An H_∞–observer at the boundary of an infinite dimensional system

D. Vries and K.J. Keesman,
Wageningen University
P.O. Box 17,
6700 AA Wageningen, NL,
{dirk.vries,karel.keesman}@wur.nl

H. Zwart
University of Twente,
P.O. Box 217,
7500 AE Enschede, NL,
h.j.zwart@utwente.nl

Abstract

We design and analyze an H_∞–observer which works at the boundary of an infinite dimensional system with scalar disturbances. The system is a model of a UV disinfection process, which is used in water treatment and food industry.

Keywords

Robust filter design, observers, boundary control theory, H_∞–optimization

1 Introduction

In many (control) applications where (bio)chemical reactions and transport phenomena occur, measurement and control actions take place at the boundaries. While a theoretical framework already exist ([1] and references therein), there is little attention to apply this theory in practice, as far as we know.

In [2], the analysis and design of a Luenberger observer for a UV disinfection example is explored. In this paper, we analyze a robust Luenberger-type observer for the same system with boundary inputs and boundary outputs, see [2] for physical background,

$$\frac{\partial x}{\partial t}(\eta,t) = \alpha\frac{\partial^2 x}{\partial \eta^2}(\eta,t) - v\frac{\partial x}{\partial \eta}(\eta,t) - bx(\eta,t), \quad x(\eta,0) = 0 \tag{1}$$

$$x(0,t) = w_1(t), \quad \frac{\partial x}{\partial \eta}(1,t) = 0, \quad y(t) = x(\eta_1,t) + w_2(t), \tag{2}$$

on the interval $\eta \in (0,1)$. Furthermore, α, v, and b are positive constants and corresponding to the diffusion constant, constant flow velocity and micro-organism light susceptibility constant, respectively. The signals $u(t)$, $w_1(t)$, $w_2(t)$ and $y(t)$ represent a scalar input, disturbance (or error) at the inlet boundary ($\eta = 0$), disturbances or errors on the output and a scalar output, respectively.

We design a dynamic Luenberger-type observer,

$$\frac{\partial \hat{x}}{\partial t}(\eta,t) = \alpha \frac{\partial^2 \hat{x}}{\partial \eta^2}(\eta,t) - v\frac{\partial \hat{x}}{\partial \eta}(\eta,t) - b\hat{x}(\eta,t), \quad \hat{x}(\eta,0) = 0 \tag{3}$$

$$\hat{x}(0,t) = 0, \quad \frac{\partial \hat{x}}{\partial \eta}(1,t) = K(t) * (y(t) - \hat{y}(t)), \quad y(t) = \hat{x}(\eta_1,t), \tag{4}$$

with K to be designed, and $*$ denotes the convolution product. As a consequence, the dynamics for the error $\varepsilon(\eta,t) = x(\eta,t) - \hat{x}(\eta,t)$ is written as

$$\frac{\partial \varepsilon}{\partial t}(\eta,t) = \alpha \frac{\partial^2 \varepsilon}{\partial \eta^2}(\eta,t) - v\frac{\partial \varepsilon}{\partial \eta}(\eta,t) - b\varepsilon(\eta,t), \quad \varepsilon(\eta,0) = 0 \tag{5}$$

$$\varepsilon(0,t) = w_1(t), \quad \frac{\partial \varepsilon}{\partial \eta}(1,t) = K(t) * (\varepsilon(\eta_1,t) + w_2(t)). \tag{6}$$

Please notice that the correction to possible disturbances w takes place at the boundary.

2 H_∞–filter problem

The aim is now to design a K such that the disturbances w_1 and w_2 have hardly any influence on $\varepsilon(1,t)$. This would enable us to predict the value of x at $\eta = 1$ accurately. Since the future of the output cannot be used, we see that K must be causal. We can write this problem as a standard H_∞–filtering problem, i.e. ,

$$\inf_{K \text{ causal}} \sup_{w} \frac{\|\varepsilon(1)\|_2}{\|w\|_2}$$

with $w(t) = \begin{pmatrix} w_1(t) & w_2(t) \end{pmatrix}^\top$.

In [2], we already explored the exponential stability for the error dynamics (5)–(6) with constant gain. In the talk we shall further outline the procedure of how K is designed for the UV-disinfection example.

References

[1] R.F. Curtain and H. Zwart. *An Introduction to Infinite Dimensional Linear Systems Theory.* Springer-Verlag, New York, 1995.

[2] D. Vries, K. Keesman, and H. Zwart. A Luenberger observer for an infinite dimensional bilinear system: a UV disinfection example. *Accepted for SSSC'07, Foz de Iguassu, Brazil.*