

Quadratic Risk Programming for whole-farm planing

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ABSTRACT

Income in the agricultural sector tend to be unstable as a result of many factors that affect production and prices. Farmers themselves use risk management strategies such as diversification or adopting less risky production methods, and share risk with others in the form of farm financing and buying insurance. This paper suggests farmers how to quantify the effect of alternative risk management strategies by using a mathematical model to predict actions under different risk management strategies. The model could be applied for different types of farms. It is based on the portfolio theory, which includes the main types of risk on the farm and risk-avoiding instruments (the presented example is yield insurance). For the optimisation of the model the quadratic risk programming method is used. The result shows the relationship between standard deviation and expected profit. The standard deviation presents how high the risk level is. So, now it is up to the farmer if he or she prefers profit stabilisation or maximisation. This means respectively lower but more stable profits with risk-avoiding instruments or higher profits without any security for the next day.

KEY WORDS

Risk management, portfolio, modelling, risk-avoiding, insurance.

INTRODUCTION

Risks are pervasive in agriculture (Hardaker et al., 1997, page 5-6) and include production risks, price and market risks, institutional risk, human or personal risk, business risk and financial risk. **Risk management** involves the selection of methods for countering for all types of risks in order to meet the decision maker's goal taking into account his risk-attitude. Thus, it is important to account for the risk-return trade-off in designing risk management strategies. **The portfolio modelling approach** is often used to present the fundamentals to balance risk strategies. Several aspect of the portfolio concept are discussed in literature. Classical description of portfolio is described by Markowitz (1956) like "security selection", at the same time he footnoted that a good portfolio is more than a long list of good stocks and bonds. A portfolio starts with information concerning individual activities.

The word **portfolio** refers to a mix, or combination of assets enterprises or investments. (Brealey and Myers, 2000). The gains in risk reduction from diversification increase as the correlation between activities decline and as the number of activities in a portfolio increases (Barry, 2000, page 222). It is often used to describe holdings of financial assets such as stocks

and bonds. In application of **risk analysis** to agricultural businesses, the concept of an asset is broadened to include crop and livestock enterprises; acquisition of machinery, buildings, and land; hiring labour; financing alternatives; consumption and tax activities; and investments in financial assets. A broader set of constraints is specified as well, including limitations on resource availability, borrowing capabilities, cash flows, and accounting and tax requirements. Various forms of diversification may characterise the risk-efficient solutions to the risk-programming model, and other risk management methods may be analysed as well. However, it can also be applied to holdings of tangible assets such as grain inventories, growing crops, livestock, machines, land and farm buildings.

Various types of information concerning farm activities can be used as input of a portfolio analysis. One source of information is the past performance of individual activities. A second source of information is the beliefs of one or more activities concerning future performance.

Some methods of managing risks are feasible for all types of farms. Others are only feasible for certain sizes and types of farms, qualities of management, financial structures, and other characteristics. The methods can be categorised in terms of production, marketing, and financial organisations of farm business.

In conclusion, the merit of adding any risky process into an existing farm business cannot be assessed without considering the potential impact on the risk-efficiency of net returns from the whole portfolio of farm-specific risky projects. With quadratic risk programming the expected income of a risk averse decision maker subject to a set of resource and other constraints could be optimised. Unfortunately, these methods have rarely been applied to whole-farm risk analysis and within the farm-specific production circumstances.

Therefore the objectives of this paper are:

- To develop an optimisation model of farm management based on portfolio theory, which captures the main types of risk and risk attitudes on the farm level.
- The model will be applicable for several types of farm (as for crop as for livestock farms).
- To design a model which helps the farmer to choose the optimal set of risk management strategies. This means that, for example, the technologies, which is used on the farm, insurance or contract possibility, etc.

LITERATURE REVIEW OF RISK PROGRAMMING TECHNIQUES

Optimisation programs

The farm optimisation model could be based on **linear and non-linear programming methods**.

Mathematical programming (MP) methods are very well adapted for just such problems. **Linear risk programming (LP)** is a widely applied MP method used for farm planning. It may be used to maximise expected profit subject to the farm resource constraints and other restrictions without taking into account risk factors. The advantages of linear risk programming models over non-linear ones were important in the past when reliable non-linear computer codes were less widely available.

One of the often used way of MP is to define the incorporating risk (different types of risks and their influence of each other). More secure plans may involve producing less of risky enterprises, diversifying into a greater number of enterprises to spread risks, using established technologies rather than venturing into new technologies and, in the case of small scale farmers, growing larger of family food requirement. Ignoring risk-averse behaviour in farm planning models often leads to results that are unacceptable to the farmer, or that bear little relation to the decisions he actually makes. To resolve this problem, several techniques for incorporating risk-averse behaviour in mathematical programming models have been

developed in recent years. Several aspects of different MP process are discussed in the literature. A brief overview is presented of widely circulated mathematical programming, which can be used for the model optimisation.

Quadratic Risk Programming (QRP) (Hazel and Norton, 1986)

The efficiency frontier set of expected value and the variance of outcomes of farm can be derived by means of quadratic programming. In this case the coefficients used in the model could be non-stochastic, the costs are constant and income distribution of farm plan is totally specified by the total gross margin distribution. Based on the farm activities the variance – covariance matrix has to be denoted.

$$V = \sum_j \sum_k X_j X_k \sigma_{jk} \quad (1)$$

Where: V is income variance, X_j, X_k are denoted like the level of j or k activity, and σ_{jk} denotes the covariance of these activities.

Equation (1) shows that the variance of total gross margin is an aggregate of the variability of individual enterprise returns, and of the covariance relationship between them. Covariances are fundamental for efficient diversification among farm enterprises as a means of hedging against risk (Markowitz, 1959). Combinations of activities that have negatively covariate gross margins will usually have a more stable aggregate return than the return from more specialised strategies. Also, a crop that is risky in terms of its own variance of returns may still prove attractive if its returns are negatively covariates with other enterprises in the farm plan.

To obtain the efficient set of expected value and the variance of outcomes it is required to minimise variance – covariance set for each possible level of expected income, while retaining feasibility with respect to the available resource constraints.

MOTAD programming

The application of the MOTAD approach (Hardaker et al., 1997, page 181-204) entails use of the same technical input-output tableau as for the LP and QRP models, but augmented with additional constraints (like absolute deviation of revenue, income deviation or probabilities) for the calculation of deviations for each state together with an additional constraint to calculate the mean absolute deviation. The deviations of the activity net revenues by state are calculated from the adjusted gross margins by deducting the corresponding expected gross margin from each. Also added to the tableau are further activities to calculate the negative deviations for each state. The model is then solved with mean absolute deviation set to an arbitrarily high value which is then progressively reduced until no further solutions of interest are found.

Target MOTAD

The Target MOTAD is modification of MOTAD in that it entails a constraint on income deviations, this time from a target level of income. Target MOTAD involves three parameters: expected profit, deviation from the target and target income. Efficient set of solutions is obtained for a given value of the target income. The main advantage is that the solutions are second-degree stochastically dominant (regardless of the distribution of income), and so are efficient for risk-averse decision makers.

The model usually is solved maximising profit for a relatively large number of combinations of target income and deviation from the target, making the results rather extensive, and a little more difficult to interpret.

Direct maximisation of expected utility (EU)

Non-linear programming is straightforward to set up a risk-programming model to maximise expected utility. The utility function will be monotonic and concave for a risk

averse decision maker, a good non-linear algorithm will find the global optimum. Should a non-linear algorithm be unavailable, the problem can also be efficiently solved by linear segmentation of the utility functions. The linear segmentation of the utility functions for each state can be adjusted iteratively after a solution has been obtained. Using this method, each solution will show the approximate level of net income for each state of nature, so the segmentation can be made finer in this region. Usually, a very good approximation can be obtained by this means within a very few iterations.

The utility function is defined for a measure of coefficient of absolute risk aversion, that is varied.

This function is popular because it is one of the few reasonably flexible functional forms that shows decreasing risk aversion. Variation of a implies variation in the coefficient of absolute risk aversion between g when a is zero and h when a is large.

The LP tableau could be extended to include the matrix of activity net revenues by state.

The solution to the UE programming model can be graphed as **cumulative distribution functions (CDF)**. This form of presentation emphasises the link between EUI programming and stochastic efficiency analysis

Discrete Stochastic Programming (DSP)

Discrete Stochastic Programming (DSP) is represented as decision trees, all such problems have a tendency to explode into what called “bushy messes” (Hardaker et al., 1997), meaning that the problem grows to have too many branches to be drawn easily or at all, and may be hard or impossible to solve if specified in all its detail. Another name for this phenomenon is “the curse of dimensionality”.

In the stochastic programming the main key assumption is that some decisions are made after the date of nature is observed. Thus, the farmer has scope for avoiding problems with infeasibilities or under-utilised resources that might otherwise arise. A farmer has to commit some resources for planting crops at the beginning of the growing season, but application of many inputs such as fertiliser, pesticides, of irrigation water typically accrue after the farmer has had time to gain new information about the season. As such, he can adjust the use of these inputs in an optimal way.

The major problem difficulty of the approach is its hearty appetite for data, and the fact that models rapidly becomes very large because of the need to have separate rows and columns for many resources and activities in every state of nature. Discrete stochastic programming models require the farm plans to be feasible for those resources in all states of nature. DSP may be incompatible with a farmer’s risk-aversion behaviour. This is most likely to happen is the farmer has some recourse for resolving infeasibilities, but these options cannot be fully specified in the model.

MATERIAL AND METHODS

The current model is developed on microeconomic level and thus applicable for on farm use. The data could be derived by two ways: statistically or Monte Carlo simulation. Both methods have advantages and disadvantages. Statistical data could be unpractical because of the data unreliability (lack and data mistakes) and of the time factor. From year to year the technologies, the techniques in agriculture are changing, thus the risks also are changing. It means the good database can also be not available. The disadvantage of Monte Carlo is a strict mathematical interpretation without an experience basis.

A hypothetical example is presented which is based on QRP to determine the optimal portfolio. The possible set of activities includes farm activities and off-farm activities such as insurance.

Portfolio analysis

From the expected revenue the distribution table can be simulated by @Risk software and hereby the variance – covariance matrix can be developed.

QRP

The variance-covariance matrix, the constraints on the farm and insurance possibility optimised by GAMS software. The meaning of the QRP model is the variance of income objective function, which has to be minimised. So, in this case the aim of the model is not to optimise or to maximise the income, but to minimise the value of variance-covariance matrix. Therefore the income has to be determined like a constant parameter, which we know before that would be reached on the farm any way. The value of income within the farm can be accounted by few ways: past performance of individual activities, beliefs of one or more activities analysts concerning future performance and mathematical approach, for example, like linear programming.

Assume a farm with a size of total area is 45 ha (all additional information about this farm are depicted in tables 1, 2 and 3). The analysed hypothetical farm can produce four crops, namely potato, wheat, sugar beet and onion. The aim of optimisation is to present the risk analysis within a farm in production by the traditional way and with risk-avoid instruments: in this example is used just one risk-avoiding instrument is yield insurance. The standard deviation of the yield and price distributions are 20% and 10% respectively (table 1). Additional constraints are depicted in table 2. Correlation between crop yields is 0.25 while correlation between yield and price (per crop) is -0.50 (table 3).

Assume, that the yield insurance for each crop is a new activity in the farm. In that way, now we have instead of four, eight kinds of activities (potato, wheat, sugar beet and onion production and production of yield insurance of each crop). The assumed yield insurance schemes for every crop include a deductible 25% insured sum and a premium of 5 euro per 1000 insured sum.

TABLE 1: Yield and price variables of crops

		<i>MEAN</i>	<i>VARIANCE</i>	<i>REVENUE</i>
Potato	Yield (kg/ha)	45000	9000	4084.20
	price (euro/kg)	0.09076	0.009076	
Winter wheat	Yield (kg/ha)	8600	1720	1170.72
	price (euro/kg)	0.13613	0.013613	
Sugar beet	Yield (kg/ha)	59500	11900	3293.92
	price (euro/kg)	0.05536	0.005536	
Onion	Yield (kg/ha)	56000	11200	4701.20
	price (euro/kg)	0.08395	0.008395	

TABLE 2: Availability and costs of labour for the example problem

<i>Month</i>	<i>Max own labour (h)</i>	<i>Max hired labour (h)</i>	<i>Cost of hired labour (euro/h)</i>
May	290	60	30
June	230	0	0
July	220	0	0
August	240	120	10
September	240	120	10
October	300	60	30

TABLE 3: Correlation matrix

		<i>Potatoes</i>		<i>Winter wheat</i>		<i>Sugar beet</i>		<i>Onion</i>	
		yield	price	yield	price	yield	price	yield	price
Potato	yield (kg/ha)	1	-0.5	0.25		0.25		0.25	
	price (euro/kg)		1						
Winter wheat	yield (kg/ha)			1	-0.5	0.25		0.25	
	price (euro/kg)				1				
Sugar beet	yield (kg/ha)					1	-0.5	0.25	
	price (euro/kg)						1		
Onion	yield (kg/ha)							1	-0.5
	price (euro/kg)								1

Before optimising the model by using QRP, the default results were obtained LP. The default results reflect the maximum income which could be achieved on this farm without any risk-avoiding instruments (i.e., without insurance possibility).

The derived results are depicted in table 4. The expected maximum income amounts 125.429 euro per year. The third column of the table shows the shadow price of activity. It means by changing the activity with one unit, for how many units the objective function will change. In this case there is no activity, which should not be changed.

TABLE 4: Obtained default results by LP

Activity	<i>LAND USE</i>	
	Opt. Level (ha)	Objective function changing with one unit increasing (euro)
Potato	9.00	0
Wheat	18.83	0
Sugar beet	11.25	0
Onion	5.92	0

The QRP part of the model optimisation was formulated in General Algebraic Modelling System (GAMS). GAMS is widely used across the world among economists, because various kinds of economic models can be written down: system equations, non-linear optimisation (also with variance-covariance matrix), equilibrium modelling, including time factor, different technologies and one of the main advantages is visual presentation of the results (Delink et al., 2001). The main parts of GAMS are sets, parameter, variables and equations. There are short description of every one of them follows below.

Three main SETS (set of all activities, land use type and month) were defined.

The next step is declaration and definition of the PARAMETERS. The parameters give exogenous fixed values of the model. In example 12 the different parameters are defined: price, yield, land use, the own and hired labour use, costs of labour use, costs of insurance and indemnity for every activity.

The distribution table (it is not presented in this paper because of its huge size) was defined with eight activities within hypothetical farm by @Risk software. But in future, the marginal income has to be used instead a revenue. From the distribution data the variance-covariance matrix was derived (see table 5) and also defined like a parameter.

TABLE 5: Variance – Covariance matrix

	<i>Pot</i>	<i>Wheat</i>	<i>Beet</i>	<i>Onion</i>	<i>Y-In-Pot</i>	<i>Y-In-Wh</i>	<i>Y-In-Be</i>	<i>Y-In-On</i>
<i>Pot</i>	510104.59	44992.90	140860.47	237643.93	481795.41	40113.65	129120.17	216245.01
<i>Wheat</i>	44992.90	38633.83	35764.83	61523.74	42062.82	35878.52	33754.37	57169.40
<i>Beet</i>	140860.47	35764.83	320075.98	179146.17	139047.73	33833.75	296190.22	161154.60
<i>Onion</i>	237643.93	61523.74	179146.17	667968.18	221094.33	56371.76	163764.73	615484.05
<i>Y-In-Pot</i>	481795.41	42062.82	139047.73	221094.33	468070.24	37410.31	127485.97	202768.96
<i>Y-In-Wh</i>	40113.65	35878.52	33833.75	56371.76	37410.31	35089.22	32008.18	52439.17
<i>Y-In-Be</i>	129120.17	33754.37	296190.22	163764.73	127485.97	32008.18	284010.64	148481.49
<i>Y-In-On</i>	216245.01	57169.40	161154.60	615484.05	202768.96	52439.17	148481.49	592886.87

Positive covariation means that high profits in one activity are associated with high profit in another activity. Negative covariation means that high profits in one activity are associated with low profits in another activity. Zero covariation means that there is no statistical association between the variations, the activities are statistically independent.

Separate attention has to be paid for the INCOME (minimum income) parameter. To put risk instruments (or variance-covariance matrix) into the GAMS, the variance of income objective function has to be minimised (Rosenthal et al., 1998). So, in this case, the aim of the model is not to optimise the income, but to minimise the value of variance-covariance matrix. Therefore, the income has to be determined like a constant parameter, which is known before that would be reached on the farm anyway. The value of a fixed income could be accounted by farmer experiences from year to year or by MP. In this case the result is how much optimum risk/insurance value and constraints if we have to reach approximately known result. The INCOME value is taken from LP optimisation, thus it is 125.000 euro.

The VARIABLES indicate the calculated activities in the model (“endogenous variables”). There are five variables: objective function of variance income, area use, annuity of production, hired labour in each period and cost of insurance. These values have to be the output of this model. The definition of the type of variable is positive (e.g. area use, annuity of production and hired labour in each period) and free (e.g. objective function of variance income).

There are eight EQUATIONS in the model, which generate the algebraic relationships between the parameters and variables, and constraints in the model: variance of income, constraints of all land use and land use for each crop, labour use in each period, insurance cost and income constraints. The VARIANCE equation is presented to calculate the variance of income. It includes variable of annuity production of each activity and variance-covariance matrix. The variance – covariance matrix indicates how different combinations of activities may reduce an investor’s risk more than having a traditional farm management. Portfolio analysis is used to determine the risk reduction by the number of present activities, correlation (or covariation) between the expected returns of the individual activities, the possible changes in the levels of costs and returns per unit of activity. The dominating (lower variance) asset or portfolio generally is preferred by risk-averse farmers to the dominated (higher variance) asset of portfolio. Finally, by using the non-linear programming, the objective function has to be minimised.

ANALYSIS OF RESULTS

Despite the insurance is an expenditure for the farmer, the optimisation problem presented that there is profit in the producing of insured crops (see table 6 and graph 1).

TABLE 6: Obtained results

	<i>Expected profit (in '000 euro)</i>						
	100	105	110	115	120	125	126
St. deviation of income	7.35	7.37	7.60	8.06	8.61	9.25	9.28
Insurance	502.50	414.40	294.90	198.20	149.90	5.30	
	Land use (ha)						
Potato			6.50	9.00	9.00	9.00	Infeasible solution
Wheat		8.80	14.26	14.26	13.50	13.50	
Beet				7.16	11.25	11.25	
Onion					1.26	10.90	
Yield-ins-potato	9.00	9.00	2.50				
Yield-ins-wheat	14.26	5.46					
Yield-ins-beet	11.25	11.25	11.25	4.09			
Yield-ins-onion	10.49	10.49	10.49	10.49	9.99	0.35	
Sum	45.00	45.00	45.00	45.00	45.00	45.00	

The result reflects the relationship between standard deviation and expected profit. Taking into account all activities and constraints on the farm, the maximum profit without risk optimisation is 125.000 euro. The standard deviation in this point is 9.3. This is maximum profit value that could be reached on this farm by using all activities without any risk management instrument. This number can be accounted by LP or by the farmer experience.

The minimum point of standard deviation is 7.3. It shows the level of expected value (in this example it equals 100.000 euro) with the same level of activity use, but with taking into account maximum use of insurance. So, with increasing the amount of standard deviation the amount of expected profit also increases and in the same time the risk level is reducing. The less amount of standard deviation means that less risk has been perceived and the less profit has been expected. In every point of standard deviation the percent of insured activity is known.

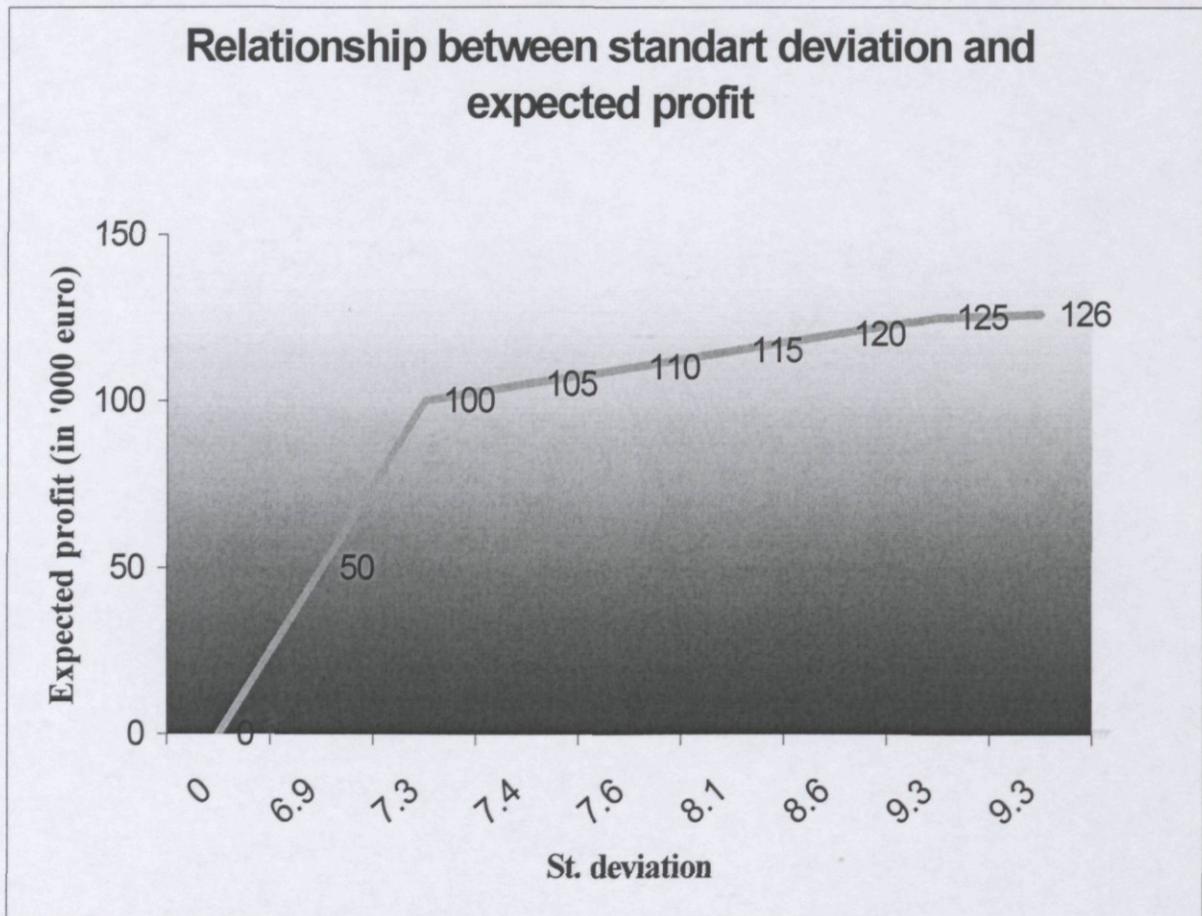
Validity measure of the model

“Validity is the extent to which the indicators “accurately” measure what they are supported to measure” (Hair, 1987).

For the model, validity estimation is created by putting, for example, the expected profit value higher than “logical” maximum among into the model. The result is infeasible solution. Thus, higher than 125.439 euro expected profit could not be reached by current conditions within the farm.

When the expected profit value is reduced below 100.000 euro then the model demonstrates that some of the activities are not used in the production. It means the level of expected profit reached lower than 100.000 euro, the farmer does not use all capacity activities.

GRAPH 1



CONCLUDING REMARKS

This paper develops and demonstrates a model of the farmer decision making under risk which incorporates insurance contracts. The model can be used to predict input-output responses to different insurance contracts, whether different contracts will be accepted, and the level of expected indemnities. This information should be of benefit to farmer, insurers and policy analysis.

On basis on this example in the near future a model will be constructed which can be used to identify and evaluate the effects of different insurance policies concerning price, yield, price and yield, or revenue (Kaylen et al., 1989). Agricultural insurance policies could radically affect crop acreage decision. Yield insurance may encourage "gambling" on yields since the farmer is protected from the effects of low yields.

Also the model has to demonstrate how much the premium has to be in view of market policy, condition on the farm, etc. Thus if the insurance indemnity is too high it could be that the farmer is not interested in producing more, because he is willing to get worse yield (or price) because of the insurance.

The example serves to identify what is necessary to apply the microeconomic model for the "real world" situations: the stochastic production functions and the joint probability.

While developing the model it seems better to use the marginal income and marginal cost data, because it does not complicate the optimisation and it helps to make the model more average, more adapted for different kind of farms.

EXPECTED PROBLEMS

The expected problem of joining together yield and price insurance conditions into the model is that in real life, bad yield year goes together with high price of products. Thus, a farmer could get more income in spite of low yield. Therefore, a very precise correlation matrix of relationship between yield and price is necessary.

It would be interesting to compare the obtained results with another simulations and to try to use goal multiple theory.

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