DRAINAGE PRINCIPLES AND APPLICATIONS

I INTRODUCTORY SUBJECTS

II THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

III SURVEYS AND INVESTIGATIONS

IV DESIGN AND MANAGEMENT OF DRAINAGE SYSTEMS

Edited from:
Lecture notes of the
International Course on Land Drainage
Wageningen

INTERNATIONAL INSTITUTE FOR LAND RECLAMATION AND IMPROVEMENT
P.O. BOX 45 WAGENINGEN THE NETHERLANDS 1973

- deel 2

PREFACE

This book is the second of four Volumes containing the edited lecture notes of the International Course on Land Drainage, which is organized annually in Wageningen, The Netherlands. In the Course an effort is made to cover, as completely as possible and within a period of three months, the basic principles of land drainage and their application. As mentioned in the Introduction to Volume I, the authors – all specialists in their particular fields – do not profess to have treated their subject matter exhaustively; within the limited time available, it is impossible for them to discuss all details of their subjects.

This second Volume presents the basic principles of land drainage by gravity and wells. It also deals with salt balances, leaching requirements, effects of irrigation on drainage, field drainage criteria, and mathematical models for different types of groundwater flow and for watershed runoff. The book can be used independently of the other Volumes although, to avoid repetition, reference is often made to their chapters. Volume I, issued in August 1972, treats basic elements, physical laws governing groundwater flow, and concepts of the plant-soil-water system in which the processes of land drainage take place. The forthcoming Volumes III and IV will discuss the various surveys and investigations required to determine the parameters of the plant-soil-water system which are to be introduced into the drainage design computations; and will also treat the design and dimensioning of drainage systems, some of the main engineering features, and aspects of operation and maintenance.

The reasons why the lecture notes of the Course are being published have been explained in the Preface and Introduction in Volume I. It was mentioned in that Preface that, after the original Editorial Committee under the chairmanship of Mr. P. J. Dieleman had broken up, a Working Group was formed to finish the job. This group consisting of members of the Institute's staff, has made no substantial changes in the work programme and the principles laid down by the Editorial Committee for the publication of these lecture notes. The members of the Working Group who contributed to the editing of Volume II were:

Mr. J. Kessler, Chairman, Chief Editor
Mr. N. A. de Ridder, Editor
Mr. M. G. Bos, Editor
Mr. R. H. Messemaeckers van de Graaff, Editor
Mr. T. Beekman, Production
Mr. J. Stransky, Subject index
Mrs. M. F. L. Wiersma-Roche, Translator

To our deep regret Mr. Kessler died suddenly in August 1972. Before his death, he had been able to complete most of the editorial work not only for Volume I but also for Volume II. His last contribution to the work was the preparation of a complete new draft of Chapter 11: Field Drainage Criteria. Mr. J. W. van Hoorn, Mr. J. H. Boumans and Mr. C. L. van Someren made editorial changes in this chapter.

Mr. Kessler's task as chairman of the Working Group has been taken over by Mr. N. A. de Ridder. I have full confidence that under his capable guidance the job of issuing the last two Volumes will be completed satisfactorily.

Wageningen, April 1973

Ch. A. P. Takes
Acting Director (1971–'72)
International Institute for
Land Reclamation and Improvement

LIST OF SUBJECTS AND AUTHORS OF VOLUMES I–IV

MAIN TABLE OF CONTENTS

PURPOSE AND SCOPE

Principles and applications of some generally used equations for subsurface flow to a system of parallel ditches or pipe drains under both steady and non-steady state conditions are discussed.

CONTENTS

8.1 INTRODUCTION

Until recently, all over the world, the only common practice of controlling the water table was by a system of open ditches. In modern agriculture many of these systems have been, or are now being, replaced by pipe drains (Chap.27, Vol.IV).

In any system of drains one may distinguish between (Fig.1):
- field drains or field laterals, usually parallel drains whose function is to control the groundwater depth;
- collector drains, whose function is to collect water from the field drains and to transport it to the main drains;
- main drains, whose function is to transport the water out of the area.

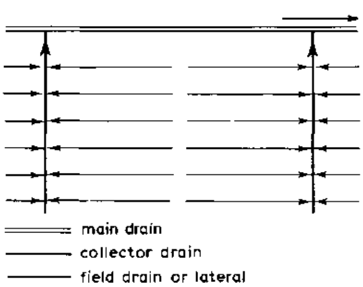


Fig.1. Drain functions.

There is not always a sharp distinction between the functions of the drains. For instance all field and collector drains also have a transport function, and all the collector and main drains also control the groundwater depth to some extent.

The discussion in this chapter will be restricted to parallel field drains. Figure 2 shows a cross-section of the laterals in Fig.1. The water table is usually curved, its elevation being highest midway between the drains. The factors which influence the height of the water table are:
- precipitation and other sources of recharge
- evaporation and other sources of discharge
- soil properties
- depth and spacing of the drains
- cross-sectional area of the drains
- water level in the drains

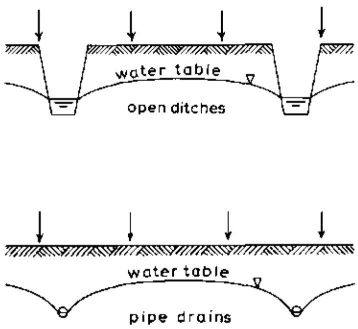


Fig.2. Cross-section of laterals showing a curved water table under influence of rainfall.

In this chapter the above factors are interrelated by drainage equations, based on two assumptions, viz.:
- two-dimensional flow, i.e. the flow is identical in any cross-section perpendicular to the drains;
- a uniform distribution of the recharge, steady or non-steady, over the area between the drains.

Most of the equations discussed in this chapter are moreover based on the Dupuit-Forchheimer assumptions (Chap.6, Vol.I). Consequently they have to be considered as approximate solutions only. Such approximate solutions, however, are generally accepted as having such a high degree of accuracy that their application in practice is completely justified.
A distinction is made between steady state and non-steady state drainage formulas. The steady state formulas (Sect.8.2) are derived under the assumption that the recharge intensity equals the drain discharge rate and consequently that the water table remains in position. The non-steady state drainage equations (Sect. 8.3) consider the fluctuations of the water table with time under influence of a non-steady recharge.

8.2 STEADY STATE DRAINAGE EQUATIONS

8.2.1 HORIZONTAL FLOW TO DITCHES REACHING AN IMPERVIOUS FLOOR

It is recalled from Chap.6, Vol.I that under the assumptions of one-dimensional horizontal flow, implying parallel and horizontal streamlines, the flow to vertically walled ditches reaching an impervious floor (Fig.3a) can be described by the so-called Donnan equation (DONNAN, 1946)

$$R = q = \frac{4K(H^2 - D^2)}{L^2} \tag{1}$$

where

R = recharge rate per unit surface area (m/day)

q = drain discharge rate per unit surface area (m/day)

K = hydraulic conductivity of the soil (m/day)

H = height above the impervious floor of the groundwater table midway between two drains (m)

D = height above the impervious floor of the water level in the drains = thickness of aquifer below drain level (m)

L = drain spacing (m)

which has also been derived by HOOGHOUDT (1936).

Equation 1 may be rewritten as

$$q = \frac{4K(H+D)(H-D)}{L^2} \tag{2}$$

Setting (Fig.3a) h = H-D and H+D = 2D+h, where h is the watertable height above drain level at midpoint, i.e. the hydraulic head for subsurface flow into drains (m), Eq. 2 then changes into

$$q = \frac{8K(D+\frac{1}{2}h)h}{L^2} \tag{3}$$

The factor $D+\frac{1}{2}h$ in Eq.3 can be considered to represent the average thickness of the soil layer through which the flow takes place (aquifer), symbolised by $\overline{D}$. Introducing $\overline{D}$ into Eq.3 yields

$$q = \frac{8K\overline{D}h}{L^2} \tag{4}$$

where $K\overline{D}$ = transmissivity of the aquifer (m^2/day).

Equation 3 can be written as follows

$$q = \frac{8KDh + 4Kh^2}{L^2} \tag{5}$$

Setting D = 0 gives

$$q = \frac{4Kh^2}{L^2} \tag{6}$$

Equation 6 apparently represents the horizontal flow above drain level. This equation is known as the Rothe equation. It seems to have been derived as early as 1879 by Colding in Denmark.

If D is large compared with h, the second term in the numerator of the right hand side of Eq.5 can be neglected against the first term, giving

$$q = \frac{8KDh}{L^2} \tag{7}$$

Equation 7 and the first term of Eq.5 apparently represent the horizontal flow below drain level.

The above considerations permit the conception of a two-layered soil with interface at drain level. Accordingly Eq.5 may be rewritten as

$$q = \frac{8K_bDh + 4K_ah^2}{L^2} \tag{8}$$

where

K_a = hydraulic conductivity of the layer above drain level (m/day)

K_b = hydraulic conductivity of the layer below drain level (m/day)

8.2.2 PRINCIPLES OF THE HOOGHOUDT EQUATION

If the ditches do not reach the impervious floor, the flow lines will not be parallel and horizontal but will converge towards the drain (radial flow). In this region the flow system cannot be simplified to a flow field with parallel and horizontal streamlines without introducing large errors.

The radial flow causes a lengthening of the flow lines. This lengthening causes a more than proportional loss of hydraulic head since the flow velocity in the vicinity of the drains is larger than elsewhere in the flow region. Consequently, the elevation of the water table will be higher when the vertically walled ditches are replaced by pipe drains, the drain level remaining the same.

HOOGHOUDT (1940) derived a flow equation for the flow as presented in Fig.3b, in which the flow region is divided into a part with horizontal flow and a part with radial flow.

If the horizontal flow above drain level is neglected, the flow equation for a uniform soil reads

$$h = \frac{qL}{K} F_H \tag{9}$$

and

$$F_H = \frac{(L-D\sqrt{2})^2}{8DL} + \frac{1}{\pi} \ln \frac{D}{r_o\sqrt{2}} + f(D,L) \tag{10}$$

where

r_o = radius of the drains

$f(D,L)$ = a function of D and L, generally small compared with the other terms in Eq.10; it can therefore usually be ignored (LABYE, 1960).

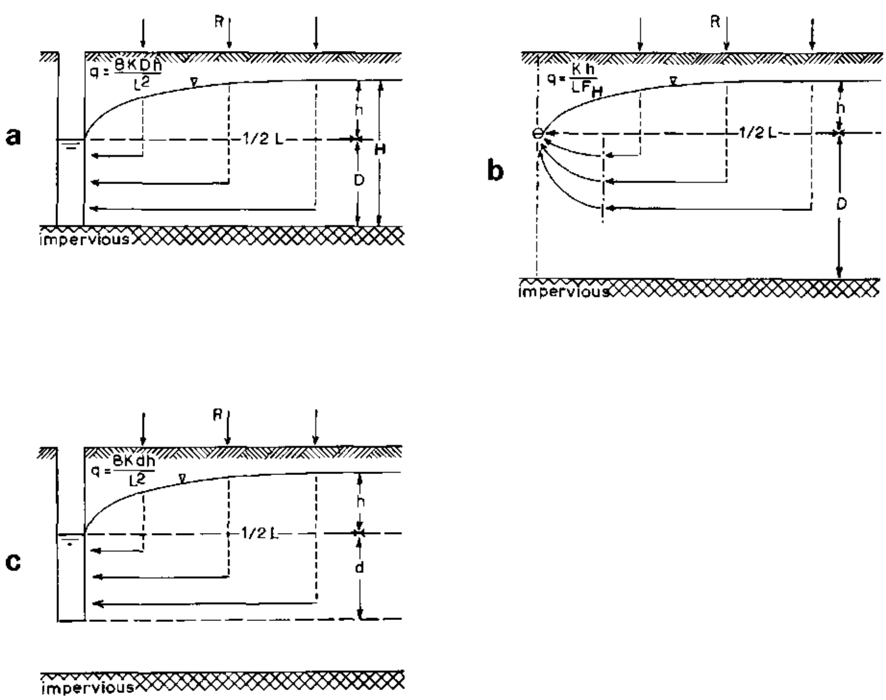


Fig.3. The concept of the equivalent depth to transform a combination of horizontal and radial flow into an equivalent horizontal flow.

The first term of the right hand member of Eq.10 pertains to horizontal flow, the second and the third term to radial flow.

Instead of working with Eqs.9 and 10, HOOGHOUDT considered it more practical to have a formula similar to the equations given in the previous section. To account for the extra resistance caused by the radial flow, he introduced a reduction of the depth D to a smaller equivalent depth d. By so doing, the flow pattern is replaced by a model with horizontal flow only (Fig.3c). If we consider only the flow below drain level, Eq.7 is reduced to

$$q = \frac{8Kdh}{L^2} \tag{11}$$

where d < D. This equation must be made equivalent to Eq.9. Solving the latter equation for q and equating the result with Eq.11 results in the equation for the equivalent depth

$$d = \frac{L}{8F_H} \tag{12}$$

The factor d is like F_H a function of L, D and r_o, as may be seen from Eqs.10 and 12. Values of d for r_o = 0.1 m and various values of L and D are presented in Table 1. For other drain diameters Fig.14 can be used, which will be explained in Sect.8.2.9.

In order to take radial flow into account the d-value can be introduced into all equations of Sect.8.2.1. When introduced in Eq.8 it yields

$$q = \frac{8K_b dh + 4K_a h^2}{L^2} \tag{13}$$

Equation 13 is called the Hooghoudt equation.

Discussion

In Eq.10 the first term in the right hand member pertains to the horizontal flow region. Comparison with Eq.7 proves that the horizontal flow is taken over a distance $L-D\sqrt{2}$ instead of L, and that the radial flow consequently is taken over a distance of $\frac{1}{2}D\sqrt{2}$ to both sides of the drains.
If we neglect f(D,L) in Eq.10 and set

$$F_h = \frac{(L-D\sqrt{2})^2}{8DL} \tag{14}$$

and

$$F_r = \frac{1}{\pi} \ln \frac{D}{r_o\sqrt{2}} \tag{15}$$

Eq.10 may be written as

$$F_H = F_h + F_r$$

Consequently Eq.9 changes into

$$h = \frac{qL}{K} F_h + \frac{qL}{K} F_r = h_h + h_r \tag{16}$$

Thus the total hydraulic head is the sum of the hydraulic heads h_h and h_r required for horizontal and radial flow respectively.

Table 1. Values for the equivalent depth d of Hooghoudt (r_o = 0.1 m, D and L in m)

L→	5 m	7.5	10	15	20	25	30	35	40	45	50
D											
0.5 m	0.47	0.48	0.49	0.49	0.49	0.50	0.50				
0.75	0.60	0.65	0.69	0.71	0.73	0.74	0.75	0.75	0.75	0.76	0.76
1.00	0.67	0.75	0.80	0.86	0.89	0.91	0.93	0.94	0.96	0.96	0.96
1.25	0.70	0.82	0.89	1.00	1.05	1.09	1.12	1.13	1.14	1.14	1.15
1.50		0.88	0.97	1.11	1.19	1.25	1.28	1.31	1.34	1.35	1.36
1.75		0.91	1.02	1.20	1.30	1.39	1.45	1.49	1.52	1.55	1.57
2.00			1.08	1.28	1.41	1.5	1.57	1.62	1.66	1.70	1.72
2.25			1.13	1.34	1.50	1.69	1.69	1.76	1.81	1.84	1.86
2.50				1.38	1.57	1.69	1.79	1.87	1.94	1.99	2.02
2.75				1.42	1.63	1.76	1.88	1.98	2.05	2.12	2.18
3.00				1.45	1.67	1.83	1.97	2.08	2.16	2.23	2.29
3.25				1.48	1.71	1.88	2.04	2.16	2.26	2.35	2.42
3.50				1.50	1.75	1.93	2.11	2.24	2.35	2.45	2.54
3.75				1.52	1.78	1.97	2.17	2.31	2.44	2.54	2.64
4.00					1.81	2.02	2.22	2.37	2.51	2.62	2.71
4.50					1.85	2.08	2.31	2.50	2.63	2.76	2.87
5.00					1.88	2.15	2.38	2.58	2.75	2.89	3.02
5.50						2.20	2.43	2.65	2.84	3.00	3.15
6.00							2.48	2.70	2.92	3.09	3.26
7.00							2.54	2.81	3.03	3.24	3.43
8.00							2.57	2.85	3.13	3.35	3.56
9.00								2.89	3.18	3.43	3.66
10.00									3.23	3.48	3.74
∞	0.71	0.93	1.14	1.53	1.89	2.24	2.58	2.91	3.24	3.56	3.88

Table 1. (cont.)

L →	50	75	80	85	90	100	150	200	250
D									
0.5	0.50								
1	0.96	0.97	0.97	0.97	0.98	0.98	0.99	0.99	0.99
2	1.72	1.80	1.82	1.82	1.83	1.85	1.00	1.92	1.94
3	2.29	2.49	2.52	2.54	2.56	2.60	2.72	2.70	2.83
4	2.71	3.04	3.08	3.12	3.16	3.24	3.46	3.58	3.66
5	3.02	3.49	3.55	3.61	3.67	3.78	4.12	4.31	4.43
6	3.23	3.85	3.93	4.00	4.08	4.23	4.70	4.97	5.15
7	3.43	4.14	4.23	4.33	4.42	4.62	5.22	5.57	5.81
8	3.56	4.38	4.49	4.61	4.72	4.95	5.68	6.13	6.43
9	3.66	4.57	4.70	4.82	4.95	5.23	6.09	6.63	7.00
10	3.74	4.74	4.89	5.04	5.18	5.47	6.45	7.09	7.53
12.5		5.02	5.20	5.38	5.56	5.92	7.20	8.06	8.68
15		5.20	5.40	5.60	5.80	6.25	7.77	8.84	9.64
17.5		5.30	5.53	5.76	5.99	6.44	8.20	9.47	10.4
20			5.62	5.87	6.12	6.60	8.54	9.97	11.1
25			5.74	5.96	6.20	6.79	8.99	10.7	12.1
30							9.27	11.3	12.9
35							9.44	11.6	13.4
40								11.8	13.8
45								12.0	13.8
50								12.1	14.3
60									14.6
∞	3.88	5.38	5.76	6.00	6.26	6.82	9.55	12.2	14.7

As can be seen from Table 1, the value of d increases with D until D ≃ ¼L. For larger values of D the equivalent depth d remains approximately constant. Apparently the flow pattern is then not affected by the depth of the impermeable layer (Fig.4).

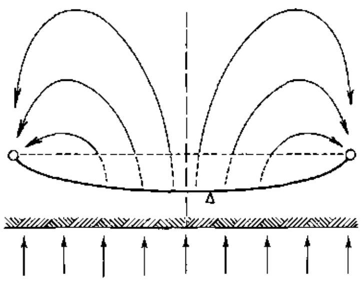


Fig.4.
Flow pattern in case of a deep uniform soil.

8.2.3 APPLICATION OF THE HOOGHOUDT EQUATION

The Hooghoudt equation is commonly used to calculate the drain spacing L, if the factors q, h, K, D and r_o are known. The formula can also be used to calculate the soil constants K and D, if q, h, L and r_o are known (Chap.26, Vol.III). Since the drain spacing L depends on the equivalent depth d, which in turn is a function of L, the formula cannot be given explicitly in L. Its use therefore as a drain-spacing formula involves a trial and error procedure. The trial and error method can be avoided by making use of nomographs examples of which are given in Figs.6 and 7.

<u>Example 1</u>

For the drainage of an irrigated area drain pipes with a radius of 0.1 m will be used. They will be placed at a depth of 1.8 m below the soil surface. A relatively impermeable soil layer was found at a depth of 6.8 m below the soil surface. From augerhole tests the hydraulic conductivity above this layer was estimated at 0.8 m/day (Fig.5).
Suppose that an irrigation is applied approximately once in 20 days. The average irrigation losses, which recharge the already high groundwater table, amount to 40 mm per 20 days so that the average discharge of the drainage system amounts to 2 mm/day.

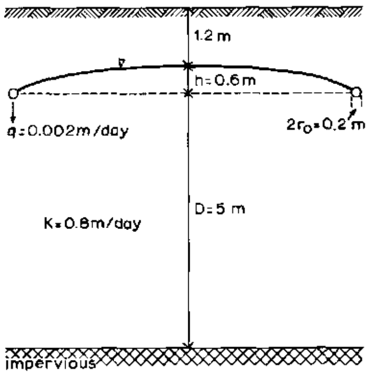


Fig.5. Drainage conditions in Example 1.

What drain spacing must be applied when an average water-table depth of 1.20 m below the soil surface is to be maintained?
From the above information we have

r_o = 0.1 m
q = 0.002 m/day
$K_a = K_b$ = 0.8 m/day
D = 5 m
h = 0.6 m

Substitution of the above values into Eq.13 gives

$$L^2 = \frac{8K_b dh^2 + 4K_a h^2}{q} = \frac{8 \times 0.8 \times 0.6 \times d + 4 \times 0.8 \times 0.6^2}{0.002}$$

$$L^2 = 1920d + 576$$

Trial 1
Take L = 80 m and read from Table 1: d = 3.55 m.
$L^2 = 1920\ d + 576 = 1920 \times 3.55 + 576 = 7392\ m^2$.
This is not in agreement with $L^2 = 80^2 = 6400\ m^2$.
Therefore L = 80 m is apparently too small.

Trial 2
Take L = 87 m and read from Table 1: d = 3.63 m.
$L^2 = 1920\ d + 576 = 1920 \times 3.63 + 576 = 7546\ m^2$.
This is sufficiently close to $L^2 = 87^2 = 7569\ m^2$.

Conclusion: The drain spacing required to satisfy the above conditions is $L = 87$ m.

Note:
In the equation $L^2 = 1920\ d + 576$, the term 576, representing the flow above drain level is comparatively small.
Neglecting it one obtains

$$L = \sqrt{1920\ d} = \sqrt{1920 \times 3.58} = 83 \text{ m.}$$

<u>Example 2</u>

To illustrate the use of nomographs of Figs.6 and 7 consider again the previous example.

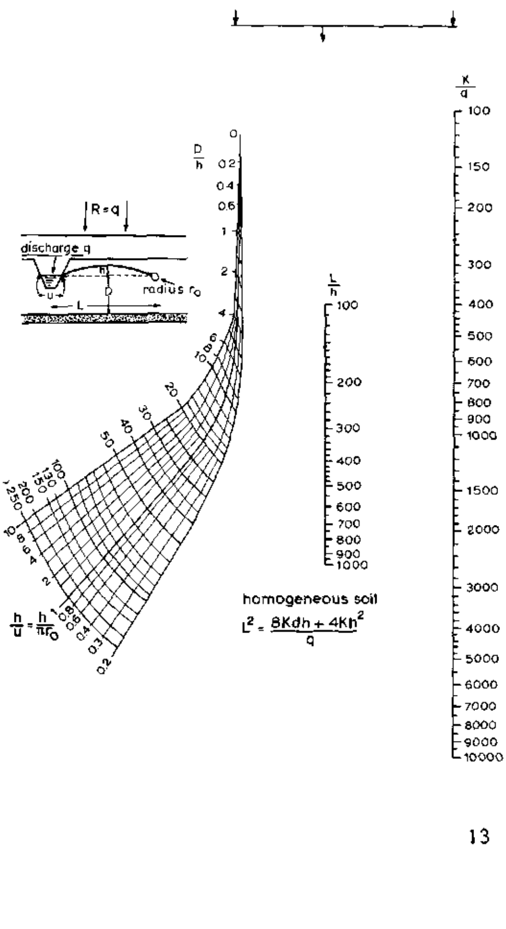


Fig.6.
Nomograph for the determination of drainspacing if $\frac{L}{h} < 100$.
(BOUMANS, 1963).

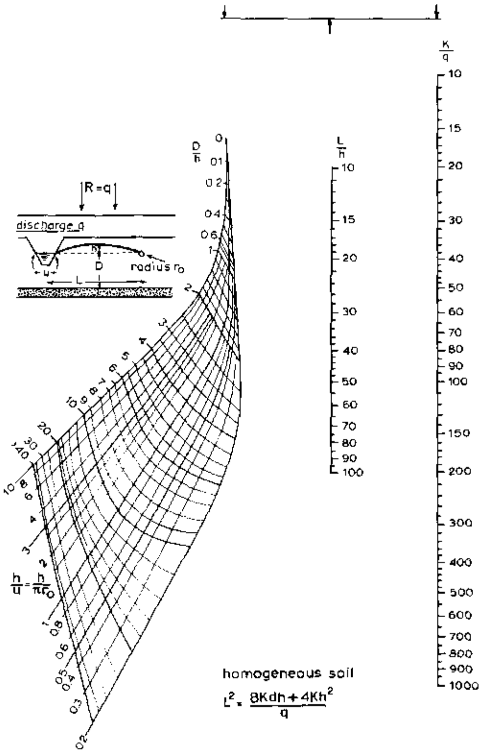


Fig.7.
Nomograph for the determination of drainspacing if $\frac{L}{h} > 100$ (BOUMANS, 1963).

Calculate $\frac{D}{h} = \frac{5}{0.6} = 8.3$ and $\frac{h}{\pi r_o} = \frac{0.6}{\pi \times 0{,}1} = 1.9$

Fix the intersection point of the corresponding curve in the left hand part of Fig.7. Calculate $\frac{K}{q} = \frac{0.8}{0.002} = 400$. Fix this point on the right hand scale and connect it with the above intersection point by a straight line. Read at the intersection of the straight line and the middle scale that $\frac{L}{h} = 140$. Calculate finally $L = 140\ h = 140 \times 0.6 = 84$ m.
The same graphs may be used for open ditches by setting $u = \pi r_o$, where u is the wet perimeter of the drain (Sect.8.2.7).

8.2.4 PRINCIPLES OF THE KIRKHAM EQUATION

KIRKHAM (1958) gives an analytical solution for a problem similar to Hooghoudt's, viz. two-dimensional flow, a regularly distributed rainfall over the area, and drains not reaching an impervious floor. If the flow above the drains is ignored, Kirkham's solution can be written in a form similar to Eq.9

$$h = \frac{qL}{K} F_K \tag{17}$$

and

$$F_K = \frac{1}{\pi} \left[\ln \frac{L}{\pi r_o} + \sum_{n=1}^{\infty} \frac{1}{n} \left(\cos \frac{2n\pi r_o}{L} - \cos n\pi\right)\left(\coth \frac{2n\pi D}{L} - 1\right) \right] \tag{18}$$

Values of F_K are given in Table 2. It is found that the F_K values of Kirkham are very close to the F_H values of Hooghoudt, so that both the Hooghoudt and the Kirkham equations give almost identical results (WESSELING, 1964).

Table 2. Values of F_K according to Toksöz and Kirkham.

L/D → D/2r_o	100	50	25	12.5	6.25	3.125	1.5625	0.78125
8192	-	-	-	-	-	-	-	2.654
4096	-	-	-	-	-	-	2.65	2.43
2048	-	-	-	-	-	2.66	2.43	2.21
1024	-	-	-	-	2.84	2.45	2.21	1.99
512	-	-	-	3.40	2.63	2.23	1.99	1.76
256	-	-	4.76	3.19	2.40	2.01	1.76	1.54
128	-	7.64	4.53	2.96	2.19	1.78	1.54	1.32
64	13.67	7.43	4.31	2.74	1.96	1.57	1.32	1.10
32	13.47	7.21	4.09	2.52	1.74	1.35	1.10	0.88
16	13.27	6.99	3.86	2.30	1.52	1.13	0.88	0.66
8	13.02	6.76	3.64	2.08	1.30	0.90	0.66	0.44
4	12.79	6.54	3.42	1.86	1.08	0.68	0.44	-
2	12.57	6.32	3.20	1.63	0.85	0.46	-	-
1	12.33	6.08	2.95	1.40	0.62	-	-	-
0.5	12.03	5.77	2.66	1.11	-	-	-	-
0.25	11.25	5.29	2.20	-	-	-	-	-

In the solution represented by Eq.17 the flow in the upper region has been neglected (Fig.8). In a later paper KIRKHAM (1960) reported that, if vertical flow is assumed in this region, the hydraulic head should be multiplied by $(1-q/K)^{-1}$. Since this term relates to the flow in the layer above drain level, the general equation for a two-layer problem is (WESSELING, 1964)

$$h = \frac{qL}{K_b} \frac{1}{1-q/K_a} F_K \tag{19}$$

where K_a is the hydraulic conductivity above drain level and K_b below that drain level. The boundary between the two layers must, as in the Hooghoudt solution, coincide with the drain level (Fig.8).

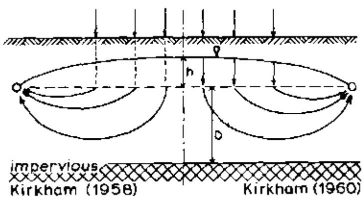


Fig.8.
Two-dimensional flow pattern according to the analytical solutions of KIRKHAM (1958, 1960).

8.2.5 APPLICATION OF THE KIRKHAM EQUATION

A graphical solution of Kirkham's equation is presented in Fig.9 (modified after TOKSÖZ and KIRKHAM, 1961). An application of the graphical solution will be given below.

Example 3

The data of Example 1 (Sect.8.2.3) will be used. We have

r_o = 0.10 m D = 5 m

q = 0.002 m/day h = 0.6 m

$K_a = K_b$ = 0.8 m/day

Take on the vertical axis of Fig.9 the value

$$\frac{h}{D}\left(\frac{K_b}{q} - \frac{K_b}{K_a}\right) = \frac{0.6}{5}\left(\frac{0.8}{0.002} - \frac{0.8}{0.8}\right) = 48$$

Go from this point in horizontal direction till the line marked $D/(2r_o)$ = $5/(2 \times 0.1) = 25$, which is found by interpolation between the lines marked 16 and 32. Go from this point vertically downwards and read on the axis L/D = 17. With D = 5 m, L = 5 × 17 = 85 m.

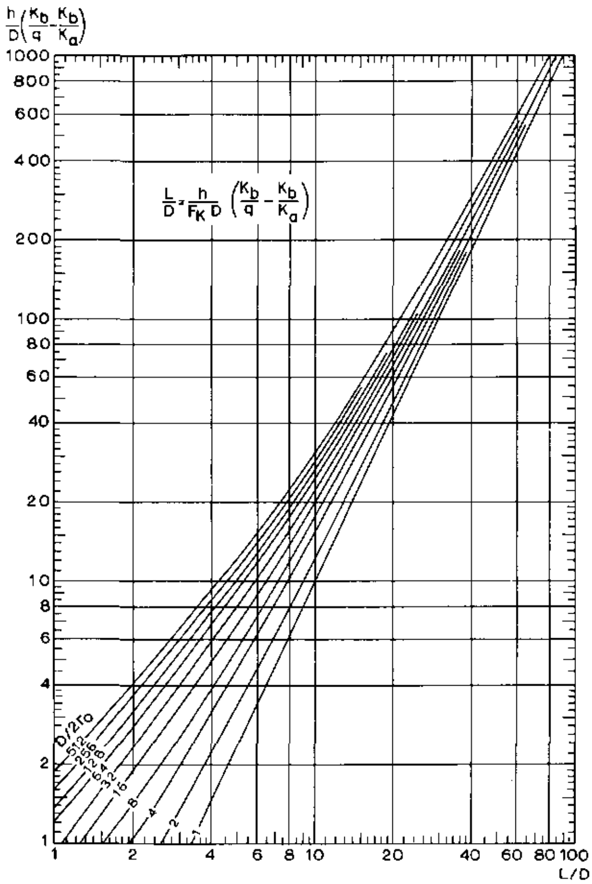


Fig.9.
Nomograph for the determination of drainspacing (modified after TOKSÖZ and KIRKHAM, 1961).

8.2.6 PRINCIPLES AND APPLICATION OF THE DAGAN EQUATION

Analogous to the method of Hooghoudt, DAGAN (1964) thought the flow to be composed of a radial flow in the area between the drain and a distance $\frac{1}{2}D\sqrt{2}$ away from the drain, and an intermediate, though mainly horizontal, flow in the area between the $\frac{1}{2}D\sqrt{2}$ plane and the midplane between the drains.
The Dagan equation, in a form similar to the Hooghoudt and Kirkham equations, reads

$$h = \frac{qL}{K} F_D \tag{20}$$

The expression for F_D is

$$F_D = \frac{1}{4}\left(\frac{L}{2D} - \beta\right) \qquad (21)$$

where $\beta = \frac{2}{\pi} \ln(2\cosh \frac{\pi r_o}{D} - 2)$ (22)

In Fig.10 the term β has been presented as a function of $\frac{\pi r_o}{D}$. Note that β-values are negative. With the aid of this figure the application of Dagan's equation is easy.

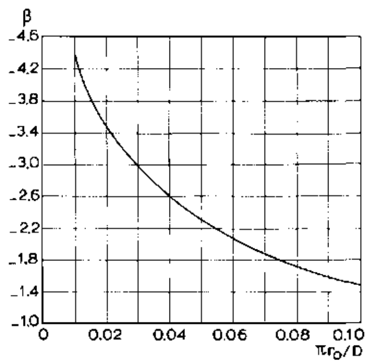


Fig.10.
Nomograph for the determination of β in the Dagan equation (DAGAN, 1964).

Example 4

The data of Example 1 (Sect.8.2.3) will be applied. We have

r_o = 0.10 m   D = 5 m
q = 0.002 m/day   h = 0.6 m
K = 0.8 m/day

Read from Fig.10 with $\frac{\pi r_o}{D} = 3.14 \times \frac{0.1}{5} = 0.06$ that $\beta = -2.1$.
Substitution of β into Eq.21 gives

$$F_D = \frac{1}{4}\left(\frac{L}{2D} - \beta\right) = \frac{1}{4}\left(\frac{L}{2D} + 2.1\right).$$

Substitution of F_D into Eq.20 yields

$$h = \frac{qL}{K} F_D = \frac{qL}{4K}\left(\frac{L}{2D} + 2.1\right)$$

Inserting the given information and rearranging yields

$$L^2 + 21\ L - 9600 = 0$$

and

$$\frac{-\ 21 \pm \sqrt{441 + 4 \times 9600}}{2} = \frac{21 \pm 197}{2}\ \mathrm{m}$$

Since $L > 0$, we find $L = 88$ m.

8.2.7 PRINCIPLES OF THE ERNST EQUATION

The Ernst equation is applicable to two-layered soils. It offers an improvement on the former formulas insofar as the interface between the two layers can be either above or below drain level. It is especially useful when the upper layer has a considerably lower hydraulic conductivity than the lower layer.

Like the Hooghoudt equation, the Ernst equation is found as the sum of the hydraulic heads required for the various flow components in which the flow towards the drains may be schematically divided.
In analogy with Ohm's law, we may write for groundwater flow

$$q = h/w \quad \text{or } h = qw$$

where q is the flow rate, h is the hydraulic head and w is the resistance. Thus, if we divide the flow towards the drains into vertical, horizontal and radial flow, the total hydraulic head may be given by

$$h = h_v + h_h + h_r = qw_v + qLw_h + qLw_r$$

where the subscripts v, h, and r refer to vertical, horizontal and radial flow. Note that horizontal and radial flow equal qL, i.e. the drain discharge per unit length of drain, whereas vertical flow equals q, the drain discharge rate per unit surface area.
Writing out the various resistance terms, we can read Ernst's equation as (ERNST, 1956, 1962)

$$h = q\,\frac{D_v}{K_v} + q\,\frac{L^2}{8\Sigma(KD)_h} + q\,\frac{L}{\pi K_r}\ln\frac{aD_r}{u} \tag{23}$$

where

h = total hydraulic head or water-table height above drain level at midpoint (m)

q = drain discharge rate per unit surface area (m/day)

L = drain spacing (m)

K_r = hydraulic conductivity in the layer with radial flow (m/day)

K_v = hydraulic conductivity for vertical flow (m/day)

D_v = thickness of layer over which vertical flow is considered (m)

D_r = thickness of layer in which radial flow is considered (m)

$\Sigma(KD)_h$ = transmissivity of the soil layers through which horizontal flow is considered (m^2/day)

u = wet perimeter of the drain (m)

a = geometry factor for radial flow depending on the flow conditions.

The values for D_v, $\Sigma(KD)_h$, D_r, a, and u are to be determined in accordance with the soil profile and the relative position and size of the drains. The appropriate values are derived from the following data which characterize the specific drainage conditions, namely:

D_1 = average thickness below the water table of the upper layer with permeability K_1

D_2 = thickness of the lower layer with permeability K_2

D_o = thickness below drain level of the layer in which the drains are located

h = water-table height above drain level at midpoint

y = water depth in the drain; for a pipe drain $y = 0$.

The values for D_v, $\Sigma(KD)_h$, D_r, a, and u are now considered in some detail, with the help of Figs. 11a to d.

- Vertical flow takes place in the layer between the maximum water table midway between the drains and the drain bottom. Usually the thickness of the layer for vertical flow can be taken as $D_v = y+h$ for ditches, and $D_v = h$ for pipe drains. In fact this should be $\frac{1}{2}(y+h)$ and $\frac{1}{2}h$ respectively, but usually this factor is of little importance.

- Horizontal flow occurs over the whole thickness of the aquifer, thus $\Sigma(KD)_h = K_1D_1 + K_2D_2$. If the depth to the impervious layer increases, the value of K_2D_2 increases too, making $\Sigma(KD)_h$ tending to infinity and the horizontal resistance to zero. In order to prevent this, the total thickness of the layers below the drains D_o or $D_o + D_2$ is restricted to $\frac{1}{4}L$ when the impermeable layer is deeper than $\frac{1}{4}L$ below drain level.

- Radial flow is taken into account only in the layer below drain level, thus $D_r = D_o$, with the condition that for radial flow the same restriction should be applied for D_o as for horizontal flow, viz. $D_o < \frac{1}{4}L$.

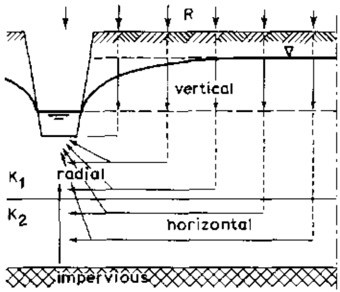


Fig.11a
Geometry of two-dimensional flow towards drains according to ERNST (1962).

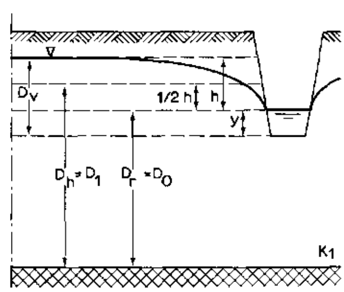


Fig.11b
Geometry of the Ernst equation for a homogeneous soil.

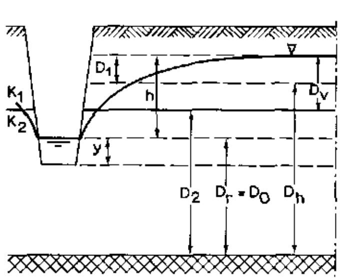


Fig.11c
Geometry of the Ernst equation for a two-layered soil with the drain in the lower layer.

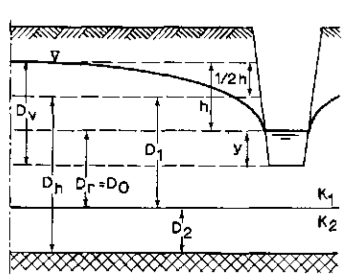


Fig.11d
Geometry of the Ernst equation for a two-layered soil with the drain in the upper layer.

- With respect to the value of a the following cases can be considered:

Homogeneous soils

In a homogeneous soil ($D_2 = 0$, Fig.11b), take a = 1. Further $D_v = y+h$, $\Sigma(KD)_h = K_1D_1$, $K_r = K_1$ and $D_r = D_o$, so that Eq.23 becomes

$$h = q\,\frac{y+h}{K_1} + q\,\frac{L^2}{8K_1D_1} + q\,\frac{L}{\pi K_1}\ln\frac{D_o}{u} \tag{24}$$

In homogeneous soils the vertical resistance is usually negligibly small. Moreover, as in most practical cases $h \ll D_o$, D_1 is usually reduced to D_o, neglecting the horizontal flow through the layers above drain level.

If the depth from drain bottom to impermeable layer, D_o, is larger than $\frac{1}{4}L$, the flow is thought not to reach beyond this depth. Since the drain spacing is not known beforehand this condition has to be checked afterwards. Actually, the calculations will lead to the same results when D_o is between $\frac{1}{4}L$ and $\frac{1}{2}L$. Beyond these limits, however, too small spacings are calculated.

Layered soils

If the drains are situated in the lower layer of a two-layered soil (Fig.11c) and $K_1 < K_2$, the vertical resistance in the second layer can be neglected against that in the first one.

From Fig.11c it can be seen that the thickness of the layer over which vertical flow must be considered equals $D_v = 2D_1$.

For the horizontal flow component we have in this case $\Sigma(KD)_h = K_1D_1 + K_2D_2$. Since $K_1 < K_2$ and $D_1 < D_2$, the first term is usually neglected and $\Sigma(KD)_h = K_2D_2$.

Radial flow is taken into account over the layer $D_r = D_o$.

For both the horizontal and the radial flow component, again the restriction is made that the thickness D_o may not exceed $\frac{1}{4}L$. The equation to be used then becomes

$$h = q\,\frac{2D_1}{K_1} + q\,\frac{L^2}{8K_2D_2} + q\,\frac{L}{\pi K_2}\ln\frac{D_o}{u} \tag{25}$$

If the drain is entirely in the upper layer of a two-layered soil (Fig.11d), the following conditions must be discerned with respect to the geometry factor a:

I $\quad K_2 > 20\,K_1$

The geometry factor a = 4 and Eq.23 becomes

$$h = q\,\frac{y+h}{K_1} + q\,\frac{L}{8(K_1D_1 + K_2D_2)} + q\,\frac{L}{\pi K_1}\ln\frac{4D_o}{u} \tag{26}$$

II $\quad 0.1K_1 < K_2 < 20\,K_1$

The geometry factor a has to be determined from the nomograph given in Fig.12, and to be introduced in Eq.23

III $0.1K_1 > K_2$

The geometry factor $a = 1$.

The lower layer can be considered impervious and the case reduces to that of a homogeneous soil underlain by an impervious boundary, so that Eq.24 is applicable.

- In the above equations the wet perimeter u of the drain occurs.
For ditches this factor is calculated as

$$u = b + 2y\sqrt{s^2 + 1} \tag{27}$$

where

b = bottom width of the ditch

y = water depth in the ditch

s = side slope of the ditch: horizontal/vertical.

For pipe drains, laid in trenches and sometimes surrounded by enveloping materials of good permeability, it is more difficult to determine an exact value for u. Under normal conditions u is determined from

$$u = b + 2 \times 2r_o \tag{28}$$

where

b = the width of the trench and

r_o = the radius of the drain.

If filter material is used, it is advisable to replace $2r_o$ by the height of the filter.

8.2.8 APPLICATION OF THE ERNST EQUATION

Drain spacings may be calculated directly or determined with the aid of the nomographs given in Figs.12 and 13 (VAN BEERS, 1965). The computation is carried out in steps to facilitate the right choice of the equations.

Step 1

Check the soil profile.

If the soil is homogeneous or if the depth of the layer in which the drain will be situated is more than $\frac{1}{4}$L, apply Eq.24. If less, go to step 2 and 3.

Step 2

Calculate the term $h_v = q\ D_v/K_v$.

Since this term is independent of L, it can be calculated directly and subtracted from h to yield Eq.29

$$h' = h-h_v = \frac{qL^2}{8(KD)_h} + \frac{qL}{\pi K_r} \ln \frac{aD_r}{u} \tag{29}$$

In most cases h_v is very small and may be ignored.

Step 3

Determine the geometry factor a.

If $K_2 > 20\ K_1$, set a = 4 and apply Eq.26.

If $0.1K_1 < K_2 < 20\ K_1$, determine a from Fig.12 and apply Eq.27.

If $K_2 < 0.1K$, set a = 1, consider the soil homogeneous and apply Eq.24.

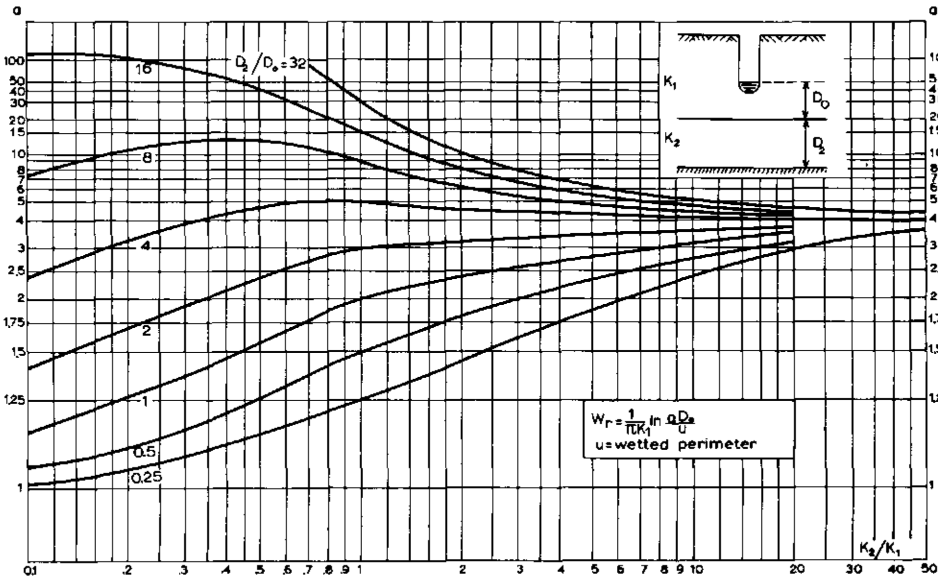


Fig.12. Nomograph for the determination of the geometry factor a for radial resistance in the Ernst equation (VAN BEERS, 1965).

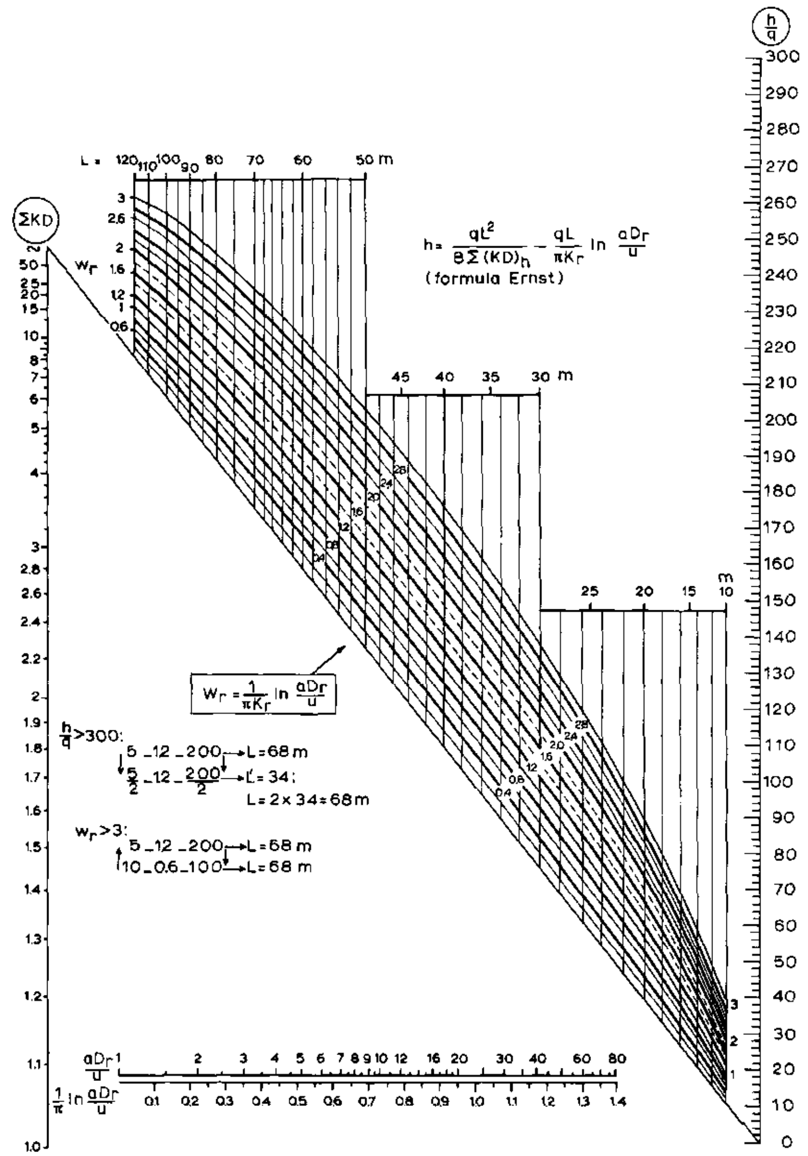


Fig.13. Nomograph for the determination of drainspacing with the Ernst equation if $D_o < \frac{1}{4}L$.

The application of the Ernst equation as a drain spacing formula will be illustrated by three examples: for a homogeneous soil ($D_o < \frac{1}{4}L$), for a two-layered soil with interface below drain level ($D_o < \frac{1}{4}L$) and for a deep soil ($D_o > \frac{1}{4}L$).

Example 5

The data of Example 1 (Sect.8.2.3) will be used. In addition to a trench width of 0.25 m, we have (see Fig.11b)

r_o = 0.1 m $\qquad$ D_o = 5 m

q = 0.002 m/day $\qquad$ h = 0.6 m

K_1 = 0.8 m/day

Since the soil is homogeneous, Eq.24 and Fig.13 are applicable. Thus we have, taking $u = 0.25 + 4 \times 0.1 = 0.65$ m,

$$h = 0.6 = \frac{qL^2}{8K_1D_1} + q\,\frac{L}{\pi K_1}\ln\frac{D_o}{u} = \frac{0.002\ L^2}{8 \times 0.8 \times 5.30} + \frac{0.002\ L}{\pi \times 0.8}\ln\frac{5}{0.65}$$

and

$$L = \frac{-\ 0.8 \pm \sqrt{0.64 + 4 \times 0.03 \times 300}}{2 \times 0.03} = \frac{-\ 0.8 \pm 6.05}{0.06}$$

Since $L > 0$ it follows that $L = 87.5$ m.

The nomogram of Fig.13 is used as follows:
Connect the point $\Sigma KD = K_1(D_o + \frac{1}{2}h) = 0.8 \times 5.30 = 4.2\ m^2/day$ on the left hand axis with the point $\frac{h}{q} = \frac{0.6}{0.002} = 300$ on the right hand axis by a straight line. Intersecting with the curve for

$$w_r = \frac{1}{\pi K_r}\ln\frac{aD_r}{u} = \frac{1}{\pi \times 0.8}\ln\frac{5}{0.65} = 0.8$$

one reads in a vertical direction on the axis that $L = 88$ m.

Example 6

A soil consists of two distinct layers. For the upper layer K_1 = 0.2 m/day and for the lower layer K_2 = 2 m/day. The interface of the two layers is at a depth of 0.50 m below the bottom of the drain ditch (Fig.11d). The thickness of the lower layer to an impermeable layer D_2 = 3 m. The ditch has a bottom width of 50 cm, side slope 1:1 and the water depth y = 30 cm. The hydraulic head is set at

$h = 1.20$ m at a steady state discharge of $q = 10$ mm/day.

From the above information (see Fig.11d)

$h = 1.2$ m $\qquad D_o = 0.5 + 0.3 = 0.8$ m

$q = 0.01$ m/day $\qquad D_1 = 0.8 + \frac{1}{2} \times 1.2 = 1.4$ m

$K_1 = 0.2$ m/day $\qquad D_2 = 3.0$ m

$K_2 = 2.0$ m/day $\qquad u = 0.5 + 2 \times 0.3\sqrt{2} = 1.35$ m

$y = 0.3$ m

Step 1

Assume $D_o < \frac{1}{4}L$ so that Eq.23 should be used.

Step 2

$$h_v = q\frac{D_v}{K_v} = q\frac{h+y}{K_1} = 0.01\frac{1.2+0.3}{0.2} = 0.075 \text{ m}$$

$h' = h - h_v = 1.2 - 0.075 = 1.125$ m.

Step 3

Since $K_2/K_1 = 10$ determine a from Fig.12.

Go from the point $K_2/K_1 = 10$ at the lower axis vertically upward to the line for $D_2/D_o = 3.0/0.8 = 3.8$ (interpolate between 2 and 4) and read on the vertical axis $a = 4$.

$$\Sigma(KD)_h = K_1D_1 + K_2D_2 = 0.2 \times 1.4 + 2 \times 3.0 = 6.3 \text{ m}^2/\text{day}$$

$$w_r = \frac{1}{\pi K}\ln\frac{aD_r}{u} = \frac{1}{\pi K_1}\ln\frac{4D_o}{u} = \frac{1}{\pi \times 0.2}\ln\frac{4 \times 0.8}{1.35} = 1.37 \text{ days/m}$$

Thus:

$$h' = 1.125 \text{ m} = \frac{qL^2}{8\Sigma(KD)_h} + \frac{qL}{\pi K_r}\ln\frac{aD_r}{u} = \frac{0.01\ L^2}{8 \times 6.3} + 0.01 \times 1.37\ L$$

or

$$0.2\ L^2 + 13.7\ L - 1125 = 0$$

and

$$L = \frac{-13.7 + \sqrt{13.7^2 + 4 \times 0.2 \times 1125}}{2 \times 0.2} = \frac{-13.7 + 33}{0.4} = 48 \text{ m}$$

This value can also be found from Fig.13.

Since $D_o = 0.8$ m the condition $D_o < \frac{1}{4}L$ is fulfilled.

Example 7

The data are as in Example 6, except that $D_o = 10$ m.

Step 1

Since it is likely that D_o will be more than $\frac{1}{4}L$, the solution for a homogeneous soil, as given by Eq.24, will be applied. This means that the second layer, whatever its permeability or thickness, has no influence on the flow to the drains. The assumption that $D_o > \frac{1}{4}L$ must be checked afterwards.
Following Example 6, Step 2, the vertical hydraulic head $h_v = 0.075$ m and $h' = 1.125$ m.
Solving now Eq.24 for a = 1, $K_1D_1 = 0.2 \times 10.6 = 2.1$ m^2/day, $D_o = 10$ m and u = 1.35 m, results in

$$1.125 \text{ m} = \frac{0.01}{8 \times 2.1} L^2 + \frac{0.01}{\pi \times 0.2} L \ln \frac{10}{1.35}$$

from which the drain spacing is calculated: L ≈ 24 m.
Since D_o(= 10 m) is indeed more than $\frac{1}{4}L$ (= 6 m) the assumption $D_o > \frac{1}{4}L$ was correct and the example could be treated as a homogeneous soil.
As D_o, introduced in the computation, is less than $\frac{1}{2}L$ (= 12 m) the solution obtained will also be correct.
This can be checked by taking $D_o = 6$ m. Solving Eq.24 now results in

$$1.25 \text{ m} = \frac{0.01}{8 \times 1.3} L^2 + \frac{0.01}{\pi \times 0.2} L \ln \frac{6}{1.35}$$

from which once again a drain spacing of 24 m is calculated.

8.2.9 GENERALIZED NOMOGRAPHS

For a homogeneous soil, with $D < \frac{1}{4}L$ and without regard to head losses due to vertical and horizontal flow above drain level Eq.24 reads

$$h = \frac{qL^2}{8KD} + \frac{qL}{\pi K} \ln \frac{D}{u}$$

The corresponding Hooghoudt equation writes as

$$h = \frac{qL^2}{8Kd}$$

Equating the above expression for h yields, after rearrangement

$$d = \frac{D}{1 + \frac{8D}{\pi L} \ln \frac{D}{u}} \qquad (30)$$

This expression for the equivalent depth d is presented graphically in Fig.14.

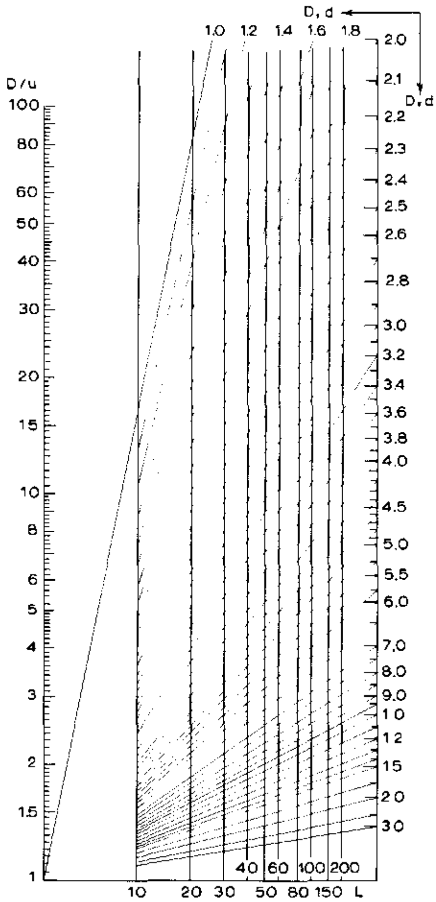


Fig.14.
Nomograph for the determination of the equivalent depth d (VAN BEERS, unpublished).

The nomograph of Fig.14 has the advantage that d can be determined for all values of r_0 or u, whereas in Table 1, d is given for a fixed value of r_0 only. An example of the use of the nomograph is given in Fig.14. When D/u = 15, D = 10 m and L = 40 m, d = 3.7 m.

VAN BEERS (in press) expressed the drain spacing for a homogeneous soil with negligible flow above drain level and $D < \frac{1}{2}L$ as

$$L = L_o - C \tag{31}$$

where

$L_o = \sqrt{8KDh/q}$

$C = D \ln \frac{D}{u}$

When the expression for L_o is compared with the Hooghoudt equation, it is readily seen that L_o represents the drain spacing for horizontal flow. For the radial resistance a subtraction C is applied. This is in contrast to the Hooghoudt solution where a reduction of D to d is used to account for radial flow.

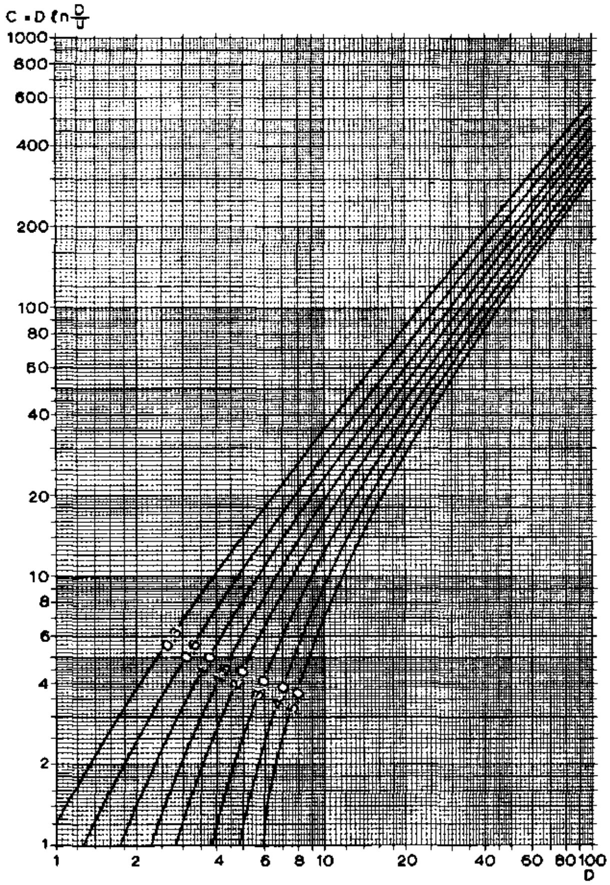


Fig.15.
Nomograph for the calculation of the subtraction C in the generalized equation $L = L_o - C$ (VAN BEERS, unpublished).

To calculate the subtraction C, the nomograph of Fig.15 may be used. This nomograph has the advantage of also being applicable to solve the non-steady state Glover-Dumm equation.
To compute C, take the relevant value of D on the horizontal lower axis. From this point go vertically upward to the value of u and read D ln(D/u) on the vertical axis.

8.3 NON-STEADY STATE DRAINAGE EQUATIONS

8.3.1 INTRODUCTION

In areas with periodic irrigations or high intensity rainfall, the assumption of a steady recharge is no longer justified. Under these conditions non-steady state solutions of the flow problem must be applied. Non-steady state solutions are indispensable when actual, non-steady water table elevations and drain discharges, as obtained from field data, must be evaluated (Chap.26, Vol. III).

It is recalled from Chap.6 (Vol.I) that the differential equation for non-steady state flow, as derived on the basis of the Dupuit-Forchheimer assumption, can be written as

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} - R \qquad (32a)$$

or, when the recharge rate R equals zero

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} \qquad (32b)$$

where
KD = transmissivity of the aquifer (m^2/day)
R = recharge rate per unit surface area (m/day)
h = hydraulic head as a function of x and t (m)
x = horizontal distance from a reference point, e.g. ditch (m)
t = time (days)
μ = drainable pore space (dimensionless, m/m)

8.3.2 PRINCIPLES OF THE GLOVER-DUMM EQUATION

DUMM (1954) used a solution for Eq.32b found by Glover who assumed an initial horizontal groundwater table at a certain height above the drain level. The solution describes the lowering of the groundwater table - which does not remain horizontal - as a function of time, place, drain spacing and soil properties.
The initial horizontal water table is thought to have been the result of an instantaneous rise caused by rainfall or irrigation, which instantaneously recharged the groundwater. Later DUMM (1960) assumed that the initial water table is not completely flat but has the shape of a fourth degree parabola, which resulted in a slightly different formula.

Figure 16 depicts the condition before and just after an instantaneous rise of a horizontal groundwater table. The initial and boundary conditions for which Eq. 32b must be solved are:

$t = 0, \quad h = R_i/\mu = h_o, \quad 0 < x < L$ (initial horizontal groundwater table)

$t > 0, \quad h = 0, \quad x = 0, x = L$ (water in drains remains at zero level = drain level)

where

R_i = instantaneous recharge per unit surface area (m)

h_o = height above drain level of the initial horizontal water table.

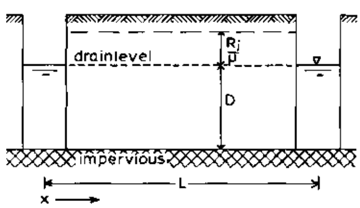


Fig.16.
Boundary conditions for the Glover-Dumm equation with initial horizontal water table.

The solution of Eq.32b for these conditions may be found in CARSLAW and JAEGER (1959)

$$h(x,t) = \frac{4h_o}{\pi} \sum_{n=1,-3,5,}^{\infty} \frac{1}{n} e^{-n^2\alpha t} \sin \frac{n\pi x}{L} \qquad (33)$$

where

$$\alpha = \frac{\pi^2 KD}{\mu L} \text{(reaction factor, day}^{-1}\text{)} \qquad (34)$$

For the height of the water table midway between the drains at any time t, $h_t = h(\frac{1}{2}L,t)$, one may substitute $x = \frac{1}{2}L$ into Eq.33 yielding

$$h_t = \frac{4}{\pi} h_o \sum_{n=1,-3,5,}^{\infty} \frac{1}{n} e^{-n^2 \alpha t} \tag{35}$$

Apparently the value of each term of Eq.35 decreases with increasing n. If $\alpha t > 0.2$ the second and next term will be comparatively small and may be neglected. Equation 35 then reduces to

$$h_t = \frac{4}{\pi} h_o e^{-\alpha t} = 1.27\ h_o e^{-\alpha t} \tag{36}$$

Under the assumption of an initial water table having the shape of a fourth degree parabola, Eq.36 changes into (DUMM, 1960)

$$h_t = 1.16\ h_o e^{-\alpha t} \tag{37}$$

The only difference between Eq.36 and Eq.37 is a change of the shape factor $\frac{4}{\pi} = 1.27$ in 1.16.

Substituting Eq.34 into Eq.37 and solving for L yields

$$L = \pi \left[\frac{KDt}{\mu}\right]^{\frac{1}{2}} \left[\ln\ 1.16\ \frac{h_o}{h_t}\right]^{-\frac{1}{2}} \tag{38}$$

which is called the Glover-Dumm equation.

As the Glover-Dumm equation does not take into account a radial resistance of flow towards drains not reaching an impermeable layer, the thickness of the aquifer D is often replaced by the d-value of Hooghoudt to account for the convergency of the flow in the vicinity of the drains. This substitution is justified since the flow paths for steady and non-steady flow may be considered at least similar, although not exactly identical.

Thus Eq.34 becomes

$$\alpha = \frac{\pi^2 Kd}{\mu L^2}\ (\mathrm{day}^{-1}) \tag{39}$$

and Eq.38 changes into

$$L = \pi\left[\frac{Kdt}{\mu}\right]^{\frac{1}{2}} \left[\ln 1.16 \frac{h_o}{h_t}\right]^{-\frac{1}{2}} \tag{40}$$

This may be called the modified Glover-Dumm equation.

8.3.3 APPLICATION OF THE GLOVER-DUMM EQUATION

The Glover-Dumm equation is particularly used to calculate the drain spacing in irrigated areas. It requires the determination of the soil properties K, D, and μ, the geometry of the drains and a drainage criterion. Compared with steady state formulas the Glover-Dumm equation requires a water table drawdown criterion in a certain time (h_o/h_t), instead of a water table elevation-discharge criterion (Chap.11, Vol.II). Moreover, the drainable pore space μ, is only required in non-steady state drain spacing formulas.

The calculation of the drain spacing L from Eq.40 requires a trial and error procedure, because due to the introduction of the equivalent depth $d = f(L,D,u)$ the quantity L cannot be given explicitly. With the help of Fig.15, the trial and error procedure may be avoided.

Example 8

Water is applied in an irrigated area every 10 days. The field application losses which percolate to the groundwater are 25 mm each irrigation and are regarded as an instantaneous recharge, R_i = 0.025 m. With an effective porosity μ = 0.05 the recharge causes an instantaneous rise of the water table, $\Delta h = R_i/\mu$ = 0.025/0.05 = 0.5 m.

The maximum permissible height of the water table is set at 1 m below the soil surface. The drain level is chosen at 1.8 m below the soil surface. We then have h_o = 1.8 - 1.0 = 0.8 m.

The water level must be lowered by Δh = 0.5 m in the next 10 days or else with the next irrigation, it will rise to above 1.0 m below ground surface. Therefore we have $h_{10} = h_o - \Delta h$ = 0.8 - 0.5 = 0.3 m. If the depth to an impervious layer is found at 9.5 m below the soil surface, if K = 1m/day, and if the radius of the pipe drains is 10 cm, calculate the drain spacing.

From the above information we have

$K = 1.0$ m/day $\quad t = 10$ days

$D = 7.7$ m $\quad r_o = 0.1$ m

$m = 0.05$

$h_o = 0.8$ m

$h_{10} = 0.3$ m

Substituting the above data into Eq.40, gives

$$L = \pi \left[\frac{1.0 \times d \times 10}{0.05}\right]^{\frac{1}{2}} \left[\ln \frac{1.16 \times 0.8}{0.3}\right]^{-\frac{1}{2}} \text{ m}$$

or

$L = 41.8 \sqrt{d}$ m

1st trial: $L = 80$ m.

Read from Fig.14, with $\frac{D}{u} = \frac{D}{\pi r_o} = \frac{7.7}{\pi \times 0.1} = 25$ and $D = 7.7$ m, that $d = 4.4$ m.

Substitution gives: $41.8 \sqrt{4.4} = 88$ m.

This is more than 80 m and L should be estimated at more than 88 m.

2nd trial: $L = 100$ m.

Read from Fig.14 that $d = 4.8$ m. Thus: $41.8 \sqrt{4.8} = 92$ m. This is less than 100 m and L should be estimated less than 92 m.

3rd trial: $L = 90$ m.

Read from Fig.14 that $d = 4.7$ m. Thus: $41.8 \sqrt{4.7} = 90$ m, and since the estimate was 90 m this is the correct drain spacing.

The solution with the nomograph of Fig.15 proceeds as follows:

Calculate (Eq.38)

$$L_o = \pi \left[\frac{KDt}{\mu}\right]^{\frac{1}{2}} \left[\ln 1.16\ h_o/h_t\right]^{-\frac{1}{2}}$$

$$= \pi \left[\frac{1.0 \times 7.7 \times 10}{0.05}\right]^{\frac{1}{2}} \times \left[\ln \frac{1.16 \times 0.8}{0.3}\right]^{-\frac{1}{2}} = 116 \text{ m}$$

Determine $C = D \ln \frac{D}{u}$ from Fig.15 by taking on the lower axis the point $D = 7.7$ m. Go from there vertically upward to intersect the curve for $u = \pi r_o = 0.3$ m. Read on the vertical axis that $C = 25$ m.

Compute $L = L_o - C = 116 + 25 = 91$ m.

8.3.4 DISCUSSION OF THE GLOVER-DUMM EQUATION

Time averaged hydraulic head

For various reasons, e.g. to account for the average horizontal flow above drain level or to apply steady state equations, it may be required to compute a time averaged hydraulic head, $\bar{h}$, between h_o and h_t or between h_{t_1} and h_{t_2} during tail recession.

One could during tail recession take the arithmetic mean $\frac{1}{2}(h_{t_2} + h_{t_1})$ but then $\bar{h}$ will be overestimated since h_t changes according to an exponential function.

The average $\bar{h}$ may be defined as

$$\bar{h} = \frac{1}{t}\int_o^t h_t dt = \frac{1}{t}\int_o^t 1.16\ h_o e^{-\alpha t} dt$$

which yields upon integration and rearranging

$$\bar{h} = \frac{1.16\ h_o}{\alpha t}\ (1-e^{-\alpha t}) = \frac{1.16\ h_o - h_t}{\ln(1.16\ h_o/h_t)} \qquad (41)$$

Another possibility is to use the geometric mean giving

$$\bar{h} = \sqrt{h_{t_1} h_{t_2}} \qquad (42a)$$

or

$$\log \bar{h} = \frac{1}{2}(\log h_{t_1} + \log h_{t_2}) \qquad (42b)$$

Flow above the drains should be taken into account if h is relatively large or D is small.

Eq.40 then reads

$$L = \pi\left[\frac{K(d+\frac{1}{2}\bar{h})}{\mu}t\right]^{\frac{1}{2}} \left[\ln\ 1.16\ h_o/h_t\right]^{-\frac{1}{2}} \qquad (43)$$

Non-steady discharge

The discharge of the drains at time t, when expressed per unit surface area, can be found from Darcy's law

$$q_t = \frac{-\ 2KD}{L}\left[\frac{dh}{dx}\right]_{x=0} \ (\text{m/day}) \qquad (44)$$

Differentiating Eq.33 with respect to x and substituting x=0, gives for Eq.44

$$q_t = \frac{8}{\pi^2}\ \alpha R_i \sum_{n=1,3,5,}^{\infty} e^{-n^2\alpha t} \qquad (45)$$

Neglecting all the terms except the first gives

$$q_t = \frac{8}{\pi^2}\ \alpha R_i e^{-\alpha t} \qquad (46)$$

Substituting $R_i = h_o\mu$ and for h_o the expression given in Eq.36 yields

$$q_t = \frac{2}{\pi} \alpha\mu h_t \tag{47}$$

Substituting the value of α from Eq.39 gives

$$q_t = \frac{2\pi Kd}{L^2} h_t \tag{48}$$

which is similar to the Hooghoudt equation except that the factor 2π is now obtained instead of 8.

From Eqs.36 and 46 it can be deduced that, during tail recession

$$\frac{q_{t_2}}{q_{t_1}} = \frac{h_{t_2}}{h_{t_1}} = e^{-\alpha(t_2-t_1)} \tag{49}$$

According to Eq.49 a plot of q_t or h_t on a logarithmic scale and time (t) on a linear scale will result in a straight line.
This relation is of importance to determine α from field data of drained plots (Chap.26, Vol.III).

8.3.5 PRINCIPLES OF THE KRAIJENHOFF VAN DE LEUR-MAASLAND EQUATION

Both KRAIJENHOFF VAN DE LEUR (1958) and MAASLAND (1959) derived solutions for non-steady state groundwater flow to drains. The solution is based on a steady recharge over any time period t instead of an instantaneous recharge as assumed by Glover-Dumm.
The applicable differential equation is Eq.32a. Starting with a flat water table at drain level at t = 0 and assuming a recharge intensity R (m/day) from the moment t = 0 on, yields the following initial and boundary conditions:

$h = 0$ for $t = 0$ and $0 < x < L$	(initial horizontal groundwater table at drain level at $t = 0$)
$h = 0$ for $t > 0$ and $x = 0$, $x = L$	(water in drains remains at zero level = drain level)
R = constant for $t > 0$	(constant recharge R starts at $t = 0$).

For the above boundary conditions the height of the water table midway between parallel drains ($x = \frac{1}{2}L$) at any time t is

$$h_t = \frac{4}{\pi}\frac{R}{\mu} j \sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3} (1-e^{-n^2 t/j}) \tag{50}$$

where $j = \frac{\mu L^2}{\pi^2 KD}$ is called the reservoir coefficient. (51)

It is remarked that convolution of Eq.35 with R/μ yields Eq.50 (Chap.15, Vol.II). The factor $\alpha = \frac{1}{j}$, used by DUMM (1954) and DE ZEEUW (1966) is a "reaction factor" which expresses the drainage intensity (Chap.16, Vol.II).

The discharge intensity q_t (m/day) of a parallel drainage system at any time t is found in a way similar to that given for Eq.45

$$q_t = \frac{8}{\pi^2} R \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}(1-e^{-n^2 t/j}) \tag{52}$$

The equations 50 and 52 are only valid as long as the constant recharge rate R continues. When such a recharge rate occurs long enough, the flow conditions must become steady too. For $t \to \infty$, Eq.52 changes into

$$q = \frac{8}{\pi^2} R \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} = \frac{8}{\pi^2} R \frac{\pi^2}{8} = R \tag{53}$$

which gives the steady state condition where the discharge intensity q equals the recharge intensity R.
For $t \to \infty$, Eq.50 becomes

$$h = \frac{4}{\pi} \frac{R}{\mu} j \sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3} = \frac{4}{\pi} \frac{R}{\mu} j \frac{\pi^3}{32} = \frac{\pi^2}{8} \frac{R}{\mu} j \tag{54}$$

Substitution of j from Eq.51 and rearranging gives

$$h = \frac{RL^2}{8KD} \tag{55}$$

The latter equation is similar to the Hooghoudt equation with the exception that no radial flow is taken into account.
When introducing the equivalent depth d of Hooghoudt instead of D, to account for the convergence of streamlines in the vicinity of drains not reaching an impermeable layer, Eq.51 changes into

$$j = \frac{1}{\alpha} = \frac{\mu L^2}{\pi^2 Kd} \quad \text{(days)} \tag{56}$$

The justification of the substitution of the equivalent depth d is based on the same grounds as for Eq.39.

8.3.6 APPLICATION OF THE KRAIJENHOFF VAN DE LEUR-MAASLAND EQUATION

The Kraijenhoff van de Leur-Maasland equation is not used for routine drain spacing computations, which are usually based on an assumed steady or instantaneous recharge. The equation however proves very useful when changes in water table elevation and discharge rate must be known for chosen drainage conditions and in response to a changing recharge pattern. Such calculations are usually computerized.

The Kraijenhoff van de Leur-Maasland equation will be applied in order of increasing complexity: constant and continuous recharge, constant recharge during a restricted period, and intermittent recharge.

constant and continuous recharge

Equations 50 and 52 can be written as

$$h_t = \frac{R}{\mu} j \, c_t \qquad (57)$$

where

$$c_t = \frac{4}{\pi} \sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3} (1-e^{-n^2 t/j}) \qquad (58)$$

and

$$q_t = R \, g_t \qquad (59)$$

where

$$g_t = \frac{8}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} (1-e^{n^2 t/j}) \qquad (60)$$

The factors c_t and g_t depend only on time t and on reservoir coefficient j, so that they can be tabulated (Table 3).

Example 9

Assume a drainage system with j = 5 days. The soil has an effective porosity μ = 0.04. There is a constant recharge of 10 mm/day (R = 0.01 m/day). The value for $\frac{R}{\mu}$ j will then be 1.25 m.
For the computation of the water-table height h_t or the discharge q_t at any time Table 3 can be used, as is illustrated below.

Table 3. c_t and g_t coefficients for the Kraijenhoff van de Leur-Massland equation.

t/j	g_t	c_t	t/j	g_t	c_t	t/j	g_t	c_t
0.01	0.072	0.010	0.48	0.497	0.447	1.10	0.730	0.809
0.02	0.102	0.020	0.50	0.507	0.463	1.15	0.743	0.830
0.03	0.125	0.030	0.52	0.518	0.477	1.20	0.756	0.850
0.04	0.143	0.039	0.54	0.528	0.492	1.25	0.767	0.869
0.05	0.161	0.049	0.56	0.537	0.507	1.30	0.779	0.887
0.06	0.176	0.060	0.58	0.546	0.521	1.35	0.790	0.903
0.07	0.190	0.070	0.60	0.554	0.535	1.40	0.800	0.920
0.08	0.203	0.080	0.62	0.563	0.549	1.45	0.810	0.935
0.09	0.215	0.090	0.64	0.572	0.563	1.50	0.819	0.950
0.10	0.227	0.100	0.66	0.580	0.576	1.55	0.828	0.964
0.12	0.249	0.120	0.68	0.588	0.588	1.60	0.836	0.977
0.14	0.269	0.139	0.70	0.597	0.602	1.65	0.844	0.989
0.16	0.288	0.159	0.72	0.605	0.614	1.70	0.852	1.002
0.18	0.305	0.179	0.74	0.612	0.627	1.75	0.859	1.012
0.20	0.321	0.199	0.76	0.620	0.638	1.80	0.866	1.023
0.22	0.337	0.218	0.78	0.628	0.650	1.85	0.872	1.033
0.24	0.352	0.238	0.80	0.636	0.661	1.90	0.879	1.044
0.26	0.367	0.257	0.82	0.643	0.672	1.95	0.885	1.052
0.28	0.380	0.275	0.84	0.650	0.683	2.00	0.990	1.061
0.30	0.393	0.294	0.86	0.657	0.695	2.10	0.901	1.078
0.32	0.406	0.312	0.88	0.663	0.706	2.20	0.910	1.093
0.34	0.419	0.329	0.90	0.670	0.717	2.30	0.919	1.107
0.36	0.430	0.347	0.92	0.677	0.727	2.40	0.927	1.118
0.38	0.442	0.364	0.94	0.683	0.737	3.00	0.960	1.171
0.40	0.454	0.381	0.96	0.689	0.746	4.00	0.985	1.210
0.42	0.465	0.398	0.98	0.696	0.756	5.00	0.995	1.226
0.44	0.476	0.415	1.00	0.702	0.765	∞	1.000	$\frac{\pi^2}{8} = 1.232$
0.46	0.487	0.431	1.05	0.715	0.787			

time	t/j	c_t (Table 3)	g_t (Table 3)	$h_t = \frac{R}{\mu} j\, c_t$ (m)	$q_t = R\, g_t$ (m/day)
4 hrs = 1/6 day	0.033	0.033	0.131	0.041	0.00131
8 hrs = 1/3 day	0.067	0.067	0.184	0.084	0.00184
12 hrs = 1/2 day	0.100	0.100	0.227	0.125	0.00227
16 hrs = 2/3 day	0.133	0.133	0.262	0.166	0.00262
20 hrs = 5/6 day	0.166	0.166	0.292	0.208	0.00292
24 hrs = 1 day	0.200	0.199	0.321	0.249	0.00321
48 hrs = 2 day	0.400	0.381	0.454	0.476	0.00454
72 hrs = 3 day	0.600	0.535	0.554	0.669	0.00554
96 hrs = 4 day	0.800	0.661	0.636	0.827	0.00636
120 hrs = 5 day	1.000	0.765	0.702	0.956	0.00702
∞	∞	$\frac{\pi^2}{8} = 1.232$	1.000	1.540	0.01000

constant recharge during a restricted period

Consider a drained area with irrigation or rainfall occurring during one single day followed by a dry period. In order to compute the water-table heights on days subsequent to the irrigation or rainfall, we assume (Fig.17) that the recharge R of the first day continues throughout the following days, but from the second day onwards an equal negative recharge, -R, is taken into account so that the total recharge is equal to zero (principle of superposition).

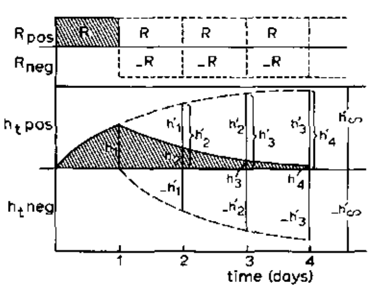


Fig.17.
Principle of superposition of recharge (R) and water-table elevation (h) for the Kraijenhoff van de Leur-Maasland equation.

For the water-table height at the end of the first day (t = 1) we then have according to Eq.57

$h = h_1' = \frac{R}{\mu} j\, c_1$

At the end of the second day we have had a positive recharge R over two days, hence

$$h_2' = \frac{R}{\mu} j c_2$$

from which we have to subtract the effect of a negative recharge over 1 day, equalling

$$h_1' = \frac{R}{\mu} j c_1$$

so that

$$h_2 = h_2' - h_1' = \frac{R}{\mu} j(c_2 - c_1)$$

Similarly, at the end of the third day, we have

$$h_3' = \frac{R}{\mu} j c_3$$

$$h_2' = \frac{R}{\mu} j c_2$$

so that

$$h_3 = h_3' - h_2' = \frac{R}{\mu} j(c_3 - c_2)$$

and at the end of the t^{th} day

$$h_t = h_t' - h_{t-1}' = \frac{R}{\mu} j(c_t - c_{t-1})$$

The height of the water table during the recession period can thus be computed with the aid of Table 3.

Example 10

Consider an area with pipe drainage at a depth of 1.00 m below soil surface and the impermeable layer at a depth of 1.20 m below the drains. The drain diameter is 0.20 m and the drain spacing is 20 m, so that d = 1.0 m (Table 1). The hydraulic conductivity of the soil K = 0.5 m/day and the effective porosity μ = 0.05.

From the above information we have

K = 0.5 m/day	L = 20 m
μ = 0.05	r_o = 0.1 m
D = 1.2 m	d = 1.0 m

Substituting the above data into Eq.56 yields

$$j = \frac{\mu L^2}{\pi^2 K d} = \frac{0.05 \times 20^2}{\pi^2 \times 0.5 \times 1.0} = 4 \text{ days}$$

Suppose that the initial water table was at drain level and that during the first day a total amount of 30 mm of percolation water (from irrigation or rain) reaches the groundwater. There is no percolation in the following days.
What will be the height of the water table midway between the drains during the days subsequent to the irrigation or rainfall?
The calculation is given in the following table:

time days	t/j	c/t	c_{t-1}	$c_t - c_{t-1}$	$\frac{R}{\mu} j$	water-table height $h_t = \frac{R}{\mu} j\ (c_t - c_{t-1})$
1	0.25	0.248	0.000	0.248	2.4 m	0.60 m
2	0.50	0.463	0.248	0.215	2.4 m	0.52 m
3						
4	1.00	0.765	0.633	0.132	2.4 m	0.32 m

<u>intermittent recharge</u>

The above method can be worked out in a more general way for intermittent recharge. Since, in general, hydrologic data are available per day only the following examples are worked out with days as the time unit. The theory however holds for any time length.
Suppose that we wish to compute the height of the water table or the discharge at the end of any arbitrary day. Let us choose the m^{th} day (Fig.18).

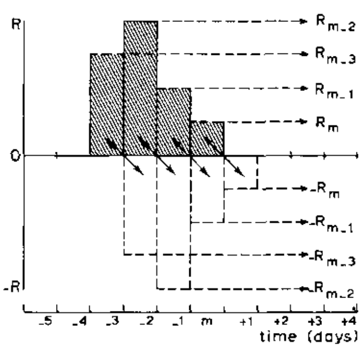


Fig.18.
Superposition of intermittent recharge for the Kraijenhoff van de Leur-Maasland equation.

Both the height of the water table and the discharge rate are influenced by the percolation during each of the preceding days. So we have to take into account:

- the recharge R_m over 1 day;
- the recharge R_{m-1} over 2 days minus the recharge R_{m-1} over 1 day;
- the recharge R_{m-2} over 3 days minus R_{m-2} over 2 days, etcetera.

The height of the water table is thus given by

$$h_m = \frac{j}{\mu}\left[R_m c_1 + R_{m-1}(c_2-c_1) + R_{m-2}(c_3-c_2) + \ldots + R_1(c_m-c_{m-1})\right] \tag{62}$$

Setting $C_1 = c_1 j$, $C_2 = (c_2-c_1)j$, $C_m = (c_m-c_{m-1})j$ we obtain

$$h_m = \frac{1}{\mu}\left[C_1 R_m + C_2 R_{m-1} + C_3 R_{m-2} + \ldots + C_m R_1\right] \tag{63}$$

Similarly, the discharge rate is given by

$$q_m = G_1 R_m + G_2 R_{m-1} + G_3 R_{m-2} + \ldots + G_m R_1 \tag{64}$$

where

$$G_1 = g_1, \qquad G_2 = (g_2-g_1), \qquad\qquad G_m = (g_m-g_{m-1}) \tag{65}$$

The factors C_1, C_2, etc. and G_1, G_2 etc. are found in Tables 4 and 5 as a function of $\alpha = 1/j$. The use of these tables will be explained in some examples.

Example 11

A drainage system with $\alpha = 0.25$ days^{-1} ($j = 4$ days) in a soil with an effective porosity $\mu = 0.05$ receives the following groundwater recharge:

February	15	16	17	18	19	20
recharge (mm)	5	20	10	5	0	0

What heights of water table and discharges will occur if on February 14 the water level was horizontal and at drain depth.

Taking successively February 20, 19, 18, 17, 16 and 15 as the m^{th} day, we obtain the following C_t-values:

date	recharge (m)	20	19	18	17	16	15	(C_t-values)
20	0	0.99	-	-	-	-	-	
19	0	0.86	0.99	-	-	-	-	
18	0.005	0.68	0.86	0.99	-	-	-	
17	0.010	0.53	0.68	0.86	0.99	-	-	
16	0.020	0.41	0.53	0.68	0.86	0.99	-	
15	0.005	0.32	0.41	0.53	0.68	0.86	0.99	

Table 4. $C_t \times 10^2$ values for the computation of unsteady waterlevels with the Kraijenhoff van de Leur-Maasland equation

$\alpha \rightarrow$ / t	0.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	99
2	100	100	100	100	100	100	100	100	100	99	99	98	97	97	96	95	94	93	92	91
3	100	100	100	100	100	99	98	97	96	95	93	91	90	88	86	84	82	80	79	77
4	100	100	100	100	98	97	95	93	90	88	85	83	80	78	75	72	70	68	65	63
5	100	100	100	98	96	93	90	87	84	80	77	74	71	68	65	62	59	57	54	52
6	100	100	99	96	93	89	85	81	77	73	69	66	62	59	56	53	50	47	45	42
7	100	100	98	94	90	85	80	75	71	66	62	58	55	51	48	45	42	40	37	35
8	100	100	96	91	86	80	75	70	65	60	56	52	48	45	41	38	36	33	31	28
9	100	99	94	89	82	76	70	64	59	54	50	46	42	39	36	33	30	28	25	23
10	100	98	92	86	79	72	65	59	54	49	45	41	37	34	31	28	25	23	21	19
11	100	97	90	83	75	68	61	55	50	45	40	36	33	29	26	24	21	19	17	16
12	100	96	88	80	71	64	57	51	45	40	36	32	29	25	23	20	18	16	14	13
13	100	95	86	77	68	60	53	47	41	37	32	28	25	22	20	17	15	13	12	11
14	99	93	84	74	65	57	49	43	38	33	29	25	22	19	17	15	13	11	10	9
15	99	92	81	71	62	53	46	40	35	30	26	22	19	17	14	12	11	9	8	7
16	99	91	79	68	59	50	43	37	32	27	23	20	17	15	12	11	9	8	7	6
17	99	89	77	66	56	47	40	34	29	24	21	18	15	13	11	9	8	7	6	5
18	98	88	75	63	53	44	37	31	26	22	19	16	13	11	9	8	6	5	5	4
19	98	86	73	61	50	42	35	29	24	20	17	14	12	10	8	7	5	5	4	3
20	98	85	71	58	48	40	33	27	22	18	15	12	10	8	7	6	5	4	3	3

Table 4. (cont.)

$\alpha \rightarrow$	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33	.34	.35	.36	.37	.38	.39	.40
t																				
1	99	99	99	99	99	99	98	98	98	98	98	97	97	97	97	96	96	96	96	95
2	90	89	88	87	86	85	84	83	82	80	79	78	77	76	75	74	73	72	71	70
3	75	73	72	70	68	66	65	63	62	60	59	57	56	55	53	52	51	50	48	47
4	61	59	57	55	53	51	50	48	46	45	43	42	40	39	38	36	35	34	33	32
5	50	47	45	43	41	40	38	36	35	33	32	30	29	28	26	25	24	23	22	21
6	40	38	36	34	32	31	29	27	26	25	23	22	21	20	19	18	17	16	15	14
7	33	31	29	27	25	24	22	21	19	18	17	16	15	14	13	12	12	11	10	10
8	26	24	23	21	20	18	17	16	15	13	12	12	11	10	9	9	8	7	7	6
9	21	20	18	17	15	14	13	12	11	10	9	8	8	7	7	6	6	5	5	4
10	17	16	14	13	12	11	10	9	8	7	7	6	6	5	5	4	4	3	3	3
11	14	13	11	10	9	8	7	7	6	5	5	4	4	4	3	3	3	2	2	2
12	11	10	9	8	7	6	6	5	5	4	4	3	3	3	3	2	2	2	1	1
13	9	8	7	6	6	5	4	4	3	3	3	2	2	2	2					
14	7	7	6	5	4	4	3	3	3	2	2	2	1	1						
15	6	5	5	4	3	3	3	2	2	2	1	1								
16	5	4	4	3	3	2	2	2	1	1										
17	4	3	3	2	2	2	1	1												
18	3	3	2	2	2	1														
19	3	2	2	1	1															
20	2	2	1	1																

Table 4. (cont.)

α →	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60
t																				
1	95	95	95	94	94	94	93	93	93	92	92	92	91	91	91	90	90	90	90	89
2	69	68	67	66	65	64	63	62	62	61	60	59	58	57	56	56	55	54	53	53
3	46	45	44	43	42	41	40	39	38	37	36	35	34	33	33	32	31	30	30	29
4	31	29	28	28	27	26	25	24	23	22	22	21	20	20	19	18	18	17	16	16
5	20	19	19	18	17	16	16	15	14	14	13	12	12	11	11	10	10	9	9	9
6	13	13	12	11	11	10	10	9	9	8	8	7	7	7	6	6	6	5	5	5
7	9	8	8	7	7	6	6	6	5	5	5	4	4	4	4	3	3	3	3	3
8	6	5	5	5	4	4	4	4	3	3	3	3	2	2	2	2	2	2	2	1
9	4	4	3	3	3	3	2	2	2	2	2	2	1							
10	3	2	2	2	2	2	1	1												
11	2	2	1																	

Table 5. $G_t \times 10^3$ values for the computation of unsteady discharges with the Kraijenhoff van de Leur-Maasland equation

α → / t	0.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	72	102	124	144	161	176	190	203	216	227	238	249	259	269	278	287	296	305	313	321
2	30	47	52	60	66	73	79	84	89	94	99	103	107	111	115	119	123	126	130	133
3	23	32	40	46	51	56	60	64	68	72	76	79	82	85	88	91	94	96	98	100
4	19	27	33	38	43	47	51	54	58	61	64	66	69	71	73	75	77	78	80	81
5	17	24	29	34	38	42	45	48	51	53	55	58	60	61	62	63	64	65	66	66
6	15	22	26	31	34	38	40	43	45	47	49	50	52	53	53	54	54	54	54	54
7	14	20	24	28	31	34	37	39	41	42	44	45	45	46	46	46	46	45	45	44
8	13	18	23	26	29	32	34	36	37	38	39	40	40	40	40	39	38	38	37	36
9	12	17	21	25	27	30	32	33	34	35	35	35	35	34	34	33	32	32	31	30
10	12	16	20	23	26	28	29	30	31	31	31	31	31	30	29	28	27	26	25	24
11	11	16	19	22	24	26	27	28	28	28	28	28	27	26	25	24	23	22	21	20
12	10	15	18	21	23	24	25	26	26	26	25	24	24	23	22	21	20	18	17	16
13	10	14	18	20	22	23	24	24	24	23	22	22	21	20	19	18	16	15	14	13
14	10	14	17	19	21	22	22	22	22	21	20	19	18	17	16	15	14	13	12	11
15	9	13	16	18	20	20	21	20	20	19	18	17	16	15	14	13	12	11	10	9
16	9	13	16	18	19	19	19	19	18	17	16	15	14	13	12	11	10	9	8	7
17	9	12	15	17	18	18	18	17	16	15	14	13	12	11	10	9	8	7	7	6
18	8	12	15	16	17	17	17	16	15	14	13	12	11	10	9	8	7	6	6	5
19	8	12	14	16	16	16	16	15	14	13	12	10	10	8	8	7	6	5	5	4
20	8	11	14	15	15	15	14	14	13	12	10	9	8	7	7	6	5	4	4	3

Table 5. (cont.)

α →	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33	.34	.35	.36	.37	.38	.39	.40
t																				
1	329	337	344	352	359	366	373	380	387	393	400	406	413	419	425	431	437	443	448	454
2	136	139	142	145	148	151	154	156	159	161	164	166	168	170	172	174	176	178	180	182
3	103	104	106	108	110	111	112	114	115	116	116	117	118	118	119	119	120	120	120	120
4	82	83	84	84	85	85	85	86	86	85	85	85	85	84	84	83	83	82	81	80
5	66	66	66	66	66	66	65	64	64	63	62	62	61	60	59	58	57	56	55	54
6	54	53	53	52	51	50	50	49	48	47	46	45	44	43	42	40	39	38	37	36
7	44	43	42	41	40	39	38	37	36	35	34	32	31	30	29	28	27	26	25	24
8	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
9	29	28	26	25	24	23	22	21	20	19	18	17	16	15	14	14	13	12	12	11
10	23	22	21	20	19	18	17	16	15	14	13	12	12	11	10	10	9	8	8	7
11	19	18	17	16	15	14	13	12	11	10	10	9	8	8	7	7	6	6	5	5
12	15	14	13	12	11	11	10	9	8	8	7	6	6	6	5	5	4	4	3	3
13	12	11	10	10	9	8	8	7	6	6	5	5	4	4	3	3				
14	10	9	8	8	7	6	6	5	5	4	4	3	3	3						
15	8	7	7	6	5	5	4	4	4	3	3	2								
16	6	6	5	5	4	4	3	3	3	2										
17	5	5	4	4	3	3	2	2												
18	4	4	3	3	2	2														
19	4	3	3	2	2															
20	3	2	2	2																

Table 5. (cont.)

α →	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60
t																				
1	460	465	471	476	482	487	492	497	502	507	512	517	522	527	532	536	541	546	550	555
2	183	185	186	188	189	190	191	192	193	194	195	196	197	198	198	199	200	200	201	201
3	120	120	120	120	119	119	119	118	118	117	117	116	116	115	114	113	113	112	111	110
4	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	60
5	53	52	51	50	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33
6	35	34	33	32	31	30	29	28	27	26	25	24	24	23	22	21	20	20	19	18
7	23	22	21	20	20	19	18	17	17	16	15	14	14	13	13	12	12	11	10	10
8	15	15	14	13	12	12	11	11	10	10	9	9	8	8	7	7	6	6	6	5
9	10	10	9	8	8	8	7	7	6	6	5	5	5							
10	7	6	6	5	5	5	4	4												
11	4	4	4																	

Multiplying the C_t-values of each column with the corresponding recharges, adding the results, and dividing the sum by μ, gives the height of the water table at the date for which the column was taken:

date	height of the water table (m)	
20	20(0.68 × 0.005 + 0.53 × 0.01 + 0.41 × 0.02 + 0.32 × 0.005)	= 0.37 m
19	20(0.86 × 0.005 + 0.68 × 0.01 + 0.53 × 0.02 + 0.41 × 0.005)	= 0.48 m
18	20(0.99 × 0.005 + 0.86 × 0.01 + 0.68 × 0.02 + 0.53 × 0.005)	= 0.60 m
17	20(0.99 × 0.01 + 0.86 × 0.02 + 0.68 × 0.005)	= 0.61 m
16	20(0.99 × 0.02 + 0.86 × 0.005)	= 0.48 m
15	20(0.99 × 0.005)	= 0.10 m

A similar method is followed for the discharge rate. Hence:

date	recharge (m)	20	19	18	17	16	15
				G_t-values			
20	0	0.359	-	-	-	-	-
19	0	0.148	0.359	-	-	-	-
18	0.005	0.110	0.148	0.359	-	-	-
17	0.010	0.085	0.110	0.148	0.359	-	-
16	0.020	0.066	0.085	0.110	0.148	0.359	-
15	0.005	0.051	0.066	0.085	0.110	0.148	0.359

Multiplying the G_t-values of each column with the corresponding recharges and adding the results, yields the total discharge rate at the appropriate date:

date	discharge rate (m/day)	
20	0.110 × 0.005 + 0.085 × 0.01 + 0.066 × 0.02 + 0.051 × 0.005	= 0.0030 m/day
19	0.148 × 0.005 + 0.110 × 0.01 + 0.085 × 0.02 + 0.066 × 0.005	= 0.0039 m/day
18	0.359 × 0.005 + 0.148 × 0.01 + 0.110 × 0.02 + 0.085 × 0.005	= 0.0059 m/day
17	0.359 × 0.01 + 0.148 × 0.02 + 0.110 × 0.005	= 0.0059 m/day
16	0.359 × 0.02 + 0.148 × 0.005	= 0.0079 m/day
15	0.359 × 0.005	= 0.0018 m/day

The above calculation seems to be rather tedious, but it is quite easy if a calculator is used. It can be seen from Tables 4 and 5 that the larger α is, the fewer terms have to be used. Therefore the method is especially useful for large α-values. For smaller α-values another computation method has been worked out by DE ZEEUW (see Chap.16).

Example 12

The data for this example are derived from the example in Sect.8.3.3. Instead of an instantaneous recharge (R_i = 25 mm) however, it is assumed that the percolation from irrigation is divided over two days at a rate R = 12.5 mm/day or 0.0125 m/day, after which it is nil for eight days, followed by another two days of percolation at a rate of R = 12.5 mm/day due to a new irrigation, and once again no percolation for eight days, etcetera. The further data are: L = 90 m, d = 4.7 m, K = 1.0 m/day and μ= 0.05, from which we can derive that

$$j = \frac{\mu L^2}{\pi^2 Kd} = \frac{0.05 \times (90)^2}{9.9 \times 1 \times 4.7} = 8.7 \quad \text{or} \quad \alpha = 0.115 \text{ days}$$

At the end of the second day the height of the water table is then

$$h_2 = \frac{R}{\mu}(C_1 + C_2) = \frac{0.0125}{0.05}(1.00 + 0.98) = 0.495 \text{ m}$$

At the end of the second irrigation, so at the end of the 12th day, we find for the height of water table

$$h_{12} = \frac{R}{\mu}(C_1 + C_2 + C_{11} + C_{12}) = \frac{0.0125}{0.05}(1.00 + 0.98 + 0.38 + 0.34) = 0.675 \text{ m}$$

Similarly at the end of the third irrigation gift, i.e. at the end of the 22nd day

$$h_{22} = \frac{R}{\mu}(C_1 + C_2 + C_{11} + C_{12} + C_{21} + C_{22})$$

C_{21} and C_{22} values are not given in Table 4, but can be found from Tables 3 and Eq.62

$$C_{21} = j(c_{21} - c_{20})$$
$$C_{22} = j(c_{22} - c_{21})$$

Hence

t/j = 20/8.7 = 2.30 $\qquad c_{20}$ = 1.107

t/j = 21/8.7 = 2.41 $\qquad c_{21}$ = 1.119

t/j = 22/8.7 = 2.52 $\qquad c_{22}$ = 1.129

Thus:

$$C_{21} = 8.7(1.119 - 1.107) = 0.104$$

$$C_{22} = 8.7(1.129 - 1.119) = 0.087$$

and

$$h_{22} = \frac{0.0125}{0.05}(1.00 + 0.98 + 0.38 + 0.34 + 0.10 + 0.09) = 0.72 \text{ m}$$

As can be seen from this example, the water table builds up slowly to reach an ultimate value of slightly less than $h_t = 0.80$ m, which was the value taken as the criterion in the example of Sect. 8.3.3 where the total percolation was applied instantaneously. The water table rise is apparently less when the recharge is divided over a longer period.

8.3.7 DISCUSSION OF THE KRAIJENHOFF VAN DE LEUR-MAASLAND EQUATION

In the previous section some examples of computations of the height of the water table were given. Computation of the discharge is done along the same lines.

In analyzing discharge hydrographs or water table hydrographs from experimental fields, it is often necessary to apply certain simplifications. From Eqs.50 and 52 it can be found that the infinite series is converging, the rate of convergency depending on the value of t/j. It is often found that, according to a simple exponential function, both the water table and the discharge change with time some time after the recharge has stopped, i.e. as soon as the second, third, etc. term of Eqs. 50 and 52 can be neglected in comparison with the first term. This stage is called tail-recession (KRAIJENHOFF VAN DE LEUR, 1958).

Suppose that recharge stops at a certain moment $t = t_r$. The height of the water table is then

$$h_b = \frac{4}{\pi}\frac{R}{\mu} j \left[\sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3}\left(1-e^{-n^2 t_r/j}\right) \right]$$

For computation of the water-table height after $t = t_r$, a negative value of R has to be added as explained earlier. For any time $t > t_r$ one obtains the water table height from

$$h_t = \frac{4}{\pi}\frac{R}{\mu} j \left[\sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3}\left(1-e^{-n^2 t/j}\right) - \sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3}\left(1-e^{-n^2 (t-t_r)/j}\right) \right]$$

or

$$h_t = \frac{4}{\pi}\frac{R}{\mu} j \left[\sum_{n=1,-3,5,}^{\infty} \frac{1}{n^3}\left(e^{n^2 t_r/j}-1\right) e^{-n^2 t/j} \right] \qquad (66)$$

When t is large enough the second and further terms of the infinite series of Eq.66 become very small and are therefore negligible.
According to KRAIJENHOFF VAN DE LEUR (1958) tail recession may be assumed as soon as the second term of the series becomes smaller than 1% of the first. Eq. 66 then reduces to

$$h_t = \frac{4}{\pi}\frac{R}{\mu} j(e^{t_r/j} - 1)e^{-t/j} \tag{67}$$

Substituting two values $t = t_1$ and $t = t_2$ one obtains

$$h_{t_1}/h_{t_2} = e^{-(t_1-t_2)/j} \tag{68}$$

This relation has also been found from the Glover-Dumm equation (Eq.49). Similarly Eqs. 36, 46, and 48, derived from the Glover-Dumm equation, can be found from the Kraijenhoff van de Leur-Maasland equation. Thus, during tail recession both equations are identical.

LITERATURE

BOUMANS, J.H. 1963. Een algemene nomografische oplossing van het stationaire ontwateringsvraagstuk met toepassingen voor isotroop en anisotroop doorlatende gronden. Polytechn. Tijdschr. 18(14B): 545-551.

CARLSLAW, H.S. and J.C. JAEGER, 1959. Conduction of heat in solids. Oxford University Press (2nd ed.) 510 pp.

DAGAN, G. 1964. Spacings of drains by an approximate method. J.Irr. and Drainage Div. Proc. ASAE Paper 3824, 41-46.

DE ZEEUW, J.W. 1966. Analyse van het afvoerverloop van gebieden met hoofdzakelijk grondwaterafvoer. Meded. Landbouwhogeschool Wageningen 66-5. 139 pp.

DONNAN, W.W. 1946. Model tests of a tile-spacing formula. Proc. Soil Sci. Soc. of Am. 11: 131-136.

DUMM, L.D. 1954. Drain spacing formula. Agric. Engng. 35: 726-730.

DUMM, L.D. 1960. Validity and use of the transient flow concept in subsurface drainage. Paper presented at ASAE meeting, Memphis, Tenn. Dec. 4-7.

ERNST, L.F. 1956. Calculation of the steady flow of groundwater in vertical cross sections. Neth. J. Agric. Sci. 4: 126-131.

ERNST, L.F. 1962. Grondwaterstromingen in de verzadigde zone en hun berekening bij aanwezigheid van horizontale evenwijdige open leidingen. Versl. Landbouwk. Onderz. 67-15. 189 pp.

HOOGHOUDT, S.B. 1936. Bepaling van den doorlaatfaktor van den grond met behulp van pompproeven (z.g. boorgatenmethode). Versl. Landb. Onderz. 42: 449-541.

HOOGHOUDT, S.B. 1940. Bijdragen tot de kennis van enige natuurkundige grootheden van de grond. No.7. Versl. Landbouwk. Onderz. 46: 515-707.

KIRKHAM, D. 1958. Seepage of steady rainfall through soil into drains. Trans. Am. Geophys. Union 39: 892-908.

KIRKHAM, D. 1960. An upper limit for the height of the water table in drainage design formulas. 7th Int. Congress of Soil Sci. Madison I: 486-492.

KRAIJENHOFF VAN DE LEUR, D.A. 1958. A study of non-steady groundwater flow with special reference to a reservoir-coefficient. De Ingenieur 40: 87-94.

LABYE, Y. 1960. Note sur la formule de Hooghoudt. Bull. tech. du Gênie Rural, Min. de l'Agriculture de la Rép. Française, Paris No.49-1, 21 pp.

MAASLAND, M. 1959. Watertable fluctuations induced by intermittent recharge. J. Geophys. Res. 64: 549-559.

TOKSÖZ, S. and D. KIRKHAM, 1961. Graphical solution and interpretation of a new drain spacing formula. J. Geophys. Res. 60: 509-516.

VAN BEERS, W.F.J. 1965. Some nomographs for the calculation of drain spacings Bull. No.8. Int. Inst. Land Recl. and Improvement, Wageningen. 48 pp.

VAN BEERS, W.F.J. (in press). A generalized method and graph for computing drain spacings.

WESSELING, J. 1964. A comparison of the steady state drain spacing formulas of Hooghoudt and Kirkham in connection with design practice. J. Hydrology 2: 25-32.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

9. SALT BALANCE AND LEACHING REQUIREMENT

W. H. van der Molen
Professor
Department of Land Drainage and Improvement
University of Agriculture, Wageningen

Lecturers in the Course on Land Drainage

J. H. Boumans (1962)
Land Improvement and Reclamation Company, Zeist

W. H. van der Molen (1963–1970)
University of Agriculture, Wageningen

J. W. van Hoorn (1971–1972)
University of Agriculture, Wageningen

PURPOSE AND SCOPE

The salt balance of highly and slightly soluble salts in the soil under influence of leaching with irrigation water is discussed. The leaching requirement to maintain a favourable salt balance is calculated and applied to various irrigation conditions for equilibrium and fluctuating salt storage conditions.

CONTENTS

9.1 SALINIZATION AND DRAINAGE

Irrigated soils receive considerable quantities of dissolved salts, supplied partly by the irrigation water itself, and partly by inflowing groundwater. Irrigation water, even if it is of excellent quality, is a major source of soluble salts. An annual application of 1,000 mm water containing only 250 mg/l (p.p.m.) dissolved salts will add 2,500 kg salts to each hectare each year. If these salts are not removed from the rootzone, salinization is inevitable.
The other source of salts, a high groundwater level, is often encountered in irrigated areas. It originates either from natural hydrological conditions or from the inevitable losses of irrigation water to the groundwater reservoir. Capillary rise may cause the groundwater to reach the rootzone - or even the soil surface, where it evaporates leaving salts behind. If the groundwater reservoir is replenished over short periods only, the water table does not remain at a high level and the process of salinization comes to a standstill; in such cases the soil salt content is seldom high enough to be harmful to crops. If, however, the groundwater in an area is fed by seepage from elsewhere during the greater part of the year, the process of salinization continues and severe accumulation of salts will occur. Seepage is a wide-spread phenomenon; some typical examples are shown in Fig.1. In irrigated areas, seepage usually affects pieces of land which are temporarily not irrigated, e.g. during periods of fallow.

A certain amount of leaching is needed to counteract the process of salinization: an excess of water is supplied to the soil surface, and the salts are washed down and out of the rootzone. This water will replenish the groundwater but, if natural drainage is sufficient, it will be discharged without unduly raising the water table. Natural drainage, however, is frequently unable to cope with these excessive quantities of water and a drainage system has to be installed. Thus, in arid regions, drainage serves two purposes. The first, as with drainage in humid regions, is to maintain a favourable water balance in the rootzone. The second, contrary to drainage in most humid areas, is to maintain a favourable salt balance in the rootzone.

9.2 THE SALT BALANCE

9.2.1 THE WATER AND SALT BALANCE OF THE ROOTZONE

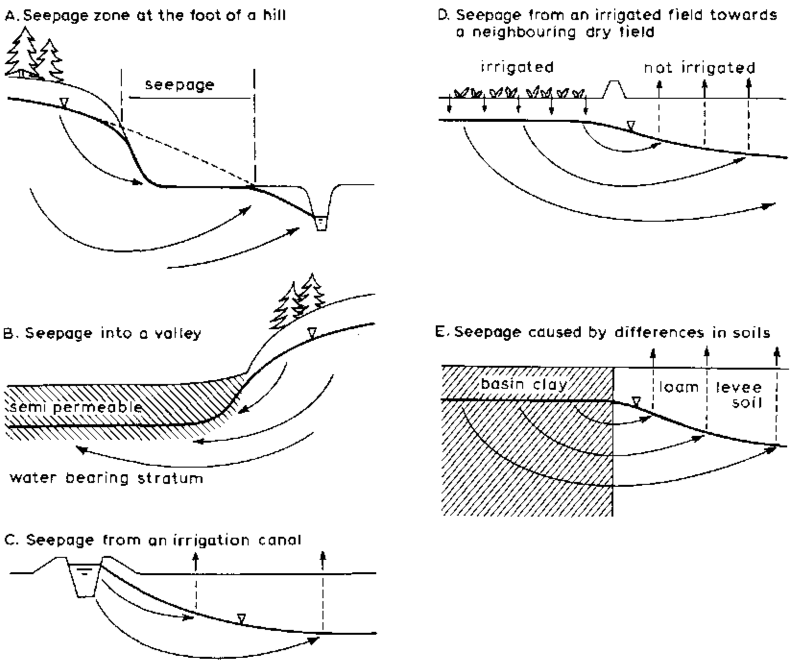
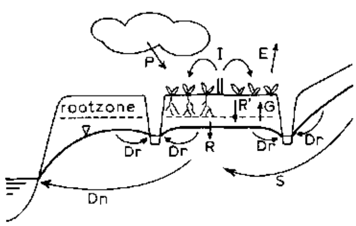


Fig.1. Seepage phenomena.



Dr = drainage
Dn = natural drainage
S = seepage

Fig.2. Water balance of an irrigated soil.

The water balance of an irrigated soil is illustrated in Fig.2. The water balance in the rootzone reads

$$I + P + G = E + R + \Delta W \tag{1}$$

where

I = *effective amount of irrigation water*

P = effective amount of precipitation

G = amount of capillary rise of groundwater
E = amount of evapotranspiration
R = amount of deep percolation
ΔW = change in amount of moisture stored.

The length of the time period over which the balance terms are taken is immaterial, as long as the period is equal for each of the terms.
It is convenient to express all terms in mm or l/m^2. I and P are defined here as effective quantities as they relate to quantities that actually infiltrate into the soil. For irrigation water this is the supply to the field, less surface runoff and evaporation. The storage ΔW may be positive (increase) or negative (decrease). Over longer periods (say one year or more) ΔW is considered negligible.

The concept of effective precipitation is likely to differ according to one's point of view. In Chapter 15, Vol.II, which deals with catchment hydrology, the definition of effective precipitation encompasses "all precipitation that eventually becomes runoff" and hence includes surface runoff, which is excluded from the water balance model above.

To arrive at a salt balance, we assume that all salts are highly soluble and that they do not precipitate. The salt balance of the rootzone then reads

$$IC_i + PC_p + GC_g = RC_r + \Delta Z' \tag{2}$$

where
C = salt concentration in meq/l
i = suffix denoting irrigation water
p = suffix denoting precipitation
g = suffix denoting groundwater
r = suffix denoting deep percolation water
$\Delta Z'$ = change in salt content of the rootzone in meq/m^2.

Since the amount of salts supplied to irrigated soils by rainfall is negligible compared with the amount supplied by irrigation water, PC_p may be taken as zero. Furthermore it is assumed that under equilibrium conditions $C_g = C_r$. Hence, Eq.2 reduces to

$$IC_i = R^* C_r + \Delta Z' \quad (3)$$

where R^* is the net deep percolation, equating (R - G). In terms of gains and losses of salts we find that the net deep percolation R^* may equal the so-called leaching requirement.

If the salt balance of the soil is in equilibrium, $\Delta Z'$ will be zero. If it is not in equilibrium, the quantity of salts in the rootzone at the beginning of the period under consideration (Z'_1) will differ from that at the end (Z'_2)

$$\Delta Z' = Z'_2 - Z'_1 \ (\text{meq/m}^2) \quad (4)$$

9.2.2 THE LEACHING EFFICIENCY

We can regard the amount of salt in the rootzone (Z') as being dissolved in the soil moisture. As downward water and salt movements in the rootzone generally take place at moisture contents near field capacity, we may logically consider Z' to be dissolved in an amount of moisture W_{fc}, which is the soil moisture content at field capacity in the rootzone expressed in mm or l/m^2. W_{fc} can be determined from

$$W_{fc} = \frac{w_{fc}}{100} D \quad (5)$$

where

w = soil moisture content in volume %

fc = suffix denoting field capacity

D = depth of rootzone in mm.

At field capacity, the salt concentration (C_{fc}) of the soil moisture in the rootzone is

$$C_{fc} = \frac{Z'}{W_{fc}} \quad (6)$$

If we consider a period in which Z' changes from Z'_1 to Z'_2, the average salt concentration ($\overline{C}_{fc}$) of the soil moisture at field capacity during that period is

$$\overline{C}_{fc} = \frac{Z'_1 + Z'_2}{2W_{fc}} = \frac{Z'_1}{W_{fc}} + \frac{\Delta Z'}{2W_{fc}} \quad (7)$$

For the salt concentration (C_r) of the water percolating below the rootzone, we can make the following three assumptions, each describing a different model

$$C_r = C_{fc} \quad \text{or} \tag{8}$$

$$C_r = fC_{fc} \quad \text{or} \tag{9}$$

$$C_r = fC_{fc} + (1 - f)C_i \tag{10}$$

where f is the leaching efficiency ($0 < f < 1$).

As will be explained in Section 9.5, Eq.8 describes the complete mixing in a reservoir without bypass (Fig.5B), whereas Eqs.9 and 10 refer to a reservoir with a bypass (Fig.5C). More precisely, Eq.9 describes the concentration of the outflow if the irrigation water has a zero salt concentration, whereas Eq.10 considers the more general case where the salt concentration of the irrigation water is not negligible. Equation 10 will be used in the considerations that follow.

The leaching efficiency, f, is higher in light-textured soils than in heavy clays, probably due to the presence of cracks in the latter. Apart from soil texture, f appears to depend largely on the method of irrigation. Under basin or border irrigation the leaching efficiency is considerably higher than under furrow irrigation, whereas the highest efficiencies of all are found if the soil is leached by rain or sprinkling of low intensity. In the soil profile, f usually increases with depth.
Tentatively, the following f-values may be used:

silt loam, sandy loam	$f = 0.5 - 0.6$
silty clay loam, sandy clay loam, loam	$f = 0.4 - 0.5$
clay	$f = 0.2 - 0.3$.

Whenever possible, f should be determined from leaching experiments or by the analysis of field leaching data (DIELEMAN, 1963; UNESCO, 1970).

9.2.3 THE SALT EQUILIBRIUM EQUATION AND THE LEACHING REQUIREMENT

Areas with well-designed irrigation and drainage systems will have no salt accumulation in their rootzone. Hence, the storage terms ΔW and $\Delta Z'$ in the water and salt balance (Eqs.1 and 3), if taken over longer periods of time - say one year - may be neglected. Remembering that the leaching R^* represents the net result of downward percolation R and capillary rise G, we can write the water balance as

$$I + P = E + R^* \tag{11}$$

The salt balance reduces to

$$I\overline{C}_i = R^*\overline{C}_r \tag{12}$$

where $\overline{C}_i$ and $\overline{C}_r$ denote the average concentrations over the period considered. Solving for I in both equations, and equating, yields

$$R^* = (E - P)\,\frac{\overline{C}_i}{\overline{C}_r - \overline{C}_i} \tag{13}$$

Substituting C_r from Eq.10, taking the long-term average $\overline{C}_r$, $\overline{C}_i$ and $\overline{C}_{fc}$ and rearranging gives

$$R^* = (E - P)\,\frac{\overline{C}_i}{f(\overline{C}_{fc} - \overline{C}_i)} \tag{14}$$

In the salt equilibrium equation (Eq.14), R^* is usually referred to as the leaching requirement, which can be calculated when a maximum value has been fixed for the salt concentration of the soil moisture ($\overline{C}_{fc}$). The corresponding irrigation requirement follows directly from the water balance (Eq.11)

$$I = E - P + R^* \tag{15}$$

in which R^* is found from Eq.14.

9.2.4 THE SALT STORAGE EQUATION

In the above it was assumed that there was no difference between the amounts of salt stored in the rootzone at the beginning and at the end of the period under consideration. Though this may be true for long periods - say one year - the amounts will change within such a period because of seasonal variations in climate, crops, water application, and water quality. Such short-term changes in soil salt content - say over a season or a month - can be calculated with the following equations.

Substituting the value of C_r from Eq.10 into Eq.3 and solving for $\Delta Z'$ yields

$$\Delta Z' = IC_i - (1 - f)R^*C_i - fR^*C_{fc} \tag{16}$$

If the period under consideration is short enough, the salt concentration of the irrigation water can be considered constant. The concentration of the soil moisture, however, is not constant and therefore C_{fc} in this formula is replaced by $\bar{C}_{fc}$ from Eq.7, giving

$$\Delta Z' = \frac{IC_i - (1 - f)R^*C_i - \dfrac{fR^*Z_1'}{W_{fc}}}{1 + \dfrac{fR^*}{2W_{fc}}}$$

If in this equation the following substitutions are made

$$K = IC_i - (1 - f)R^*C_i$$

$$L = \frac{fR^*}{W_{fc}} \qquad \text{and}$$

$$M = 1 + 0.5\ L$$

it can be rewritten in a much abbreviated form

$$\Delta Z' = \frac{K - LZ_1'}{M} \tag{17}$$

Equation 17 is referred to as the salt storage equation. If the initial salt content of the rootzone, Z_1', is known - e.g. from soil sampling - $\Delta Z'$ can be calculated directly. Equation 17 may then be used to predict the desalinization of saline soils under the influence of irrigation water. Usually, however, we shall be interested in the seasonal deviations in salinity with respect to the long-term equilibrium soil salt content. In that case Z_1' is unknown, and the only condition is that the sum of the quantites $\Delta Z'$ should be zero over a long period. In practice, $\Delta Z'$ is usually calculated over monthly periods and it is assumed that over periods of one year the changes in salinity are negligible

$$\sum_{n=1}^{n=12} \Delta Z'_n = 0 \tag{18}$$

There are two general methods to solve this problem:

- Starting with an estimated initial value for Z'_1 (which may be zero), Eq.17 is used for a large number of successive periods, until finally equilibrium is reached and the condition expressed in Eq.18, is satisfied. A similar process occurs in nature: non-saline soils, when irrigated, will build up a certain equilibrium salinity. On the other hand, however, saline soils will approach the same equilibrium if leached.

- A few trial values of Z'_1 are used. If $\Sigma\Delta Z'_n$ appears to be positive, Z'_1 is given a higher value; if $\Sigma\Delta Z'_n$ appears to be negative Z'_1 is given a lower value. The process is repeated until $\Sigma\Delta Z_n$ is close enough to zero for practical purposes.

9.2.5 THE SALT EQUILIBRIUM AND STORAGE EQUATIONS EXPRESSED IN TERMS OF ELECTRICAL CONDUCTIVITY

Hitherto, the salt concentration of water (C) has been expressed in milli-equivalents per litre. A more usual way is to express salinity in terms of electrical conductivity (EC), which is roughly proportional to C (RICHARDS et al., 1954)

$$EC \simeq \frac{C}{12} \tag{19}$$

where EC is expressed in mmhos/cm at 25 °C and C in meq/l.

The electrical conductivity of soil samples is usually determined in their saturation extract (EC_e). The relation between EC_e and EC_{fc} is

$$EC_e = \frac{w_{fc}}{w_e} EC_{fc}$$

where w_{fc} and w_e are the soil moisture contents, in volume percent, at field capacity and in the saturated paste respectively.

For medium textured soils (sandy loam, silt loam, clay loam) $w_e \simeq w_{fc}$ and therefore in the rootzone $W_e \simeq 2W_{fc}$. Hence, also making use of Eq.6

$$EC_e \simeq 0,5\ EC_{fc} \simeq \frac{C_{fc}}{24} = \frac{Z'}{24W_{fc}} \tag{20}$$

If the computations are carried out with EC values instead of C values, the values and units for Z' and ΔZ' change into Z and ΔZ respectively with

$$Z = \frac{Z'}{12} \quad \text{and} \quad \Delta Z = \frac{\Delta Z'}{12} \tag{21}$$

in which Z and ΔZ are expressed as the product of mmhos/cm and mm. For the sake of convenience we will hereafter write EC mm instead of the physically correct notation of (mmhos/cm)mm.

The salt equilibrium equation (Eq.14) and the salt storage equation (Eq.17), when expressed in terms of electrical conductivity, change respectively into

$$R^{*} = (E - P)\,\frac{\overline{EC}_i}{f(2\overline{EC}_e - \overline{EC}_i)} \tag{22}$$

$$\Delta Z = \frac{K - LZ_1}{M} \tag{23}$$

where

$$K = I\ EC_i - (1 - f)R^{*}EC_i$$

$$L = \frac{fR^{*}}{W_{fc}}$$

$$M = 1 + 0.5\ L$$

If in Eq.23, W_{fc}, I, and R^{*} are expressed in mm (I and R^{*} as totals over the period considered), then Z and ΔZ are obtained in EC mm. Furthermore, the electrical conductivity of the soil moisture at field capacity EC_{fc}, is found from

$$EC_{fc} = \frac{Z}{W_{fc}} \tag{24}$$

Likewise, the electrical conductivity of the saturation extract is approximately

$$EC_e = \frac{Z}{2W_{fc}} \tag{25}$$

Table 1. Salt and water balances for a permanently irrigated soil, all salts remaining in solution.

Part I	BASIC INFORMATION														
1	General data		W_{fc} = 300 mm; $EC_e = 0.5 \times EC_{fc}$; f = 0.5; no capillary rise							all salts remain in solution $\overline{EC}_e = 6$; $EC_{e\ max} = 8$.					
2	Period		Year	Oct.	Nov.	Dec.	Jan.	Feb.	March	Apr.	May	June	July	Aug.	Sept.
3	Land use							Irrigated Fodders							
4	E	mm	1260	100	80	70	70	70	90	100	120	140	150	140	130
5	P	mm	430	50	50	60	70	50	40	30	30	10	10	10	20
6	E − P	mm	830	50	30	10	0	20	50	70	90	130	140	130	110
7	EC_i	mmhos/cm	3.1	3	3	2	1	2	2	2	3	3	3	4	4
Part II	CONSTANT PERCOLATION		$\Delta W = 0$; $R^* = 48 - 49$ mm per month.												
8	R^*	mm	580	48	49	48	48	48	49	48	48	49	48	48	49
9	I	mm	1410	98	79	58	48	68	99	118	138	179	188	178	159
10a	Z_1	EC mm		5000	4829	4616	4325	4015	3788	3638	3539	3595	3762	3946	4234
11a	ΔZ	EC mm	−460	−171	−215	−289	−310	−227	−150	−99	+56	+167	+184	+288	+306
12a	Z_2	EC mm		4829	4616	4325	4015	3788	3638	3539	3595	3762	3964	4234	4540
10b	Z_1	EC mm		3000	2983	2909	2752	2564	2452	2406	2402	2546	2794	3093	3447
11b	ΔZ	EC mm	+697	−17	−74	−157	−188	−112	−46	−4	+144	+248	+259	+354	+250
12b	Z_2	EC mm		2983	2909	2752	2564	2452	2406	2402	2546	2794	3093	3447	3697
10c	Z_1	EC mm		4100	3998	3846	3615	3360	3186	3083	3026	3122	3326	3544	3864
11c	ΔZ	EC mm	−18	−102	−152	−231	−255	−174	−103	−57	+96	+204	+218	+320	+218
12c	Z_2	EC mm		3998	3846	3615	3360	3186	3083	3026	3122	3326	3544	3846	4082
13	EC_e	mmhos/cm	5.8	6.8	6.6	6.4	6.0	5.6	5.3	5.1	5.0	5.2	5.5	5.9	6.4

Table 1. (continued)

Part III CONSTANT IRRIGATION I = 117 – 118 mm per month															
14	I	mm	1410	117	118	117	118	117	118	117	118	117	118	117	118
15	E – P	mm		50	30	10	0	20	50	70	90	130	140	130	110
16	ΔW	mm	0	+40	0	0	0	0	0	0	0	-13	-22	-13	+8
17	R^*	mm	580	27	88	107	118	97	68	47	28	0	0	0	0
18a	Z_1	EC mm		3000	3173	2945	2581	2073	1888	1846	1888	2108	2459	2813	3281
19a	ΔZ	EC mm	+753	+173	-228	-364	-408	-185	-42	+42	+220	+351	+354	+468	+472
20a	Z_2	EC mm		3173	2945	2581	2073	1888	1846	1888	2108	2459	2813	3281	3753
18b	Z_1	EC mm		5000	5085	4594	3661	3305	2935	2781	2753	3932	3283	3637	4105
19b	ΔZ	EC mm	-423	+85	-491	-633	-656	-370	-154	-28	+179	+351	+354	+468	+472
20b	Z_2	EC mm		5085	4594	3961	3305	2935	2781	2753	2932	3283	3637	4105	4577
18c	Z_1	EC mm		4200	4320	3934	3409	2851	2549	2436	2434	2628	2979	3333	3801
19c	ΔZ	EC mm	+73	+120	-386	-525	-558	-302	-113	-2	+194	+351	+354	+468	+472
20c	Z_2	EC mm		4320	3934	3409	2851	2549	2436	3424	2628	2979	3333	3801	4273
21	EC_e	mmhos/cm	5.5	7.2	7.4	6.7	5.8	4.9	4.3	4.1	4.1	4.5	5.0	5.6	6.4
Part IV CONSTANT SALINITY EC_e = 6 mmhos/cm EC_{fc} = 12 mmhos/cm															
22	R^*	mm	593	33	20	4	0	8	20	28	60	87	93	130	110
23	I	mm	1423	83	50	14	0	28	70	98	150	217	233	260	220

9.3 EXAMPLES OF CALCULATION

9.3.1 PERMANENTLY IRRIGATED SOILS, NO CAPILLARY RISE

An example of the application of the salt equilibrium and storage equations to permanently irrigated soils is given in Table 1. This table contains four parts:

I Basic information

II Constant percolation

III Constant irrigation

IV Constant salinity.

Part I: basic information

The basic data supplied and assumptions to be made in advance are given in Part I, Lines 1 to 7. A considerable variation is apparent in the salinity of the irrigation water, but its quality is generally poor, especially in summer and autumn. The weighted mean

$$\overline{EC}_i = \frac{\Sigma\ EC_i\ (E - P)}{\Sigma(E - P)} = 3.1$$

will be taken as the annual average electrical conductivity of the irrigation water. The required net annual percolation R^* and the required effective annual irrigation I are found from Eqs.22 and 15 respectively as $R^* = 580$ mm and I = 1410 mm (Lines 8 and 9). These annual totals may be divided over the year in different ways. Parts II, III and IV show three - rather theoretical - approaches.

Part II: constant percolation

The irrigation applications are distributed in such a way that the net downward percolation is the same each month, viz. 48-49 mm (Line 8). Since the volume of water supplied each month considerably exceeds the amount lost by evaporation (compare Lines 9 and 6), the monthly changes in moisture content of the rootzone (ΔW) may be taken as zero. The depths of water to be applied (Line 9) are calculated according to Eq.15. To calculate the monthly increase in salt content, ΔZ, an estimate has to be made of the initial salt content, Z_1, of the rootzone. This is done by applying the following reasoning.

During the growing season the average value of the electrical conductivity of the saturation extract (EC_e) should not exceed 6 mmhos/cm, to preserve desirable agronomic conditions. Hence $EC_{fc} \leq 12$ (Eq.20) and consequently $\overline{Z} = \overline{EC}_{fc} W_{fc} \leq 12 \times 300$

= 3600 EC.mm. As the computation starts with the month of October, at the end of the dry season, the initial value Z_1 may be taken higher than $\overline{Z}_1$, for instance at Z_1 = 5000 EC.mm (Line 10a). With this value the change in salt storage (ΔZ) over October is found to be - 171 (Line 11a). The salt storage at the end of October (Z_2, Line 12a) is therefore 5000 - 171 = 4829. This value is then regarded as the initial salt storage (Z_1) in November (Line 10a). Continuing the computations in this way, one finds that Z_2 at the end of September is 4540 ($\Sigma\Delta Z$ = - 460). This value does not agree with the starting value Z_1 = 5000 for October, which has apparently been chosen too high.

Starting again, with a Z_1 value of 3000 (Line 10b), one obtains Z_2 = 3697 (Line 12b) in September and $\Sigma\Delta Z$ = + 697. Obviously, the value of Z_1 = 3000 is too low. Linear interpolation between the two pairs of values (5000, 4540) and (3000, 3697) so as to obtain a pair of equal values, yields (4100, 4100). Checking the value $Z_1 \approx$ 4100 (October, Line 10c) by repeating the salt storage calculations yields Z_2 = 4082 (September, Line 12c), which is sufficiently close to the starting value.
The electrical conductivity of the saturation extract of the soil (EC_e), calculated according to Eq.25, varies between 6.8 (at the beginning of October, Line 13) and 5.0 (at the beginning of May), values which are lower than the maximum permissible EC_e = 8 (Line 1). The average EC_e is 5.8, which is lower than the maximum value $\overline{EC}_e$ = 6 specified for desirable agronomic conditions.

The system of "constant" percolation requires depths of irrigation water which vary considerably from one month to another, making high demands on the irrigation system. Since the percolation is evenly distributed over the year, however, this system makes relatively low demands on the drainage system.

Part III: constant irrigation

The total irrigation requirement of 1410 mm is now evenly distributed over the year, with irrigation gifts of 117 - 118 mm per month. In winter this value is higher than (E - P) and there is an excess of water (Lines 14 and 15). In summer, however, starting in June, the volume of water applied is smaller than (E - P). The calculations of the water balance, therefore, can also best be started in June. In this month I - (E - P) = - 13 mm; therefore, no percolation is to be expected, and the deficit is supplied by the soil moisture reservoir (ΔW = - 13 mm, Line 16). At the end of August the total soil moisture extraction amounts to 48 mm. In September an excess of irrigation water I - (E - P) = 8 mm reduces the soil moisture depletion to 40 mm. In October an excess of 67 mm of irrigation water is partly (40 mm) used to restore the soil moisture reservoir to field capacity, the remainder (27 mm, Line 17) percolates below the rootzone. From the end of October till the end of May the soil is at field capacity and the net deep percolation R^* equals I - (E - P), which has a positive value.

As in Part II, the monthly salt balance is calculated with the salt storage equation. The initial value of Z_1 = 3000 in October (Line 18a) results in an end value of Z_2 = 3573 in September (Line 20a), so Z_1 = 3000 is too low an estimate. The initial value of Z_1 = 5000 in October (Line 18b) results in an end value of Z_2 = 4577 in September (Line 20b), so Z_1 = 5000 is too high an estimate. Linear interpolation similar to that done in Part II, yields Z_1 = 4200. A final check, with Z_1 = 4200, results in an end value of Z_2 = 4273, which is close enough for practical purposes. The salinity figures EC_e, calculated according to Eq.25, appear to vary between 7.4 (November) and 4.1 (April). This variation is higher than in the case of constant percolation, but well within the limits set.

From the standpoint of irrigation design, the regular distribution of the irrigation applications has certain advantages. The percolation, however, is limited to the winter months and reaches higher values (R = 118 mm in January) than for the first system (constant percolation). Therefore, the demands on the drainage system are higher.

Part IV: constant salinity

The constant salinity is fixed at EC_e = 6 mmhos/cm or EC_{fc} = 12 mmhos/cm for each month. Both irrigation and percolation demands (calculated with Eqs.22 and 15), are low in winter, but high in summer: in August they are 260 mm and 130 mm resp., which is higher than those in Parts II and III. Therefore, the method is highly unpractical.

Discussion of the results of Table 1

Irrigation, in practice, is never a matter of constant irrigation or constant percolation as assumed above. It is often practical - especially in connection with the design of borders, furrows, and canal capacities - to apply the same water depths at intervals which vary with the evapotranspiration, specific crop requirements, allowed moisture depletion, etc. Therefore, the calculations of the

As a rule, evaporation decreases considerably when the upper layers become dry (mulch effect). Since weed growth may greatly promote the loss of moisture, soil tillage is very helpful in conserving soil moisture and in preventing resalinization by capillary rise.
If the groundwater reservoir is not replenished from elsewhere (seepage), capillary rise is usually restricted to 20 - 50 mm during the fallow period, even in very dry climates. The desiccation of the soil profile, however, may be considerable and can amount to 100 and even 200 mm. The best way to obtain data on desiccation and capillary rise under local conditions is by sampling the soil at the beginning and end of the fallow period. The capillary rise during the fallow period may be regarded as negative percolation. For this period also, Eq.23 is valid, with I and EC_i equal to zero (no irrigation), R^* being negative. For the period of capillary rise f may be taken as 1.

Table 2 illustrates the conditions in a soil that is cropped and irrigated during winter and remains fallow for the period from April to October. The desiccation of the fallow soil is assumed to be 100 mm, the capillary rise 40 mm. This, together with a rainfall of 110 mm during this period, leads to an evapotranspiration of 250 mm. The annual percolation required is calculated from Eq.22 using the weighted mean of EC_i (2.5 mmhos/cm) as a basis and is found to be 191 mm. As E - P is 210 mm for the year, the irrigation required is 191 + 210 = 410 mm. This amount is divided as follows: 101 mm for October and 60 mm for each of the other winter months. This is in accordance with current irrigation practices, in which a large amount of water is given prior to sowing in order to moisten the soil and to remove salts that have accumulated near the surface.

It is reasonable to make a distinction between a desiccation of the rootzone (ΔW_r) and a desiccation of the subsoil (ΔW_s). The latter is assumed to occur between the lower boundary of the rootzone and the groundwater table.

When water is applied, it is assumed that the soil moisture reservoir in the rootzone is replenished first. Only when the rootzone is at field capacity will the deeper layers be wetted.

In October the entire excess of irrigation water over evapotranspiration I - (E - P) = 96 mm is used for replenishing the soil moisture reservoir in the rootzone. (ΔW_r = + 96, Line 11). In November the excess of irrigation water over eva-

potranspiration I - (E - P) = 30 mm is partly used to replenish the soil moisture reservoir in the rootzone (i.e. until it reaches field capacity, 4 mm and partly to restore subsoil storage 26 mm, Line 12).
It is only in December that the drainage process starts. If Dr stands for drainage, then $Dr = I - (E - P) - \Delta W_s = 60 - 10 - 14 = 36$ mm. Using Eq.23, the monthly salt balances are calculated with starting October values of $Z_1 = 2000$ and and $Z_1 = 4000$, which yield for September $Z_2 = 2142$ and $Z_2 = 3698$ respectively (Lines 14-16). Linear interpolation and checking shows that $Z_1 = 2600$ provides best starting salinity.

Applying Eq.25 to calculate the electrical conductivity of the saturation extract, we find that EC_e varies between 4.1 and 4.9, which is acceptable, though the average annual value of 4.4 is slightly above the limit set. This difference is due to the assumption that f is higher during the fallow period than during the cropping season (f = 1.0 and f = 0.5 respectively), a fact not accounted for in the salt equilibrium equation (Eq.22) with which the leaching requirement is calculated.

9.3.3 SODIUM HAZARD AND LEACHING

The structure of soils is dependent on the type of exchangeable cations. In general, bivalent cations such as Ca^{++} and Mg^{++} promote a good soil structure, whereas monovalent cations such as K^+, and especially Na^+, have a deteriorating effect causing, amongst other things, a poor permeability (Chap.3, Vol.I). In normal soils Na and K occupy only about 5% of the exchange capacity, the remainder being occupied mainly by Ca and Mg ions, and in acid soils also by Al ions. If the percentage of adsorbed Na rises above 10, sodium problems may be expected. The adverse effect of Na is the more pronounced as more clay of the swelling type is present and as the total salt concentration in the soil moisture is less. Therefore the exchangeable sodium percentage (ESP) should not exceed 10 for clayey soils of low salinity (EC_e = 4 or less). For moderately saline soils (EC_e = 6 - 8) an ESP of 15 can be tolerated. The degree of saturation of the exchange complex with sodium depends on the composition of the soil solution and is related to the sodium adsorption ratio, SAR by

$$SAR = Na/\sqrt{\tfrac{1}{2}(Ca + Mg)}, \qquad (26)$$

where Na, Ca and Mg are concentrations in me/l. The relation between the ESP of

$CaCO_3$, each meq of Ca takes away one meq of HCO_3 (stoichiometric precipitation). As an example, if P_{CO_2} = 0.01 atm, C_t = 100 meq/l and the initial difference is 2 meq/l in favour of HCO_3, one finds 6.2 meq/l HCO_3 and 4.2 meq/l Ca.

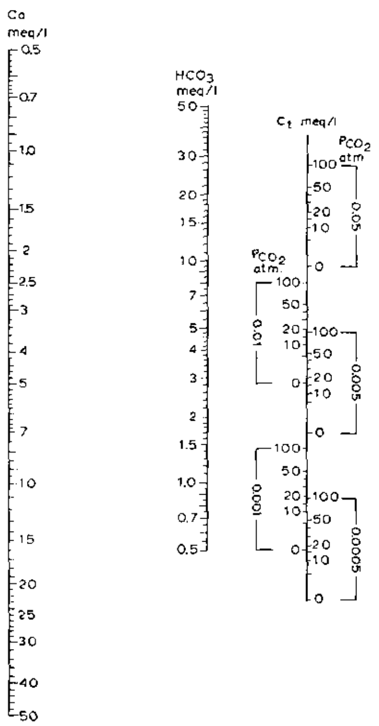


Fig.4. Solubility of $CaCO_3$ at 25°C (modified from BOWER et al., 1965).

Much less is known about the solubility of $MgCO_3$, but we may assume that this compound is at least as soluble as $CaCO_3$. Probably, however, Mg will precipitate together with Ca and form the double salt $CaCO_3.MgCO_3$ (dolomite). A solution of the compound in water shows about equal concentrations of Ca and Mg (expressed in meq/l). Therefore, if solid $MgCO_3$ is present in addition to solid $CaCO_3$, the total concentration of Ca + Mg may be tentatively put at twice the values for Ca obtained from Fig.4.

The solubility of gypsum is strongly dependent on the concentration of other salts. In general it may be assumed that the solubility of $MgCO_3 + CaCO_3 + CaSO_4 . 2 H_2O$ equals approximately 40 meq/l, which corresponds to an electrical conductivity of 3.3 mmhos/cm. In highly saline soils the solubility is higher.

Precipitation of the slightly soluble salts has two important effects:
- a favourable effect on total salinity: the total concentration will be lower than it would have been if all the salts had remained in solution
- an unfavourable effect on the sodium hazard: the relative concentration of Na increases as does the SAR-value.

9.4.2 CLASSIFICATION OF IRRIGATION WATER WITH RESPECT OT BICARBONATES AND GYPSUM

The following classification of irrigation water may help to evaluate the leaching requirement when slightly soluble salts are present:

Class I	$Mg + Ca < HCO_3 + CO_3$
Class II	$Mg + Ca > HCO_3 + CO_3$
	$Ca < HCO_3 + CO_3 + SO_4$
Class III	$Mg + Ca > HCO_3 + CO_3$
	$Ca > HCO_3 + CO_3 + SO_4$
Class IV	$HCO_3 + CO_3 + SO_4$ negligible

When water of Class I is concentrated in the soil, its Mg and Ca will precipitate as carbonates. All Na and K salts remain in solution. The sodium hazard increases with increasing RSC-value (residual sodium carbonate value), i.e. with the difference $(HCO_3 + CO_3) - (Mg + Ca)$.

With water of Class II, part of Ca and Mg will precipitate as carbonates and gypsum. As gypsum is more soluble than bicarbonates, there will usually be a fair amount of Mg + Ca remaining in the soil solution. The sodium hazard, therefore, is less than with waters of Class I. Moreover, all Na and K salts remain in solution.
As a first approximation, the amount of highly soluble sulphates can be estimated at $(HCO_3 + CO_3 + SO_4) - Ca$.

With water of Class III, as with that of Class II, part of Ca and Mg will precipitate as carbonates and gypsum. But here the concentration of Ca + Mg will exceed the solubility of slightly soluble salts, which means that sodium hazards are reduced.

Water of Class IV mainly contains chlorides as anions, and no precipitates are expected.

9.4.3 ADJUSTMENT OF EQUATIONS

The best way to make adjustments in the salt equilibrium and storage equations is to consider the highly and slightly soluble salts separately. Assuming that some of the slightly soluble compounds are present in the solid state, their contribution to the total salt concentration will be a constant, equal to their saturation concentrations. Consequently, if solid $MgCO_3$ and $CaCO_3$ are present in the soil, Eq.20 changes into

$$
\begin{aligned}
EC_e &= EC_{e(\text{carbonates})} + EC_{e(\text{highly soluble salts})} \\
&\simeq 0.8 + 0.5\ EC_{fc(h.s.s.)} \\
&\simeq 0.8 + \frac{C_{fc(h.s.s.)}}{24} \qquad (28)
\end{aligned}
$$

Similarly, if solid $MgCO_3$ + $CaCO_3$ + $CaSO_4.2\ H_2O$ is present in the soil, Eq.20 changes into

$$
\begin{aligned}
EC_e &= 3.3 + EC_{e(h.s.s.)} \\
&\simeq 3.3 + 0.5\ EC_{fc(h.s.s.)} \\
&\simeq 3.3 + \frac{C_{fc(h.s.s.)}}{24} \qquad (29)
\end{aligned}
$$

As a result, the equilibrium equation (22) and the storage equation (23) can be applied in the normal way for the highly soluble salts, after which the corrections for the slightly soluble salts can be introduced.

It should be noted that Eqs.28 and 29 are approximations only, since - as discussed above - the solubility of the slightly soluble salts is rather variable. As the solubility of gypsum increases with increasing salinity of the soil, Eq.29

tends to underestimate EC_e if $EC_{fc(h.s.s.)}$ is high. On the other hand, it tends to overestimate the actual effect of gypsum on plant growth. This is explained by the fact that under field conditions the concentrations of highly soluble salts are at least twice as high as in the saturation extract, whereas the concentrations of slightly soluble salts - when these salts are also present in the solid state - remain unchanged. As the underestimation and overestimation counteract each other, Eq.29 will give a fair enough description of the actual situation for practical purposes.

9.4.4 EXAMPLE OF IRRIGATION WATER CONTAINING GYPSUM

Table 4 presents the monthly salt and water balance of a soil irrigated with water in which gypsum predominates. The yearly average concentrations of salts are given in Line 2. The high values of Ca and SO_4 indicate that the water is nearly saturated with gypsum. Since Mg + Ca > HCO_3 and Ca < HCO_3+ SO_4 this water belongs to Class II (Sect.9.4.2). In solution remain all chlorides (3 meq/l) and all bicarbonates and sulphates not bound to Ca, estimated at 8 meq/l (i.e. HCO_3 + CO_3 + SO_4 - Ca). The total concentration of highly soluble salts in the irrigation water is therefore 11 meq/l so that $EC_{i(h.s.s.)} = \frac{11}{12} = 0.9$ mmho/cm.

Irrigation water is applied at a rate of 124 mm per month (Line 4). The percolation pattern (Line 7) follows from the irrigations (Line 4), the E - P values (Line 5), and the changes in moisture storage (Line 6). It can be seen that the soil is leached only during the months from November to May. The monthly salt storage is now calculated for the highly soluble salts with Eq.23 in a way similar to that explained in Sect.9.4.1 (Lines 8, 9 and 10). Next, the EC_e of the highly soluble salts is determined for the beginning of each month, with the aid of Eq.25 (Line 11). The conductivity of a saturated solution of Mg and Ca carbonates and gypsum (3.3 mmhos/cm) is added to the $EC_{e(h.s.s.)}$ to obtain the total electrical conductivity of the saturation extract (Line 13). The maximum value, found at the beginning of November, is the upper limit tolerated by most crops.

It can be seen from the balance sheet (Table 4) that Ca + Mg precipitated in the soil as follows. The Ca + Mg supply equals the product of irrigation supply and its Ca + Mg concentration (1488 $l/m^2 \times 38$ meq/l = 56500 meq/m^2). The removal of Ca + Mg is at most equal to the product of leaching water and its saturated concentration of Ca + Mg carbonates and gypsum (396 $l/m^2 \times 40$ meq/l = 14800 meq/m^2). The difference between Ca + Mg supply and removal (56500 - 14800 = 41700 meq/m^2)

Table 4. Constant irrigation with water in which gypsum predominates.

1	General	Units	Land use: Fodder throughout the year												
			$W_{f.c.}$ = 300 mm			EC_e = 0.5 $EC_{f.c.}$ for highly soluble salts only (h.s.s.)									
			f = 0.5			EC_i = 0.9 mmhos/cm for highly soluble salts only (h.s.s.)									
2	ions in irrigation water	-	Na	Mg	Ca	total cations	HCO_3	Cl	SO_4	total anions					
	concentration	mg/l	72	96	608	-	182	101	1660						
	equivalent weight	-	23	12	20	-	61	35.5	48						
	concentration	meq/l	3	8	30	41	3	3	35	41					
3	period	-	year	Oct	Nov	Dec	Jan	Feb	March	April	May	June	July	Aug	Sept
4	I	mm	1488	124	124	124	124	124	124	124	124	124	124	124	124
5	E - P	mm	1119	80	55	34	30	42	71	91	127	150	168	157	114
6	ΔW_r	mm	0	+44	+52	0	0	0	0	0	-3	-26	-44	-33	+10
7	R^*	mm	369	0	17	90	94	82	53	33	0	0	0	0	0
8	Z_1 (h.s.s.)	EC mm		2200	2312	2351	2114	1870	1703	1656	1662	1774	1886	1998	2110
9	ΔZ (h.s.s.)	EC mm	+22	+112	+39	-237	-244	-167	-47	+6	+112	+112	+112	+112	+112
10	Z_2 (h.s.s.)	EC mm		2312	2351	2114	1870	1703	1656	1662	1774	1886	1998	2110	2222
11	EC_e (h.s.s.)	mmhos/cm	3.3	3.7	3.9	3.9	3.5	3.1	2.8	2.8	2.8	3.0	3.1	3.3	3.5
12	EC_e (gypsum)	mmhos/cm	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
13	EC_e (total)	mmhos/cm	6.6	7.0	7.2	7.2	6.8	6.4	6.1	6.1	6.1	6.3	6.4	6.6	6.8

represents the precipitation of Ca + Mg in the soil. As this will occur mainly in the form of gypsum (equivalent weight $CaSO_4.2\ H_2O \approx 86$), it is estimated that an annual amount of $41700 \times 86 \times 10^{-6} = 3.6$ kilogram of gypsum is precipitated per m^2 of soil. This precipitate is harmless to plant and soil. Soils irrigated with water containing gypsum will become rich in gypsum and carbonates of Ca and Mg and, after centuries of use, may even largely consist of such precipitates.

9.4.5 EXAMPLE OF IRRIGATION WATER CONTAINING CALCIUM BICARBONATE

The effect of precipitation of bicarbonates will be demonstrated by gradually concentrating irrigation water of the $Ca(HCO_3)_2$ type and of excellent quality (EC_i = 0.45 mmhos/cm, see Table 5). A 10-fold increase in concentration (Line 2) will cause Ca to precipitate as carbonate. The equilibrium concentration of Ca and HCO_3 can then be approximated with the help of Fig.4, keeping in mind that the difference in concentration of Ca (38 meq/l) and HCO_3 (36 meq/l) will remain constant. Putting tentatively C_t = 20 for the concentration after precipitation, the nomogram of Fig.4 yields Ca = 5.8 meq/l and HCO_3 = 3.8 meq/l. The total concentration, found by adding the concentrations of cations, is now C_t = 15 meq/l (Line 3). Figure 4 shows that if C_t = 15 meq/l is used the concentrations of Ca and HCO_3 differ only very little from those found if C_t = 20 meq/l is used.

Table 5. Concentrations of irrigation water in which $Ca(HCO_3)_2$ predominates.

	ionic composition in meq/l; EC in mmhos/cm; P_{CO_2} = 0.01 atm.								
		Na	Ca	HCO_3	Cl	SO_4	C_t	EC	SAR
1	irrigation water	0.9	3.8	3.6	0.6	0.6	4.8	0.45	0.65
2	10-fold concentration, assuming no precipitation of salts	9	(38)	(36)	6	6	(48)	(4.5)	-
3	10-fold concentration, after precipitation of salts	9	6	4	6	6	15	1.2	5.2
4	20-fold concentration, assuming no precipitation of salts	18	(76)	(72)	12	12	96	(9.0)	-
5	20-fold concentration, after precipitation of salts	18	8	4	12	12	26	2.2	9.0

The SAR-value, after a 10-fold concentration, is as low as 5.2. Even a 20-fold concentration (Lines 4 and 5) of the irrigation water will result in a fairly low salinity (EC ≃ 2.2 mmhos/cm) and a reasonably low SAR-value of 9.0.

9.4.6 EXAMPLE OF IRRIGATION WATER CONTAINING SODIUM BICARBONATE

Table 6 shows irrigation water of the $NaHCO_3$ type, of low concentration and seemingly of excellent quality (EC = 0.48 mmhos/cm, SAR = 2.3). The dominance of HCO_3 over Ca, however, makes this water less suitable. If P_{CO_2} = 0.05 atm (5% CO_2 in the soil air), a five-fold increase in concentration is still acceptable, but a 10-fold increase, would give rise to a high SAR. A safe basis on which to calculate the leaching requirement is a 5-fold increase in concentration. If, for example, E = 1500 mm/year, P = 500 mm/year and f = 0.5, then Eq.14 gives

$$R^{*} = \frac{(1500 - 500)C_i}{0.5(5\ C_i - C_i)} = \frac{1000}{0.5 \times 4} = 500 \text{ mm/year}$$

The equilibrium electrical conductivity of the soil moisture EC_{fc} will be only 1.5 mmho/cm, the EC_e value equals approximately 0.5 EC_{fc} = 0.75 mmho/cm. In this case it is therefore the SAR instead of the EC_e that governs the leaching requirement.

Table 6. Concentration of irrigation water in which $NaHCO_3$ predominates.

	ionic composition in meq/l; EC in mmhos/cm at 25° C; P_{CO_2} = 0.05 atm.								
		Na	Ca	HCO_3	Cl	SO_4	C_t	EC	SAR
1	irrigation water	2.5	2.4	3.3	1.3	0.5	5.0	0.42	2.3
2	5-fold concentration, assuming no precipitation of salts	12.5	(12.0)	(16.5)	6.5	2.5	(25.0)	(2.1)	-
3	5-fold concentration, after precipitation of salts	12.5	4.5	9.0	6.5	2.5	17.5	1.5	8.3
4	10-fold concentration, assuming no precipitation of salts	25	(24)	(33)	13	5	(50)	(4.2)	-
5	10-fold concentration, after precipitation of salts	25	3	12	13	5	29	2.4	20.4

If leaching is not adequate, this $NaHCO_3$ type of irrigation water will cause strongly sodic soils. In the upper layer the $NaHCO_3$ may even lose CO_2 to the atmosphere and turn into Na_2CO_3.
If magnesium is present in amounts large enough to cause precipitation of $MgCO_3$ together with $CaCO_3$, the concentration of Ca + Mg may be put tentatively at twice the values obtained for Ca. This results in a decreased sodium hazard and leaching requirement.

9.5 THEORY OF LEACHING

When salty soils are leached during reclamation, or when excess irrigation water is applied to maintain a low salt content after reclamation, there is always an encroaching fluid which displaces the soil solution with which it is supposed to be completely miscible. The following theoretical models may serve to illustrate the process of solute movement through porous materials (Fig.5):
- single reservoir
- single reservoir with bypass
- series of reservoirs
- continuous column.

It is assumed that there is no chemical or physical interaction between solute, solution and soil.

9.5.1 THE SINGLE RESERVOIR

Consider an open reservoir of volume V filled with sea water of concentration C_o. The salt water is gradually replaced by fresh water (C_i) and during this process the level of the reservoir is kept constant. (Such a situation occurred in The Netherlands when Lake IJssel was separated from the sea in 1932; within a few years, the lake became fresh under the influence of the IJssel River.) We may distinguish two extreme leaching conditions: one in which no mixing of the fresh water with the sea water occurs, and the other in which complete mixing takes place (Fig.6).
If no mixing occurs, the sea water is merely displaced by fresh water at a rate Q (piston flow, Fig.6A). At the time $T = V/Q$, when all the sea water has been replaced by fresh water, the actual effluent concentration (C_u) will abruptly change from $C_u = C_o$ to $C_u = C_i$. Under natural conditions piston flow will seldom occur.

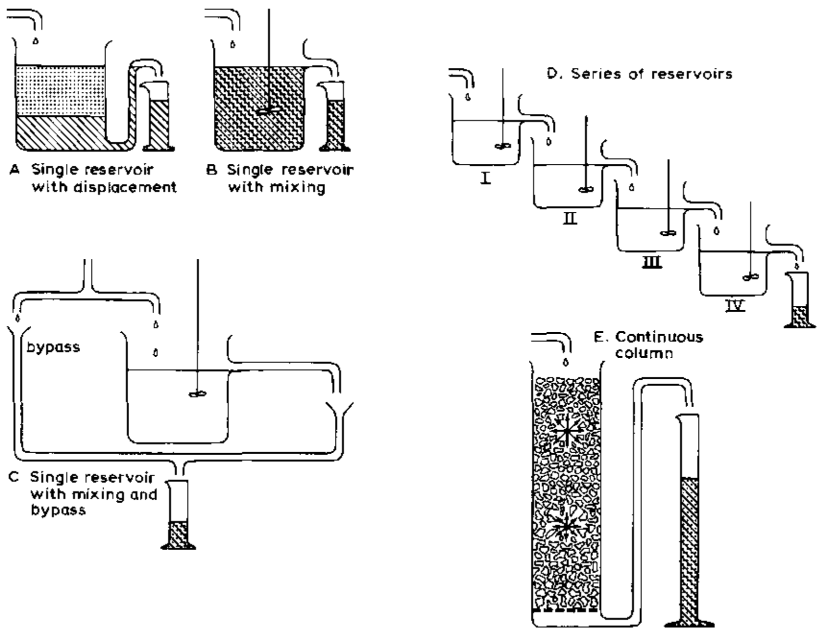


Fig.5. Theoretical models to illustrate the process of solute movement through porous materials

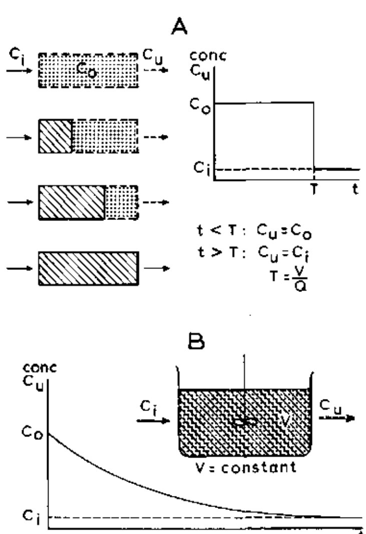


Fig. 6. Desalinization of a reservoir

If complete mixing takes place in the reservoir and if the volume of water in the reservoir is constant the salt balance reads (Fig.6B)

$$C_i Q dt = C_u Q dt + V dC \tag{30}$$

where

C = the average salt concentration of the reservoir solution

C_i = the salt concentration of the influent

C_u = the salt concentration of the effluent.

When mixing is complete, $C_u = C$, and Eq.30 becomes, after rearrangement

$$\frac{dC}{C - C_i} = -\frac{Q}{V}\, dt$$

Integration between the limits $C = C_o$ at time $t = 0$ and C at time t yields the solution

$$C_u = C = C_i + (C_o - C_i)e^{-t/T} \tag{31}$$

where C_o is the original salt concentration of the reservoir solution, and $T = V/Q$.

This result is in fair agreement with observations in some shallow lakes (like Lake IJssel). Equation 31 applies equally well to the rootzone of a soil being subjected to leaching, if the rootzone is considered a single reservoir with complete mixing.

9.5.2 THE RESERVOIR WITH BYPASS

In soils, irrigation or rain water is unlikely to mix completely with the soil solution. Part of it may move through the large pores (cracks, root holes) and arrive at the lower boundary of the rootzone without any mixing. This is expressed by

$$C_u = fC + (1 - f)C_i \tag{32}$$

which states that a fraction (f) of the incoming water will flow out of the rootzone with the concentration C of the soil solution and that a fraction (1 - f) will have the concentration C_i of the influent. Combining Eqs.32 and 30 yields, with $C = C_o$ when $t = 0$

$$C = C_i + (C_o - C_i)e^{-ft/T} \tag{33}$$

in which f is referred to as the leaching efficiency. For $C_i = 0$, Eq.33 becomes

$$C = C_o e^{-ft/T} \tag{34}$$

This model of a single reservoir was used in preceding sections as a basis for the salt balance studies.

9.5.3 THE SERIES OF RESERVOIRS

If the process of leaching is examined more closely, it will be clear that complete mixing over the entire depth of the rootzone (often 1 m or more) is not very probable. To account for the limited range over which mixing is effective, we may suppose the soil to consist of different reservoirs, e.g. corresponding with the soil layers 0 - 20, 20 - 40, 40 - 60 and 60 - 80 cm depth. Each reservoir receives the outflow from the overlying one; in each reservoir mixing is complete (Fig.5D). For irrigation water with salt concentration C_i and for a leaching efficiency f, the following expressions are found for the salt concentrations in successive reservoirs of equal volume (Fig.7):

1st reservoir : $C_I = C_i + (C_o - C_i)e^{-ft/T}$

2nd reservoir : $C_{II} = C_i + (C_o - C_i)\ (1 + \frac{ft}{T})e^{-ft/T}$

3rd reservoir : $C_{III} = C_i + (C_o - C_i)\ (1 + \frac{ft}{T} + \frac{f^2t^2}{2T^2})e^{-ft/T}$

4th reservoir : $C_{IV} = C_i + (C_o - C_i)\ (1 + \frac{ft}{T} + \frac{f^2t^2}{2T^2} + \frac{f^3t^3}{6T^3})e^{-ft/T}$

N-th reservoir : $C_N = C_i + (C_o - C_i)e^{-ft/T} \ .\ \sum\limits_{n=0}^{n=N-1} \ (1 + \frac{f^n t^n}{n!T^n})$ (35)

where $n! = 1 \times 2 \times 3 \times \ldots\ldots\ldots \times n$.

9.5.4 CONTINUOUS COLUMNS

The soil profile is in fact not made up of several separate reservoirs, but forms a continuous column. Mixing takes place at every depth, but is effective over a limited range only (Fig.5E). GLUECKAUF (1949) developed a theory about the behaviour of such columns. For desalinization of a soil, he found the following expression (Fig.8)

$$C = \tfrac{1}{2}C_o\left[\operatorname{erfc}\left(\frac{v - ax}{2v}\sqrt{\frac{v}{ak}}\right) - e^{x/k}\operatorname{erfc}\left(\frac{v + ax}{2v}\sqrt{\frac{v}{ak}}\right)\right] \qquad (36)$$

where

C_o = original salt concentration in soil moisture
v = depth of water percolated since the beginning of leaching
a = volume fraction of soil filled with water
x = depth
2k = effective mixing length

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z) = 1 \frac{2}{\sqrt{\pi}}\int_o^z e^{-z^2}\,dz$$

Error functions (erf) and complementary error functions (erfc) are treated in Chap.13.3, Vol.II.

Putting p = v/ax and N = x/2k, Eq.36 changes into

$$C = \tfrac{1}{2}C_o\left[\operatorname{erfc}\left(\frac{p - 1}{\sqrt{2p}}\sqrt{N}\right) - e^{2N}\operatorname{erfc}\left(\frac{p + 1}{\sqrt{2p}}\sqrt{N}\right)\right] \qquad (37)$$

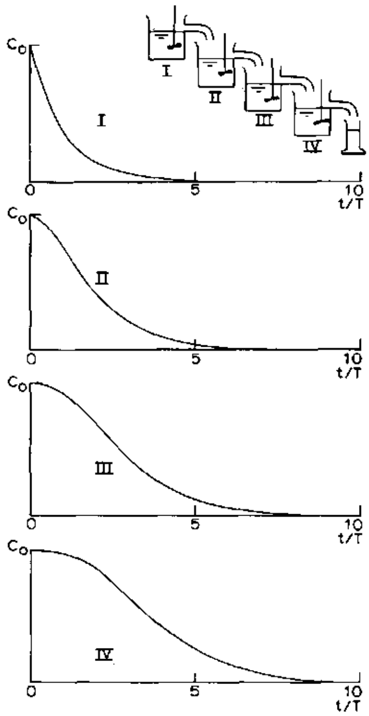
A comparison of Eq.37 with Eq.35 shows that N is the same (number of "reservoirs" above depth x). The product pN is comparable with t/T in Eq.35. As appears from Figs.7 and 8, the differences between the two methods are negligible in practice.

In more complicated cases numerical methods may be employed, e.g. when the value of the leaching efficiency (f) or the effective mixing length (2 k) are not constants, but vary with depth. In methods of this kind, the soil profile is, as it were, divided into a number of separate reservoirs, but of a volume that is proportional to the mixing length. Moreover, these reservoirs are provided with suitable bypasses to account for variations in f. By taking small increments in time or in volume of water added, all changes in the system can be calculated

9.5.5 EXAMPLE OF CALCULATION

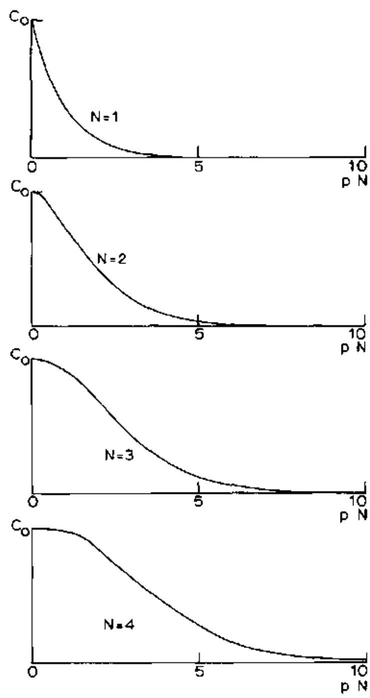
As an example we will calculate the desalinization of the following soil profile by rainfall:

Layer in cm	0 - 25	25 - 50	50 - 75	75 - 100
EC_e in mmhos/cm	12	18	24	28



$$C_N = C_i + (C_o - C_i)e^{-t/T} \sum_{n=0}^{n=N-1} (1 + \frac{t^n}{n! T^n})$$

Fig.7. Desalinization of 4 reservoirs in series.



$$C = \frac{C_o}{2} \left[\mathrm{erfc}\left(\frac{p-1}{\sqrt{2p}}\sqrt{N}\right) - e^{2N}\mathrm{erfc}\left(\frac{p+1}{\sqrt{2p}}\sqrt{N}\right) \right]$$

Fig.8. Desalinization of a continuous soil column.

Since the salt concentration is not the same over the entire depth of the soil profile, we may calculate the leaching process with the aid of Eq.35, applying the principle of superposition

$$C_I = C'_o e^{-t/T}$$

$$C_{II} = C'_{II} + C''_{II} = C'_o \ (1 + t/T)e^{-t/T} + (C''_o - C'_o)e^{-t/T}$$

etc.

in which C_o' is the concentration of the salt solution in the first layer, C_o'' in the second layer, etc.
The term $(C_o'' - C_o')e^{-t/T}$ takes the sign of the value $(C_o'' - C_o')$. Since in this example the salt concentration increases with depth, all terms are positive.

We want to know for example the desalinization after every 80 mm of rainfall. The leaching efficiency f is supposed to be 1. Assuming $w_{fc} = 0.5$, the total amount of soil moisture in a layer of 25 cm equals 125 mm. Since $t/T = Qt/W$, we can calculate the values t/T from the rainfall, Qt and the total amount of soil moisture in a layer of 25 cm, W

Rainfall in mm	80	160	240	320	400	480	560	640
t/T	0.64	1.28	1.92	2.56	3.20	3.84	4.48	5.12

Table 7 shows the process of calculation (see p.97).

We may also calculate the desalinization by a numerical method. Since leaching starts with a mixing of irrigation or rain water at concentration C_i, with the moisture of the first soil layer at concentration C_{s1}, the concentration of the soil solution of the first layer after mixing C_{x1} can be calculated in the following way:
a mm of influent water $\times C_i$ + b mm of soil water $\times C_{s_1} = (a + b) \times C_{x_1}$.

If the amount of water retained from the rainfall in the first layer is equal to c mm, an amount (a - c) having a concentration C_{x_1} percolates in depth and mixes with the moisture of the second layer. The concentration of the soil solution of the second layer after mixing C_{x_2} can be calculated in the same way

$$(a - c)C_{x_1} + dC_{s_2} = (a - c + d)C_{x_2}$$

In order to simplify the calculations and to assume the same conditions as in the case calculated above with Eq.35, we suppose that
- $C_i = 0$
- bulk density, so W_{fc}, is the same for all layers
- all rainfall percolates through the entire soil profile, the soil not drying out between successive periods of rainfall, so $c = 0$ and $W_{fc} = b = d$.

So we obtain:

$$\text{Layer } 0 - 25 \text{ cm:} \quad 125 \times 12 = (80 + 125)C_{x_1} \rightarrow C_{x_1} = 7.3$$
$$\text{Layer } 25 - 50 \text{ cm:} \quad 80 \times 7.3 + 125 \times 18 = (80 + 125)C_{x_2} \rightarrow C_{x_2} = 13.8$$

Table 8 shows the results of the calculations made (see p.98),
- with the aid of Eq.35
- with the numerical method, taking steps of 20 mm
- with the numerical method, taking steps of 80 mm.

As appears from this table, the smaller the steps, the better the results of the numerical method approach those obtained with Eq.35. In practice the differences between the two methods are almost negligible.

Table 7. Example of calculation with Eq.35.

$$C_{I} = C_o' e^{-t/T}$$

$$C_{II} = C_o'(1 + t/T)e^{-t/T} + (C_o'' - C_o')e^{-t/T}$$

$$C_{III} = C_o'(1 + t/T + t^2/2T^2)e^{-t/T} + (C_o'' - C_o')\ (1 + t/T)e^{-t/T} + (C_o''' - C_o'')e^{-t/T}$$

$$C_{IV} = C_o'(1 + t/T + t^2/2T^2 + t^3/6T^3)e^{-t/T} + (C_o'' - C_o')\ (1 + t/T + t^2/2T^2)e^{-t/T}$$

$$+ (C_o''' - C_o'')\ (1 + t/T)e^{-t/T} + (C_o'''' - C_o''')e^{-t/T}$$

1	C_o'	12.0							
2	$C_o'' - C_o'$	6.0							
3	$C_o''' - C_o''$	6.0							
4	$C_o'''' - C'''_o$	4.0							
5	t/T	0.64	1.28	1.92	2.56	3.20	3.84	4.48	5.12
6	$t^2/2T^2$	0.21	0.82	1.84	3.28	5.12	7.37	10.04	13.11
7	$t^3/6T^3$	0.04	0.35	1.18	2.80	5.46	9.44	14.98	22.37
8	$1+t/T$	1.64	2.28	2.92	3.56	4.20	4.84	5.48	6.12
9	$1+t/T+t^2/2T^2$	1.85	3.10	4.76	6.84	9.32	12.21	15.52	19.23
10	$1+t/T+t^2/2T^2+t^3/6T^3$	1.89	3.45	5.94	9.64	14.78	21.65	30.50	41.60
11	$e^{-t/T}$	0.527	0.278	0.147	0.077	0.0408	0.0215	0.0113	0.006
12	C_I = 1×11	6.3	3.3	1.8	0.9	0.5	0.3	0.1	0.07
13	C_{II} = 1×8×11	10.4	7.6	5.1	3.3	2.1	1.3	0.7	0.4
14	C_{II}'' = 2×11	3.2	1.7	0.9	0.5	0.2	0.1	0.1	0.1
15	C_{II} = 13+14	13.6	9.3	6.0	3.8	2.3	1.4	0.8	0.5
16	C_{III}' = 1×9×11	11.7	10.4	8.4	6.3	4.6	3.2	2.1	1.4
17	C_{III}'' = 2×8×11	5.2	3.8	2.6	1.6	1.0	0.6	0.3	0.2
18	C_{III}''' = 3×11	3.2	1.7	0.9	0.5	0.2	0.1	0.1	0.1
19	C_{III} = 16+17+18	20.1	15.9	11.9	8.4	5.8	3.9	2.5	1.7
20	C_{IV}' = 1×10×11	12.0	11.5	10.4	8.9	7.2	5.6	4.1	30
21	C_{IV}'' = 2×9×11	5.9	5.2	4.2	3.1	2.3	1.6	1.1	0.7
22	C_{IV}''' = 3×8×11	5.2	3.8	2.6	1.7	1.0	0.6	0.4	0.2
23	C_{IV}'''' = 4×11	2.1	1.1	0.6	0.3	0.2	0.1	0.1	0.0
24	C_{IV} = 20+21+22+23	25.2	21.6	17.8	14.0	10.7	7.9	5.7	3.9

Table 8. Leaching of a soil profile by rainfall.

1. EC_e-values calculated with Eq.35

Layer in cm	Before leaching	After leaching with							
		80 mm	160 mm	240 mm	320 mm	400 mm	480 mm	560 mm	640 mm
0 - 25	12.0	6.3	3.3	1.8	0.9	0.5	0.3	0.1	0.07
25 - 50	18.0	13.6	9.3	6.0	3.8	2.3	1.4	0.8	0.5
50 - 75	24.0	20.1	15.9	11.9	8.4	5.8	3.9	2.5	1.7
75 - 100	28.0	25.2	21.6	17.8	14.0	10.7	7.9	5.7	3.9

2. EC_e-values calculated with the numerical method (steps of 20 mm)

Layer in cm	Before leaching	After leaching with							
		80 mm	160 mm	240 mm	320 mm	400 mm	480 mm	560 mm	640 mm
0 - 25	12.0	6.7	3.8	2.1	1.2	0.7	0.4	0.2	0.1
25 - 50	18.0	13.7	9.6	6.5	3.9	2.5	1.6	1.0	0.6
50 - 75	24.0	20.1	16.0	12.1	8.9	6.2	4.3	2.9	1.9
75 - 100	28.0	25.2	21.7	18.0	14.5	11.1	8.3	6.1	4.4

3. EC_e-values calculated with the numerical method (steps of 80 mm)

Layer in cm	Before leaching	After leaching with							
		80 mm	160 mm	240 mm	320 mm	400 mm	480 mm	560 mm	640 mm
0 - 25	12.0	7.3	4.5	2.7	1.7	1.0	0.6	0.4	0.2
25 - 50	18.0	13.8	10.1	7.3	5.6	3.8	2.6	1.7	1.1
50 - 75	24.0	20.0	16.2	12.7	10.0	7.6	5.7	4.1	2.9
75 - 100	28.0	24.9	21.6	18.1	15.0	12.1	9.6	7.5	5.7

LITERATURE

BOWER, C.A., WILCOX, L.V., AKIN, G.W. and KEYES, Mary G. 1965. An index of the tendency of the $CaCO_3$ to precipitate from irrigation waters. Soil Sci. Soc. Am. Proc. 29: 91-92.

DIELEMAN, P.J. (Ed.). 1963. Reclamation of salt affected soils in Iraq. Publication No.11, International Inst. Land Recl. and Impr., Wageningen. 175 pp.

GLUECKAUF, D. 1949 Activity coefficients in concentrated solutions containing several electrolytes. Nature 163: 414-415.

RICHARDS, L.A. (Ed.). 1954. Diagnosis and Improvement of saline and alkali soils. Agr. Handbook 60, U.S.D.A., Washington D.C. 160 pp.

UNESCO. 1970. Research and Training on irrigation with saline waters. Techn. report of UNDP project Tunisia 5.

<u>Suggestions for further reading</u>

General

ALLISON, L.E. 1964. Salinity in relation to irrigation. Advances in Agronomy 16: 139-179.

BERNSTEIN, L. 1966. Soil salinity and crop productivity. Span (Shell Public Health and Agric. News) 9: 6-79.

FAO/UNESCO. In press. International source book on irrigation and drainage of arid land in relation to salinity and alkalinity. 663 pp.

KELLY, W.P. 1951. Alkali soils, their formation, properties and reclamation. Rheinhold Publ. Corp., New York. 176 pp.

Leaching under arid conditions

HULSBOS, W.C. and BOUMANS, J.H. 1960. Leaching of saline soils in Iraq. Neth. J. Agric. Sci. 8: 1-10.

Leaching under humid conditions

VAN DER MOLEN, W.H. 1957. The exchangeable cations in soils flooded with sea-water. Staatsdrukkerij 's Gravenhage. 167 pp.

VERHOEVEN, B. 1953. Over de zout- en vochthuishouding van geïnundeerde gronden. Versl. Landbk. Ond. 67.17. Staatsdrukkerij 's Gravenhage. 202 pp.

Bicarbonate effects

BOWER, C.A. and WILCOX, L.V. 1965. Precipitation and solution of calcium carbonate in irrigation waters. Soil Sci. Soc. Am. Proc. 29: 93-94.

Theory of leaching

GARDNER, W.R. and BROOKS, R.H. 1957. A descriptive theory of leaching. Soil Sci. 83: 295-304.

MILLER, R.J., BIGGAR, J.W. and NIELSEN, D.R. 1965. Chloride displacement in Panoche clay loam in relation to water movement and distribution. Water Resources Research 1: 63-73.

VAN DER MOLEN, W.H. 1956. Desalinization of saline soils as a column process. Soil Sci. 81: 19-27.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

10. EFFECTS OF IRRIGATION ON DRAINAGE

J. NUGTEREN
Professor
Department of Irrigation and Civil Engineering
University of Agriculture, Wageningen

Lecturers in the Course on Land Drainage

J. F. Graadt van Roggen (1962–1968)
Dwars, Heederik en Verhey, Consulting Engineers, Amersfoort

J. Nugteren (1969–1971)
University of Agriculture, Wageningen

K. Roscher (1972)
University of Agriculture, Wageningen

PURPOSE AND SCOPE

A brief discussion of the interrelationships between irrigation and drainage.

CONTENTS

10.1 INTERRELATION OF IRRIGATION AND DRAINAGE

Irrigation in the world today covers approximately 160 million ha, excluding areas under natural flooding. About half of this total is found in the arid and semi-arid subtropical zones. It was particularly in these zones that the special drainage measures demanded by irrigation came to be recognized, as over the years those areas with insufficient drainage began to show rising groundwater tables and increasing salinity.
About two thirds of the total irrigated area has been brought under irrigation since the beginning of this century, but only recently has it become generally accepted that the installation or improvement of drainage forms an indispensable part of any irrigation project.

In ancient times, as in the days of the various Babylonian kingdoms, salinity and water logging gradually reduced the productivity of the land. Market records have shown that in such a situation the cultivation of wheat - a crop sensitive to soil salinity - gave way to the more tolerant barley, but that finally large areas had to be abandoned and the farmers moved to new land. The rise and fall of the various kingdoms in Mesopotamia were evidently closely related to this changing state of agriculture.
The Imperial Valley in California, comprising 200,000 ha, was brought under irrigation about 1910. Only fifteen years later the productivity of this area was severely threatened since no provisions had been made for the discharge of the superfluous irrigation water and the salts that were brought to the area at a rate of 800 kg per ha with each irrigation application. Large parts of the valley went out of production and it was this catastrophe that provided the impulse for research into proper methods of re-establishing and maintaining sufficiently low salt concentrations in the soil. Due to the work of the U.S. Salinity Laboratory at Riverside, California, and that of other institutions, the remedy for drainage problems in irrigated lands is at present well-known, but only in a minor part of the affected, subtropical areas have the necessary works been carried out. It is estimated that approximately 50 million ha of irrigated lands still do not have the required drainage facilities.

The particular effects of irrigation on the criteria and the design of a drainage system are more dominant when rainfall is of lesser importance for the growth of crops than irrigation is.

In order to distinguish these effects clearly, it will be assumed in the following that during the irrigation season the contribution of rainfall to the production of crops and to the drainage discharge can be neglected. This assumption will be valid in deserts and in most of the steppe climates. However, in the latter climatic zones, rainfall during the winter period may amount to 300 mm or more, and irrigation in this period will then be of the same order as precipitation; consequently, the drainage design in such areas must be based on the combination of these two sources of supply.

The main aspect of drainage, in so far as it is necessitated by irrigation, is that its discharge capacity should correspond to that quantity of irrigation water supplied in excess of the crop requirements. This discharge capacity consists of two components: the surface runoff and the subsurface discharge. The subsurface discharge must be correlated with a minimum depth to the groundwater table, or a maximum rise of the groundwater table above the drain pipes of the water surface in the drains (see Chap.8, Vol.II). The excess supply of irrigation water is primarily needed to cover losses that occur either in conveyance or during field application. Moreover, in zones of negligible or limited rainfall, a further surplus may be required to maintain an acceptable salinity level in the rootzone. This amount will depend, among other things, on the quality of the irrigation water as expressed by its salt concentration. Thus the drainage requirements are dependent on both the net quantity and the quality of the irrigation water.

As losses by evaporation - except those by evaporation from the ground surface, which are included in the evapotranspiration - usually represent only a very small fraction of the total supply, it can be stated with a reasonable approximation that the drainage discharge D_A over a given irrigation season and for the entire area is

$$D_A = V - E \tag{1}$$

where
D_A = drainage discharge of irrigated area
V = total irrigation supply
E = evapotranspiration of crop
The total efficiency of the irrigation system, e_p, expresses the ratio between

the quantity effectively used for evapotranspiration and the total quantity supplied

$$e_p = \frac{E}{V} \tag{2}$$

The overall efficiency can be considered the product of the conveyance efficiency, e_c, (being the ratio between the quantity reaching the fields and the total supply) and the field application efficiency, e_a, (being the ratio between the quantity reaching the fields and the evapotranspiration of the crop), so that

$$e_p = e_c\ e_a \tag{3}$$

From Eqs.1 and 2 it follows that

$$D_A = (1 - e_p)V \tag{4}$$

D_A, V, and E can be expressed in mm per given period of time.

As the drainage discharge of an irrigated area is due partly to surface runoff and partly to groundwater flow, these two components must be estimated before a system can be designed (Chap.15, Vol.II). The field losses, both as surface and subsurface discharge, will be more or less evenly distributed over the area, but within a single field they may show flow concentrations during and after applications. The conveyance losses consist of canal seepage, which depends on the soil properties or the quality of the linings, and the operational losses, which are excess quantities spilled into the drainage system.

Since the conveyance losses cause additional drainage discharge in the vicinity of canals only, the field drainage requirements for any portion of the area, excluding the effects of the canals, are

$$D_a = (1 - e_a)e_cV \tag{5}$$

where

D_a = field drainage discharge of sub-area

e_cV = the relevant volume reaching the sub-area.

The sub-area considered should be of such a size that local peak discharges due to the irrigation of a single field have no effect on the discharge of the sub-area; that is to say, field irrigation within such a sub-area can be considered evenly distributed at any time. The drainage discharges of smaller sub-areas, and finally, of a farm, expressed in depth of water (mm/day) or per surface unit (l/sec/ha) will have mean values over a longer period corresponding with the discharge according to Eq.5. During short periods, however, peaks will occur - particularly in the surface runoff - which will determine the capacities of smaller collector drains, and of farm and field drains.

10.2 CONTROL OF IRRIGATION

The overall irrigation efficiency as defined by Eq.2 will show whether the irrigation and drainage systems are functioning effectively. It represents the quality of the operation of both conveyance and field irrigation, and determines the magnitude of the drainage capacity demanded by irrigation. By Eqs.2, 4, and 5

$$D_A = \left(\frac{1}{e_p} - 1\right)E \qquad (6)$$

$$D_a = \left(\frac{1}{e_a} - 1\right)E \qquad (7)$$

A high value of the field efficiency is not always advisable or realistic. Large field losses through percolation or surface runoff should be prevented by proper field lay-out and appropriate stream size on the fields; but, as will be explained later, some field application methods appear to have a certain percentage of unavoidable losses. Reducing these losses below a certain limit - which is determined by the topography and the lay-out - would result in local irrigation deficiencies. Moreover, in arid and semi-arid zones part of the losses can be regarded as being beneficial in view of maintaining an acceptable salinity level in the soil. Consequently, both an upper limit and a lower limit are applicable to the field efficiencies of various methods; beyond these, either under-irrigation or unnecessary losses will occur.

When rainfall is neglected, the water balance of the field is

$$e_c V = E + \alpha E + R' + S$$

where

αE = leaching requirement, i.e. the additional quantity required to remove the salts left behind by the evapotranspiration of quantity E

R' = excess percolation to the subsoil due to non-uniform application

S = surface runoff on sloping fields

Setting $R' + S = \beta(E + \alpha E)$ yields

$$e_c V = (1 + \beta)\ (1 + \alpha)E$$

Since

$$e_c\ e_a\ V = E$$

it follows that

$$e_a = \frac{1}{(1 + \alpha)\ (1 + \beta)} \tag{8}$$

and

$$e_p = \frac{e_c}{(1 + \alpha)\ (1 + \beta)} \tag{9}$$

From Eqs.6 and 7 it follows that

$$D_A = \left[\frac{(1 + \alpha)\ (1 + \beta)}{e_c} - 1\right]\ E \tag{10}$$

and

$$D_a = \left[(1 + \alpha)\ (1 + \beta) - 1\right]\ E \tag{11}$$

The limits of the efficiency for the various field irrigation methods and those of the ratios D_A/E and D_a/E, which can be obtained with reasonable control of supply and application, are determined by the values of e_c, α, and β for each specific case.

10.3 CONVEYANCE LOSSES

As stated earlier, conveyance losses consist of losses by percolation and operational losses in the distribution system.
Percolation from unlined canals will depend on the permeability of the soil and the depth to the groundwater table. If the groundwater table is far below the canal bed, a predominantly vertical flow will develop under mainly saturated conditions. Table 1 gives an indication of seepage for various soils.

Table 1. Seepage losses per m^2 of wet canal perimeter (POIREE and OLLIER, 1968).

Surrounding soil	Loss ($m^3/m^2/day$)	Loss per km canal length as percentage of flow (%) ')
clay	0.09	0.07
loamy clay	0.18	0.14
sandy clay	0.20 - 0.40	0.15 - 0.31
sand	0.50	0.38
sand-gravel	0.75	0.58
gravel	1.00 - 1.80	0.77 - 1.39

') assumed average water depth: 1.50 m; assumed average stream velocity: 1 m/sec.

If the soil surrounding the canal contains different layers, or if a pervious or semi-pervious lining is applied, the flow is governed by the least permeable layer. After a period of percolation, this layer and those above will become saturated, whereas the soil underneath will remain unsaturated (Fig.1).
When the groundwater table in the area near the canal is high, the flow will be governed by the available head between the groundwater and the water surface in the canal, and by the horizontal permeability of the soil (Fig.2). In such a case the tendency exists for the groundwater table to reach the ground surface at the outer slope of the embankment, which may result in pools of stagnant water or in surface runoff due to seepage. The bulk of the flow, however, will contribute to the subsurface discharge.
With a deep groundwater table the seepage losses will be larger than with a higher water table in the same soil, but the higher water table will have a much more severe effect on the top soil and the crops. Serious salinity along main

irrigation canals is a well-known feature in arid climates and is caused by capillary rise from the groundwater table and evaporation at the surface.

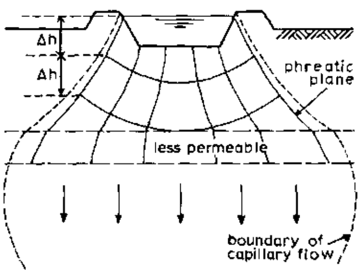


Fig.1. Canal seepage with deep groundwater table.

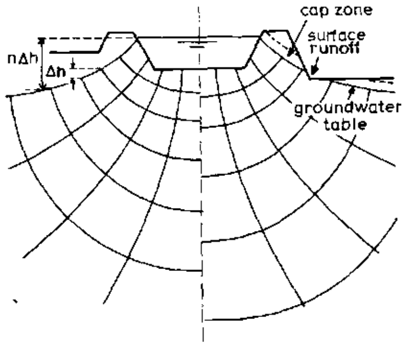


Fig.2. Canal seepage with shallow groundwater table at different depths.

Canal linings can be made of various materials (see Chap.29, Vol.IV). Completely tight linings do not exist, but concrete lining, if well constructed, can reduce the seepage to less than 0.2% of the conveyed quantity per km length of canal. As farm ditches are often used intermittently, losses due to saturation of the surroundings after an off-period may be considerable. In less permeable soils or in lighter soils with adequate canal linings, seepage losses in a medium-sized system will usually not be more than 5-10% of the total supply.

The total operation losses, being the positive difference between supply and demand, is difficult to assess. If the water is distributed among the farms according to a rotation schedule, the tendency exists to supply larger quantities than the average requirement so as to cover any extreme meteorological conditions. As a result, during part of the time a substantial surplus is spilled. This happens, for instance, when the rotation is based on a constant interval between field applications, corresponding with the period of highest evapotranspiration (Section 10.4). During the periods of lesser evapotranspiration, the farmers receive a quantity which is more than they need to replenish the rootzone, so they will divert part of the supply to the farm drains.
These operational losses can, in principle, be reduced by adjusting the operation to the average requirements at a given moment. The delivery according to the farmer's demand, however, requires that both farmers and irrigation officials have a good knowledge of crop requirements, and, furthermore, that the canal system be flexible and adequately provided with measuring and regulating devices.

The change-over from a rather simple rotational system to the better, adaptable-demand system is usually a complicated process requiring a staff of qualified technicians. Only if operational losses under the existing method are substantial, if the cost of the necessary works is economically justified, and if the farmers have sufficient training in irrigation, should such a change-over be considered. For an estimate of operational losses in a specific area, intensive flow measurements must be taken and a quantitative analysis made of the present operations. Existing systems show widely ranging percentages of losses: for rotation supply an average may be of the order of 20%.

10.4 APPLICATION OF IRRIGATION WATER

To maintain a sufficient air content in the soil, water is usually applied to the fields at regular intervals. The one important exception is rice, which thrives in saturated soil and here water is usually applied continuously, thus keeping a water layer on the ground surface. For all other crops the soil moisture is replenished when it has diminished to such an extent that the evapotranspiration commences to differ substantially from its potential value.

Assuming that the depth of the rootzone is D (mm), and θ_{fc} and θ_o are the moisture content in volume percentage at field capacity and at the accepted lower moisture limit respectively, the quantity W (mm) added to the rootzone during irrigation is

$$W = \frac{\theta_{fc} - \theta_o}{100} D \tag{12}$$

The amount of water replenishing the soil moisture is equal to the amount of soil moisture taken up by the plants between two irrigation applications. If, during the interval of n days between irrigations, the average daily actual evapotranspiration is $\overline{E}_a$ (mm.day^{-1})

$$W = n\overline{E}_a = \frac{\theta_{fc} - \theta_o}{100} D \tag{13}$$

Under controlled irrigation, the application takes place when the lower limit θ_o has been reached; the moisture content is then brought to field capacity. The variation in the size of application is proportional to the wetted depth, which should correspond with the depth of the rootzone at the moment of irrigation.

The interval between irrigations must depend on the day-to-day evapotranspiration and the rainfall during that period (Fig.3).

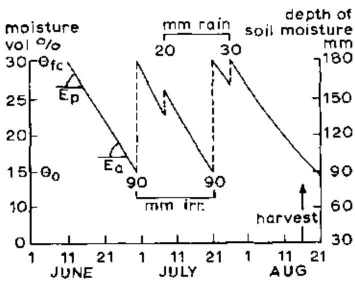


Fig.3. Diagram of soil moisture variation in a rootzone of 60 cm.

In practice the farmer will very often not be able to have the quantity he requires delivered on the exact day that the soil moisture is depleted to its lower limit. Under a rotational system, as explained in Section 10.3, he will receive his supply at a predetermined date, which will usually be somewhat earlier than required. Irrigation then takes place with a soil moisture content above θ_o, and, if the depth W according to Eq.12 is applied, this will result in wetting a zone beyond the roots. As already stated, the farmer may, if he is aware of the higher moisture content, shorten this application time, and spill part of the flow to the drains.

The infiltration of the water from the surface into the soil profile generally has a rate decreasing with time. For many soils, this rate I_{inst} (mm/min) as a function of the time of application t, can be expressed as

$$I_{inst} = a\ t^b \tag{14}$$

where a is a coefficient depending on the type of soil and on the moisture content at the beginning of the infiltration and b, which also depends on the soil, ranges between -0.5 and 0.
From Eq.14 the cumulative infiltration I_{cum} over the period t can be derived

$$I_{cum} = \frac{a}{b+1}\ t^{b+1} \tag{15}$$

10.5 FIELD IRRIGATION METHODS

The many different field irrigation methods can be broadly divided into four groups:
stagnant flooding (basin irrigation)
flow irrigation
subsurface irrigation
overhead (sprinkler) irrigation.

Stagnant flooding and flow irrigation are known as surface irrigation.
For continuous irrigation, stagnant flooding or subsurface irrigation are the most suitable methods. For intermittent application, flow irrigation, stagnant flooding, or overhead irrigation are suitable.

Subsurface irrigation is based on a controlled groundwater table near the groundsurface. The soil moisture is supplied from the groundwater table by capillary action in accordance with the demands made by the evapotranspiration. This method can only be applied under specific hydrological and climatological conditions, which restricts the extent of its utilization.
Overhead irrigation by stationary or rotating sprinklers, supplied by pressure lines under 3-5 atmospheres, is a highly efficient system. Losses due to surface runoff, deep percolation or other factors can be small if the system is properly designed and operated.

In view of their effects on drainage, flood and flow irrigation need closer consideration. If surface irrigation is applied, the choice between stagnant flooding and flow irrigation depends on the general slope of the fields. For horizontal or nearly horizontal areas, the flooding method is widely used, both for continuous irrigation (rice) and for intermittent supply (cover crops, small grains). As fields are rarely completely horizontal, the slight differences in elevation are compensated for by low levees or checks. Thus basins are formed which are filled separately and in which the required depth of water is ponded. For paddy the supply is usually continuous, but for intermittent irrigation, once the proper quantity is applied, the infiltration ends when this quantity is absorbed by the soil. Paddy fields on sloping land have small basins (or sawahs) - sometimes with dimensions of 10 m or less - perpendicular to the contours in order to keep the depth of the water layer approximately uniform. Since too many field ditches would reduce the cropped areas, the supply in such cases is passed from one basin to another. For intermittent irrigation, basins are only practical on flat land, and there all basins are supplied directly from a field ditch (Fig.4).

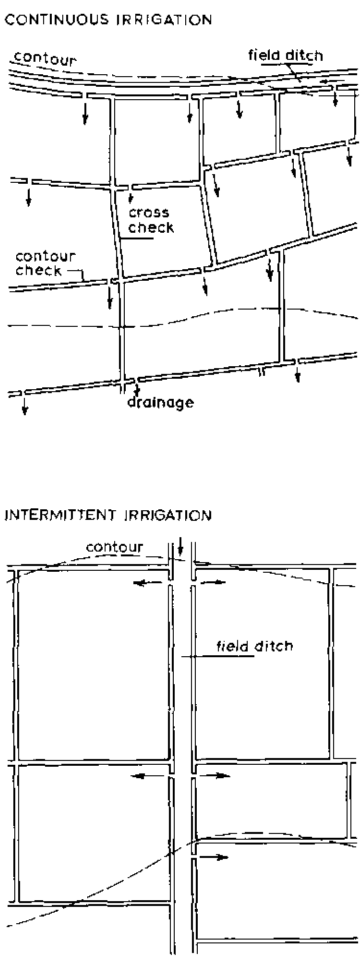
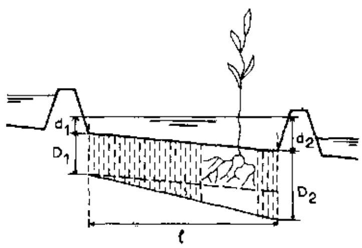


Fig.4. Lay-out of basins for continuous and intermittent irrigation.

If the soil is less permeable so that with intermittent irrigation the water remains on the field for a considerable number of hours, a certain loss may occur through evaporation. On a horizontal field, this will be the only loss if the right amount is supplied. Usually, however, the land has a slight grade or the surface is somewhat irregular, and differences in depth of application will then occur. If a sufficient cumulative infiltration is to be guaranteed at the highest points, excess infiltration will take place in the lower areas, resulting in a loss by deep percolation (Fig.5). If these percolation losses are considerable, irregularities in the ground surface should be smoothed out and slopes inside the basins should be levelled (Fig.6). Such work can be done by special land levelling equipment or, if necessary adjustments are only slight, the land can simply be ploughed in the right direction.
Apart from systematic over-irrigation caused by poor understanding of water control, it will be evident from the above that for basin irrigation the losses due to percolation depend on uniformity of application and, therefore, on the slope or the irregularities of the ground surface within the basin.

The slope of the basin does not have much relevance to percolation losses in paddy fields. Here a steady flow through the saturated zone



$$D_1 = \frac{100\ d_1}{\theta_{fc} - \theta_o} \qquad D_2 = \frac{100\ d_2}{\theta_{fc} - \theta_o}$$

$$e_a = \frac{\ell\ D_l}{\frac{1}{2}\ \ell(D_1 + D_2)} = \frac{2\ D_1}{D_1 + D_2} = \frac{2\ d_1}{d_1 + d_2}$$

D_l : depth rootzone

D_1, D_2 : depth wetted zone

θ : moisture content (vol.%) at field capacity (θ_{fc}) and initially (θ_o)

e_a : field application efficiency due to losses by irregular distribution

Fig.5. Intermittent basin irrigation on slightly sloping ground surface.

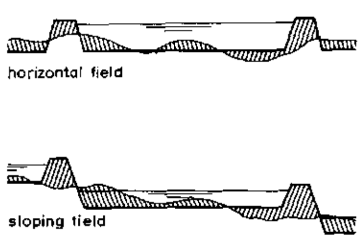


Fig.6. Landlevelling for basin irrigation (vertical scale exaggerated with respect to horizontal scale).

occurs, and losses are determined by the permeability of the soil layers. Nevertheless a horizontal basin floor is important for the optimum submergence of the developing plants as a uniform depth of water, varying with the growth stage of the paddy, should be maintained on the field. The effective control of the water depth on sawahs on rather steep slopes requires considerable quantities of earthmoving, and sometimes the construction of retaining walls (Fig.7).

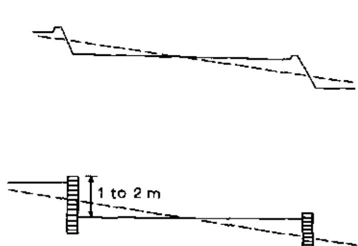


Fig.7. Cross-section of sawahs in upland areas.

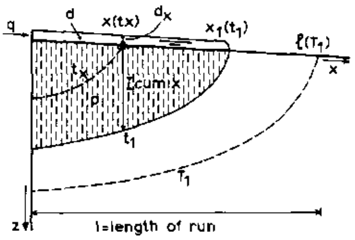
The losses on paddy fields depend primarily on the permeability of the controlling layer in the profile. In cohesive soils the farmers reduce these losses by puddling their fields (which is, in fact, a method of destroying the soil structure by wet ploughing and harrowing) before transplanting the paddy from the nursery fields. Other losses may occur through surface runoff from a basin complex due to a certain degree of over-irrigation, which is sometimes recommended: slowly flowing water lowers the water temperature and prevents the growth of algae.

For fields with slopes steeper than 0.2 or 0.3% and intermittent application, some form of flow irrigation will be applied. Flow irrigation covers all methods of field irrigation in which water moves under gravity till it reaches the point of infiltration. The different types of flow irrigation depend on the degree and the kind of remodelling done on the sloping natural ground surface.
For wild flooding - the primitive form of flow irrigation - only very limited land levelling, or none at all, is carried out. The water is released from a field ditch along a contour and another field ditch at the end of the run collects the runoff. As no lateral control exists and the fields remain irregular, flow concentrates at the lower spot and a deficiency occurs at the higher. Further, serious erosion may result. Losses are high due to both percolation and surface runoff. Wild flooding should not be applied in areas where a reasonable water control with acceptable field efficiency is being aimed at.

Controlled flow irrigation can be obtained by applying either border-strip or furrow irrigation.
For border-strip irrigation the field is divided into strips, 3 to 10 m wide, running in the direction of the slope. The strips are separated by low border checks, which prevent the flow from concentrating at low lying spots. The checks are usually 10-15 cm high and are constructed by a grader. The performance of the borders, from the water efficiency point of view, can be improved by levelling the slope to obtain a uniform gradient, and by cross-levelling the borders if a side slope necessitates this operation. Borders are suitable for crops like alfalfa, wheat, and sorghum, as well as for pastures.
Furrow irrigation is applied for vegetables, fruit trees, sugarcane and sugarbeet. Furrows are constructed according to the spacing of the plants and water infiltrates from the furrows both vertically and horizontally. The infiltration rate depends not only on the soil properties and the moisture content, but also on the size and shape of the furrow and the depth of the flow.

Border-strips and furrows can be applied on slopes of 0.3 to 2.0 or 4.0%, depending on the erosion sensitivity.

The principle of flow irrigation includes the fact that the depth of wetting over the length of the run will not be uniform. After the commencement of irrigation, the front of the water layer advances along the run and infiltration starts subsequently in the various parts of the run. The total advance time being T_1, it is only after that period that infiltration begins at the lower end of the run. This has to last for the required infiltration time T_i to T_3 (Figs. 8 and 9). The supply is cut off at the moment T_2 which, if properly chosen, allows the water to disappear from the surface at the moment T_3. This recession always takes less time than the advance: consequently the contact time at most of the length of run is in excess of T_i. The infiltration during the excess time represents a loss of water; since this quantity will finally percolate to the subsoil and reach the groundwater table, it will contribute to the drainage discharge. In order to keep the advance time as short as possible, and thus reduce the difference between advance and recession time, the supply per border or furrow should be as large as possible without causing erosion (Fig.9).



q : flow supplied to border per unit width
d : volume on surface per unit width
u : volume infiltrated into soil per unit width

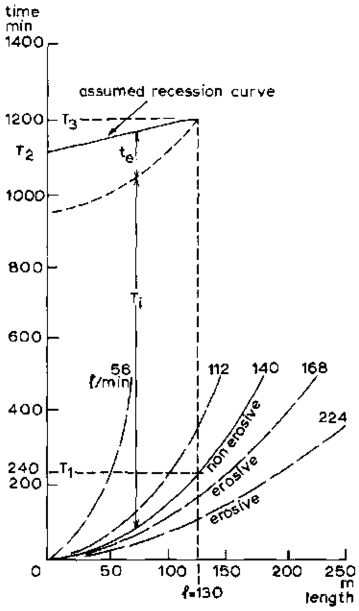
Fig.8. Advancing front of border irrigation.

As a rule of thumb, T_1 should be restricted in relation to T_i according to

$$T_1 \leq \frac{T_i}{4} \tag{16}$$

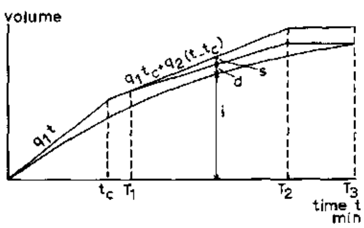
This restriction limits the length of run on a given field and with a maximum supply. On the other hand it is not recommended that the length of run be shortened much beyond this limit, since the field application would then require more man-hours.

Usually the supply is too large at the moment the flow reaches the lower end of the field, since no new surface is to be covered and the infiltration rate gradually decreases. Although a certain surface drainage will be unavoidable, it can be kept to a minimum by reducing the supply at the upper end, the so-called cut-back (Fig.10).



furrow slope $20^o/oo$
medium textured soil
depth of rootzone 120 cm
furrow spacing 105 cm
Q = 56 - 224 l/min
erosion limit 140 l/min
T_1 = advance time for selection of length of run ℓ
T_2 = supply time
T_3 = completion time
t_e = excess contact time
$d = D(\theta_{fc} - \theta_o) = 90$ mm
$T_i = 960$ min
$T_1 = \frac{1}{4} T_i = 240$ min
$\ell = 130$ m

Fig.9. Selection of stream size and length of run for furrow irrigation (CRIDDLE et al., 1956).



T_1 : advance time to end of run
T_2 : supply time
T_3 : completion of runoff
t_c : time of cut back of supply
i : infiltrated volume per unit width
d : volume on surface per unit width
s : runoff volume per unit width
q : supply to border per unit width

Fig.10.
Volumes of border irrigation as function of time.

As with basin irrigation, the efficiency of field application by borders and furrows can be improved by levelling, since the water will be more evenly distributed. The length of run is another important aspect: the shorter the run, the shorter the advance time, and the less difference in contact time between the upper and the lower part.
If T_i is the required infiltration time for the depth d

$$d = \frac{\theta_{fc} - \theta_o}{100} D = I_{cum}$$

According to Eq.15

$$T_i = \left[\frac{b + 1}{a} d \right]^{\frac{1}{b + 1}}$$

If $T_i \leqslant \frac{T_1}{4}$ the percolation losses can usually be kept within 15 - 20% of the applied quantity, so β will not exceed 0.25, and in most cases will be less. This value may increase rapidly if the indicated time limit is exceeded or if longer runs are installed, although these are, of course, labour saving for the farmer.

The surface losses for borders and furrows commence at the moment the tail end of the run is reached. As stated above, these losses can be reduced by cutting back the incoming flow at the proper time. It must be noted, however, that this reduction only represents a real improvement in field efficiency if the quantity thus saved is used at some other place. This may cause organizational difficulties for the farmers, who would in this way have to control various groups of runs, all with different starting and finishing times. For this reason cutting back the stream size is not common practice. Nevertheless, with moderate slopes and reasonably limited lengths of run, the surface runoff, even without cut-back, will only be 10 to 15% in most cases.

Losses in farm and field ditches are usually relatively low. To judge the total water management on the farm, the efficiency of conveyance by these ditches and of the application on the fields is taken together and defined as the farm efficiency. The farm efficiencies obtained with different field irrigation methods on various types of soil and with varying degrees of land preparation are given in Table 2.

Table 2. Average farm irrigation efficiencies (in %) for various field irrigation methods (KELLER, 1965).

Site	Borders	Furrows	Basins
1. Sandy soils			
well graded	60	40 - 50	70
insufficiently graded	40 - 50	35	n.a.
rolling or steep	n.a.	20 - 30	n.a.
2. Medium textures deep			
well graded	70 - 75	65	70
insufficiently graded	50 - 60	55	n.a.
rolling or steep	n.a.	35	n.a.
3. Medium textures shallow			
well graded	65	50	60
insufficiently graded	40 - 50	35	n.a.
rolling or steep	n.a.	30	n.a.
4. Heavy soils			
well graded	60	65	60
insufficiently graded	40 - 50	55	n.a.
rolling or steep	n.a.	35 - 45	n.a.

n.a. = not applicable

10.6 BENEFICIAL EXCESS IRRIGATION

Irrigation in excess of the requirements of evapotranspiration (and land preparation), but serving to control the plant environment, is not a loss of water in the strict sense. Included in environmental control are such things as: the limitation of the salt concentration in the soil, the protection of crops against night frost, the sustained saturation of the topsoil of rice fields, the flooding of fields as a protection against mice or weeds, or the conveyance of fertilizing or protective agents dissolved in water. These excess quantities, except for a usually small percentage that may evaporate, will contribute to the drainage dis-

charge. The amounts involved depend on the type and degree of control required, and should be estimated in each particular case, together with the respective components of surface and subsoil runoff.
As discussed in Chapter 9, Vol.II, the quantity of excess irrigation required for the regular leaching of soils in arid or semi-arid zones can be estimated fairly accurately provided sufficient data have been collected on trial fields. Irrigated areas without sufficient drainage have become seriously affected by salinization. Reclamation of such areas requires the installation of adequate drainage, followed by a leaching operation that may consume a considerable depth of water and may last one year or more. Neither the drainage and irrigation systems, nor the water resources, can be expected to cope if this operation is carried out over a substantial part of the area at the one time, and a gradual reclamation will therefore take place. The design criteria will consequently not be affected by this reclamation. When the level of salinization in the rootzone has been reduced to an acceptable value, or if no prior salinization has taken place due to the absence of irrigation, the quality of the soil moisture must be maintained by the application of additional percolating quantities at regular intervals.

In accordance with Eq.14 of the preceding chapter, the allowance α to be added to the net crop requirements for control of salinization over a given period is

$$\alpha = \frac{EC_i}{f(EC_{fc} - EC_i)} \tag{17}$$

where EC_{fc} is the electrical conductivity at field capacity (mmhos/cm at 25 ^{o}C) related to the electrical conductivity of the saturation extract, EC_s by

$$EC_{fc} = 2\ EC_s \tag{18}$$

The value of α in a given situation depends on the salt concentration in the irrigation water, EC_i, the leaching efficiency, f, and the salt tolerance, EC_s, of the crop (Fig.11). It is evident that in arid and semi-arid zones, where the effective rainfall is negligible and the evapotranspiration during part of the year is as high as 300 mm per month, any α value above 0.5 would involve a considerable drainage capacity.
The efficiency of the field application, disregarding any surface runoff, will be at its best when the percolation "loss" is in accordance with the required allowance of αE. Therefore this efficiency is maximum

$$e_{a\ max} = \frac{E}{E + \alpha E} = \frac{1}{1 + \alpha} \qquad (19)$$

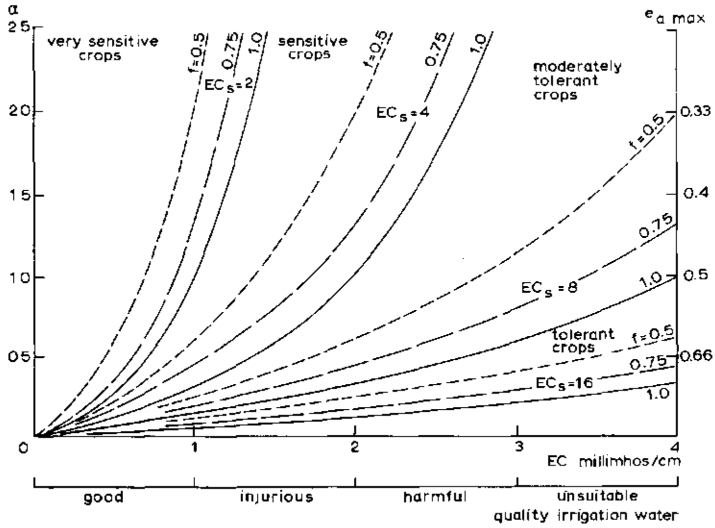


Fig.11. Leaching requirement as function of water quality, leaching efficiency and salt tolerance of crops.

As $e_{a\ max}$ depends on the same parameters as α, the values are shown at the right-hand side of Fig.11.

In practice it is not necessary to apply the additional fraction at each irrigation application. During periods of peak irrigation requirements and with limited water resources, the leaching can usually be postponed for some months, provided the assumed EC_s-values show some margin with respect to crop tolerance. After such a period, or after the harvest, one complete irrigation must be used to make up the deficiency in the leaching allowance.
The tolerance of crops under saline soils moisture conditions, as expressed in the assumed value of EC_s, is still subject to discussions. Most crops are moderately tolerant; some, like fruit trees and white clover, are sensitive. Sugarbeet and sugarcane are tolerant except for the germination stage, when they fall within the sensitive category. Barley and cotton are not affected by rather high soil salinity as far as their yields are concerned, although their vegetative growth may be hampered. On the other hand, the yields of rice and wheat are severely reduced by high soil salinity in the fruiting stage.

An important question is whether the more or less unavoidable field losses by deep percolation can be utilized for salinity control. As far as intermittent irrigation is concerned, we have seen in Section 10.5 that percolation losses occur because of non-uniform application depth. Under stagnant flooding this non-uniformity is caused by sloping or irregular ground surfaces. Under flow irrigation it is due to differences in contact time between the higher and the lower parts of the field. In both cases the minimum applied depth, without excess irrigation, corresponds with the wetting of the rootzone. A practical solution, although not an ideal one, is to add only a portion of the depth required for salinity control, or non at all, in such a way that a certain deficit is accepted over a restricted area. Thus part of the percolation losses will be used for salt drainage. The areas where the deficits will occur - and where some reduction in yields may be expected - will be located, for a stagnant flooding system, around the highest parts of the basin, and for a flow irrigation system, near the lower end of the runs (Figs.12 and 13).

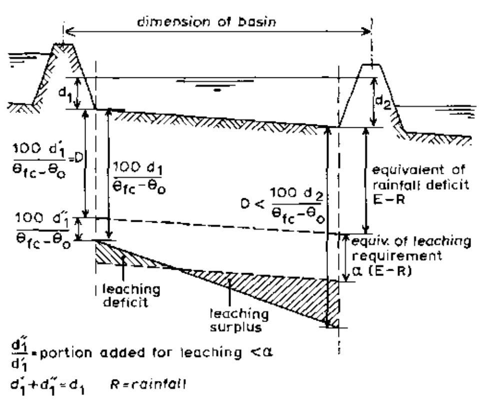


Fig.12. Leaching under sloping basin.

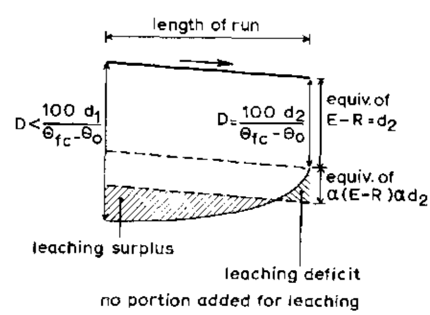


Fig.13. Leaching under border strip.

Under continuously flooded paddy fields, percolation will usually be in excess of the quantity required to balance the salt content of the irrigation water, while any salt deposits will have been removed by the puddling and submergence of the field before planting.

If the solution mentioned above is accepted - that of reducing the α-value and making part of the β-losses beneficial - strips of land may gradually become saline. In such a case the lay-out of the fields should be changed after some years. The new lay-out should be such that the affected strips will have excess percolation: with flow irrigation, for instance, affected strips would then be located at the upper end of the runs. In this way the salt content of the soil moisture can quickly be reduced to the accepted limit.

10.7 RE-USE OF DRAINAGE WATER

In many places where there is a substantial and continuous drainage discharge, the drainage water is used to irrigate an area adjacent to the region where it originates from. Depending on the slope of the land and the depth of the open drains, the water level can be brought above the ground surface at a shorter or longer distance from the drainage area. Raising the water level relative to the ground surface can be accomplished by a diversion structure in the main drain in combination with flat gradient channels, or by pumping. In this way the final effect of the sometimes considerable losses in one irrigation area can, at least partly, be reduced by applying this so-called return flow to another area in the vicinity.

In deciding whether the re-use is acceptable, the first question is: what is the quality of the drainage water? As with any other irrigation supply, the salt concentration should not surpass a certain limit, depending on the soil type, the crops, and the evapotranspiration-rainfall ratio (Section 10.6). In arid zones, where the limited rainfall will not contribute to the leaching of salts, this limit will be lower than in more humid areas.
The quality of the drainage water will depend on the quality of the original irrigation water, the portion that has passed the soil by deep percolation, and the salinity of the soil; the larger that portion and the higher the soil salinity, the less suitable will be the drainage water for re-use. The return flow is therefore generally fit for irrigation if a substantial part of it originates from surface runoff. The surface runoff, whether from rainfall, from field irrigation, or from excessive water escaping from the canal system, is much more irregular in quantity than subsoil runoff.
It follows that only a small part of the total drainage flow can be considered a dependable supply for irrigation. Furthermore, to avoid uncontrolled irrigation in the area of re-use, the peak drainage flow must pass the diversion site unobstructed.

Summarizing these different aspects, one can say that even if the major part of the discharge is surface runoff a direct connection between the drainage system of one area and the irrigation system of another is not recommendable in view of the sudden and substantial changes in flow.
The trend in modern designs is to convey the drainage flow out of an area by means of a main drain and to carry it back to a river, where the salts are diluted. Assuming that its quality for re-use is acceptable, the drainage water is then applied in irrigation further downstream.

10.8 DRAINAGE DISCHARGE CAUSED BY IRRIGATION

As to the magnitude of the drainage discharge as a consequence of irrigation, it should be noted that the most important and, in relation to different areas, most varying factors are α and β in Eqs.8 and 9.

For more or less humid zones α will be zero, while in arid and semi-arid zones, as stated in Section 10.6, $\alpha = 0.5$ will be a practical upper limit beyond which the feasibility of the irrigation project - with particular reference to the combination of soils, water quality, and crops - must be seriously reconsidered. For semi-arid zones, where, because of their available water resources, the bulk of subtropical irrigation is located, an average of $\alpha = 0.25$ can be assumed. Field losses as expressed by β vary widely owing to local differences. For basin irrigation on slightly sloping surfaces with reasonable control, β values from 0.2 to 0.3 can be assumed. For flow irrigation on borders or in furrows, and with the requirements fulfilled for lengths of run and controlled supply as discussed in Section 10.4, the value of β may range from 0.30 to 0.50.

The field efficiency e_a, resulting from the combination α- and β-values can be derived from the diagram of Fig.14. For the above limits of β for basin irrigation and $\alpha = 0.25$, e_a will vary between 0.6 and 0.7, and for flow irrigation between 0.5 and 0.6. From Section 10.3 a practical upper limit of 0.9 for e_c can be derived, assuming a flexible supply system with small operational losses. In this case the overall efficiency for basin and flow irrigation will be varying between 0.45 and 0.65, assuming the above indicated α- and β-values. However, if the operational losses are of the order of 20% of the total supply, as often happens, the overall efficiency may decrease to values as low as 0.35 or 0.5.

A reliable assessment of the values of the different factors involved, particularly α and β, can only be made by appropriate field experiments and trials on farmers' fields.

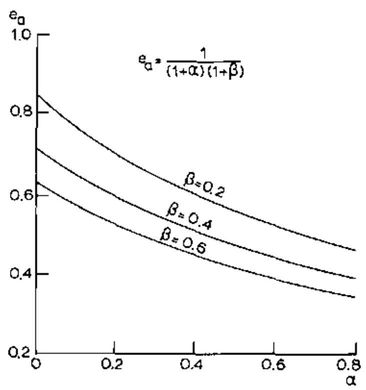


Fig.14. Relation between α, β and e_a.

From the above indicative values of the field and overall efficiencies, and from Eqs.6 and 7, it is evident that the drainage discharges in depth of water layer or per unit area are of the same order as the irrigation supply. It should be noted that the assumptions made in the foregoing only apply if the contribution made by rainfall is negligible or of minor importance.

In the case where field efficiency varies from 0.5 to 0.7, the drainage discharge, excluding canal losses, will range

$$0.45\ E < D_a < E$$

The discharge for a large area, including that portion contributed by conveyance losses in the canals and ditches, and assuming an overall efficiency from 0.45 to 0.65, results in an order of magnitude

$$0.5\ E < D_A < 1.2\ E$$

However, as has been stated, the overall efficiency may be as low as 0.35, particularly when the rotational supply system is poorly adjusted to crop requirement, or if field irrigation practices are below reasonable standards.

In such cases the overall drainage discharge over a specific period of time, e.g. the growing season, will be twice the net irrigation requirement.

In some areas the natural drainage capacity may be such that it can meet the needs of the drainage flow caused by subsoil runoff, whereas the surface drainage will require a system of field and collector drains, which must be constructed at the same time as the irrigation canals.

LITERATURE

CRIDDLE, W.D., DAVIS, S., PAIR, C.H. and SHOCKLEY, D.G. 1956. Methods for evaluating irrigation systems. Handbook No.82, U.S.D.A., Washington D.C., 24 pp.

KELLER, J. 1965. Effects of irrigation method on water conservation. J. Irr. and Drainage Div. ASCE *91*: 61-72.

POIREE, M. and OLLIER, Ch. 1968. Irrigation, les réseaux d'irrigation: théorie, technique et économie des arrosages. Editions Eyrolles, Paris, 405 pp.

Suggestions for further reading

HAGAN, R.M., HAISE, H.R. and EDMINSTER, T.W. (eds.). 1967. Irrigation of Agricultural lands. Am. Soc. Agron. Publ., Madison, Wisc. U.S.A., 1180 pp.

HOUK, I.E. 1951. Irrigation engineering. Vol.I: Agricultural and hydrological phases. Wiley, Chapman and Hall, New York, 545 pp.

HOUK, I.E. 1956. Irrigation engineering. Vol.II: Projects, conduits and structures. Wiley, Chapman and Hall, New York, 531 pp.

ISRAELSON, D.W. and HANSEN, V.E. 1962. Irrigation principles and practices. Wiley and Sons, Inc., New York, 467 pp.

RAADSMA, S. and SCHRALE, G. 1971. Annotated bibliography on surface irrigation methods. Int. Inst. for Land Recl. and Improvement, Bibliography No.9, 72 pp.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

11. FIELD DRAINAGE CRITERIA

J. KESSLER†

Land and Water Management Specialist
International Institute for Land
Reclamation and Improvement
revised by
J. W. VAN HOORN
C. L. VAN SOMEREN
J. H. BOUMANS

Lecturers in the Course on Land Drainage

J. Wesseling (1962–1966)
Institute for Land and Water Management Research

P. J. Dieleman (1967–1969)
International Institute for Land Reclamation and Improvement

J. Kessler (1970–1971)
International Institute for Land Reclamation and Improvement

J. H. Boumans (1972)
Land Improvement and Reclamation Company, Zeist

PURPOSE AND SCOPE

The drainage criteria for both rain-fed and irrigated areas are formulated in terms of required drain discharge, water-table depth control, and salinity control. The approach is based on both steady and unsteady state conditions.

CONTENTS

11.1 INTRODUCTION; FORMULATING THE DRAINAGE CRITERIA

The objective of field drainage is to prevent the occurrence of an excessive moist condition in the rootzone which, either directly or indirectly, has a harmful effect on the growth of crops and, moreover, to do so on a sound economic basis. In arid areas, a further objective is to prevent the accumulation of salts in the rootzone or to leach accumulated salts out of the soil profile.

Most crops require that the soil in the rootzone remains unsaturated. Controlling the groundwater level by drainage is a generally effective means of ensuring this condition.
It should be realized, however, that in soils with a restricted vertical water percolation, due to the occurrence of dense, impermeable soil, the problem may be an accumulation of excess water on the ground surface or a perched water table at some depth in the profile. In such conditions a soil cannot be adequately drained by simply lowering the groundwater level. This restricted effect of subsurface drainage on the moisture and salinity conditions in the rootzone should be kept in mind when field drainage criteria are being discussed.

Formulation of the drainage criteria

For steady state groundwater conditions the drainage formulas discussed in Chap. 8 Vol.II can be written in the general form:

$$L^2 = 8\ KD\ \frac{h}{q} \tag{1}$$

where KD stands for the soil medium, characterized by hydraulic conductivity, thickness, and position relative to drain level of the various layers discerned, and where the ratio h/q stands for the chosen combination of groundwater level and drain discharge required to prevent the occurrence of excess water in the rootzone.
The term h/q is thus the drainage criterion for steady state groundwater conditions.
For unsteady state groundwater conditions the drainage criteria cannot be expressed in terms of a fixed water table elevation with a corresponding fixed drain discharge. Instead, the criteria are formulated in terms of a required rate at which the groundwater table must be lowered. This can be seen by writing the modified Glover-Dumm drainage equation, discussed in Chap.8, Vol.II, as:

$$L^2 = \pi^2 \frac{Kd}{\mu} \frac{t}{\ln(1.16\ h_o/h_t)} \qquad (2)$$

where Kd/μ, characterizes the soil medium, and the term $t/\ln(1.16\ h_o/h_t)$ stands for the drainage criterion for unsteady state groundwater conditions.

Note: The symbol h in drainage formulas always refers to the groundwater elevation relative to drain level (available head), while the critical groundwater depth is defined relative to ground surface. The drain level therefore must implicitly be taken into account when a drainage criterion is chosen.

The appropriate choice of drainage criterion will depend on the following set of conditions:

- hydrological conditions, which determine the quantity of excess water to be drained within a specified time;
- agronomic conditions, which, depending on the crops and specific soil conditions, determine the permissible upper limit of the rootzone's soil moisture content and its duration;
- soil conditions, which determine the relations: between aeration and moisture content, groundwater level and soil moisture content, and groundwater level and capillary rise;
- economic conditions, which determine the cost-benefit ratio, i.e. the ratio between the costs of installing a drainage system and the benefits derived from less frequent and less severe yield depressions.

The complexity of the interrelation between all these conditions means that a drainage criterion should be regarded as no more than an attempt - although one based on empirical knowledge and theoretical reasoning - to express the aims of a future drainage system in a single value, e.g. h/q, which can be handled mathematically.

11.2 DESIRABLE DEPTH OF THE WATER TABLE

Before deciding on the required watertable control, one must first consider what objectives are being aimed for under the given specific conditions.

Broadly, the end in view will be one of the following (VAN BEERS, 1966):

- prevention of waterlogging outside the main growing season; its effect on crop growth will be indirect, and we might call it "soil drainage" or "off-season drainage";
- prevention of waterlogging during the main growing season; this will have a direct effect on crop growth and we shall refer to it as "crop-season drainage";
- prevention of salinization of the soil by irrigation or by capillary rise of groundwater, which will be referred to as "salt drainage".

11.2.1 OFF-SEASON DRAINAGE

In temperate, marine climates, precipitation excess usually occurs during the winter season only. This fact is obvious from the data on average monthly precipitation and evaporation for the central part of The Netherlands as shown in Fig.1.

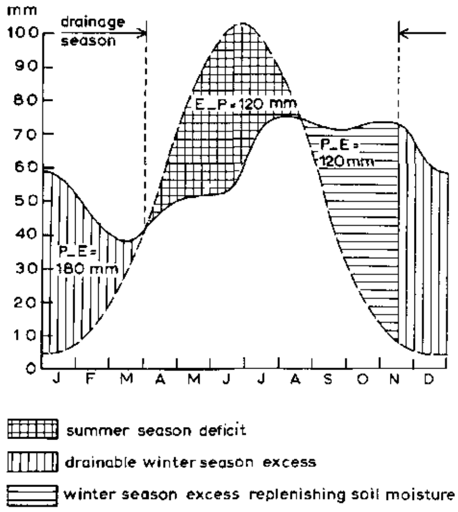


Fig.1. Average monthly precipitation and evaporation in The Netherlands

During the period from April through August the average evaporation exceeds precipitation by about 120 mm, while from September through March average precipitation exceeds evaporation by about 300 mm. Assuming a soil moisture storage capacity of about 120 mm after the summer deficit season and no irrigation, the average total quantity of water to be drained during the winter season will be 300 - 120 = 180 mm. Under the average climatic conditions in The Netherlands, an accumulated precipitation excess of 120 mm will be reached about mid-November, indicating the start of the drainage season.

Average climatic conditions, however, are enough to characterize the climate.

Frequency studies show that once in 5 years the summer precipitation deficit in The Netherlands is about 50 mm and almost nothing once in 10 years. In wet years this causes the drainage season to extend from roughly the end of August to the beginning of May, which is still outside the main summer growing period. High intensity rainfall does indeed occur in the summer, e.g. 70 mm over 5 days once in 5 years, but the soil moisture storage capacity is usually large enough to prevent an impermissible rise of the water table.
The conclusion then is that high groundwater tables occur during the off-season only, and the question may thus be raised why drainage is necessary. Chapter 4, Vol.I, presents a number of arguments in favour of drainage, such as the effect on land trafficability, on soil structure, on soil temperature, on nitrification and other microbiological activities.
Quantitative off-season data, from which a desirable watertable depth for different crops and soil types could be deduced, are not in plentyful supply. SIEBEN (1963) reporting on investigations on tile drained plots in part of the Lake IJssel polders in The Netherlands, related the yields of various crops, both autumn-sown and spring-sown, with the winter watertable depths expressed in SEW-30 values (Fig.2). SEW-30 means the sum of the daily values by which the groundwater table exceeds, during winter, a level of 30 cm below ground surface, midway between the drains, expressed in cm days. Sieben found that no harm is done to crop, soil, or general farm management if the groundwater table rises no higher than 30 cm below ground surface during winter. (Summer water tables were deep and did not interfere with yields, confirming the observations on a groundwater level experimental field published by VAN HOORN (1958).
Expressed in SEW-30 a value of 200 is given as a limit, below which no damage is found; for the Lake IJssel polders this value was equivalent to a steady state drainage criterion h/q equal to 30/7 (see Eq.1). Higher SEW-values had noticeably harmful effects, the damage depending on both frequency and duration of the exceedance (see Fig.2).

The following factors are listed to explain the effect of the off-season water table on crop development during the summer:

- high water tables mean that the soil is cold and wet; as a result, the seedbed is prepared under unfavourable conditions in early spring and/or planting is delayed;
- high water tables lead to some deterioration of soil structure and consequently to reduced aeration;
- insufficient aeration and relatively low temperatures mean that there is insuf-

ficient mineralization and nitrification, and nitrogen is thus in short supply (this effect can be partly compensated for by heavy nitrogen fertilization);
- uptake of other ions may be impaired, e.g. manganese.

It should be noted that the critical watertable depth of 30 cm found from the studies of Sieben refers to newly reclaimed sea-bottom soils of the Zuiderzee polders, which are ripened to a shallow depth only (see Chapter 32, Vol.IV). Reports from comparable studies on other soil types are unknown.

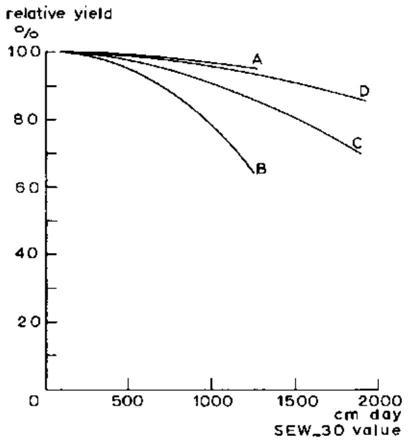


Fig.2. Relative yields of seed and the SEW-30 value on plots provided with tile drainage (after SIEBEN, 1963)

11.2.2 CROP-SEASON DRAINAGE

During the growing season the root system is developed to a much greater depth and is more vigorous than during the off-season. As a consequence adequate aeration in the rootzone is of primary importance and constitutes the first objective of drainage. Aeration requirements of crops and physical conditions of soils relative to aeration should now determine the desirable watertable depth during the various growth stages of the crop. Although many investigations have been carried out, only few data are available that can be used to quantify the drainage criteria.

From available data in Hungary SALAMIN (1957) compiled a tentative table giving the percentage yield reductions for a variety of crops when waterlogging occurs for periods of 3, 7, 11, and 15 consecutive days in any month. It is evident from his data that the yield reduction is much more severe if the period of waterlogging occurs in the middle of the growing season, than if it happens during the off-season. However, as waterlogging was defined by Salamin as complete inundation of the soil and no variation in soil conditions was taken into account, the data cannot lead to a formulation of the drainage criterion.

Experimental work in The Netherlands

VAN HOORN (1958) describes the results of a groundwater level experimental field in The Netherlands, with arable crops on a heavy marine clay soil. Over a period of 8 years the winter water table was kept permanently at 40 or 30 cm depth, whereas water tables were maintained in steps, ranging from 40 to 150 cm below ground surface. The yields of most of the crops on shallow summer water tables showed a decrease, due to insufficient aeration of the rootzone leading to poor root development and inadequate nitrification. Further, the soil structure of plots with a high summer water table gradually deteriorates, a condition disadvantageous for tillage operations. The same high water table during the off-season produced no adverse effects on the soil structure. Hence it is the combination of a high water table and cultivation operations that affects the structural stability of the soil, making the top layer sensitive to compaction and so leading to damage.

The effect of different groundwater levels on grassland, was investigated by MINDERHOUD (1960). His trials covered a period of 4 years and were conducted on an experimental field laid out on heavy river clay. Throughout the year the water tables in the various plots were kept constant at levels ranging from 40-150 cm below ground surface. The investigation made clear that on this soil type there is not one single depth at which a water table can be considered optimal for grassland throughout the year. Instead the best results are obtained with varying levels, depending on the prevailing weather conditions. In a dry summer a high groundwater level of only a few decimeters will be optimal for heavily grazed grassland. In a wet summer the same shallow depth may mean a reduction in net profit, not due to any decrease in the gross production of grass (which can indeed be rather good) but instead, due to losses resulting from the poor quality of the grass and from the deterioration in the soil's structural stability (compaction, puddling, poor trafficability) and especially in grazing and utilization of the grass. With these possible losses in mind, it will be clear that a watertable depth of 100 cm or more is to be preferred during a wet season. Recognizing that weather conditions can vary greatly from one year to another, Minderhoud came to the conclusion that the water table of intensively used grassland should be at least 60 cm deep in summer, whereas during winter a depth of

20-30 cm is acceptable. This choice means a compromise between optimal production and low management costs, while at the same time it is closely allied with the natural fluctuations in the groundwater level during the year. HOOGERKAMP and WOLDRING (1965) reported also on the relation between crop production and groundwater level from data collected at this experimental field. Their conclusions for the plots under grassland were the same as those published by Minderhoud. For arable crops they found the optimal groundwater table depth during the growing season to be 100-110 cm.

Lacking more conclusive data but basing our suggestions on generally accepted empirical values, we list in Table 1 the desirable watertable levels for grassland and field crops during the growing season.

Table 1. Recommended depth of groundwater table for Dutch conditions

Soil texture	Watertable depth which should not be exceeded for more than brief periods	
	Grassland	Field crops
Coarse	0.4 - 0.6 m	0.6 - 0.9 m
Medium	0.6 - 0.9 m	0.9 - 1.2 m
Fine	0.6 - 0.9 m	1.2 - 1.5 m

Although these values are valid for most crops, modifications can be introduced compatible with the specific tolerance of certain crops to different aeration conditions.

Note: The results obtained from experimental fields in The Netherlands show that the benefit of drainage is primarily attributable to its positive effect on tillage conditions and trafficability, rather than to any direct effect on crop production during the growing-season. In fact very often a farmer's decision to drain his field is based entirely on avoiding problems of practical management during autumn and spring, when harvesting and tillage operations and grazing may be impeded because of waterlogging. It is clear that management factors, though fundamental in assessing the economic results of drainage, are as difficult to quantify as the direct relation between crop yield and depth of groundwater table, this relation often being considered the only one that matters in the evaluation of the drainage.

11.2.3 SALT DRAINAGE

Moisture stress in the rootzone of irrigated land is an inevitable periodical occurrence. It will be encountered during a fallow period or some time after an application of irrigation water. This stress will cause an upward movement of water to the rootzone, bringing with it a certain amount of salts which add to those supplied by the irrigation water. This upward moisture and salt transport is related to the depth of the groundwater table.

Under neutral drainage conditions which means no natural drainage nor any underground supply of water emanating from adjacent high-lying areas, the groundwater table will drop rapidly in the fallow season as a result of evapotranspiration. This drop will occur to that depth at which the vertical moisture and salt transport becomes practically zero. This depth is called the critical depth.
Drainage projects, however, are often situated in relatively low-lying areas receiving a net subsurface inflow, which is generally salty and which in fact is drainage water of neighbouring soils. Under these conditions the critical depth will not be reached by evapotranspiration, and, if the water table is not kept at or below the critical depth by a drainage system, the upward transport of moisture and salt will continue throughout the whole non-irrigation season. It is for this reason that in irrigated areas drains should be installed at great depth, i.e. below the critical depth. The critical depth, which could roughly be defined as the depth at which the capillary upward transport becomes less than 0.5 mm/day

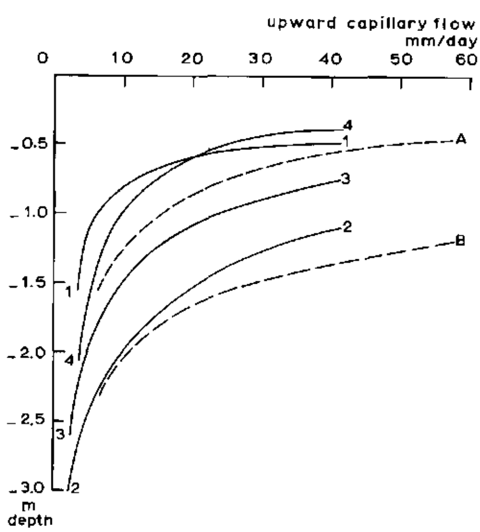


Fig.3. Relation between watertable depth and upward capillary flow

is not the same for all soils. Medium textured soils, with the relatively high unsaturated conductivity, have a greater upward transport than do coarse and heavy textured soils.
The relation between capillary salinization and watertable depth was studied by TALSMA (1963) for various soils in Australia (Fig.3). Other work in this field has been reported by WIND (1955), KOVDA (1961), and MARSHALL (1959).

11.3 CRITERIA FOR RAIN-FED AREAS

11.3.1 GENERAL CONSIDERATIONS

Although in the preceding sections we have spoken of the desired depth of the water table, it will of course be obvious that the water table may occasionally rise to much higher levels after excessive rainfall or an irrigation application. There are, therefore, two ways in which the drainage criterion can be formulated. It can be expressed either in terms of steady state flow, as the drain discharge rate required when the water table has risen to a certain depth below ground surface, or expressed in terms of unsteady state flow, as the fall of the water table required within a certain period after the water table has risen to near the surface.

The criterion depends upon the excess of water to be expected, upon soil conditions, crops, and the cost-benefit ratio of the drainage system. The benefits of a drainage system are difficult to calculate as drainage affects not only crop yield but also tillage conditions on arable land and grazing possibilities on grassland, all factors that come under the general heading of farm management. With the benefits of a drainage system already difficult to calculate, how much more true is this for the benefits of a change in the drainage criterion. What benefit for instance, would be derived in a certain case if the required discharge rate were increased from 7 to 10 mm/day?

For this reason drainage criteria have generally been established on the basis of field observations and farmers' experience. In this way Hooghoudt in The Netherlands observed that tillage conditions and yields were satisfactory on arable land that was tile drained at a depth of about 1 m and where discharge rates of about 5 mm/day were measured in combination with a water table at a depth of 50 cm. He then suggested that the drainage criterion for arable land be a discharge rate of 5 mm/day and a watertable depth of 50 cm, which for a drain depth of 1 m means a ratio h/q of 100 days.

Actually the following drainage criteria, expressed in terms of steady state flow, are utilized in The Netherlands (Table 2).

Table 2. Drainage criteria used in The Netherlands

Land use	Discharge rate q in m/day	Watertable depth in m	h in m (drain depth 1 m)	Ratio h/q in days
grassland	0.007	0.30-0.40	0.70-0.60	100-85
arable land	0.007	0.40-0.50	0.60-0.50	85-70
newly reclaimed Lake IJssel polders	0.007-0.010	0.30	0.70	100-70
orchards	0.007	0.50-0.70	0.50-0.30	70-40
bulb fields	0.010	0.50	0.50	50
vegetables	0.007	0.60-0.70	0.40-0.30	60-40
glasshouses	0.020-0.030	0.40	0.60	30-20

These criteria are widely applied in The Netherlands, without taking an area's specific topography into account. However, when it is evident that there will be seepage inflow into the area and this will not be (fully) intercepted by the system of open watercourses, such inflow will be kept in mind when the discharge criterion is being chosen. On the other hand, seepage losses from the area can be a reason for accepting somewhat lower values for the criterion.

A drainage system based on the above criteria will result in water tables which are at or below drain depth during the growing season April-September. In the winter period November-March the water table will be above drain depth and discharge of excess water will take place. The water table may rise to a shallow depth for short periods without harming the crop or interfering with good farm management. It is clear from this table that with a drain depth of 1 m a drainage criterion comprising a discharge rate of 0.007 m/day when the water table is 0.50 m deep expressed the same degree of groundwater control as a criterion comprising a discharge rate of 0.010 m/day with a water table depth of 0.30 m, since the ratios h/q are the same.

France, Belgium, and North-West Germany apply virtually the same drainage criteria as those listed in Table 2. England works on the basis of the drainage criteria which only serve to calculate the drain diameter, and are related to the annual rainfall (Note Min. of Agriculture United Kingdom, 1967). (See Tab.3).

Table 3. Design Drainage Rates for subsurface drainage in relation to precipitation in the United Kingdom

Mean precipitation mm/year	Design Drainage Rate (q) for underground drainage			h/q in days
	mm/day	inch/day	specification	h = 80 cm
2000	25	1.0	normal sites	30
1500	19	0.75	normal sites	40
1000	13	0.5	normal sites	60
875	10	0.4	normal upland sites	80
<875	7.5	0.3	watertable control areas (other than peat soils)	105
<875	6.5	0.25	deep peat fens (> 60 cm peat)	120

No depth criteria are given, the discharge requirements are the maximum quantities to be evacuated by the drainage system, i.e. for a groundwater table at or near to the surface. A comparison of the English norms for the less than 875 mm precipitation class with the Dutch criteria, shows that the h/q values are of the same order for h=80 cm, i.e. a drain depth of 1 m and a water table at 20 cm below the surface.

Drainage criteria as presented in Table 2, are not used in other parts of Germany and some countries of Eastern Europe; the approach to drainage design rests on the purely empirical basis of a direct correlation between soil type (mainly characterized by its texture classes), land use and topography, as against the required drain depth and drain spacing. Usually this design basis refers to soils with a poor structure and with a limited infiltration capacity as compared with the rainfall intensity. Such soils will have a considerable surface drainage, thus reducing the need for groundwater drainage.

As already stated, the criteria listed in Table 2 are used almost everywhere in The Netherlands, without modifications being made for differences in drain depth or in the soils or drainable pore space. Although using one and the same criterion for different drain depths and drainable pore spaces will theoretically result in different watertable elevations, these differences may in practice be rather small. To illustrate, if we take the drainage criterion comprising a discharge rate of 0.007 m/day with a watertable depth of 0.40 m and two drain depths, 0.80 m and 1.20 m, the ratio h/q for the 80 cm drain depth will be two times as small, which means a sharper drainage criterion. Counterbalancing the advantage of a sharper drainage criterion, however, is the fact that the amount of water which can be stored in the soil profile between drain depth and a depth of 40 cm is also two times as small.

In areas with a continental climate characterized by high intensity summer showers, such as the central states of the U.S.A., temporarily high water tables, even reaching the ground surface, cannot be prevented. The drainage criteria are then formulated as the required rate of fall of the water table after the occasional occurrence of high intensity rainfall and a water table rise to near the ground surface. A much used criterion is that the water table should drop from the ground surface to at least 30 cm in 24 hrs and to 50 cm in 48 hrs (KIDDER and LYTLE, 1949). When the water table rises to some 15 cm below ground surface, it should drop to 35-40 cm in one day (NEAL, 1934; WALKER, 1952).

It is interesting to compare the steady state criteria applied in The Netherlands with the falling watertable criteria applied in the U.S.A.

For the drainage criterion comprising a discharge rate of 0.007 m/day when the watertable depth is 0.50 m, the hydraulic head (h) will be 50 cm if the drain depth is 1 m. If the water table rises to the surface, both the discharge rate and the hydraulic head will increase two times[+)], becoming resp. 0.014 m/day and 100 cm. For clay soils with a drainable pore space of 4-5%, this would result in a drop of the water table from ground surface to about 30 cm in one day, which corresponds to the U.S.A. standard. For medium textured and sandy soil, however, with a drainable pore space of more than 5%, the two criteria are no longer comparable: the U.S.A. standard would require a higher discharge rate than the Dutch.

+) This ratio is based on the simplified linear relation given in Eq.1. Actually the relation shows an increase of the discharge rate, which is more than linear proportional with the rise of the water table. This is due to the second term in the formula (see Chap.8, Vol.II) and the generally increasing value of the hydraulic conductivity in the more shallow soil layers.

11.3.2 USE OF THE UNSTEADY STATE APPROACH IN ESTABLISHING DRAINAGE CRITERIA

After formulas for unsteady state flow became available, it was possible to calculate water table and discharge hydrographs for a statistically determined design rainfall or for the actual rainfall records covering many years, and then to draw conclusions from these hydrographs concerning the frequency of watertable elevations and the required discharge rates.

VAN HOORN (1960) made use of a design rainfall for a selected critical rainfall period, which was derived from the rainfall depth-duration-frequency curves as determined from a statistical analysis of the rainfall record (Chap.18, Vol.III). An example of such depth-duration-frequency curves is given in Fig.4.

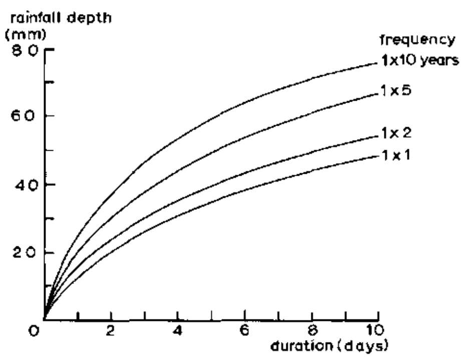


Fig.4. Example of rainfall depth-duration-frequency curves

From the frequency curve of once in 5 years, it can be seen that over a period of 1, 2, 3 and 4 days a rainfall total of respectively 20, 30, 38 and 44 mm can be expected. From this a daily rainfall sequence of 20 mm for the first day, 10 mm for the second day, 8 mm for the third day, and 6 mm for the fourth day could be deduced; but any other distribution, making up a total of 44 mm in 4 days, could be assumed too.

Using the unsteady state formula of Kraijenhoff van de Leur (see Chap.8, Vol.II) van Hoorn calculated the watertable elevations for:

- seven-day rainfalls occurring with a frequency of once in 1, 2, 5, and 10 years, preceded and followed by a constant rainfall of 2 mm/day;
- a drainable pore space of 0.035, which was appropriate for the basin clay soil under study;
- an average drain depth of 90 cm, being determined by the presence of a good permeable layer at that depth;

- various drain spacings L as they would have been calculated with a steady state formula using drainage criteria of respectively 2, 3, 5, 7, 9 and 11 mm/day discharge when the watertable depth is 20 cm.

The water table hydrographs - an example of which is presented in Fig.5 - showed that in the case of the seven-day rainfalls the following water tables would be attained:

Discharge rate in mm/day of drainage criterion	Seven-day rainfall having a frequency of 1× year	1×2 years	1×5 years	1×10 years
2-3	0 cm	0 cm	0 cm	0 cm
5	20	10	0	0
7	32	22	8	0
9	40	32	20	0
11	48	40	22	10

On experimental plots and tile-drained fields it was observed that a drainage criterion comprising a discharge rate of 7 to 9 mmm/day with a watertable depth of 20 cm corresponded with good grazing possibilities on grassland during wet periods. According to the data in the table above, such a criterion expressed in steady state terms also corresponded on this soil with a water table rising into the turf layer about once every two to five years.

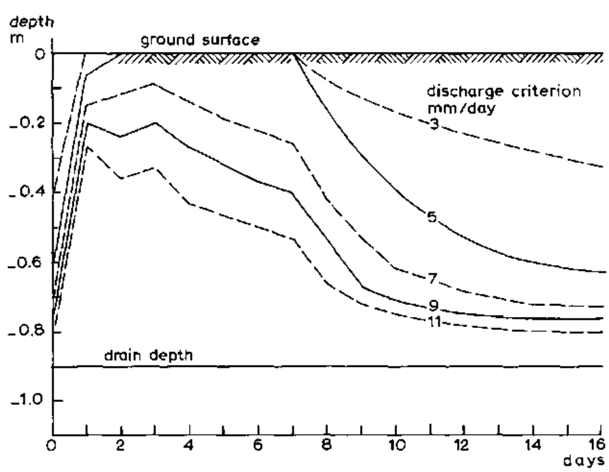


Fig.5. Example of watertable hydrographs computed for different discharge criteria in case of a seven-day's precipitation having a frequency of 1×5 years (after VAN HOORN, 1960)

SEGEREN and VISSER (1971) described the effect of different drainage intensities on apple orchards on clay loam soils in the Lake IJssel polders. They found that the roots of apple trees can withstand high groundwater levels of 6 weeks duration during winter, but in summer noticeable damage occurs after even one week of submergence. This damage consists of the decay of young roots and a decreased uptake of nutrients. An adequate criterion is that after a period of excess rainfall in the growing season, the groundwater should drop below the rootzone (about 1 m-ground surface) within one week. High, short-duration groundwater levels of up to 40 cm-g.s. appear to have no appreciably damaging effect. The unsteady state criterion can be expressed as a fall from 40 cm to 100 cm within 7 days, the drain depth being 110 cm below surface. This corresponds to a steady state criterion comprising a discharge rate of 10 mm/day with a watertable depth of 60 cm, the ratio h/q being 50 days.

A disadvantage of statistically determined design rainfall is that the characterization of precipitation by depth-duration-frequency curves is incomplete because it lacks information about preceding and subsequent rainfall, and because it does not specify the actual distribution of daily rainfall within the period of heavy precipitation.
The availability of computers nowadays makes it possible to overcome this disadvantage and to obtain watertable hydrographs from long-term rainfall records and for specified conditions of drainable pore space, drain depth, and the reservoir coefficient, j, in the unsteady state formula (remember that $j = \mu L^2/\pi^2 Kd$). The daily watertable elevations can subsequently be subjected to a statistical analysis. In this way VAN SCHILFGAARDE (1965) and DE JAGER (1965), obtained frequency distributions of predicted watertable elevations.
Van Schilfgaarde found, for example, that for a certain soil in North-Carolina and for a certain reservoir coefficient, the water table can be expected to rise 51 times in 25 years (about twice per year) to 45 cm above drain level for 48 hours or more, and about three times per year for 24 hours or more.
After a drain depth has been selected, these watertable elevations can be translated into watertable depths. Then it is up to the agronomist and the economist to tell the design engineer which hydrograph or frequency distribution is acceptable from the standpoint of crop production and farm economics. Unfortunately, however, little is known at present of crop response to varying watertable elevations. The approach taken by SIEBEN (1963) to characterize the varying watertable elevations by a single value (see Chap.4, Vol.I, and Sect.11.4.1 of the present chapter) may lead to practical results, as shown by BOUWER (1969).

Recently WESSELING (1969) elaborated the results of the frequency analysis of watertable elevations computed by de Jager from the rainfall records of the winter period 1913 to 1963 at De Bilt, The Netherlands. These computations were made with a computer for various reservoir coefficients, j, introduced in the unsteady state equation derived by Kraijenhoff.

Wesseling's study allows the following conclusions to be drawn:

- The drainage criterion of 7 mm/day drain discharge with a water table depth of 50 cm results in watertable depths that are reached or exceeded with varying probabilities, depending on the drainable pore space. If we take, for example, the watertable depths of respectively 50 cm, 25 cm, and 0 cm (i.e. at ground surface), we find, for a drain depth of 1 m and different values of the drainable pore space, the following probabilities of exceedance:

drainable pore	watertable depth		
space μ	50 cm	25 cm	0 cm
0.02	10 × year	5 × year	2 × year
0.05	5 × year	1 × year	1 × 5 years
0.10	2 × year	1 × 6 years	1 × 20 years

- When we assume as a representative value for the drainable pore space of the soils in The Netherlands, $\mu = 0.05$, the drainage criterion can apparently also be defined as a water table at 25 cm below ground surface being reached or exceeded with a probability of once per year (1 × year). To meet this requirement the drain-discharge rate, q, of the steady state drainage criterion should be modified according to the actual drainable pore space as follows:

μ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
q mm/day	19.0	12.0	9.5	8.0	7.0	6.5	6.0	5.5	5.0	4.5

The effect of drainable pore space on required discharge rate is already known from practice, where the discharge rate in sandy soils in sometimes reduced to 4 à 5 mm per day.

- The influence of the drainable pore space can be explained by the proportional change in available storage of groundwater in the soil. For the same reason the drain depth will have an influence on the drainage criterion.

It can be deduced from the data presented by Wesseling that for a soil with a drainable pore space of 0.05, the probability that a watertable depth of 50 cm will be reached or exceeded decreases from 10 × year to respectively 5 × year, 2 × year and 1 × year when the drain depth is increased from 0.7 m to respectively 1.0 m, 1.5 m and 2.0 m. For all four drain depths the same drainage criterion (7-50) was used. If it is decided that the probability of exceedance of a depth of 50 cm should not be more than 5 × year, as is the case with the drain depth of 1.0 m, the conclusion is that the drain-discharge rate could actually be decreased for drain depths of more than 1 m, and should be increased for drain depths of less than 1 m, according to the following estimates:

drain depth (m)	0.7	1.0	1.5	2.0
q mm/day	10.5	7.0	5.5	4.5

Although lower values can be taken for the discharge rate if either drainable pore space or drain depth increases, these differences in discharge rate will have relatively little influence on the drain spacing.

To illustrate, let us take discharge rates of 9, 7, and 5 mm/day, the value of 7 mm/day corresponding to the drainage criterion for average conditions. Since the drain spacing is inversely proportional to the square root of the discharge rate, an increase of the discharge rate from 7 to 9 mm/day means a drain spacing which equals 0.88 times the spacing corresponding to a discharge rate of 7 mm/day. A decrease of the discharge rate from 7 to 5 mm/day means a drain spacing which equals 1.18 times the spacing corresponding to a discharge rate of 7 mm/day.

It is evident that differences in drain spacing due to a change in the discharge rate of the drainage criterion as a result of a deviation from average conditions are relatively small in comparison with differences due to the heterogeneity in the KD-value of the soil.

When the unsteady state approach is used in establishing drainage criteria, the following procedure may be suggested:

1) Calculate by computer the actual groundwater storage above drain level from the original record of daily rainfall over many years and for a number of alternative "j" values which characterize alternative drain intensities ($j = \mu L^2/\pi^2 Kd$).

Any reduction in the rainfall reaching the groundwater, due to surface runoff or storage in the soil moisture reservoir, should be introduced before the data are fed to the computer.

2) Translate the calculated changes of groundwater storage into fluctuations of groundwater height above drain level - i.e. groundwater table hydrographs - by introducing the appropriate value for the drainable pore space (μ).

3) Characterize for the critical drainage period the relation between groundwater level and crop yields, tillage conditions, or grazing possibilities, in the way Sieben has suggested, by means of the Sum Exceedance Values of a chosen groundwater depth.
It is also conceivable that the relation be characterized by a permissible frequency of exceedance of a certain groundwater depth (e.g. 1 × year for a groundwater depth of 25 cm, or 1 × 5 years for a groundwater table reaching the ground surface). It should be remembered that such a characterization also has to take the soil type into account.

4) Find for any selected drain depth and from the groundwater-table hydrographs produced by the computer, which "j" value meets the conditions of groundwater-depth control as given under 3).
When required, this "j" value can be expressed in the ratio h/q, by substituting the relevant value of μ into the equation $\pi^2 j/8\mu = h/q$. The ratio h/q can then be introduced as drainage criterion in the available steady state drainage formula.

11.4 CRITERIA FOR IRRIGATED AREAS

11.4.1 DISCHARGE RATE AND WATER TABLE

In arid areas irrigation practices determine the volume and rate of groundwater recharge. When irrigation water is applied, this is always coupled with water losses. Such losses can be divided into intentional and unavoidable losses.

Intentional losses, which are required to maintain a favourable salt balance in the root zone, percolate through the root zone to the groundwater and have to be removed by sub-surface drainage.
Unavoidable losses result from canal seepage and from field applications, which cannot be made with a 100% efficiency. The canal seepage or conveyance losses and part of the field losses will recharge the groundwater. Any losses resulting from spill or surface waste will be taken care off by a surface drainage system.

This chapter being restricted to subsurface drainage, the losses that have to be considered in its context are: conveyance losses, unavoidable percolation losses, and intentional losses to meet the leaching requirements.

Conveyance losses

The magnitude of the conveyance losses may range from 5% for lined canals to as much as 50% for unlined earthen canals used intermittently in a rotation system. Conveyance losses can be measured in areas already being irrigated, while for new irrigation-drainage schemes they can be estimated on the basis of a comparison with already irrigated areas or on calculations of the expected infiltration rate through the canal bottoms and side slopes. If an impermeable layer is found at shallow depth, the bulk of the losses may be intercepted by an interceptor drainage system running parallel to the irrigation canals. If no impermeable layer is present, the conveyance losses should be regarded as a steady recharge averaged over the whole irrigated and drained area.

If conveyance losses are high, e.g. more than 20%, canal lining should be seriously considered, as it will reduce, or may even solve, the drainage problem; the improved water economy and the beneficial effect on the soil's salt balance are further arguments in favour of canal lining.
Exemplifying these arguments is the Beni Amino irrigation scheme in the Tadla region of Morocco, where shallow water tables and waterlogged conditions were found to be mainly caused by excessive canal losses, which were as high as 50% in certain sections. Lining the canals solved the drainage problem entirely, as the natural drainage conditions were sufficient to cope with the normal field losses and the leaching requirements (TADLA report, 1964).

Percolation losses

Field application losses include surface and percolation losses. Percolation losses vary considerably depending on the soil type, the degree of levelling, the lay-out of the scheme, and the skill of the operator. Of the total amount of irrigation water supplied during a cropping season, 30 to 40% may be considered a reasonable estimate of losses for gravity irrigation; for sprinkler irrigation this percentage may be taken at 25%.

These losses are not uniformly distributed over the growing season, the percentage being higher during the initial growing stages and lower during later growing periods when the root system is well developed. This results in a more or less constant value of the monthly percolation losses expressed in water depth. The daily rate of those losses to be accounted for in the discharge design criterion can therefore be expressed as: 30 to 40% of the total crop irrigation divided by the number of drainage days in the case of gravity irrigation. The field application efficiency (e_a) is the ratio between the amount of water stored in the rootzone and required for evapotranspiration (E) and the amount of water delivered to the field (I_d):

$$e_a = \frac{E}{I_d}$$

The total water losses are:

$$I_d - E = I_d(1 - e_a)$$

The total losses consist of surface waste and deep percolation (R) below the rootzone. If α is the fraction lost by deep percolation, then

$$R = \alpha\ I_d(1 - e_a)$$

If for average conditions $e_a = 0.50$ and $\alpha = 0.7$, then

$$R = 0.35\ I_d$$

and surface waste will be in the order of 0.1 I_d.

Intentional losses for leaching

The leaching requirement R^*, can be calculated with the formulas and procedure expounded in Chapter 9.

Subsoil supply of foreign water

Irrigation areas are often situated in valleys and basin areas whose groundwater is continuously or seasonally fed by underground flow from higher-lying areas. In fact, this underground flow represents the natural drainage of the higher areas, resulting from rainfall or irrigation. This foreign drainage water, which is often salty, should be accounted for in the discharge criterion; it should be added to the discharge rate already required for losses and leaching.

Except in those areas on the receiving end, natural drainage, even in very limited quantities, is a favourable phenomenon. It reduces the required discharge rate and eliminates the risk of resalinization during the non-irrigation season.

A quantitative evaluation of subsoil supply of foreign water or of natural drainage is difficult to assess without very intensive and costly hydrogeological investigations. Estimates can be made on the basis of differences in groundwater depth, preferably maximum depth at the end of the dry season, and of differences in groundwater salinity. Shallow depths and high salinity indicate usually subsoil supply of foreign water; great depths and low salinity may indicate natural drainage.

Usually the amount of irrigation water required for leaching will be less than the percolation losses. So the question arises as to the effectiveness of the percolation losses in leaching the soil. In other words, is it the larger of the two losses that determines the recharge of excess water to the groundwater or is it the sum of the two?
Percolation losses will constitute effective leaching only if they occur uniformly over the field. This may be so in basin irrigation if the stream size is well attuned to the soil infiltration rate, but a uniform distribution will usually not occur in borderstrip flooding, furrow irrigation, and other surface irrigation systems (ISRAELSEN and HANSEN, 1962).
The general practice, however, is to omit the intentional additional water application for leaching when the unavoidable deep percolation losses are already of the same order as the leaching requirement (see also Chap.10, Vol.II).

Watertable depth during irrigation period

During the growing season, the water table should in general be kept below the rootzone of the crops, which for well-developed annual crops means about 1 m below surface.
A distinction should be made between the permissible depth of the water table when steady state formulas are used and the permissible depth of the water table when an unsteady state approach is used. In the first case we are concerned with the average water table during the irrigation season and one should choose as permissible depth the depth of the rootzone. In the second case a somewhat higher level may be chosen immediately after irrigation as the water table does not remain at this level for long.

Watertable depth during fallow period

If there is no subsoil supply of foreign water the groundwater table will drop during the fallow period to drain depth or, as a result of evaporation, to the critical groundwater depth if this depth be the greater. No special groundwater depth requirements have to be met under these conditions. If there is a subsoil supply of foreign water, however, the water table will not drop automatically to the critical depth during the fallow period but will have to be kept at or near this depth by means of the drainage system, in order to reduce to a minimum value the continuous capillary rise of groundwater to the surface. Under these conditions the drainage design has to meet not only the crop season criteria, but likewise the fallow season criteria, the latter being the required groundwater depth at or near to the critical depth and a discharge rate equal to the rate of underground supply of foreign water.

When considering the choice of discharge rate and hydraulic head for the drainage criterion, one should not forget that the drain spacing is inversely proportional to the square root of the discharge rate and proportional under normal conditions to the square root of the hydraulic head. Taking into account the approximation in the formulas, the inaccuracy of the KD-value and the fact that in practice it is a matter of calculating an order of magnitude for the drain spacing, a change in the discharge rate or in the hydraulic head within certain limits is of relatively little importance.

Examples of steady state drainage requirements applied in irrigation projects

Tunisia, Medjerda valley (MEDJERDA report, 1961)
2 mm/day, watertable depth 1 m.
Crops and cropping intensity adapted to high salinity of irrigation water,
EC_i 3 à 4 mmho/cm.

Algeria, Habra valley (HABRA report, 1971)
General requirement 2 mm/day, depth 0.80 m.
The discharge correction for natural drainage and supply of foreign groundwater, this correction being related to depth and salinity of groundwater, is in this already irrigated area as follows (see Table 4).

Morocco, Sebou valley (SEBOU report, 1970)
Medium and light textured soils 1.8 mm/day and 1 m depth.
Heavy textured soils 1 mm/day and 1 m depth.
Reduction heavy soils related to lower irrigation intensity and low infiltration rate.

Imperial valley, U.S.A. (ISRAELSEN, 1950)
1.6 mm/day, no depth mentioned.

Nile Delta, Egypt (U.A.R. report, 1965)
Heavy clay soils, good quality water 1 mm/day, groundwater depth 0.50 m.

Table 4. Discharge correction (mm/day)

Depth class	Salinity class		EC mmhos	
water below ground level	< 4	4-8	8-16	> 16
< 0.5	0	+ 0.5	+ 1.0	+ 1.5
0.5 - 1.0	- 1.0	0	+ 0.5	+ 1.0
1.0 - 2.0	- 2.0	- 1.5	- 1.0	- 0.5
> 2.0	- 2.0	- 2.0	- 2.0	- 2.0

11.4.2 THE FORMULA FOR UNSTEADY STATE COMPUTATIONS

Drainage computations for irrigated areas can be made with steady state drainage formulas. However, the use of unsteady state formulas offers an interesting approach to what may actually happen in practice. This approach is discussed in the next paragraphs.

The modified Glover-Dumm formula as presented in Chap.8, Vol.II, will be used for the computations. Fig. 6 shows the geometry and symbols utilized. The formula is written as follows:

$$L^2 = \frac{\pi^2 \ Kd \ t}{\mu \ \ln(1.16 \ h_o/h_t)} \tag{4}$$

where:

- L = drain spacing (m)
- K = hydraulic conductivity (m/day)
- d = equivalent layer of Hooghoudt (m)
- μ = drainable pore space of the soil
- h_o = watertable height above drain level just after an instantaneous recharge due to irrigation (m)
- h_t = watertable height above drain level just before an instantaneous recharge due to irrigation (m)

t = length of period between two irrigations (days)

The values for h_o, h_t, and t specify the drainage criterion.
The maximum value for h_o can be deduced from the watertable depth requirements during the crop season and h_t is then computed from the relation:

$$h_o = h_t + R_i/\mu$$

where R_i is the instantaneous recharge (in m) occurring with a frequency of once every t days.
The nomographic solution of the relation between h_t/h_o and $Kdt/\mu L^2$ as published by DUMM (1960) is given in Fig.7.

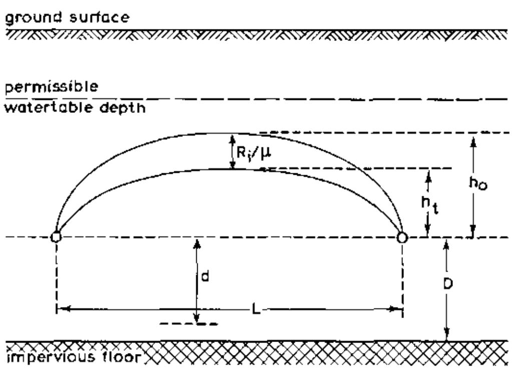


Fig.6. Geometry and symbols used in the modified Glover-Dumm formula

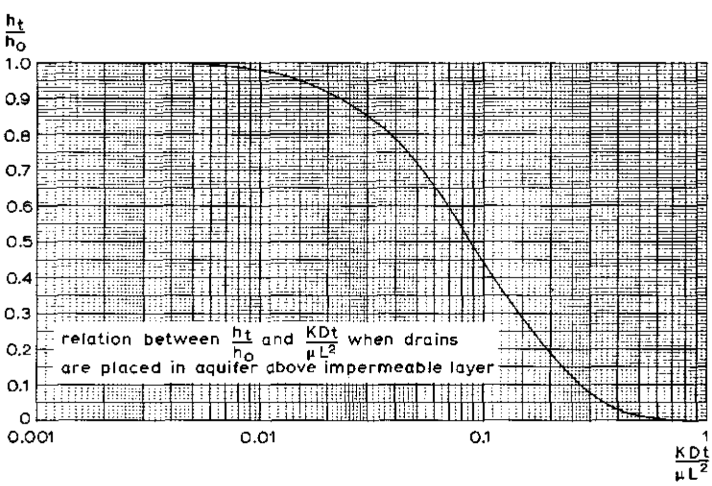


Fig.7. Nomographic solution of the modified Glover-Dumm formula (DUMM, 1960)

11.4.3 COMPUTATION FOR PEAK IRRIGATION PERIOD

The interval between two irrigations is shortest during the peak irrigation period. If we assume the same recharge to the groundwater at each irrigation, the groundwater table will consequently reach its highest elevation during this period.

We shall assume for the computation that the instantaneous recharge of each irrigation application must be drained out completely before the next irrigation is due, and that the water table reaches its permissible level after each irrigation (Fig.8).

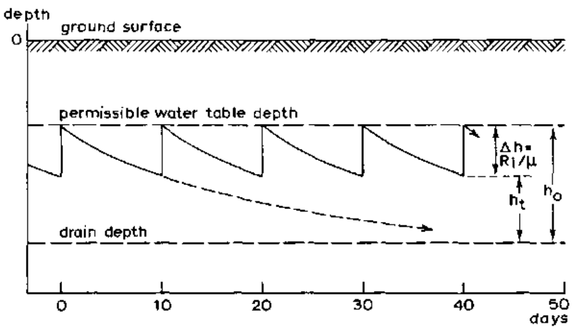


Fig.8. Watertable hydrograph during peak irrigation, with ten days between irrigations

The value for h_o to be introduced in the formula thus equals: the depth of the drain minus the permissible watertable depth.

The value for h_t then equals: $h_o - R_i/\mu$.

To illustrate how a computation is made and what data are required to do so, we give the following example.

The specific conditions and data for the area under consideration are listed below:

climate
- evapotranspiration (E) for peak season: 225 mm/month or 7.5 mm/day
- rainfall: negligible.

groundwater
- incoming foreign water (artesian, seepage): none
- natural drainage: negligible
- capillary risee: negligible during peak irrigation season.

soil

- soil texture: silty clay loam
- characteristic moisture contents (on volume basis):

  saturation percentage (w_e) = 65%

  field capacity (w_{fc}) = 36%

  wilting point (w_{wp}) = 16%
- drainable pore space (estimated): μ = 0.1 or 10%
- hydraulic conductivity:

  of the upper 4 m of soil: K = 1m/day

  below 4 m (tight clay): considered impervious
- leaching efficiency (estimated for silty clay loam): f = 0.6

irrigation

- irrigation system: check flooding (basin) system
- irrigation efficiency:

  field application efficiency: e_a = 0.7

  surface waste: none

  conveyance losses: intercepted outside the area
- quality irrigation water: concentration (EC_i) = 0.9 mmho/cm
- depth of rootzone: assumed to be 1 m
- permissible depletion of soil moisture: 50% of total available moisture

drainage

- water table to be kept below 1 m
- permissible salt concentration: EC_e = 4 mmho/cm
- drainage system: tile drains with wet perimeter (u) = 0.4 m
- depth and spacing: to be computed from the above data.

The depth and spacing of the drains are computed as follows:

The net amount of water to be supplied at each irrigation equals the amount of moisture that the soil retains between field capacity and permissible level of depletion.

Available moisture is 36% - 16% = 20%, i.e. 200 mm total available moisture if the rootzone is 1 m thick. With a permissible depletion of 50% the net amount of water to be supplied at each irrigation is 100 mm. During the peak irrigation season, when evapotranspiration amounts to 7.5 mm/day, an irrigation application will be required every 13 days (=100/7.5).

The gross amount of water to be applied to the field depends on application efficiency and the leaching requirement. With $e_a = 0.7$, the gross amount will be $\frac{100}{0.7} = 143$ mm, of which the deep percolation losses - in the absence of surface waste - amount to 43 mm every 13 days, or about 100 mm per month.

Introducing the appropriate values for EC_i, EC_e, w_e, w_{fc}, f, and E, we find the leaching requirement (Chap.9, Sect.2.5) as:

$$R^* = \frac{0.9}{0.6(\frac{65}{36} \times 4 - 0.9)} \times 225 = 53 \text{ mm/month}$$

Hence the deep percolation losses are considerably higher than the leaching requirement. Since percolation losses in basin irrigation may be assumed to be uniformly distributed, there is no need to add the leaching requirement to the unavoidable deep percolation losses.
Therefore, the design of the drain system will be based on the percolation losses only.

Deep percolation losses of 45 mm (rounded off to nearest 5 mm) will cause the water table to rise by $\frac{45}{0.1} = 450$ mm = 0.45 m.

When the permissible water table depth of 100 cm below ground surface is attained after each irrigation in the peak season, it follows that the water table depth just before irrigation will have to be 100 + 45 = 145 cm deep. So the tile drains should be laid at a minimum depth of about 1.50 m. Calculations for four depths are summarized below (Tab.5):

Table 5. Computation of drain spacing

drain depth (m below surface)	h_o (m)	h_t (m)	h_t/h_o	$\frac{Kdt}{\mu L^2}$	L (m)
(1)	(2)	(3)	(4)	(5)	(6)
1.50	0.50	0.05	0.10	0.250	31
1.65	0.65	0.20	0.31	0.135	42
1.80	0.80	0.35	0.44	0.100	49
2.10	1.10	0.65	0.59	0.068	56

Column 2: h_o = drain depth minus permissible watertable depth
= 1.50 - 1.00 = 0.50 m etc. (see also Fig.5)

Column 3: h_t = h_o minus rise of water table after irrigation
= h_o - 0.45 m

Column 5: When $\frac{h_t}{h_o}$ is known, $\frac{Kdt}{\mu L^2}$ is obtained with the curve of Fig.7.

Column 6: For drain depth 1.50 m; $\frac{Kdt}{\mu L^2} = 0.25$, or $L^2 = \frac{Kdt}{0.25\mu}$

Introducing K = 1 m/day, t = 13 days, and μ = 0.1, then L^2 = 520 d. The drain spacing L is then found by the trial- and error-procedure discussed in Chap.8, Vol.II. The d-values may be obtained from Fig.14, Chap.8.

When the calculations are made with a steady state formula, almost the same drain spacings are obtained if we introduce as hydraulic head (h) the average value for the period between two irrigations.

11.4.4 COMPUTATION BASED ON THE DYNAMIC EQUILIBRIUM CONCEPT

Sometimes the drains are laid so deep that the losses resulting from one water application do not cause the water table to rise to its highest permissible level.

Following the concept of a dynamic equilibrium over the entire season, as introduced by the U.S. Bureau of Reclamation (DUMM and WINGER, 1963), the water table is allowed to rise gradually in the course of the irrigation season in such a way that it reaches its maximum permitted height at the end of the season, or at the end of the peak period. During the next fallow or non-irrigation period the water table will fall again to approximately drain level.

The criterion in this case is apparently that the annual discharge equals the annual recharge. If not, the water table would rise in the course of some years, reaching an equilibrium level which would affect optimal crop growth. During the irrigation season, however, and especially during the peak period, drain discharge is less than the recharge, but even though the groundwater table is rising, it remains below the permissible level (Fig.9).

The computations according to the dynamic equilibrium concept are slightly different from those given for the peak irrigation period, because the values for h_o

and h_t cannot be fixed beforehand and moreover change with each irrigation.

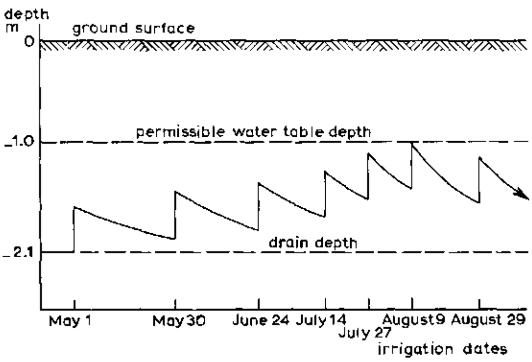


Fig.9. Watertable hydrograph during irrigation season with irregular periods between irrigations (data taken from example in text)

Finding the right drain spacing will be a trial and error procedure and will consist of assuming a drain spacing, L, and calculating from the known values for D, t, μ, and R_i/μ, the water table elevations (h_t and h_o) before and after each irrigation application for the entire irrigation season. If the water table is found to rise above the permissible depth, the computations must be repeated with a somewhat smaller drain spacing.

We shall illustrate these computations with the following example.

Example

The data and conditions are the same as given for the example in Sect. 11.4.3, but the computations are now made for the entire irrigation season, which is assumed to extend from the first of May till the end of August. The results of the computations are shown in Table 6. The number of irrigation applications and dates at which they are required, are found from evapotranspiration data over this period (Columns 1 and 2 of Tab.6). Each irrigation will cause the water table to rise over R_i/μ; with R_i = 45 mm and μ = 0.1, R_i/μ = 0.45 m (Column 3).
The water-table height, h_o, will then be: $h_o = R_i/\mu + h_t$ (Column 4); prior to the first irrigation (May 1) it is assumed that the water table is at drain level, thus $h_t = 0$.

The length of the period till the next irrigation is due, t (Column 5), is called the drain-out period, during which the water table falls from h_o to h_t.
The columns 6-9 are required to calculate the water-table height (h_t) at the end of each drain-out period from the relation between $Kdt/\mu L^2$ and h_t/h_o as given in the nomograph of Fig.7.

K and μ are known from the soil data; t is obtained from Column 5; d is found as a function of D, u, and the estimated drain spacing L; and L must be estimated.

Table 6. Computation of water-table height during the irrigation season

No. irrig. period	Date	R_i/μ (mm)	h_o (m)	t (days)	d (m)	$\frac{Kdt}{\mu L^2}$	h_t/h_o	h_t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
								0.00
1	May 1	0.45	0.45	29	1.67	0.134	0.31	0.14
2	May 30	0.45	0.59	25	1.67	0.116	0.37	0.22
3	June 24	0.45	0.67	19	1.67	0.088	0.48	0.32
4	July 14	0.45	0.77	13	1.67	0.061	0.64	0.49
5	July 27	0.45	0.94	12	1.67	0.056	0.68	0.64
6	August 9	0.45	1.09	20	1.67	0.093	0.47	0.51
7	August 29	0.45	0.96	60	1.67	0.276	0.08	0.08

Basic data for computation:

K = 1m/day	drain depth = 2.10 m
L (estimated) = 60 m	permissible water-table depth = 1.00 m
μ = 0.10	D = 4.0 - 2.1 = 1.90 m
u = 0.4 m	maximum h_o = 2.10 - 1.00 = 1.10 m
d = f(D, u and L) = 1.67 m (from Fig.9, Chap.8)	

With the same basic data but calculating for the peak irrigation period only, a drain spacing of 56 m was found for a drain depth of 2.10 m (see Tab.5). Calculating on the basis of dynamic equilibrium, we now find that a drain spacing of 60 m would also have resulted in an adequate water-table control. From a dynamic equilibrium calculation for a spacing of 56 m appears that the maximum water table would have remained 11 cm below the critical depth.

The difference in drain spacings obtained with these two methods becomes more evident with greater depth, a higher value for the drainable pore space, or a more irregular irrigation schedule.

LITERATURE

BEERS, W.F.J. van, 1966. The three main functions of a subsurface drainage system. Annual Report, International Institute for Land Reclamation and Improvement. ILRI. Wageningen, pp.14-17.

BOUWER, H. 1969. Salt balance, irrigation efficiency, and drainage design. Proceedings Journal of the Irrigation and Drainage Division, ASCE IR.1. 153-170.

DUMM, L.D. 1960. Validity and use of the transition flow concept in subsurface drainage. A.S.A.E. paper No. 60-717.

DUMM, L.D., WINGER, R.J. 1963. Subsurface drainage system designed for irrigated areas using transient flow concept. A.S.A.E. paper No. 63-213.

HABRA, 1971. Etude de l'Assainissement du Périmètre de l'Habra. Rép.Algérienne Démocratique et Populaire. Grontmij Cons.Eng. pp.176-177.

HOOGERKAMP, M., WOLDRING, J.J. 1965. Ontwatering van rivierklei. Mededeling No.116. Proefstation Akker- en Weidebouw, Wageningen. 104 pp.

HOORN, J.W.van, 1960. Groundwater flow in basin clay soil and the determination of some hydrological factors in relation with the drainage system (in Dutch with English summary). Versl. Landbouwk. Onderz. No.66.10, Wageningen. 136 pp.

HOORN, J.W.van, 1958. Results of a groundwater level experimental field with arable crops on clay soils. Neth. J. of Agric. Sci. 6:1-10.

ISRAELSEN, O.W. and others, 1950. Effectiveness of gravity drains and experimental pumping for drainage. Delta Area Utah. Bulletin 345 Utah Agr.Exp.St.p.8.

ISRAELSEN, O.W., HANSEN, V.E. 1962. Irrigation principles and practices. 3rd ed. John Wiley & Sons. New York. 447 pp.

JAGER, A.W.de, 1965. Peak runoff in small river basins in The Netherlands (in Dutch with English summary). Versl. Landbouwk. Onderz. 658. Wageningen, 167 pp.

KIDDER, E.H., LYTLE, W.F. 1949. Drainage investigations in the plastic till soils of Northeastern Illinois. Agr.Eng.39:384-386.

KOVDA, V.A. 1961. Principles of the theory and practice of reclamation and utilization of saline soils in the Arid Zones. Arid Zone Research 14, Proc. Teheran Symp. UNESCO, Paris. 201-213.

MARSHALL, T.J. 1959. Relations between water and soil. Techn.Comm.50, Commonw. Bureau of Soils. Harpenden. 91 pp.

MEDJERDA, 1961. Dévéloppement Rural de la Basse Vallée de la Medjerda, Tunisie. Office de la Mise en Valeur de la Vallée de Medjerda. Grontmij Cons.Eng. Chap.5, pp.1-7.

MINDERHOUD, J.W. 1960. Growth of grass and Groundwater level. Thesis Agricultural University, Wageningen.

MINISTRY OF AGRICULTURE. United Kingdom. 1967. Design of Underdrainage Schemes. Limitations of Drain Length. Note of Land Drainage Division of M.A.F.F. 5 pp.

NEAL, J.H. 1934. Proper spacing and depth of tile drains determined by the physical properties of the soil. Min.Agric.Exp.Sta. Techn.Bull.101.

SALAMIN, P. 1957. Relations entre les irrigations et l'évacuation de l'eau. Third Congress on Irrigation and Drainage. San Francisco. R7-Q10: 117-146.

SCHILFGAARDE, J. van, 1965. Transient design of drainage systems. Proc.J.Irr.and Drain. Div., ASCE 91 IR 3:9-22.

SEBOU, 1970. Drainage de la PT_1 du projet Sebou. Rapport de Mission de J.H.Boumans. Royaume du Maroc. ORNUAG KENITRA. Rapport 70-1.

SEGEREN, W.A., VISSER, J. 1971. New requirements for the drainage of apple orchards. In Dutch, with English summary. Van Zee tot Land No.49. Aspects of research in Eastern Flevoland. 103-126.

SIEBEN, W.H. 1963. Relations between drainage conditions and crop yield for young light clay soils in the Noordoost polder (in Dutch with English summary). Van Zee tot Land No.40, The Netherlands. 117 pp.

TADLA, 1964. Etude de Mise en Valeur du Royaume du Maroc - Périmètre du Tadla 507. Grontmij Cons.Eng. Chap.6, pp.12-14.

TALSMA, T. 1963. The control of saline groundwater. Meded.63. Landbouwhogeschool, Wageningen. 68 pp.

U.A.R. 1965. UNDP-FAO pilot project for tile drainage in the UAR. ILACO Cons.Eng.

WALKER, P. 1932. Depth and spacing for drain laterals as computed from coresample permeability measurements. Agric.Eng. 33:71-73.

WESSELING, J. 1969. Storage factor and drainage criterion (in Dutch with English summary). Meded.118. Institute for Land and Water Management Research, Wageningen. 8 pp.

WIND, G.P. 1955. A field experiment concerning capillary rise of moisture in a heavy clay soil. Neth.J.of Agric.Sci. 3:60-69.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

12. FLOW TO WELLS

J. Wesseling
Head of the Hydrology Department
Institute for Land and Water
Management Research, Wageningen

Lecturers in the Course on Land Drainage

J. Wesseling (1963–1970)
Institute for Land and Water Management Research

N. A. de Ridder (1971–1972)
International Institute for Land Reclamation and Improvement

PURPOSE AND SCOPE

Discussion of well-flow equations for steady- and unsteady-state conditions in phreatic and semi-confined aquifers.

CONTENTS

12.1 INTRODUCTION

Wells play an important role in water management. They are used not only to obtain water for domestic, industrial, and irrigation purposes, but also to lower the groundwater table. Moreover, data obtained from pumping tests performed on wells can be analyzed to calculate the hydraulic properties of aquifers and confining beds. A number of well-flow equations will be discussed in this chapter. These equations may be used:

- to predict the drawdown under steady or unsteady flow conditions when water is pumped at a known discharge from an aquifer with known hydraulic properties;
- to calculate the hydraulic properties of an aquifer from pumping test data, i.e. from the measured discharge of the pumped well and the drawdown at various distances from the well (Chap.25, Vol.III);
- to calculate the required discharge of wells to be used for drainage purposes and the spacing of such wells (Chap.14, Vol.II).

A well is constructed by drilling a hole into a saturated aquifer; the hole is cased and is equipped with a screen over those parts of the aquifer that have the most favourable water-transmitting properties. The annular space around the screen is generally gravel-packed (Fig.1). The well is equipped with a pump to lift the water from the aquifer to the ground surface. During pumping the water level in the well is lowered; a hydraulic gradient is established in the surrounding area, and groundwater flows towards the well from all directions (radial flow).

The flow towards the well can be described by combining Darcy's law and the law of continuity of mass into a single differential equation. Solutions of the differential equation - often called well functions - give the relation between the pumping rate, the drawdown of the hydraulic head at any distance from the well, the hydraulic properties of the aquifer and, for unsteady flow, the pumping time. The following discussion will be restricted to flow to wells in phreatic and semi-confined aquifers because these aquifers are of particular importance to agrohydrological problems. They have been defined and described in Chap.1, Vol.I. It will be recalled that a phreatic (also called unconfined) aquifer (Fig.2A) is a pervious layer, partly filled with water, and overlying an impervious layer. Its upper boundary is formed by a free water table or phreatic surface. In a well that penetrates into a horizontal, unconfined aquifer, the water does not rise above the phreatic level.

A semi-confined aquifer (Fig.2B) consists of a completely saturated, pervious layer, which is covered by a semi-pervious layer and is underlain by a layer that is either impervious or semi-pervious. A semi-pervious layer is defined as a layer through which the horizontal flow is negligible in comparison with the flow in the underlying and/or overlying pervious strata. If the water in the aquifer is in equilibrium, its piezometric level will coincide with the phreatic level in the overlying semi-pervious layer. Any lowering of the piezometric head in a semi-confined aquifer - for example by pumping - will result in a difference in hydraulic head between the water in the aquifer and in the covering semi-pervious layer. Hence, a vertical flow of water will be generated from the semi-pervious layer into the pumped aquifer.

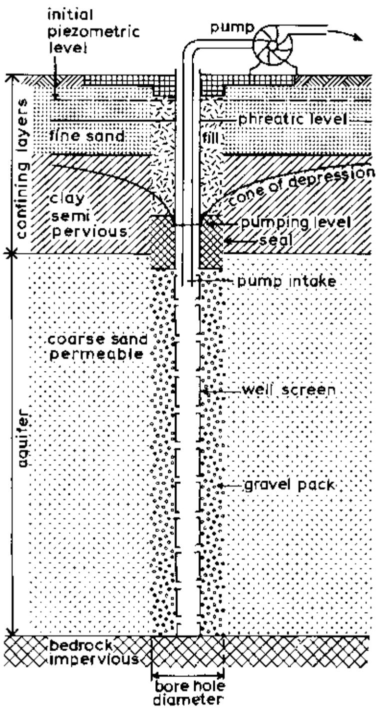


Fig.1. Schematic section through a pumped well.

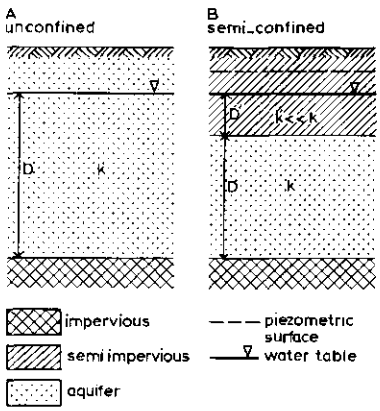


Fig.2. Schematic sections through an unconfined (A) and a semi-confined aquifer (B).

12.2 FLOW TO SINGLE WELLS IN INFINITE AQUIFERS

When a well penetrating an extensive aquifer is pumped at a constant rate, water is removed from the soil pores, and the water table is lowered from its initial position. The influence of the pumping extends radially outwards from the well with time. The water table will continue to decline as long as the aquifer is not recharged. Thus, theoretically, steady flow cannot exist in an extensive aquifer. The flow towards the well is unsteady (transient) until a source or region of recharge is intercepted. The rate of decline of the water table, however, decreases continuously as the area influenced by the pumping expands. In practice it is said that the flow has reached a steady state if the change in drawdown with time has become negligibly small, so that the hydraulic gradient has become constant.

In what follows, both steady and unsteady flow to wells will be considered. Unless otherwise specified, the following assumptions will apply:

- the aquifer is horizontal and has an infinite areal extent,
- the aquifer is homogeneous and isotropic with respect to its hydraulic properties,
- prior to pumping, the phreatic surface and/or piezometric surface are (nearly) horizontal over the area that will be influenced by the pumping,
- the aquifer is pumped at a constant rate,
- the well fully penetrates the aquifer and thus receives water by horizontal flow over the entire thickness of the aquifer.

12.2.1 STEADY FLOW TO A WELL IN A PHREATIC AQUIFER

Figure 3 shows a well fully penetrating a phreatic aquifer and discharging at a constant rate Q. Applying the Dupuit-Forchheimer assumptions (Chap.6, Vol.I), we find that the flow through any arbitrary cylinder with radius r coaxial with the well is

$$Q = 2\pi rKh \frac{dh}{dr} \tag{1}$$

where

Q = well discharge ($m^3 day^{-1}$),

r = distance from the well (m),

K = hydraulic conductivity of the aquifer ($m\ day^{-1}$),

h = hydraulic head (m).

Integration between the limits $r = r_1$, $h = h_1$ and $r = r_2$, $h = h_2$ gives

$$Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2/r_1)} \qquad (2)$$

or

$$Q = \frac{\pi K(h_2 + h_1)(h_2 - h_1)}{\ln(r_2/r_1)} \qquad (3)$$

When the drawdown, Δh, (i.e. the change in head due to pumping) is small in comparison with the thickness D of the saturated part of the aquifer, we may write $h_2 + h_1 \approx 2D$. Since $h_1 = D - \Delta h_1$, and $h_2 = D - \Delta h_2$ (Fig.3), Eq.3 becomes

$$\Delta h_1 - \Delta h_2 = \frac{Q}{2\pi KD} \ln(r_2/r_1) \qquad (4)$$

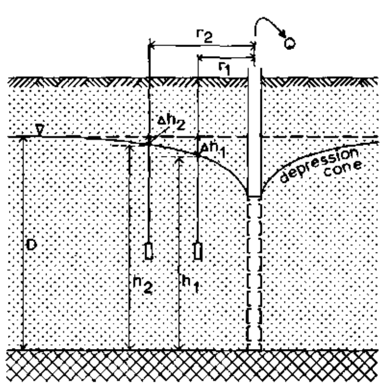


Fig.3. Schematic cross-section of a pumped phreatic aquifer.

Equation 4 makes it possible to calculate the transmissivity of the aquifer, (KD), from pumping test data by substitution of the measured well discharge and "steady"-drawdown. Conversely, for a given discharge the drawdown at any distance from the well may be calculated if the transmissivity, KD, is known and it is assumed that there is no change in head at a distance r_e, i.e. $\Delta h_2 = 0$ for $r_2 > r_e$, where r_e is termed the radius of influence of the well. Equation 4 then becomes

$$\Delta h_r = \frac{Q}{2\pi KD} \ln(r_e/r) \qquad (4a)$$

Example 1

An unconfined aquifer having a transmissivity KD = 1200 m^2/day is pumped by a fully penetrating well at a rate of 1500 m^3/day. It is assumed that the radius of influence r_e = 500 m. The well has an effective radius r_w = 0.30 m. a) What is the drawdown in the well? b) What is the drawdown at 30 m from the well?

a) At the well, Eq.4a is written as

$$\Delta h_w = \frac{Q}{2\pi KD} \ln(r_e/r_w)$$

Substituting the above values gives

$$\Delta h_w = \frac{1500}{2 \times 3.14 \times 1200} \times 2.3 \log (500/0.3) = 1.48 \text{ m}$$

b) At 30 m from the well, Eq.4 gives

$$\Delta h_{30} = \frac{1500}{2 \times 3.14 \times 1200} \times 2.3 \log (500/30) = 0.56 \text{ m}$$

12.2.2 UNSTEADY FLOW TO A WELL IN A PHREATIC AQUIFER

Unsteady flow occurs from the moment pumping starts and continues until a recharge boundary is intercepted. In practice the flow towards a well is considered to be unsteady as long as the changes in drawdown with time are measurable, or in other words, as long as a change in the hydraulic gradient can be measured. In Chap.6, Vol.I, the differential equation for two-dimensional unsteady flow in a phreatic aquifer was given as

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{KD}\frac{\partial h}{\partial t}$$

where S replaces the symbol μ. S designates the storage coefficient which, in phreatic aquifers, is considered equal to the effective porosity μ.
In polar coördinates this equation reads

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{KD}\frac{\partial h}{\partial t} \qquad (5)$$

With the initial and boundary conditions

$h = h_o$ for $t = 0$ and $o < r < \infty$ (flat water table around the well before pumping),
$h = h_o$ for $r = \infty$ and $t \geq o$ (no influence of pumping at infinite distance from the well),
the solution of the differential equation is (THEISS, 1935)

$$\Delta h = h_o - h = \frac{Q}{4\pi KD} W(u) \tag{6}$$

where

$$u = \frac{r^2 S}{4KDt} \tag{7}$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy \tag{8}$$

y = dummy variable
t = elapsed time.

$W(u)$ is the exponential integral (JAHNKE and EMDE, 1945), which is known as the Theiss well function.
Equation 6 makes it possible to calculate the drawdown at any distance r from the well at any time, if Q, S, and KD are known. Conversely, the value of S and KD can be calculated if Q, Δh, and t are known.
The exponential integral of Eq.8 cannot be solved analytically. It may be expanded into a convergent series to give

$$W(u) = (-\ 0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \ldots\ldots) \tag{9}$$

If u is small ($u < 0.01$), the third and following terms of the series become negligible (COOPER and JACOB, 1946), and Eq.6 may be written as

$$\Delta h = \frac{Q}{4\pi KD} (-\ 0.5772 - \ln u) \tag{10}$$

Substituting u gives

$$\Delta h = \frac{Q}{4\pi KD} (\ln \frac{4KDt}{r^2 S} - 0.5772)$$

Table 1. Values of the Theis well function W(u), and the modified Bessel function of the first kind and zero order, $K_o(x)$.

u	W(u)	x	$K_o(x)$
0.0001	8.63	0.01	4.72
0.0002	7.94	0.02	4.03
0.0004	7.25	0.04	3.34
0.0006	6.84	0.06	2.93
0.0008	6.55	0.08	2.65
0.001	6.33	0.10	2.43
0.002	5.64	0.20	1.75
0.004	4.95	0.40	1.11
0.006	4.54	0.60	0.777
0.008	4.26	0.80	0.565
0.01	4.04	1.0	0.421
0.02	3.35	1.2	0.318
0.04	2.68	1.4	0.244
0.06	2.30	1.6	0.188
0.08	2.03	1.8	0.146
0.10	1.82	2.0	0.114
0.20	1.22	2.2	0.0893
0.40	0.702	2.4	0.0702
0.60	0.454	2.6	0.0554
0.80	0.311	2.8	0.0438
1.0	0.219	3.0	0.0347
1.2	0.158	3.2	0.0276
1.4	0.116	3.4	0.0220
1.6	0.0863	3.6	0.0175
1.8	0.0647	3.8	0.0140
2.0	0.0489	4.0	0.0112
2.5	0.0249	4.2	0.0089
3.0	0.0131	4.4	0.0071
3.5	0.00697	4.6	0.0057
4.0	0.00378	4.8	0.0046

or

$$\Delta h = \frac{Q}{4\pi KD} 2.3 \log \frac{2.25KDt}{r^2 S} \tag{11}$$

which is an equation often used in analysing pumping test data (Chap.23, Vol.III).

Example 2

We shall once again consider the unconfined aquifer of Example 1 and assume further that S = 0.1. a) What is the drawdown at 30 m from the well after two days of continuous pumping? b) What is the time needed to reach a steady-state drawdown of 0.56 m at 30 m from the well? c) What is the distance at which $\Delta h = 0$ at t = 9.3 days?

a. According to Eq.11

$$\Delta h = \frac{1500}{4 \times 3.14 \times 1200} 2.3 \log \frac{2.25 \times 1200 \times 2}{900 \times 0.1} = 0.41$$

b. Substitution of the appropriate values into Eq.11 yields

$$0.56 = \frac{1500}{4 \times 3.14 \times 1200} 2.3 \log \frac{2.25 \times 1200 \times t}{900 \times 0.1}$$

$$t = 9.3 \text{ days}$$

c. Once again substituting the appropriate values into Eq.11 gives

$$\Delta h = 0 = \frac{1500}{4 \times 3.14 \times 1200} 2.3 \log \frac{2.25 \times 1200 \times 9.3}{r^2 \times 0.1}$$

Since

$$\frac{2.3 \times 1500}{4 \times 3.14 \times 1200} \neq 0$$

the argument of the logarithm must be equal to 1 (log 1 = 0), and it follows that

$$r = 500 \text{ m}$$

12.2.3 STEADY FLOW TO A WELL IN A SEMI-CONFINED AQUIFER

As stated in Sect.1, pumping from a semi-confined aquifer will generate a vertical flow of water from the semi-pervious layer towards the aquifer (Fig.4).

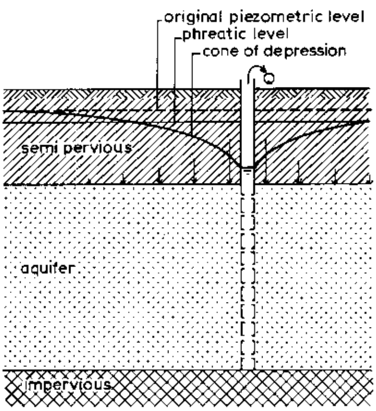


Fig.4. Schematic cross-section of a pumped semi-confined aquifer.

We shall now assume that the phreatic level remains in its initial position due to a continuous recharge of the semi-pervious layer from open water courses. The head difference between the phreatic water and the semi-confined water in the aquifer will then everywhere be equal to the drawdown of the hydraulic head, and the recharge rate will be proportional to the head difference.
According to Darcy's law, the vertical flow can be expressed as

$$v_z = K' \frac{h - h'}{D'} = \frac{\Delta h}{c} \tag{12}$$

where

v_z = rate of vertical flow (m day^{-1}),

c = D'/K' = resistance of the semi-pervious layer to vertical flow (days),

D' = thickness of the saturated part of the semi-pervious layer (m),

K' = hydraulic conductivity of the semi-pervious layer for vertical groundwater flow (m days^{-1}),

h = hydraulic head of the groundwater confined within the aquifer (m),

h' = phreatic level, relative to a datum plane (m),

Δh = $h - h'$ = drawdown (m).

When water is pumped from the aquifer, the drawdown will increase and the cone of depression will expand with time. Hence the rate at which the aquifer is recharged by water from the confining layer will increase as well. At a certain time, the recharge will equal the discharge of the well and steady-state conditions will occur.

The differential equation for steady flow towards a well in an infinite semi-confined aquifer reads

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} - \frac{(h - h')}{KDc} = 0 \tag{13}$$

With the boundary conditions

$$h = h', \qquad r = \infty$$

$$h' = \text{constant}, \qquad 0 < r < \infty$$

$$Q = \left(\frac{\partial h}{\partial r}\right)_{r=r_w}$$

the following solution was obtained by DE GLEE (1930)

$$\Delta h = \frac{Q}{2\pi KD} K_o \left(\frac{r}{\sqrt{KDc}}\right) \tag{14}$$

where K_o is a modified Bessel function (Hankel function) of the first kind and zero order. Numerical values for this function are given in Table 1 (page 173). It can be shown (HANTUSH, 1956) that for $\frac{r}{\sqrt{KDc}} < 0.05$, Eq.14 may be approximated by

$$\Delta h = \frac{Q}{2\pi KD} \ln 1.12 \frac{\sqrt{KDc}}{r} \tag{15}$$

Example 3

A semi-confined aquifer has a transmissivity KD = 2500 m^2/day. The semi-pervious layer covering it has a saturated thickness D' = 11 m, and a hydraulic conductivity for vertical flow K' = 0.02 m/day. The pumping rate Q equals 1800 m^3/day.

a. What is the drawdown in the aquifer at a distance of 50 m from the well?

b. At the same distance from the well, what is the rate at which the aquifer is recharged from the confining layer?

c. What is the radius of influence of the well?

a. The drawdown of the piezometric level can be calculated by using Eq.14 or Eq.15. The hydraulic resistance of the top layer c = 11/0.02 = 550 days and $\sqrt{KDc} = \sqrt{2500 \times 550}$ = 1170 m. For r = 50 m, $r/\sqrt{KDc}$ = 0.043. Table 1 gives K_o (0.043) = 3.26. Substituting this value and the given values of Q and KD into Eq.14 yields Δh = 0.37 m.
Substituting r = 50 and $\sqrt{KDc}/r$ = 23.4 into Eq.15 yields Δh = 0.37 m.

b. At 50 m from the well the recharge rate from the confining layer can be calculated by using Eq.12

$$v_z = \frac{\Delta h}{c} = \frac{0.37}{550} = 0.67 \times 10^{-3} \text{m/day}$$

c. The radius of influence, i.e. the value of r for which Δh = 0, is derived from Eq.15. If Δh = 0, then $1.12\ \sqrt{KDc}/r = 1$ and r = 1310 m.

12.2.4 UNSTEADY FLOW TO A WELL IN A SEMI-CONFINED AQUIFER

For the unsteady flow the differential equation is

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} - \frac{(h - h')}{KDc} = \frac{S}{KD}\frac{\partial h}{\partial t}$$

It should be noted that the storage coefficient S of the aquifer is not the same as the effective porosity μ since it depends on the elastic properties of both the aquifer material and the water. Its numerical value is of the order of magnitude of 10^{-4}, whereas the effective porosity is of the order of magnitude of 10^{-2}.
With the boundary conditions

$h(r,t) = h'$ for t = 0 and r > 0 (initial hydraulic head),
$h(r,t) = h'$ for $r = \infty$ and t > 0 (no influence at infinite distance),
h' = constant

the following solution was obtained by HANTUSH and JACOB (1955)

$$\Delta h = \frac{Q}{4\pi KD}\ 2K_o\left(\frac{r}{\sqrt{KDc}}\right) - \int_{u'}^{\infty} \frac{1}{y} \exp\left(- y - \frac{r^2}{4KDcy}\right)dy \qquad (16)$$

where y = a dummy variable.

$$u' = \frac{r^2}{4KDc} \frac{1}{u} = \frac{t}{Sc}$$

$$u = \frac{r^2 S}{4KDt}$$

For large values of t, the second term of the right-hand member of Eq.16 tends to zero, and the steady-state solution given in Eq.14 is obtained. For practical application, Eq.16 is generally written in the form

$$\Delta h = \frac{Q}{4\pi KD} W(u', \frac{r}{\sqrt{KDc}}) \qquad (17)$$

where $W(u', r/\sqrt{KDc}$ is referred to as the well function for semi-confined aquifers. Numerical values of this function can be found in HANTUSH (1956) and in KRUSEMAN and DE RIDDER (1970).

Drawdown values at various times and for various distances from the pumped well can be calculated in a way similar to that explained for the steady-state solution. First, relevant values for u' and $r/\sqrt{KDc}$ are computed, and W-values are read from the tables. Substituting these values and those of Q and KD yields the drawdown Δh. Recharge rates can be computed, using Eq.12. One should take into account that Δh is a function of t and is therefore subject to changes. As a result, the recharge changes as well.

12.3 OTHER WELL FLOW PROBLEMS

12.3.1 WELLS IN AQUIFERS WITH STRAIGHT HYDRAULIC BOUNDARIES

In the preceding sections it was assumed that the aquifer had an infinite areal extent. If, however, its well is located near a canal or river, this condition is not satisfied and the equations previously derived are no longer valid.

For an aquifer with straight boundaries, a solution for the differential equation can be found by applying the principle of superposition. This principle enables a flow system to be split up into two or more elementary sub-systems, the sum of which is hydraulically equivalent to the original flow system. Hydraulically, a canal is a line of constant hydraulic head, i.e. a line of zero drawdown (Fig.5). We shall therefore imagine an infinite aquifer and try to find in it a system of wells that would induce a zero-drawdown at the place where in reality the canal(s) is (are) located (MUSKAT, 1937).

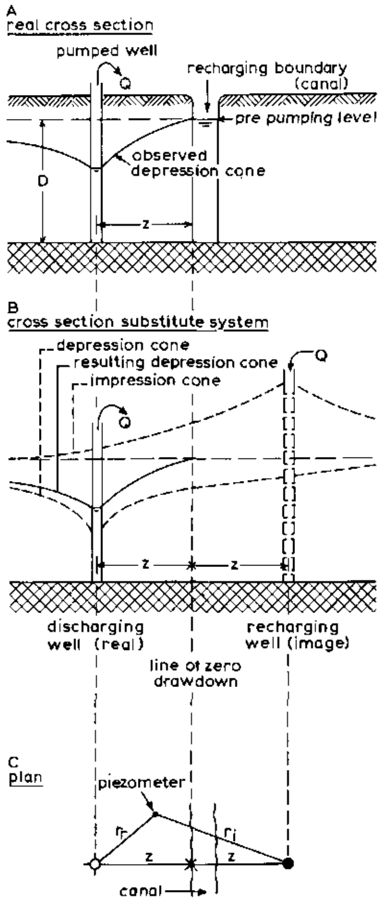


Fig.5. Plan of a pumped phreatic aquifer with straight hydraulic boundary

If there is only one canal, such a system consists of one real discharging well (called sink) and one imaginary recharging well (called source) located at a point where it would be if the real well were reflected, the canal acting as a mirror plane. The discharges of both wells are assumed equal but with opposite signs. Therefore, the (imaginary) recharging well will cause an (imaginary) negative drawdown.

At any point of an infinite aquifer, the real well will cause a drawdown

$$\Delta h_r = \frac{Q}{2\pi KD} \ln \frac{r_e}{r_r}$$

and the imaginary source a drawdown

$$\Delta h_i = \frac{-Q}{2\pi KD} \ln \frac{r_e}{r_i}$$

where

r_r = distance from the considered point to the real well (sink),
r_i = distance from the considered point to the image well (source),
r_e = radius of influence.

Adding up, one obtains

$$\Delta h = \Delta h_r + \Delta h_i = \frac{Q}{2\pi KD} \left[\ln \frac{r_e}{r_r} - \ln \frac{r_e}{r_i} \right]$$

$$\Delta h = \frac{Q}{2\pi KD} \ln \frac{r_i}{r_r} \tag{18}$$

Along the canal $r_r = r_i$, and

$$\Delta h = \frac{Q}{2\pi KD} \ln 1 = 0 \tag{19}$$

so that the condition of zero-drawdown along the canal is satisfied.
Equation 18 is usually written in cartesian coördinates. Taking the y-axis along the canal and the x-axis parallel to the line through the centre of the wells (Fig.6A), we get

$$r_i = \sqrt{(x_i + x_w)^2 + (y_i - y_w)^2}$$

$$r_r = \sqrt{(x_i - x_w)^2 + (y_i - y_w)^2}$$

and the drawdown Δh at any arbitrary point with coördinates (x_i,y_i) is

$$\Delta h = \frac{Q}{2\pi KD} \left[\frac{1}{2} \ln \frac{(x_i + x_w)^2 + (y_i - y_w)^2}{(x_i - x_w)^2 + (y_i - y_w)^2} \right] \tag{20}$$

The factor between brackets is denoted by the symbol G(x,y) (Green's function)

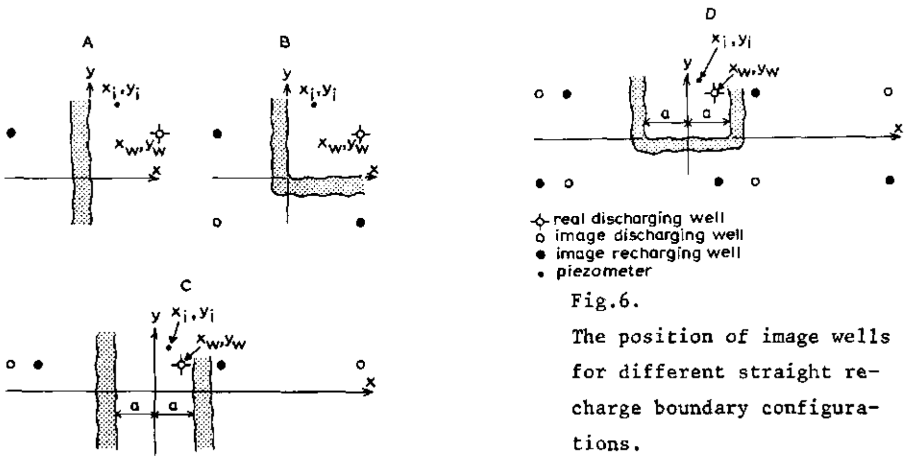


Fig.6.
The position of image wells for different straight recharge boundary configurations.

and the general equation for this type of solution is

$$\Delta h = \frac{Q}{2\pi KD}\, G(x,y)$$

For a well located near two canals perpendicular to each other, the solution requires three image wells (Fig.6B), and Green's function becomes (MUSKAT, 1937)

$$G(x,y) = \frac{1}{2} \ln \frac{\{(x_i - x_w)^2 + (y_i + y_w)^2\}\ \{(x_i + x_w)^2 + (y_i - y_w)^2\}}{\{(x_i - x_w)^2 + (y_i - y_w)^2\}\ \{(x_i + x_w)^2 + (y_i + y_w)^2\}} \qquad (21)$$

When the well is located between two parallel canals (Fig.6C), the pattern of image wells repeats itself into infinity, although of course the influence of image wells at a great distance ($r_i > 100\ r_r$) becomes negligible.
Green's function becomes

$$G(x,y) = \frac{1}{2} \ln \frac{\cosh\{\pi(y_i - y_w)/2a\} + \cos\{\pi(x_i + x_w)/2a\}}{\cosh\{\pi(y_i - y_w)/2a\} - \cos\{\pi(x_i - x_w)/2a\}} \qquad (22)$$

where a = half the distance between the parallel canals, and the angles are expressed in radians.
For a strip of land bordered by a canal that intersects two parallel canals at right angles (Fig.6D), Green's function reads

$$G(x,y) = \frac{1}{2} \ln \frac{\cosh\{\pi(y_i - y_w)/2a\} + \cos\{\pi(x_i + x_w)/2a\}}{\cosh\{\pi(y_i - y_w)/2a\} - \cos\{\pi(x_i - x_w)/2a\}} \times$$

$$\times \frac{\cosh\{\pi(y_i + y_w)/2a\} - \cos\{\pi(x_i - x_w)/2a\}}{\cosh\{\pi(y_i + y_w)/2a\} + \cos\{\pi(x_i + x_w)/2a\}} \qquad (23)$$

Example 4

Suppose that the unconfined aquifer of Example 1 (KD = 1200 m^2/day and Q = 1500 m^3/day) is now cut by a canal 100 m from the pumped well with radius r_w = 0.30 m. a) What is the drawdown in the well? b) What is the drawdown at a point with coördinates (200,0)?

a. The drawdown at any point may be calculated, by using Eq.18 or Eq.19. For the boundary of the real well r_w = 0.3 m and r_i = 200 m, the drawdown in the well, according to Eq.18, is

$$\Delta h_w = \frac{1500}{2 \times 3.14 \times 1200} \times 2.3 \log \frac{200}{0.3} = 1.29 \text{ m}$$

which is 0.19 less than if there were no canal (Δh = 1.48, in Example 1).

b. Taking the x-axis along the line through real and image well, and the y-axis along the canal, we find x_w = 100 and y_w = 0. The points (x_i,y_i) = (0,0) and (x_i,y_i) = (200,0) are both 100 m from the well but at different sides of it. At the point (0,0) the drawdown is zero. At the point (200,0) the drawdown, according to Eq.20, is

$$\Delta h = \frac{1500}{2 \times 3.14 \times 1200} \frac{2.3}{2} \log \frac{(200 + 100)^2}{(200 - 100)^2} = 0.22 \text{ m}$$

12.3.2 INTERMITTENT PUMPING

Suppose a well in an infinite phreatic aquifer is pumped daily for 8 hours at a rate Q m^3/day and is shut off for the rest of the day. In this situation the equations derived in Sect.12.2 are not applicable because they assume a constant pumping rate. A solution can be found by using the principle of superposition. Imagine that the pumping is continuous. When in reality pumping is stopped, a continuous negative pumping rate is added; when the pumping is resumed, a positive pumping rate is added, and so on (Fig.7).

Hence, after n days of intermittent pumping (each day during 1/m-th part of the day at a rate Q m^3) the drawdown at a point at distance r from the pumped well is the sum of the drawdowns caused by each of the pumping and recharge rates. The drawdown is calculated by using Eq.11: after n days the drawdown caused by the pumping that started at t = 0 is

$$\Delta h = \frac{Q}{4\pi KD} 2.3 \log \frac{2.25KDn}{r^2S}$$

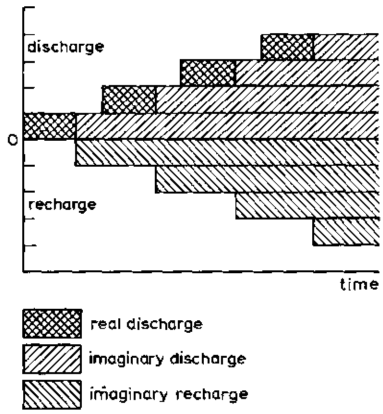


Fig.7.
Discharge and recharge pattern of intermittent pumping.

The drawdown at t = n days, caused by the assumed continuous recharge that started when the pumping was stopped for the first time at t = 1/m days, is

$$\Delta h = \frac{-Q}{4\pi KD} 2.3 \log \frac{2.25KD(n - 1/m)}{r^2S}$$

The drawdown at t = n days, caused by the pumping that was resumed the second day, i.e. at t = 1, is

$$\Delta h = \frac{Q}{4\pi KD} 2.3 \log \frac{2.25KD(n - 1)}{r^2S}$$

The drawdown at t = n days, caused by the recharge that started when pumping was interrupted on the second day, is

$$\Delta h = \frac{-Q}{4\pi KD} 2.3 \log \frac{2.25KD(n - 1 - 1/m)}{r^2S}$$

The pumping that started on the n-th day caused, at t = n, a drawdown

$$\Delta h = \frac{Q}{4\pi KD} 2.3 \log \frac{2.25KD\,\{n - (n - 1)\}}{r^2S}$$

and the recharge that started when pumping was interrupted on the n-th day caused, by the end of that day, a drawdown

$$\Delta h = \frac{-Q}{4\pi KD} 2.3 \log \frac{2.25KD \{n - (n-1) - 1/m\}}{r^2 S}$$

The real drawdown at the end of the n-th day is the sum of the drawdown caused by each of the pumping and recharge periods

$$\Delta h_{t=n} = \frac{Q}{4\pi KD} 2.3 \quad \log \frac{2.25KD}{r^2 S} n + \log \frac{2.25KD}{r^2 S} (n-1) +$$

$$+ \ldots\ldots + \log \frac{2.25KD}{r^2 S} \{n - (n-1)\} - \log \frac{2.25KD}{r^2 S} (n - 1/m) -$$

$$- \log \frac{2.25KD}{r^2 S} (n - 1 - 1/m) - \ldots\ldots\ldots$$

$$- \log \frac{2.25KD}{r^2 S} \{n - (n-1) - 1/m\}$$

$$= \frac{Q}{4\pi KD} 2.3 \log \frac{n \times (n-1) \times \ldots \times \{n - (n-1)\}}{(n - 1/m) \times (n - 1 - 1/m) \times \ldots \{n - (n-1) - 1/m\}}$$

$$= \frac{Q}{4\pi KD} 2.3 \log \frac{1 \times 2 \times \ldots \times n}{(1 - 1/m) \times (2 - 1/m) \times \ldots \times (n - 1/m)} =$$

$$= \frac{Q}{4\pi KD} 2.3 \log \frac{n!}{(n - 1/m)!} \tag{24}$$

12.3.3 STEADY FLOW TO A WELL IN A PHREATIC AQUIFER WITH VERTICAL REPLENISHMENT

In the preceding sections it was assumed that the pumped phreatic aquifers were not replenished by percolating rain or irrigation water. It is now assumed that the phreatic aquifer is replenished at a constant rate, R, expressed as a volume per unit surface per unit of time (m^3/m^2 day = m/day), see Fig.8.

The steady flow through an arbitrary cylinder at a distance r from the well is given by

$$Q_r = \pi(r_e^2 - r^2)R = 2\pi r h K \frac{dh}{dr} \tag{25}$$

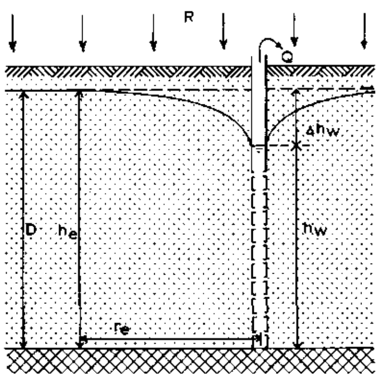


Fig.8.
Schematic cross-section of a pumped phreatic aquifer with vertical replenishment.

Since, in steady state, the discharge of the well (Q_w) equals the recharge of the area within the radius of influence

$$Q_w = \pi r_e^2 R \tag{26}$$

It follows that

$$Q_r = Q_w - \pi r^2 R = 2\pi r h K \frac{dh}{dr}$$

or

$$\left(\frac{Q_w}{r} - \pi r R\right) dr = 2\pi K h \, dh$$

Integration between the limits $r = r_w$, $h = h_w$ and $r = r_e$, $h = h_e$ yields

$$Q_w \ln(r_e/r_w) - \frac{1}{2}\pi R(r_e^2 - r_w^2) = Q_w = \pi K(h_e^2 - h_w^2) \tag{27}$$

The quantity $\frac{1}{2}\pi R r_w^2$ is very small in comparison with $\frac{1}{2}\pi R r_e^2$ and can be neglected. If, moreover, the drawdown in the well is small in comparison with the original hydraulic head, the right-hand member of Eq.27 may be expressed as (PETERSON et al., 1952)

$$\pi K(h_e + h_w)(h_e - h_w) \simeq 2\pi K D \, \Delta h_w$$

Since, according to Eq.26

$$r_e^2 = \frac{Q_w}{\pi R}$$

Eq.27 may be written as

$$\Delta h_w = \frac{Q_w}{2\pi KD} \left(2.3 \log \frac{r_e}{r_w} - \frac{1}{2}\right) \tag{28}$$

If $r_e/r_w > 100$, and if we accept an error of 10 per cent, the term $-\frac{1}{2}$ in this equation can be neglected.

Example 5

An irrigated area of 1000 × 1000 m is drained by a well in its centre. The average deep percolation losses resulting from the application of excess irrigation water amount to 2 mm per day. The hydraulic conductivity of the aquifer material is K = 25 m/day; the thickness of the water-bearing layer is D = 25 m. The radius of the well r_w = 0.1 m. What is the drawdown in the well?
If we take r_e = 500 m, being a reasonable estimate, the recharge (which in steady state equals the pumping rate)

$$Q_w = 0.002 \times 3.14 \times 500^2 = 1570 \text{ m}^3/\text{day}$$

Substituting this value into Eq.28 gives

$$\Delta h_w = \frac{1570 \times 2.3 \log \left(\frac{500}{0.1}\right) - 785}{2 \times 3.14 \times 25 \times 25} = 3.2 \text{ m}$$

LITERATURE

COOPER, H.H. and JACOB, C.E. 1946. A generalized graphical method for evaluating formation constants and summarizing well-field history. Trans. Am. Geophys. Union, 27: 526-534.

DE GLEE, G.J. 1930. Over grondwaterstromingen bij wateronttrekking door middel van putten. J. Waltman, Delft, 175 pp.

HANTUSH, M.S. 1956. Analysis of data from pumping tests in leaky aquifers. Trans. Am. Geophys. Union, 37: 702-714.

HANTUSH, M.S. and JACOB, C.E. 1955. Nonsteady radial flow in an infinite leaky aquifer. Trans. Am. Geophys. Union, 36: 95-100.

JAHNKE, E. and EMDE, F. 1945. Tables of functions with formulas and curves. Dover Publ. New York, 306 pp.

KRUSEMAN, G.P. and DE RIDDER, N.A. 1970. The analysis and evaluation of pumping test data. Bull.11. Int. Inst. for Land Reclam. and Improv., Wageningen. 200 pp.

MUSKAT, M. 1937. The flow of homogeneous fluids through porous media. McGraw-Hill Book Comp., New York, 763 pp.

PETERSON, D.F., ISRAELSON, O.W. and HANSEN, V.E. 1952. Hydraulics of wells. Techn. Bull.351. Utah Agric. Exp. Stn. Logan, Utah, 48 pp.

THEIS, C.V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. Trans. Am. Geophys. Union, 16: 519-524.

Suggestions for further reading

ANONYMUS 1964. Steady flow of groundwater towards wells. Proc. Inf. Comm. Hydrol. Res. TNO 10: 179 pp. The Hague.

DAVIS, S.N. and DE WIEST, R.J.M. 1966. Hydrology. John Wiley and Sons Inc., New York, 463 pp.

DE WIEST, R.J.M. 1965. Geohydrology. John Wiley & Sons Inc., New York, 366 pp.

HANTUSH, M.S. 1964. Hydraulics of wells. Adv. Hydroscience I: 281-432. Academic Press, New York.

HARR, M.E. 1962. Groundwater and Seepage. McGraw-Hill Book Comp., New York, 315 pp.

JACOB, C.E. 1950. Flow of groundwater. In H. Ronse(ed.). Engineering Hydraulics, John Wiley and Sons Inc., New York, 321-386.

POLUBARINOVA - KOCHINA, P.Ya. 1962. The theory of groundwater movement (English translation). Princeton University Press, Princeton, N.J., 613 pp.

TODD, K.D. 1959. Groundwater hydrology. John Wiley and Sons Inc., New York, 336 pp.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

13. SEEPAGE

J. WESSELING
Head of the Hydrology Department
Institute for Land and Water
Management Research, Wageningen

Lecturers in the Course on Land Drainage

J. Wesseling (1964–1970, 1972)
Institute for Land and Water Management Research

N. A. de Ridder (1971)
International Institute for Land Reclamation and Improvement

PURPOSE AND SCOPE

A discussion of some groundwater flow problems specific to drainage.

CONTENTS

13.1 INTRODUCTION

The term seepage is generally defined as the flow of water through soils. It is also used for the flow of water leaving the soil (seepage through an earth dam, or seepage into a ditch), or entering the soil (seepage from canals or ditches into underlying permeable layers). In this chapter we shall discuss some seepage problems specific to drainage. This discussion is far from complete, because it is restricted to those problems that can be analyzed from data obtained in a rather simple way.

To explain the behaviour of water tables or hydraulic heads, it is often necessary to describe the groundwater flow system. In the past, solutions to numerous groundwater flow problems have been derived and are described in literature. These solutions all have in common that, if the boundary conditions for which they have been derived are fulfilled, and if the aquifer characteristics are known, the flow of groundwater can be predicted. This implies that, if a solution is available for a certain flow problem, any changes in the groundwater flow due to changes in the boundary conditions can be computed by substituting the proper values of the aquifer characteristics into the equations.

Often, however, the values of the region's aquifer characteristics are unknown. It may happen that hydrological data are available from other sources, e.g. from hydrogeological investigations or from pumping tests, but that these data have to be verified. It should be recalled that pumping tests are rather expensive and that the required equipment is not always available. Collecting data on hydraulic heads, however, is a fairly simple matter, and these data often allow the flow conditions of a region to be described. Such flow conditions can be compared with theoretical solutions, which may then be used to analyse the collected data, enabling the aquifer characteristics to be computed. This approach often allows hydrological data obtained by other methods to be checked.

It should be noted that the theoretical solutions we shall discuss have been derived for ideal conditions, e.g. isotropic and homogeneous aquifers and confining layers, and often relatively simple or idealized boundary conditions. These conditions will seldom be met in nature. It should therefore be borne in mind that far greater errors are caused by differences between the actual boundary conditions and those assumed in theory than those attributable to anisotropy or non-homogeneity of the material. It is of the utmost importance that when applying the solutions, whether for predicting situations or for analysing observation data, one should choose the solution that best fits the existing boundary conditions.

13.2 SEEPAGE FROM CHANNELS INTO SEMI-CONFINED AQUIFERS

It will be recalled (Chap.1, Vol.I) that a semi-confined aquifer consists of a water-bearing layer covered by a layer with a low, though measurable, hydraulic conductivity. Since the hydraulic conductivity of the horizontal covering layer is low, compared with that of the aquifer, we may neglect the horizontal flow in this layer (see also Chap.6, Vol.I).

It will be assumed that the aquifer is cut over its whole thickness by a straight channel (river, canal) having a constant water level. Under equilibrium conditions, both the hydraulic head of the water confined within the aquifer and the phreatic level will coincide with the water level in the channel (Fig.1).

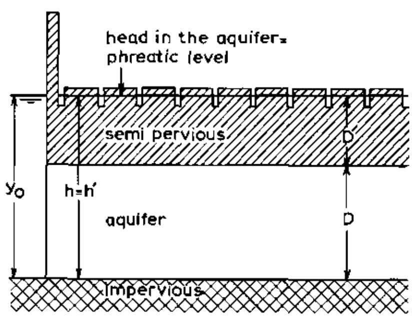


Fig.1. Semi-confined aquifer cut by a channel: equilibrium conditions.

In practice this will seldom happen. Evapotranspiration will withdraw water from the top soil and consequently lower the phreatic level, while it may also happen that the phreatic level is artificially (shallow ditches) kept at a lower level than the hydraulic head in the aquifer (Fig.2).

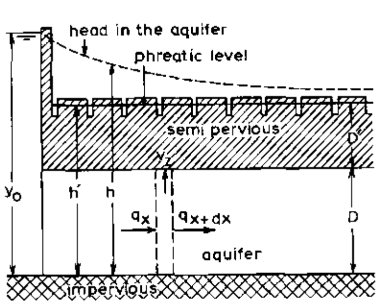


Fig.2. Semi-confined aquifer cut by a channel: seepage flow.

On the other hand, after heavy precipitation and insufficient drainage the phreatic level may rise above the hydraulic head (Fig.3).

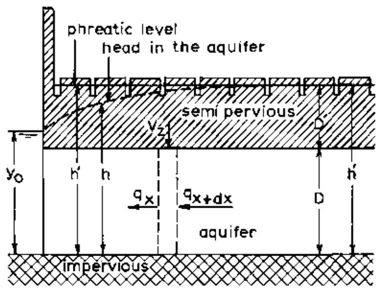


Fig.3. Semi-confined aquifer cut by a channel: drainage flow.

As soon as there is a difference between hydraulic head and phreatic level, a vertical flow of water will occur. When the phreatic level drops below the hydraulic head, the flow will be vertically upward; when the phreatic level rises above the hydraulic head, the flow will be downward. In the following discussion we will restrict ourselves to upward flow only.
The flow from the aquifer into the covering layer will cause the hydraulic head to be lowered and a gradient will occur, generating a flow from the channel into the aquifer.

The hydraulic head in the aquifer is denoted by h and the phreatic head in the covering layer by h'. It is assumed that the water in the channel is kept at a constant level, y_o, that the phreatic level is kept constant at h'_o, and that the flow is in steady state (i.e. the hydraulic gradients are constant).
The change in the horizontal flow rate is due to vertical flow v_z into the covering layer. If v_z is taken positive for upward flow, the continuity equation reads (Fig.2)

$$v_z = - \frac{dq_x}{dx} \qquad (1)$$

The horizontal flow in the aquifer can be expressed by

$$q_x = - KD \frac{dh}{dx} \qquad (2)$$

According to Darcy's law the vertical flow is

$$v_z = K' \frac{h - h'_o}{D'} = \frac{h - h'_o}{c} = \zeta(h - h'_o) \tag{3}$$

where

K' = the hydraulic conductivity of the covering layer (m day^{-1}),

D' = the thickness of the saturated part of the covering layer (m),

c = D'/K' = resistance of the covering layer to vertical flow (day),

ζ = $1/c$ = leakage coefficient (day^{-1}).

Rewriting Eq.3 gives

$$h = v_z c + h'_o \tag{4}$$

and substituting Eq.1 into Eq.4 yields

$$h = - c \frac{dq_x}{dx} + h'_o \tag{5}$$

Since h'_o is constant,

$$\frac{dh}{dx} = - c \frac{d^2q_x}{dx^2} \tag{6}$$

Substituting Eq.6 into Eq.2 gives

$$q_x = \quad KDc \frac{d^2q_x}{dx^2} \tag{7}$$

13.2.1 INFINITE AQUIFER

For an aquifer extending infinitely beyond the channel, the boundary conditions for which Eq.7 has to be solved are

$$h = y_o \text{ at } x = 0$$

$$h = h' \text{ at } x = \infty \; (h' = \text{constant})$$

The differential equation can be solved by assuming that

$$q_x = e^{ax}$$

where a is a constant. Substituting this form into Eq.7 yields $a = \pm 1\sqrt{KDc}$, so that the general solution becomes

$$q_x = C_1\, e^{x/\sqrt{KDc}} + C_2\, e^{-x/\sqrt{KDc}} \tag{8}$$

where C_1 and C_2 are constant coefficients, and $\sqrt{KDc} = \lambda$ is the leakage factor with the dimension of a length.

From the boundary conditions it follows that h has to remain finite for $x = \infty$, whence $C_1 = 0$. Substituting further $q_x = q_o$ at $x = 0$ yields

$$C_2 = q_o$$

Hence,

$$q_x = q_o\, e^{-x/\sqrt{KDc}} \tag{9}$$

This equation allows the seepage rate to be calculated at any distance x from the channel, if the seepage rate q_o at the border of the channel and the leakage factor are known.
To find an expression for h, Eq.9 is differentiated with respect to x

$$\frac{dq_x}{dx} = -\frac{q_o}{\sqrt{KDc}}\, e^{-x/\sqrt{KDc}} \tag{10}$$

Substitution of Eq.10 into Eq.5 gives

$$h - h'_o = q_o \sqrt{c/KD}\, e^{-x/\sqrt{KDc}} \tag{11}$$

Substituting the boundary condition $h = y_o$, $x = 0$ into Eq.11 yields

$$q_o = (y_o - h'_o)\sqrt{KD/c} \tag{12}$$

which is an expression for the seepage rate at the border of the channel.
Substitution of Eq.12 into Eq.11 eliminates q_o

$$h - h'_o = (y_o - h'_o)e^{-x/\sqrt{KDc}} \tag{13}$$

or

$$\sqrt{KDc} = \frac{x}{2.30\ \{\log(y_o - h'_o) - \log(h - h'_o)\}} \qquad (14)$$

Equation 14 allows an analysis of observed data. Let us suppose that the hydraulic head in the aquifer is measured in a row of piezometers perpendicular to the channel, hence at different distances x. Let us further assume that the (constant) phreatic level is known from a number of shallow observation wells in the covering layer. Now Eq.14 shows that plotting the observed data of $(h - h'_o)$ against the distance x on single-logarithmic paper (h - h' on the logarithmic axis) gives a straight line (Fig.4) whose slope equals 2.30 $\sqrt{KDc}$, from which $\sqrt{KDc}$ can be computed.
If the KD-value is known from other investigations, the value of c can be calculated, and the converse is also true.
In practice there will nearly always be a deviation from a straight line relationship in the neighbourhood of the channel. This is because the channel generally penetrates only part of the aquifer, which means that one has to take a certain radial flow into account near the channel. The resistance caused by the radial flow can be expressed either in metres horizontal flow (Fig.4) or in a reduction of (h - h') into an effective value.

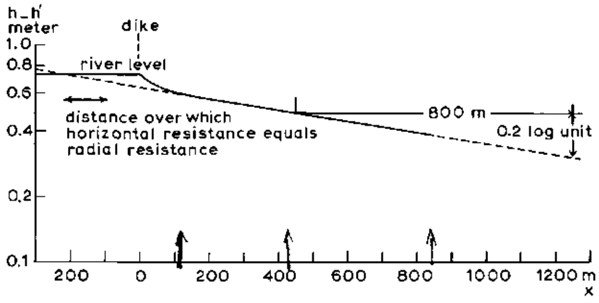


Fig.4.
Analysis of piezometric data in a semi-confined aquifer cut by a channel.

It should be noted that for the analysis it is not particularly necessary to measure the water level in the channel. The analysis can be carried out with any arbitrary measuring point as a reference.

If the value of $\sqrt{KD/c}$ is known, the amount of flow can be calculated from Eq.11. The flow rate per unit length of channel, q_o, is given at x = 0 while, for each value of x, a value of q_x can be calculated. The rate of vertical flow per unit width between x_1 and x_2 is given by $q_{x_1} - q_{x_2}$.

Example 1

Figure 5 shows the geological profile of the polder 'Dalem' along the river Waal (The Netherlands). The relatively coarse sandy aquifer is covered by a 12 m thick, semi-pervious layer of fine sand, clay, and peat. Three piezometers have been placed in a row perpendicular to the river at 120, 430, and 850 m from the dike. The water table may be considered constant at zero level. Figure 4 shows a plot of the difference between the head in the aquifer and the phreatic level versus distance on single logarithmic paper, while the river level is also shown.

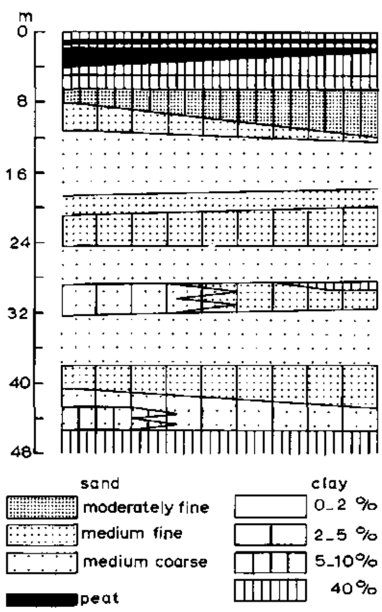


Fig.5.
Geological profile of the polder Dalem, The Netherlands (semi-confined aquifer, DE RIDDER et al., 1962).

The slope of the straight line through the plotted points equals $\frac{800}{0.2}$. Hence

$$2.30\sqrt{KDc} = \frac{800}{0.2}$$

or

$$KDc = \left(\frac{800}{2.30 \times 0.2}\right)^2 = 3.02 \times 10^6 \text{ m}^2$$

According to Fig.4 the point where the extended straight line intersects the river level lies at a distance of 215 m outside the dike. Hence the radial resistance due to the river's partial penetration of the aquifer is equal to the horizontal flow over a distance of 215 m.

13.2.2 FINITE AQUIFER

In the previous section it was assumed that the aquifer extended infinitely beyond the dike. Normally, however, the aquifer will have a restricted extension (Fig.6).

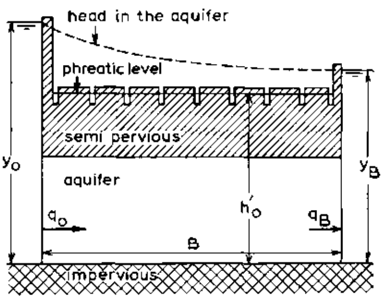


Fig.6.
Semi-confined aquifer of finite length cut by a channel.

If B denotes the extension of the aquifer (MAZURE, 1936) and $\sqrt{KDc}$ the leakage factor, the solution of Sect. 13.2.1 can be used, provided

$$B > 3\sqrt{KDc}$$

For narrower strips a solution may be obtained by assuming

$$q_x = C_1 \cosh \frac{x}{\lambda} + C_2 \sinh \frac{x}{\lambda}$$

where C_1 and C_2 are constants to be determined and $\lambda = \sqrt{KDc}$. If the phreatic level has a constant elevation h'_o and the width of the strip equals B, the general solution becomes

$$q_x = \sqrt{KD/c}\left[(y_o - h'_o)\frac{\cosh(B/\lambda - x/\lambda)}{\sinh B/\lambda} - (y_B - h'_o)\frac{\cosh x/\lambda}{\sinh B/\lambda}\right] \qquad (15)$$

where y_B is the hydraulic head in the aquifer at the far end of the strip. From this equation the flow entering the aquifer is found to be

$$q_o = \sqrt{Kd/c}\left[\frac{(y_o - h'_o)}{\tanh(B/\lambda)} - \frac{(y_B - h'_o)}{\sinh(B/\lambda)}\right] \quad (16)$$

and the amount leaving the aquifer at x = B

$$q_B = \sqrt{KD/c}\left[\frac{(y_o - h'_o)}{\sinh(B/\lambda)} - \frac{(y_B - h'_o)}{\tanh(B/\lambda)}\right] \quad (17)$$

The difference $q_o - q_B$ equals the seepage rate per metre channel over the whole width B of the aquifer.

13.2.3 CIRCULAR BOUNDARY CONDITIONS

Let us now assume a large circular polder with radius R, surrounded by an area whose land surface and groundwater table are higher than those in the polder. Hence there is a radial flow of groundwater into this polder from outside (Fig.7).

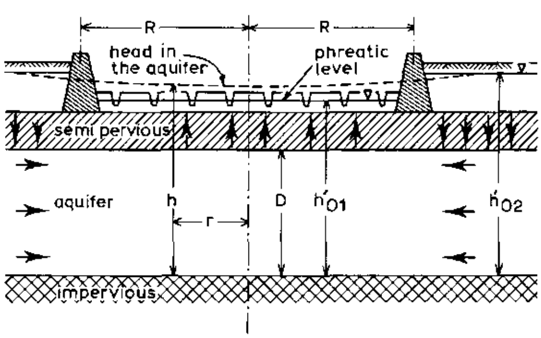


Fig.7.
Seepage into a circular polder.

The horizontal flow in the semi-confined aquifer with constant thickness D and hydraulic conductivity K can be expressed by

$$Q = -\ 2\pi KDr\,\frac{dh}{dr} \quad (18)$$

and the vertical flow through the overlying layer with saturated thickness D' and hydraulic conductivity for vertical flow K' by

$$v_z = \frac{K'(h - h')}{D'} = \frac{h - h'}{c} \quad (19)$$

where c is the hydraulic resistance of the overlying layer.
Continuity requires that

$$\frac{dQ}{dr} + 2\pi r v_z = 0 \tag{20}$$

Substituting the above expressions for Q and v_z into Eq.20 and dividing by $-2\pi KD$, yields

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} - \frac{h - h'}{KDc} = 0$$

In the case of a constant level $h' = h'_o$ in the polder, we may write $h - h'_o = y$, and if we write $r = x\sqrt{KDc}$, this equation takes the standard form

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - y = 0 \tag{21}$$

which is a second order linear differential equation frequently met with when axially symmetric flow problems are being considered.
The general solution of Eq.21 is a linear combination of two independent solutions (MAZURE, 1936)

$$y = C_1\, I_o(x) + C_2\, K_o(x) \tag{22}$$

where C_1 and C_2 are arbitrary constants, and $I_o(x)$ and $K_o(x)$ are modified Bessel functions of the first (I_o) and second kind (K_o), and of zero order.
The derivatives of $I_o(x)$ and $K_o(x)$ are denoted by $I_1(x)$ and $-K_1(x)$ respectively, and are called modified Bessel functions of the first order. Values of I_o, K_o, I_1, and K_1 for different values of x can be found in tables, e.g. VERRUIJT (1970).
For $r = x\lambda$, with $\lambda = \sqrt{KDc}$ (leakage factor) and $\beta = \sqrt{KD/c}$, the solution of the problem becomes

$$h - h'_o = C_1\, I_o(r/\lambda) + C_2\, K_o(r/\lambda) \tag{23}$$

and from Darcy's law

$$Q = 2\pi\beta r\left[-C_1\, I_1(r/\lambda) - C_2\, K_1(r/\lambda)\right] \tag{24}$$

Because of the different values of h'_o inside and outside the polder (h'_{01} for $0 < r < R$, and h'_{02} for $r \geq R$), two sets of constants (C_{11}, C_{21} and C_{12}, C_{22}) have to be found.
The values of C_{11} and C_{21} can be found from the conditions within the polder, where

$$0 < r < R \qquad h'(r) = \text{constant } h'_{01}$$

Since $Q = 0$, for $r = 0$, it follows that $C_{11} = 0$, so that

$$Q_1(r) = 2\pi\beta r\ C_{21}\ K_1(r/\lambda) \tag{25}$$

and

$$h_1(r) - h'_{01} = C_{21}\ K_o(r/\lambda) \tag{26}$$

The values of C_{12} and C_{22} can be found from the boundary conditions in the surrounding area, where

$$R < r < \infty : \quad h'(r) = \text{constant} = h'_{02}$$

Because of the finite value of h for $r = \infty$, it follows that $C_{22} = 0$. Hence

$$Q_2(r) = -\ 2\pi\beta r\ C_{12}\ I_1(r/\lambda) \tag{27}$$

and

$$h_2(r) - h'_{02} = C_{12}\ I_o(r/\lambda) \tag{28}$$

At the boundary of the polder $r = R$, it is required that $h_1(R) = h_2(R)$ and $(dh_1/dr)_R = (dh_2/dr)_R$. Substituting these expressions into Eqs.25-28 gives for C_{12} and C_{21} the conditions

$$h'_{02} - h'_{01} = C_{21}\ K_o(R/\lambda) - C_{12}\ I_o(R/\lambda) \tag{29}$$

and

$$C_{21}\ K_1(R/\lambda) = -\ C_{12}\ I_1(R/\lambda) \tag{30}$$

Elimination of C_{12} yields

$$h'_{02} - h'_{01} = C_{21}\left[K_o(R/\lambda) + \frac{K_1(R/\lambda)}{I_1(R/\lambda)} I_o(R/\lambda)\right] \tag{31}$$

Substituting this expression into Eq.25 yields the following result for the inflow into the polder (at r = R)

$$Q(R) = 2\pi\beta r \frac{I_1(R/\lambda)K_1(R/\lambda)}{I_1(R/\lambda)K_o(R/\lambda) + I_o(R/\lambda)K_1(R/\lambda)} (h'_{02} - h'_{01}) \tag{32}$$

To find a distribution of the seepage water inside the polder, values of $r < R$ can be substituted into Eq.32.

13.2.4 SEEPAGE DISTRIBUTION

In the previous sections it was assumed that both aquifer and confining layer were uniform, i.e. that they had the same KD- and c-values everywhere. In practice KD-values, but especially c-values, may change over rather short distances. If this is so, the distribution of seepage intensities can easily be derived by analyzing groundwater maps. This method will be explained in Chap.21, Vol.III.

13.3 FLOW FROM OR TOWARDS DITCHES IN PHREATIC AQUIFERS

13.3.1 AFTER AN INSTANTANEOUS CHANGE IN WATER LEVEL

In the foregoing section we discussed some seepage problems in semi-confined aquifers. Similar problems may occur in unconfined aquifers. For example, what effect will an instantaneous rise or fall of the water level in a ditch have on the water table in the neighbouring areas?

Let us suppose that the water level in a ditch penetrating the whole thickness of a horizontal unconfined aquifer with infinite extent is y_o at $t = 0$. Let us further suppose that the water table in the neighbouring areas is in static equilibrium, i.e. that the water table has the same level $h = y_o$ (Fig.8).
When the water level in the ditch is lowered (or raised) instantaneously by an amount Δy at $t = 0$, water will flow out of (or into) the adjoining area till the water table and the ditch level are again in equilibrium. The influence of the change in the water level in the ditch is given by

$$\frac{\partial h}{\partial t} = \frac{KD}{\mu} \frac{\partial^2 h}{\partial x^2} \qquad (33)$$

provided that $\Delta y << D$, so that D is not affected too much by the rising water table, and that the flow in the aquifer is horizontal, so that the Dupuit assumptions are valid. The boundary conditions for which this equation can be solved are then

$$h = y_o,\ o < x < \infty,\ t = 0$$

$$h = y_o - \Delta y,\ x = 0,\ t > 0.$$

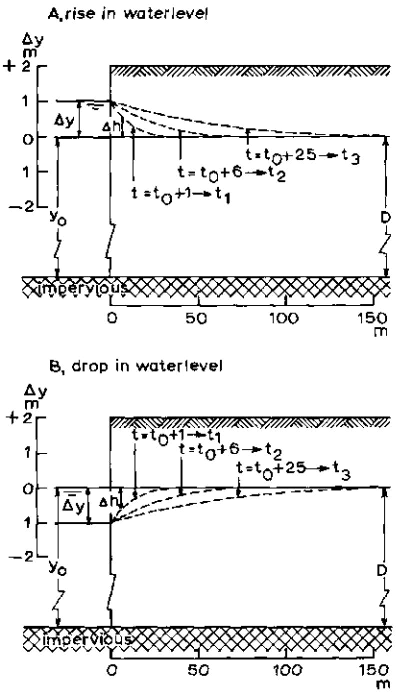


Fig.8.
Changing water table in an unconfined aquifer. A) after an instantaneous rise of the water level in the ditch, and B) after an instantaneous drop of the water level in the ditch.

EDELMAN (1947) showed that the solution of Eq.33 can be expressed in terms of two new variables

$$T = \frac{KD}{\mu} t \qquad (34)$$

$$u = \frac{x}{2\sqrt{T}} = \frac{x}{2\sqrt{(KDt/\mu)}} \qquad (35)$$

The solution then becomes

$$\Delta h = (y_o - h) = - \Delta y \text{ erfc}(u) = - \Delta y\ f_o(u),\ \Delta y > 0 \tag{36}$$

where erfc(u) = 1 − erf(u) is the complementary error function (JAHNKE and EMDE, 1945). Values of the function $f_o(u)$ are given in Table 1.

The flow per unit length of ditch at any distance x can be found by differentiating Eq.36 with respect to x and substituting the result in Darcy's equation, which yields

$$q_x = KD(\Delta y)T^{-\frac{1}{2}} \frac{e^{-u^2}}{\sqrt{\pi}} \tag{37}$$

for x = 0, Eq.37 reduces to

$$q_o = \frac{KD(\Delta y)}{\sqrt{\pi}} T^{-\frac{1}{2}} \tag{38}$$

which gives the amount of water flowing into one side of the ditch. The total inflow into the ditch therefore equals $2q_o$, provided that the aquifer extends infinitely at both sides of the ditch.

If the water level in the ditch is raised instantaneously over a distance Δy, a rise of the water level in the soil may be expected. Since a rise can be regarded as a negative fall, Δy is taken negative so that Eq.36 must be read as

$$\Delta h = + \Delta y\ f_o(u),\ \Delta y < 0 \tag{36a}$$

because $f_o(u)$ is negative.

The equations can be used to calculate either the change in water table elevation in the soil (if the hydraulic properties of the aquifer are known) or to calculate the hydraulic properties (if the change in water table elevation has been measured in a row of piezometers).

Example 2

We shall suppose an unconfined aquifer with a saturated thickness D = 10 m, a hydraulic conductivity K = 1 m/day, and an effective porosity μ = 0.1. This

Table 1. Values for $f_o(u)$, $f_1(u)$ and $f_2(u)$.

u	$\frac{1}{\sqrt{\pi}} e^{-u^2}$	$f_o(u)$	$f_1(u)$	$f_2(u)$
0.000	0.5642	- 1.0000	1.1284	- 1.0000
0.025	0.5639	- 0.9717	1.0794	- 0.9448
0.050	0.5628	- 0.9436	1.0312	- 0.8920
0.075	0.5611	- 0.9155	0.9849	- 0.8416
0.100	0.5586	- 0.8875	0.9397	- 0.7935
0.125	0.5555	- 0.8596	0.8960	- 0.7476
0.150	0.5517	- 0.8320	0.8537	- 0.7039
0.200	0.5421	- 0.7773	0.7732	- 0.6227
0.250	0.5300	- 0.7237	0.6982	- 0.5497
0.300	0.5157	- 0.6714	0.6285	- 0.4829
0.350	0.4992	- 0.6206	0.5639	- 0.4232
0.400	0.4808	- 0.5716	0.5042	- 0.3699
0.450	0.4608	- 0.5245	0.4495	- 0.3222
0.500	0.4394	- 0.4795	0.3993	- 0.2799
0.550	0.4169	- 0.4367	0.3534	- 0.2423
0.600	0.3936	- 0.3961	0.3119	- 0.2090
0.650	0.3698	- 0.3580	0.2741	- 0.1798
0.700	0.3457	- 0.3222	0.2402	- 0.1540
0.750	0.3215	- 0.2888	0.2097	- 0.1315
0.800	0.2975	- 0.2579	0.1824	- 0.1120
0.850	0.2740	- 0.2293	0.1581	- 0.0949
0.900	0.2510	- 0.2031	0.1364	- 0.0803
0.950	0.2288	- 0.1791	0.1173	- 0.0677
1.000	0.2076	- 0.1573	0.1005	- 0.0568
1.050	0.1874	- 0.1376	0.0857	- 0.0476
1.100	0.1683	- 0.1198	0.0729	- 0.0396
1.150	0.1504	- 0.1039	0.0617	- 0.0329
1.200	0.1337	- 0.0897	0.0520	- 0.0273
1.250	0.1183	- 0.0771	0.0438	- 0.0224
1.300	0.1041	- 0.0660	0.0366	- 0.0184
1.350	0.0912	- 0.0562	0.0307	- 0.0148
1.400	0.0795	- 0.0477	0.0253	- 0.0122
1.450	0.0698	- 0.0403	0.0209	- 0.0100
1.500	0.0595	- 0.0390	0.0172	- 0.0081
1.600	0.0436	- 0.0237	0.0114	- 0.0055
1.700	0.0314	- 0.0162	0.0076	- 0.0032
1.800	0.0221	- 0.0109	0.0050	- 0.0020
1.900	0.0153	- 0.0072	0.0031	- 0.0012
2.000	0.0104	- 0.0047	0.0020	- 0.0007
2.100	0.0069	- 0.0030	0.0012	- 0.0005
2.200	0.0045	- 0.0019	0.0007	- 0.0003
2.300	0.0029	- 0.0011	0.0005	- 0.0001
2.400	0.0018	- 0.0007	0.0003	
2.500	0.0011	- 0.0004	0.0002	

aquifer is cut by a canal. At $t < 0$ the water level in the canal and the water table in the aquifer have the same elevation. At $t = 0$ the water level in the canal is raised 1 metre, i.e. $\Delta y = 1$. What will be the rise of the water table at various distances from the canal after 25 days? The transmissivity of the aquifer $KD = 1 \times 10 = 10\ m^2/day$ is assumed to be constant, although with the rise of the water table the value of D, and hence of KD, changes on the average from 10 to 10.5.

According to Eq.34

$$T = \frac{KD}{\mu} t = \frac{10}{0.1} t = 100\ t$$

With t = 25 days, Eq.35 yields

$$u = \frac{x}{2\sqrt{T}} = \frac{x}{2\sqrt{100 \times 25}} = 0.01\ x$$

For various values of x, the value of u is calculated and the corresponding value of $f_o(u)$ is read from Table 1.
Substitution of these values into Eq.36 yields the rise of the water table after 25 days at the selected distances from the canal (Table 2, Fig.8A).

Table 2. Calculation of rise of water table of Example 2.

Distance in metres	u	$f_o(u)$ from Table 1	watertable rise (in metres)
10	0.1	- 0.8875	0.89
20	0.2	- 0.7773	0.78
40	0.4	- 0.5716	0.57
60	0.6	- 0.3961	0.40
80	0.8	- 0.2579	0.26
100	1.0	- 0.1573	0.16

Example 3

The analysis of the change in water table caused by a sudden rise or fall of the water level in a canal makes it possible to determine the hydraulic properties of the aquifer. For this purpose the effect on the water table is measured in a row of piezometers perpendicular to the canal. We shall suppose that $\mu = 0.1$ and that the piezometers are located at distances of 10, 20 and 40 metres from the canal. At $t < 0$ the water table has the same elevation as the water level in the canal. At $t = 0$ the water level in the canal is raised over a distance $\Delta y = 0.5$ m. Water table measurements are then made at different times, yielding the results given in Table 3.

Table 3. Observed rise of the water table Δh in three piezometers.

Distance (metres)	Time since rise (days)				
	$t = 0.5$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
10	0.25	0.29	0.32	0.34	0.35
20	0.13	0.19	0.25	0.26	0.27
40	0.035	0.065	0.125	0.165	0.19

From the data in Table 3 we get

for $x = 10$ metres

$\frac{x}{\sqrt{t}}$	14.2	10	7.1	5.8	5
$\frac{\Delta h}{\Delta y}$	0.50	0.58	0.64	0.68	0.70

for $x = 20$ metres

$\frac{x}{\sqrt{t}}$	28.2	20	14.2	11.6	10
$\frac{\Delta h}{\Delta y}$	0.26	0.38	0.50	0.52	0.58

for $x = 40$ metres

$\frac{x}{\sqrt{t}}$	56.8	40	28.4	23.2	20
$\frac{\Delta h}{\Delta y}$	0.047	0.13	0.25	0.33	0.38

We then plot the values of $\Delta h/\Delta y$ against $x/\sqrt{t}$ on double logarithmic paper (observed data curve).
Next we prepare a master chart by plotting on the same type of paper $f_o(u)$ versus u. We then match the observed data curve and the master chart curve (Fig.9). As match point we select the point z with master chart coördinates u = 0.1, $f_o(u) = -1.0$, giving $x/\sqrt{t} = 4$ and $\Delta h/\Delta y = 0.8$.

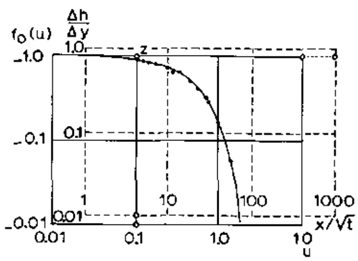


Fig.9.
Observed data curve $\Delta h/\Delta y$ versus $x/\sqrt{t}$ superimposed on type curve $f_o(u)$ versus u.

Substitution of these values and the value of $\mu = 0.1$ into Eq.35 yields

$$\frac{KD}{\mu} = \frac{x}{2\sqrt{t}}\,\frac{1}{u} = \frac{4}{2} \times \frac{1}{0.1} = 20$$

Hence, $KD = 400 \times 0.1 = 40\ m^2/day$.

According to Eq.36a

$$\frac{\Delta h}{\Delta y} = f_o(u)$$

If $f_o(u) = -1$, $\Delta h = \Delta y$. This occurs at the canal border only where x = 0. However, from the coördinates of the match point it follows that for $f_o(u) = -1$, $\Delta h/\Delta y = 0.8$. This means that $\Delta h_o = 0.8\Delta y = 0.8 \times 0.5 = 0.4$ metre. The difference $\Delta y - \Delta h_o = 0.5 - 0.4 = 0.1$ m is the head loss due to radial flow, which occurs because the canal does not cut the whole aquifer.

According to Eq.38, the loss of water from one side of the canal per metre length is

$$q_o = KD\,\frac{\Delta h_o}{\sqrt{\pi}}\,T^{-\frac{1}{2}} = \frac{\Delta h_o}{\sqrt{\pi}} \times \sqrt{\frac{KD\mu}{t}} = \frac{0.4}{\sqrt{3.14}} \times \sqrt{\frac{40 \times 0.1}{t}}$$

It must be remarked that neither water losses by evapotranspiration nor additional supplies from rainfall have been taken into account. A field experiment, therefore, should not last longer than say two or three days in order to prevent errors caused by rainfall or evaporation.

13.3.2 AFTER A STEADY CHANGE IN WATER LEVEL

In the foregoing section a solution was given for the flow out of, or into, an unconfined aquifer after an instantaneous change in the water level of a ditch or canal penetrating the whole thickness of the aquifer. In this section a solution will be given for the situation where the change in the water level is proportional to time, in other words, the level changes at a linear rate, denoted by α. Hence

$$\Delta y = \alpha t \tag{39}$$

so that the initial and boundary conditions for which Eq.33 must be solved are

$$h = y_o, \quad 0 < x < \infty, \quad t < 0$$
$$h = y_o - \alpha t, \quad x = 0, \quad t > 0$$

Introducing again the variables T and u (Eqs.34 and 35) and moreover

$$\alpha' = (\mu/KD)\alpha \tag{40}$$

so that

$$\Delta y = \alpha t = \alpha(\mu/KD)T = \alpha' T$$

the boundary conditions read

$$h = y_o - \alpha' T, \text{ at } x = 0 \text{ and } T > 0$$

The solution then becomes

$$\Delta h = y_o - h = -\alpha' T\, f_2(u), \ \Delta y > 0 \tag{41}$$

$$q_x = \alpha' T^{\frac{1}{2}} KD\, f_1(u) \tag{42}$$

where $f_1(u) = df_o(u)/du$ and $f_2(u) = df_1(u)/du$. As with Eq.36a, this solution is also valid for a rising water table. In that case Δy must be taken negative and the solution reads

$$\Delta h = \alpha' T\, f_2(u), \quad \Delta y < 0 \tag{41a}$$

For $x = 0$, $u = 0$ and $f_1(u) = 1.13$, so that Eq.42 reduces to

$$q_o = 1.13\ \alpha' T^{\frac{1}{2}} KD \tag{43}$$

giving the flow per unit length out of, or into, one side of the canal.

Example 4

Let us suppose that in the situation described in Example 2 the water level in the canal had not been raised instantaneously at $t = 0$ but in such a way that a rise of 1.00 metre was reached after 25 days. Supposing again that $KD/\mu = 100$, we find from Eq.39

$$\alpha = \Delta y/t = 1/25 = 0.04$$

and from Eq.40

$$\alpha' = \alpha \frac{\mu}{KD} = 0.04 \times \frac{0.1}{10} = 0.0004$$

The rise of the water table at, say, a distance of 25 metres from the canal is found (Table 4) by computing u for various values of t and reading the corresponding values of $f_2(u)$ from Table 1. Substituting this value into Eq.41 gives the rise of the water table.

Table 4. Rise of water table at x = 25 m.

Time since t_0 (in days)	$F = 100t$	$u = \frac{25}{2\sqrt{T}}$	$f_2(u)$	$\alpha' F$	Δh
1	100	1.25	- 0.0224	0.04	0.00
5	500	0.56	- 0.2357	0.20	0.05
10	1 000	0.40	- 0.3699	0.40	0.15
15	1 500	0.32	- 0.4589	0.60	0.28
20	2 000	0.28	- 0.5089	0.80	0.41
25	2 500	0.25	- 0.5497	1.00	0.55

According to Eq.43, the water losses from one side of the canal per metre length on the fifth day are

$$q_o = 1.13\ \alpha' T^{\frac{1}{2}} KD = 1.13 \times 0.0004\ \sqrt{500} \times 10 = 0.1\ m^2/day$$

In a way similar to that illustrated in Example 3, the hydraulic properties of the aquifer can be calculated by matching a master chart curve u versus $f_2(u)$, and an observed data curve $\Delta h/\Delta y$ versus $x/\sqrt{t}$.

13.4 TRANSMISSION OF WAVES

The water level in a body of open water sometimes shows a regular change in the form of a train of sinusoides (e.g. tidal waves). If such an open water body is in direct contact with an aquifer of finite extent and constant thickness, the sinusoidal movement of the open water level will be propagated into the aquifer and piezometric readings will show a similar movement. However,

- the amplitude of the sinusoides diminishes with increasing distance from the open water (damping), and
- there is a certain time lag with which the highest and lowest levels are recorded (phase shift). The time lag increases with the distance from the open water.

It is clear that there must be a relationship between the damping and the phase shift on one side and the hydraulic characteristics on the other side. Therefore, the analysis of the propagation of waves allows these characteristics to be determined. The only data required for this purpose are piezometric data at various distances from the open water. The observations must cover at least half a cycle so that phase shift and damping can be determined. Preferably, several full cycles should be observed because the damping and phase shift may be different for the maximum and the minimum of the curve, and average values should then be used.

The sinusoidal movement of the open water may be described by

$$y_o = y_m + A \sin nt \tag{44}$$

where

y_o = water level with respect to a certain reference level (m),
y_m = mean height of the water level with respect to the same reference level (m),
A = amplitude of the wave (m),
n = $2\pi/T$ = frequency (radians/day),
T = time required for a full cycle (days).

The reduced sinusoidal movement of the hydraulic head in an aquifer at a distance x from the open water and at a time t can, according to STEGGEWENTZ (1933), be described as

$$h(x,t) = h_m + A\ e^{-ax} \sin(nt - bx) \tag{45}$$

where
$h(x,t)$ = hydraulic head in the aquifer at distance x and at time t (m),
h_m = mean hydraulic head in the aquifer at distance x (m),
bx = phase shift (m),
e^{-ax} = amplitude reduction factor (dimensionless).

Both amplitude reduction and phase shift depend on the distance x (x is taken zero at the boundary of the open water).
Substitution of the above-mentioned quantities into the differential equation describing the groundwater flow yields a relation between the constants a and b and the hydraulic characteristics of the aquifer.

13.4.1 WAVE TRANSMISSION IN PHREATIC AQUIFERS

STEGGEWENTZ (1933) found for the relationship between a, b, and the hydraulic characteristics in a phreatic aquifer

$$a = b = \left(\frac{\mu n}{2KD}\right)^{\frac{1}{2}} \tag{46}$$

It should be noted that in a phreatic aquifer the damping and the phase shift are the same. If this is not so, the aquifer must be semi-confined.

13.4.2 WAVE TRANSMISSION IN SEMI-CONFINED AQUIFERS

BOSCH (1951), taking into account the compressibility of both water and aquifer material, showed that in a semi-confined aquifer with constant h' the following relations hold

$$a^2 - b^2 = \frac{1}{KDc} \tag{47}$$

$$2ab = \frac{nS}{KD} \tag{48}$$

where S = the storage coefficient of the aquifer defined by

$$S = \rho gD \left(\frac{\varepsilon}{E_w} + \frac{1}{E_S}\right) \tag{49}$$

where

ρ = density of the water

g = acceleration of terrestrial gravity

D = thickness of the aquifer

ε = porosity of the aquifer

E_w = modulus of elasticity of water

E_S = modulus of elasticity of the aquifer material

Example 5

In a semi-confined aquifer along the river Waal (The Netherlands), which is influenced by the tide of the North Sea, the groundwater fluctuations caused by the tidal movement in the river have been measured. The hydrographs of some of the piezometers are shown in Fig.10. From these hydrographs the amplitude is read and, by comparing the hydrographs of the piezometers with the hydrograph of the river, the time lag for each piezometer is determined. To express the phase shift in radians, the time lag t is multiplied by $2\pi/T$.

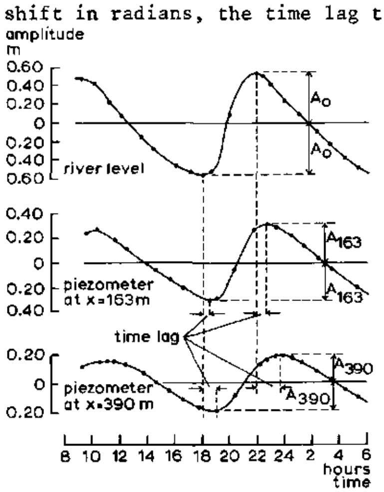


Fig.10.
Hydrographs of the river Waal, The Netherlands, and a row of piezometers, showing the transmission of tidal waves (DE RIDDER et al., 1962).

It should be noted that the time lag after low-tide is less than that after high tide. The average time lag and the average amplitude are used in the calculations.

From Eq.45 it is clear that the amplitude A_o at $x = 0$ and the amplitude A_x at any arbitrary value of x are related by

$$A_x = A_o\ e^{-ax}$$

$$\frac{A_x}{A_o} = e^{-ax}$$

or

$$2.30\ \log\ (\frac{A_x}{A_o}) = -\ ax \qquad (50)$$

Hence the value of a may be found as the slope of a straight line, which is obtained by plotting A_x/A_o versus x on single-logarithmic paper (A_x/A_o on the logarithmic scale). Theoretically, this straight line should pass through the origin, but this scarcely ever happens due to the influence of entrance resistances near the river. In the example shown in Fig.11 the difference, Δx of x per log cycle of A_x/A_o, is 800 m. Hence, according to Eq.50

$$a = \frac{2.30}{800} = 2.87 \times 10^{-3}$$

The phase shift $\frac{2\pi}{T}t$ is plotted versus x on linear paper, giving a straight line from which b can be determined. The value of b is the ratio between the phase shift and an arbitrarily chosen distance.
In the example of Fig.11

$$b = \frac{0.9}{600} = 1.5 \times 10^{-3}$$

With a and b known, it is possible to calculate $\lambda = \sqrt{KDc}$ from Eq.47

$$\lambda = \frac{1}{a^2 - b^2} = \frac{1}{(2.87 \times 10^{-3})^2 - (1.5 \times 10^{-3})^2} \simeq 410\ \text{m}$$

and to calculate S/KD from Eq.48

$$\frac{S}{KD} = \frac{2ab}{n} = \frac{2 \times 2.87 \times 10^{-3} \times 1.5 \times 10^{-3}}{2 \times 3.14/0.5} \simeq 0.68 \times 10^{-6}\ \text{day/m}^2$$

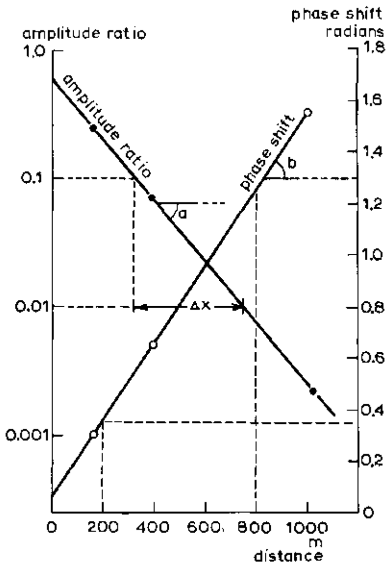


Fig.11.
Analysis of the amplitude and phase shift data (DE RIDDER et al., 1962).

13.5 LOSSES OF WATER FROM DITCHES TOWARDS A GROUNDWATER TABLE

13.5.1 LOSSES OF WATER TOWARDS A DEEP GROUNDWATER TABLE

In an irrigation ditch the water level is often high compared with the water table in the surrounding soil, and losses of water are thus inevitable. We will consider here the situation in which the ditch lies in a soil with a relatively low permeability ($0.5 < K < 2$) and a deep water table (Fig.12). It should be noted that if the hydraulic conductivity of the soil is very low, the water table will rise till it equals the water level in the ditch; if the hydraulic conductivity is very large the losses will be so great that the ditch will run dry.

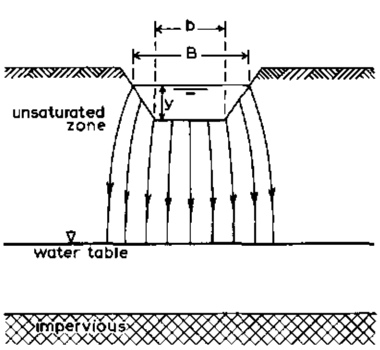


Fig.12.
Losses from a ditch to a deep water table.

WEDERNIKOW, as cited by MUSKAT (1937, p.331), showed that

$$q = \frac{\pi s y}{\cos^{-1} k^{*}} = K(B + 2y \frac{I}{I'}) \tag{51}$$

where

q = loss of water per unit length of ditch ($m^3\ m^{-1}\ day^{-1}$),

y = height of water level in the ditch (m),

B = width of ditch at water level (m),

b = bottom width of ditch (m),

$s = \frac{B - b}{2y}$ = side slope of the ditch (horizontal/vertical),

K = hydraulic conductivity (m day^{-1}),

I and I' are complete elliptic integrals of the first kind with moduli k^{*} and $\sqrt{1 - k^{*2}}$ respectively, and

$$k^{*} = \sin \frac{\pi}{q} (\frac{q}{2} - \frac{B}{2} + \frac{b}{2}) = \cos \frac{\pi s y}{q} \tag{52}$$

A simple solution of Eq.51 can be obtained by using the diagram of Fig.13.

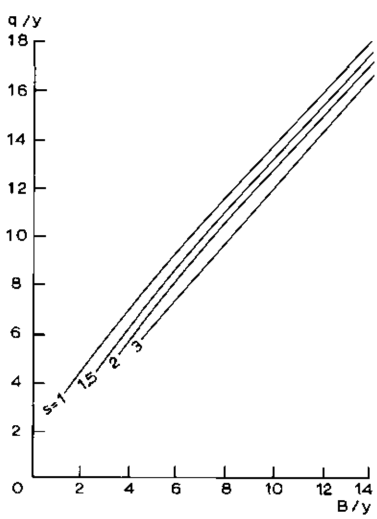


Fig.13.
Diagram for the analysis of water losses from a ditch to a deep groundwater table.

The procedure to be followed in constructing this diagram is:

- Choose values of q/y and calculate k^{*} for given values of s using Eq.52,
- Read the values of I and I' from a table of these functions (DWIGHT, 1947),
- Calculate the corresponding value of B/y, which for this purpose is written as

$$\frac{B}{y} = \frac{q}{y} - 2\frac{I}{I'}$$

(Note that K = 1 m/day)

- Plot, for given values of s, q/y against B/y.

With Eq.51 the losses are calculated if K is known, or the value of K is calculated if q is known.

Example 6

An irrigation ditch with top width B = 4 m, bottom width b = 2 m, and a water depth y = 1 m, lies in a soil that has a hydraulic conductivity of 0.8 m/day. What is the loss of water per unit length of ditch?
From these data we calculate

$$s = \frac{B - b}{2y} = \frac{4 - 2}{2 \times 1} = 1 \qquad \text{and} \qquad \frac{B}{y} = 4$$

From the diagram we find the corresponding value of q/y = 6.75. Hence, for K = 1 m/day, q = 6.75 × 1 = 6.75 m^3/day per metre of ditch; for K = 0.8 m/day, q = 0.8 × 6.75 = 5.40 m^3/day per metre of ditch.

Example 7

To maintain the water level in a 10 m long ditch at 1 m above the bottom (hence y = 1 m), water is pumped into this ditch from a nearby canal through a pipe line with a built-in water meter and valve. The top width and bottom width of the ditch are respectively B = 4 m and b = 2 m. The volume of water per day, necessary to maintain the water level is measured by reading the water meter, Q = 33.7 m^3. What is the hydraulic conductivity of the soil?
For Q = 33.7 m^3/day, it follows that q_{obs} = Q/10 = 3.37 m^3/day per metre ditch. Since B = 4 m and b = 2 m, it follows that s = 1 and B/y = 4, whence for K = 1, q/y = 6.75 or q = 6.75. Hence

$$K = \frac{6.75}{q_{obs}} = \frac{6.75}{3.37} = 0.5 \text{ m/day}$$

13.5.2 LOSSES OF WATER TOWARDS A SHALLOW GROUNDWATER TABLE

Consider a ditch with a water level higher than the water table in the adjoining area (Fig.14) but at shallower depth than that considered in Fig.12. To find a solution to this problem, MUSKAT (1937) divided the flow region into two parts, I and II (Fig.14).

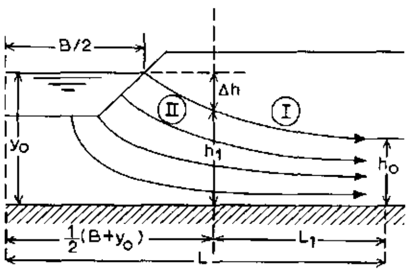


Fig.14.
Water losses from a ditch to a shallow water table.

In Region I he considered the flow to be horizontal. Hence

$$\frac{q}{2} = K \left(\frac{h_1 - h_o}{L_1}\right) \left(\frac{h_1 + h_o}{2}\right) \tag{52}$$

where q/2 is the flow per unit length from one side of the ditch. In Region II the flow may be written as

$$\frac{q}{2} = K\ (y_o - h_1)f \tag{53}$$

where f is a factor depending on the geometry of both canal and aquifer.

Elimination of the unknown h_1 from Eqs.52 and 53 gives

$$\frac{q}{2} = Kf \left[y_o + fL_1 - \{(y_o + fL_1)^2 - y_o + h_o^2\}^{\frac{1}{2}} \right] \tag{54}$$

To obtain the value of f, the flow in Region II is replaced by the flow from a finite line source of length B/2. For this flow the distribution of the potentials and the streamlines is given by

$$h + i\Psi = \log(\sinh z + \sqrt{\sinh^2 z - \sinh^2 f}$$

where $z = x + iy$, h the potential and Ψ the stream function (Chap.6, Vol.I). By choosing various values of B/2 and y_o, the potential distribution and the streamline pattern can be computed and the corresponding values of h_1 and f can be read.

The result is given here in curves for constant f values and variable B/y_o and h_1/y_o (Fig.15). A distinction has been made between shallow ditch cross-sections ($B/u > 0.9$) and deep cross-sections ($B/u < 0.9$). The diagrams give a complete solution to the problem. Since the flow in both regions of Fig.14 must be the same, a certain value of h_1 is chosen. The procedure to be followed is then

- Compute B/u and choose the proper diagram,
- Choose a value of h_1 and compute h_1/y_o and B/y_o,
- Read the appropriate value of f from Fig.15,
- Substitute the values of h_o, h_1, y_o, f and L_1 into Eqs.52 and 53 and solve for q,
- If different q values are found, repeat the procedure with the adjusted h_1 value.

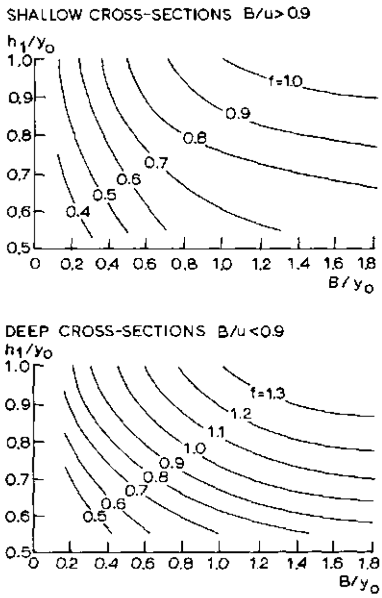


Fig.15.
Diagram for the analysis of water losses from a ditch to a shallow water table.

Example 8

Assume a ditch with top width B = 3 m, bottom width b = 1 m, water depth y_o = 1 m, and h_o = 4 m above an impermeable layer. At a distance L = 54 m, the water table height h_o is 4 m above the impervious layer.

- Compute the wetted perimeter $u = 1 + 2 \times 1.41 = 3.82$ m. Hence

$B/u = 3/3.82 = 0.76$ m

$(B + y_o)/2 = (3 + 5)/2 = 4$ m

$B/y_o = 3/5 = 0.6$ m

$L_1 = L - (B + y_o)/2 = 54 - 4 = 50$ m.

- Assume $h_1 = 4.8$ m. Then $h_1/y_o = 4.8/5 = 0.96$ m.
- Read from the diagram in Fig.15 $(B/u > 0.9) f = 1.08$.
- Substitute f into Eqs.52 and 53 giving

$$\frac{q}{2} = K \left(\frac{4.8 - 4.0}{5.0}\right) \left(\frac{4.8 + 4.0}{2}\right) = 0.0704 \text{ K}$$

$$\frac{q}{2} = K(5.0 - 4.8)1.08 = 0.216 \text{ K}$$

Apparently h_1 has been chosen too low, giving too high a value of q/2 in Region II. Therefore, choose $h_1 = 4.9$ m, giving $h_1/y_o = 0.98$ and $f = 1.1$. Substitution then gives

$$\frac{q}{2} = 0.0801 \text{ K}$$

$$\frac{q}{2} = 0.11 \text{ K}$$

Although closer, the result is not yet satisfactory, so the procedure should be repeated with $h_1 = 4.95$ m.

LITERATURE

BOSCH, H. 1951. Geohydrologisch onderzoek Bergambacht (unpublished research report).

DE RIDDER, N.A., BLOK, T. and COLENBRANDER, H.J. (editors). 1962. De waterbehoefte van de Tielerwaard-West, Interimrapport van werkgroep I van de Commissie ter bestudering van de waterbehoefte van de Gelderse landbouwgronden, 135 p.

DWIGHT, H.B. 1947. Tables of integrals and other mathematical data, 3rd. edition: MacMillan, New York, 288 p.

EDELMAN, J.H. 1947. Over de berekening van grondwaterstromingen, Thesis Delft (mimeographed).

JAHNKE, E. and EMDE, F. 1945. Tables of functions, 4th ed. Dover Publ., New York, 304 p.

MAZURE, J.P. 1936. Geohydrologische gesteldheid van de Wieringermeer. Rapp. en Meded. Zuiderzeewerken 5: 67-131.

MUSKAT, M. 1937. The flow of homogeneous fluids through porous media. McGraw-Hill Book Comp., New York, 763 p.

STEGGEWENTZ, J.H. 1933. De invloed van de getijbeweging van zeeën en getijrivieren op de stijghoogte van het grondwater. Thesis Delft, 138 p.

Suggestions for further reading

ARAVIN, V.I. and NUMEROV, S.N. 1965. Theory of fluid in undeformable porous media. Israel Program for Scientific Translations, Jerusalem, 511 p.

BAER, J., ZASLAVSKY, D. and IRMAY, S. 1968. Physical principles of water percolation and seepage. Arid Zone Research XXIX, UNESCO, Paris, 465 p.

BOUWER, H. 1969. Theory of seepage from open channels. Adv. Hydro Science, Editor Ven te Chow. Acad. Press, New York, 5: 121-173.

DE RIDDER, N.A. and WIT, K.E. 1967. Seepage flow analysis of a small polder in the SW part of The Netherlands. Journ. Hydrol., 5: 47-57.

EDELMAN, J.H. 1972. Groundwater hydraulics of extensive aquifers. Int. Inst. Land Recl. and Improvement, Bull. 13. 216 p.

HARR, M.E. 1962. Groundwater and seepage. McGraw-Hill Book Comp., New York, 315 p.

POLUBARINOVA-KOCHINA, P.Ya. 1962. Theory of groundwater movement. Princeton Univ. Press., Princeton, 613 p.

WESSELING, J. 1959. The transmission of tidal waves in elastic artesian aquifers. Neth. Journ. Agr. Science 7: 22-32.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

14. DRAINAGE BY MEANS OF PUMPING FROM WELLS

N. A. DE RIDDER
Geohydrologist
International Institute for
Land Reclamation and Improvement

Lecturers in the Course on Land Drainage

J. H. Edelman (1968–1971)
Consulting Engineer, Grenoble

N. A. de Ridder (1972)
International Institute for Land Reclamation and Improvement

PURPOSE AND SCOPE

Well drainage can be regarded as an alternative to gravity drainage. Some theoretical and practical aspects of well drainage are outlined.

CONTENTS

14.1 INTRODUCTION

The usual method of draining land is by gravity drainage, i.e. a system of field drains (open ditches or pipe drains), transport canals, and, if the water cannot be disposed of by gravity, a pumping station at the outlet. An alternative method of drainage is to lower the water table by pumping from wells (Chap. 12, Vol. II). The use of this method, however, is much more restricted by the geohydrological conditions of the area, and it cannot be regarded merely as a substitute for gravity drainage.

Unlike gravity drainage, which has been practised in various forms for hundred of years, the technique of well drainage is a comparatively recent development, and the number of projects where well drainage is being applied is still small. Notable examples are found in California (U.S.A.) where some projects date back to as early as 1918 (PETERSON, 1957). Other examples are the Indus Plain in West Pakistan, where multiple well systems have been installed for water table and salinity control (ANONYMOUS, 1964), the Hunger Steppe, Uzbekistan (USSR), where the first wells were sunk in 1926 but did not give good results (MICHAELSON, 1967), and the Ararat Plain (USSR), where some 2000 hectares are being drained by 25 wells (ANANIAN et al., 1969).

Some aspects of well drainage will be discussed in this chapter; these include its advantages and disadvantages, the problem of the distribution of hydraulic head when more than one well pumps the same aquifer, well spacing and drainage criteria, different aquifer conditions, and such limiting factors as hydraulic characteristics of aquifers and confining layers.

14.2 ADVANTAGES OF WELL DRAINAGE

Well drainage has certain advantages over gravity drainage. These are:

- On undulating land with local depressions not having natural outlets, the pumped water is generally disposed of through pipe lines connecting the various wells. Excessive earth-moving is thus avoided, as no deep canals need be dug through topographic ridges. Also, without such canals and ditches more efficient farming operations can be introduced.
- The cost of maintaining the pipe line system may be considerably less than for open drains and transport canals.
- Well drainage enables the groundwater table to be lowered to a much greater depth than does gravity drainage. This means that a greater portion of the

excess water can be stored before it has to be removed, whilst in arid and semi-arid regions a deeper groundwater table reduces salinisation of the soil.

- The deeper layers, or substrata, may be much more pervious than the layers near the surface (Chap. 1, Vol. I). Pumping from these layers may reduce the artesian pressure that is often present, creating instead a vertical downward flow through the upper layers. If the pervious substrata are found at a depth of 5 m or more, it is only with well drainage that full benefit can be derived from these favourable geohydrological conditions.
- If the water in the pumped aquifer is of good quality, it can be used for irrigation. The drainage water then has an economic value and this fact may contribute considerably to the economic feasibility of the venture.

14.3 DISADVANTAGES OF WELL DRAINAGE

Well drainage also has certain disadvantages when compared with gravity drainage. To mention a few:

- A pumped well is a more complex engineering structure than an open drain or tile line and is therefore more difficult and costly to construct, maintain, and operate.
- The energy required for operating a multiple well system must be purchased as electricity or fuel.
- Legal regulations may sometimes forbid the use of pumped wells for drainage of land; pumping from wells may reduce the pressure in aquifers to such an extent that existing domestic wells cease flowing.
- Unlike gravity drainage, well drainage is not economically feasible on small areas because too large a portion of the water drained out of the area then consists of "foreign" water, i.e. groundwater inflowing from the surrounding areas.
- If, during the growing season, the water table rises to the land surface (due for instance to a heavy rainstorm after irrigation), it must be lowered rapidly because most crops can only stand waterlogging for a limited time. This implies a high drainage rate, i.e. a dense network of wells. (Of course, the high investment costs of installing such a dense network of wells can be reduced by spacing the wells farther apart and pumping them continuously, but this in turn will raise operation and maintenance costs of the wells.)
- Well drainage can only be applied successfully if the aquifer characteristics are favourable, i.e. if the transmissivity of the aquifer is fairly high; only

then can the wells be widely spaced. If the aquifer is semiconfined, (i.e. an upper layer of clay overlying a sandy aquifer) an additional criterion is the value of the hydraulic resistance of the upper clay layer. This value must be low enough to ensure an adequate percolation rate. Hence, a decision in favour of well drainage should only be taken after a careful hydrogeological investigation has proved that its application is practicable.

- Well drainage may not be technically and economically feasible in those areas where the artesian pressure in the aquifer to be pumped is too high or seepage is excessive.

14.4 WATER TABLE AND DISCHARGE CRITERIA

In discussing drainage by means of wells, it may be useful to recall the water table and discharge criteria for arable land (see also Chap. 11, Vol.II). During the off-season the water table should be maintained at a depth of at least 0.50 m below the ground surface, though no great harm will be done if it rises incidentally to higher levels and stays there for a few days. In the planting period the water table should be at a depth of at least 0.75 m below the ground surface. During the greater part of the growing season the water table should be deep enough to prevent it from rising into the rootzone of the crops after irrigation or rainfall. If, nevertheless, it does rise into the rootzone, it must be lowered at a rate indicated roughly in Fig.1.

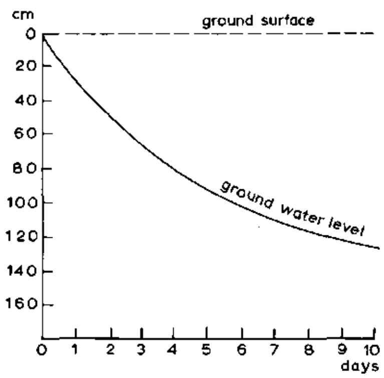


Fig. 1. Rate of water table lowering required for effective drainage of land.

Let us consider a hypothetical case where the water table has risen to the ground surface. If the effective porosity of the soil is 10 per cent, 30 mm of water must be discharged during the first day to induce the required 0.30 m drop in water table. Within two days 50 mm of water must be removed to lower the water table to the required depth of 0.50 m at the end of the second day. It is clear that a very dense network of wells would be necessary to satisfy these heavy criteria.

A much more favourable situation occurs if, after heavy rainfall, the water table does not rise to the land surface. Let us consider the case that the water table rises to, say, 0.80 m below the ground surface. Then, as Fig. 1 shows, it should be lowered approximately 0.23 m within two days after the cessation of rain, corresponding to a discharge rate of 23 mm per two days. A less dense network of wells will be required to satisfy these criteria. Hence maintaining the water table at an average level deep enough to create sufficient storage in the upper soil layers allows for a wider spacing of the wells.

14.5 INTERFERENCE OF WELLS

When a well in an extensive aquifer is pumped, the flow to this well is in an unsteady state: the drawdown (cone of depression) expands with time. The flow is said to approach a quasi-steady state if no appreciable additional drawdown is observed beyond a certain distance, r_e, from the well. This distance is called radius of influence of the well. If, however, a source or region of replenishment is intercepted, the flow becomes steady as soon as recharge and discharge balance each other.
Unless the periods of continuous pumping are relatively short and/or the spacing of the wells in a multiple well system is so great that their individual zones of influence do not overlap, the discharge and the drawdown of each well in the system will be affected by those of neighbouring wells. This is called interference of wells. To calculate the drawdown induced by pumping from a multiple well system, the method of superposition may be applied.

14.5.1 METHOD OF SUPERPOSITION

The differential equation describing the two-dimensional flow of ground water in the x, y - plane reads

$$\frac{\delta^2 h}{\delta x^2} \quad \frac{\delta^2 h}{\delta y^2} = 0 \tag{1}$$

where h represents the hydraulic head. For the present considerations it is more convenient to rewrite the differential equation in terms of drawdown s, defined as $s = h_e - h$, where h_e represents the constant value of the hydraulic head when the water is at rest. Thus Eq.1 can be written as

$$\frac{\delta^2 s}{\delta x^2} + \frac{\delta^2 s}{\delta y^2} = 0 \tag{2}$$

The problem is to find a function satisfying Eq. 2 at all points in a certain region, and also satisfying certain conditions at the boundary of this region. Equation 1 is linear because the dependent variable, s, appears in it only to the power one. The equation is also homogeneous because s appears in each term. For such differential equations the principle of superposition applies, which states that a linear combination of the solutions to the equation is also a solution. In other words, if s_1 is a solution and s_2 is another solution, then the linear combination

$$s = C_1 s_1 + C_2 s_2$$

is also a solution (C_1 and C_2 being arbitrary constants). Some examples will be given below.

14.5.2 DRAWDOWN OF INTERFERING WELLS IN AN UNCONFINED AQUIFER

If N fully penetrating wells pump an unconfined aquifer, the drawdown at point P can be found as the sum of the drawdowns due to pumping the individual wells (Fig.2). If the drawdown in the well is small compared with the saturated thickness of the aquifer, the Dupuit-Forchheimer assumptions may be applied (Chap.6, Vol.I). Hence, the drawdown for conditions of steady flow is given as

$$h_e^2 - h^2 = \sum_{i=1}^{N} \frac{Q_i}{\pi K} \ln (r_{e,i}/r_i) \tag{3}$$

where

Q_i = constant discharge of the i-th well (m^3/day),

r_i = distance from P to the i-th well (m),

$r_{e,i}$ = radius of influence of the i-th well (m),

K = hydraulic côndnctivity of the aquifer (m/day),

h_e = undisturbed hydraulic head (m),

h = hydraulic head during pumping (m).

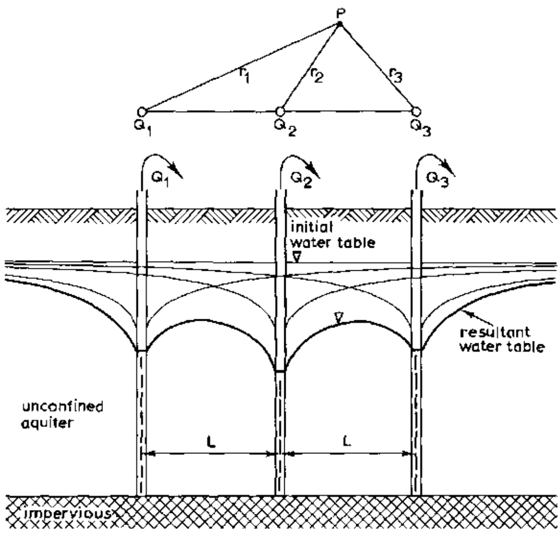


Fig. 2. Individual and composite water tables for three equally spaced pumping wells fully penetrating an unconfined aquifer and set in a straight line. No recharge on the ground surface is assumed.

If $Q_i = Q_N$ = 1/N-th of the total extraction rate (Q) and if all wells have the same radius of influence, $r_{e,i} = r_e$, then Eq. 3 becomes (BEAR et al. 1968)

$$h_e^2 - h^2 = \frac{Q}{\pi K} \ln (r_e/\bar{r}) \tag{4}$$

where

$\bar{r} = (r_1 r_2 r_3 \ldots r_n)^{1/N}$ = equivalent distance from point P.

If two wells fully penetrate an unconfined aquifer, if they are spaced a distance L apart, discharge simultaneously over the same period of time t, and have the same diameter $2r_w$ and drawdown s_w, then their discharges Q_1 and Q_2 can be expressed as (HANTUSH, 1964):

$$Q_1 = Q_2 = \frac{2\pi K(h_e^2 - h_w^2)}{W(r_w^2\mu/4KDt)+W(L^2\mu/4KDt)} \tag{5}$$

where

W = well function for unsteady flow (Chap.12, Vol.II)

h_w = water level in pumped well at time t since pumping started (m)

and other symbols are as defined before.

Similarly, for three wells forming an equilateral triangle with sides L (Fig.3)

$$Q_1 = Q_2 = Q_3 = \frac{2\pi K(h_e^2 - h_w^2)}{W(r_w^2\mu/4KDt) + 2W(L^2\mu/4KDt)} \qquad (6)$$

If the pumping time is long enough, so that $L^2\mu/4KDt < 0.05$, then Eqs.5 and 6 may be replaced by, respectively,

$$Q_1 = Q_2 = \frac{\pi K(h_e^2 - h_w^2)}{\ln(2{,}25\ KD/L\mu r_w)} \qquad (7)$$

and

$$Q_1 = Q_2 = Q_3 = \frac{\pi K(h_e^2 - h_w^2)}{\ln(R^3/L^2 r_w)} \qquad (8)$$

where

$$R = 1.5(KDt/\mu)^{\frac{1}{2}}$$

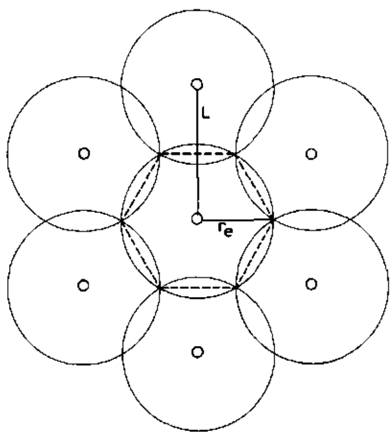


Fig. 3. Wells located in a pattern of equilateral triangles. Well spacing $L=r_e\sqrt{3}$

If four wells are arranged in a square with sides L (Fig.4), and the condition $L^2\mu/2KDt < 0.05$ is satisfied, then the discharge of each of the four wells can be expressed by

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{\pi K(h_e^2 - h_w^2)}{\ln(R^4/r_w L^3\sqrt{2})} \qquad (9)$$

If three wells are spaced a distance L apart along a straight line (Fig.2) and the condition $L^2\mu/KDt < 0.05$ is satisfied, the discharge of each of the outer wells is given by

$$Q_1 = Q_3 = \frac{\pi K(h_e^2 - h_w^2)\ \ln\ (L/r_w)}{2\ \ln\ (R/L)\ \ln\ (L/r_w) + \ln\ (L/2r_w)\ \ln\ (R/r_w)} \quad (10)$$

and the discharge of the inner well by

$$Q_2 = \frac{\pi K(h_e^2 - h_w^2)\ \ln\ (L/2r_w)}{2\ \ln\ (R/L)\ \ln\ (L/r_w) + \ln\ (L/2r_w)\ \ln\ (R/r_w)} \quad (11)$$

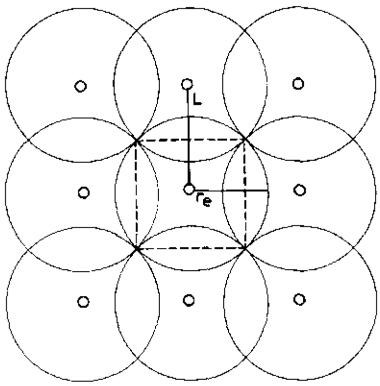


Fig. 4. Wells located in a pattern of squares. Well spacing $L = r_e\sqrt{2}$

Example 1

Seven wells, located at random, fully penetrate an unconfined aquifer. Each well yields 70 l/s and has a radius of influence of 300 m. The hydraulic conductivity of the aquifer K = 40 m/day and its saturated thickness $D = h_e$ = 50 m. The distances from a point P to the wells are r_1 = 50 m, r_2 = 70 m, r_3 = 100 m, r_4 = 60 m, r_5 = 200 m, r_6 = 80 m, and r_7 = 50 m. What is the drawdown at point P if the flow towards the wells has reached a steady state?

The equivalent distance from point P is, according to Eq.4,

$\bar{r} = (50 \times 70 \times 100 \times 60 \times 200 \times 80 \times 50)^{1/7} = {}^1/_7 \log 168 \times 10^{11} = 77.50$ m

Applying Eq. 4, and substituting, we obtain,

$$50 \quad - h^2 = \frac{7 \times 6048}{3.14 \times 40} \ln\ (300/77.50)$$

$$h^2 = 2044.38\ m^2$$

$$h = 45.20\ m$$

The overall effect of the pumping results in a drop in water table at point P of 50 - 45.20 = 4.80 m

Example 2

Three wells fully penetrate an unconfined aquifer whose transmissivity $KD = 2000\ m^2/day$. The wells are located on a straight line, a distance $L = 100$ m apart (Fig.2). Prior to pumping, the horizontal water table is found at 50 m above the impervious base of the aquifer. The effective porosity of the aquifer $\mu = 10$ per cent. The wells have the same radius $r_w = 0.25$ m. After 20 days of continuous pumping, a drop in water table of 3 m is observed in each well. What are the discharges of the inner well and the outer wells?
The condition $L^2\mu/KDt < 0.05$ is satisfied; $(100^2 \times 0.1)/2000 \times 20 = 0.025$, so that Eqs. 10 and 11 are valid. $R = 1.5\ (2000 \times 20/0.1)^{\frac{1}{2}} = 948.7$ m. Applying Eq.11 and substituting, we find the discharge of the inner well as

$$Q_2 = \frac{3.14 \times 40(50^2 - 47^2)\ \ln\ (100/0.5)}{2 \times \ln\ (948.7/100)\ \ln\ (100/0.25) + \ln\ (100/0.5)\ \ln\ (948.7/0.25)}$$

$$= \frac{193431.5}{70.5} = 2745\ \ m^3/day = 32\ l/s$$

The discharge of the outer wells is found from Eq. 10

$$Q_1 = Q_3 = \frac{3.14 \times 40(50^2 - 47^2)\ln\ (100/0.25)}{2 \times \ln\ (948.7/100)\ \ln\ (100/0.25) + \ln\ (100/0.5)\ \ln\ (948.7/0.25)}$$

$$= \frac{218743.1}{70.5} = 3103\ \ m^3/day = 36\ l/s$$

14.6 DEVELOPMENT OF THE HYDRAULIC HEAD DURING SHORT PUMPING PERIODS

If a well in a homogeneous and isotropic aquifer with a horizontal water table is pumped, the cone of depression expands with time. The hydraulic head around the well develops according to Theis's formula (Chap.12, Vol.II).

We shall now consider the case in which a system of wells is to be installed in such an aquifer and that these wells will pump simultaneously but only for a short period. Such a situation may occur in areas where the average water table is deep enough although it may rise incidentally (far) into the rootzone due to heavy rainfall or irrigation losses. It will then be necessary to pump for a certain period to remove the excess water from the rootzone. As was noted earlier, the rate at which this water must be removed depends, among other things, on the height to which the water table has risen into the rootzone. If the wells are to be pumped for a short time only, the question arises what should be their spacing if the required drop rate is to be met.

The problem of the distribution of head when more than one well pumps the same aquifer for a certain period has been investigated by MUSKAT (1934, 1937). He studied the problem for different well patterns: three wells forming an equilateral triangle, four wells in a square pattern, a battery of wells set in a circle, and several other more complex cases. He found that if the wells are not too widely spaced and if they pump simultaneously from a homogeneous unconfined aquifer, the hydraulic head can be expressed as follows

$$h = \frac{Q_o}{2\pi KD}\left[\frac{3}{4} + \log (r/r_e) - \frac{1}{2}(r/r_e)^2 - (2KDt/\mu r_e^2) + 2\sum_{n=1}^{\infty} \frac{I_o(\alpha_n \frac{r}{r_e})\, e^{-\alpha_n^2 (KDt/\mu r_e^2)}}{\alpha_n^2\, I_o^2(\alpha_n)}\right] \quad (12)$$

where

- h = hydraulic head (m),
- Q_o = constant well discharge since time $t = 0$ (m^3/day),
- KD = transmissivity of the aquifer (m^2/day),
- μ = effective porosity of the aquifer (dimensionless),
- r = radial distance from a well, or the distance from a well to the centre of a group of wells (m),
- r_e = radius of influence of a well (m),
- I_o = Bessel function of zero order (dimensionless),
- α_n = value to be found from $I_1(\alpha_n) = 0$, in which I_1 is Bessel function of the first order,
- α_1 = first positive square root,
- α_2 = second positive square root, etc.

In studying the same problem, ERNST (1970) gave a graphical representation of Eq. 12 and this is shown in Fig.5. It can be seen from this diagram that for $\tau = KDt/\mu r_e^2 < 0.1$ Theis's formula for unsteady flow is valid, whereas for $\tau > 0.3$ there is a uniformly descending depression cone of constant form. It can also be seen that the logarithmic development of the cone only occurs within a relatively short distance of the pumped well.

For some specific values of r/r_e, ERNST (1970) also showed how the water table behaves according to Eq.12 (see Fig.6). From this diagram it can be seen that, for $r = r_e$ at $\tau > 0.3$, the following linear equation is valid with good approximation

$$h(r_e) = \frac{Q_o}{2\pi KD} (1/4 - 2\tau) \tag{13}$$

By substituting $\tau = KDt/\mu r_e^2$ and introducing the drawdown s instead of the hydraulic head h, Eq. 13 can be written as follows

$$\frac{\pi \mu r_e^2}{Q_o} s = t - \frac{\mu r_e^2}{8KD} \tag{14}$$

where

- t = time required to induce the desired drawdown (days),
- r_e = radius of influence (m),
- s = drop in water table at time t (m),
- Q_o = constant well discharge since t = 0 (m^3/day),
- KD = transmissivity of the aquifer (m^2/day).

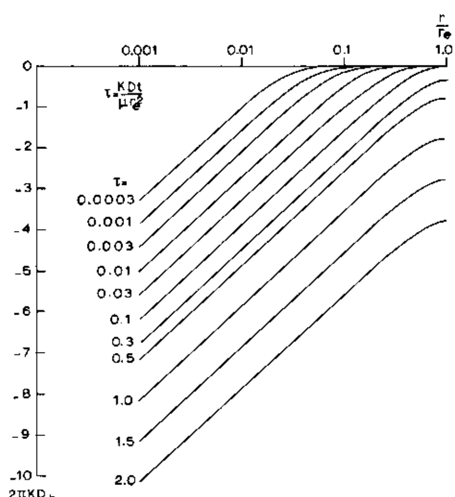


Fig.5. Graphical representation of MUSKAT's formula (after ERNST 1970).

Note that for wells arranged in a square or triangular pattern, their circular zones of influence overlap slightly (Figs.3 and 4). This overlap is larger for wells arranged in a square pattern than for those forming an equilateral triangle. The well spacing is given by, respectively, $L = r_e\sqrt{2}$ and $L = r_e\sqrt{3}$.

<u>Example 3</u>

An unconfined, homogeneous aquifer of large lateral extent has the following hydraulic characteristics: $KD = 3000$ m^2/day and $\mu = 0.10$. Let us assume that heavy rain causes the water table to rise to the land surface. Then, as Fig.1 shows, 50 mm of water must be removed within two days to induce the required 0.50 m drop in water table. If wells are used, each well yielding 100 l/s, what should be their spacing in a triangular pattern? Substitution of the assumed values into Eq.14 gives

$$\frac{3.14 \times 0.1 \times r_e^2}{8640}\, 0.50 = 2 - \frac{0.1 \times r_e^2}{8 \times 3000}$$

r_e = 300 m and L = 300 × 1.73 ≃ 520 m.

One well can drain a surface area

$$\pi r_e^2 = 3.14 \times (300)^2 = 28 \text{ ha}$$

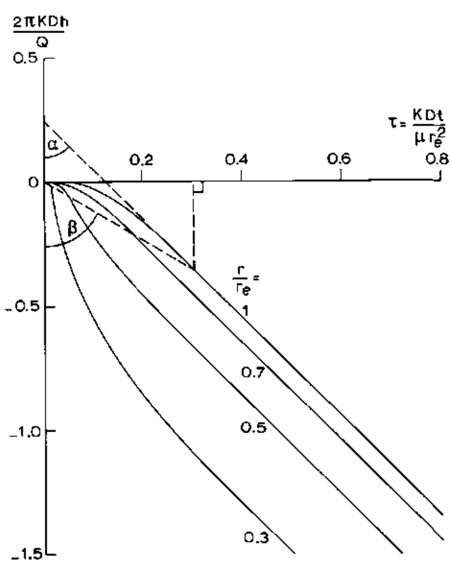


Fig. 6. Behaviour of the water table according to MUSKAT's formula for specific values of r/r_e (after ERNST, 1970).

Example 4

When rainfall in the above example causes the water table to rise to, say, 0.80 m below the land surface, then, as Fig.1 shows, the water table should be lowered approximately 0.23 m within two days after the cessation of rain, corresponding to a discharge rate of 23 mm/day. What should the well spacing be in this case? Assuming all other values to be the same as in Example 3, we find on substitution

$$\frac{3.14 \times 0.1 \times r_e^2}{8640} 0.23 = 2 - \frac{0.1 \times r_e^2}{8 \times 3000}$$

$$r_e = 400 \text{ m and } L = 400 \times 1.73 \approx 690 \text{ m}$$

One well can drain a surface area

$$\pi r_e^2 = 3.14 \times (400)^2 = 50 \text{ ha}$$

The above examples show, that a deep initial water table, by providing a greater storage in the upper soil layers, reduces the required discharge rate and allows the wells to be spaced farther apart.

Equation 14 can also be used to formulate the discharge criterion for a system of wells if these wells are to induce a drop in water table Δs at the rim of their zone of influence ($r = r_e$) during a pumping period Δt. For this purpose we can rewrite Eq.14, and the discharge of each well should then satisfy the following condition (ERNST, 1970)

$$Q_o > \frac{\pi KD\Delta s}{(KD/\mu r_e^2)\Delta t - 1/8} \tag{15}$$

It should be noted that Eqs. 13 and 15 are not valid for small values of τ. Anyway, it would not be practical to apply the formulas because of the unfavourable ratio between the induced drop in water table at r_e and the volume of pumped water. Even for $\tau = 0.3$, this ratio is not more than 59 per cent of the most favourable value obtained for $\tau \to \infty$ (see ratio cot β/cot α in Fig.6). Nor will very large values of τ be used in practice because of the corresponding small values of r_e.

Finally, the above formulas can also be applied for variable well discharges, provided the pumping periods are not too short (preferably $\tau > 0.3$) and that the discharge is constant throughout each pumping period.

14.7 DRAINAGE WELLS IN SEMI-CONFINED AQUIFERS

Up to now, our discussion has only been concerned with the drainage of unconfined aquifers. Yet in many agricultural areas suffering from high water tables, semi-confined aquifers occur, i.e. an aquifer covered by a semi-pervious layer and bounded below by an impervious layer (Fig.7). The hydraulic head of the water confined within the aquifer is often found above the head of the water table in the upper layer, thus causing an upward flow from the aquifer into that layer.

14.7.1 ARTESIAN WELLS

If the water in the aquifer shown in Fig.7 is under artesian pressure, i.e. its hydraulic head, h, is far above the height of the water table, h', in the upper confining layer, an upward seepage flow occurs. Wells are sometimes used to relieve this artesian pressure, resulting in a reduction of the upward seepage. If these relief wells, which are free flowing, are fully penetrating, the following equation may be used to predict the decrease in hydraulic head at any distance, r, from a well.

$$h_2 - h_1 = \frac{Q}{2\pi KD} \ln (r_2/r_1) \tag{16}$$

If r_e and r_w are, respectively, the radius of influence and the radius of the well, and h_e and h_w are, respectively, the hydraulic head at a distance r_e and at the well face, Eq. 16 may be written as

$$h_e - h_w = \frac{Q}{2\pi KD} \ln (r_e/r_w) \tag{17}$$

This equation allows the radius of influence to be calculated if the well discharge, Q, the transmissivity, KD, and the drawdown at the well face, $h_e - h_w$, can be estimated with reasonable accuracy.

14.7.2 INTERCEPTOR WELLS

The piezometric surface of the water in a semi-confined aquifer is not always level. In polder districts or areas along embanked rivers with high water levels, the piezometric surface may have a slope that will be indicated by α. A flowing well installed in an artesian aquifer will intercept

$$Q = 2r_e KD\alpha \tag{18}$$

Eliminating r_e from Eqs. 17 and 18 and rewriting gives the following equation obtained by PETERSON (1957)

$$\frac{Q}{KD(h_e - h_w)} = \frac{2}{2.303 \log \left[\frac{1}{2} \left(\frac{Q}{KD(h_e - h_w)} \right) \left(\frac{(h_e - h_w)/r_w}{\alpha} \right) \right]} \tag{19}$$

which may be applied to estimate the steady-state discharge for a well intercepting an artesian aquifer. The equation cannot be explicitly solved. PETERSON (1957) solved it graphically (Fig.8). From this diagram Q can be read if the slope of the piezometric surface, α, the drawdown at the well face $s = (h_e - h_w)$, the well radius, r_w, and the transmissivity, KD, are known.

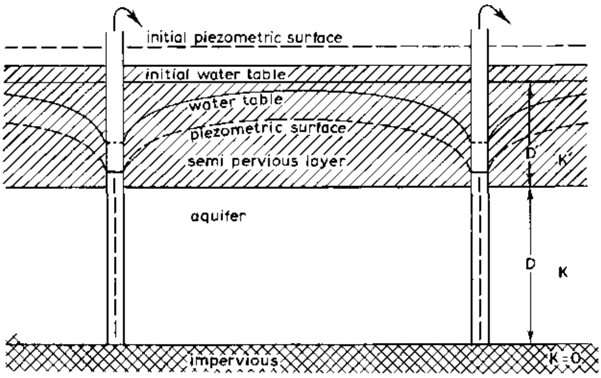


Fig.7. Wells in a semi-confined aquifer.

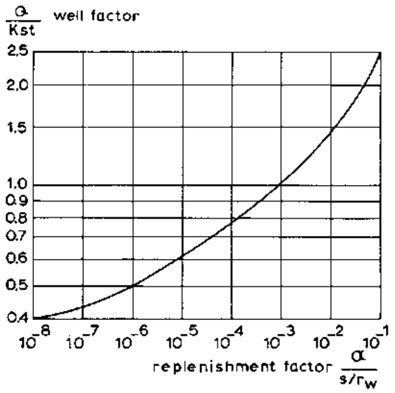


Fig. 8. Discharge parameters for horizontally replenished artesian well (after PETERSON, 1957).

14.7.3 WELLS IN AN EXTENSIVE SEMI-CONFINED AQUIFER

Figure 9 shows a semi-confined aquifer replenished by percolating rain or excess irrigation water at a rate R. The recharge at the land surface causes the water table in the upper clay layer to rise above the head in the underlying aquifer. Hence a downward flow through the clay layer to the aquifer occurs. The question arises whether drainage wells installed in the underlying aquifer can be used to lower the water table in the upper clay layer.

The hydraulic characteristics defining this problem are the hydraulic resistance of the upper clay layer, c, the transmissivity of the aquifer, KD, and for unsteady flow the effective porosity of the upper clay layer, μ, and the storage coefficient of the aquifer, S.

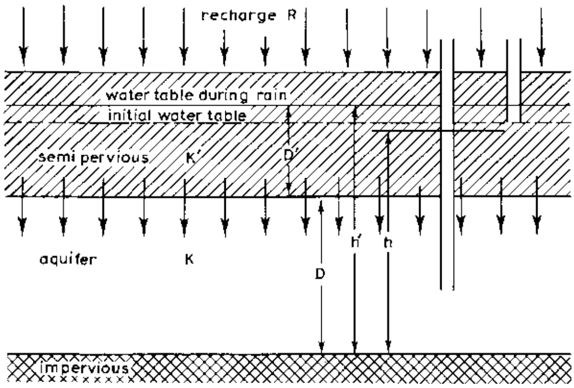


Fig.9. Semi-confined aquifer uniformly recharged by percolating rain.

If we assume a steady recharge, R, from rain or excess irrigation water on the ground surface, the rate of vertical downward flow through the upper clay layer towards the aquifer is defined by

$$R = v_z = \frac{h' - h}{c} \tag{20}$$

where

h' = height of water table (m),

h = hydraulic head in the aquifer (m),

c = D'/K' = hydraulic resistance of the saturated part of the clay layer (day).

In semi-confined aquifers, head differences of the order of a few centimetres to, say, 1 or 1.5 m are rather common. Usually, the water table is rather shallow and does not occur deeper than a few metres below the land surface. Hence head differences of many metres are unrealistic. Head differences of a few centimetres to, say, 10 cm are so small that they can be neglected. Assuming a head difference of 1 m and taking two extreme values for R = 1 mm/day and 10 mm/day, we find from Eq. 20 that the value of c varies between 100 and 1000 days. A value twice as high (c = 2000 days) requires a head difference twice as high as was assumed in order to have the same percolation rate maintained. For a percolation rate of 10 mm/day this would result in a head difference of 20 m, which is impossible.

These tentative calculations show clearly that particular attention should be given to the upper limit of the hydraulic resistance of the upper clay layers when well drainage in semi-confined aquifers is being considered as an alternative to gravity drainage. For values of c much larger than 1000 days, drainage by wells will not be a suitable solution to the problem.

In a similar way, the transmissivity, KD, of the aquifer must have a value that is sufficiently large for well drainage to be technically and, in particular, economically feasible. If we assume for conditions of steady flow that the rate of extraction from a well equals the recharge from percolating rain or irrigation water, then we may write

$$Q_o = R\ A \tag{21}$$

where $A = \pi r_e^2$ = area drained by the well.

If the wells are set in a regular pattern (squares, triangles, hexagons), and if the flow towards the wells has attained a steady state, i.e. the discharge of the wells equals the percolation rate R, the drawdown at the face of a well is given by (see also Chap. 12, Vol.II)

$$h_e - h_w = \frac{Q_o}{2\pi KD} \ln\ (r/r_e) - 1/2 \tag{22}$$

with the symbols as defined earlier.
For $r_e/r_w > 100$, and if errors not larger than 10 per cent are admitted, this equation may be replaced by

$$h_e - h_w = \frac{Q_o}{2\pi KD} \ln\ (r_e/r_w) \tag{23}$$

where Q_o = R A (Eq. 21).

Since the well discharge is a fixed value, depending on the percolation rate, and the drawdown in the well should not exceed a certain maximum value (to prevent flow velocities at the well screen from becoming too high), it can easily be seen from these equations that the lower the transmissivity of the aquifer, the smaller the radius of influence, r_e, of the well, and thus the smaller the well spacing, L (Figs.3 and 4). Too low transmissivity values would make the well spacing so close that drainage by wells would not be economically feasible.

The formulas discussed so far apply only to wells forming square, triangular, or hexagonal patterns. They are not applicable to wells sited in parallel lines a distance B apart, the spacing of the wells along the lines being L, where L is considerably smaller than B (Fig. 10). In such a situation, if the recharge on the land surface from rain or irrigation water is uniform, and if the flow towards the wells has attained a steady state, the discharge of each well can be written

$$Q_o = R\ B\ L \tag{24}$$

where Q_o is the extraction from each well.

Since parallel lines of wells show a certain analogy with parallel ditches or canals, EDELMAN (1972) derived an approximate solution for the drawdown at the face of each well. In both cases the water table is lowered along a line which is the axis of either the line of wells or ditch. Hence the line of wells may be replaced by canals from which a quantity q_o is extracted per unit length, so that

$$q_o = R\ B$$

The maximum water table height occurs in the *symmetry* axis C - C'. The difference in hydraulic head (i.e. the difference between the maximum water table elevation midway between the canals and the water level in the canals, also called available head) is given by (see also Chap. 6, Vol.I)

$$\Delta h_1 = \frac{RB^2}{8KD} \qquad (25)$$

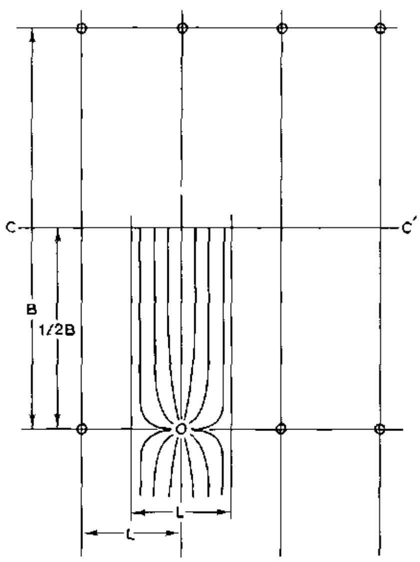


Fig.10. Wells in parallel series a distance B apart. Well spacing within the series is L (L<<B) (after EDELMAN, 1972).

In reality, of course, the extraction does not take place from canals or ditches, but from parallel lines of wells. As a consequence the hydraulic head midway between the lines of wells (in the symmetry line C - C') is not constant. Deviations from the average value of the head may, however, be neglected because it was assumed that the distance B between the lines is much greater than the well spacing L along the lines. Hence the head midway between the lines of wells may be considered a constant, h_e. Secondly, the hydraulic head in a well, h_w, is lower than the head in the canal. The energy losses are concentrated in the vicinity of the well, where the flow is radial. For radial flow the head loss may be expressed as

$$h_e - h_w = \Delta h_2 = \frac{Q_o}{2\pi KD} \ln (r_e/r_w) \tag{26}$$

The method of superposition may be applied to find the difference between the head at the well face and that midway between the lines of wells. Adding Eqs.25 and 26 gives

$$h_e - h_w = \frac{RB^2}{8KD} + \frac{Q_o}{2\pi KD} \ln (r_e/r_w) \tag{27}$$

Taking for r_e such a value that the circumference of a circle with radius r_e is equal to the length of the section through which the water flows towards the well from both sides:

$$2\pi r_e = 2L$$

and replacing the well discharge Q_o at steady flow by RBL, we can rewrite Eq.27 as

$$h_e - h_w = \frac{RB^2}{8KD} + \frac{RBL}{2\pi KD} \ln (L/\pi r_w) \tag{28}$$

As can be seen from this equation, if the discharge of each well remains constant while the rate of recharge is four times greater, the well spacing will be half its initial value. If the transmissivity, KD, is four times less, both the well spacing and well discharge will be one quarter of their initial values.

LITERATURE

ANANIAN, A.K., MKRTCHIAN, S.M., MKHITARIAN, G.M., SIMONIAN, G.A. and KARAPETIAN, S.G. 1969. Research for design of vertical drainage on sodic solonchaks of the Ararat Plain. Int. Symp. on Reclamation of sodic soils. Yerevan (U.S.S.R.), 31 p.

ANONYMOUS. 1964. Report on Land and Water Development in the Indus Plain. The White House, Washington. U.S. Gov. Printing Office, 454 p.

BEAR, J., ZASLAVSKY, D. and IRMAY, S. 1968. Physical principles of water percolation and seepage. Arid Zone Research XXIX, Unesco, Paris, 465 p.

EDELMAN, J.H. 1972. Groundwater hydraulics of extensive aquifers. Techn.Bull.13, Int. Inst. for Land Recl. and Improvement, Wageningen, 216 p.

ERNST, L.F. 1970. Drainage van een dik, homogeen doorlatend grondpakket door middel van putten. Nota 575, Inst. for Land and Water Management Res., Wageningen, 7 p.

HANTUSH, M.S. 1962. Drainage wells in leaky water-table aquifers. Proc. Am. Soc. Civil Engrs. 88, HY 2, 123-137.

HANTUSH, M.S. 1964. Hydraulics of wells. Advances in Hydroscience, I. Acad. Press, New York, 281-432.

HUISMAN, L. 1972. Groundwater recovery. Macmillan, London, 336 p.

IRMAY, S. 1960. Calcul du rabattement des nappes aquifères. VI[es] journées de l'hydraulique, Nancy. La Houille Blanche, I, 7, 61-70.

JAHNKE, E. and EMDE, F. 1945. Tables of functions with formulae and curves. Dover Publ. New York, 304 p.

MICHAELSON, B.A. 1967. Vertical drainage for improvement of saline land resulting from irrigation. Int. Seminar on Irrig. and vertical drainage. Tashkent, 1967, II, FAO, Rome, 245-252.

MUSKAT, M. 1934. The flow of compressible fluids through porous media and some problems in heat conduction. Physics, 5, 71-94.

MUSKAT, M. 1937. The flow of homogenous fluids through porous media. McGraw-Hill Book Co., New York, 736 p.

PETERSON, D.F. 1957. The theory of drainage by pumping from wells. Drainage of Agricultural Lands. Editor: J.N.LUTHIN. Madison, Wisconsin, 181-215.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

15. RAINFALL-RUNOFF RELATIONS AND COMPUTATIONAL MODELS

D. A. KRAIJENHOFF VAN DE LEUR
Professor
Department of Hydraulics
University of Agriculture, Wageningen

Lecturers in the Course on Land Drainage

D. A. Kraijenhoff van de Leur (1962–1972)
University of Agriculture, Wageningen

PURPOSE AND SCOPE

An introduction to catchment hydrology is given. The estimation of runoff volume, the unit hydrograph, and model synthesis are discussed for both direct runoff and baseflow as a "systems problem".

CONTENTS

15.1 INTRODUCTION

It is a drainage engineer's business to prevent excess water from doing harm. He will consequently be interested in the source and magnitudes of the discharges he will have to cope with when planning culverts, bridges, retention reservoirs, drainage schemes for waterlogged areas, or the reclamation of flood plains. A drainage engineer should therefore have an understanding of the principles governing the flow of surface and subsurface water before it reaches defined channels and also of the principles, magnitudes, and fluctuations in streamflow which together determine the runoff process.

Runoff comprehends the flow of water through channels on the surface of the earth. It has its origin in the precipitation of atmospheric moisture, which in turn is chiefly evaporated from the oceans and carried over the continents as a part of the general air mass circulation. In a general sense, runoff is the residual of precipitation that is drained from the land after the demands of evaporation have been met. Over a long period the total volume of runoff must indeed equal the difference between precipitation and evaporation. Over shorter periods, however, the rainfall-runoff relation will be further governed by a great number of intermediate reservoirs or storages of various nature inherent to the specific local conditions as regards vegetal, soil, geologic, and topographic factors.

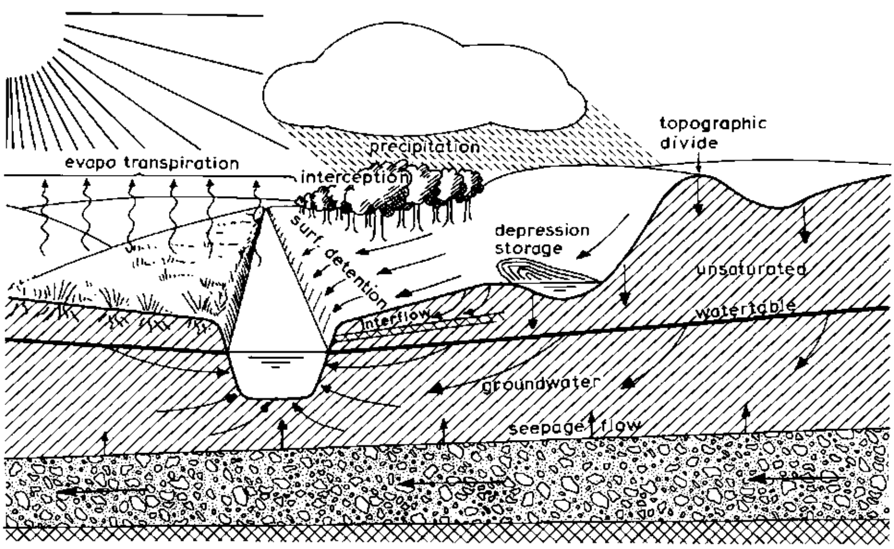


Fig.1. The hydrologic cycle.

Part of the rain will be temporarily stored on the vegetation; this interception will eventually evaporate or reach the soil as stem flow. Rain reaching the soil may infiltrate into it, and part of it will merely become soil moisture, only to be lost again by transpiration or direct evaporation. The soil moisture excess will percolate down to the groundwater table and replenish the groundwater storage, to be discharged ultimately as groundwater flow into the channel system. When the rainfall exceeds the soil's infiltration capacity (maximum possible infiltration capacity rate at a certain moment), the rainfall excess will first build up depression storage, filling depressions and holes in the surface, from which it will ultimately infiltrate or evaporate after the end of rainfall. When the depressions begin to overflow, overland flow has set in and the water reaches the channel system via small rills and rivulets. The volume of water that is on its way to the channel system as overland flow is called the surface detention, which again acts as a reservoir. The next and last reservoir is the channel system where channel storage is being built up after the arrival of the first overland flow. It follows that there are two main paths by which water moves from the soil surface to the stream, namely along the soil surface and through the groundwater reservoir. Short circuits must, however, be expected. Water that has already penetrated into the soil may move over a shallow layer of low permeability to be forced out again at a lower point of the slope where it changes into overland flow. This is called subsurface stormflow or interflow. On the other hand, water moving along the soil surface may still become groundwater when it gets to a surface with a higher infiltration capacity and it consequently infiltrates into the soil.

Overland flow becomes surface runoff after it has arrived safely in the channel system and is transported through the outlet of the drainage basin. Surface runoff together with interflow make up the direct runoff, which moves swiftly through the drainage basin to the outlet. This direct runoff is the major cause of flood waves.

The discharge from the groundwater reservoir into the channel system responds relatively slowly to an additional supply of infiltrating water from rain or snowmelt. It makes up the groundwater runoff, or base flow, and its contribution to most floods is small, although in many areas groundwater runoff represents the greatest percentage of the annual runoff volume and is the only source of stream flow during protracted dry spells. In areas with deep highly permeable soils, runoff may not occur at all, even after rainstorms of the highest intensities. In these cases floodwaves are caused exclusively by groundwater flow and some interflow over less permeable soil strata along the channel network.

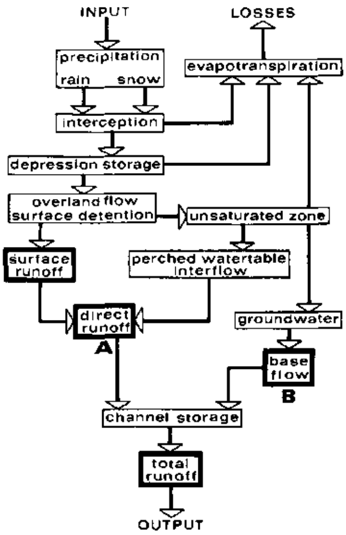


Fig.2. Flow chart of the hydrologic cycle.

The hydrographs representing such floodwaves consequently show a rather smooth appearance and the crest flow, or peak flow, expressed in surface inches or millimeters will be lower and of longer duration than floodwaves having an appreciable contribution from surface runoff. In highly permeable regions the relationship between discharge and rainfall minus evapotranspiration is mainly governed by the extent of the groundwater reservoirs that are feeding the channel system.

15.2 THE DRAINAGE BASIN

The drainage basin (also called drainage area, catchment, or watershed) is the entire area drained by a stream in such a way that all streamflow originating in the area is discharged through a single outlet. The topographic divide, or watershed line, which encloses the drainage basin, designates the area in which overland flow will move towards the drainage system and ultimately become surface runoff at the outlet. As the phreatic divide does not always coincide with the topographic divide, the groundwater flow may not conform to surface drainage boundaries and watershed-leakage may occur (Fig.3).

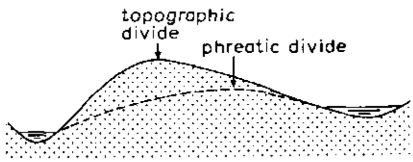


Fig.3.
Topographic and phreatic divide.

In areas where the basement rocks are almost exclusively calcareous, karst regions should be expected, in which subterranean channels cross the topographic divides freely. Under such circumstances only a very thorough survey can determine the areas that contribute to the discharge of a certain outlet. The drainage basin, with all its specific characteristics, can be regarded as the intermediate agent that turns precipitation on the basin into runoff at the outlet. So if climatic conditions are similar for two drainage basins, their characteristics will determine their "handwriting", as expressed by a continuous graph of the runoff at the outlet, the so-called hydrograph of discharge (Fig.4).

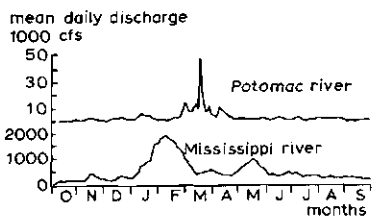


Fig.4.
Hydrographs of the Potomac river Maryland, U.S.A., and the Mississippi river at Memphis, Tenessee, U.S.A. (LINSLEY et al., 1949).

We shall now specify some of these characteristics.

15.2.1 THE SOIL

A deep and permeable soil is most favourable for infiltration. This means that the precipitation minus the evapotranspiration will replenish the groundwater reservoir. This storage has a smoothing effect on the maximum and minimum flow and therefore the hydrograph of discharge will present a rather sluggish appearance. The other extreme is a bare rock-surface, which turns practically all rainfall into overland flow and offers hardly any opportunity for storage. Here the hydrograph of discharge will show sharp peaks and prolonged periods of very low flow or no flow at all (intermittent streams). In between these extremes, many intermediate situations will exist, such as shallow soils with or without

various types of vegetation.
Vegetation - and the litter underneath - protects the open structure of the soil against the splashing and puddling action of raindrops, an action that usually affects the infiltration capacity rate of a bare soil. Cultivation of arable land strongly affects runoff conditions: freshly ploughed fields may prevent all overland flow, whereas harvesting operations may leave the fields bare and with a compacted soil surface. In these areas surface runoff will vary considerably with the season. Moreover, biologic activity in the soil varies seasonally, having its effects on soil structure and porosity. Soil as a factor conditioning runoff will be further affected by frost, moisture content, and swelling colloids.

To summarize: the role of the soil as an intermediate factor in the precipitation-runoff relationship is determined by seasonal factors (vegetation, cultivation, and biologic activity), by factors that are partly seasonal and partly incidental (evaporation and soil frost), and finally by factors that are mainly incidental (antecedent precipitation and temperature).

15.2.2 THE AREA

The basin's size affects the runoff characteristics in the following ways:

- All other factors (including depth and intensity of rainfall) being the same, two basins, regardless of their size, will produce the same total runoff expressed in surface inches or millimeters depth. However, the larger the basin, the longer it takes for the total runoff to pass through the outlet; consequently the base of any hydrograph of flood will broaden out as the area of the basin increases, whereas the peak flow expressed in surface inches or millimeters will decrease.
- It was assumed above that the depth of rainfall is the same on both the small and the large basin. However, the average depth of precipitation that is likely to occur with a given frequency decreases with the area of the basin. This is due to the limited areal extension of storms of high intensity. Consequently storms with the same frequency of occurrence will cause crest flows expressed in surface inches or millimeters that are lower for large than for small basins.

The approximate relationship is such that the envelope curve of maximum crest flows varies inversely with the square root of the size of the drainage area, other factors being the same (Fig.5). Such a relation is a useful tool because it enables an approximate insight into the runoff relations of an unmeasured drainage basin to be obtained through comparison with a basin of similar type for which rainfall and runoff data are available.

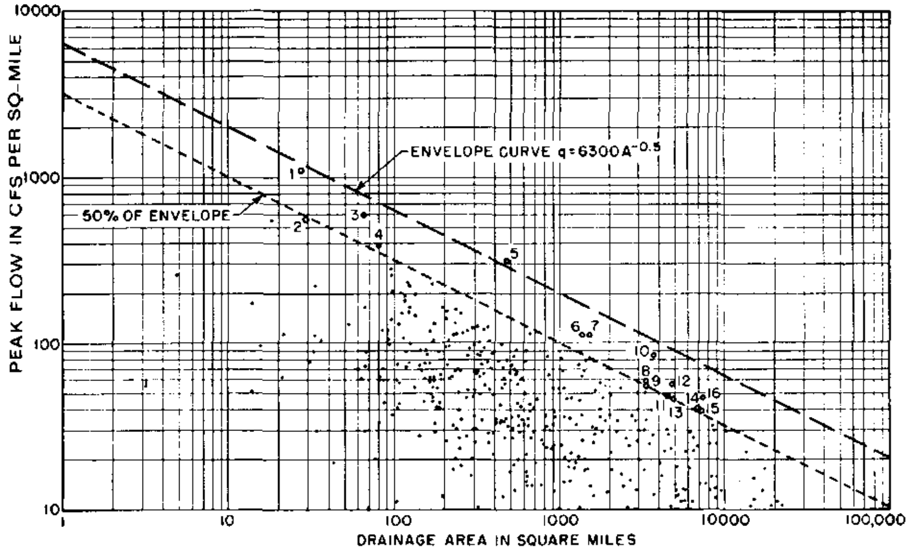


Fig.5. Envelope curve of extreme floods for the South Atlantic and Eastern Gulf of Mexico Drainage areas (LINSLEY, et al., 1949).

15.2.3 THE SLOPE

In a drainage basin two types of slopes can be distinguished:
- overland slope
- channel slope.

overland slope

The overland slope influences the velocity of overland flow and may thus be important in small basins where the time spent in overland flow is an appreciable part of the total time required for the water to reach the outlet. The overland slope, however, is not the only factor determining the travel time of flow; the type of vegetation or the direction in which the farmer ploughs his field may be more significant than the overland slope as measured from a topographic map. Moreover, strip-cropping and contour ploughing are practised on a wider scale as the overland slope increases. As the area of a drainage basin increases, so, normally, does the number of channels guiding intermittent and perennial flow, which means that the relative importance of overland slope decreases with respect to the total travel time to the gauging station at the outlet. Because of the rapid development of ephemeral streams, there is no sharp division between overland flow and channel flow.

channel slope

Other things being equal, the steeper the channel slope, the greater the velocity of flow, thus the shorter the time required for the total volume of runoff to reach the outlet and consequently the more peaked the hydrograph of runoff will be. In other words, the channel storage will be correspondingly small and will thus cause less delay and attenuation of the "wave" of precipitation that is moving towards the outlet. The channel slope is derived from the stream profile which is a plotting of elevation versus the horizontal distance along the main stream. If the stream profile is curved, the equivalent uniform slope is found by drawing a straight line through the downstream end so as to have the same area under the line as is under the profile (Fig.6).

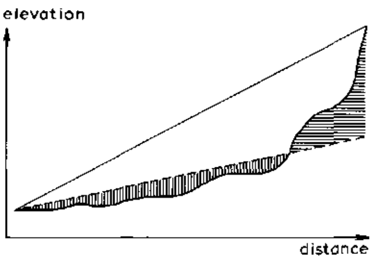


Fig.6. Determination of the equivalent uniform slope.

It was found for seven small agricultural basins, extending in size from 1.25 to 112 acres, that a factor $\frac{L}{\sqrt{s}}$ determined the time of concentration

$$T_c = a\left(\frac{L}{\sqrt{s}}\right)^n$$

where a and n are constants

L = length of travel

s = channel slope

T_c = time of concentration. (The time of concentration is the time required for a particle of water from the most remote part of the basin to reach the outlet.) These basins were all located on the same farm in Tennessee and all were under the same cultivation (Fig.7).

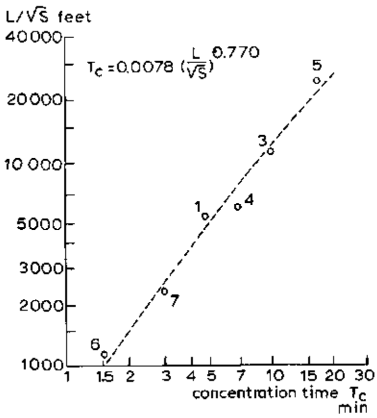


Fig.7. Relation between the concentration time, T_c, and basin characteristics (KIRPICH, 1940).

The time of travel of a floodwave will usually not be equal to the ratio of channel length and velocity at crest flow. This can be understood by considering the translation of a monoclinal wave through a channel that is already subject to a certain initial discharge Q_1. This uniformly progressive wave (Fig.8) travels down the channel at a constant velocity v_w. An observer running along with the same velocity v_w may regard the wave as being stable and as taking in a steady discharge $Q_o = (v_w - v_1)A_1$ (A = wetted cross-section) at the front while leaving an equal steady discharge $Q_o = (v_w - v_2)A_2$ behind (Fig.8).

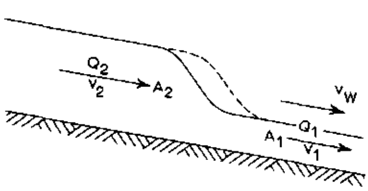


Fig.8. Propagation of a monoclinal wave.

Equating both values

$$(v_w - v_2)A_2 = (v_w - v_1)A_1$$

and solving for v_w yields

$$v_w = \frac{v_2A_2 - v_1A_1}{A_2 - A_1} = \frac{Q_2 - Q_1}{A_2 - A_1}$$

where

v = wave celerity

A = wetted cross-section.

The SEDDON law for the celerity of a floodwave is

$$c = v_w = \frac{dQ}{dA} \tag{1}$$

For ordinary channel sections in which the velocity increases as the wetted cross-section increases, the curve representing the relation between Q and the wetted cross-sectional area A is usually concave upward (Fig.9).

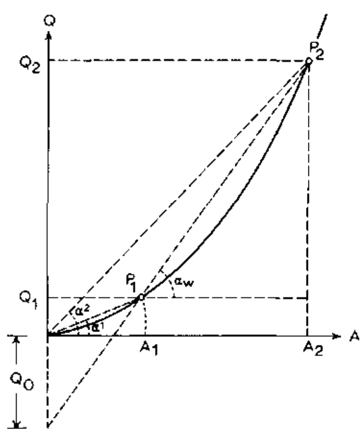


Fig.9. A rating curve.

The picture shows that the wave velocity or celerity v_w must then be greater than the maximum flow velocity v_2 because of the initial flow Q_1

$$v_w = \mathrm{tg}\ \alpha_w > v_2 = \mathrm{tg}\ \alpha_2$$

In a wide rectangular channel

$$\bar{v} = C\, y^{1/2}\, s^{1/2}$$

$$A = B\, y$$

so that

$$Q = C\, B\, y^{3/2}\, s^{1/2}$$

and

$$c = \frac{dQ}{dA} = \frac{1}{B}\frac{dQ}{dy} = \frac{3}{2}\, C\, y^{1/2}\, s^{1/2} = \frac{3}{2}\, \bar{v}$$

where $\bar{v}$ = average velocity of flow at the crest of the wave
c = wave celerity
C = Chézy's coefficient
y = water depth in the channel
s = slope of the channel
B = width of the channel.

In a similar way it can be shown for a triangular cross-section that $c = \frac{5}{4}\,\bar{v}$.

The time of travel of the monoclinal wave over a distance L is

$$t = \frac{L}{v_w} = \frac{L(A_2 - A_1)}{Q_2 - Q_1} = \frac{\Delta S}{\Delta Q} \qquad (2)$$

where S = storage.

So the time of travel equals the ratio of the increase of channel storage and the increase of discharge. While travelling down the main channel, floodwaves originating from channel branches in the upstream areas of a drainage basin will be joined by floodwaves from other tributaries, and the total of all these contributions will ultimately determine the hydrograph of runoff at the outlet. Obviously this normal case will deviate considerably from the simplified picture of a uniformly progressive wave given above. Nevertheless the general notions for within-bank flow - that initial flow causes a floodwave's speed of travel to exceed the maximum velocity at the flood crest, and that the time of travel over a certain distance is related to the ratio of storage and discharge - can be maintained. There are indications that in the design of closed systems - the type most commonly used in municipal storm drainage - the time regarded as being critical is, for flat areas, the time required to fill the storage in the system, and, for steep areas, the time of flood travel in the collecting system. Such critical rainfall periods will be used in the rational method (see Sect.6). It seems advisable not to use the term "time of concentration" when a floodwave overruns initial flow; then the term "time of travel" seems to be more relevant. When the latter term is used for the whole process of transformation of a "wave" of rainfall into a floodwave at the outlet, the terms "basin lag", "time of travel" and "average delay time" have rather similar meanings. Though not everyone uses the same definition, we shall follow the recent trend and define the basin lag as the time from the centre of area of the graph of rainfall excess to the centre of area of the resulting graph of direct runoff (Fig.10).

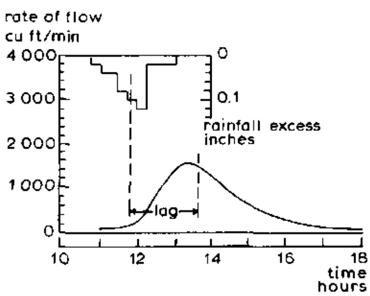


Fig.10. The basin lag.

15.2.4 THE CONFIGURATION OF THE CHANNEL SYSTEM

The following features can be distinguished:
- channel storage
- the density of the channel network
- the stream pattern
- the condition of the channels.

channel storage

With two channels of equal slope, the one with the larger cross-section has more storage capacity per unit of length. The general effect of storage on floodwaves is twofold: lag and attenuation (flattening). To illustrate this point we consider a detention reservoir (Fig.11).

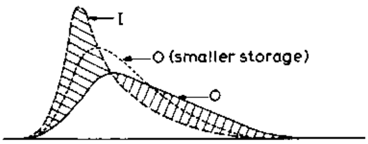


Fig.11. Inflow, I, and outflow, O, of a detention reservoir.

In the figure, I represents the hydrograph of inflow and O stands for the hydrograph of outflow from the reservoir. As the inflow proceeds, both storage and outflow will increase, the outflow being only a function of storage. The maximum volume of storage is represented by the shaded area to the left of the point of intersection of the in- and outflow hydrographs. At that time the outflow rate must also reach its maximum. In the subsequent period of storage depletion the rate of outflow must exceed the inflow rate. The figure shows that both the time lag and the degree of attenuation will increase with the storage capacity. Though channel storage differs from a detention reservoir in its effects on streamflow, it will also produce time lag, and normally also attenuation (Fig.12).

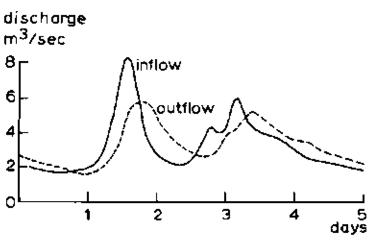


Fig.12. Inflow and outflow graphs of a channel reach.

the density of the channel network

The following types of streams can be distinguished:
- ephemeral streams, which carry surface runoff only; they have no well-defined channels but follow slight depressions in the natural contour of the ground surface;
- intermittent streams, which stop flowing when the groundwater level falls below the bottom of the channel;
- perennial streams, which always carry stream flow.

In drainage basins with relatively steep slopes, a greater density of the channel network will mean shorter length and time of overland flow; basin lag will be shorter and crest flow higher. In relatively flat areas, on the other hand, a denser network means more storage, which will counteract the above effect of more rapid concentration into the channel system.

the stream pattern

A fan-shaped area with streams converging more or less towards a common point suggests the possibility of synchronized peaks from the constituent sub-areas, whereas an elongated area traversed by a major stream with more or less uniformly spaced tributaries suggests the possibility of a slower and less pronounced rise and recession. This point should be considered in relation to the condition of the channel.

condition of the channel

When a winding tributary, blocked by growth of weeds, is straightened and cleaned, the total resistance to flow will be considerably diminished and floodwaves will pass through it at a higher speed and with less attenuation because of decreased storage in flood plains (Fig.13). If this tributary runs through the lower part of the drainage basin and joins the main stream close to the outlet, its amelioration will effect crest flows favourably because its own floodwave will have passed through the outlet by the time waves originating in upstream areas arrive there. On the other hand amelioration works in the upstream parts of a drainage basin may cause considerable damage in the downstream area when the result is a congestion of floodwaves in the lower parts. Obviously amelioration work should always proceed in the upstream direction. If the amelioration of the channel system also entails a lower groundwater table in an initially water-logged area, the total effect may be a decrease of peak floods. This is caused by an increased storage and delay in the unsaturated zone. Such increased storage capacity may even suppress all direct runoff.

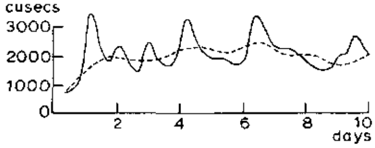


Fig.13. Actual post-drainage hydrograph (solid curve) and calculated pre-drainage hydrograph (dotted line) (O'KELLY, 1955)

15.3 THE SYSTEMS APPROACH IN HYDROLOGY

15.3.1 THE HYDROLOGIC SYSTEM

The preceding general description of the various features of a drainage system was meant to provide a basic understanding of the various features governing the runoff process that converts precipitation into outflow. The picture we have thus obtained is of a mainly qualitative nature and we must now proceed to express the relationship between precipitation and runoff in more quantitative terms. In other words we will have to analyse the "drainage system" and try to determine the system's operation that converts the inputs such as solar radiation and precipitation into outputs such as evaporation losses and streamflow through the outlet. Figure 14, borrowed from DOOGE (in press), illustrates the available sources of information on the system's operation.

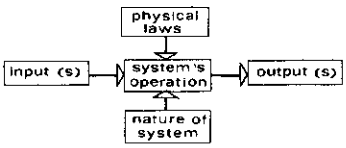


Fig.14. DOOGE's diagram of systems approach

15.3.2 THE PHYSICAL APPROACH

The vertical in the diagram of Fig.14 represents the available physical formation on the nature and the structure of the system, and on the laws governing its behaviour. If this information were complete it could be used to construct a mathematical model that would express the transformation of inputs into outputs. This can be illustrated by the following extremely simple storage and drainage system: A vertical cylinder with diameter D, drains through a capillary (length L, diameter d). The law of Poisseuille reads

$$Q = \frac{g}{\nu} \frac{\pi}{128} d^2 \frac{\Delta h}{L} \tag{3}$$

where Q = discharge

g = acceleration due to gravity

Δh = height of the water column which equals head loss over capillary

ν = kinematic viscosity

The height of the water column Δh can be expressed in the storage S and the diameter D

$$\Delta h = \frac{4S}{\pi D^2} \tag{4}$$

The combination of Eq.3 and Eq.4 yields

$$S = kQ, \tag{5}$$

where $k = \dfrac{\nu \; 32 \; LD^2}{gd^4}$

k being the characteristic time for this system.

Substituting Eq.5 into the continuity equation

$$P = q + \frac{dS}{dt} \tag{6}$$

where P denotes the amount of precipitation which has to be discharged, yields the mathematical model of the system's operation

$$P = q + k\,\frac{dq}{dt} \tag{7}$$

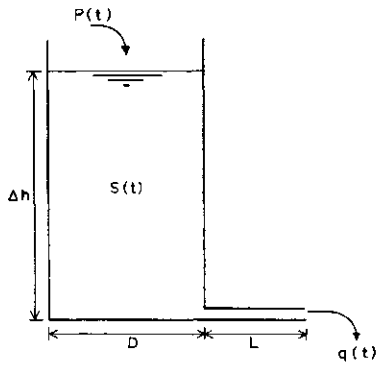


Fig.15. A linear reservoir

The physical characteristics of the system determine the value of the parameter k in this mathematical model. We find that not only does the physical approach along the vertical in DOOGE's diagram reveal the system's operation but also that the mathematical model derived from it is applicable to any other analogous system. In hydrology, however, such an ideal situation never occurs "because on the one hand the physical laws are impossible to determine or too complex to apply, or on the other hand the geometry of the system is too complex or the lack of homogeneity too great..." (DOOGE, in press). So the physical approach on its own will not lead to a complete solution. But appropriate physical research may enable us to produce a structurally sound model and provide us with some ideas about where to look for the physical characteristics that determine the model parameters.

15.3.3 THE EMPIRICAL APPROACH

Empirical information (the horizontal in Fig.14) on the system's operation can be obtained by measuring series of inputs and corresponding series of outputs. The methods available for the analysis of these input and output data are usually counted either among the statistical methods or among the methods of parametric hydrology, which comprise system's analysis and model synthesis. Before discussing these two groups of methods, some further information must be given on the types of systems that are to be studied.

First a distinction is made between static and dynamic systems. Strictly one should distinguish between the static and dynamic states of a system. A beam bending under a load reaches the static state almost immediately. The value of the output variable, the deflection, is then fully determined by the simultaneous input, which is the load on the beam. A static system is "memoryless". A dynamic system, however, such as the example in Sect.3.2, has a memory. It provides temporary storage for past inputs. Therefore such antecedent inputs also determine the actual state of the system and consequently also influence the output values.

One further speaks of lumped systems - as distinguished from distributed systems - with lumped input(s) and output(s). Although the movement of effective precipitation, which eventually becomes runoff at the catchment's outlet, is always a spatially varied and complicated process, its overall effect is transformation of input(s) into output(s), and therefore the system's operation may be considered one lumped operation. Input and output variables like precipitation, snowmelt, evaporation, etc., vary not only with time but also in space. Nevertheless one may feel justified in "lumping" such variables and speaking of catchment precipitation or catchment evaporation. Actually these lumped variables are indices or weighted averages of the true non-uniformly distributed hydrologic variables. Uniformly distributed inputs cause no difficulties with regard to lumping. Even non-uniform distributions may be expressed in one index, i.e. may be lumped, if they are characterized by a more or less stable non-uniform areal distribution (orographic effect).

Of course this lumping of variables is no longer warranted as soon as changes of the proportional distribution pattern have an appreciable effect on the system's function and on other input and output variables of interest. This limitation implies that the study of lumped input-output relations is only applicable to relatively small hydrologic systems. In many cases it is possible to subdivide a distributed system into sufficiently small sub-systems where the input and output variables can be measured, so that each sub-system is open to the direct empirical approach.

15.3.4 STATISTICAL METHODS

Measured series of inputs and outputs and corresponding time series of outputs can be used for correlation studies, both graphical and computational, in an attempt to describe the system's operation. In Section 4.2 a graphical correlation analysis will be presented which describes the amount of effective precipitation or runoff that will be caused in one catchment by a rainstorm of certain depth and duration under certain moisture conditions in the catchment as determined by antecedent precipitation and seasonal effects. An example of computational correlation is the prediction of the amount of spring flow from snowmelt. Here a number of input variables can be used, such as last autumn's rainfall, the winter precipitation, temperatures in preceding months, etc. In a purely empirical approach any combination of input variables can be chosen to operate in any correlation model, the sole criterion being the best fit of observed and computed outputs. In the total absence of physical information, the best statistical methods may produce false suggestions as to the relevant input variables and the structure of the system. This is due to errors in the observed outputs and errors in the computed outputs because of inaccurate input values. Eventually the statistician will produce a correlation model that provides a close enough description of the input-output relation of the available data. Once such a successful correlation model has been obtained the hydrologist may be tempted to draw conclusions from the structure of this model with regard to the structure of the system. These conclusions he would like to apply to other similar hydrological systems in order to avoid the necessity of starting the same labour of data acquisition all over again. Here ANDERSON's (1966) warning seems to be in place: *Unless the correlation model is based on accurate and complete data and/or physical model*, "such conclusions are bound to be contrived nonsense".

Most hydrologic systems are essentially dynamic and obviously the memory of a dynamic system is a reason for auto-correlation to occur in the output variable under consideration. For instance, the correlation of today's outflow from a lake (or catchment) with yesterday's outflow may be so strong that it obscures the outflow's correlation with other variables, such as the inflow of river water and groundwater, the precipitation, and the evaporation. Corrections for auto-correlation in order to bring forward the influence of the other variables may be successful if these corrections are based on the true relation between storage and lake outflow. If this knowledge is lacking, as in any purely empirical approach, the interval of study must be so chosen that the effect of carry-over from one period into the other becomes negligible compared with other quantities over these intervals. It means that either the effect of memory, to be expressed in some characteristic time (see k in Sect.3.2), should be small in proportion to the chosen interval, or the effect of storage on the output should be the same at the beginning and at the end of the interval. The latter solution is usually chosen for studying the correlation of the amounts of evaporation with precipitation, radiation, etc., for so-called hydrological years (November 1st - October 31st) because the variation of moisture and groundwater conditions on these dates in successive years is negligible compared with that of other relevant yearly quantities.

Obviously, for such long-duration intervals the lumping of the system and its variables is far less restricted than for the short term variations that occur in the usual rainfall-runoff study. To summarize, it can be stated that correlation models are effective for describing the operation of essentially static systems. Considerable difficulties arise, however, as soon as the dynamic character of a hydrologic system has to be taken into consideration. For a correct evaluation of a purely empirical correlation model it is essential to realize that the model can only describe the system's operation of converting inputs into outputs. The model cannot be expected, however, to provide definite information on the structure of the system. In terms of systems analysis (Sect.3.5) the system remains a "black box", only to be opened by insight into its structure and into the physical laws governing it.

It seems appropriate to close this brief discussion of the role of statistics in hydrology with a quotation from MORONEY's "Facts from Figures"(1956): "..... at no point are statistical methods more of a sausage machine than in correlation analysis. The problem of interpretation is always very much more difficult to deal with than the statistical manipulations, and for this side of the work there is no substitute for detailed practical acquaintance with every aspect of the problem. The statistician can only help out the specialist in the field, not replace him".

15.3.5 THE ANALYSIS OF LINEAR SYSTEMS

Along with statistical methods, other tools for reaping the fruits of empirical research have been introduced in hydrology under the name of linear systems analysis. The meaning of the word "linear" is that the principle of superposition is assumed to apply to the system's operation: if an input $x_1(t)$ causes the system to produce an output $y_1(t)$ and an input $x_2(t)$ gives rise to an output $y_2(t)$, then a linear system converts an input $x_1(t) + x_2(t)$ into an output $y_1(t) + y_2(t)$. Consequently if $x_2(t) = a\,x_1(t)$, then $y_2(t) = a\,y_1(t)$ (see Fig.16).

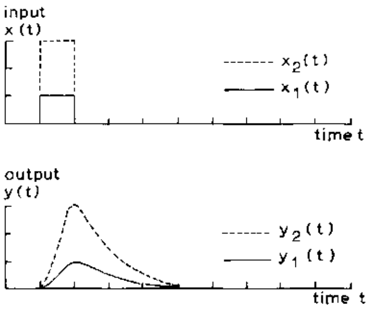


Fig.16. A linear system. If the input $x_1(t)$ causes the linear system to produce the output $y_1(t)$, then the input $x_2(t)$ = a $x_1(t)$ results in an output $y_2(t)$ = a $y_1(t)$.

For a linear system, which is also time-invariant, the output is always the same for a certain input x(t), regardless of the time at which x(t) is applied.

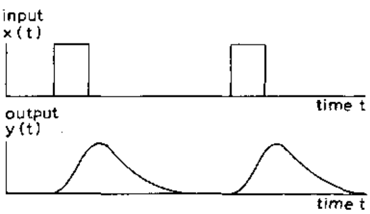


Fig.17. *A time invariant system.*

Any input can be regarded as being composed of a number of unit elements. If the characteristic response of the linear and time-invariant system to such a unit element is known, the output may be found by applying the principle of superposition.

Since SHERMAN's concept of the unit hydrograph (Sect.5) was recognized as such a characteristic response, an impressive array of applications of linear mathematics has been introduced in hydrology.

The three types of problems to be tackled with linear systems analysis are shown in Table 1 (DOOGE, in press).

Table 1. Type of problems to be tackled with linear systems analysis.

	Type of problem	input	system	output
Analysis	prediction	known	known	?
	identification	known	?	known
	detection	?	known	known

In hydrology the ultimate aim of systems analysis is the prediction of basin discharge, either for purposes of forecasting (design flood), or for "hindcasting", which is the reconstruction of the hydrograph of outflow for periods when only rainfall data were collected. These reconstructed hydrographs may be used for frequency studies. For prediction, however, one must also know the system's operation, so that the actual problem to be solved is the problem of identification; it is the problem of finding the system's characteristic response from given records of past inputs and corresponding outputs. It will be shown in Sect.5 that the unit hydrograph method follows this line of identification and subsequent prediction.

The third type of problem in systems analysis is the problem of detecting the input that has caused a certain measured output from a system of which the characteristic response is known. If one knows the discharge of a catchment, as well as the characteristic response, then the input, which is the amount and the distribution in time of excess rainfall that caused the discharge, can be found. Another, more general, problem of detection is the evaluation of measured data. Measurements contain random and systematic errors, due to the procedure that is used. This measuring procedure may be regarded as a system with measured data as an output (known), and the true values of the physical variable as input (unknown). For instance a stage hydrograph (output) written by a stage recorder in a stilling well shows a more or less distorted picture of the true water-level variations (input), not only because of the detention storage effect of the stilling well but also because of friction and other imperfections of the mechanism which together determine the system's operation. (Of course in this example the system may not be linear.)

An essential feature of systems analysis is that it also aims at the overall input-output relation of a certain linear time-invariant system. Because no attention is paid to either the structure or the governing physical laws, it is also called linear black-box analysis. So far, linear black-box analysis has been restricted to lumped systems as distinguished from distributed systems, and to lumped inputs and lumped outputs. Unfortunately, strict linearity and time invariance do not occur in hydrology. Many hydrologic systems, however, can be closely approximated by linear and time invariant systems for certain ranges of variation of the variables and for certain periods. Keeping these limitations well in mind, the powerful techniques of linear systems analysis can be employed to acquire information on the system's behaviour under both normal and extreme conditions. When applied to sub-systems, they will afford an insight into the structure of the total system and into the relative importance of its component sub-systems. The limited scope of this presentation does not allow any further discussion of these linear techniques. A comprehensive treatment of the subject has been given by DOOGE (in press).

15.3.6 MODEL SYNTHESIS OR SIMULATION

It seems appropriate to begin this discussion of model synthesis and simulation with a few words on the context in which the words "system" and "model" are being used here. Out of the many definitions of a "system" that literature provides, the following has been chosen: "If some part of the real world is considered separately from its environment, it can be called a system. Inputs and outputs connect the system with its environment". In hydrology this system concept can be applied to a catchment. Here we can distinguish inputs and outputs, such as precipitation, incoming heat flow and radiation, evaporation, and flow through the outlet. Such inputs and outputs connect the catchment with the atmosphere, adjacent catchments, etc. But as soon as the hydrologist starts to describe and discuss a system like a catchment, introducing notions like distributed systems and sub-systems, he in fact is replacing the catchment as a separate part of reality by some conceived model of similar but simpler structure (ROSENBLUETH, 1945).
Similar to correlation models, such conceptual models in parametric hydrology may be either based on some general idea of the overall structure of the runoff process in a catchment, or they may have been developed out of relatively detailed physical information on some specific hydrologic system. Between these two extremes, obviously the colour of the box which represents the system's operation may vary from completely black through numerous shades of grey, depending on the amount of essential physical information on the hydrologic system that has been worked into the conceptual model. An example of the other end of the range is the "white box" of Sect.3.2, which will, however, turn grey as soon as any difficulty arises with the measurement of the physical characteristics that make up the compound parameter k and when the lack of physical information must be compensated by empirical information. Most classical models belong to the black-box category. They are essentially so because they are based on general notions and not on actual physical information on a specific system or group of systems under study. In Section 6 a crude division will be made between groups of conceptual models as they are based on different general notions, such as "the reservoir approach", "the translation approach", and "the combined approach".

As these models have a simple and linear structure, the system's operation can be expressed in a linear mathematical model such as the one developed in Sect. 3.2. In this mathematical model the input-output relation is further defined by one or more parameters.

Like a factory-tailored suit that has been made without any specific information on the individual who is to wear it, the success of any conceptual model depends on its design (the structure) and on the possibilities (the parameters) of adapting the size and shape to the body it will be made to fit (the system). In hydrology the parameters of the chosen conceptual model must be optimized in order to produce the best possible fit to the catchment's precipitation-runoff system. The "goodness of fit" may be judged by some objective criterion such as the least sum of squares of the deviations between the observed outputs of the system and the model outputs generated from the corresponding inputs. For systems that can be considered linear and time invariant, this optimization can be achieved by using the techniques of systems analysis. Then the synthesized characteristic response of the model is compared with the actual system's response, as derived from the empirical input and output data, by the methods of identification of the system's operation (Sect.3.5).Obviously a good conceptual model (a good design) only needs a small number of optimized parameters to provide a good fit for many individual systems. On the other hand a great number of parameters can easily obscure the quality of the design or the relevance of the model's structure. Moreover, when judging conceptual models it should be remembered that in electronics two systems of totally different structure can be built, producing exactly the same input-output relation (within a certain range). Therefore an excellent fit of calculated and observed outputs may be an indication, but is no proof, of an analogy between the structures of model and system. Such proof can only be derived from physical information about the system. Although the above remarks on conceptual models have been made in connection with parametric models as distinguished from statistical models, they nevertheless apply to both categories.

When the two groups are compared, it appears that parametric hydrology specializes in dynamic-state systems and is actually focused on the system's memory. On the other hand statistical correlation methods can handle a number of simultaneous inputs, but there the system's memory seems to be a major stumbling block. The methods appear to be complementary. There is even an overlap with regard to certain linear techniques for finding the system's characteristic response from statistics of the input and output series. Apparently both the statistical and the parametric methods of handling empirical data, as represented by the horizontal line in Fig.14, will render their best services in close coöperation with the physico-analytical approach as indicated by the vertical line. To adapt a Dutch saying: "The physico-analytical cripple should guide the empirical blind man", and to quote again the mathematician, ANDERSON (1966): "The strength in understanding natural systems though comes from close observation, and in the field workers, as in the Infantry, lies our ultimate strength".

For the sake of completeness, this somewhat formal introduction to model synthesis in hydrology has, at a number of places, run ahead of the material to be discussed in the following sections. The reader is therefore advised to come back to this introduction after he has seen some actual models. It is hoped that he will then recognize the systems approach as an indispensable tool for clarifying the underlying concepts of hydrological practices and for providing a scientific background for the design and evaluation of hydrologic models. Clearly a number of practices in engineering hydrology owe their popularity rather to oversimplification and to the lack of reliable data to verify their results than to the soundness of the underlying concepts. It is the hydrologist's responsibility to analyse such practices and decide whether the underlying concepts are sound and whether they lead to realistic conclusions with regard to the runoff process under study.

In many drainage basins the losses occur in early stages of the runoff process when the bulk of evaporation losses are drawn from interception and soil moisture (Fig.2). This reasoning leads to a division of the catchment system into the two sub-systems of Fig.18.

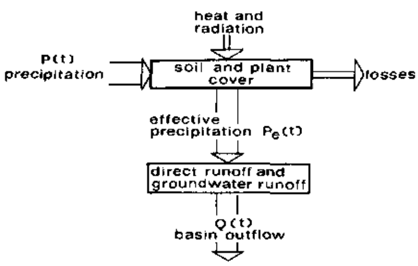


Fig.18. Division of the catchment system into two sub-systems.

Of course the two sub-systems are in reality interrelated. Clearly the condition of the soil plays a role both in the evaporation process and in the distribution of the effective precipitation over the branches of direct runoff and groundwater runoff; a high moisture content is a necessary condition for a high evaporation rate, but at the same time it is often an incentive to higher rates of direct runoff because of the corresponding reduction of the infiltration rates into the soil. Nevertheless it is common practice to accept this double role of the soil and to separate the two sub-systems so that the effective precipitation, which is an output of the first sub-system, is regarded as the input to the second sub-system, where the hyetograph of effective precipitation (time-intensity diagram) is subsequently transformed into the hydrograph of outflow. Before these two sub-systems are discussed, mention must be made of developments such as the STANFORD model, the BALEK model, and the model of DAWDY and O'DONNELL. These are more detailed and possibly more realistic computer simulations of the catchment system, but the optimization of the correspondingly great numbers of parameters requires big computers, which at the present stage are not generally accessible to the practising hydrologist. This discussion will therefore be restricted to the (factory-tailored) conceptual models of the above bipartite design.

If the area of the drainage basin is not too large, changes in the proportional distribution of precipitation will not be significant as to their effect on the system's operation, so that the precipitation on the catchment or basin can be referred to as basin precipitation, a lumped input variable. This basin precipitation can be measured by means of rain gauges and some weighting procedure such as the THIESSEN method or the isohyet method (Chap.18, Vol.III). At the other end of the total system the discharge through the basin outlet represents the output variable, provided that no appreciable leakage or deep percolation occurs and all water leaves the basin via this outlet. The outlet is chosen at a location where a measuring station can be installed.

For the same reasons as given for the basin precipitation, the net influx of heat and radiation can also be considered a lumped input variable. Unfortunately all efforts directed towards the computation of loss rates on the basis of measurements of this input variable have as yet met with little success. The study of losses caused by the basin evaporation has been called "the most desperate part of the desperate science of hydrology". The determination of these loss rates is one of the weakest links in any precipitation-runoff model because apparently the hydrograph of effective precipitation, which expresses the output variable of the first sub-system, cannot be determined by straightforward subtraction of computed loss rates from basin precipitation. The available physical information on the "soil and plant cover" sub-system is mainly of a qualitative nature so that the box is essentially black. Unfortunately precipitation is the only measurable input. It will be shown in Sect.4 how some methods make use of the fact that no water is added or lost in the second sub-system. By considering relatively long intervals, these methods get around the essentially dynamic character of this second sub-system and use the basin outflow to pick up quantities of effective precipitation that correspond with isolated rain storms. In some cases, such as indicated in Fig.19, the hyetograph of precipitation and the corresponding hyetograph of outflow thus supply some information on the magnitude of the losses and how they are distributed in the period considered. In this way some empirical information on the first sub-system's operation is obtained. The next step is to incorporate into a model the little physical information we have on the evaporation process, and finally to optimize its parameters to obtain the best possible fit between the "observed" effective precipitation and the output which the model generates from the corresponding basin precipitation. The obvious non-linearity, the neglect of dynamic effects, and the presence of several variables suggest the use of correlation models. In the following, attention will be paid to some of these models for the first sub-system. It will also be indicated that the concept of losses is changing. The above view on losses through evaporation implies that the total runoff, including groundwater runoff, must be dealt with in the second sub-system. This opinion is not shared by the traditional hydrologist, who - in imitation of the American pioneers - considers baseflow as a negligible quantity, so that all infiltration into the soil should be considered a loss to the runoff process. The unit hydrograph method, a method of analysis of the second sub-system, was developed for direct runoff only, thus neglecting the base flow. In Sect.5 the method will be discussed, and in Sect.7 it will be shown that its application is not limited to direct runoff only.

15.4 RAINFALL AND EFFECTIVE RAINFALL, DETERMINATION OF LOSSES

The transformation of rainfall into effective rainfall, which eventually leaves the drainage basin as outflow, and the losses through evapotranspiration will be discussed as a separate system (the first sub-system of Section 3.6). As was stated in Sect.3.6, the output of the first sub-system, the excess precipitation, is not measurable as such. Its amount and distribution in time has to be detected from the output of the second sub-system, the basin outflow. This can easily be done for separate storms falling on basins with a relatively "short memory", a small storage. In Fig.19 the storage of the second sub-system is depleted before the second storm starts. Figure 13 shows the output of a system with a "relatively long memory". Here it is more difficult to discern, in the hydrograph of outflow, the separate effects of each element of the rainfall input. This would imply an intelligent guess with regard to the transformation in the second subsystem. In Fig.13 the essential difficulty of dividing a system with a long memory into two separate sub-systems is illustrated. Nevertheless the usual approach of determining the losses and the subsequent transformation of the effective rainfall will be followed.

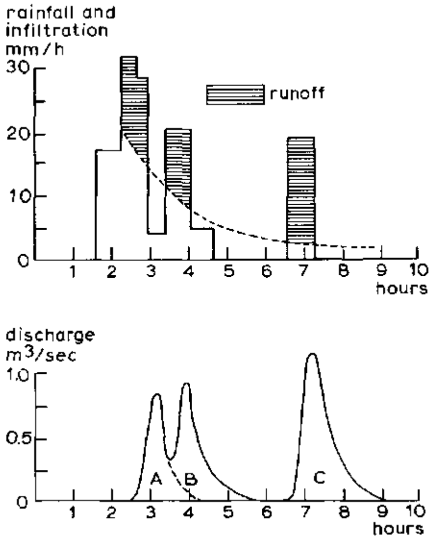


Fig.19.
Variation in infiltration capacity rate during a period of rain.

15.4.1 INFILTRATION APPROACHES

Let us consider a small drainage area on which a storm occurs and let P be the equivalent uniform depth of precipitation over the area (Fig.20). Now (a) part of the rainfall remains on leaves and blades of grass and later evaporates (interception); (b) part of it fills depressions at the soil surface or is used to neutralize soil moisture deficiency in the upper soil horizons, where it either evaporates or is drawn into the root systems of plants and is transpired through their leaves; (c) part of it percolates down into the groundwater reservoir, raising the level of the water table; (d) part of it will infiltrate and run along horizontal strata to come out again as interflow; and (e) part of it moves overland to the main stream. As regards runoff, (a) and (b) are total "losses". The portions of (c), (d), and (e) that reach the stream gauge form the runoff Q, caused by the precipitation P. This runoff can further be split up into groundwater runoff, or base flow, caused by (c), and direct runoff caused by (d) and (e). Surface runoff and interflow together make up the direct runoff, which is the main cause of floods, and in this respect percolation (c) into the groundwater reservoir, ultimately causing base flow, may also be considered a loss. The flow chart of the runoff process now takes on the form of Fig.21.

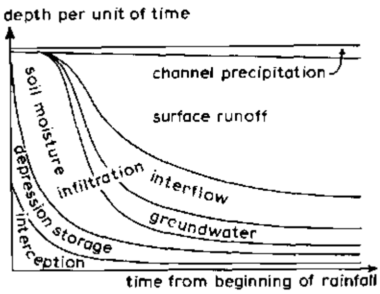


Fig.20. Distribution of the precipitation over various storages (LINSLEY et al., 1958).

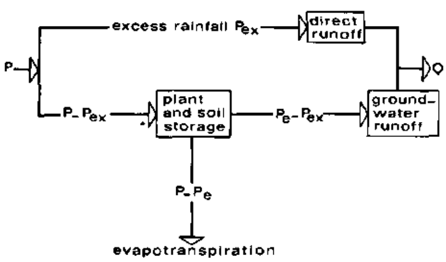


Fig.21. The rainfall, P, which eventually becomes outflow, may be divided into excess rainfall, P_{ex}, which is transformed into direct runoff and rainfall minus excess rainfall, which, in turn, is transformed into groundwater runoff.

In Fig.21, P_{ex} stands for precipitation excess, the direct "spill-over" of the drainage basin causing surface runoff and interflow. Therefore P_{ex} must be distinguished from the effective precipitation P_e, which includes all precipitation that eventually becomes runoff, including groundwater runoff.

Viewed from the angle of direct runoff, the losses can be regrouped as follows:

interception (a), direct infiltration (b+c), depression storage (b), and losses from overland flow after the end of rainfall. Of these losses, only the direct infiltration can be determined locally with a certain degree of accuracy, with the aid of infiltrometers and rain simulators on small test plots. Such local measurements of the infiltration rate supply values that are comparative only, considering the numerous complexes of soil, cover, and condition that make up a drainage basin. Moreover, the impact of drops has a noticeable effect upon the rate of infiltration into a soil and it is therefore important that rainfall simulators indeed closely simulate actual rainfall. Small-sized rainfall simulators can be used in a quick survey of the drainage area to collect estimates of infiltration capacities of each of the complexes.

We, however, are interested in more than just the direct infiltration since we want to know the total capacity of the drainage basin to retain water under the existing conditions. These losses can be found by comparing a hyetograph, or rainfall intensity diagram, with the resulting hydrograph of direct runoff at the outlet of the drainage basin (Fig.19). This procedure can be applied to a small drainage basin, where a relatively close succession of short rainstorms produce a number of separate floodwaves in the hydrograph. When two hydrographs overlap, they can be split up with reasonable accuracy by drawing a recession line parallel with the next line of recession. The areas below the graphs A, B, and C represent the volumes of rainfall excess in the three separate storms in the hyetograph. An infiltration capacity-rate curve should be so drawn that the shaded areas above this line represent corresponding volumes of rainfall excess. Strictly speaking, this line should be called a "retention capacity rate curve" because it not only represents the infiltration at capacity rate but it also includes interception, depression storage, and losses from overland flow after the end of rainfall.
On the other hand the retention does not include the interflow, i.e. that part of the infiltration yet to join the direct runoff after some detention in the top layer.

The infiltration capacity of the soil is not a constant, but is subject to seasonal variations. Moreover, it will normally decrease during the storm because of splashing raindrops, swelling colloids, and increasing soil moisture content. The total result of the initial soil conditions and the initial demands of interception and depression storage is an infiltration capacity rate curve that starts at a high value f_o and then falls rapidly during the early stages of a storm, finally levelling off and approaching a constant value f_c. It should be remembered that this method can only be applied on small, relatively homogeneous drainage basins, where successive, uniformly distributed rainstorms only cause small overlap of hydrographs and yet should fall within a relatively short period so that the infiltration capacity rate cannot be restored by intermediate evapotranspiration. Furthermore, each infiltration capacity rate curve is derived for certain conditions of soil and vegetation in the drainage basin. As such curves must be used to split up volumes of rainfall into rainfall excess and losses, the procedure of deriving sufficient curves for various conditions is rather involved. Many engineers prefer a simpler method and use infiltration indices. These indices are based on the assumption that for a specific storm with given initial conditions the rate of basin recharge or retention rate (rainfall minus direct runoff) remains constant throughout the storm period (Fig.22).

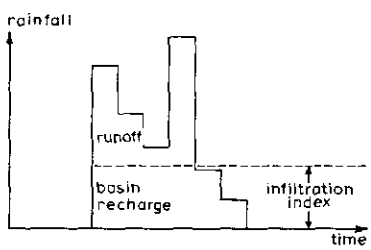


Fig.22. Infiltration index (LINSLEY et al., 1949).

The figure shows that the index equals total basin recharge divided by the duration of rainfall, provided that the rainfall intensity is continuously in excess of the index. The area above the index line represents the total runoff.

For studies concerning maximum flood possibilities, a minimum index is used, which corresponds with very wet conditions. In both infiltration approaches - the infiltration capacity rate curve and the infiltration index - it is essential that runoff be considered as rainfall minus losses. This is more realistic and therefore superior to the use of runoff coefficients that are mere ratios of runoff to rainfall. The error of assuming that the infiltration index should represent an actual infiltration rate should be avoided. For a complex drainage basin the difference between total rainfall and direct runoff also depends on the portion of the area that produces runoff throughout the rain. This portion increases with the rainfall intensity and consequently the infiltration index will also increase. It should always be kept in mind that neither the infiltration index nor the infiltration capacity rate is a constant for a certain area. Both undergo changes depending on antecedent rainfall and seasonal conditions.

Table 2. Infiltration indices at Ralston Creek near Iowa City, USA, (3 sq. miles) for 56 storms over 8 years (inches/hour).

	J	F	M	A	M	J	J	A	S	O	N	D
a)	0.00	0.17	0.12	0.12	1.30	1.57	1.36	0.75	1.11	0.60	0.11	0.00
b)	0.00	0.00	0.03	0.10	0.24	0.81	0.32	0.47	1.92	0.25	0.00	0.00

a) storms following 2 or more rainless days

b) storms following 1, or less than 1, rainless day

Although the above procedure is basically simple, in application a number of genuinely difficult questions arise:

(a) What is the effect of antecedent conditions?

(b) At what rate does infiltration capacity recover in periods between rains?

(c) What is the effect of season?

(d) What correction should be made for surface storage effects (cultivation)?

Such factors are accounted for by the graphical coaxial correlation analysis.

15.4.2 THE GRAPHICAL COAXIAL CORRELATION ANALYSIS

This method was originally developed by the U.S. Weather Bureau. A full discussion of the subject may be found in "Hydrology for Engineers", LINSLEY et al. (1958).

DOOGE (1967) pointed out that the coaxial correlation procedure should explicitly involve the assumption of a specific model of drainage basin behaviour. It follows that different types of drainage basins will give rise to different models and therefore to different types of coaxial diagrams. BECKER (1967, 1968) has worked along these lines and he modified the diagrams as developed by the U.S. Weather Bureau in order to bring them into agreement with certain concepts of the transformation of precipitation into runoff. The following discussion of the graphical coaxial correlation method is based on recent work by BECKER.

A first approximation of the average moisture condition of the soil and the plant cover in the basin may be expressed by the Antecedent Precipitation Index

$$API_n = P_n + cP_{n-1} + c^2P_{n-2} + \dots c^iP_{n-1} \quad (c < 1) \tag{8}$$

API_n is the value of the antecedent precipitation index at the end of the time interval number n; P_{n-1} stands for the precipitation during the last but one interval, etc. When the expression is written as follows

$$API_n = P_n + P_{n-1}\, e^{-1/k} + P_{n-2}\, e^{-2/k} + \dots P_{n-1}\, e^{-i/k} \tag{9}$$

it shows that the API may be regarded as a storage of precipitation, which is directly proportional to its rate of depletion (API = kq) and which is replenished at the end of each interval by the precipitation observed during that interval. The API does not give a true idea of moisture conditions in the soil since the direct runoff is not subtracted from the incoming precipitation; nevertheless the API has been found to be a useful indicator of initial soil moisture conditions.

A second factor to be introduced should represent the various seasonal changes that occur on the surface (vegetation, cultivation, etc.), in the soil (structure, biological activity, etc.), and in the rate of soil moisture depletion through evaporation. The reflection of the API against the appropriate season line in the first quadrant of Fig.23 indicates, on the horizontal axis, the recharge capacity of the drainage basin for a rainfall of great depth and very low intensity. This implies the assumption of a complete saturation of the recharge capacity possessed by the basin after a certain antecedent precipitation in a given time of the year.
In the second quadrant this recharge capacity is subsequently reduced for the actual duration of rainfall. Here the same great depth of rain is supposed to fall over a period of duration T_r. The intensity of rainfall may be in excess of the capacity of the basin to absorb water, and the actual possible recharge R_T is determined by this capacity rate of recharge f.
Apparently f is some function of the actual recharge capacity D, which may be visualized as a moisture deficit in soil and plant cover. BECKER assumes a proportionality of f and D

$$f = \frac{f_{max}}{D_{max}}\, D = -\frac{dD}{dt} \tag{10}$$

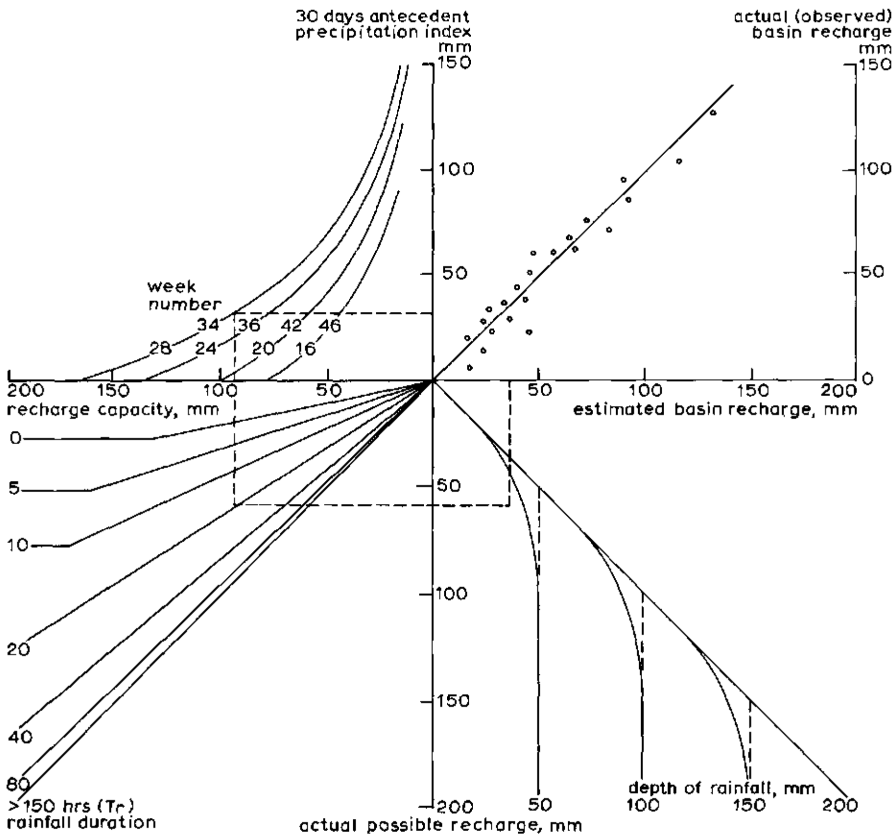


Fig.23. Coaxial rainfall-recharge relation diagram.

where $\frac{f_{max}}{D_{max}}$ is a constant ratio.

At the beginning of rainfall t = 0, D = D_{max} (the entry value from the first quadrant) and f = f_{max}.

For this initial condition the solution of the above equation reads:

$$D = D_{max} e^{-\frac{f_{max}}{D_{max}}(T_R + T_o)}$$

where T_o is the duration of surface runoff after the rain has stopped, so that $T_R + T_o$ is the duration of basin recharge.

The basin recharge caused by a rainfall of great depth and duration T_r is

$$R_T = D_{max} - D = D_{max}\left[1 - e^{-\frac{f_{max}}{D_{max}}(T_R + T_o)}\right] \tag{11}$$

This relationship is shown in Fig.24.

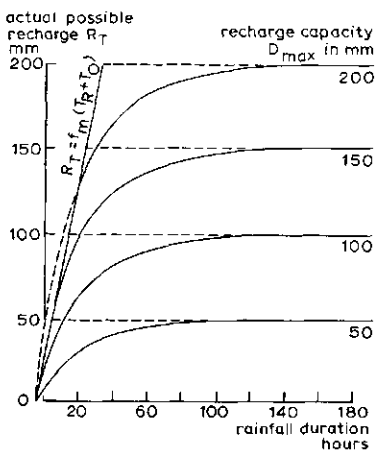


Fig.24. Actual possible recharge as determined by recharge capacity and rainfall duration.

The duration of rainfall T_R determines the value of the expression between brackets. This involves a straight-line relationship of R_T and D_{max} for a chosen value of rainfall duration T_R. It also follows that these lines pass through the origin.
Whereas R_T cannot exceed the available recharge capacity D_{max}, BECKER has introduced a second threshold in his model for the moisture recharge: For short duration rainfalls the possible rate of infiltration f_m into the soil may be less than the possible recharge rate of soil moisture. This limitation is expressed in Fig.24 by a straight line $R_T = f_m\ (T_R + T_o)$. Its effect in the second quadrant of Fig.23 shows in horizontal segments of the lines of lower rainfall durations.

Finally the actual depth of rain determines which part of the recharge capacity is actually used for recharge. Figures 25A and B show the development towards the right-hand bottom diagram of Fig.23. Graph A is drawn in accordance with the "threshold concept", which indicates that all precipitation is turned into basin recharge as long as the recharge capacity exceeds the depth of precipitation.

All precipitation in excess of this limit becomes runoff. This simplified concept, however, does not hold as soon as the recharge capacity rate or the recharge capacity itself are no longer evenly distributed over the drainage basin. Therefore the true relationship between recharge capacity and actual recharge as depending on rainfall depth will produce a diagram as shown in Graph B. Finally Graph C is obtained through replotting Graph B; here the runoff that equals precipitation minus recharge is plotted on the horizontal axis. This presentation is used for direct derivation of runoff or effective precipitation from the corresponding volumes of precipitation. The broken lines indicate the relationship in accordance with the threshold concept.

The basin recharge thus determined in this multiple graphical correlation procedure should agree with the actual basin recharge. The actual recharge is determined as explained for the derivation of an infiltration capacity rate curve (Fig.19). The optimization of the regression lines is obtained by trial and error. This procedure is fully explained in "Hydrology for Engineers", Section 8.7, LINSLEY et al. (1958).

15.4.3 THE CURVE NUMBER METHOD

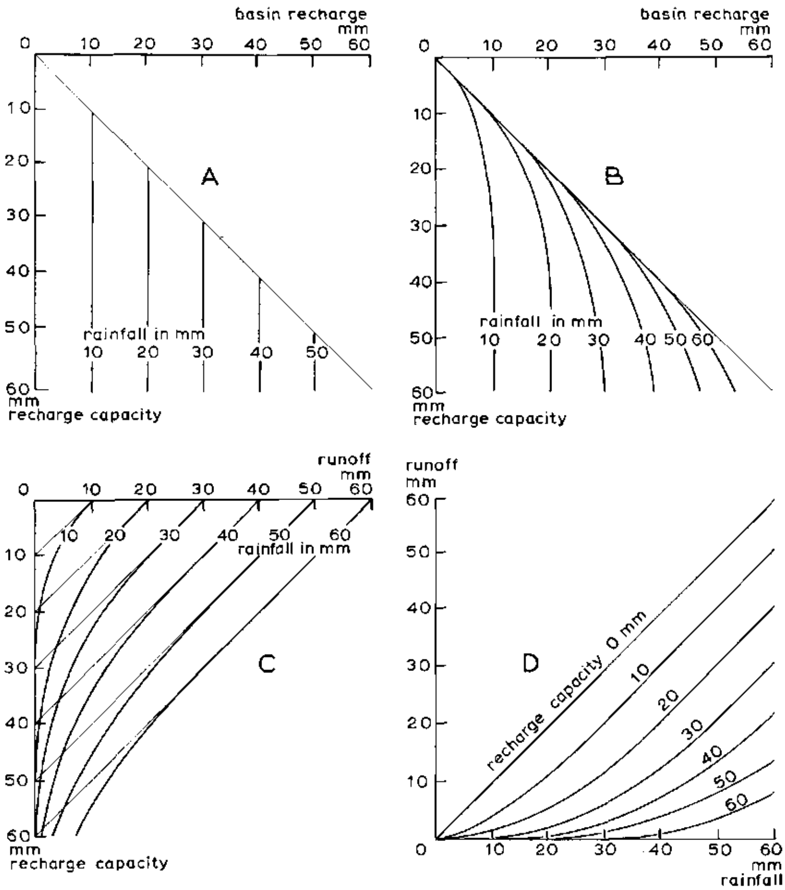
The concept of a limited recharge capacity, which is determined by antecedent moisture conditions and by the physical characteristics of the drainage basin, has been elaborated by the U.S. Soil Conservation Service to a preconceived multiple correlation model in which the partial correlations are expressed in tabular form. This method is described in the S.C.S. National Engineering Handbook (1964; SCHULZE, 1966). It takes its name from the curve number

$$CN = \frac{1000}{10 + S} \qquad (12)$$

where S is the recharge capacity or "potential maximum retention" at a certain time. The curve number is found from tables as a function of antecedent rainfall, land use, density of plant cover, soil type, and conservation practices. These tables have been developed for U.S. conditions and are not readily applicable in other parts of the world. If used outside the U.S.A., they will first have to be adjusted to local conditions.

The underlying concept of the model is:

$Ia = 0.2\ S$ is an initial quantity of interception, depression storage, and initial infiltration that must be satisfied by any rainfall before runoff can occur.



A. Basin recharge as a function of recharge capacity and rainfall depth according to the threshold capacity.

B. Basin recharge as a function of recharge capacity and rainfall depth abandoning the simplifying threshold concept.

C. Runoff as a function of recharge capacity and rainfall depth (rearrangement of Fig.25B).

D. Runoff as a function of recharge capacity and rainfall (rearrangement of Figs. 25B and 25C).

Fig.25. The relation between basin recharge, runoff, recharge capacity and rainfall depth.

The ratio of direct runoff Q and the precipitation minus the initial loss P - Ia equals the ratio of the actual recharge minus initial loss, P - Q - Ia and the recharge capacity S. Literature provides no physical reasoning on which this mathematical model could be based.

$$\frac{Q}{P - Ia} = \frac{P - Q - Ia}{S} \tag{13}$$

or

$$Q = \frac{(P - Ia)^2}{P - Ia + S}$$

and since Ia = 0.2 S it follows that:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \tag{14}$$

The curve (Fig.26) expressing the relation of Q and P depending on the parameter S (and Ia = 0.2S) is only a variant of Fig.25D and can be directly derived by using the recharge capacity as a parameter and plotting runoff against precipitation. S is the only parameter in this model that determines the relationship between the amount of rainfall in one day and the corresponding daily amount of rainfall excess that will subsequently be transformed into direct runoff.

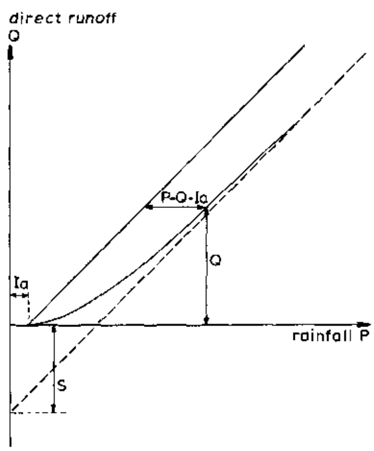


Fig.26. Curve number method:
Relationship rainfall-direct runoff as dependent on the recharge capacity.

A heterogeneous basin may be divided into sub-areas with different curve numbers. The total rainfall excess is then obtained by adding up the amounts that have been computed for the sub-areas. The basic assumption, which has been expressed in Eq.13, is certainly open to criticism. For high values of P and Q the left hand side of Eq.13 approaches unity, whereas the right hand side cannot exceed the value of 0.8, unless the actual recharge $P - Q$ exceeds the recharge capacity S. This, of course, is in contradiction with the concept of recharge capacity. Substitution of $Q = P - S$ in Eq.14 shows that the limit is reached for $P = 4.2$ S. Therefore the U.S. Soil Conservation Service introduced the limits $P > Ia$ and $S > Ia + F$, where $F = P - Ia - Q$. It follows that $S > P - Q$. For high curve numbers which go with a small recharge capacity, this could imply a definite restriction to the method's applicability.

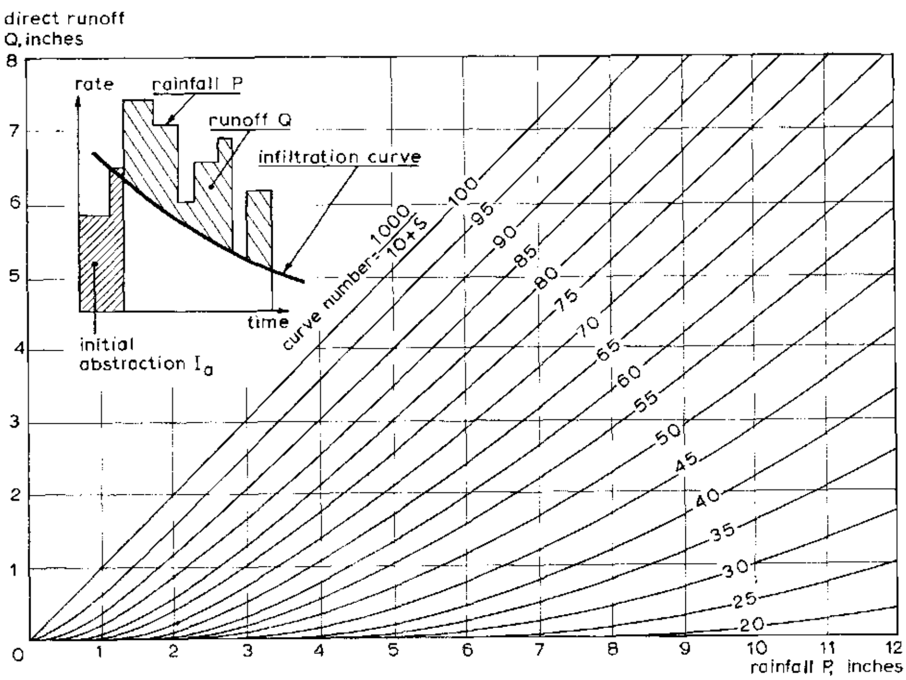


Fig.27. Solution of Eq.14 for various values of the recharge capacity, S. (U.S. Soil Conservation Service, 1964).

Although the underlying concept is not quite sound, the method is presented here because much work has been done to correlate the one parameter S with antecedent rainfall, seasonal effects, and certain characteristics of the soil surface and plant cover.

15.4.4 COMPARISON OF THE GRAPHICAL COAXIAL CORRELATION METHOD AND THE CURVE NUMBER METHOD

Common features of the graphical coaxial correlation method and the curve number method are:
Limited recharge capacity determined by antecedent precipitation and seasonal effects.

The differences are:

graphical coaxial method

The lines in Fig.23 that relate recharge capacity, rainfall, and recharge (such as the lines in the first and second quadrants) are found by trial and error, using measured (or estimated) input and output data. In other words the optimization of the model in the way of model synthesis may lead to a fair degree of precision. But the model is then only applicable to the drainage basin under consideration.

curve number method

This one-parameter model is a purely synthetic model, which can be adjusted to any drainage basin within the range of cases that have been studied for the establishment of the correlations expressed in various tables. The method can thus be used for ungauged drainage basins, but the quality of the results must necessarily be relatively poor. The use of infiltration capacity rate curves, infiltration indices or a rainfall-runoff correlation analysis leads to an estimate of the volume of water that would be delivered to a stream as the result of rain or melting snow. It will be explained in the following section how this runoff volume is transformed by the second sub-system of Fig.18 into a hydrograph of flow at a point in the main channel.

15.5 THE UNIT HYDROGRAPH METHOD

15.5.1 PRINCIPLES

In 1932 L.K. SHERMAN introduced the unit hydrograph as an important tool to be used in the transformation of a hyetograph of excess rainfall into a hydrograph of outflow from a drainage basin. The unit hydrograph method is a typical example of the linear "black box analysis", as applied to the second sub-system of

Sect.3, the basic assumption being that the system is linear and time invariant. The characteristic response of the second sub-system is the unit hydrograph of the drainage basin. The derivation of this unit hydrograph is therefore an identification problem. Subsequently, using this unit hydrograph, a chosen design storm is transformed into a design flood. In most regions of the world, flood danger is almost exclusively caused by overland flow and subsequent surface runoff; for such conditions the unit hydrograph was originally developed. Here this line of thought will be followed although it will be shown in Sect.7 that the concept of the unit hydrograph can be applied in a wider field that may also include groundwater flow.

It has been explained that any flood-period hydrograph may be considered a hydrograph of direct runoff superposed on a hydrograph of groundwater runoff. It is also clear that such fluctuations as may exist in groundwater discharge are of a different character and usually of a lower order of magnitude than are the fluctuations in surface runoff, since they are caused by different types of flow. It is thus logical to attempt a separation of a flood-period hydrograph into two parts, so that the phenomenon of direct runoff may be analyzed independently (Fig.28). We shall consider a single crested floodwave in the hydrograph of runoff. The preceding dry period is typified by a groundwater depletion curve, and the rather sharp departure at point (a) designates the arrival of direct runoff at the outlet where the stream-flow is gauged.

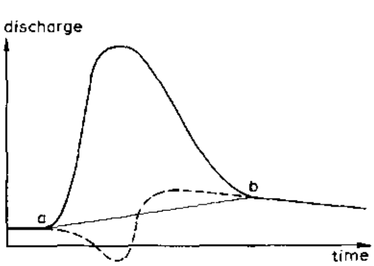


Fig.28. Separation of direct runoff and base flow.

The first problem to be solved is how to locate the end (b) of direct runoff on the falling limb. It should be realized that the falling limb is a recession curve representing the depletion of surface detention, channel storage and ultimately groundwater storage. Therefore the falling limb must merge into the groundwater depletion curve. When surface detention and channel storage have been depleted, groundwater flow continues (Fig.29). The combination of a number

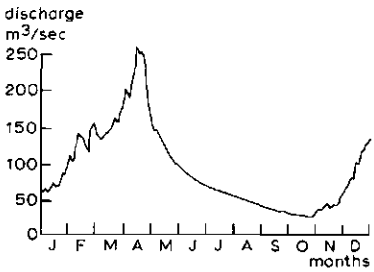


Fig.29. Groundwater recession curve of the Lualaba River, Congo (Kinshassa).

of such recession curves yields a groundwater depletion curve by allowing for the flow created by direct runoff to have passed the gauging station at the outlet (Fig.30). This curve typifies the extent and depth of the groundwater reservoir. In the case represented in the picture it is an important tool for predicting the minimum flows to be expected. This curve can now be fitted to the falling limb in order to trace the tail of the depletion curve backward into time.

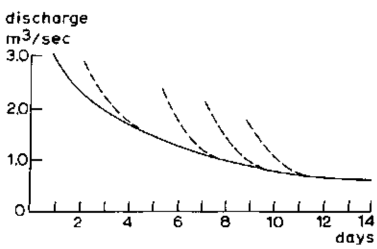


Fig.30. The groundwater recession or depletion curve composed of individual storm recessions.

Thus point (b) (cf. Fig.28) is found, representing the time when both surface detention and channel storage have been depleted and direct runoff has come to an end. The next problem - how to draw a line between (a) and (b) for marking off the base flow - is hard to solve accurately. It is logical to suppose that the rising water table in the channel system represses the inflow of groundwater, to be followed by an increase during the fall, and that consequently the line of separation may be curved as shown in the picture. The exact shape of the line is, however, uncertain. Fortunately groundwater or base flow in many cases only makes up a minor part of the total flood flow and then a reasonable approximation is attained by drawing a straight line between (a) and (b). Whatever method is chosen in separating direct runoff and base flow it is essential that a consistent procedure be followed. The time interval (a) - (b) designates the duration of direct runoff and is called the base length of the hydrograph of direct runoff. By plotting the figure above the line (a) - (b) separately, the hydrographs of direct runoff are obtained. The area underlying this curve represents the total volume of direct runoff, which is the sum of surface runoff and interflow.

It was found empirically that rainstorms of uniform intensity, causing equal durations of rainfall excess on one drainage basin produce hydrographs which fit rather closely the following properties:

a. The duration of direct runoff and therefore the base length (a) - (b) is essentially constant, regardless of differences in the intensities of the flood-producing rains and the total volume of direct runoff.
b. If two storms of uniform intensity and the same duration produce different volumes of direct runoff, then the rates of direct runoff at corresponding times after the beginning of each storm are in the same proportion to each other as the total volumes of direct runoff.
c. The time distribution of direct runoff from a given storm is independent of concurrent runoff from antecedent storm periods.

These properties are those of a linear time invariant system. It has been further found that for every drainage basin there is a certain unit storm period for which the shape and duration of the hydrograph are not significantly affected by changes of the distribution of a certain volume of rainfall excess over this unit storm period. This means that the time that elapses from the beginning of direct runoff until the hydrograph reaches its crest is essentially the same for all storms producing rainfall excess of a duration that is shorter than the unit storm period. For very small drainage basins the unit storm period does not exceed the period of rise, and for drainage areas exceeding 2 sq. miles its duration is not longer than half the period of rise or one fourth the basin lag. The proposition sub c implies that hydrographs resulting from successive unit storms of different intensities have proportional ordinates and can be accumulated, which means mathematically that the phenomenon is linear and the principle of superposition applies (Fig.31).

The unitgraph, or unit hydrograph, is the hydrograph of direct runoff resulting from a rainfall excess of one surface inch or millimeter, uniformly distributed over the whole basin and of a duration equal to, or shorter than, the unit storm period. The method of deriving the unitgraph for a certain drainage basin is based on the simultaneous analysis of:

- Continuous hyetographs of basin rainfall, obtained from the records of a sufficient number of stations in or near the basin.
- The hydrograph of runoff from the drainage basin, based on continuous gauging at the outlet and covering the same period as the rainfall data.

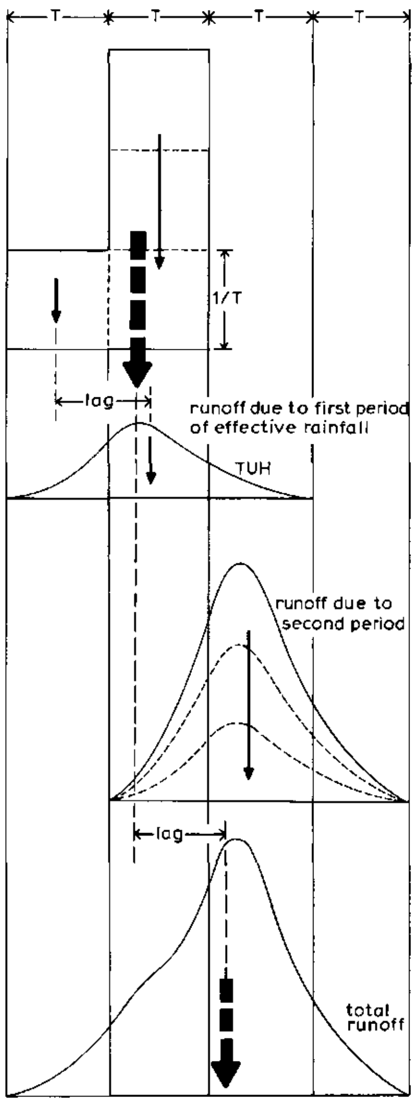


Fig.31.
The principle of superposition.

A number of isolated floodwaves caused by short periods of rainfall excess are selected from the hydrograph of runoff. Next the base flow is deducted from these flood hydrographs, using the method already explained. For each wave the area enclosed under the obtained hydrograph of direct runoff designates the total volume of direct runoff. This volume is divided by the area of the basin. thus expressing the total direct runoff in surface inches or millimeters. All ordinates of the hydrograph are then divided by this number of surface inches, yielding one shape of the unit hydrograph.
Because of inaccuracies in the basic data, of non-uniform distribution of storms, and of departures of drainage basin performance from unitgraph theory, it is not to be expected that all unitgraphs thus derived from a number of isolated flood periods will be identical (Fig.32). It is a common practice to derive a number of such graphs and to plot them on a single set of coördinates, shifting individual graphs slightly to the right or left to make their crests more or less coincide in time. The mean of the crests may then be taken as the best value for the crest of the composite unitgraph, and the remainder of the graph may be sketched in by the eye, adjustments being made to insure that the area under it totals unity. The base length for all unitgraphs should be taken as the average of the various lengths, indicated by application of the depletion curve to the individual hydrographs. It should be noted that the choice of a short base length will make the unitgraph relatively high, which in most design problems is not objectionable since the unitgraph will be used for the transformation of a design storm into a critical floodwave.

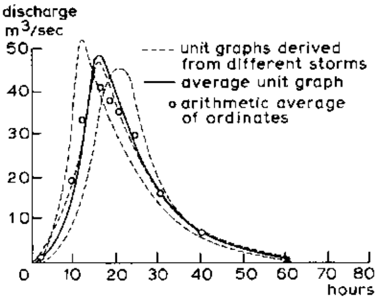


Fig.32. Determination of the average unit hydrograph (LINSLEY et al., 1958).

15.5.2 FINDING A DESIGN FLOOD WITH THE AID OF THE UNIT HYDROGRAPH (PREDICTION)

The planning of any hydraulic structure, channel, or detention reservoir implies an estimate of the highest discharge the structure will have to cope with. If a reliable long-period hydrograph is available, probability-theory can be applied in order to find the highest discharge that is not likely to be exceeded in a certain period. Generally, however, hydrographs - when available - only cover short periods, whereas rainfall data can often be obtained for periods of many years, either within the area itself or from stations located elsewhere in the same meteorologically homogeneous region.

The first step in determining the design flood is the selection of a design storm from the rainfall records. The next step is to make an estimate of the probable total volume of rainfall excess, by applying either the infiltration approach or some rainfall-runoff correlation method. Thus the time distribution of rainfall excess as caused by the design storm is found. The period of rainfall excess is subsequently divided into unit storm periods, thus yielding a succession of unit storms.

The available composite unit hydrograph is converted into a distribution graph by changing the ordinate scale from cubic feet per second to percentage of total runoff (Fig.33). Ordinates to the distribution graph represent volume per unit storm period rather than rate of discharge; hence the graph necessarily has a stepped form. These successive percentages can now be applied in turn to the volumes of rainfall excess in each unit storm period, yielding an equal number of overlapping hydrographs. By adding the coinciding ordinates the hydrograph of direct runoff is obtained (Fig.35).

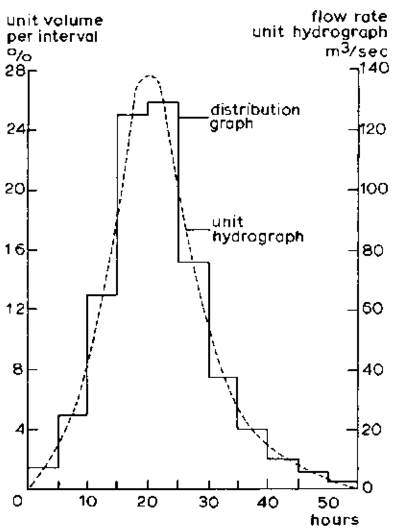


Fig.33. Unit hydrograph and distribution graph.

Because it is not always apparent which design flood will produce the highest crestflow, it may be necessary to apply the above procedure to a number of design storms.

This computation is given below in tabular form. P_1, P_2, ... are volumes of rainfall excess in the successive unit storm periods; u_1, u_2, ... are the percentages of the distribution graph; and finally Q_1, Q_2, ... stand for the volumes of outflow in each unit storm period. In number n row the distribution graph is applied to the number n input.

P_1	P_2	P_3	P_4	P_5	P_6				
u_1P_1	u_2P_1	u_3P_1	u_4P_1	u_5P_1					
	u_1P_2	u_2P_2	u_3P_2	u_4P_2	u_5P_2				
		u_1P_3	u_2P_3	u_3P_3	u_4P_3	u_5P_3			
			u_1P_4	u_2P_4	u_3P_4	u_4P_4	u_5P_4		
				u_1P_5	u_2P_5	u_3P_5	u_4P_5	u_5P_5	
					u_1P_6	u_2P_6	u_3P_6	u_4P_6	u_5P_6
Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}

so that:

$$
\begin{array}{lcl}
Q_1 & = & u_1P_1 \\
Q_2 & = & u_2P_1 + u_1P_2 \\
Q_3 & = & u_3P_1 + u_2P_2 + u_1P_3 \\
Q_4 & = & u_4P_1 + u_3P_2 + u_2P_3 + u_1P_4 \\
Q_5 & = & u_5P_1 + u_4P_2 + u_3P_3 + u_2P_4 + u_1P_5 \\
Q_6 & = & 0 + u_5P_2 + u_4P_3 + u_3P_4 + u_2P_5 + u_1P_6 \\
Q_7 & = & 0 + 0 + u_5P_3 + u_4P_4 + u_3P_5 + u_2P_6 \\
Q_8 & = & 0 + 0 + 0 + u_5P_4 + u_4P_5 + u_3P_6 \\
Q_9 & = & 0 + 0 + 0 + 0 + u_5P_5 + u_4P_6 \\
Q_{10} & = & 0 + 0 + 0 + 0 + 0 + u_5P_6
\end{array}
$$

The general expression is $$Q_n = \sum_{i=1}^{i=n} u_i P_{n-(i-1)} = \sum_{i=1}^{i=n} P_i u_{n-(i-1)} \qquad (15)$$

Equation 15 is the summation form of the convolution integral, which will be discussed in Sect.5.3.

When the above method has yielded the hydrograph of surface runoff, the most probable hydrograph of base flow must be added to obtain the design flood. Conversely to the above example, where the outflow was calculated from a given series of rainfalls P and a given distribution graph (a unit hydrograph with the ordinate expressed as a percentage of total runoff volume), the unit hydrograph may be calculated from a given series of rainfalls P and outflow Q, by solving Eq.15 for u.
There are several solution-techniques available, which are all rather cumbersome if no digital computer can be used. A suitable method for digital electronic computing of the unit hydrograph is the matrix inversion:
Equation 15 can be seen as the multiplication of the matrix (P) by the vector (u)

$$(P)\ .(u) = (Q) \tag{16}$$

Programmes for the solution of a matrix equation like Eq.16 are normally available in each programme library. It should be noted that u in Eq.15 is overdetermined and if the set of equations which make up Eq.15 is incompatible, which is always the case with real data, the matrix technique automatically includes a "best fit" procedure.

In larger watersheds many complications may arise because of major differences in duration, distribution, and intensity of rainfall and because of varying soil conditions. Major floods will then frequently be the result of high rates of overland flow in only a portion of the basin. A flood routing procedure may then be necessary. Whether the unitgraph method can be applied is mainly a matter of judgment concerning local distribution of intense rains.
Storm movement may also affect unitgraph proportions, and this obviously plays a more important role in larger drainage basins. As this chapter is only meant to introduce the basic notions underlying some simple hydrological techniques, we cannot enlarge upon the many problems to be solved in the analysis of larger watersheds.
At the end of this section one most important point must be stressed: The study of runoff is only on a sound basis when it rests on actual measurements of flow.

15.5.3 MATHEMATICAL TOOLS OF LINEAR SYSTEMS ANALYSIS APPLIED TO THE UNIT HYDROGRAPH METHOD

The discussion of the unit hydrograph method has shown that a system having excess rainfall as an input, and runoff as an output, may be considered a linear time invariant system. This system transforms a unit volume block input of excess rainfall of duration T into a T-hour unit hydrograph (TUH). The duration T is a determining factor for the shape of the unit hydrograph. Decrease of this duration, implying an increase in intensity, makes the unit hydrograph more skewed (Fig.34). In other words: when T decreases, the unit hydrograph merges gradually into its limiting shape, the instantaneous unit hydrograph (IUH). This IUH is the result of an instantaneous unit volume input. For practical purposes the TUH for a certain duration input T_o is sufficiently close to the final IUH. This duration T_o may then be considered the unit storm period.

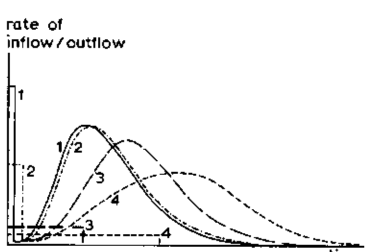


Fig.34. Shape of the unit hydrograph for various durations of excess rainfall.

In linear systems analysis the response of a system to a unit volume of instantaneous input (unit impulse) is known as the impulse response. Consequently the instantaneous unit hydrograph is the impulse response of the system.

In the application of the unit hydrograph method described in Sect.5.2, the time distribution of excess rainfall is broken up into intervals of steady input rates of a duration T_o, so that the corresponding T_oUH only slightly deviates from the IUH. The T_oUH itself is also broken up into intervals of duration T_o. So the resulting hydrograph of outflow, such as the design flood of Sect.5.2, takes the shape of a step function ("quantized discrete time" output). The input, however, can also be considered a continuous function, consisting of a succession of infinitesimal instantaneous inputs of volume $x(\tau)\, d\tau$ with intensity $x(\tau)$ and duration $d\tau$. (See Fig.35).
If the IUH is expressed as $u(o, t)$, then the input $x(\tau)\, d\tau$, applied at time τ contributes to the output y, at time t

$$dy(t) = u(o,\ t - \tau)\ x(\tau)\ d\tau$$

Consequently the output y, at time t, caused by a succession of inputs $x(\tau)\ d\tau$, is

$$y(t) = \int_{\tau=o}^{\tau=t} x(\tau)\ u(o,\ t - \tau)\ d\tau \tag{17}$$

The operation performed by the integral of Eq.17 is known as a convolution and is essentially the same as the tabular computation given in Sect.5.2.

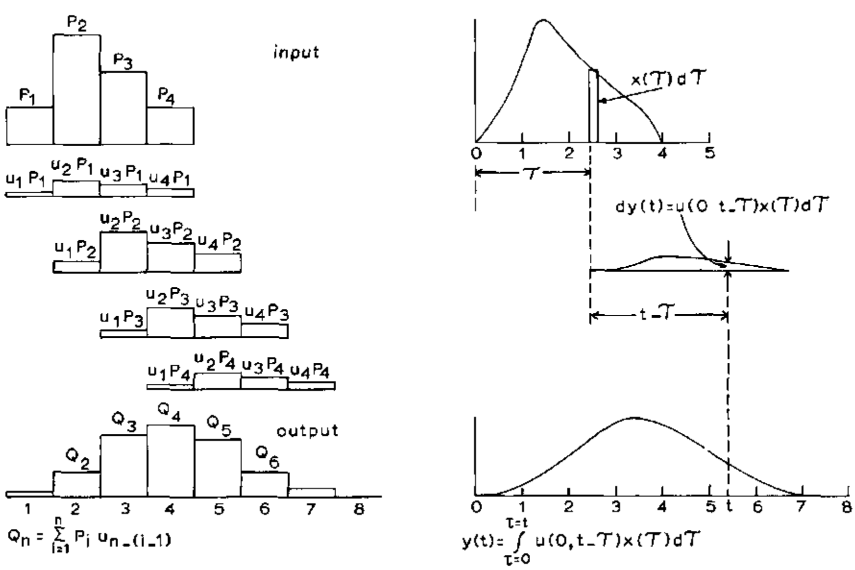


Fig.35. The convolution of IUH.

The convolution integral permits the output to be computed from any input if the IUH is known. The IUH may be found by graphical means, as was explained in Sect.5.1, or by solving Eq.15 for u, which could be done, for example, by inversion via matrices (See Sect.5.2). Both methods derive the shape of the unit hydrograph directly from actual time series of excess rainfall and corresponding outflow. They are typical for a "linear black-box" approach.

A third possibility is the use of a preconceived expression for the IUH, based on some concept of the runoff process as has been discussed in Sect.3.4. The parameter values in such a conceptual model are derived from the actual inputs and outputs of the runoff system. These conceptual models will be discussed in Sect.6: Model Synthesis.

Along with the impulse response, another characteristic response is used in linear systems analysis: the S-curve. An S-curve is the response to a unit step input; in other words, an S-curve pictures the growth of the outflow rate to its final unit value as caused by a constant unit intensity input

$$S_t = \int_{\tau=o}^{\tau=t} u(o,\ t-\tau)\ d\tau \tag{18}$$

where $x(\tau) = 1$ for $\tau > o$.

Substituting $t - \tau = \sigma$ and $d\tau = -\,d\sigma$

$$S_t = -\int_{\sigma=t}^{\sigma=o} u(o,\ \sigma)\ d\sigma = \int_{\sigma=o}^{\sigma=t} u(o,\ \sigma)\ d\sigma \tag{19}$$

An S-curve starting at the time T can be expressed by

$$S_{t-T} = \int_{\sigma=o}^{\sigma=t-T} u(o,\ \sigma)\ d\sigma \tag{20}$$

It follows that a block input of duration T and intensity $\frac{1}{T}$ causes the T-hour unit hydrograph

$$u(T,\ t) = \frac{1}{T}\left\{\int_{o}^{t} u(o,\ \sigma)\ d\sigma - \int_{o}^{t-T} u(o,\ \sigma)\ d\sigma\right\} = \frac{1}{T}\left(S_t - S_{t-T}\right) \tag{21}$$

$$= \frac{1}{T}\int_{t-T}^{t} u(o,\ \sigma)\ d\sigma \tag{22}$$

(valid for $t > T$. For $t < T$ the lower limit becomes o).

So the TUH is found by subtracting two S-curves: the one starting at t=o, minus the one starting at t=T (See Fig.37). Multiplication with a factor $\frac{1}{T}$ is necessary to maintain a unit volume.
Equation 22 also shows that the ordinate of a T-hour unit hydrograph at any time

is the average ordinate of the IUH during a period of T-hours before that time.

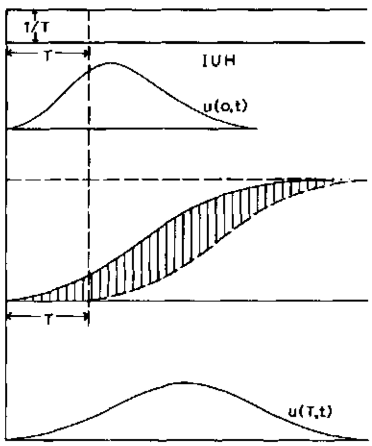


Fig.36. The determination of a TUH through subtraction of two S-curves: the one starting at t=o minus the one starting at t=T.

The peak rate Q_p is the average ordinate of the largest possible T-hour block of the IUH and occurs at a time t_p at the end of this block (See Fig.37). For a small value of T the block is narrow and centred under the crest of the IUH.

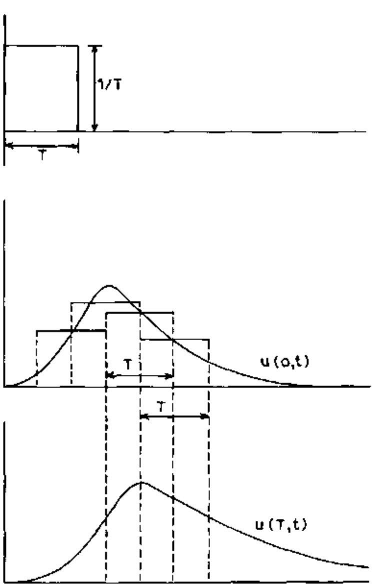


Fig.37. Graphical construction of a TUH from an IUH.

Doubling the period T will only cause a slight decrease of the average ordinate, which is the peak value of u(T, t). In this example (Fig.38), 2T could be taken as the unit storm period because the peak value Q_p of u(2T, t) is practically the same as the peak value of u(T, t).

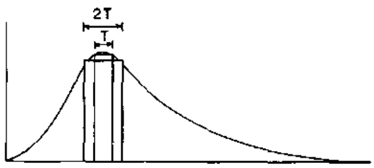


Fig.38. Effect of T on crest flow.

15.6 MODEL SYNTHESIS

A basic concept of the runoff system may lead to the synthesis of a model for the system's operation. In the history of hydrology a considerable number of rainfall-runoff models have been developed. The drainage basin transforms excess rainfall into outflow. This transformation implies both lag and attenuation of the time distribution of excess rainfall before it finally becomes the hydrograph of outflow. Figure 11 shows lag and attenuation as caused by a pure detention reservoir. In this case attenuation prevails, although lag also occurs. The translation of a monoclinal wave (see Fig.8) was used as a simplified illustration of a floodwave's progress through the channel system. This translation caused time lag only and no attenuation, and it was found that the time of travel was related to the ratio of storage and discharge above certain initial values. When the various methods of runoff routing are reviewed, three approaches can be discerned:
- The reservoir approach (Sect.6.1)
- The translation approach (Sect.6.2)
- The combination of reservoir and translation approach (Sect.6.3).

15.6.1 THE RESERVOIR APPROACH

In the description of the routes followed by overland flow and sub-surface flow, attention was given to the various forms of storage that flow has to pass through on its way to the outlet: surface runoff passes through surface detention and channel storage, while interflow - the other component of direct runoff - must, in addition, pass through storage in upper soil layers.

J.M. LYSHEDE (1955) indicated this "reservoir effect" and he described the hydrograph as a sum of exponential functions that could be the expression of a number of linear reservoirs. He states, however, that "any curve can be fairly accurately described as a sum of several exponential functions" and therefore the physical meaning of the model's structure should not be overestimated.
In a linear reservoir the outflow rate is proportional to storage

$$S = kQ \tag{23}$$

We first consider the IUH of a linear reservoir. The IUH is defined as the reaction in outflow to the instantaneous input of unit volume, $S = 1$ at $t = 0$.
At $t > 0$ the reservoir will be emptied according to

$$Q(t) = -\frac{dS(t)}{dt} = \frac{1}{k}\, S(t) \tag{24}$$

Hence

$$\frac{dS(t)}{S(t)} = -\frac{1}{k}\, dt \tag{25}$$

The general solution of this differential equation is

$$\ln S(t) = -t/k + C \tag{26}$$

With the initial condition

$$S(t) = 1 \text{ at } t = 0$$

and, since $\ln 1 = 0$, it follows that $C = 0$. Hence

$$S(t) = e^{-t/k} \tag{27}$$

$$Q(t) = \frac{1}{k}\, e^{-t/k} \tag{28}$$

Because the input was an instantaneous input of a unit volume, it follows that

$$u(o,\ t) = Q(t) = \frac{1}{k}\, e^{-t/k} \tag{29}$$

The S-curve for a linear reservoir is the outflow that is caused by a unit step input. This is a block input of unit intensity and of infinite duration beginning at t = 0.

$$S_t = \int_o^t u(o,\sigma)\, d\sigma \qquad \text{or} \qquad u(o,t) = \frac{dS}{dt} \tag{30}$$

$$S_t = \int_o^t \frac{1}{k}\, e^{-\sigma/k}\, d\sigma = - e^{-\sigma/k} \Big|_o^t = - e^{-t/k} + 1$$

$$S_t = (1 - e^{-t/k}) \tag{31}$$

The T-hour unit hydrograph for a linear reservoir is the outflow that is caused by a block input of duration T and of intensity 1/T

$$u(T,\ t) = \frac{1}{T} \int_{t-T}^{t} u(o,\ \sigma)\ d\sigma \tag{32}$$

$$= \frac{1}{T}\,(- e^{-t/k} + e^{-\frac{t-T}{k}})$$

$$= \frac{1}{T}\,(e^{T/k} - 1)\ e^{-t/k} \tag{33}$$

With a constant inflow rate P_1 from t = 0 till t = 1, the outflow rate will be (see Eq.31)

$$Q_1 = P_1\ (1 - e^{-1/k})$$

This is the result of the convolution P_1 with the IUH.
Suppose the constant inflow rate from t = 1 till t = 2 is P_2, then convolution of P_2 with the IUH yields

$$Q_2' = P_2\ (1 - e^{-1/k})$$

But at t = 2 there is still outflow from the first period with inflow rate P_1. This contribution Q_2'' to the total outflow rate Q_2 can be found from Eq.33 for t = 2, T = 1 and the inflow rate P_1 instead of $\frac{1}{T}$

$$Q_2'' = P_1(e^{1/k} - 1)\ e^{-2/k}$$

$$= P_1(1 - e^{-1/k})\ e^{-1/k}$$

$$= Q_1 e^{-1/k} \tag{34}$$

Hence

$$Q_2 = Q_2' + Q_2'' = Q_1 e^{-1/k} + P_2(1 - e^{-1/k}) \tag{35}$$

Thus for a simple linear storage typified by its proportionality factor k, the outflow rate at the end of an interval can be derived from the outflow rate at the end of the former interval and the inflow during the considered interval. In general

$$Q_t = Q_{t-1} e^{-1/k} + P_t(1 - e^{-1/k}) \tag{36}$$

The time lag of the IUH of a linear storage can be determined by computing the first moment with respect to the origin, giving the centroid of the area

$$\text{lag} = \frac{\int_o^\infty \frac{1}{k}\ te^{-t/k}dt}{\int_o^\infty \frac{1}{k}\ e^{-t/k}dt} = \frac{-k\int_o^\infty tde^{-t/k}}{\int_o^\infty e^{-t/k}dt} \tag{37}$$

$$= -k\ \frac{te^{-/tk}\Big|_o^\infty - \int_o^\infty e^{-t/k}dt}{\int_o^\infty e^{-t/k}dt}$$

$$= -k\ \frac{0 - \int_o^\infty e^{-t/k}dt}{\int_o^\infty e^{-t/k}dt} = -k\ \frac{0-1}{1} = k$$

It can be proved for a linear reservoir that the distance in time between the centres of area of the time distribution of excess rainfall and the resulting hydrograph must always equal k, the proportionality factor of the reservoir. It follows then that the lag of n reservoirs in series must be equal to nk (See also Fig.31).

In 1956 SUGAWARA and MARUYAMA presented a hydraulic model of glass cylinders, emptying themselves through capillary tubes. These linear reservoirs, arranged both in parallel and in series, thus imitated the system of reservoir effects in the drainage basin. For two different reservoirs in series, the instantaneous IUH of the first reservoir constitutes the input for the second reservoir, so that the IUH of the total model can be derived as follows

$$u(o,t) = \int_{\tau=o}^{\tau=t} \frac{1}{k_1} e^{-\tau/k_1} \frac{1}{k_2} e^{-\frac{t-\tau}{k_2}} d\tau \tag{38}$$

$$= \frac{1}{k_1 k_2} e^{-t/k_2} \int_{\tau=o}^{\tau=t} e^{\frac{k_1-k_2}{k_1 k_2}\tau} d\tau$$

$$= \frac{1}{k_1-k_2} e^{-t/k_2} \left[(e^{t\frac{k_1-k_2}{k_1 k_2}} -1) \right]$$

$$= \frac{1}{k_1-k_2} \left[(e^{-t/k_1} - e^{-t/k_2}) \right] \tag{39}$$

The expression for the IUH shows that the sequence of the two successive operations does not affect the result: k_1 and k_2 in Eq.39 can be interchanged.

For two equal reservoirs the IUH reads

$$u(o,\ t) = \int_{\tau=o}^{\tau=t} \frac{1}{k} e^{-\tau/k} \frac{1}{k} e^{-\frac{t-\tau}{k}} d\tau$$

$$= \frac{1}{k^2} te^{-t/k} \tag{40}$$

A series of three equal reservoirs has the IUH:

$$u(o,\ t) = \frac{t^2}{k^3}\ \frac{1}{2}\, e^{-t/k} \tag{41}$$

NASH (1958), through a more elegant and direct derivation, found for a series (cascade) of n-equal reservoirs:

$$u(o,\ t) = \frac{t^{n-1}}{k^n}\ \frac{1}{(n-1)!}\ e^{-t/k} = \frac{1}{k\Gamma(n)}\ e^{-t/k}\left(\frac{t}{k}\right)^{n-1} \tag{42}$$

where $\Gamma(n) = (n-1)!$ for integer values of n.

This is NASH's expression for the IUH of a drainage basin. In analogy with Eq.30

$$u(o,\ t) = \frac{dS}{dt}$$

The TUH can be expressed in the finite difference form

$$u(T,\ t) = \frac{S_t - S_{t-T}}{T} = \frac{1}{T}\, S_t - \frac{1}{T}\, S_{t-T}$$

In other words the TUH can be derived by convolving a step input of infinite duration and intensity 1/T beginning at t=o with the unit hydrograph u(o, t) and subtracting the result of a similar convolution when the step input begins at t=T (Fig.37)

$$u(T,\ t) = \frac{1}{T}\left(\int_o^t u(o,\ \sigma)\, d\sigma - \int_o^{t-T} u(o,\ \sigma)\, d\sigma\right) = \frac{1}{T}\,(S_t - S_{t-T})$$

Note that

$$u(T,\ t) = \frac{1}{T}\,\frac{1}{\Gamma(n)}\left[\int_o^{t/k} e^{-\sigma/k}(\sigma/k)^{n-1}\, d(\sigma/k) - \int_o^{\frac{t-T}{k}} e^{-\ /k}(\sigma/k)^{n-1}\, d(\sigma/k)\right]$$

$$u(T,\ t) = \frac{1}{T}\left[I(n,\ t/k) - I(n,\ \frac{t-T}{k})\right] \tag{43}$$

I(n, t/k) is the incomplete gamma function of order n at t/k. These incomplete gamma functions have been tabulated (PEARSON's Tables of Incomplete Gamma Functions).

Applying the theory of statistical moments to this gamma distribution (Poisson distribution), NASH succeeded in correlating n and k empirically with physical characteristics of the drainage basin

$$\text{lag} = nk = 20\ L^{0.3}\ EA^{-0.33}\ \text{hr} \tag{44}$$

where EA = equivalent uniform slope (see Fig.6) in parts per 10,000

L = length of the main channel in km.

The number of storages n is

$$n = \frac{L^{0.1}}{0.41} \tag{45}$$

The lag time of the IUH of a cascade of n equal linear reservoirs is found by computing the centre of area of the IUH, the first moment about the origin which equals nk (compare Eq.37). The second moment about this centre of area (the variance of the IUH) equals nk^2.

15.6.2 THE TRANSLATION APPROACH (rational method)

According to DOOGE (1959), it was MULVANEY who in 1851 proposed a method that is known as the rational method. This method is based on the assumption that the effect of rainfall on the most remote part of the basin takes a certain period, the time of concentration T_c, to arrive at the outlet. This time of concentration can either be derived from correlations with basin characteristics or it can be computed from the times of flow in successive "bank-full" reaches of the main channel. It is further assumed that a constant intensity of excess rainfall CP occurs, uniformly spread over the area A, where C is a runoff coefficient. If this rate of input, a step function, continues until the time of concentration T_c has expired, the excess rainfall that fell on the remotest point of the drainage basin will just begin to cause a reaction at the outflow, so that the latter will have reached its ultimate and maximum rate $Q = CPA$. (46)

If it is decided that the design flow rate Q may be exceeded on an average of once in N years, rainfall intensity/duration formulas or graphs are used to find the average rainfall rate P for the period T_c to be exceeded with an average return interval of N years (Fig.39).

One fundamental weakness of this method emerges when the growth of Q over the period T_c to its final value Q = CPA is considered. This growth can be represented by an S-curve, the ordinates of which have been multiplied by CPA. The shape of this curve is determined by the basin's geometry and topography.

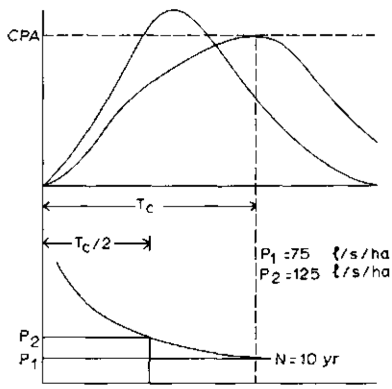


Fig.39. Rational method.

Figure 39 shows the T_c hydrograph and the 1/2 T_c hydrograph, both caused by rainfall intensities of the same probability 1/N. Obviously in this example the average rainfall rate P_2 with the same recurrence interval of N years, but for a period of 1/2 T_c, will result in a higher outflow rate because this rate P_2 is considerably higher than the rate P_1 for the total time of concentration. A number of finite period TUH's are now tried and their ordinates are multiplied by the appropriate rates from the rainfall intensity/duration curve in order to find the highest peak flow value (Fig.39). This method certainly shows a marked improvement over the rational method. The modified rational method, or time area method, can be regarded as the next step in the translation approach (Fig.40).

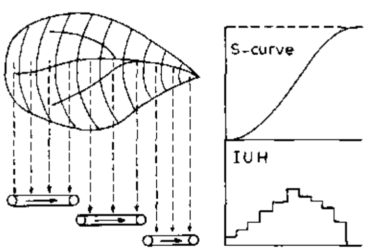


Fig.40. Modified rational method.

Using the hydraulic features of the "bank-full" channel system, the travel times to the outlet are determined for a number of points in the drainage basin, and time contour lines with equal time intervals are drawn. If it is assumed that an instantaneous excess rainfall of unit depth occurs simultaneously on all points of the basin, the excess rainfall on the elementary area between the time contour lines t and t+1 will arrive at the outlet between t and t+1 and will be represented by the appropriate part of the instantaneous hydrograph situated over this interval. This hydrograph can be called the time-area diagram or curve. Dividing all ordinates by the number of surface units A will yield the IUH according to the modified rational method. Of course, the method is not restricted to a constant input over the critical period and any design storm can be transformed to an outflow hydrograph. The topography of the basin may indicate that a certain pattern of areal distribution, instead of a uniform rain, must be considered critical. For that case the elementary areas between the time contour lines should be weighted accordingly and this will result in a time area diagram that is adjusted for the variation in rainfall intensity. The lag of this linear translation model is the distance in time between the origin and the centre of area of the time area diagram.

Within the scope of this presentation of runoff models with linear elements, it is relevant to note that in both the rational method and the modified rational method the translation of excess rainfall is supposed to occur through a system of linear channels (A wave passing through a linear channel is "translated" only, not attenuated). In these channels the travel times are independent of discharge rates. The channel system can be represented by a system of conveyor belts, each moving with its own constant speed independent of the load that is dumped on it. To simplify the picture further the system of conveyors can be replaced by one string of conveyors along the main channel. Each elementary area between two time contour lines dumps its load of excess rainfall onto the line of conveyors at the point where it crosses this elementary area. The local translation on the line is slower as the time contour lines are closer together and it follows from continuity reasons that "congestions" of storage will occur at these points. To return to the runoff process, this would mean that there is more storage in regions where the velocity of propagation is relatively low. This seems to be natural, but it must be added that the assumption of a constant velocity independent of the discharge rate is not realistic in most cases, since usually the one increases with the other.
NASH (1958) applied the modified rational method to a number of natural drainage basins where actual time distribution of excess rain and outflow rates were available. Comparison of computed and observed hydrographs, however, showed a serious overestimate of flood peaks.

15.6.3 THE COMBINED APPROACH

In a series of papers (1934, 1936, 1937) ZOCH presented a runoff model which consisted of one linear storage that was fed by a rectangular block input of uniform excess rain. He also presented solutions for triangular and elleptic inputs.
These inputs can be regarded as the effect of translation in particular basins (with the appropriate shape and topography) on an instantaneous excess rainfall. In that case the input diagrams represent the respective time area curves.
Indeed CLARK (1954) uses this same idea and presented an IUH that was obtained by routing the time-area curve through a single linear storage. He first calculated translation times and then drew the time contour lines in order to find the time-area curve. This curve is usually approximated by a bar diagram (Fig.41) and the successive flow rates of this diagram can be routed through the linear storage by the use of the routing Eq.36.

O'KELLY (1955) concluded from his study of a number of Irish drainage basins that the smoothing effect of storage on the time-area curve was so great that the latter could be replaced by an isosceles triangle without loss of accuracy. The base of this triangle was the time of concentration T_c and its area represented the unit depth of input. O'KELLY routed this input through one linear storage in order to find the IUH.
DOOGE (1959) presented a general theory for the linear runoff model. It is based on the assumption that the composite effect of storage and translation in a linear drainage basin can be represented by the transformation performed by a cascade of linear channels connecting equal linear storage elements. The rainfall excess from the elementary areas between successive contour lines is fed into this cascade and subsequently routed through the appropriate length of linear channel and the corresponding number of equal linear storage elements. DOOGE shows that CLARK's and NASH's methods are special cases of his generalized model. It should be noted that DOOGE's time area concentration curve represents translation effects that include the delay time due to overbank storage, whereas the classical method of computing travel times to the outlet is based on the assumption of a bankful channel system.

SINGH (1964) presented a model where the time-area curve is routed through the two linear storages, representing the effects of overland flow and of channel flow. Both the second storage parameter k_2 and the time of concentration T_c vary with the "equivalent instantaneous rainfall excess", which is the ratio of the reconstructed peak discharge and the peak ordinate of the IUH used in reconstructing the discharge hydrograph. Since this ratio determines the IUH, it is a trial and error procedure, which introduces a non-linear element into the model.

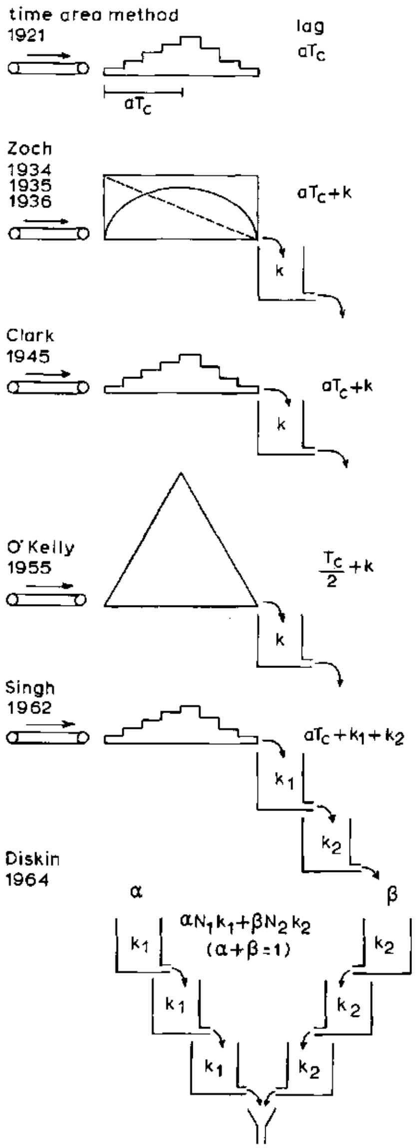


Fig.41.
Combined translation and storage models.

This and a number of other models have been reviewed by VEN TE CHOW (1964). LAURENSON (1962) discussed a number of runoff models and called particular attention to the fact that separation of translation from attenuation is unreal since any storage produces both. An underlying misconception is to apply the time of travel concept to a "drop of water" whereas the true implication, or lag, is the time it takes for the effect of an element of rainfall excess to reach the outlet. LAURENSON also studied the effect of non-linearities on the relation between rainfall excess and discharge from a drainage basin.

15.7 PARALLEL DEVELOPMENTS IN THE NETHERLANDS

In The Netherlands - with its flat topography, its deep soils, and long lasting rains of relatively low intensity - surface runoff is not a common phenomenon in natural drainage basins. This was the reason why attention was given primarily to the hydrograph of groundwater flow. Little thought was given to unit hydrograph theory, since groundwater flow has been explicitly excluded from practical unit hydrograph studies.

To arrive at rules that expressed the relation between rainfall and groundwater runoff, efforts were directed towards finding mathematical expressions for the flow system. Considering that the subsoil in this country has been deposited in horizontal layers and the fact that straight parallel drains are frequent, the linearized two-dimensional DUPUIT-FORCHHEIMER model was expected to provide a reasonable approximation (Chap.6, Vol.I)

$$\left.\begin{array}{l} q = -\,KD\,\dfrac{\partial h}{\partial x} \\ \\ R = \mu\,\dfrac{\partial h}{\partial t} + \dfrac{\partial q}{\partial x} \end{array}\right]\ \mu\,\frac{\partial h}{\partial t} = R + KD\,\frac{\partial^2 h}{\partial x^2} \qquad (47)$$

where h is the elevation of the phreatic level, μ the effective porosity, R the recharge to the groundwater per unit surface area, and KD the transmissivity. According to this model, non-steady groundwater flow to drains is analogous to one-dimensional heat flow and, after the example of BOUSSINESQ, a number of mathematical techniques that had been developed in this field were applied with advantage to the study of groundwater flow.

When the classification suggested by AMOROCHO and HART (1964) is applied, the study of groundwater runoff might be said to belong under physical hydrology since it attempts to give a quantitative description of a natural hydrologic system based on the laws of hydrodynamics.

It should be noted that such a model of groundwater flow is simple when compared with any model that describes, with a reasonable degree of accuracy, the intricate process of direct runoff. The complete runoff process is a system of interconnected component processes with complicated interactions, and it is not yet susceptible to a full quantitative description. Therefore, if this diffusion-type model is applied to the complete runoff process, it belongs to the field of systems approach to hydrology - what has been called "parametric" hydrology - which is aimed solely at finding an input-output relationship that can be used for the reconstruction of past events or the prediction of future events (See Sect.15.3).

Dutch hydrologists have so far been reluctant to leave the safe ground of physical hydrology: they try to stretch the solutions they obtain from simplified models to fit hydrologic situations that deviate considerably from their simple basic models. It would seem that in this "fitting process", which includes both model synthesis and systems analysis, an amount of subjective judgment is used, based on qualitative and semi-quantitative insight into the role of a number of complicating factors. The main object of hydrologic research in this country has been the improvement of this insight through studies of nature and models.

The original unit hydrograph method clearly belongs to the domain of parametric hydrology and moreover it deals exclusively with direct runoff, giving hardly any attention to groundwater flow. For these reasons the theoretical implications of the unit hydrograph method as brought forward by NASH, DOOGE, O'DONNELL and others, at first went by unheeded until it was discovered that the basic assumptions of linearity and invariance which underlie the unit hydrograph method are in complete accord with the nature of the simplifying assumptions that have been accepted in order to find analytical solutions for the equations describing the flow of groundwater.

At this moment of discovery, it was found that concepts developed in physical groundwater hydrology also played important roles in parametric hydrology. It appeared that these concepts had been developed systematically in parametric hydrology and the results could be used with advantage in the study of groundwater flow from polders and natural drainage basins.

KRAIJENHOFF (1966) reviewed a number of Dutch models for rainfall-runoff studies and exposed their structure in terms of parametric hydrology. The following sections are quoted from this review.

15.7.1 THE EDELMAN MODEL

EDELMAN (1947) developed equations for the two-dimensional free surface flow of groundwater from an infinite stretch of land into a channel, where specified level variations or rates of withdrawal occur (Fig.42a). He also noted that the approximating assumption of a constant transmissivity between the free groundwater surface and the impermeable layer causes the water level variations in the channel to have the same (computed) effect on groundwater flow as do appropriate rates of rainfall and evaporation, which cause variations of the groundwater level while the water in the channel remains at the same level.

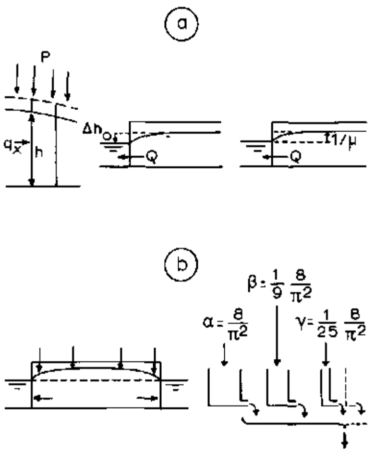


Fig.42.
Models for non-steady groundwater runoff:
a. EDELMAN and
b. KRAIJENHOFF (1958, 1966).

Although EDELMAN repeatedly used the superposition principle in his linearized model, he derived separate analytical solutions from his equation for instantaneous and gradual lowering of the water level in the channel. Through the use of the convolution integral, the latter solution can be derived simply from the former. This will be shown in the following application of linear model concepts to the flow of groundwater to a channel with a fixed level, a flow caused by percolation of rain into the phreatic zone.
EDELMAN's equation for one-side flow to a unit length of channel, following an instantaneous lowering Δh_o of the water level in the channel, is

$$Q(t) = \Delta h_o \frac{1}{\sqrt{\pi}} \sqrt{KD\mu}\; t^{-\frac{1}{2}} \qquad (\Delta h_o \ll D)$$

An instantaneous supply of unit depth of percolation causes the water table to rise $1/\mu$. The resulting flow to the unit length of channel is

$$u(o,\ t) = \frac{1}{\sqrt{\pi}} \sqrt{\frac{KD}{\mu}}\ t^{-\frac{1}{2}}$$

We can now apply the convolution integral in order to find the expression for the increase of groundwater flow caused by a step input of constant rate R of percolation into the phreatic zone

$$Q(t) = - \int_{\tau=o}^{\tau=t} \frac{R}{\sqrt{\pi}} \sqrt{\frac{KD}{\mu}}\ (t-\tau)^{-\frac{1}{2}}\ d(t-\tau) = R \frac{2}{\sqrt{\pi}} \sqrt{\frac{KD}{\mu}}\ t^{\frac{1}{2}} \qquad (48)$$

In order to apply this equation to flow from a drainage basin, flow from two sides into a channel must be considered; this means multiplication by a factor 2. Then allowance must be made for the fact that a unit length of channel in a drainage basin only drains a limited stretch of land. The average length of these stretches is the reciprocal of the drainage density $L = A/\Sigma l$, where A = basin area and Σl = the total length of channels in the basin.
The flow to the channel system expressed as flow per unit area is

$$Q_t = \frac{4}{\sqrt{\pi}} R \sqrt{\frac{KD}{\mu L^2}\ t} \qquad (49)$$

Since the underlying EDELMAN equation was derived for flow from an infinite stretch of land, this formula is only valid as long as flow to one channel is not being influenced by the presence of the other channels in the system. For a system of equidistant parallel channels this influence can be neglected until a period

$$j = \frac{1\ \mu L^2}{\pi^2 KD} \qquad (50)$$

has expired since the beginning of percolation to a horizontal water table (Fig.43). All factors that determine the nature of the soil and the nature and density of the drainage network are incorporated in this "reservoir coefficient", which typifies the drainage situation (KRAIJENHOFF, 1958). In Fig.43, Equation (α) is identical with Eq.49, and (β) represents the outflow from a stretch of land which has a limited width between two parallel channels (to be discussed in the next section).

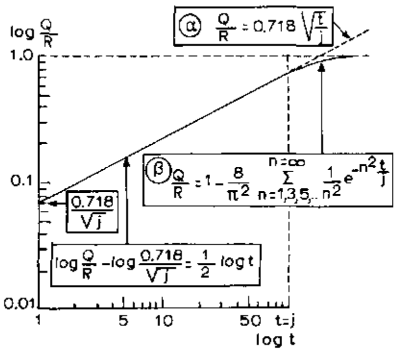


Fig.43. Growth of outflow rates caused by
a. step function of inflow according to
b. EDELMAN and
c. KRAIJENHOFF (1958, 1966).

Introduction of the reservoir coefficient into Eq.49 yields

$$Q_t = \frac{4}{\pi\sqrt{\pi}} R \sqrt{t/j} = 0.718 R \sqrt{t/j} \tag{51}$$

If j is expressed in unit intervals, the rate of outflow at the end of the third interval, for example, must be

$$Q_3 = \frac{0.718}{\sqrt{j}} \left[R_1\sqrt{3} + (R_2 - R_1)\sqrt{2} + (R_3 - R_2)\sqrt{1} \right]$$

$$= \frac{0.718}{\sqrt{j}} \left[R_1 (\sqrt{3} - \sqrt{2}) + R_2 (\sqrt{2} - \sqrt{1}) + R_3 \sqrt{1} \right]$$

Because of its restricted applicability, this simple formula can be used only to calculate groundwater flow caused by intensive short-duration inputs of recharge.

15.7.2 THE KRAIJENHOFF MODEL

GLOVER (1954) studied the falling groundwater table between equidistant parallel ditches or drains, following an instantaneous recharge R_i of excess irrigation water (Chap.8, Vol.II)

$$h(x, t) = \frac{R_i}{\mu} \frac{4}{\pi} \sum_{n=1,3,5..}^{n=\infty} \frac{1}{n} e^{-n^2 t/j} \sin\frac{n\pi x}{L} \tag{52}$$

where j is given by Eq.50.

KRAIJENHOFF (1958) derived from this equation the instantaneous hydrograph of flow to the drainage channels. It can be expressed by

$$u(o,\ t) = \frac{8}{\pi^2}\ \frac{1}{j} \sum_{n=1,3,5\ldots}^{n=\infty} e^{-n^2\ t/j} \tag{53}$$

In analogy with the technique of influence lines, this "influence function" was integrated in order to find the expression for flow caused by a continuous rate of steady percolation. It is apparent that here the concepts of the IUH and the convolution integral were used.
To pursue this parallel, Eq.53 can be written as follows

$$u(o,\ t) = \frac{8}{\pi^2}\ \frac{1}{j}\ e^{-t/j} + \frac{1}{9}\ \frac{8}{\pi^2}\ \frac{9}{j}\ e^{-9t/j} + \frac{1}{25}\ \frac{8}{\pi^2}\ \frac{25}{j}\ e^{-25t/j} + \ldots\ldots$$

Substituting $k_1 = j$, $k_2 = j/9$ and $k_3 = j/25$ etc.

$$u(o,\ t) = \frac{8}{\pi^2}\ \frac{1}{k_1}\ e^{-t/k_1} + \frac{1}{9}\ \frac{8}{\pi^2}\ \frac{1}{k_2}\ e^{-t/k_2} + \frac{1}{25}\ \frac{8}{\pi^2}\ \frac{1}{k_3}\ e^{-t/k_3} + \ldots \tag{54}$$

It can be shown that Eq.54 expresses the impulse response of a model that consists of parallel linear storages of decreasing magnitude, the respective storages being fed with decreasing parts of the input (Fig.42b). It should be noted that $8/\pi^2\ (1 + \frac{1}{9} + \frac{1}{25} + \ldots) = 1$.

In order to find the lag of this model it should be realized that the various parts of input passing through the respective linear storages each undergo their appropriate lag. It follows from the first moment about the origin

$$\text{lag} = \frac{8}{\pi^2} k_1 + \frac{1}{9}\frac{8}{\pi^2} k_2 + \frac{1}{25}\ \frac{8}{\pi^2} k_3 + \ldots = \frac{8}{\pi^2}\ j \left[\ 1 + \frac{1}{3^4} + \frac{1}{5^4} + \ldots \right] =$$

$$= \frac{8}{\pi^2}\ \frac{\pi^4}{96}\ j = \frac{\pi^2}{12}\ j \tag{55}$$

DE JAGER (1965) used this model for the synthesis of flood hydrographs of basins in alluvial soils. In flat areas that were well drained by a system of parallel drains he obtained excellent fits with observed hydrographs. Here the drainage situation corresponded closely with the physical basis of the model. With a number of natural basins the agreement proved to be good. In some cases two parallel models were used, one with a relatively small and the other with a relatively large reservoir coefficient.

15.7.3 THE DE ZEEUW MODEL

In his search for hydrological characteristics for a polder area, HELLINGA (1952) found an approximately constant ratio between the daily quantities of water pumped out of the polders and the amounts of rainfall excess that still remained to be pumped out. In other words this is an approximate proportionality of outflow rate and storage (Fig.44).

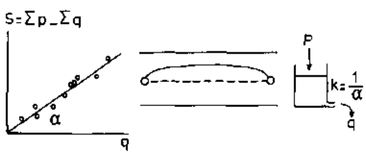


Fig.44. Quasi-steady model of DE ZEEUW and HELLINGA (1952, 1958).

DE ZEEUW and HELLINGA (1958) observed that storage in a polder area is mainly the groundwater stored below the groundwater table between the parallel tile drains or ditches. The mathematical expression for the ratio between outflow rate and storage was found from a combination of the continuity equation and the steady-state relationship between the rate of flow to parallel drains and the storage below a groundwater table of elliptic shape (Fig.15).

$$q = \alpha S \tag{56}$$

and

$$\alpha = 10 \frac{KD}{\mu L^2} \tag{57}$$

Equation 56 is the expression for a single linear storage with a proportionality factor $k = 1/\alpha$. Consequently the lag of this model is $1/\alpha$ and the IUH can be expressed by

$$u(o, t) = \alpha e^{-\alpha t}$$

DE ZEEUW and HELLINGA (1958) were the first to use one compound hydrologic factor to typify a drainage situation. By its very nature this quasi-steady solution is appropriate to describe relatively slow variations of flow.
In his more recent models for natural drainage basins DE ZEEUW (1966) sometimes uses two or three parallel linear storages, whereas in other cases he places KRAIJENHOFF's model parallel to one or two linear storages. The contributions

from these parallel storages to the total outflow are functions of the flow rate from the biggest storage, which represents groundwater flow from higher grounds. Here a non-linear element of feed-back is introduced and consequently neither an IUH nor a constant time lag can be indicated. These models developed by DE ZEEUW (1966) are considered in Chap.16, Vol.II.

LITERATURE

AMOROCHO, J. and HART, W.E. A critique of current methods in hydrologic systems investigations. Trans. Am. Geophys. Union, 25 : 307-321.

ANDERSON, D.V. 1966. Review of basic statistical concepts in hydrology. Proc. 5th Canadian Hydrology Symp., McGill Univ., Montreal, 5 : 3-27.

ASELTINE, J.A. 1958. Transform method in linear system analysis. McGraw-Hill New York, 300 pp.

BECKER, A. 1967. Threshold considerations and their general importance for hydrologic system investigations. Proc. Intern. Hydrology Symp., IASH, Ft. Collins, Colorado, 1 : 94-102.

BECKER, A. 1968. Modellkonzeption zur gesetzmässigen Erfassung der Niederschlag-Abfluss-Beziehungen. Wasserwirtschaft-Wassertechnik 18(1) : 16-21.

BUREAU OF RECLAMATION 1953. Water Measurement Manual. U.S. Department of the Interior, Washington, 1953.

CLARK, C.O. 1965. Storage and the unit hydrograph. ASCE Trans. 100 : 1416-1446.

CHOW, V.T. 1964. Handbook of Applied Hydrology. McGraw-Hill Book Co., London, 1418 pp.

DE JAGER, A.W. 1965. Hoge afvoeren van enige Nederlandse stroomgebieden. Centrum voor Landbouwpublikaties en landbouwdokumentatie, Wageningen, 167 pp.

DE ZEEUW, J.W. and HELLINGA, F. 1958. Neerslag en afvoer. Landbouwkundig Tijdschrift 70 : 405-421.

DE ZEEUW, J.W. 1966. Analyse van het afvoerverloop van gebieden met hoofdzakelijk grondwaterafvoer. Doctor's Thesis, Agric. University, Wageningen, 139 pp.

DISKIN, M.H. 1964. A basic study of the linearity of the rainfall runoff process in watersheds. Ph.D. Thesis Univ. of Illinois, Urbana, Illinois, 160 pp.

DOOGE, J.C.I. 1959. A general theory of the unit hydrograph. J. Geophys. Res. 64 : 241-256.

DOOGE, J.C.I. 1967. Lectures in the Workshop on Linear Theory of Hydrologic Systems. University of Maryland, Maryland, in press.

DOOGE, J.C.I. Linear Theory of Hydrologic Systems. In press.

EDELMAN, J.H. 1947. Over de berekening van grondwaterstromingen. Doctor's Thesis, Technical University, Delft. 147 pp.

EDSON, C.G. 1951. Parameters for relating unit hydrograph to watershed characteristics. Trans. Am. Geophys. Union 32 : 591-596.

GLOVER, see DUMM, L.D. 1954. Drain spacing formula. Agric. Engin. 35 : 726-730.

HELLINGA, F. 1952. Enige aspecten van de inrichting van polders. De Ingenieur, 64 : B33-B36.

HOLTAN, H.N., MINSHALL, N.E. and HAROLD, L.L. (eds.) 1962. Field manual for research in agricultural hydrology. Agriculture Handbook No.224, U.S.D.A., Washington, D.C. 215 pp.

JOHNSTONE, D. and CROSS, W.P. 1949. Elements of Applied Hydrology. The Ronald Press Cy., New York, 276 pp.

KIRKHAM, D. and DE ZEEUW, J.W. 1952. Field measurements for tests of soil drainage theory. Soil Science Soc. Am. Proc. 16 : 286-293.

KIRPICH, Z.P. 1940. Time of concentration of small agricultural watersheds. Civ. Eng. (N.Y.) 10 : 362.

KRAIJENHOFF VAN DE LEUR, D.A. 1958. A study of non-steady groundwater flow with special reference to a reservoir coefficient. De Ingenieur 70 : B87-B94.

KRAIJENHOFF VAN DE LEUR, D.A. 1966. Runoff models with linear elements. In: Recent trends in hydrograph synthesis. Versl. en Meded. Comm. Hydrol. Onderz. T.N.O. 13, The Hague, 107 pp.

LAURENSON, E.M. 1962. Hydrograph synthesis by runoff routing. The University of New South Wales, Water Research Laboratory Report No.66.

LINSLEY, R.K., KOHLER, M.A. and PAULHUS, J.L. 1949. Applied Hydrology. McGraw-Hill Book Comp., 689 pp.

LINSLEY, R.K., KOHLER, M.A. and PAULHUS, J.L. 1958. Hydrology for Engineers. McGraw-Hill Book Comp., 340 pp.

LYSHEDE, J.M. 1955. Hydrologic studies of Danish watercourses. Folia Geographica Danica, Tome VI, 155 pp.

MORONEY, M.J. 1956. Facts from figures. Pelican Book Company, London, 472 pp.

NASH, J.E. 1957. The form of the instantaneous unit hydrograph. IASH Assemblée Générale de Toronto, Tome III, 114-121.

NASH, J.E. 1958. Determining runoff from rainfall. Proc. Inst. Civ. Engs. 10 : 163-184.

NASH, J.E. 1958. The effect of flood-elimination works on the flood frequency of the river Wandle. Proc. Inst. Civ. Engs. 13 : 317-338.

NASH, J.E. 1960. A note on investigation into two aspects of the relations between rainfall and storm runoff. IASH Publication No.51, 568-578.

O'KELLY, J.J. 1955. The employment of unit hydrographs to determine the flows of Irish drainage channels. Proc. Inst. Civ. Engs. 4 : 365-412.

PEARSON, K. 1965. Tables of the incomplete gamma-function. Cambridge University Press, 164 pp.

REMENIERAS, G. 1960. L'hydrologie de l'ingénieur. Eyrolles, Paris. 423 pp.

ROSENBLUETH, A. and WIENER, N. 1945. The role of models in science. Philosophy of Science, 12 : 316-321.

SCHULZE, F.E. 1966. Rainfall and rainfall excess. In: Recent trends in hydrograph synthesis. Versl. en Meded. Comm. Hydrol. Onderz. T.N.O. 13, The Hague, 107 pp.

SHERMAN, L.K. 1932. Streamflow from rainfall by the unit graph method. Eng. News-Rec. 108 : 501-505.

SINGH, K.P. 1964. Non-linear instantaneous unit-hydrograph theory. Hydr. Div. ASCE 90 : 313-347.

Soil Conservation Service 1964. National engineering handbook, Section 4, Hydrology, U.S. Department of Agriculture, Washington, D.C.

SUGAWARA, M. and MARUYAMA, F. 1956. A method of prevision of the river discharge by means of a rainfall model. L'Ass. Intern. d'Hydr., Publ. No.42, Tome III : 71-76.

WISLER, C.D. and BRATER, E.F. 1949. Hydrology. John Wiley & Sons. New York, 419 pp.

WISLER, C.D. 1958. Hydrology. John Wiley & Sons, New York, 408 pp.

ZOCH, R.T. 1934. On the relation between rainfall and stream flow. Monthly Weather Rev. 62 : 315-322.

ZOCH, R.T. 1936. On the relation between rainfall and stream flow. Monthly Weather Rev. 64 : 105-121.

ZOCH, R.T. 1937. On the relation between rainfall and stream flow. Monthly Weather Rev. 65 : 135-147.

THEORIES OF FIELD DRAINAGE AND WATERSHED RUNOFF

16. HYDROGRAPH ANALYSIS FOR AREAS WITH MAINLY GROUNDWATER RUNOFF

J. W. DE ZEEUW
Research Hydrologist
Department of Land Drainage and Land Improvement
University of Agriculture, Wageningen

Lecturers in the Course on Land Drainage

J. W. de Zeeuw (1962–1972)
University of Agriculture, Wageningen

PURPOSE AND SCOPE

A procedure is described to derive calculation models for the precipitation - runoff relation from observed discharge hydrographs of areas in which groundwater runoff predominates.

CONTENTS

16.1 MODELS OF GROUNDWATER RUNOFF

16.1.1 INTRODUCTION

This chapter will describe how the parameters of a precipitation-runoff model can be derived from observed discharge hydrographs of different catchment areas in The Netherlands (DE ZEEUW, 1966). The basic concepts of the runoff process which apply to this model have already been discussed in Chap.15.

The fundamental idea is that the discharge hydrograph of an area necessarily shows the hydrologically characteristic properties of that area and will thus yield the parameters of the model. Consequently, all parameters in the model are derived from the hydrograph and it is inadvisable to consider the field conditions in the area too closely beforehand, in order to be able to perform the analysis objectively. This way of tackling the problem prevents the introduction of superfluous complications into the model and avoids the omission of essential elements. The only assumption made is that discharge reacts according to a simple exponential function.

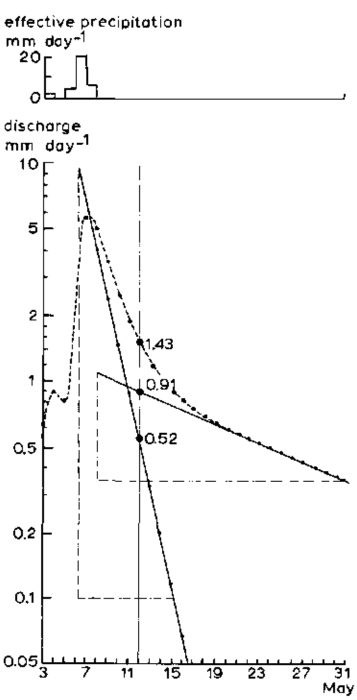


Fig.1.
Example of the analysis of a hydrograph.

The hydrograph is usually analyzed by the method indicated in Fig.1 (to be discussed in Section 16.3). By plotting the hydrograph on semi-logarithmic paper, the slopes are revealed of the relevant components of which the total hydrograph is composed. These components may be regarded as the outputs of two or more parallel linear reservoirs (Sect. 16.2). In some rather exponential cases the steepest component, indicating the fastest discharge reaction, fits the series of the Boussinesq reservoir (Sect. 16.2) better than it does the linear reservoir. This will be explained later.
Each of the hydrograph components in Fig.1 can be interpreted as the discharge of a distinct, hydrologically homogeneous, part of the basin under consideration. From the slope of the straight line, the parameter is found in the formula that describes the reaction of the linear reservoir, corresponding to a homogeneous area. This parameter is called the reaction factor. It is related to the drainage characteristics of the area, as will be discussed later.

16.1.2 THE PURPOSE OF THE MODELS

When a water-control structure is being planned, it is essential to know the discharge that the structure will have to cope with. This discharge is often called the specific discharge of the upstream area, or the design discharge. Its value should be such that the volume of water stored in any part of the upstream area will only infrequently exceed a predetermined level. High discharges from a given area show a characteristic frequency distribution. The frequency of high discharges, however, is small, justifying the risk that is taken by choosing a lower design discharge than the highest that can be expected.
The hydrologist has to provide the characteristic runoff frequency distribution. Basic data from which such a distribution can be established are often lacking, in which case the discharge pattern will have to be reconstructed from the available historical rainfall records.
This can be done, for example, by applying the Unit Hydrograph Method (see Chap. 15), which was originally developed for those conditions where direct runoff plays a major role. With this method the flood produced by a particular storm can easily be traced, enabling the unit hydrograph to be determined. Statistical frequency analyses may then be applied to storm (or precipitation) occurrences in order to determine a design rainfall from which the design discharge can be derived.

The Unit Hydrograph Method, however, cannot be applied in areas where groundwater

runoff predominates. Precipitation of weeks or even months ago may substantially influence the discharge intensity originating from groundwater flow. As a result, groundwater flood waves are much flatter than direct runoff waves and are less easily separated from one another. The necessity of using long term rainfall records has led to the use of precipitation-runoff models.
The appropriate procedure to develop a model is to measure continuously for one or more years the discharge of, and the precipitation on, the basin under consideration and to find from these data the model and its parameters that reproduces the runoff hydrograph when the measured precipitation data are fed into the model.
When it has been proved that the model is satisfactorily composed, i.e. when calculated data fit the measured data well, the model can be used to reconstruct historical discharge hydrographs. These can extend as far back as equally reliable precipitation data are available, preferably obtained from the same unchanged rainfall stations in the basin.

16.1.3 CONCEPT OF THE RUNOFF PROCESS IN THE NETHERLANDS

According to Chap.15 the runoff process can be divided into two stages:
- the transformation of measured precipitation into effective precipitation
- the transformation of effective precipitation into runoff.

The transformation of measured precipitation into effective precipitation

Effective precipitation is here defined as that part of the precipitation which leaves the drainage basin in a liquid form. The remaining part, which is eventually evaporated, is considered a "loss". The most important loss is the evapotranspiration from the soil moisture reservoir. Under the conditions prevailing in The Netherlands, nearly all rainfall, except that part which is intercepted by the vegetation, will infiltrate into the soil because of the soil's relatively high intake rate, the low intensity of rainstorms, and the flatness of the country. Almost no overland flow takes place.
As a consequence nearly all rainfall either replenishes the soil moisture reservoir to field capacity, making up for the evaporation losses, or percolates to the groundwater. This leads to the assumption - used in our model and known as the threshold concept - that no groundwater recharge will take place as long as the soil moisture reservoir is not completely replenished.
The effective precipitation is then equal to the measured precipitation, minus a calculated soil moisture deficit. To calculate this deficit, use is made of a

year-round standard evaporation for ten-day periods which was derived in 1897 from the water balance of a large polder. The actual evapotranspiration will deviate from the standard values, but this deviation is considered to be within the limits of error in the rainfall and discharge data.

The transformation of effective precipitation into (groundwater) runoff

The transformation of a hyetograph of effective precipitation into a hydrograph of discharge results from the passage of water through various reservoirs in which the runoff water is temporarily stored before it reaches the outlet of a drainage basin (see also Chap.15). These reservoirs are:
- surface reservoirs
- soil moisture reservoirs
- groundwater reservoirs
- channel reservoirs.

Surface reservoirs

Storage in local depressions is considered of minor importance on highly permeable soils. However, solid precipitation (snow) may be temporarily stored on the surface. Since accurate data on the melting of snow are usually lacking, two calculations are made, based on different assumptions: the first considering snow as normal rainfall and the second assuming that all snow melts on the last day of the period with snow cover. Reality lies between the two extremes.

Soil moisture reservoirs

The effective precipitation replenishes the soil moisture reservoir from which, in turn, the groundwater reservoir is recharged by percolation. Because of this transition from soil moisture into groundwater, the recharge pattern is flattened in comparison with the hyetograph of effective precipitation. This flattering effect, however, has only limited consequences on the discharge computations, because a certain flattening of the recharge pattern is already introduced by taking daily precipitation data instead of the actual rainfall distribution within the day. In areas with shallow groundwater tables, i.e. those having a fast discharge reaction, the flattening thus introduced is a sufficient approximation of the flattening effect that would have resulted from the soil moisture reservoir. In areas with deep groundwater tables, on the other hand, the discharge reaction is so slow that the relative influence of any flattening of the recharge pattern would have only a negligible influence on the computed discharges.

Groundwater reservoirs

The groundwater reservoir causes an important lag and attenuation, the value of which depends on the size of the reservoir (spacing of natural or artifical channels), on the effective porosity, and on the transmissivity. These properties are lumped together in the reaction factor. Often the body of groundwater of an area is subdivided into a great number of small reservoirs. Every strip of land between two valleys, open channels, or even artificial ditches and tile drains is in essence an individual reservoir. An area characterized by individual reservoirs of about the same dimensions, transmissivity, etc., is considered one single reservoir with an overall reaction factor reflecting the average conditions in the area.

Channel reservoirs

The groundwater discharge part of the hydrograph is generally not perceptibly influenced by channel storage. This is due to the fact that groundwater runoff tends to occur rather equally distributed over a whole area, which results in a nearly planparallel rising and falling of the open water level in the entire channel system. The hydrograph determined by recording this rising and falling at the outlet point of the area therefore has a shape corresponding to the progress of the groundwater outflow.

When the transport capacity of ditches is inadequate and open water levels rise so high that groundwater runoff is reduced, the same applies but the (then smaller) reaction factor is no longer determined by the physical properties of the profile. Later on this case will be indicated as "marshland discharge".

Channel storage does, however, affect the surface runoff component of the hydrograph in the sense that reaction factors for surface runoff tend to be smaller for larger areas. This is caused by the fact that, at least in the cases we are dealing with, surface runoff occurs locally, evoking real discharge waves that will attenuate to a larger degree, the longer their way through the channel system (Chap.15).

16.2 THE MATHEMATICAL EXPRESSIONS FOR THE MODEL TRANSFORMING EFFECTIVE PRECIPITATION INTO (GROUNDWATER) RUNOFF

Since the runoff process is a function of time, the transformation of effective precipitation into (groundwater) runoff has to be described by an expression for nonsteady flow from a reservoir. In this section the linear reservoir and the Boussinesq or Kraijenhoff reservoir will be discussed.

16.2.1 THE LINEAR RESERVOIR

A reservoir is called a linear reservoir when the outflow is directly proportional to the dischargeable storage. Such a linear reservoir will have all the resistance to flow concentrated at the outflow point. The flow and continuity equations for a linear reservoir are

$$\text{flow equation: } q = \alpha S \tag{1}$$

$$\text{continuity equation: } P_e = q + \frac{dS}{dt} \tag{2}$$

where

q = discharge per unit surface area in mm/day

S = storage per unit surface area in mm

α = reaction factor in day^{-1}

P_e = effective precipitation per unit surface area in mm/day

A combination of Eqs.1 and 2 results in a differential equation which has as solution

$$q_n = q_{n-1}\, e^{-\alpha(t_n - t_{n-1})} + P_{e,n}(1 - e)^{-\alpha(t_n - t_{n-1})} \tag{3}$$

where q_n is the discharge and $P_{e,n}$ the depth of P_e during interval t_{n-1} to t_n.

The same equation has been found in Chap.15 by the convolution of the instantaneous unit hydrograph of a linear reservoir

$$u(o,t) = \alpha e^{-\alpha t} \tag{4}$$

When t and α are expressed in the same unit of time, say days, $t_n - t_{n-1}$ reduces to 1. Moreover, $e^{-\alpha}$ is a constant for a certain value of α. Values for α, $e^{-\alpha}$ and $(1 - e^{-\alpha})$ are given in Table 1.

In a tabular form, Eq.3 reduces to

Table 1. Exponential function.

α	$e^{-\alpha}$	$1-e^{-\alpha}$	α	$e^{-\alpha}$	$1-e^{-\alpha}$
0.001	0.9990	0.0010	0.250	0.7788	0.2212
0.005	0.9950	0.0050	0.260	0.7711	0.2289
0.010	0.9900	0.0100	0.270	0.7634	0.2366
0.015	0.9851	0.0139	0.280	0.7558	0.2442
0.020	0.9802	0.0198	0.290	0.7483	0.2517
0.025	0.9753	0.0247	0.300	0.7408	0.2592
0.030	0.9705	0.0295	0.320	0.7261	0.2739
0.035	0.9656	0.0344	0.340	0.7118	0.2882
0.040	0.9608	0.0392	0.360	0.6977	0.3023
0.045	0.9560	0.0440	0.380	0.6839	0.3161
0.050	0.9512	0.0488	0.400	0.6703	0.3297
0.055	0.9465	0.0535	0.420	0.6570	0.3430
0.060	0.9418	0.0582	0.440	0.6440	0.3560
0.065	0.9371	0.0629	0.460	0.6313	0.3687
0.070	0.9324	0.0676	0.480	0.6188	0.3812
0.075	0.9278	0.0722	0.500	0.6065	0.3935
0.080	0.9231	0.0769	0.520	0.5945	0.4055
0.085	0.9185	0.0815	0.540	0.5827	0.4173
0.090	0.9139	0.0861	0.560	0.5712	0.4288
0.095	0.9094	0.0906	0.580	0.5599	0.4401
0.100	0.9048	0.0952	0.600	0.5488	0.4512
0.110	0.8959	0.1042	0.620	0.5379	0.4621
0.120	0.8869	0.1131	0.640	0.5273	0.4727
0.130	0.8781	0.1219	0.660	0.5169	0.4831
0.140	0.8694	0.1306	0.680	0.5066	0.4934
0.150	0.8607	0.1393	0.693	0.5000	0.5000
0.160	0.8521	0.1479	0.700	0.4966	0.5034
0.170	0.8437	0.1563	0.800	0.4493	0.5507
0.180	0.8353	0.1647	0.900	0.4066	0.5934
0.190	0.8270	0.1730	1.000	0.3679	0.6321
			1.100	0.3329	0.6671
0.200	0.8187	0.1813	1.200	0.3012	0.6988
0.210	0.8106	0.1894	1.400	0.2466	0.7534
0.220	0.8025	0.1975	1.600	0.2019	0.7981
0.230	0.7945	0.2055	1.800	0.1653	0.8347
0.240	0.7866	0.2134	2.000	0.1353	0.8647
			2.303	0.1000	0.9000
			2.996	0.0500	0.9500
			4.605	0.0100	0.9900
			5.298	0.0050	0.9950
			6.908	0.0010	0.0090
			7.601	0.0005	0.9995
			9.210	0.0001	0.9999
			∞	0.000	1.000

$$q_1 = q_o c + P_{e,1}(1 - c)$$

$$q_2 = q_1 c + P_{e,2}(1 - c)$$

$$q_2 = q_2 c + P_{e,3}(1 - c) \tag{5}$$

etc.

where c stands for the exponential factor $e^{-\alpha}$.

16.2.2 THE BOUSSINESQ OR KRAIJENHOFF RESERVOIR

The outflow from this type of reservoir is not directly proportional to the dischargeable storage; proportionality, however, is fairly approximated during tail recession. The Boussinesq series is derived under the assumption that the resistance in the vicinity of the outflow point equals zero; in other words, that the internal horizontal resistance is the only one existing (BOUSSINESQ, 1904). Introducing the parameter j, being the reciprocal of Boussinesq's α, Kraijenhoff developed a complete set of formulas for this situation (KRAIJENHOFF VAN DE LEUR, 1958). See also Chap.8 and Chap.15, Vol.II. Of this reservoir, only the instantaneous unit hydrograph will be given here

$$u(o,t) = \frac{8}{\pi^2}\alpha \sum_{n=1,3,5}^{n=\infty} e^{-n^2\alpha t} \tag{6}$$

where $\alpha = \frac{1}{j}$.

A plot of the u(o,t) of the linear reservoir on semi-log paper shows a straight line (Fig.2).

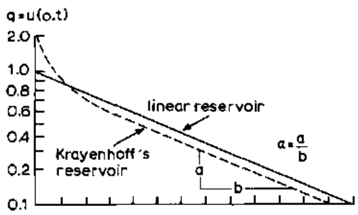


Fig.2.
The outflow from a linear reservoir and a Kraijenhoff reservoir following an instantaneous recharge.

The line of Kraijenhoff's reservoir is curved in the beginning and approaches a straight line after some time. The curvature in this line indicates a higher initial discharge due to the series of e-functions in Eq.6, but very soon the first term of Eq.6 becomes large in comparison with the sum of the other terms, and the series behaves as a single e-function, yielding a straight line (Fig.2).

In plots on semi-log paper of observed hydrographs under natural conditions, this upward curvature of Kraijenhoff's model is not often found. This can be understood when the resistance in the immediate vicinity of the outflow point, i.e. a channel or drain, is taken into account. In Fig.3 three assumptions are compared:

- all resistance is concentrated in the immediate vicinity of the channel (Fig.3a), which leads to the linear model,
- no resistance in the immediate vicinity of the channel (Fig.3c), which results in the Kraijenhoff model, and
- a situation intermediate between the above two (Fig.3b).

Fig.3a.
All resistance in the immediate vicinity of the drain (linear reservoir).

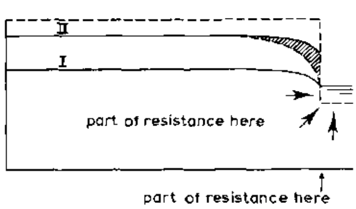


Fig.3b.
Intermediate situation (field condition).

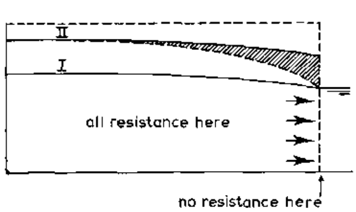


Fig.3c.
No resistance in the immediate vicinity of the drain (Kraijenhoff reservoir).

Fig.3. Comparison of a linear reservoir and a Kraijenhoff reservoir.

In Fig.3 the water table before an instantaneous recharge is indicated by I and the water table immediately after recharge by II.
Neither Fig.3a nor Fig.3c depicts the real situation. Under field conditions both types of resistance will always be present (Fig.3b). From mathematical derivations and from field data it has become clear that only in rare cases, i.e. when the resistance in the immediate vicinity of the outflow point is relatively unimportant, does the model of Fig.3c give the better approximation. This is the reason why the model used in the examples of this chapter is based on the assumption of linear reservoirs only.

16.2.3 THE REACTION FACTOR

The reaction factor of a reservoir is, as was stated before, a parameter for the drainage characteristics of the area with which the reservoir corresponds. According to Chap.8, Vol.II the reaction factor α (= $\frac{1}{j}$ of the Kraijenhoff reservoir) may be written

$$\alpha = \frac{\pi^2 KD}{\mu L^2} = \frac{10KD}{\mu L^2} \tag{7}$$

where
K = hydraulic conductivity in m/day
D = thickness of the phreatic aquifer in m
μ = effective porosity
L = drain spacing in m.

For a purely linear reservoir the factor π^2 has to be replaced by 8 because in this case the flow toward the drain increases in proportion to the distance from the point midway between the drains (compare Hooghoudt's formula for steady-state groundwater conditions).
Hence the expression for the reaction factor becomes

$$\alpha = \frac{8KD}{\mu L^2} \tag{8}$$

where the thickness of the phreatic aquifer D is replaced by the equivalent layer of thickness d (according to Hooghoudt), to account for the radial resistance.

16.3 THE ANALYSIS

16.3.1 THE PROCEDURE OF ANALYSIS

Any hydrograph can be approximated by juxtaposition of parallel linear reservoirs, each characterized by a reaction factor (α). Besides, when the analysis is performed systematically, the set of parameters found exclusively from the discharge hydrograph generally turns out to be interpretable in terms of the drainage conditions that prevail in the catchment area.
The set of parameters characterizing the drainage conditions of an area includes firstly the watershed leakage, secondly the α-values and the areal fractions occupied by the discerned groundwater reservoirs, and thirdly the divider according to which the division of precipitation between surface runoff and groundwater discharge is made.

Watershed leakage

The starting point for a hydrograph analysis is always the calculation of the water balance for the period which is being analyzed in order to eliminate possible groundwater losses to, or gains from, adjacent watersheds. The difference between the total measured discharge Σq and the total calculated effective precipitation ΣP_e, plus the difference in groundwater storage ΔS between the beginning and end of the analyzed period, indicates whether there is any watershed leakage and if so, whether it is positive or negative

$$\Sigma q + \Delta S - \Sigma P_e < 0 \rightarrow \text{negative watershed leakage (loss)}$$
$$\Sigma q + \Delta S - \Sigma P_e > 0 \rightarrow \text{positive watershed leakage (gain)}$$

The value of ΔS can only be determined after the analysis has been completed, for only then can the water storage ($S = q/\alpha$) of the different reservoirs be computed. The practical solution is to make ΔS approximately zero, by choosing a period of analysis which is as long as possible and for which the discharge q of the area has similar values at the beginning and end. It is imperative to get rid of the watershed leakage effect before the analysis is started.
Fortunately, watershed leakage is characterized by such a small reaction factor, caused by a large value for L in Eq.8, that time variations hardly occur and that as a sufficient approximation its value can be considered constant in time.
Total watershed leakage, divided by the number of days in the period being considered, gives the mean daily leakage loss or gain. In case of loss, this con-

stant value has to be added to the observed hydrograph, and in case of gain it has to be deducted from the hydrograph. Only when the hydrograph is corrected in this way it is possible to obtain consistent parameters.

Finding the reaction factors

The procedure of unravelling a discharge hydrograph is shown in Fig.4.

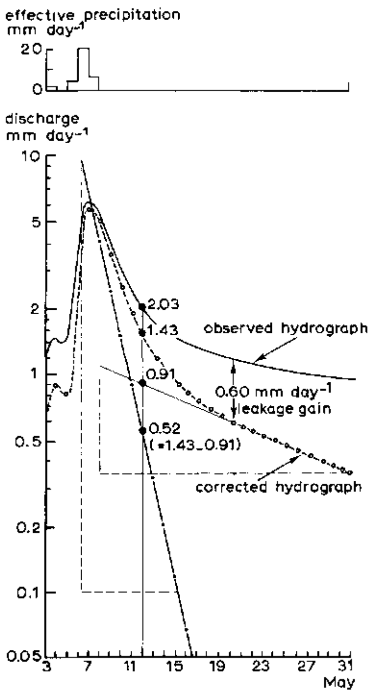


Fig.4.
Example of the analysis of a hydrograph with a correction for watershed leakage gain (compare Fig.1).

The observed hydrograph is plotted on semi-log paper. The water balance revealed a watershed leakage gain of 0.60 mm/day. The hydrograph is accordingly lowered to its corrected position and now shows a straight tail recession. From the slope of the tail end of the hydrograph, the smaller one of two reaction factors is found. The other reaction factor is found by projecting the straight part backward in time, replotting the daily difference between the projected line and the corrected hydrograph, and determining the slope of the resulting line.
The reaction factor α, i.e. the absolute value of the slope of the recession line plotted on semi-log paper, is computed as follows.

When $P_e = 0$ Eq.3 reduces to

$$q_2 = q_1 e^{-\alpha(t_2-t_1)}$$

$$\log q_2 = \log q_1 - \frac{\alpha(t_2-t_1)}{2.30}$$

$$\alpha = 2.30 \frac{\log q_1 - \log q_2}{t_2 - t_1} \tag{9}$$

If the recession periods with zero effective precipitation occurring between flood peaks are too short to permit the reservoir with the smallest but one reaction factor to become fully exhausted, the reaction factor of the slowest reacting reservoir cannot be found from the hydrograph. An estimation of the smallest α can then be furnished by Eq.8 only, after estimating or determining KD, μ, and L as averages for the slowest reacting area. It will often prove difficult to find correct values for the factors mentioned because of inhomogeneities in the area. Even so, this method might at least produce an order of magnitude that can be expected for the smallest α.
The reaction factor may also be determined from Eq.1, $q = \alpha S$, when discharge q and water storage S can be found concurrently. This method is especially suited for the analysis of artificially drained areas (polders). There, discharge occurs intermittently and only (daily) pumped out quantities are known, instead of discharge intensities at certain moments. The storage from day to day is found from the water balance: $S = \Sigma P_e - \Sigma q - \Sigma E$. The discharge (mm/day) may be approximated as mean values per day, i.e. equal to the daily pumped amounts per unit area. The latter are plotted against the mean storage, being the means of each two successive S-values.

Division of precipitation between reservoirs

To be sure of getting a consistent result, the division of precipitation between the discerned reservoirs has also to be based on the shape of the hydrograph. First the surface runoff peaks are separated from the observed hydrograph and the runoff volume is compared with the measured precipitation they originate from. This leads to a divider setting apart the portion of every precipitation that makes up the surface runoff. A small number of slightly different dividers may prove necessary, dependent upon the antecedent precipitation.
The rest of the daily precipitations enters the groundwater reservoirs after being transformed into effective precipitation along the lines discussed in

Section 16.1.3.

In the model, i.e. in the calculations according to Eq.3, each one of the discerned groundwater reservoirs receives the full effective precipitation, in accordance with actual field conditions. This means that every single groundwater reservoir in the model would yield a total discharge volume which equals the total effective precipitation. So the computed discharges have to be multiplied by reduction factors, the sum of these necessarily being unity.

This reduction can be interpreted as the translation of computed discharges expressed per unit reservoir area into the discharge per unit total area of the considered basin; in other words, the reduction factors, which are derived exclusively from the shape of the hydrograph, stand for the areal fractions occupied by the separate reservoirs.

An important feature of the areal fractions is that, though their sum must remain unity, they need not be constants. The explanation is that in many regions ditches run dry in summer, but are water-bearing in wet periods. Areas with dry ditches (like areas with no ditches at all) react with a small α (large L in Eq.8), while areas with water-bearing ditches have rather high reaction factors. Accordingly as ditches become dry, the areas occupied by the slower reacting reservoirs will increase, and vice-versa. This shifting of reservoir area limits is revealed from the hydrograph analysis because different values for the reduction factors will be found, when the analysis is carried out for different periods.

It appears that the variations in the value of the reduction factors in the model can be related to the unreduced computed q_s of the slowest reacting reservoir. This is explained by the fact that the computed q_s is proportional to the storage (Eq.1), and that a bigger storage means a higher groundwater level. Thus there will be more water-bearing ditches, which results in an extension of the area occupied by the rapidly reacting reservoir at the expense of the slower ones.

16.3.2 THE ELEMENTS NEEDED FOR THE RECONSTRUCTION OF HISTORIC DISCHARGES

For an assumed area with some surface runoff, indicated by a subscripted r, and two groundwater reservoirs, the one indicated by a subscripted s for slow and the other by f for fast, the runoff model needed to reconstruct historic discharges should include the following elements:

- the exact delimination of the considered area;
- the type and location of the rain gauge(s) (note that equally reliable rain data should be available for the period of reconstruction);
- the evaporation sequence used in the analysis (same remark);

- the watershed leakage;
- the reaction factor α_r for surface runoff;
- the divider for separating the portions of individual precipitations that are discharged as surface runoff;
- the reaction factors discerned in the groundwater part of the hydrograph: α_s and α_f;
- the areal fractions m_s and m_f, and their relations with the computed values of q_s;
- the balance equation from which the sequence of discharge intensities of the whole area, q_a, is computed

$$q_a = m_s q_s + m_f q_f + q_r + \text{watershed leakage (gain or loss)} \qquad (10)$$

which equation combines the hydrologically relevant components of the area, and is therefore called the area discharge characteristic.

16.3.3 THE ORDER OF MAGNITUDE OF REACTION FACTORS

The value of the reaction factor for surface runoff has been found to range from 200 day^{-1} (for 0.5 ha) to 0.3 day^{-1} (for 100.000 ha), the most frequent range being 1 to 3 day^{-1}. For small urban areas (0.16 to 0.40 ha) values up to 700 day^{-1} occur (VIERSMAN, 1966).
The value of the reaction factor α_f for well-drained agricultural lands varies from 0.3 to 0.7 day^{-1}. For areas with inadequate drainage the value of the reaction factor is often found to be of the order of 0.05 day^{-1}.
In areas with a deep groundwater table and a large groundwater reservoir, values of the reaction factor α_s as small as 0.001 day^{-1} may be observed.

16.4 NUMERICAL EXAMPLE OF HYDROGRAPH ANALYSIS

Relatively long series of rainfall and discharge data are needed for the hydrograph analysis of groundwater discharge (at least one complete year, but preferably several years), to make sure that the resulting area discharge characteristic (Eq.10), gives reliable results for varying weather conditions. As the analysis of data from such a lenghty period would be too cumbersome to serve as an example, a fictitious hydrograph has been composed to explain the principles of analysis. Discrepancies in the data are thus excluded, so that the analysis may be confined to a short period with one important discharge peak only. In reality, a longer period with many discharge peaks would have to be analyzed. The procedure, however, is the same.

It is assumed that discharges have been continually registered for the fictitious catchment area of "Fluvius River" for four weeks of May in the year 2000. In the catchment area one can distinguish a valley bottom and higher lying areas. The valley bottom is well-drained by a rather dense system of ditches, while in the higher land ditches also occur but are more widely spread.
The fictitious hydrograph is shown in Fig.5.

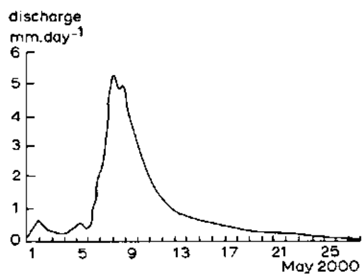


Fig.5.
The observed hydrograph (numerical example of hydrograph analysis).

Listed in Table 2 are the basic data on measured precipitation (Column 2) and evapotranspiration from the "standard sequence" (Column 3). Table 3 shows the discharge values (observed and corrected values).
The period chosen for analysis extends from the beginning of the day May 1st till the end of the day May 23rd, because at these moments the observed discharges are equal, so that it may be expected that the water storages will be approximately the same too.

16.4.1 UNRAVELLING THE HYDROGRAPH

In unravelling the hydrograph, we apply the following procedure:
1. We first calculate the effective precipitation (Table 2, Column 6). This is done by computing for each day the difference between daily precipitation and daily evapotranspiration (Column 4). Accumulated negative values are noted in Column 5, which represents the soil moisture deficit, and positive values in Column 6, which represents the effective precipitation.

Table 2. Basic data of the catchment of the river "Fluvius".

1	2	3	4	5	6
-1					
1	3.8	1.8	+ 2.0		+ 2.0
2	0.8	1.8	- 1.0	- 1.0	
3	1.2	1.8	- 0.6	- 1.6	
4	5.4	1.8	+ 3.6		+ 2.0
5	-	1.8	- 1.8	- 1.8	
6	7.6	1.8	+ 5.8		+ 4.0
7	21.8	1.8	+20.0		+20.0
8	7.8	1.8	+ 6.0		+ 6.0
9	1.0	1.8	- 0.8	- 0.8	
10	0.2	1.8	- 1.6	- 2.4	
11	-	2.1	- 2.1	- 4.5	
12	-	2.1	- 2.1	- 6.6	
13	5.7	2.1	+ 3.6	+ 3.0	
14	1.1	2.1	- 1.0	- 4.0	
15	3.6	2.1	+ 1.5	- 2.5	
16	0.3	2.1	- 1.8	- 4.3	
17	-	2.1	- 2.1	- 6.4	
18	-	2.1	- 2.1	- 8.5	
19	-	2.1	- 2.1	-10.6	
20	-	2.1	- 2.1	-12.7	
21	-	2.5	- 2.5	-15.2	
22	0.6	2.5	- 1.9	-17.1	
23	-	2.5	- 2.5	-19.6	
24	-	2.5	- 2.5	-22.1	
25	-	2.5	- 2.5	-24.6	
26	-	2.5	- 2.5	-27.1	
27	-	2.5	- 2.5	-29.6	+
					34.0

Column
1: date
2: measured precipitation in mm
3: evapotranspiration in mm according to the standard sequence
4: daily precipitation minus daily evapotranspiration
5: calculated moisture deficit in mm
6: effective precipitation in mm

Table 3. Discharge rate of the watershed of the "Fluvius" river.

1	2	3	4	5
-1	0.12			
1	0.60			
2	0.28			
3	0.21			
4	0.48			
5	0.40			
6	2.20			
7	5.32			
8	4.67	5.07	1.12	3.95
9	3.03	3.43	1.06	2.37
10	2.03	2.43	1.01	1.42
11	1.41	1.81	0.96	0.85
12	1.02	1.42	0.91	0.51
13	0.77	1.17	0.87	0.30
14	0.61	1.01	0.83	0.18
15	0.50	0.90	0.79	0.11
16	0.41	0.81	0.75	0.06
17	0.35	0.75	0.71	0.04
18	0.29	0.69		
19	0.25	0.65		
20	0.22	0.62		
21	0.18	0.58		
22	0.15	0.55		
23	0.12	0.52		
24	0.10	0.50		
25	0.07	0.47		
26	0.05	0.45		

1: date
2: Observed discharge rate at the end of the nth-day
3: discharge rate during recession corrected for groundwater loss
4: readings from graph for smallest reaction factor
5: discharge rate during recession of faster reacting reservoir.

2. We then compute the total runoff volume from the discharge frequency distribution as derived from the hydrograph (Table 4).
Column 1 of Table 4 gives the class boundaries of the discharge, Column 2 the number of days that the discharge exceeds each class boundary and Column 3 the values of Column 2 as a percentage of the period length.
Column 4 gives the class intervals, being the difference of two consecutive values of Column 1, while Column 5 gives the days of exceedance, being the average of two consecutive values of Column 2.
Column 6 is the result of the multiplication of the values of Column 4 and Column 5, and represents the discharge volume per class. The sum of the values of Column 6 equals the total runoff volume.

3. From the balance of the total discharge volume (sum of the values of Column 6 of Table 4) and the total effective precipitation (sum of the values of Column 6 of Table 2) we find a watershed leakage loss: 24.8 - 34.0 = - 9.2, which is $-\frac{9.2}{23} = -0.4$ mm/day.

4. Next, we plot the falling limb (recession) of the hydrograph (Column 2 of Table 3) on semi-log paper, starting with the moment the effective precipitation came to an end, i.e. May 8th (Fig.6).

Length of period 23 days

1	2	3	4		5		6
Class boundaries	exceeded during	%	class interval	×	mean number of		content per class 4 × 5
mm/day	days	%	mm/day		days		mm
0.0	23.0						
			0.10	×	23.00	=	2.30
0.1	23.0	100					
			0.10	×	21.40	=	2.14
0.2	19.8	86					
			0.20	×	16.30	=	3.26
0.4	12.8	56					
			0.20	×	10.90	=	2.18
0.6	9.0	39					
			0.20	×	8.20	=	1.64
0.8	7.4	32					
			0.20	×	7.00	=	1.40
1.0	6.6	29					
			0.40	×	6.00	=	2.40
1.4	5.4	23.5					
			0.40	×	5.00	=	2.00
1.8	4.6	20					
			0.40	×	4.20	=	1.68
2.2	3.8	16.5					
			0.40	×	3.50	=	1.40
2.6	3.2	14					
			0.49	×	2.90	=	1.16
3.0	2.6	11.5					
			1.00	×	2.10	=	2.10
4.0	1.6	7					
			1.00	×	1.10	=	1.10
5.0	0.6	2.5					
			0.32	×	0.30	=	0.10
5.32 (peak)	0.0	0					
							24.86

Table 4. Computation of the discharge-frequency distribution

The downward curvature of this plot proves the existence of a leakage loss.

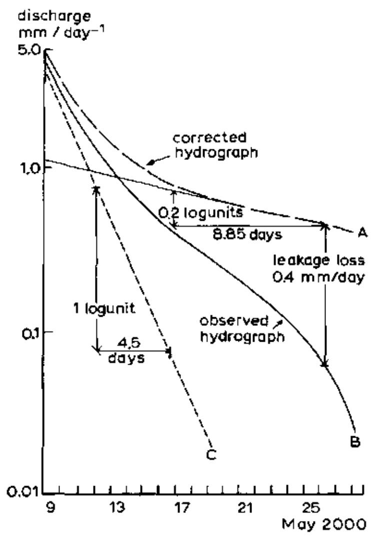


Fig.6.
Plot of the recession part of the hydrograph on semi-log paper (numerical example of hydrograph analysis).

5. In Column 3 of Table 3 the leakage loss (0.4 mm/day) is added to the observed discharge and the falling limb of the thus corrected hydrograph is replotted on semi-log paper. The lower end of the falling limb now plots as a straight line.

6. From the slope of this straight line, we can then find the reaction factor for the reservoir with the slowest reaction:

$$\alpha_s = 2.30 \frac{\log q_1 - \log q_2}{t_2 - t_1}$$

$$\alpha_s = \frac{2.30 \times 0.2}{8.85} = 0.05 \text{ day}^{-1}$$

7. If we now extend the straight line of Fig.6 to the left, we can determine the daily differences between the extended line and the corrected discharge hydrograph (Column 5 of Table 3). These differences may be considered the outflow from one or more reservoirs with faster reactions.

8. We now replot the values of Column 5 of Table 3 on semi-log paper (Fig.6). This turns out to be a straight line, indicating one fast reacting reservoir only. The reaction factor determined from the slope of this straight line is

$$\alpha_f = 2.30 \frac{\log q_1 - \log q_2}{t_2 - t_1}$$

$$\alpha_f = \frac{2.30 \times 1}{4.5} = 0.51 \text{ day}^{-1}$$

In this example it is clear that only two reaction factors are needed to describe the discharge hydrograph.

16.4.2 DETERMINING REDUCTION FACTORS AND RECONSTRUCTING THE HYDROGRAPH

The discharge hydrograph of the example yielded two linear groundwater reservoirs with reaction factors $\alpha_s = 0.05$ and $\alpha_f = 0.51$. The first reaction factor is of the order of magnitude that is expected for an insufficiently drained area, the output of which may be called "marshland discharge" (furtheron indicated by the subscript, m). The second is of the order of magnitude of an area with well-drained fields, yielding "field discharge" (subscript f).
We shall now recalculate both "marshland discharge" and "field discharge" as if each were the output for the entire catchment area.

First we need the starting values ($q_{m,o}$ and $q_{f,o}$), i.e. the reservoir discharges that are still running on the first day of the calculation period, due to preceding precipitations. Long periods are involved especially with small reaction factors. It would be a cumbersome calculation the ordinary way and a shortcut is desirable. This is presented by the formula

$$q_o = \frac{1 - e^{-\alpha}}{e^{-\alpha}} \Sigma e^{n\alpha} P_{e,n}$$

It has been proved that a sufficiently accurate result is obtained when daily precipitations, with α in day^{-1}, are used for only 20 preceding days. Rainfall further in the past can be handled with 10 day-period mean values, using α in (10 days)$^{-1}$. After a total of six 10 day-periods monthly values are appropriate up to a total of six months. For still earlier periods, quarterly means can be applied.
This may be clarified by the following example. From the data given in Table 5, $q_{o,m}$ for the marshland discharge ($\alpha_m = 0.05$ day^{-1}) is computed in Table 6. It turns out that the precipitations of six previous months influence the $q_{m,o}$-value, which comes to 0.99 mm/day.

Table 5. Effective precipitation of period prior to May 1st, 2000, i.e. measured precipitation reduced for evaporation in the usual way. Days not mentioned have zero effective precipitation.

n = 0 for May 1st.

daily values for April 2000

date	n	mm/day
22	− 9	3
20	− 11	5
18	− 13	8
14	− 17	5
12	− 19	2
11	− 20	2
8, 7, 6, 3	− 3 (10 days)	2, 3, 1, 2 → 0.8 mm/day

10 day-periods for March 2000

	n	mm/day
3rd per.	− 4	0.1
2nd per.	− 5	1.5
1st per.	− 6	0.4

months prior to six 10 day-periods

	n	mm/day
Febr.	− 3	2.3
Jan.	− 4	1.4
Dec.	− 5	2.0
Nov.	− 6	1.7
Oct., Sept., Aug.	− 3 (quarter)	1.0, 0.4, 0.1 → 0.5 mm/day

For the field discharge the computation is exactly the same, only far shorter because of the higher reaction factor (0.51 day^{-1}). It reads:

n	$-n \times \alpha$	$P_{e,n}$	$e^{n\alpha}$	$e^{n\alpha} \times P_{e,n}$	q_o
− 9	4.59	3	0.010	0.030	
− 11	5.61	5	0.004	0.020	
− 13	6.63	8	0.001	0.008	

because $e^{n\alpha}$ becomes very small,

rest estimated to be about $\underline{0.001}$ +

$$\frac{1 - e^{-\alpha}}{e^{-\alpha}} \Sigma(\text{days}) = \frac{0.40}{0.60} \times 0.059 = \quad 0.04 \text{ mm/day}$$

So the starting values for the recalculations are:

$q_{m,o}$ = 0.99 mm per day

$q_{f,o}$ = 0.04 mm per day

To determine the reduction factors, and reconstruct the hydrograph, we proceed as follows:

1. We first calculate the marshland discharge (q_m) and the field discharge (q_f) with Eq.3. The result is shown in Table 7.
Line 2 of Table 7 gives the effective precipitation.
Introducing the reaction factor for marshland discharge (α_m = 0.05 days^{-1}) into Eq.3 we obtain

$$q_{m,n} = q_{m,n-1}e^{-0.05} + P_{e,n}(n - e^{-0.05})$$

$$q_{m,n} = 0.095\ q_{m,n-1} + 0.05\ P_{e,n}$$

On line 3 of Table 7, $P_{e,n}$ is multiplied by 0.05 and on Line 4 $q_{m,n-1}$ is multiplied by 0.95.
The sum of lines 3 and 4 is given on Line 5 and represents the calculated marshland discharge in mm/day per unit reservoir area.
Lines 6, 7, and 8 represent the same procedure for the field discharge.

2. We then determine the reduction factors m_m and m_f of the reservoirs for marshland discharge and field discharge, respectively. Both reservoirs together represent the whole basin area, so $m_m + m_f = 1$.
In order to reduce the influence of observational inaccuracies, the reduction factors can best be determined from a period with a relatively high discharge. There are two unknowns, so two equations are needed. The first one reads $m_m + m_f = 1$ and the second is adopted from the area discharge characteristic (Eq.10), and reads

$$m_f q_f + m_m q_m = q_a - \text{watershed leakage} = q_{corrected}$$

For the 9th of May the second equation reads (calculated values from Lines 5 and 8 of Table 7 and Column 3 of Table 3)

$$2.12\ m_m + 4.74\ m_f = 3.43\ \text{mm/day}$$

Table 6. Computation of q_o

n	$-n\times\alpha$	$P_{e,n}$	$e^{n\alpha}$	$e^{-n\alpha}\times P_{e,n}$
days		mm/day		
- 1		-		
- 2		-		
- 3		-		
- 4		-		
- 5		-		
- 6		-		
- 7		-		
- 8		-		
- 9	0.45	3	0.64	1.92
- 10		-		
- 11	0.55	5	0.58	2.90
- 12		-		
- 13	0.65	8	0.52	4.16
- 14		-		
- 15		-		
- 16		-		
- 17	0.85	5	0.43	2.15
- 18		-		
- 19	0.95	2	0.39	0.78
- 20	1.00	2	0.37	0.74 +

$\alpha = 0.05\ \text{day}^{-1}$
$= 0.5\ (10\ \text{days})^{-1}$
$= 1.5\ \text{month}^{-1}$
$= 4.5\ \text{quarter}^{-1}$

$e^{-\alpha}$ for
days = 0.95
(10 days) = 0.61
months = 0.22
quarters = 0.01

$$\frac{1-e^{-\alpha}}{e^{-\alpha}}\ \Sigma(\text{days}) = \frac{0.05}{0.95} \times 12.65 = 0.67\ \text{mm/day}$$

10 day periods

- 3	1.50	0.8	0.224	0.179
- 4	2.00	0.1	0.135	0.014
- 5	2.50	1.5	0.082	0.123
- 6	3.00	0.4	0.050	0.020 +

$$\frac{1-e^{-\alpha}}{e^{-\alpha}}\ \overset{10}{\Sigma}(\text{days}) = \frac{0.39}{0.61} \times 0.336 = 0.21\ \text{mm/day}$$

months

- 3	4.50	2.3	0.0111	0.0255
- 4	6.00	1.4	0.0025	0.0035
- 5	7.50	2.0	0.0006	0.0012
- 6	9.00	1.7	0.0001	0.0002 +

$$\frac{1-e^{-\alpha}}{e^{-\alpha}}\ \Sigma(\text{months}) = \frac{0.78}{0.22} \times 0.0304 = 0.11\ \text{mm/day}$$

quarters

- 3	13.5	0.5	0.0000014	
- 4			so small,	
- 5			that all the	
- 6			rest becomes zero	
etc.				——— +

$$\frac{1-e^{-\alpha}}{e^{-\alpha}}\ \Sigma(\text{quarters}) = \text{———} \times \text{zero} = 0.00\ \text{mm/day} \ +$$

$$q_o = 0.99\ \text{mm/day}$$

Table 7. Determination of reduction factors and reconstruction of hydrograph

Reconstruction of hydrograph

River: F L U V I U S

Month: May

Year : 2000

$\alpha_m = 0.05\ \text{day}^{-1}$ $\alpha_f = 0.51\ \text{day}^{-1}$

$e^{-\alpha} = 0.9512 = 0.95$ $e^{-\alpha} = 0.6005 = 0.6$

$1 - e^{-\alpha} = 0.0488 = 0.05$ $1 - e^{-\alpha} = 0.3995 = 0.4$

$q_{m,o} = 0.99\ \text{mm day}^{-1}$ $q_{f,o} = 0.04\ \text{mm day}^{-1}$

line																								
1 n (date)	−1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
2 $P_{e,n}$	–	2.0	–	–	2.0	–	4.0	20.0	6.0	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
3 0.05 $P_{e,n}$		0.10			0.10		0.20	1.00	0.30															
4 0.95 $q_{m,(n-1)}$		0.94	0.99	0.94	0.89	0.94	0.89	1.04	1.94	2.12	2.02	1.91	1.82	1.73	1.65	1.57	1.49	1.42	1.34	1.28	1.21	1.15	1.09	1.04
5 $q_{m,n}$	0.99	1.04	0.99	0.94	0.99	0.94	1.09	2.04	2.24	2.12	2.02	1.91	1.82	1.73	1.65	1.57	1.49	1.42	1.34	1.28	1.21	1.15	1.09	1.04
6 0.4 $P_{e,n}$		0.80			0.80		1.60	8.00	2.40															
7 0.6 $q_{f,(n-1)}$		0.02	0.49	0.29	0.17	0.58	0.35	1.17	5.50	4.74	2.84	1.70	1.02	0.61	0.37	0.22	0.13	0.08	0.05	0.03	0.02	0.01	0.01	0.00
8 $q_{f,n}$	0.04	0.82	0.49	0.29	0.97	0.58	1.95	9.17	7.90	4.74	2.84	1.70	1.02	0.61	0.37	0.22	0.13	0.08	0.05	0.03	0.02	0.01	0.01	0.00
9		Marshland: $m_m = 0.5$ and Field: $m_f = 0.5$ Watershed leakage = − 0.40 mm/day																						
10 0.5 $q_{m,n}$	0.50	0.52	0.50	0.47	0.50	0.47	0.55	1.02	1.12	1.06	1.01	0.96	0.91	0.87	0.83	0.79	0.75	0.71	0.67	0.64	0.61	0.58	0.55	0.52
11 0.5 $q_{f,n}$	0.02	0.41	0.25	0.15	0.49	0.29	0.97	4.58	3.95	2.37	1.42	0.85	0.51	0.30	0.18	0.11	0.06	0.04	0.02	0.01	0.01	0.00	0.00	
12 S U M	0.52	0.93	0.75	0.62	0.99	0.76	1.52	5.60	5.07	3.43	2.43	1.81	1.42	1.17	1.01	0.90	0.81	0.75	0.69	0.65	0.62	0.58	0.55	0.52
− 0.40 =																								
13 q_n^{co}	0.12	0.53	0.35	0.22	0.59	0.36	1.12	5.20	4.67	3.03	2.03	1.41	1.02	0.77	0.61	0.50	0.41	0.35	0.29	0.25	0.22	0.18	0.14	0.12

and, upon substitution of the first equation,

$$2.12\ m_m + 4.74\ (1 - m_m) = 3.43\ \text{mm/day}$$

which gives

$$m_m = 0.5 \quad \text{and} \quad m_f = 0.5$$

In this example only one short period is analyzed and reconstructed. When analyzing a longer period with a number of discharge peaks, different values of the reduction factors are often found. These values should then be related to the groundwater conditions in the basin, i.e. to the calculated discharge of the slowest reacting reservoir, q_s, which generally indicates the groundwater conditions fairly well. It should be remembered that reduction factors are interpreted as the areal fractions occupied by the separate reservoirs and that the variations in areal fractions originate from ditches that are alternately dry and water-bearing, so that the adjacent area periodically belongs either to a slower or a faster reacting reservoir.

3. We now multiply the calculated marshland discharge (Line 5 of Table 7) and the calculated field discharge (Line 8 of Table 7) by the areal fractions. The results are shown on Lines 10 and 11 of Table 7. From the sum of Lines 10 and 11, presented on Line 12, the watershed leakage loss is subtracted on Line 13. Line 13 represents the reconstructed discharge hydrograph.

4. After the completion of both analysis and reconstruction, we still have to check whether the water-balance equation from which the value of watershed leakage was calculated was sufficiently correct. Calculations in Table 7 (Lines 5, 8 and 13) show that the values 0.12 found for q_o^{co} and q_{23}^{co} are not equivalent. They contain different portions of both types of discharge. Applying Eq.1 ($S = q/\alpha$), we find that the increase of the water storage for the marshland reservoir is $m_m \frac{(1.04 - 0.99)}{0.05} = 1.00\ m_m = 0.50$ mm. In the same way the field discharge reservoir shows a decrease of $m_f \frac{(0.04 - 0.00)}{0.51} = 0.08\ m_f = 0.04$ mm. So, during the period of analysis, 0.46 mm over 23 days or 0.02 mm per day remains in the area. Hence the correct value for the watershed leakage loss amounts to 0.40 - 0.02 = 0.38 mm per day. Fortunately, the difference between this and the original 0.40 mm is so small that there is no need to reconsider the analysis.

Evaluation

The goodness of fit of the reconstructed hydrograph may be judged visually by plotting the observed and the reconstructed hydrograph (Fig.7).

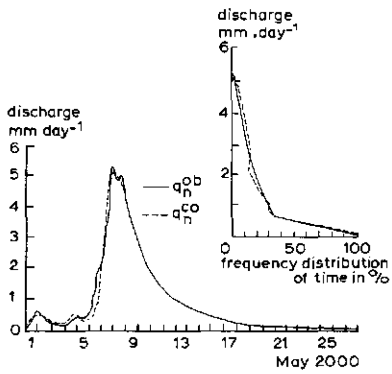


Fig.7.

Comparison of the observed and constructed hydrograph (numerical example of hydrograph analysis).

Differences between the observed hydrograph and the reconstructed hydrograph are unavoidable. Some errors are due to the simplification of the model, and random errors occur because the basin precipitation is calculated from point-rainfall derived from rain gauges that underestimate the real precipitation to a varying degree. The same applies for the calculated moisture deficit that is required to assess the effective precipitation.

The goodness of fit can therefore best be judged by comparing the discharge-frequency curve of the reconstructed hydrograph with the discharge-frequency curve of the observed hydrograph (see Table 4, and Fig.7).
When the discharge-frequency curve of the reconstructed hydrograph gives a good fit to the discharge-frequency curve of the observed hydrograph, the model may be considered to be precise enough for reconstruction purposes. It is advisable to check the result by calculating the discharge-frequency for a period not used for the analysis, but for which measured discharge are available.

16.5 EXAMPLES OF ANALYZED AREAS

16.5.1 THE "KLEINE DOMMEL" BASIN

The "Kleine Dommel" basin (DE ZEEUW, 1966) is situated in the southern part of The Netherlands.

General data

- area: 19.100 ha;
- relief: undulating;
- soil: sandy with some loam in the valley;
- drainage conditions: mean distance between principal water courses 2-3 km; variable ditch spacing in the valleys; no tile drainage;
- land use: pine forest and arable land on the higher grounds; grassland in the valleys and on some flat higher grounds; negligible areas of open water and metalled surfaces;
- hydrological data: discharge data over 5 years (1957-1961); mean daily precipitation data from three stations; evaporation data determined according to standard sequence.

Analysis

The water balance over the whole period showed an average watershed leakage loss of 0.30 mm/day. The observed hydrographs were corrected by adding this value to the measured daily discharge. Using the discharge data of long dry periods (i.e. discharge of slowest reacting reservoir only), the reaction factor of this so-called seepage reservoir, α_s, and its areal fraction, m_s, could be determined. Subsequently, the year-round fluctuations of the seepage discharge were calculated.
Subtracting the calculated seepage discharge from the corrected observed hydrograph yielded the discharge hydrograph of the areas with ditches, i.e. those areas having a faster reaction. Further analysis showed that this hydrograph is characterized by two reaction factors, one attributable to marshland discharge, α_m, the other to field discharge, α_f. Finally, for both types of discharge we calculated the areal fractions, m_m and m_f, which proved to be variable (see below).
For the basin "Kleine Dommel" as a whole, the area discharge characteristic, Eq.10, is:

$$q_n = m_{s,n}q_{s,n} + m_{m,n}q_{m,n} + m_{f,n}q_{f,n} - 0.30 \text{ mm/day}$$

where

$$q_{s,n} = q_{s,n-1}e^{-\alpha_s} + P_{e,n}(1 - e^{-\alpha_s})$$

$$q_{m,n} = q_{m,n-1} e^{-\alpha_m} + P_{e,n}(1 - e^{-\alpha_m})$$

$$q_{f,n} = q_{f,n-1} e^{-\alpha_f} + P_{e,n}(1 - e^{-\alpha_f})$$

and where, according to the analysis,

α_s = 0.003 day^{-1},
α_m = 0.07 day^{-1},
α_f = 0.6 day^{-1}.

The varying values of the areal fractions are related to the calculated seepage discharge (Table 8).

Table 8. Relation between areal fractions and seepage discharge in the "Kleine Dommel" basin.

$q_{s,n}$ mm/day	$m_{s,n}$	$m_{m,n}$	$m_{f,n}$
< 0.90	0.60	0.30	0.10
0.90 - 1.05	0.60	0.25	0.15
1.05 - 1.20	0.60	0.20	0.20
1.20 - 1.35	0.60	0.15	0.25
> 1.35	0.50	0.15	0.35

The frequency distributions of observed and reconstructed discharges are given in Fig.8.
Figure 9 gives details for the year 1960. The differences between the computed and the observed frequency of high discharge intensities are due to the fact that in the real situation part of the peak discharge is stored as surface storage, i.e. inundations occur, caused by the inadequate transport capacity of the existing river system. Observations revealed that overtopping of the banks occur when the water level reaches 19.35 m + NAP, corresponding with a transport capacity of 2.5 to 3.0 mm per day (see Fig.8).
Improvements to these water courses would diminish the inundations, but would cause an increase in the peak discharges. The reconstructed discharge hydrograph

is a prediction of the discharge intensities that could be expected if the discharge facilities were adequate, or in other words if the water courses had had the required dimensions.

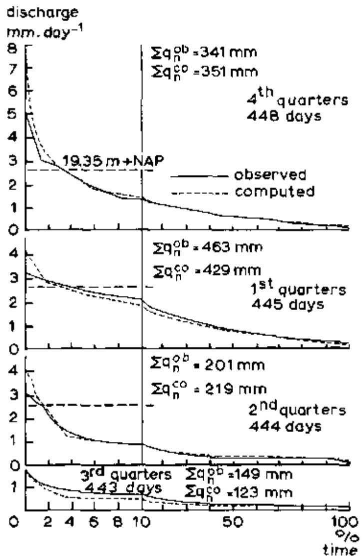


Fig.8.
Frequency distributions of observed and computed discharges of the "Kleine Dommel" for the period 10-11-'56 to 1-11-'61.

Another conclusion is that it makes a great difference whether heavy precipitation (say 50 mm/day) follows a dry or wet period. After a long dry period, the calculated seepage flow may be as low as 0.8 mm/day. Under such conditions, only 10% of the catchment will cause field discharge (Table 8, last column), which, in turn, will cause a peak of

$$q_{f,n} = m_f P_{e,n}(1 - e^{-\alpha_f}) =$$

$$= 0.10 \times 50(1 - e^{-0.60}) =$$

$$= 0.10 \times 50 \times 0.45 = 2.25 \text{ mm/day}$$

In a wet period, when the calculated seepage flow may be as much as 1.4 mm/day, 35% of the area will contribute to the field discharge peak of

$$q_{f,n} = 0.35 \times 50 \times 0.45 = 7.88 \text{ mm/day}$$

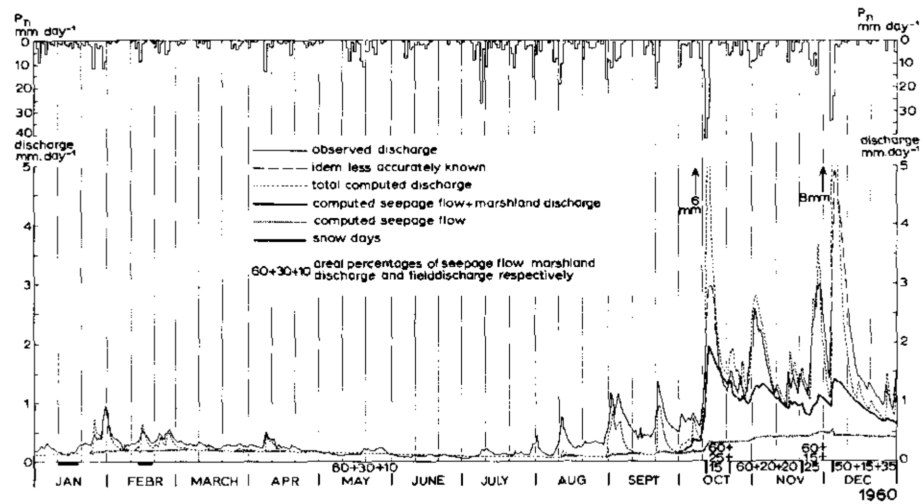


Fig.9. Hydrograph data for the "Kleine Dommel (19.100 ha)

Hence, to acquire the correct frequency distribution of discharge peaks for a groundwater runoff basin with varying areal fractions, the entire sequence of discharge must be reconstructed.

16.5.2 THE "GEUL" BASIN

The "Geul" basin (DE ZEEUW, 1966) is situated partly in the utmost south of The Netherlands and partly in Belgium and Germany.

General data

- area: 30.500 ha;
- relief: hilly, steep valley walls;
- soil: loamy; in 20% of the area, bedrock lies close to the ground surface;
- drainage conditions: rivers are far apart, surface runoff is frequently observed and there are no ditches or tile drains;
- hydrological data: discharge data over 3 years (1955-1958); mean daily precipitation data from two stations; evaporation according to standard sequence.

Analysis

The water balance of the whole period showed an average watershed leakage loss of 0.20 mm/day. The observed hydrograph was corrected by adding this value to the measured daily discharge. To obtain the hydrograph of groundwater discharge, the peaks originating from surface runoff q_r had to be separated first.
These peaks were plotted on semi-log paper to yield the reaction factor of surface runoff, α_r. Next, the portions, $P_{r,n}$, of the measured precipitation that make up the surface runoff were determined as being equal to the water volume contained in the single peaks. Rather simple relations can be established between $P_{r,n}$ and the measured precipitation, which relations differ for various preceding weather conditions. In the present case different relations have to be used when three preceding days have either an increasing evaporation surplus, a decreasing evaporation surplus or a precipitation surplus.
Values of $P_{r,n}$ deduced from the said relations are subsequently subtracted from the measured precipitation. The thus reduced precipitation enters into the calculation of the sequence of the effective precipitation, which is added to the groundwater reservoirs.
Further analysis showed that the groundwater runoff part of the hydrograph can be characterized by two reaction factors. One is attributable to marshland discharge from the very wet area with shallow bedrock, α_m, the other the seepage flow from

the rest of the basin with moderately permeable soils and long distances between river branches, α_s.
The areal fractions of the seepage reservoir, m_s, and of the marshland reservoir, m_m, were found to be constant, i.e. independent of changes in groundwater level (m_s = 0.2 and m_m = 0.8).
This is in agreement with the fact that the areal fractions are related to the geology of the area, i.e. depth to bedrock, instead of to the groundwater conditions.

For the Geul-basin, the area discharge characteristic, Eq.10, reads

$$q_n = 0.2\ q_{s,n} + 0.8\ q_{m,n} + q_{r,n} - 0.20\ \text{mm/day}$$

where

$$q_{s,n} = q_{s,n-1}e^{-\alpha_s} + P_{e,n}(1 - e^{-\alpha_s}),$$

$$q_{m,n} = q_{m,n-1}e^{-\alpha_m} + P_{e,n}(1 - e^{-\alpha_m}),$$

$$q_{r,n} = q_{r,n-1}e^{-\alpha_r} + P_{r,n}(1 - e^{-\alpha_r}),$$

$P_{r,n}$ = portion of the measured precipitation that makes up the surface runoff on the n^{th} day,
$P_{e,n}$ = effective precipitation on the n^{th} day, calculated from the remainder of measured precipitation after subtracting $P_{r,n}$,

and, according to the analysis:

α_s = 0.005 day^{-1},
α_m = 0.05 day^{-1},
α_r = 1.4 day^{-1},

The results of the reconstruction over nearly four and a half years are summarized in Fig.10 and details for the year 1956 are shown in Fig.11. For low discharge rates, the reconstructed and observed hydrographs show a good fit. For high discharge rates, the agreement is not as good, but taking into account the fast reaction due to the surface runoff, it may be considered satisfactory.

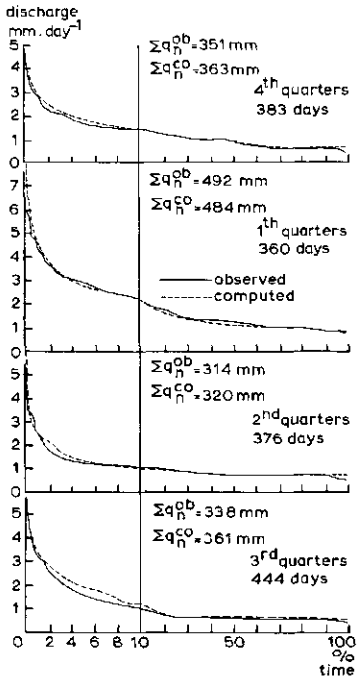


Fig.10. Frequency distributions of observed and computed hydrographs of the "Geul" for the periods 1-1-'55 to 1-11-'58 and 5-6-'59 to 15-12-'59

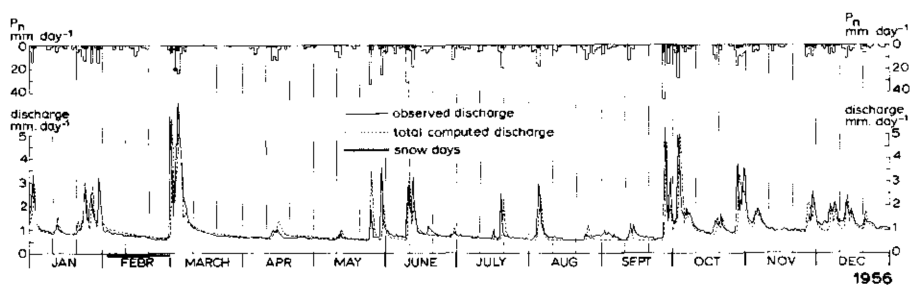


Fig.11. Hydrograph data for the "Geul" (30.500 ha) (the black parts of the precipitation pattern denote $P_{r,n}$)

The most important criterion by which the accuracy of the analysis is judged, however, is the agreement between the frequency distribution curves, and here this is quite acceptable.

16.6 FINAL REMARKS

It will be clear from the preceding sections of this chapter that it is impossible to get a perfect similarity between the reconstructed and the observed hydrograph, due to the inevitable inaccuracies in the hydrological data, mainly precipitation. Even so, the analysis does give us an insight into the runoff system of a basin.

There are two main causes of discrepancies. Firstly, rain gauges tend to underestimate precipitation, but our model is attuned to the average underestimation which results from the type and location of the rain gauge(s) used. (A rainfall record therefore becomes useless when a change in type or location of the rain gauge has occurred). The actual underestimation of separate storms, however, varies with drop size and wind speed. This means that sometimes too high discharge peaks will be computed and sometimes too low. Secondly, we measure point-rainfalls, which are also known to be either too high or too low. As a result computed discharges will sometimes be overestimated, other times they will be underestimated. Over a number of years, however, these opposite effects cancel each other. This is why the analysis must be aimed at a good agreement between the reconstructed and the observed discharge-frequency distributions (Figures 8 and 10), and not primarily on a similarity between the hydrographs (Figures 9 and 11). Fortunately, the design discharge, which the water-control structures in the basin will have to cope with, is derived from the discharge-frequency distribution (see Sect.16.1.2).

LITERATURE

BOUSSINESQ, J. 1904. Recherche théorétiques sur l'écoulement des nappes d'eau infiltrées dans le sol et sur le débit des sources. Journal de mathématiques pures et appliquées, tôme X, série 5.

DE ZEEUW, J.W. 1966. Analyse van het afvoerverloop van gebieden met hoofdzakelijk grondwaterafvoer (in Dutch with English summary). Thesis, Agric. University, Wageningen. 139 pp.

KRAIJENHOFF VAN DE LEUR, D.A. 1958. A study of nonsteady groundwater flow with special reference to a reservoir coefficient. De Ingenieur 70: 387-394.

LINSLEY, R.K., KOHLER, M.A. and PAULHUS, J.L.M. 1958. Hydrology for engineers. McGraw-Hill Book Comp., 340 pp.

VIERSMAN, JR., W̌. 1966. The hydrology of small impervious areas. Water Res. Research 2: 405-412.

Suggestions for further reading

AMOROCHO, J. and HART, W.E. 1964. A critique of current methods on hydrologic system investigations. Trans. Am. Geophys. Union 45: 307-321.

SUGAWARA, M. and MARUYAMA, F. 1956. A method of prevision of the river discharge by means of a rainfall model. Int. Ass. Sci. Hydrol. Publ. No.42: 71-76.

PRINCIPAL SYMBOLS USED IN VOLUME II

SYMBOL	DESCRIPTION	DIMENSION
A	cross-sectional area; horizontal surface area	L^2
	wave amplitude	L
a	geometric factor in Ernst's formula	dimensionless
B	width	L
b	bottom width of channel	L
bx	phase shift factor	dimensionless
C	salt concentration	meq/litre
	Chézy's coefficient	$L^{\frac{1}{2}}T^{-1}$
	runoff coefficient	dimensionless
C	correction for drain spacing	L
$C_{1,2}$	constant	dimensionless
C_t, c_t	computation functions for groundwater height	dimensionless
c	hydraulic resistance of semipervious layer	T
	celerity of wave	LT^{-1}
D	thickness of aquifer or saturated layer;	L
	thickness of layer below drain level;	
	depth of rootzone	
D_A, D_a	drainage discharge of irrigated area or sub-area	LT^{-1}
d	thickness of equivalent depth in Hooghoudt's formula, water depth on land to be irrigated	L
E	evapotranspiration	LT^{-1}
	elasticity modulus	$ML^{-1}T^{-2}$
EC	electrical conductivity	$ohm^{-1}cm^{-1}$
ESP	exchangeable sodium percentage	dimensionless
e	efficiency, base of natural (Napierian) logarithm	dimensionless
e^{-ax}	amplitude reduction factor	dimensionless
erf(u)	error function	dimensionless
erfc(u)	complementary error function	dimensionless
F_H, F_K, F_D	flow functions of Hooghoudt, Kirkham and Dagan	dimensionless
F	function	L^2
f	leaching efficiency	dimensionless
G	capillary rise of ground water	LT^{-1}
G(x,y)	Green's function	dimensionless
G_t, g_t	computation functions for drain discharge	dimensionless
g	acceleration due to gravity	LT^{-2}

SYMBOL	DESCRIPTION	DIMENSION
H	height of water table above impervious layer midway between two drains	L
h	hydraulic head; height of water table above drain level midway between two drains	L
	saturated depth	L
I	effective amount of irrigation water	LT^{-1}
I_d	amount of irrigation water delivered to the field	LT^{-1}
I_{ins}	infiltration rate	LT^{-1}
I_{cum}	cumulative infiltration	L
$I_o(x)$	modified Bessel function of the first kind and order zero	dimensionless
$I_1(x)$	modified Bessel function of the first kind and order one	dimensionless
I(nt/k)	incomplete gamma function or order n	dimensionless
IUH	instantaneous hydrograph	dimensionless
i	infiltrated volume per unit length	L^2T^{-1}
j	groundwater reservoir factor	T
K	hydraulic conductivity	LT^{-1}
$K_o(x)$	modified Bessel function of the second kind and order zero	dimensionless
$K_1(x)$	modified Bessel function of the second kind and order one	dimensionless
KD	transmissivity of water-bearing layer	L^2T^{-1}
K/D	leakage coefficient of semipervious layer (=1/c)	T^{-1}
$(KDc)^{\frac{1}{2}}$	leakage factor of semipervious layer	L
k	lag of linear reservoir	T
L	length; drain- or well-spacing; furrow length	L
m	areal fraction	dimensionless
n	time constant	dimensionless
	frequency	radians T^{-1}
P	precipitation	LT^{-1}
Q	discharge	L^3T^{-1}
q	discharge per unit width or per unit length	L^2T^{-1}
	discharge per unit surface area	LT^{-1}
R	recharge rate; deep percolation	LT^{-1}
R^+	leaching requirement	LT^{-1}
RSC	residual sodium carbonate value	meq/l
R, r	radial coordinate	dimensionless

SYMBOL	DESCRIPTION	DIMENSION
r	radius; radial distance	L
S	surface runoff	LT^{-1}
	storage coefficient of aquifer	dimensionless
	storage per unit surface area	L
S_t	ordinate of S-curve	L^3
SAR	sodium adsorption ratio	dimensionless
s	*side slope of ditch; channel slope*	dimensionless
	drawdown in well	L
T, t	time; period	T
TUH	t-hour unit hydrograph	dimensionless
t_r	recharge period	T
u	wetted perimeter of drain	L
	Boltzman factor	dimensionless
	volume infiltrated per unit width	L^2T^{-1}
u(o,t)	ordinate of IUH	L^3T^{-1}
u(T,t)	ordinate of TUH	L^3T^{-1}
V	volume of reservoir	L^3
	total irrigation water supply per unit area	LT^{-1}
v	flow velocity; apparent velocity; specific discharge (flow rate per unit of cross sectional area)	LT^{-1}
W	soil moisture volume	L
W(u)	exponential integral; Theis' well function	dimensionless
w_e	moisture content of saturation extract	dimensionless
w_{fc}	moisture content at field capacity	dimensionless
w_{wp}	moisture content at wilting point	dimensionless
w_r	resistance to radial flow	$L^{-1}T$
x, y, z	cartesian coordinates	dimensionless
y	hydraulic head of open water; water depth in channel	L
Z'	salt content of the soil	meq/m^2
Z	salt content of the soil	(mmhos/cm) mm
α	reaction factor (1/j)	T^{-1}
α	ratio indicating leaching requirement	dimensionless
α	steady change of groundwater level	LT^{-1}
αE	leaching requirement	LT^{-1}
β	ratio indicating irrigation loss	dimensionless

SYMBOL	DESCRIPTION	DIMENSION
Δ	small increment of	dimensionless
Θ	soil moisture content (volume %)	dimensionless
π	circular circumference-diameter ratio, 3.146	dimensionless
μ	effective porosity, drainable pore space	dimensionless
ρ	mass density of water	ML^{-3}
λ	leakage factor	L
ζ	leakage coefficient	T^{-1}
ψ	stream function	L^2T^{-1}
$\Gamma(\eta)$	gamma function	dimensionless
∞	infinite value	dimensionless
∂	partial derivative sign	dimensionless

SUBJECT INDEX