7

Dynamic programming to optimize treatment and replacement decisions

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Objectives

From this chapter the reader should gain knowledge of:

- the methodological aspects of treatment and replacement decisions in livestock
- the basic principles of dynamic programming to support these decisions

7.1 Introduction

Commercial livestock farms produce either products extracted from the animals over their lives (such as milk, eggs and wool), or the meat harvested at the end of the animals' lives (such as beef, pork and chicken), or both. Necessary inputs include feed and veterinary treatment. Decisions have to be made on the quality, quantity and timing of the feed and veterinary inputs. The product return to these inputs changes continuously over the life of the animals. Typically, productivity of the animals first increases and then declines with age. If the livestock enterprise is to be a continuing one, a decision must be made on when to replace breeding females.

Furthermore, in case of disease, farmers are frequently faced with the problem whether to treat or replace an affected animal. The cost-value trade-off is then important. Will the animal recover completely and will it reach its previous production level? If so, how long does it take before the animal is at its normal level again? Another important question in this respect is the repeatability of disease. All these factors have to be balanced before the farmer can make an appropriate treatment or replacement decision.

In section 2 of this chapter some methodological aspects of treatment and replacement decisions at the animal level are reviewed. In section 3, the technique of dynamic programming (DP), which can be used to optimize these multi-stage decisions, is introduced. Lastly, two DP-applications are presented, the first one being an application to dairy cows, and the second one involving sows.

7.2 Methodological aspects

The technique for determining optimal livestock replacement decisions relies on the

production function approach as explained in Chapter 2 and depends on the shape of the marginal net revenue curve (ie, the net revenue in each additional year, month or day of life), the characteristics of replacement animals, the discount rate and whether or not involuntary replacement takes place. The net revenues from not only the animals present in the herd but rather from the present and all subsequent (replacement) animals are to be maximized. This implies that an infinite planning horizon has to be considered in the marginal net revenue approach. For simplicity reasons assume that there is no discounting and involuntary replacement, and that net revenue is represented as a function of time (Figure 7.1).

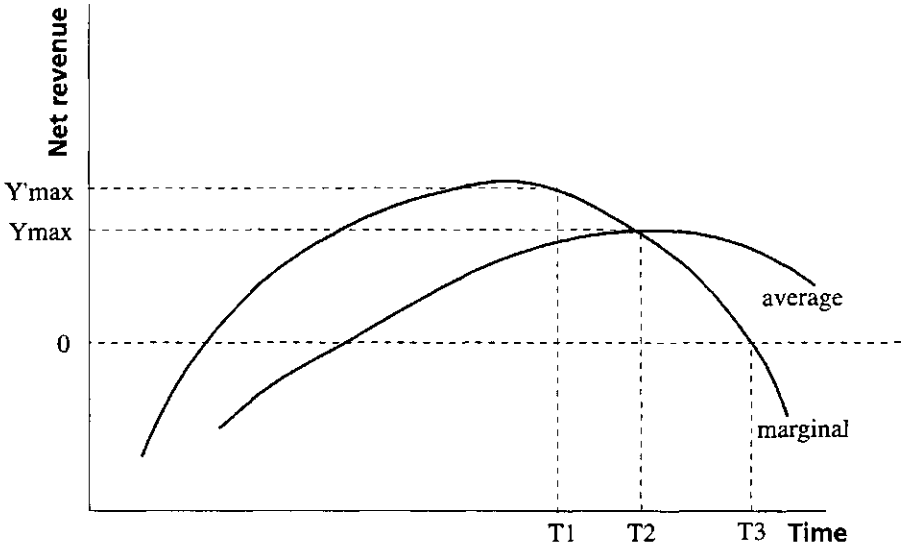

Figure 7.1 Determination of the optimal time for replacement in a situation without an alternative opportunity (T3), and in situations of identical replacement (T2) and nonidentical replacement (T1) (derived from Van Arendonk, 1985)

Furthermore, assume that the decision problem is how long the livestock unit is to be kept. The answer depends on the three opportunities available at the moment(s) at which the unit can be replaced.

- If there are no replacement animals available at the decision moment, the relevant objective is maximum net revenue, which corresponds with the optimal time for replacement T_3. This represents the situation in which there are no opportunity costs.
- If there are identical replacement animals available, the optimal time for replacement is T2. T2 corresponds with the time at which the marginal net revenue from the present animal(s) equals the expected maximum average net revenue from the subsequent replacement animal(s) (y_{max}). The maximum average net revenue from subsequent replacement animal(s), which is used to determine the optimal time, can be interpreted as the opportunity cost of postponed replacement.

- Lastly, if there are nonidentical (better) animals available, the optimal replacement time is T1. T1 corresponds with the time at which the marginal net revenue from the present animal(s) equals the expected maximum average net revenue from the subsequent nonidentical replacement animals (y'max).

When there is time preference of net revenue, comparison of expected costs and revenues should be made at the same point in time. This can be achieved by **discounting** future costs and revenues, as explained in Chapter 3 (section 5). When discounting is applied, the optimal time to replace is reached when the marginal net revenue from the present animal(s) is equal to the maximum annuity of expected net revenues from the subsequent replacement animal(s). In the latter value, the marginal net revenues and periods of time are weighed to allow for time preference. A higher discount rate can result in both later and earlier replacement, depending on the shape of the marginal net revenue curve.

The marginal net revenue technique is explained by a simple calculation model (ie, identical replacement, no discounting, but including involuntary disposal) for fictitious animals. In calculating the optimal lifespan for individual animals, the opportunity costs must be determined first. The calculation is based on the average performance of animals present in the herd, assuming this to be the best estimate for expected future net revenue of young replacement animals. Future revenues and costs are weighed with the probability of animal survival. The formula is:

$$ANR_j = (\Sigma_{i=1..j}\, p_i \times MNR_j) / (\Sigma_{i=1..j}\, p_i \times l_i)$$

where

ANR_j	= expected average net revenue per year;
i	= decision moment of retention or replacement ($1 \leq i \leq j$), which is at the end of period i;
j	= period, at the end of which an animal can be replaced;
p_i	= probability of survival until the end of period i, calculated from the moment at which the young animal starts its first production (end of period 0);
l_i	= length of period i (in years); and
MNR_i	= marginal net revenue in period i including a correction for change in slaughter value and financial loss associated with disposal.

In Table 7.1, the formula has been applied to fictitious animals. The price of a highly pregnant replacement animal is US$500. The average net revenue is maximal at the end of period 5 (at decision moment 5). The optimal moment for replacement with identical animals is also at the end of period 5: the economically optimal lifespan is the last period with a positive difference between expected marginal net revenue of the present animal and maximum average net revenue of its replacement.

Table 7.1 Calculation model for identical replacement of a fictitious animal (all monetary values in US$)

Decision moment i (yr)	Marginal net revenue[a]	Slaughter value	Financial loss at disposal	Marginal probability of disposal	Probability of survival until year i	Marginal net revenue[b]	Average net revenue	RPO
0[c]		500						0[h]
1	200	345	60	0.15	1.00	36[d]	36[f]	212[i]
2	285	380	85	0.20	0.85	303[e]	159[g]	157
3	320	390	88	0.25	0.68	308	199	86
4	325	375	90	0.30	0.51	283	213	27
5	305	350	93	0.40	0.36	243	216[j]	—
6	250	300	—	1.00	0.21+ 3.61[k]	200	215	—

[a] Between the end of period i-1 and i, excluding change in slaughter value and financial loss at disposal

[b] Between the end of period i-1 and i, including change in slaughter value and financial loss at disposal

[c] Young highly pregnant animal, about to start its first production

[d] 200 + (345 - 500) - (0.15 x 60) = 36

[e] 285 + (380 - 345) - (0.20 x 85) = 303

[f] (1.00 x 36) / 1.00 = 36

[g] (1.00 x 36 + 0.85 x 303) / (1.00 + 0.85) = 159

[h] 1.00 x (36 - 216) + 0.85 x (303 - 216) + .. + 0.36 x (243 - 216) = 0

[i] 0.85/0.85 x (303 - 216) + 0.68/0.85 x (308 - 216) + .. + 0.36/0.85 x (243 - 216) = 212

[j] opportunity cost

[k] total herd life

After an animal's optimal lifespan has been determined, the total extra profit to be expected from trying to keep her until that optimum, compared with immediate replacement, can be determined taking into account the risk of premature removal of retained animals. This total extra profit is called **Retention Pay-Off** (RPO) and is calculated as follows:

$$RPO_i = \Sigma_{j=i+1..r}\, p_j\, (MNR_j - ANR_{max} \times l_j)$$

where

RPO_i = Retention Pay-Off at decision moment i;
r = optimal moment for replacement;
p_j = probability of survival until the end of period j, calculated from decision moment i;
l_j = length of period j (in years);
MNR_j = marginal net revenue in period j; and
ANR_{max} = expected maximum average net revenue per year.

The RPO is an economic index, which makes it possible to rank animals according to their future profitability: the higher the RPO, the more valuable the animal. A value below zero means that replacement is the most profitable choice. RPO also represents the maximum amount of money that should be spent in trying to keep an animal in case of reproductive failure or health problems.

Applied to livestock, the marginal net revenue approach faces two specific problems:
- For the calculation of the opportunity cost of postponed replacement it must be assumed that all subsequent replacement animals are identical with respect to net revenues. This assumption makes it impossible to account - directly - for continuous genetic improvement and seasonal variation.
- Variation in expected performances of both present and all subsequent replacement animals is not taken into account.

Extension of the marginal net revenue approach to overcome these limitations results in what is called the **dynamic programming** (DP) technique. DP is considered a better and more flexible tool for determining treatment and replacement decisions in livestock, and is introduced in the next sections.

7.3 Brief introduction to dynamic programming

Dynamic programming (DP) is a mathematical technique which is especially of value in situations where a sequence of decisions has to be made, as is the case with livestock replace ment decisions. DP uses the repetitiveness of the decisions to save computation time. It depends on a deceptively simple but remarkably powerful principle. It is generally referred to as **Bellman's Principle of Optimality** (Bellman, 1957):

> An optimal policy has the property that, whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

In DP a **policy** is defined as a sequence of decisions taken at different **stages**. Consider the case of a finite horizon with N stages. At each stage the system can be described completely by a state variable S_n. The **states** are the various possible conditions in which the system might be (eg, pregnant or open) at that stage of the problem (eg, 3 months after parturition). The action to be taken at stage n is the decision variable, denoted by X_n. Finally, there is the **objective function**. This is defined for each stage and is the value of the function appropriate for that stage and all subsequent stages. In the deterministic dynamic programming model, where all the subsequent outcomes are known for certain, the value of the objective function is an expression of all the decision variables still to be taken together with the value of the current state variable. Suppose that $C_n(X_n)$ is the value of the objective between stages n and n+1 when action X_n is taken. Bellman's principle of optimality now permits a statement of the problem in terms of its optimal policy.

Choose X_n so that

$$f_n(S_n) = \text{Opt}\ \{C_n(X_n) + f_{n+1}(S_{n+1})\}$$

where S_{n+1} is a known function of S_n and X_n and Opt is minimizing or maximizing as appropriate.
The solution procedure in a DP-model usually begins from the most remote stage (as the form of the equations imply) and works backwards to the present. So, first $f_N(S_N)$ is determined, with N denoting the end of the planning horizon or the last stage. After that, the stage number n is decreased by one (ie, N-1), the next $f_{N-1}(S_{N-1})$ function is calculated by using the value for $f_N(S_N)$ that has just been derived during the previous iteration. This process keeps repeating until the model finds the optimal policy starting at the initial stage (n=1). The variable S_1 is the known initial state, and $f_1(S_1)$ is therefore the total objective which is to be optimized. At each stage the optimal decision is determined for all combinations of the state variables, which specify the state of the process (eg, age and production in case of livestock).
Consider the following DP-example about finding the least-cost path through the network shown in Figure 7.2.

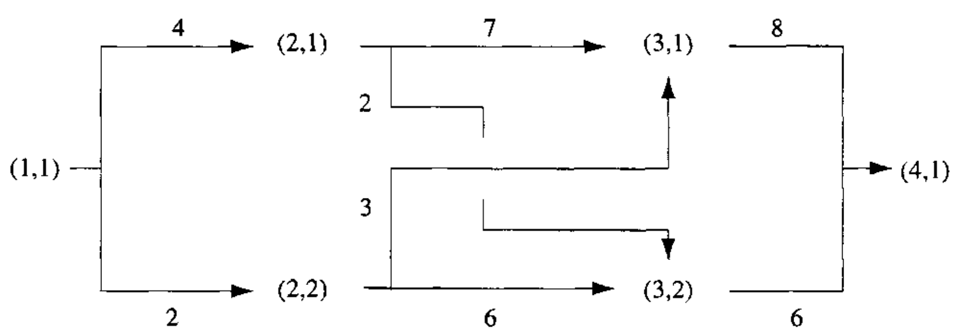

Figure 7.2 A least-cost network problem

Nodes have been designated (i,J_i), where i is the decision stage and J_i the state number. The optimal path must start at (1,1) and end at (4,1). Inter-node costs, or negative stage returns, are shown beside the linking decision arrows. A useful system for solving DP problems by hand is the preparation of a series of tables, one for each stage, starting with the final decision stage (Table 7.2). Each table has a row for each feasible state. Against each feasible state, the total cost to the end of the planning horizon is shown. Total cost is the sum of the stage costs and the optimal (least) costs to the planning horizon from the state accessed at the next stage. The last two columns of the table show optimal (least) total costs and the optimal decision associated with it. The procedure is demonstrated for the network problem of Figure 7.2 in Table 7.2.
The first row in Table 7.2 consists of the costs from node (4,1) to the end of the planning

horizon (4,1), being zero. The second row consists of the cost of linking nodes (3,1) - (4,1) and (3,2) - (4,1), being 8 and 6 respectively. The third row consists of the cost of linking nodes (2,1) - (3,1) being 7 (stage cost) plus 8 (optimal cost between (3,1) and (4,1)), which equals 15. The second possibility here consists of the cost of linking nodes (2,1) - (3,2), being 2 plus 6 equals 8. The least cost of moving from (2,1) to (4,1) therefore is the minimum value of [15,8] = 8 via (3,2), as depicted in the last two columns of the third row. Other rows are determined in the same way.

Table 7.2 DP-solution procedure for the least-cost network problem

Node	Costs to next node		Least costs	Optimal next node
(4,1)	-		0	-
(3,1)	8 (4,1)		8	(4,1)
(3,2)	6 (4,1)		6	(4,1)
(2,1)	7+8 = 15 (3,1)	2+6 = 8 (3,2)	8	(3,2)
(2,2)	3+8 = 11 (3,1)	6+6 = 12 (3,2)	11	(3,1)
(1,1)	4+8 = 12 (2,1)	2+11 = 13 (2,2)	12	(2,1)

Table 7.2 shows that the least-cost path from (1,1) to (4,1) incurs a cost of 12. The least-cost path itself is found by tracking forward through the table. The table shows that (2,1) should succeed (1,1), and (3,2) should succeed (2,1). The optimal sequence of nodes therefore is (1,1) (2,1) (3,2) (4,1), with associated cost 4 + 2 + 6 = 12.

So far all costs and demands have been assumed to be known for certain (deterministic approach). Often this is not realistic, however. In livestock production, for instance, the unpredictable nature of the data should be taken into account. **Stochastic DP** requires the same fundamental assumptions as the deterministic approach. At each stage there is an explicitly known state variable S_n. The decision variable is again denoted by X_n, but whereas in the deterministic model S_n and X_n lead to a unique state variable S_{n+1} at the next stage, in stochastic DP there is a probability distribution, dependent on S_n and X_n, over the next state variable. The cost of the stage, $C_n(X_n)$, is usually assumed to be known without error. The equivalent form of the fundamental equation replaces the term $f_{n+1}(S_{n+1})$, which would otherwise be a random variable, with its expected value. This is the weighed average of all possible values of $f_{n+1}(S_{n+1})$, where the weights are the corresponding probabilities. This is written as $E[f_{n+1}(S_{n+1})]$. Now X_n is to be chosen so that

$$f_n(S_n) = \text{Opt}\ \{C_n(X_n) + E[f_{n+1}(S_{n+1})]\}$$

DP has the advantage of placing no restrictions on the nature of the functions used to specify the structure of the system. So, linear as well as nonlinear relationships can be included. Furthermore it is possible to alter parameter values over time, offering the opportunity to include, for instance, seasonality and continuous genetic improvement. In the field of animal health economics, DP has been used most extensively in culling decisions in dairy cattle (see Van Arendonk, 1985; Kristensen, 1993; Houben, 1995) and in sows (Huirne, 1990).

7.4 Application of dynamic programming to replacement decisions in dairy cows

In the case of dairy cows, major revenues and costs differ with age and stage of lactation. Simultaneous consideration of all these - biological and economic - variables and their interrelationship is critical for making accurate replacement decisions. Decisions to replace individual animals are mainly based on economic rather than biological considerations under the condition that the size of the herd must remain constant. The farmer replaces a cow when a higher profit is to be expected from its replacement.

The simplest DP-formulation of the replacement problem has one state variable, lactation number S_n at stage n, and the decision option to keep the cow for at least one more lactation, or replace the cow with a heifer that is about to start its first lactation (Kennedy, 1986). The decision stage is the start of each lactation. Net returns over the lactation $R_n(S_n)$ depend on the cost of feed, the price of milk and the price of calves. If the decision is to keep the cow, and the lactation is successful, the state at stage n + 1 is 2. The return from the sale of the culled cow is denoted by $L_n(S_n)$, and the cost of the replacement heifer by C_n.

The lactation of the cow may be unsuccessful either because of failure due to low yield or a disease problem, or because of the death of the cow. If the lactation is unsuccessful, replacement is forced. In the case of forced or involuntary replacement because of failure, which has a probability of $PF(S_n)$, it is assumed that the stage net return is still $R_n(S_n)$. In the case of involuntary replacement because of death, which has a probability of $PD(S_n)$, it is assumed that the stage net return is also $R_n(S_n)$ but no return is realized from the sale of the cow. The probability of a successful transition from lactation S_n to $S_n + 1$ is therefore $(1 - PF(S_n) - PD(S_n))$ denoted by $PS(S_n)$. The discount factor is symbolized with δ. The recursive equation for maximization of the present value of expected net revenue is:

$$V_n(S_n) = \max [VK_n(S_n), VR_n(S_n)] \qquad (n = N-1,....,1)$$

and

$$V_N(S_N) = L_N(S_N) \qquad (n = N)$$

where

$$VK_n(S_n) = R_n(S_n) + \delta[PS(S_n)V_{n+1}(S_n+1) + \{PF(S_n) + PD(S_n)\} \times \{V_{n+1}(1) - C_{n+1}\} + PF(S_n)L_{n+1}(S_n+1)]$$

for the decision **to keep** the cow for at least one more lactation; and

$$VR_n(S_n) = L_n(S_n) - C_n + R_n(1) + \delta[PS(1)V_{n+1}(2) + \{PF(1) + PD(1)\} \times \{V_{n+1}(1) - C_{n+1}\} + PF(1)L_{n+1}(2)]$$

for the decision **to replace** the cow with another one that is about to start its first lactation. After the optimal lifespan of a cow is calculated in this way, the model can be used to determine the Retention Pay-Off (RPO) for each individual cow:

$$RPO(S_n) = VK_n(S_n) - VR_n(S_n)$$

The above-mentioned equations must be extended and reformulated to obtain a real-life DP-application. State variables additional to lactation number which may be included are stage of lactation, moment of conception, month of calving, and perhaps most importantly, milk production level during previous and present lactations. Clearly, extending the number of state variables results in an increased complexity of the DP-equations.
Results of a dairy herd replacement model are presented below. It is a stochastic DP-model in which the state variables include lactation number, stage of lactation, milk production during previous and present lactations, and time of conception. The calculated RPO-values for typical - Dutch - conditions are given in Table 7.3, calculated for cows that have just become pregnant at three months after calving.

Table 7.3 Retention Pay-Off (RPO) of cows that have just become pregnant at three months after calving (in US$)

Lactation	Relative production level of cow[a]				
	80	90	100	110	120
1	-[b]	100	350	575	825
2	50	200	500	750	1075
3	75	225	525	800	1150
4	75	200	475	750	1075
5	50	150	400	675	975
6	25	100	325	575	875
7	-	25	225	450	725
8	-	-	100	325	575

[a] Relative to herd average at Mature Equivalent (%)
[b] An RPO-value below zero

As can be seen in Table 7.3, a first calving cow with an average production level (100%) has an RPO of US$350 at three months after calving. This is the financial loss should this cow be replaced for some reason. RPO also represents the maximum amount of money that should be spent in trying to keep her. RPO increases considerably for cows with higher production levels. A cow in third lactation with a relative production level of 120% has even an RPO of US$1150. The RPO of poor-producing animals declines sharply. A first lactation cow with a production level of 80% has a negative RPO, which means that replacement is the most profitable option. Should such a cow not be culled in its first lactation and should keep producing at 80% level, then its RPO from the second lactation onwards is just high

enough to remain in the herd until its 7th lactation.
Values shown in Table 7.3 are valid for cows that become pregnant at a normal moment in lactation. When this is not so, the farmer has to make a choice between the following (bad) options: (1) to re-inseminate the cow and accept the loss due to an increased calving interval, or (2) to replace the cow and accept the loss associated with premature disposal. In Table 7.4, results are presented for three different moments of decision: three, five and seven months after calving. Moreover, two different breeding outlooks are considered: (1) an optimistic outlook that assumes that the cow will have normal probabilities of conceiving in future lactations, and (2) a pessimistic outlook that assumes that the cow's fertility problems will recur.

Table 7.4 Critical production levels below which it is not profitable to inseminate empty cows

Decision moment[a]	Minimum calving interval (months)	Production level cow in lactation[b] 1	2	3	4	5	6	7	8
Optimistic breeding outlook									
3	12	86	86	88	90	92	94	98	102
5	14	90	90	92	96	98	100	104	110
7	16	96	96	98	102	104	108	112	118
Pessimistic breeding outlook									
3	12	86	86	88	90	92	94	98	102
5	14	100	100	100	102	104	106	108	114
7	16	120	114	114	116	116	118	120	124

[a] Months after calving
[b] Relative to herd average at Mature Equivalent (%)

The results in Table 7.4 indicate that from an economic point of view cows in their first lactation that produce less than 86% of herd average should not be inseminated any more at three months after calving. Assuming a normal distribution of production, and a phenotypic intra-herd standard deviation of milk yield of 12%, this result implies that 12 to 13% of first lactation cows should be culled for insufficient production capacity. At five months after calving, the production level should be at least 90% to justify insemination of non-pregnant animals, and the limit is 96% at seven months in lactation. So, from an economic point of view young animals with a high production level can be inseminated several times. For older cows, the critical production level is higher because of various factors, including the sharply increasing probability of involuntary disposal in future lactation and the continuous genetic increase in milk production. The critical production levels are strongly increased when recurrent fertility problems are to be expected (pessimistic breeding outlook), especially at moments of decision later in lactation. The influence decreases with a higher age of the cow concerned, because the remaining expected life has decreased, and hence the expected number of future calving intervals.

7.5 Application of dynamic programming to replacement decisions in sows

The application of DP to sows is very similar to that of dairy cows and, therefore, discussed only briefly. The simplest DP-formulation of the sow replacement problem has one state variable, parity number S_n at stage n, and the decision option to keep the sow for at least one more parity, or replace her with a replacement gilt that is about to start its first parity. The decision stage is the start of each parity, ie, the moment of weaning the piglets. Net returns over parity $R_n(S_n)$ depend on the feed and the price of feeder pigs sold. The return from the sale of the culled sow is denoted by $L_n(S_n)$, and the cost of the replacement gilt by C_n. The definition and probability of forced or involuntary replacement because of failure ($PF(S_n)$), of involuntary replacement because of death ($PD(S_n)$), and of a successful transition from parity S_n to $S_n + 1$ ($PS(S_n)$) are similar to the application to dairy cows. The recursive equations for maximization of the present value of expected net revenue, as presented in the previous section, are also valid for the sow replacement problem. To obtain a real-life DP-application, these DP-equations must also be extended. Additional state variables are moment of conception and piglet production level during the last and second last parity.

The results of a replacement model for sows are discussed. The state variables include parity number, litter size (number of pigs born alive) during the last and second last parity, and moment of conception. The average RPO-values for various sows at the first time of breeding after weaning are given in Table 7.5.

Table 7.5 Retention Payoff for sows pregnant at the first moment of conception after weaning (in US$)

Parity	Pigs born alive[a]	Relative production level of sow 50%	75%	100%	115%	130%
1	9.6	20	65	110	135	165
2	10.3	-[b]	40	110	150	190
3	10.8	-	20	90	135	180
4	11.1	-	-	70	115	155
5	11.2	-	-	50	90	135
6	11.1	-	-	30	70	110
7	11.0	-	-	15	50	90
8	10.9	-	-	5	35	70

[a] Parity-specific averages in the herd (= 100%)
[b] An RPO-value below zero

As could be expected a longer herd life is especially profitable for the better-producing sows. Table 7.5 also shows that strong selection in the earlier parities is economically not worthwhile. Even sows that produce 50% below average should not be culled on strictly economic grounds before their second parity. The key factor here is the low **repeatability** of litter size as a predictor of future performance of sows.

The critical production levels below which it is not profitable to (re)breed sows that fail to conceive are presented in Table 7.6. Results consider the optimistic breeding outlook only, assuming no expected repeatability of fertility problems in future parities.

Table 7.6 Critical production levels below which it is not profitable to breed empty sows

Parity	Pigs born alive[a]	Breeding 1	Breeding 2	Breeding 3	Breeding 4
1	9.6	40[b]	40[b]	66	85
2	10.3	47	57	77	98
3	10.8	58	70	93	115
4	11.1	69	88	110	134
5	11.2	82	101	125	144
6	11.1	88	110	133	150[b]
7	11.0	98	117	142	150[b]
8	10.9	100	124	147	150[b]

[a] Parity-specific averages in the herd (= 100%)
[b] Lower (40%) and upper (150%) production level used in the model

Average-producing sows (ie, 100%) in the first and second parity can be allowed at least three rebreedings before replacement becomes more profitable. As could be expected the critical production level below which rebreeding is not profitable any more strongly increases with a higher parity number. A third rebreeding is hardly ever optimal for sows in parities six to eight (critical production level equalling - at least - 150%).

Exercise

In Chapter 19 you can find an example on dynamic programming, in which the calculation of the optimal time of sow replacement is shown step by step. You have to use these results to calculate the RPO of the sows according to the explanation in this chapter. You can then change some input values for a sensitivity analysis to see how to use such a model for specific purposes. Finally, the model is extended by taking into account genetic improvement of the sows over time. The entire exercise takes approximately 45 minutes.

7.6 Concluding remarks

Dynamic programming is a flexible mathematical technique for determining the economically optimal treatment and replacement decisions for dairy cows and sows. Major **advantages** of DP include the possibility of allowing for variation in, and possible repeatability of traits. Both the risk that a high-producing animal (cow or sow) may have a low future production and the risk that an animal may be replaced with a low-producing replacement animal can, therefore, be taken into account. However, the DP-model easily becomes very large. This results in a high memory request and high computation costs. Kristensen (1993) developed a very efficient DP-algorithm, ie, the **Hierarchic Markov Process** (HMP), which can be used to optimize relatively large DP-models. Houben (1995)

used the HMP-approach to include mastitis incidence in the replacement model.
The calculated RPO-values for individual cows and sows can serve as useful guides for making replacement decisions. In case of health problems, the RPO-value of an animal represents the maximum amount of money that should be spent in trying to get her back to previous production levels.
The repetitive nature of the DP-algorithm makes it almost impossible to include culling reasons that are difficult to quantify, such as maternal characteristics. However, these can be taken into account using expert systems that are integrated with the DP-model. First promising prototypes for such systems have been made available for sows (Huirne, 1990).

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