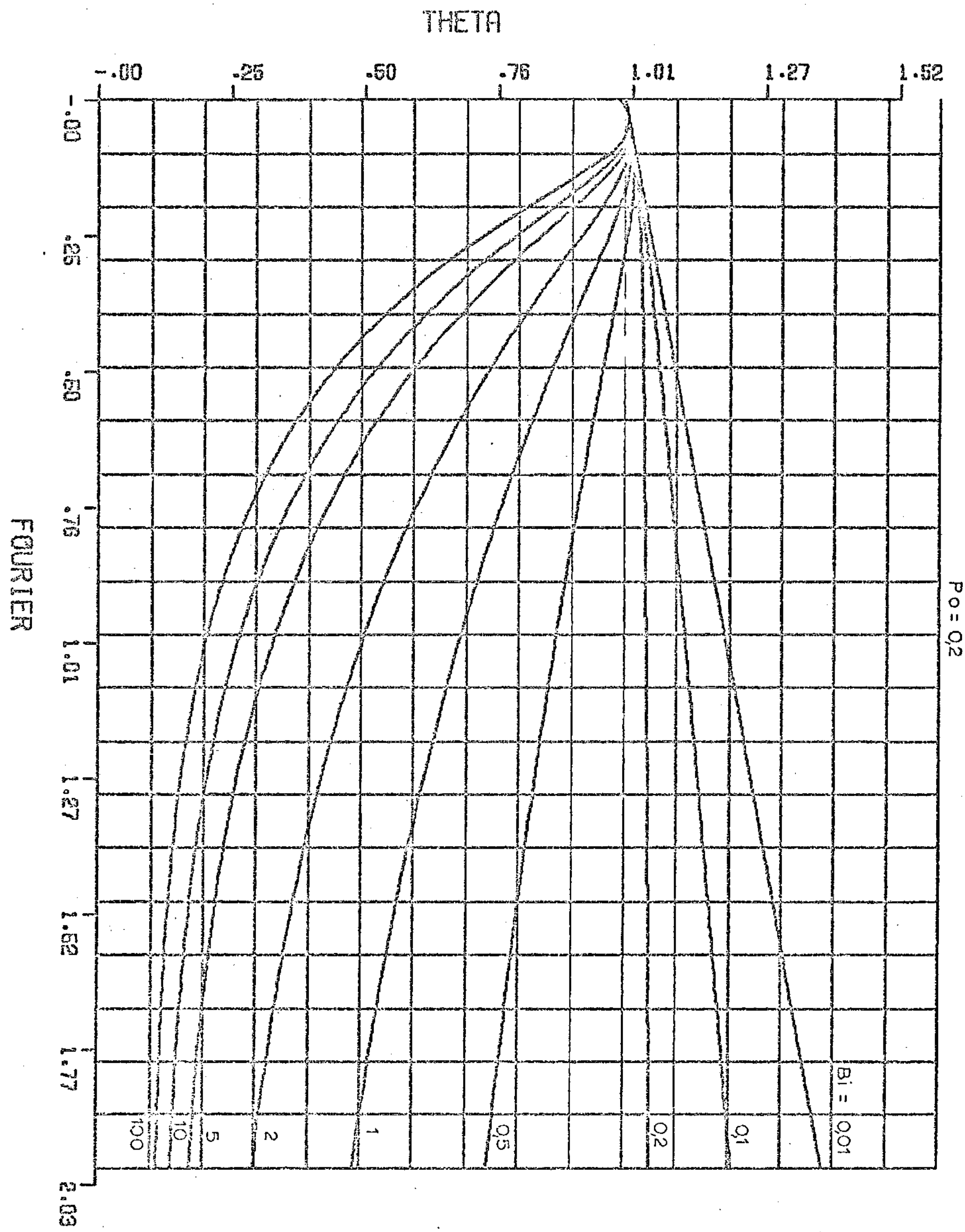
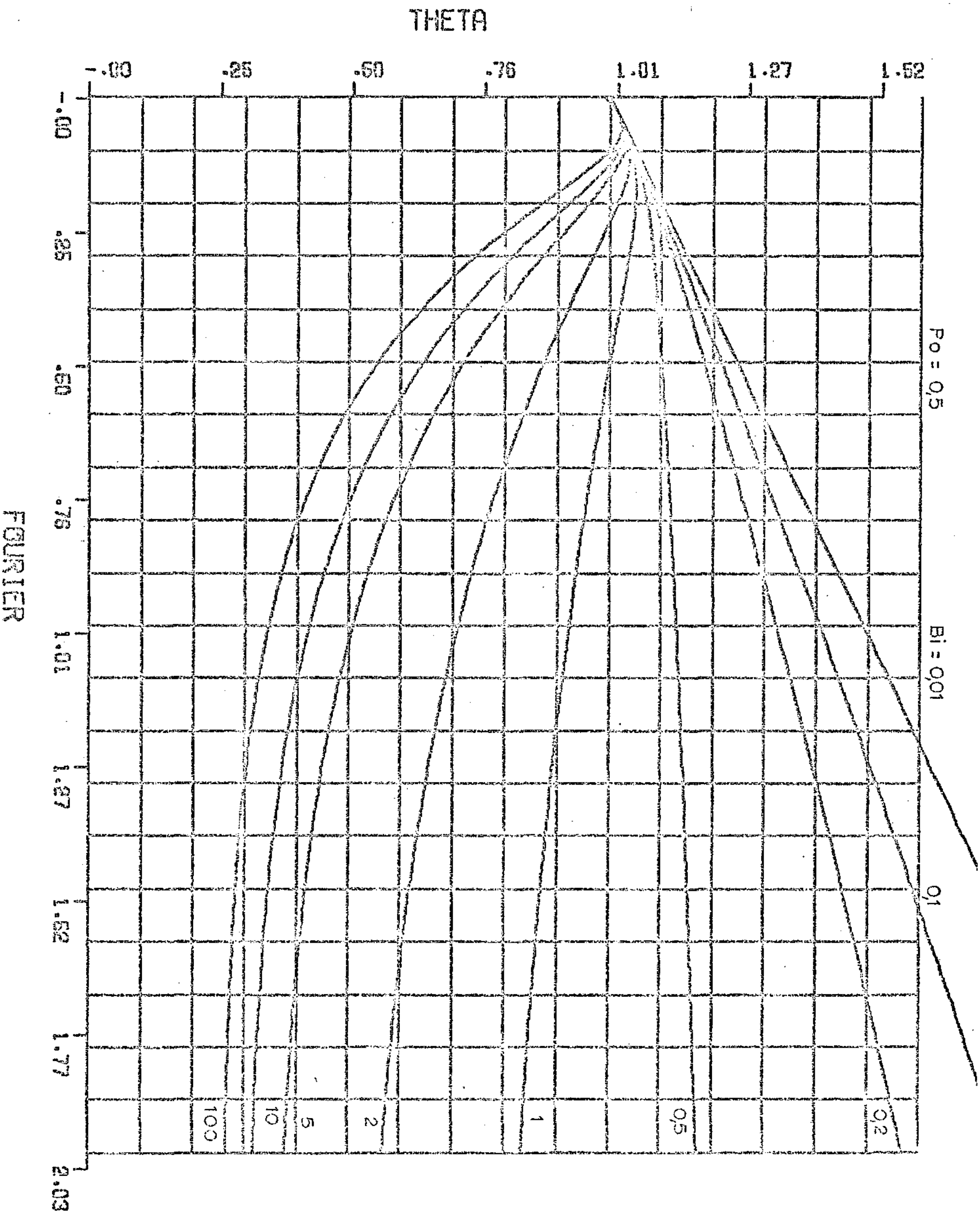
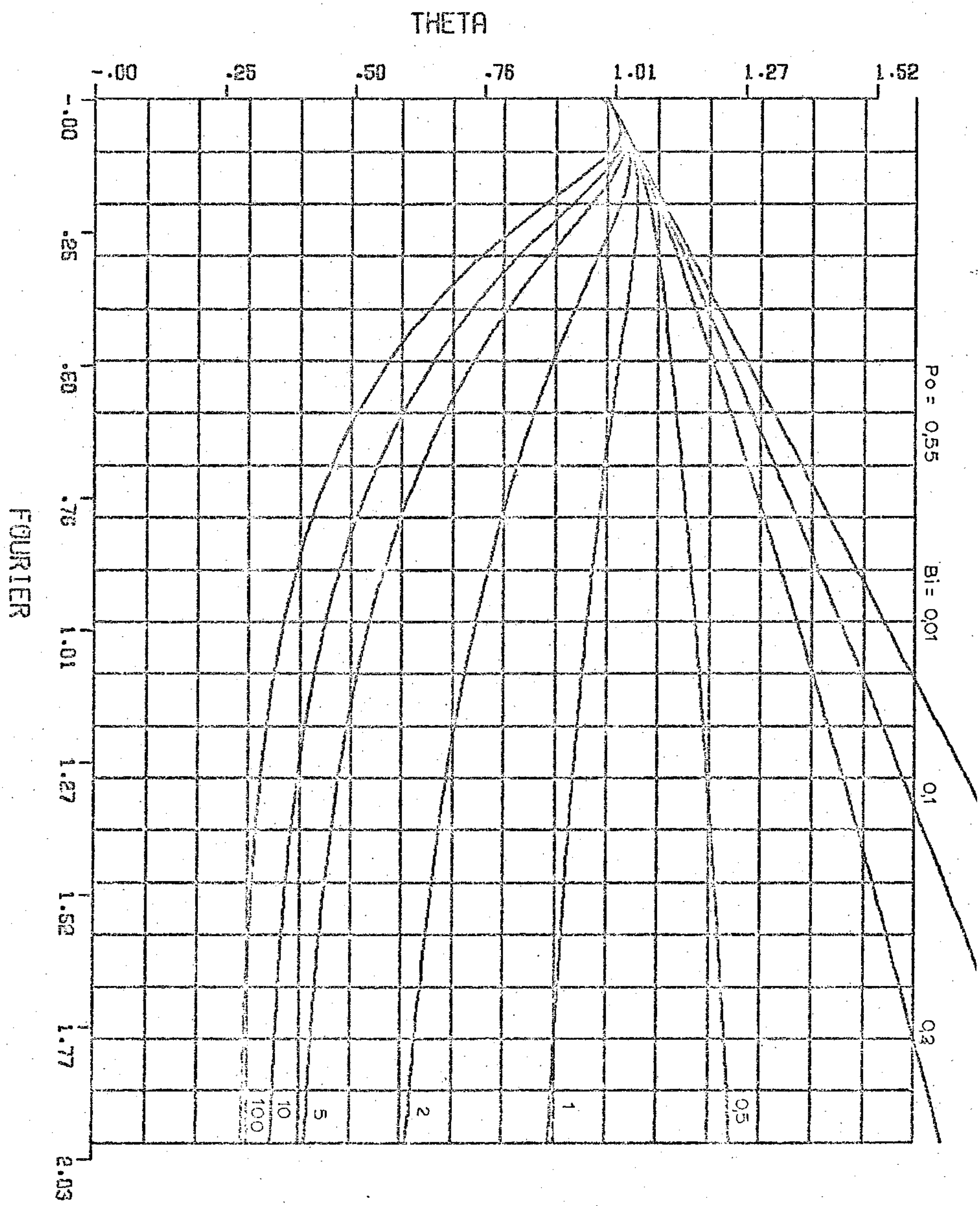
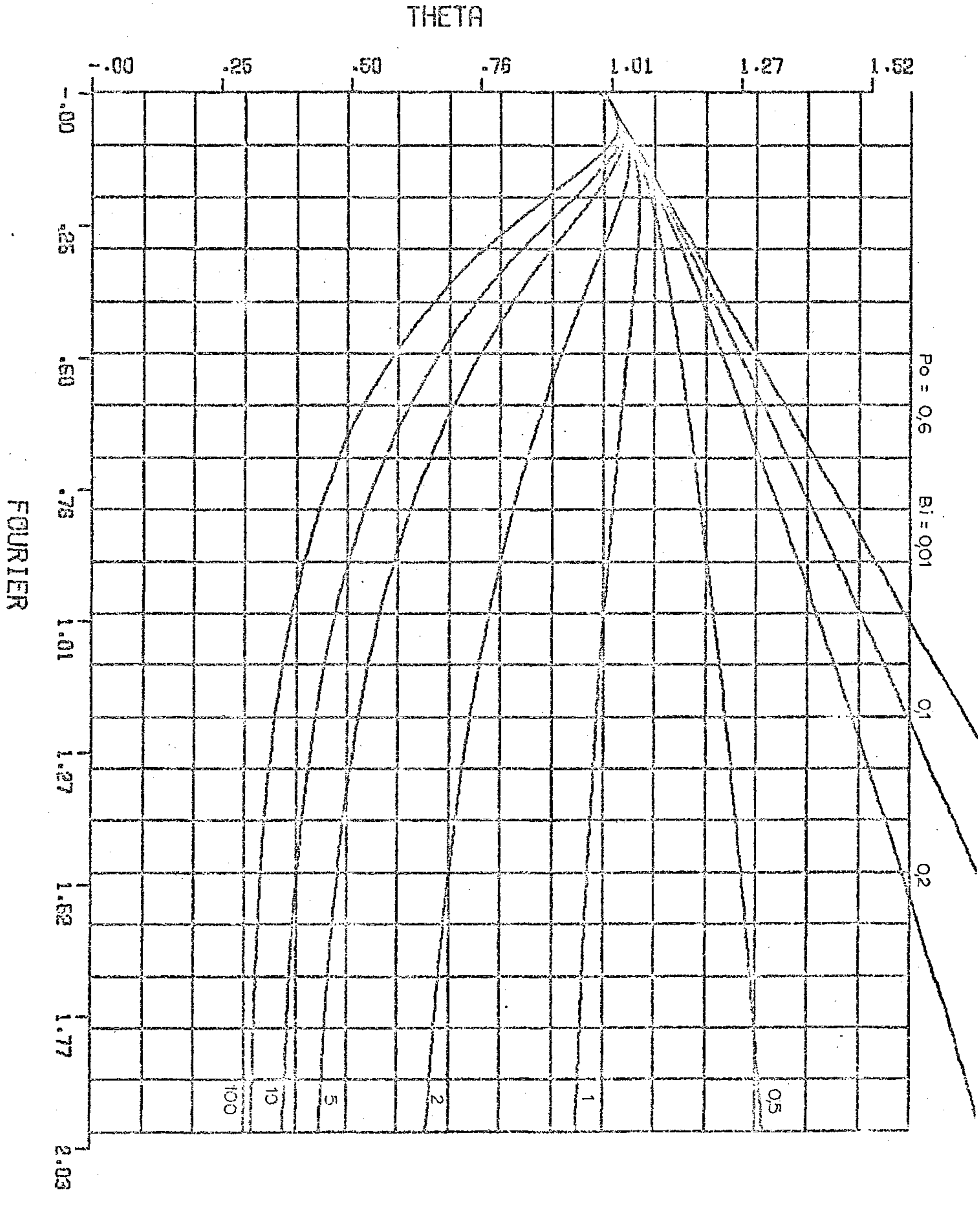
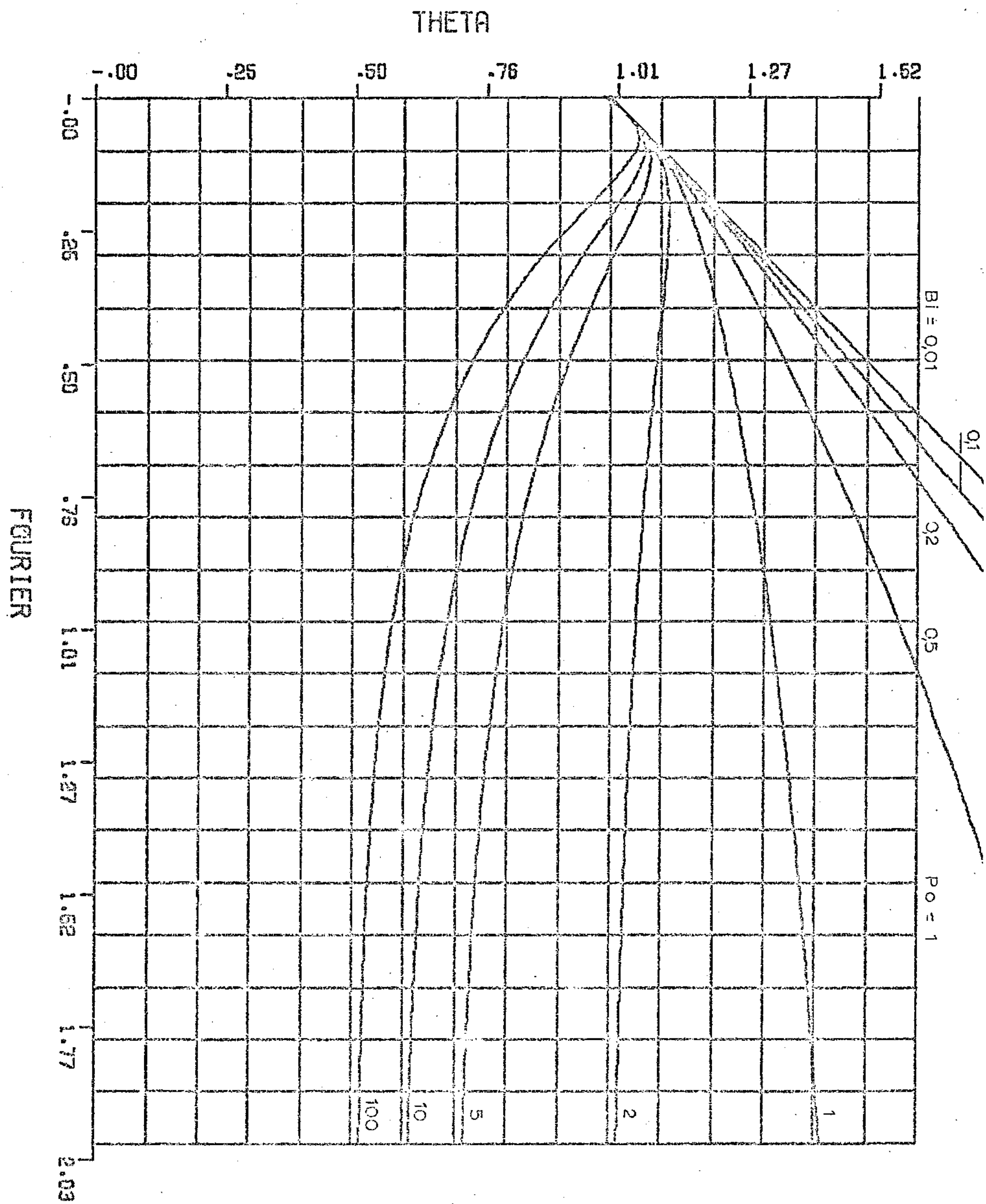
10
100


THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Po = 0,2
Bi =
100
10
5
2
1
0,5
0,2
0,1
0,01

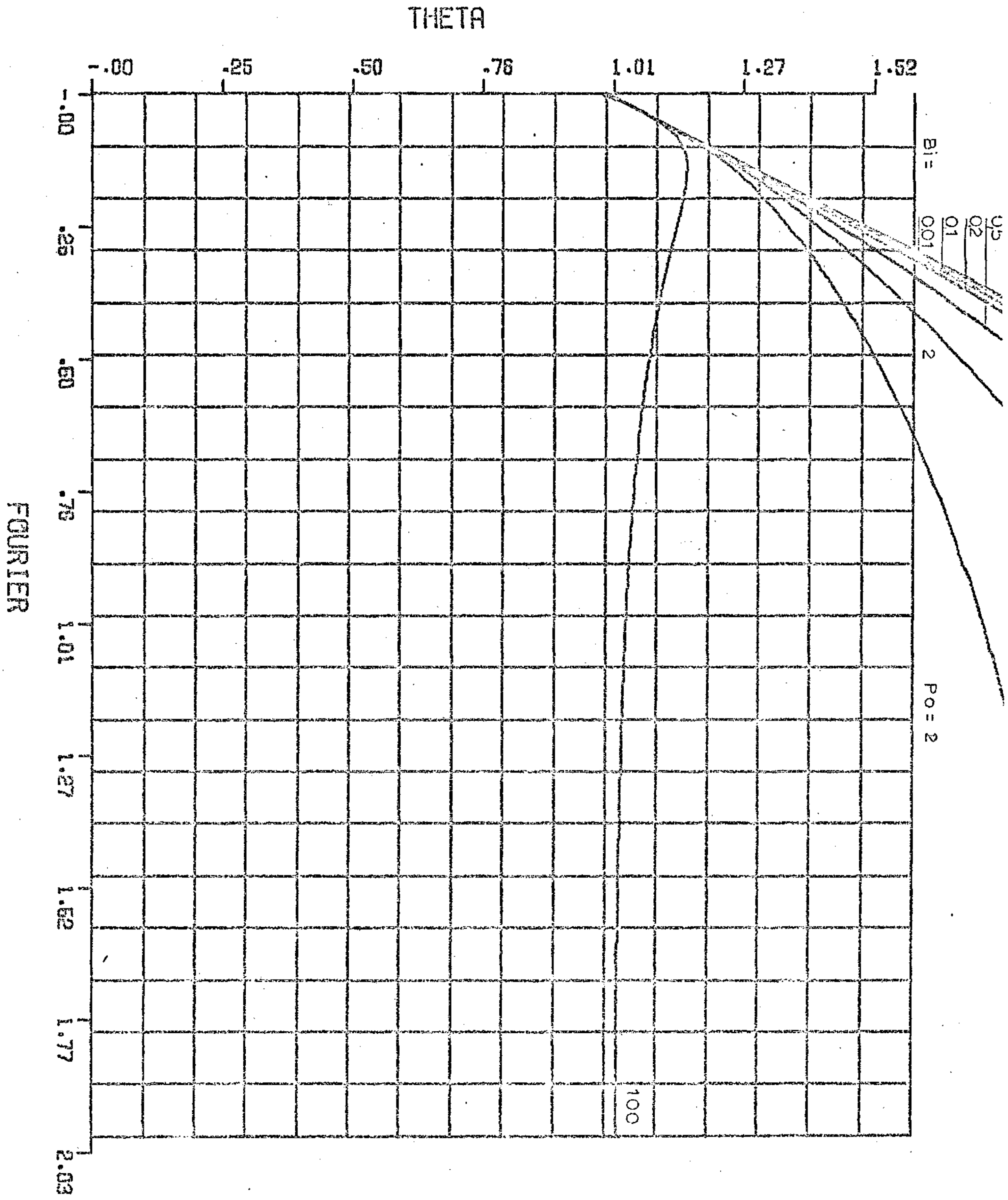

THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Po = 0,5
Bi = 0,01
0,1
0,2
0,5
1
2
5
10
100


THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Po = 0,55
Bi= 0,01
0,1
0,2
100
10
5
2
1
0,5


THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Po = 0,6
Bi = 0,01
0,1
0,2
100
10
5
2
1
0,5



THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Bi = 0,01
0,1
0,2
0,5
Po = 1
100
10
5
2
1


THETA
-.00
.25
.50
.76
1.01
1.27
1.52
FOURIER
-.00
.25
.50
.76
1.01
1.27
1.52
1.77
2.03
Bi=
.001
.01
.02
2
100
Po = 2

Aanhangsel

Uit de computerberekeningen zijn de volgende grafieken samengesteld:

1) $\Theta_{max} - 1$ vs. Po, parameter Bi

2) $\Theta_{max} - 1$ vs. Bi, parameter Po

3) $\Delta Fo/\Theta \geqslant 1$ vs. Po, parameter Bi

4) $\Delta Fo/\Theta \geqslant 1$ vs. Bi, parameter Po

De grafieken geven informatie over de te verwachten temperatuurpiek tijdens de afkoeling en de duur van deze piek.

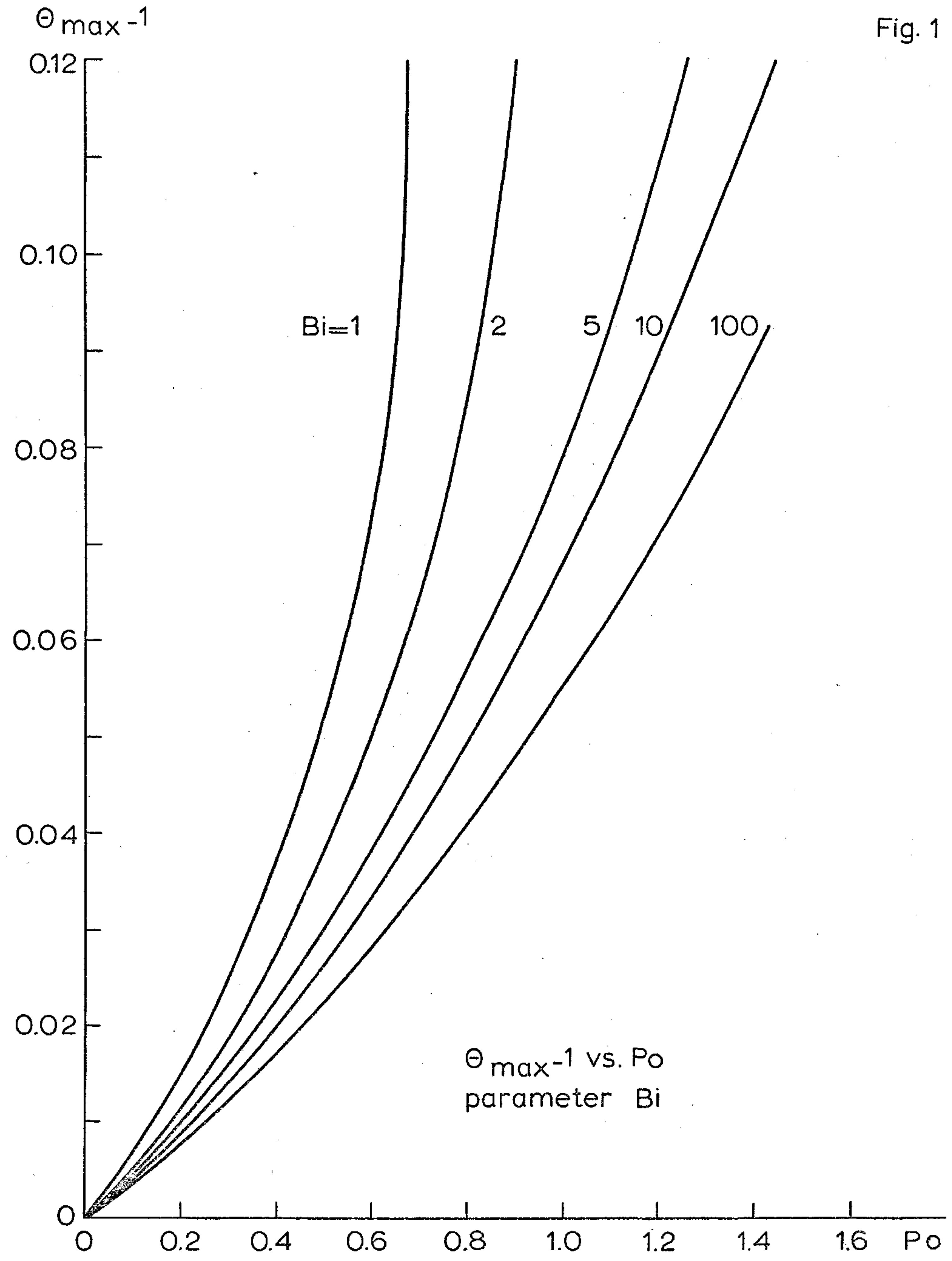
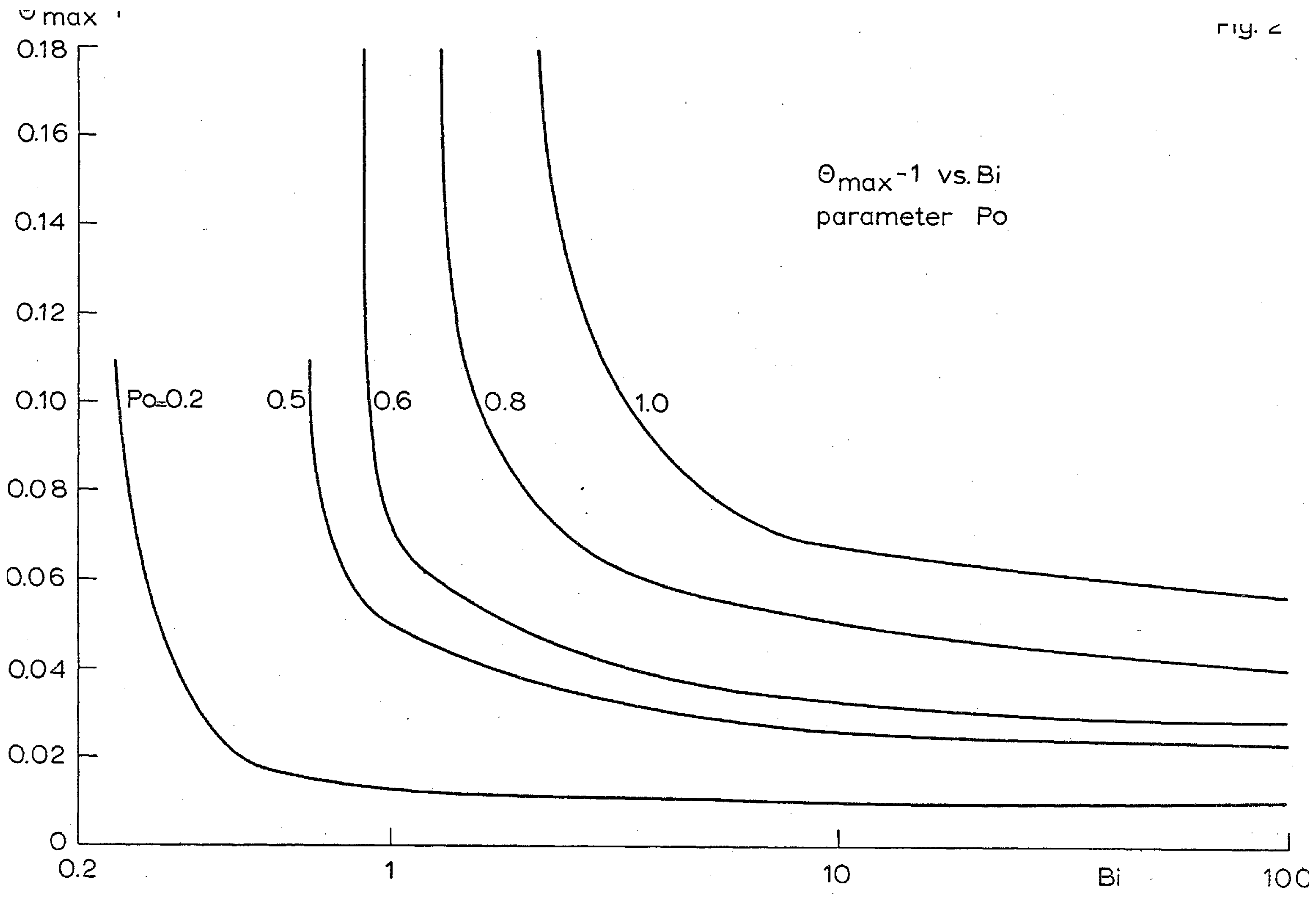


Fig. 1
Θ max -1
0.12
0.10
0.08
0.06
0.04
0.02
0
Bi=1
2
5
10
100
Θ max -1 vs. Po
parameter Bi
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Po

Fig. 2



$\Theta_{max}^{-1}$
$\Theta_{max}^{-1}$ vs. Bi
parameter Po
Po=0.2
0.5
0.6
0.8
1.0
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0.2
1
10
Bi
100

Fig. 3

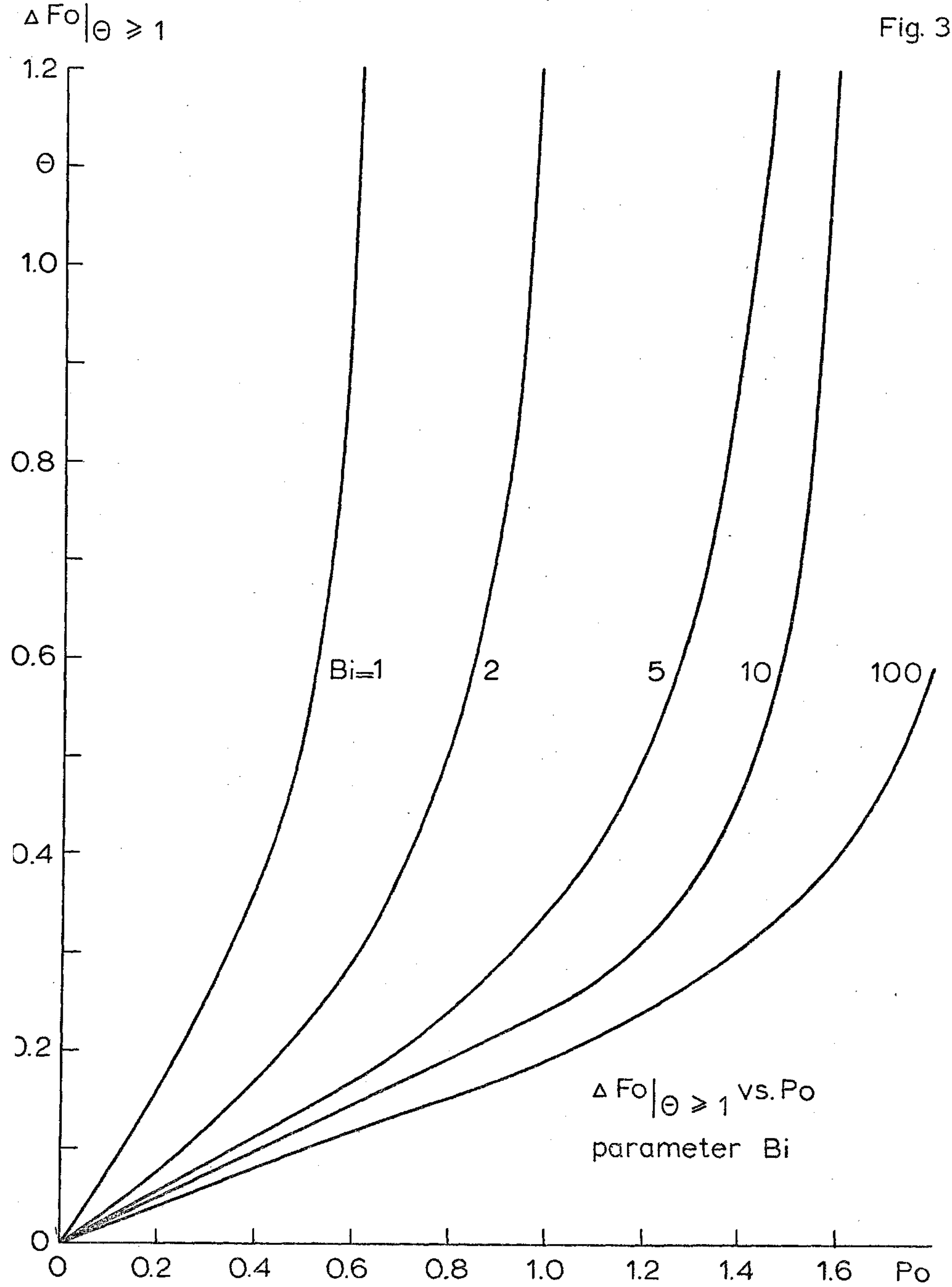
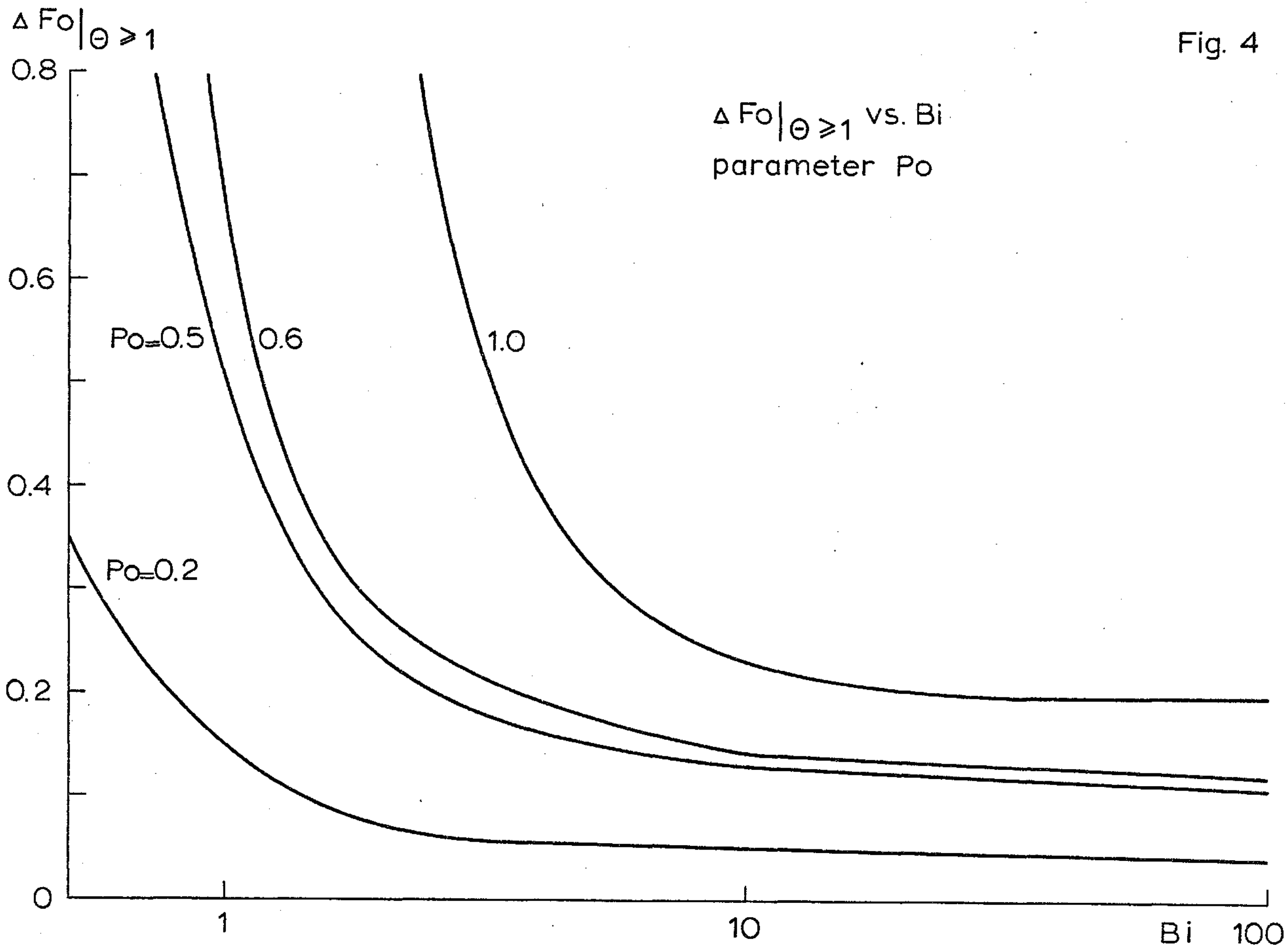

$\Delta Fo|_{\Theta \geqslant 1}$
1.2
$\Theta$
1.0
0.8
0.6
0.4
0.2
0
Bi=1
2
5
10
100
$\Delta Fo|_{\Theta \geqslant 1}$ vs. Po
parameter Bi
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Po

Fig. 4



$\Delta Fo|_{\Theta \geq 1}$
$\Delta Fo|_{\Theta \geq 1}$ vs. Bi
parameter Po
0.8
0.6
0.4
0.2
0
1
10
Bi 100
Po=0.5
0.6
1.0
Po=0.2