A Model for the Formation and Melting of Ice on Surface Waters Revisited

MSc thesis

(Wiersema, 2012)

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1. Introduction

Ice skating in the Netherlands nowadays is a very popular sport and recreation activity. Every winter season most of the Dutch people are waiting for the ditches and lakes to freeze over. When the first frost arrives and the ice is thick enough people take their ice skates and leave their houses to skate on the natural ice and make some skating tours. Although the Netherlands is controlled by maritime climate conditions (Peel et al. 2007), the average winter temperatures are close to the freezing point. This results in only a few cold spells to occur each winter. On the other hand, 20% of The Netherlands is located below sea level; therefore many ditches, canals and lakes to regulate water levels are present in the landscape (CBS, 2009). If these are frozen, one can travel large distances over the ice. The most popular skating tour is called the “Elfstedentocht” (Eleven Cities race/tour) and takes place in the north of the Netherlands in the province Friesland and covers about 200 km of ice along the eleven cities (figure 1).

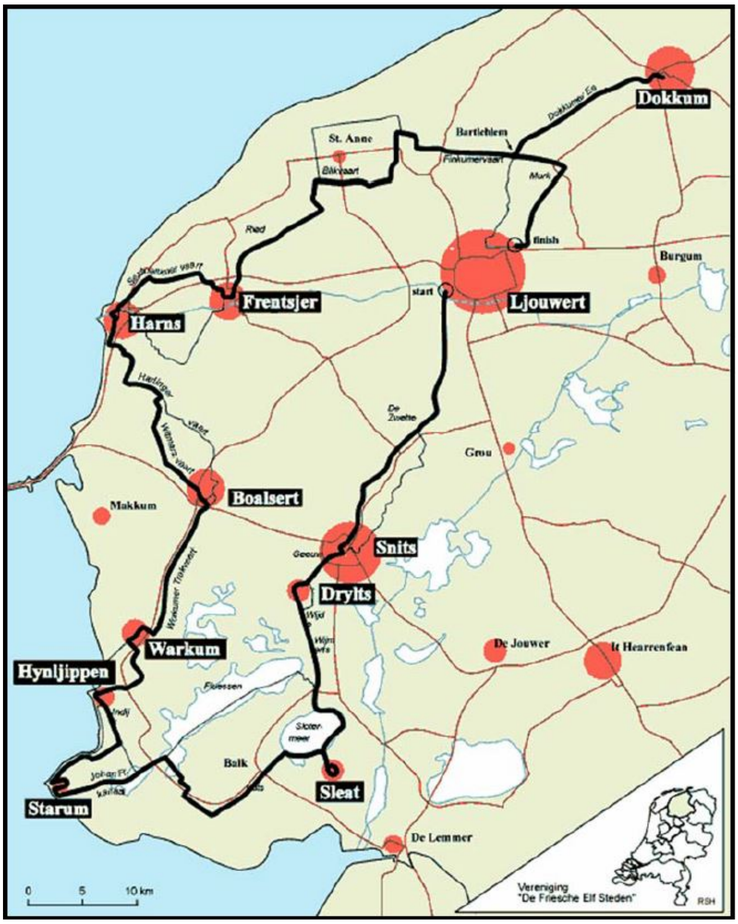

Figure 1: The province Friesland with the eleven cities and Leeuwarden as the capital of Friesland. Leeuwarden has always been the start and finish of the tour and from there the participants go to Sneek (Snits), IJlst (Drylts), Sloten (Sleat), Stavoren (Starum), Hindeloopen (Hynljippen), Workum (Warkum), Bolsward (Boalsert), Harlingen (Harns), Franeker (Frentsjer), Dokkum and back to Leeuwarden. (Elfstedentocht, 2013)

The royal association of the “Friesche Elf Steden” limits the maximum number of 16.000 ice skaters to be present on the ice (Elfstedentocht, 2013), and the minimum ice thickness to be 15 cm, therefore a number of icy days need to be present in order to let the tour go on.

Due to the popularity of ice-skating there was a need in the eighties for a forecasting model that predicts the thickness of ice. De Bruin en Wessels (1988) developed a relatively simple model, which requires standard meteorological weather data to forecast the ice thickness. The model can also be used for other purposes, such as hydrological studies of lakes and reservoirs in temperate regions and also to simulate the water temperature for water transport, management or recreation in connection with environmental questions. In summer authorities can be informed about the water temperatures and in winter ice thickness can be forecasted for recreation purposes. For this last purpose there is nowadays even a higher need for a good ice thickness model because the annual chance of organizing an ‘Elfstedentocht’ decreases according to Vissers and Petersen (2008). In figure 2 they show the annual probability for an ‘Elfstedentocht’. The tour was organised only 15 times since 1909, and the last race took place in 1997.

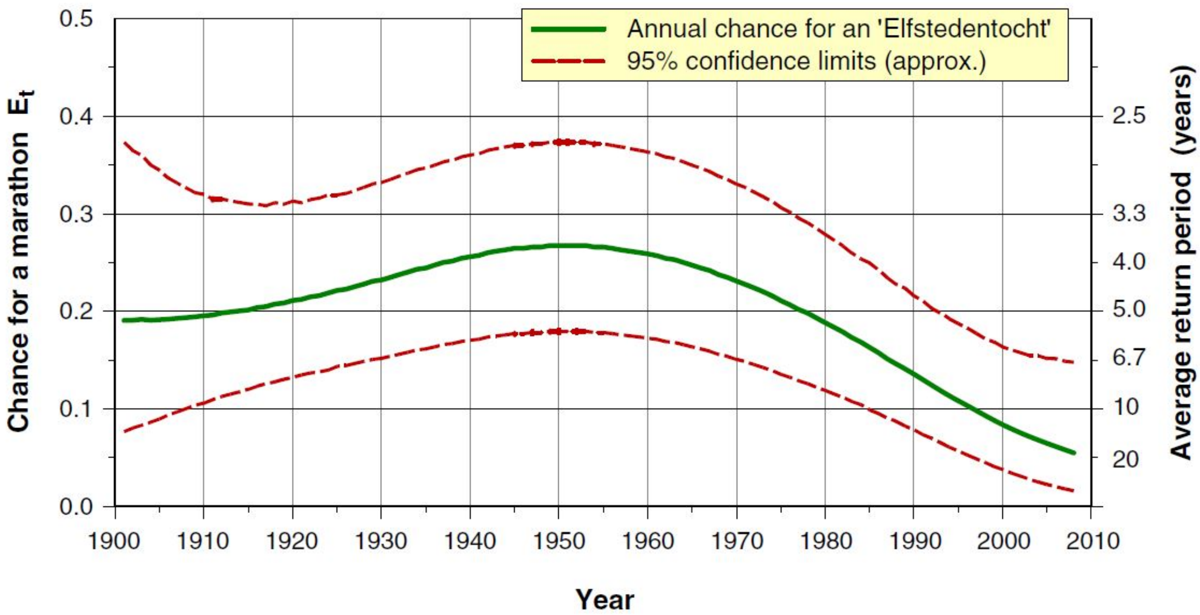

Figure 2: The Annual chance for the ‘Elfstedentocht’ E_t. On the right y-axis the average retrun period R_t is shown, which is the inverse of the annual chances. (Vissers and Petersen, 2008)

Visser (2005) and the Environmental Balance (MNP, 2005) illustrated this graph and found with a nonlinear trend that during the twentieth century the chance of an ‘Elfstedentocht’ has decreased from once every 4 years from 1950 and once every 5 years from 1901 to once every 10 years in 2004. This is also confirmed by de Vries et al. (2012) who state that in the future cold spells are projected to become around 5 Kelvin warmer. Herein cold spells are defined as a period where the temperature falls below the 10% quantile of the winter temperature distribution. Because of this warming the temperatures remain above zero degrees Celsius and have a large climatic impact on ice growth. But several other studies suggest that also the amount of cold spells will increase. Galli et al (2010) shows a decrease of the number of cold spells in the Alps on average but in about 30% of the area the behaviour is more complicated and the amount of cold spells were increasing in some occasions. Kodra et al (2011) found that despite a general warming trend, regional preparedness for extreme cold events cannot be compromised. Song et al. (2014) shows an increase of 6.4 in the frequencies of cold spells per decade since the 1980s. Gerber et al. (2014) argue that recent anomalously cold winters may not be triggered by sea ice or sea surface temperatures anomalies alone. It is therefore not certain that the chance for an eleven cities tour will become zero in the future, and thus the evaluation and further development of an ice growth model is necessary.

De Bruin and Wessels (1988) formulated a model that forecasts ice thickness at a certain time step and tested their model against observations of 1985, 1986 and 1987. The model is already in use for 25 years and used widely in the Netherlands by several authorities (e.g. Royal Dutch Meteorological Institute (KNMI), Rijkswaterstaat and Meteogroup). However those authorities declared that the model does not show the desired results. Therefore we will evaluate the model of de Bruin and Wessels (1988). We only have the results and model description from their paper; therefore we will reproduce their model and evaluate this model with the currently running model at the KNMI. To clarify the above, we will evaluate the results of 3 different models:

1. The results of the originally created model in 1988 by de Bruin and Wessels (1988) labelled as “BW88” (no model code available)
2. The currently running operational model of BW88 used by the KNMI called “KNMI-oper” (model code kindly provided by the KNMI)
3. The reproduction of BW88 by deHaan et al. (2014) labelled as “deHaan2014” (model code programmed by reading de Bruin and Wessels (1988))

We will investigate the differences of the 3 model results for 1985-1987 with the observations. For 2010-2013 we will only compare the results of KNMI-oper and deHaan2014 with the observations because BW88 only shows results for 1985-1987. A sensitivity analysis will be performed to identify the most sensitive pieces of the KNMI-oper model and the deHaan2014 model. The analysis will be done on meteorological parameters like snow, wind, cloudiness, air- and wet bulb temperatures, and for the timing of these parameters.
Due to these raised issues, the following research question with 2 sub questions will be answered:

What problems do we encounter when evaluating the results of the three different ice thickness models?

- Does the deHaan2014 and KNMI-oper model reproduce observations sufficiently well?
- What are the key differences/sensitivities between the results of BW88, KNMI-oper and deHaan2014?

In this research the following topics will be discussed: In chapter 2 we will go along several studies of the past and the history of ice growth modelling. In chapter 3 we will discuss the set-up of the models and the problems we encounter associated with the differences between the models. In chapter 4 we will discuss the methodology of our research and in chapter 5 we will present our results and the discussion of the results. In chapter 5 we will answer our research questions and conclude our research.

2. Background

There are many meteorological variables that influence ice- growth and thickness, it is therefore an interesting problem to model ice formation and melt on surface waters. As water has its highest density at 4^{o}C, colder and less dense water will rise, but warmer water also rises because it is less dense. Due to these properties, the water will mix continuously until the whole water column reaches a temperature of 4°C. From that point on, further cooling results in expansion of the space between water molecules, so that the water becomes less dense. If there is no mixing of the water by wind or currents the top layer of the water will cool to the freezing point (0°C). At the freezing point further cooling will result in ice formation at the surface. The formed ice layer will block the exchange of energy between the cold air and the warm water. The dropping of the temperature of the water below will stop and heat losses will be used for the production of ice. (Ice in lakes and rivers, 2014)
Wind is an important factor which can transfer cool or warm air by advection or it can generate turbulence which enhance the mixing of water and therefore increase the surface temperature due to higher temperatures from the bottom. The radiation and energy balance play an important role where clouds and obstacles like bridges can block the outgoing long wave radiation and therefore reduce cooling. On the other hand incoming shortwave radiation is also reduced directly on the ice, however, during the day bridges can warm up and emit radiation towards the ice. (de Bruin and Wessels, 1988)

For the Netherlands de Bruin and Wessels (1988) designed an ice growth model for operational use. The model is used for forecasting the ice thickness up to 1 week ahead in situations where ice is growing from 0 to 20 cm. The model is similar to that of Piotrovich and Deriugin (1973) or Braslawiski (1973), they suggested formulas for computing ice accretion on the lower surface, taking into account all the components of the snow ice cover heat balance (evaporation, convective heat exchange with the air, effective radiation, radiation absorption, heat output of the water) under the conditions of the stationary process of accretion. However an important difference for the transfer of heat and water vapour are the exchange coefficients. These take into account the effects of atmospheric stability. De Bruin and Wessels (1988) derived an equation for the ice thickness based on work of Keijman (1974), who investigated an estimation of the energy balance of a lake. This equation is an extension of that given by Ogura (1952).

According to Ashton (1989) the most commonly used method for predicting the ice thickness of ice is based on the “Stefan problem”, believed to be originated by Stefan (1889) which describes the temperature distribution in a homogeneous medium undergoing a phase change. The problem is accomplished by solving the heat equation with known initial temperature distribution and a boundary condition. More research done on the Stefan problem is partly summarized by Michel (1971). Research done on fresh water ice and snow is done by Ashton (2011) which includes the thinning of ice and especially in snow ice formation, which occurs when the buoyancy of the ice is smaller than the weight of the snow. Another study on ice formation in a fresh water lake has been done by Fang et al. (1996) who produced an algorithm for ice formation on a lake and tested this algorithm for a lake in Minnesota.
Other studies combine fresh and sea water like Launiainen and Cheng (1998) who developed an ice thickness model for sea ice that is also valid for fresh water. Their construction paid special attention to the parameterization of long wave radiation, air-ice turbulent flux estimation and short-wave radiation extinction in ice and snow. Sea ice has different properties than fresh water: sea ice freezes at about -2^{o}C, instead of 0^{o}C for fresh water ice and depends on the salinity level (de Bruin and Wessels, 1990), sea ice will form later than surface ice because surface water cools faster (Groen and Jilderda, 2007) and the water depth in case of sea ice is much larger; therefore it takes more time to freeze, because it takes more time to cool down due to mixing of the water. Although these

differences are present, by adjusting relevant coefficients and parameters both can be modelled according to the model of Launiainen and Cheng (1998). However in our study the effects of sea water will not be taken into account and is a different subject.

In additon, Mironov (2008) developed a model called “FLake” that is used for numerical weather prediction and incorporates vertical temperature structure and mixing conditions in fresh-water lakes of various depths on time scales from hours to years. The model calculates ice and snow thicknesses based on the temperature profiles and heat budgets of the ice and snow layers. At KNMI, the model of de Bruin en Wessels (1988) has been compared with the operational model Flake and compared for the winters of 1985, 1997 and 2012 by de Bruijn en Bosveld (2009) and de Bruijn et al (2014). They found that “FLake” does a reasonable job in the prediction of the ice thickness and is competitive with the operational model. But “FLake” overestimates the ice growth slightly because of an underestimation of the down welling shortwave radiation of the ECMWF model and of an uncertain temperature of the deepest bottom layer. They also found that small-scale variations in the water profile in shallow basins cannot be represented by both Flake and BW88 but ice- and snow thickness, both variables with a longer timescale, are quite well represented and biases are smaller than 0.01 m. However the measurements used in this study are sparse and only validated for 2 winters.

3. Set-up of the different models

3.1 Summary of BW88

In this section a summary of the paper from 1988 of the Bruin and Wessels will be given. In figure 3 the most important parameters of the model are given by a graphical representation. The model starts when little ice has formed. The ice thickness h_i (m) is calculated according to:

$$\frac{dh_i}{dt} = -\left(\frac{G+Q_w}{\rho_i \lambda_m}\right) - \left(\frac{E}{\rho_i}\right), \tag{1}$$

Where t (hours) is the time, ρ_i (kg m^{-3}) the density of ice and λ_m (J kg^{-1}) the latent heat of fusion. The ice loss due to evaporation is given by the second term of (1), therefore a minus sign is included before the second term. The heat flux density, $Q_{w,}$ (W m^{-2}), from the water to the ice layer is positive when heat is added to the ice layer and is formulated as:

$$Q_w = \frac{k_w (T_{bs} - T_{bi})}{h_w} \tag{2}$$

Where k_w is the heat flux by molecular heat conduction in water (0.6 W m^{-1} K^{-1})
T_{bs} (K) is the temperature of the bottom water and T_{bi} (K) the temperature just under the ice. The difference between these temperatures is near 4 K if the temperature profile under the ice is stable, this holds only for small canals and lakes with minor depth and not for rivers which are heavily exposed to turbulence. The depth of the water column is denoted by h_w (m).
In equation (1), G (W m^{-2}) is the amount of heat that is added to or withdrawn from the ice-water system due to radiation losses and vertical transport of heat and water vapour. G is positive when heat is added to the system and is the most significant factor that determines the ice thickness.
G is formulated as:

$$G = Q^* - H - \lambda_{sub} E, \tag{3}$$

where Q^* (W m^{-2}) is the actual net radiation, H (W m^{-2}) is the sensible heat flux and $\lambda_{sub} E$ (W m^{-2}) the latent heat flux.

$$Q^* = K^+ - K^- + L^+ - L^-, \tag{4}$$

where K^+ (W m^{-2}) is the incoming shortwave radiation and is approximated by the solar elevation ϕ and the total cloud cover *N* (octas)as follows:

$$K^+ = 1353(0.60 + 0.22\sin\phi)\sin\phi(1 - 0.7N^2). \tag{5}$$

The solar elevation is approximated with the astronomical formula mentioned by Holtslag and van Ulden (1983). The outgoing short wave radiation K^- (W m^{-2}) is calculated as follows:

$$K^- = a * K^+, \tag{6}$$

where *a* is the albedo of the snow or ice surface.
For the incoming long wave radiation the following semi-empirical parameterization by Paltridge and Platt (1976) is used:

$$L^+ = \varepsilon_s(\varepsilon_a \sigma T_a + 60N - 42(N - N')), \tag{7}$$

where ε_s and ε_a are the emissivities of the surface and air respectively, σ (Js^{-1}m^{-2}K^{-4}) is the Stefan-Boltzmann constant, T_a (K) is the air temperature and N' (octas) the cloud coverage of low and middle clouds. The outgoing long wave radiation is calculated from the surface temperature T_s (K) which is not given and is therefore calculated by an approximation. The calculation of T_s is one of the most important parts of the paper written by de Bruin and Wessels (1988). This surface temperature is unknown beforehand but determines the energy balance fluxes which are the most significant parameters of the model; see formula 17 for a description.

$$L^{-} = \varepsilon_s \sigma T_s^{\ 4} \tag{8}$$

Another important part is the calculation of the sensible and latent heat fluxes H (W m^{-2}) and LE (W m^{-2}) defined as:

$$H = \rho_a c_p (T_s - T_a) / r_a \tag{9}$$

and

$$\lambda_{sub} E = \frac{\rho_a c_p}{\gamma} (e_s - e_a) / r_a . \tag{10}$$

According to the Monin-Obukhov theory the aerodynamic resistance r_a (sm^{-1}) is a rather complicated function of the Obukhov length. The Obukhov length is related to the ratio between buoyancy and shear and a measure for the stability. However stability effects are still important because T_a can differ significantly from T_s, therefore de Bruin and Wessels (1988) searched for an empirical relationship for an exchange coefficient A (W m^{-2} K^{-1}) in which stability effects are still incorporated. This exchange coefficient is actually defined as:

$$A = 4\varepsilon_s \sigma T_i^{\ 3} + \frac{\rho c_p}{r_a}(1 + \frac{s}{\gamma}) , \tag{11}$$

where T_i (K) is the ice bulb temperature at screen height, ρ_a (kg m^{-3}) the air density, c_p (J kg^{-1}K^{-1}) the specific heat of air at constant pressure, *s* (Pa K^{-1}) the derivative of the saturation water vapour pressure with respect to ice at T=T_i and γ (Pa K^{-1}) the psychometric constant.

However due to the problems with stability mentioned above, the empirical relation for A used by the model is:

$A = 4\varepsilon_s \sigma T_i^{\ 3} + 2.5 u_{10}$ Under stable and neutral conditions when $T_s \le T_a$ (12a)

$A = 4\varepsilon_s \sigma T_i^{\ 3} + 2.5 u_{10} \sqrt{1 + \frac{10(T_s - T_a)}{u_{10}^{\ 2}}}$ Under unstable conditions when $T_s > T_a$ (12b)

The part under the root of the exchange coefficient A for unstable conditions (12b) is similar to the Richardson number and is larger than the exchange coefficient for stable conditions. They combined the sensible and latent heat flux into A, T_i and T_s as follows, for G we now read:

$$G = Q_n^* + A(T_i - T_s) \tag{13}$$

Where Q_n^* (W m^{-2}) is the same as Q^* but for L^- in (4), the surface temperature in (8) now reads the ice bulb temperature, so Q_n^* is the net radiation if T_s=T_i. In this case (8) becomes:

$$L^{-} = \varepsilon_s \sigma T_i^{\ 4} \text{ (W m}^{-2}\text{)}, \tag{14}$$

with the combination of a so called equilibrium temperature

$$T_e = T_i + Q_n^* / A \text{ (K)}. \tag{15}$$

They combined the radiation, sensible heat flux and latent heat flux into the description of:

$$G = A(T_e - T_s) \text{ (W m}^{-2}\text{)}. \tag{16}$$

The surface temperature is calculated as follows:

$$T_s = T_f + (\frac{A' h_i}{k_i})(T_e - T_f) \text{ (K)}, \tag{17}$$

where T_f is the freezing temperature and mostly 273.15 K and k_i (W m^{-1} K^{-1}) is the thermal conductivity of ice.

In their paper, BW88 then further distinguish G between the ice growth stage and the melting stage. For a clear explanation the reader is advised to read the paper of de Bruin and Wessels (1988). In figure 3 a graphical overview of the set-up of the model is given.

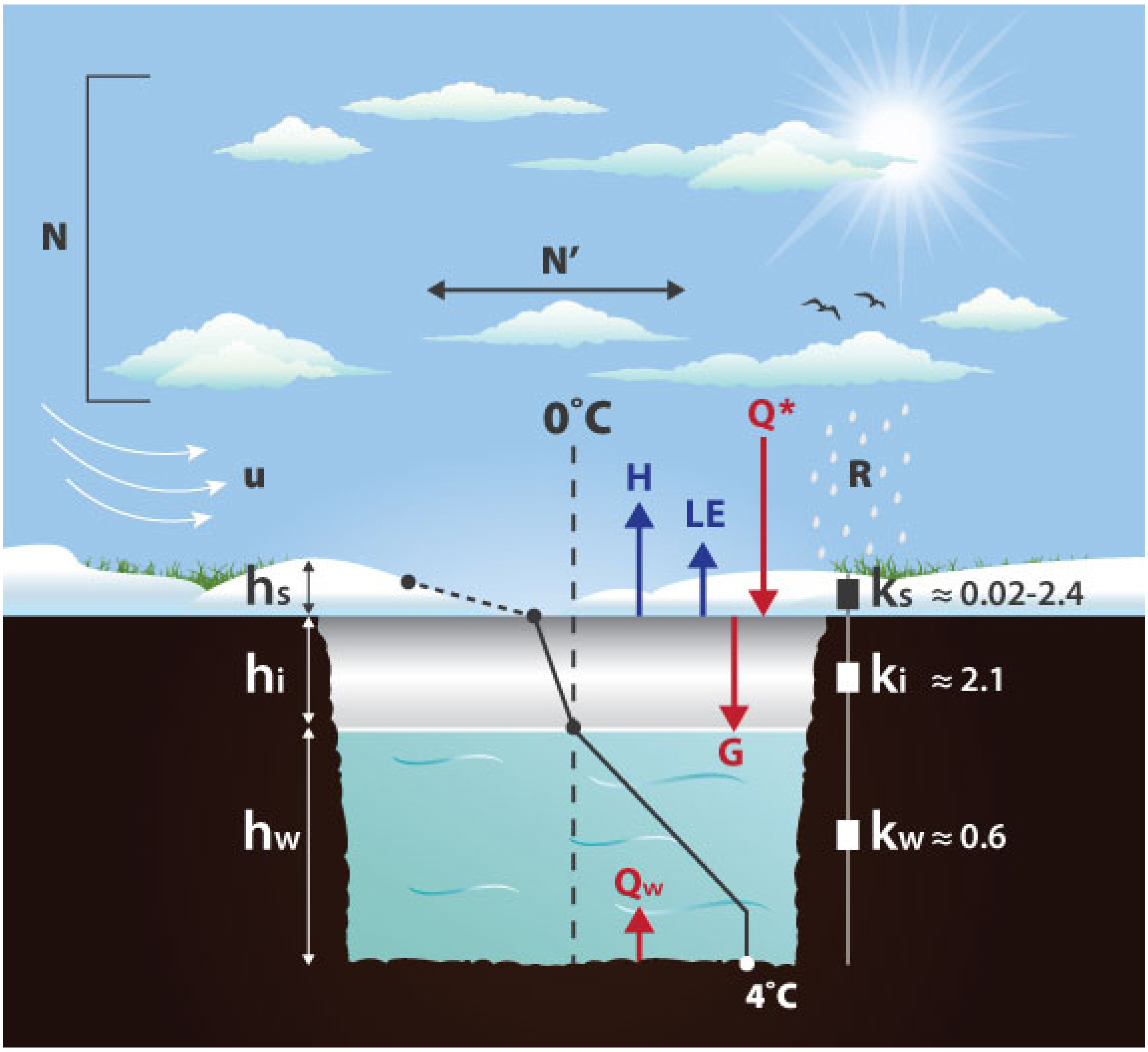

Figure 3: Graphical overview of meteorological parameters of the ice-water-snow system.

Figure 3 depicts the most important parameters of the model. Most variables are explained above or will be explained in the next sections. The black line in the middle shows the temperature profile of the ice-water-snow system. The model assumes that the water at the bottom has a temperature of 4°C and the temperature of the bottom of the ice is supposed to be 0°C. The amount of precipitation is given by R and is the water equivalent precipitation. The snow height h_s is retrieved from this water equivalent precipitation by dividing it by the snow density. For every layer, a thermal conductivity is present in W m^{-1} K^{-1} (k_s, k_i and k_w). These thermal conductivities ensure a certain resistance of the heat trapped in the ice-water-snow system to leave the system to the atmosphere. The higher the number, the less the resistance, the thermal conductivity of ice has thus the least resistance. The thermal conductivity of snow depends on the density of the snow. The density of the snow ranges from 90-900 kg m^{-3} and the corresponding thermal conductivity ranges therefore from $2.43*10^{-2}$ to 2.43 W m^{-1} K^{-1}. Fresh show can therefore largely increase the resistance that keeps the heat trapped in the system.

3.2 Practical problems encountered

In this chapter we will discuss various practical problems that we encountered during the reproduction of BW88 in the form of deHaan2014. We encountered these problems because in BW88 not every detail is elaborated, but to get a working ice growth model these details have to be clear. DeHaan2014 is thus the same as BW88 but with the elaborated details listed below.

The time steps used by de Bruin and Wessels (1988) for the input of the meteorological data is 12 hours. DeHaan2014 model uses time steps of 1 hour for a more accurate ice growth and even more accurate snow amount.

Because the ice (T_i) and wet (T_w) bulb temperatures are not measured directly, we approximate them with the combination of the Clausius Clapeyron and the Psychrometric equation that need the input of the air- and dew point temperature. We also investigated the empirical relationship for the wet bulb temperature of Stull (2011) and the ice bulb relationship from Rogers and Yau (1989). The difference between these relationships was minor and therefore we decided to use the calculation of T_i and T_w based on the Clausisus Clapeyron and the Psychrometric equation. This is the same method used by BW88 and KNMI-oper.

To calculate outgoing long wave radiation, BW88 uses a fraction of low and middle clouds: N' (octas). This factor is one number and includes both low and middle cloud coverage. N' is not measured by the KNMI weather stations; therefore deHaan2014 model uses SYNOP data where cloud coverage is present and split up in 4 layers. We decided to use the cloud cover of the 1st layer and if there was no data available the cloud cover of the 2nd layer, where the 1st and 2nd layers are the layers closest to the ground. If for both layers the data was empty, N' is set to zero. If the layers were coded with a 9, which means the clouds could not be measured due to mist or other reasons, the cloud cover was set to 8. For the years 2010, 2011 and 2012 there was no SYNOP data available directly, therefore it was chosen to put N' at a certain factor of the total cloud coverage; N'=0.7*N. This factor is based on the average of the years 1985, 1986 and 1987 of N' divided by N. We tested several factors for N' and it turned out that N' is not a sensitive parameter.

Considering precipitation, the reported precipitation at the KNMI station is converted to zero when the rain was lower than 0.05mm per hour. This is done because it is uncertain how much rain fell by such low amounts. The amount of snow is only converted from precipitation data if there was snow fallen in the previous hour and/or during the time of observation. If both rain- and snow occurrence took place then the precipitation was not converted to snow amount.

In deHaan2014 we programmed the water depth as a function of the ice thickness. This is the total water depth minus the ice thickness. Hence instead of using h_w in the denominator of formula 2, we use h_w-h_i. The influence of this implementation is however not significant.

For the latent heat of evaporation during the melting stage we used the formula from Harrison (1963):

$$\lambda_v = 10^6 (2.501 - 0.002361 T_a) \qquad (18)$$

For the latent heat of fusion we used a constant factor of $3.34*10^5$ and for the latent heat of sublimation we used the formula created by polynomial curve fitting by Rogers and Yau (1989):

$$\lambda_{sub} = 10^3 (2834.1 - 0.29 T_a - 0.004 {T_a}^2) \qquad (19)$$

The short wave incoming radiation (their formula 16 in de Bruin and Wessels (1988)) cannot be negative, therefore the radiation is set to zero if negative. De Bruin and Wessels (1988) specify the

short incoming radiation zero when the solar elevation is negative, the hour is set at 24, or when the short wave incoming radiation is zero.
These practical problems described above are assumed to not have a great impact on the ice thickness. Therefore these practical problems will not be further elaborated in the results section. According to Mureau (2014) the model is highly tuned and it is partly for this reason that we encounter these problems during evaluation of the set-up of the model. The difference in the reduction of the time step from 12 to 1 hour can influence the timing of snow and ice thickness and can therefore influence the ice thickness significantly, but more details on the timing of snowfall is given in the results section.

3.3 Modifications of KNMI-oper from BW88

From 1988 onwards the model of de Bruin and Wessels (1988) is maintained and adjusted at the KNMI. Errors were removed, modifications are made and extra options were added. Therefore the code is not the same anymore as it was in 1988. In this chapter we give a clear representation of the deviations of the model from 1988 until now. In the results section we show the impact of these deviations by means of figures. We have to make a difference between documented deviations and undocumented deviations. The documented deviations are written in the source code of the KNMI-oper model and were documented up to 1997. The undocumented deviations were discovered during the comparison of BW88 with KNMI-oper and are never documented in the paper of de Bruin and Wessels (1988) or were added later to the model but never documented, it is for these undocumented deviations unclear when the deviation from BW88 took place.

3.3.1 Documented modifications

For this comparison and further results the model version from 29 december 1989 from the author H.R.A. Wessels at the KNMI de Bilt is used. The modifications to the model are visible in Table 1 and are obtained from the programmed FORTRAN code 'IJS.f'. The deviations can therefore be too technical and should only be read with the programmed version of the code. However to explain the differences in ice thickness, these changes in table 1 require notification.

Table 1: Documented modifications by the KNMI to the model of De Bruin and Wessels (1988) from 1990 until 1997.

Date	Error/Improvement/Option	Section of model	Further specifications
31 December 1990	Error in conversion dew point	3.1	
11 February 1991	Error in evaporation/snowmelt-loop	3.4.3	In Do-loop 241
21 February 1991	Improvement option wind-clearings	3.5	
15 April 1991	Option for separating RH and Td	2	For humidity flag also 3 or 4 allowed
15 April 1991	Option for skipping of data lines	1, 2 and 3.2	
19 September 1991	Improvement of summer turbidity and albedo	3.4.1	Different version used in 1992-1993 for turbidity
19 September 1991	Option of screen print morning water temperature if higher than 4°C	4	
11 November 1991	Improvement urban effect	3.4.3	'mtown'=2-99
32 December 1992	Error: N not allowed greater than N'	3.4.1	
16 January 1997	Error in Ts (probably small effect)	3.4.2	Not clear what error

The errors, improvements and options in Table 1 have been implemented after publication of BW88. It is partly because of these modifications that differences occur between the results of the three models.

3.3.2 Undocumented modifications

When reading through the code of KNMI-oper the deviations below were discovered:

If the wet bulb temperature is greater than 0.5°C, the precipitation will be treated as rain. In the code of deHaan2014 the occurrence of rain is already specified at the input.

If the ice sheet is thinner than 5cm and the wet bulb temperature is lower than 1°C they reduce the time step from 12 hours to 3 hours. In the code of deHaan2014 the time step is already 1 hour, therefore this specification is not applied.

The treatment of snow is one of the most important differences to study, because snow is a very sensitive parameter of the model.

In our code we calculate the snow density at forehand. Every hour a factor of 0.5 kg/m^3 is added to the amount, like the paper describes.

$$\rho_j = 90 + 0.5(J - j)\Delta t \quad (20)$$

Where ρ_j is the density of snow at time step *j* and *J* is the present time step. The density of fresh snow is 90 kg/m^3 and the density added per hour is 0.5 kg/m^3. When $\rho_j \leq 900$ kg/m^3, the density is stays 900 kg/m^3. Because the density is known beforehand, we can calculate the thermal conductivity of snow, k_s which is

$$k_s = 3*10^{-6} \rho_s^{\ 2} \ \ Wm^{-1}K^{-1} \quad (21)$$

according to the BW88. Because the rain intensity is also known, we can now calculate the snow height h_s and the factor h_s/k_s. To illustrate how these factors are calculated an example will be given for further clarification.
Suppose we have 2 arrays, 1 array with rain intensities (R_j) in kg m^{-2} h^{-1} and 1 array with the densities (ρ_s)in kg m^{-3} (table 2), then the according snow height (h_s) in meters is present in table3.

Table 2: Arrays with rain intensity and corresponding density

R_j (kg m^{-2} h^{-1})	0	0.8	0.6	1.6	0
ρ_s (kg m^{-3})	90	90.5	91	91.5	92

Table 3: Snow height in meters at every hour and for every precipitation event

	1 hour	2 hours	3 hours	4 hours	5 hours	Sum
h_s (m)	0/90					0
h_s (m)	0.8/90	0/90.5				0.0089
h_s (m)	0.6/90	0.8/90.5	0/91			0.0155
h_s (m)	1.6/90	0.6/90.5	0.8/91	0/91.5		0.0332
h_s (m)	0/90	1.6/90.5	0.6/91	0.8/91.5	0/92	0.0330

In KNMI-oper model, the snow aging process is almost the same, but the following subtle, but important differences are present. In the KNMI-oper model the rain intensities are sums over the last 12 hours, whereas in deHaan2014 the rain intensities are calculated every hour. This is an important difference because rain calculated at a time step of 12 hours is seen by the model as a rain intensity at every time step of 12 hours whereas in time steps of 1 hour different intensities can take place and ice- and snow thicknesses react differently on this type of input for different time steps. For example, when all snow falls at the first time step in deHaan2014, the ice growth is already reduced in the first time step at 1 hour. But when it falls at the time step of 12 hours, the ice can grow 11 hours at a fast rate and at the 12th time step the growth is reduced. In the KNMI-oper model this is thus threated equally because they only use time steps of 12 hours.
Another difference is the aging of the snow. In the deHaan2014 model aging continues until the end of the ice event for the complete dataset. In the code of the KNMI the aging stops when 15 precipitation events did happen. When at every time-step precipitation does occur, this is at 15*12=180 hours in their code. The rain that fell during the first time step does nog age anymore when it ends up in the 15th time step. Moreover the KNMI-oper model updates the snow height when snow ages, evaporates or melts.
They specify surface conditions according to 4 categories:

0. Ice + Snow — standard
1. Clear Ice — when snow thickness is 0 mm
2. Wet Ice — when wet bulb temperature is greater than 0°C
3. Open water — when ice thickness is lower than 0 mm

The deHaan2014 model is based on BW88 where we specify the melting stage based on the surface temperature of the ice. When the surface temperature is higher or equal to 0°C, then the melting stage is present, otherwise the ice growing stage is present. For the ice growth phase, the surface temperature (Ts) and the albedo (a) we test the occurrence of snow.

There are also minor differences in the calculation of shortwave radiation. DeHaan2014 follows the paper of de Bruin and Wessels (1988) and the specified approximation of the solar elevation mentioned by Holtslag and van Ulden (1983), where we use a solar constant of 1353 W m^{-2}. In the KNMI-oper model the solar constant is not constant but varies during the day of the year from approximately 1330 – 1376 W m^{-2}.
In their formula 16 of de Bruin and Wessels (1988) the so called turbidity function has changed over time in the KNMI-oper model. In the paper of de Bruin and Wessels (1988) this factor is

$$(0.60 + 0.22\sin\phi), \tag{22a}$$

in 1992-1993 the KNMI-oper model used

$$(0.60 + 0.16\sin\phi), \tag{22b}$$

and now they use

$$(0.45 + 0.40\sin\phi). \tag{22c}$$

Another difference that needs to be mentioned is the use of local and universal time. In the KNMI-oper model they use local time whereas we use universal time. This has consequences for the calculation of the hour angle (*h*), when using universal time, a correction for longitude has to be made. The formulation of the hour angle in the code is the same, but in our code 2 extra factors are added. These factors are $-\pi$ and $-\lambda_w$ where λ_w is the western longitude of the location in radians. We did not test if there were differences in results due to these differences in formulations but expect them to be minor in difference.
A difference in the formulation of the solar declination is the use of arcsine by the paper where the KNMI-oper model uses the arctangent. The last difference in the shortwave radiation is in the formulation of the cloud cover. In the BW88 model they use,

$$(1 - 0.7N^2), \tag{23a}$$

with N the cloud cover in octas, in the KNMI-oper model they use,

$$(1 - 0.73N^2) \tag{23b}$$

if the total cloud cover is equal to the cloud cover of low and middle clouds. When these factors are not equal they use

$$(1 - 0.73(N'+1)N), \tag{23c}$$

with N' the cloud cover of low and middle clouds in octas.

The calculation of the thermal conductivity of snow in the paper is already explained as

$$k_s = 3*10^{-6}\rho^2\ Wm^{-1}K^{-1}, \tag{24}$$

but in the KNMI-oper model they use a different formulation. They base the thermal conductivity on the surface conditions and combine the thermal conductivity of snow and ice in 1 formulation. This formulation is not comparable with the paper, but is highly important for the ice thickness.

Regarding the formulation of the albedo, the KNMI-oper model is different. In both models the albedo is updated when fresh snow is added to the ice. There is a minor difference in the decrease of the albedo. In our model the albedo decreases per hour with a factor of 0.002, per 12 hours this is 0.024. In the code of the KNMI this is 0.025 per 12 hours. In addition the albedo is set to 0.30 in the paper when there is no snow or when the albedo turns lower than 0.30. In our code this value is set at 0.35 as described in de Bruin and Wessels (1988). Another difference that the KNMI uses is the dependence of the albedo of the solar elevation when there is wet ice. This formulation has changed over time but is currently formulated as:

$$a = \frac{0.22}{\phi} - 0.05, \quad (25)$$

where *a* is the shortwave reflection (albedo) and ϕ is the solar zenith angle or solar elevation angle.

There is also a slight difference in surface emissivity for snow and ice in the KNMI-oper model. For ice they use 0.95 and for snow 0.90 where we use only 0.95 for both ice and snow. The apparent emissivity of clear skies as formulated as formula 19 in de Bruin and Wessels (1988) is used by the code. Our code is based on formula 18 of de Bruin and Wessels (1988).

The exchange coefficient (A) and the surface temperature (T_s) are determined iteratively by the code of the KNMI and differs between the KNMI-oper model and BW88. In the paper the first term of A is $4\varepsilon_s \sigma T_i^3$ W m^{-2} K^{-1}, where ε_s is the emissivity of ice, σ is the Stephan Boltzmann constant and T_i is the ice bulb temperature. In the code of the KNMI this term is a constant of 4 W m^{-2} K^{-1}. This is probably because in the Netherlands the term $\varepsilon_s \sigma T_i^3 \approx 1$ at that temperature.
The exchange coefficient formulated in the paper as formula 14a is formulated different in the code of the KNMI. Here the formula is executed when T_s is lower than T_a-0.5 or when the wind speed *u* is higher or equal to 6.5 m s^{-1}.

The wet bulb temperatures and ice thickness in the KNMI-oper model are corrected for urban heating. They subtract a certain amount from the ice thickness by a specified urban heat factor. It is however not documented how this factor works. In the deHaan2014 model we do not use this urban heat factor because it is not written in BW88 and we cannot evaluate on urban data with measurements.

Another important difference is the removal of ice when the ice thickness is lower than 3mm after a 12 hour period. This difference will also be evaluated in the results section. In the results section we will only evaluate the most important differences. Most of the differences mentioned above are thus not significant for the ice thickness. We did test most of these differences and they showed limited or no sensitivity. The most important differences are the removal of ice when the ice is thinner than 3mm, the timing of the snow and ice and the formulation of the thermal conductivity of snow. Therefore these important differences will be further evaluated in the results section.

4. Methodology

To answer the first research question, the results of 1985-1987 of de Bruin and Wessels (1988) are extracted from their paper. We developed deHaan2014 according to the formulas described in their paper. The source code of KNMI-oper has been kindly provided by the KNMI. In this section we discuss the forcing of the models and the observation stations.

The meteorological stations that are used as input for the model and the measurement stations are shown in figure 4. The stations for input of the model are KNMI weather stations and the data can be found at the website of the KNMI (http://www.knmi.nl/klimatologie/uurgegevens/). The stations used for input are Leeuwarden, Stavoren and Eelde. The variables from these stations used as input for the deHaan2014 model include: the station number, the date (year, month, day and hour), the 10 min averaged wind speed (m s^{-1}), the air temperature (^{o}C), the dew point temperature(^{o}C), the hourly sum of water equivalent precipitation(mm), the pressure (hPa), the total cloud cover the air (N in octas), the relative humidity (%), the occurrence of rain and/or snow in the previous hour or during the observation (0 for no occurrence and 1 for occurrence of precipitation). The coverage by low or middle clouds for the years 1985, 1986 and 1987 are retrieved from SYNOP data from the KNMI. These are hourly data given octas for 4 different layers with their specific heights. For the other years the cloud cover of low and or middle clouds are put at 0.7*N (see section 3.2) due to lacking observations of this variable since 2002. For the KNMI-oper model the same data is used for input except for the occurrence of rain and/or snow.

Figure 4: Geographical location of KNMI meteorological input stations (orange dots) and measurements stations (black stars)

The measurement stations are Veenwouden, Workum and IJlst and provide data for the years 1985, 1986, 1987, 2010, 2012 and 2013. The measurements are provided by the royal association of the "Friesche Elf Steden" and provided by the KNMI. These measurements include an intern number, the date (year, month and day), the geographical latitude and longitude of the measurements, the water depth (m), the ice thickness (cm) and the snow height (cm) rounded to an integer.

For comparison of the deHaan2014 model with the results of the Bruin and Wessels (1988), the currently running model at the KNMI (KNMI-oper) is provided by KNMI. For clarification, this is not the exact model developed in 1988 but is the model from 1988 with improvements and corrections made through the years till 1997.

5. Results and Discussion

In this section we will discuss the most important differences by means of figures and we will test the 3 models against observations. The research questions will be answered according to the following procedures:
First we will show whether the model results of BW88 are reproducible and we will show the impact of the removal of ice thinner than 3mm for the years 1986 and 1987, with forcing of Leeuwarden data and compared with measurements at Veenwouden.
Second, we will show the impact of snow on the ice thickness for the three different models for 1985 for Leeuwarden.
Third, the impact of snow is further investigated for new cases of 2010-2013 and thereby the impact of ice and snow removal when ice is thinner than 3mm and this impact for the timing of the snow events. This is done for Leeuwarden, Eelde and Stavoren and the observations for IJlst.

5.1 Clear Ice

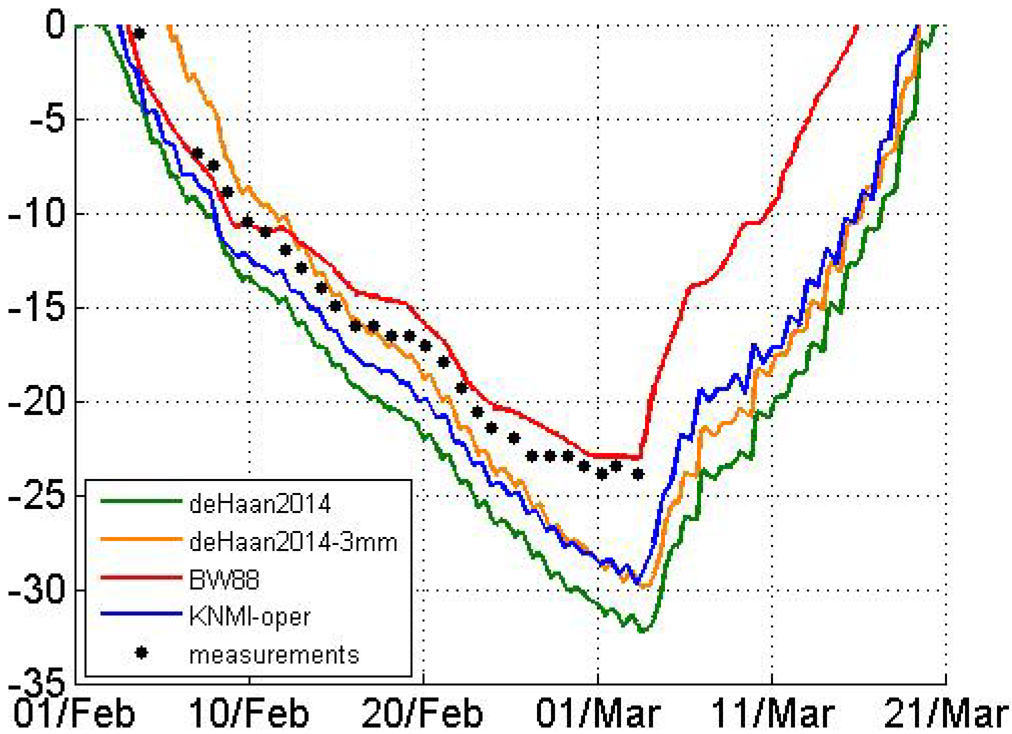

Figure 5: Ice thickness simulation during the winter of 1986 (with an Eleven Cities tour on the 26th of February) for Leeuwarden and measurements at Veenwouden. Green: the deHaan2014 model without ice removal, orange: the deHaan2014 model with ice removal of 3mm and less, red: the results of the BW88 model, blue the KNMI-oper model and black: the measurements.

In figure 5 the original measurements and ice thickness of de Bruin and Wessels (1988) for 1986 are shown for the beginning of February until the 20th of March. The KNMI-oper and the deHaan2014 model with- and without ice removal of less than 3mm thick are presented. The run is completely snow free. The data used as input for the models come from the Leeuwarden airbase. As can be seen the ice thickness in centimetres is on the y-axis, as *hi,* and the time is present on the x-axis.
The results of BW88 match closest to the observations but we cannot verify why their model gives those results because we do not have the code of this original model. Besides that, the reliability and representativity of the measurements are also unclear. The measurements are relatively uncertain because there is no metadata and it is unknown where they were measured under unknown circumstances. It is therefore hard to draw conclusions from these results.

An important finding is that the KNMI-oper model does not follow the results of BW88 but is closer to the deHaan2014 model. In addition, the BW88 model does not show fluctuations where the deHaan2014 model and the KNMI-oper model do show these fluctuations due to the diurnal cycle. These fluctuations are visible due to the strong daily cycle of radiation. During the day common values for the net radiation are 150 W m^{-2} and during the night -50 W m^{-2}. We find that the deHaan2014 model shows a consistent difference of 3cm in ice with the KNMI-oper model. This difference is due to a removal of ice that is less than 3mm thick. This removal is present in the KNMI-oper model, in the deHaan2014 model this removal of ice is optional. Therefore we plotted the same run again but now we did remove ice that is less than 3mm thick, this run is present as deHaan2014-3mm. This removal together with the difference in time-steps between the KNMI-oper model and the deHaan2014 model are significant differences in terms of ice thickness. The deHaan2014 model uses time steps of input of 1 hour whereas the KNMI-oper model uses time steps of 12 hours. It can be seen that if we do remove ice that is less than 3mm thick for both models the results of the KNMI-oper model and the deHaan2014 model are the same for the maximum ice thickness at the 4th of March.

Another result we obtained from figure 5 is that the deHaan2014-3mm model starts later with the growing phase of ice than the other runs of figure 5. The date for the ice growth is 6 February 06:00 UTC whereas the KNMI-oper model and the BW88 model have starting dates of 3 February 12:00 UTC and 4 February 00:00 UTC respectively. When comparing these results we can see that the deHaan2014 model starts about 3 days later with the growing of ice. This difference results in about 3cm less ice for the maximum ice thickness at 4 March (compare deHaan2014 with deHaan2014-3mm). Hence the removal of ice less than 3mm thick ensures a 3cm less ice thickness. A reason for the different starting times is the difference in time steps between the models. The deHaan2014 model uses time steps of 1 hour whereas the KNMI-oper model and the BW88 model uses time steps of 12 hours. When removing ice at every hour it has another influence than when removing the ice at every 12 hours. The influence is unpredictable because it depends on the time-step whether the ice will be removed. The difference in chance that ice less than 3mm thick will be removed, at a time step of 12 hours compared with a 1 hour removal, is unknown. However, we tested the removal of ice less than 3mm thick for the deHaan2014 when checking the ice thickness at time-steps of 12 hours for the year 1986. In this case the results were the same as without the removal of ice less than 3mm thick. The KNMI-oper model is calibrated for time-steps of 12 hours and it is therefore a challenging task to compare different time-steps between the models (Mureau, 2014).

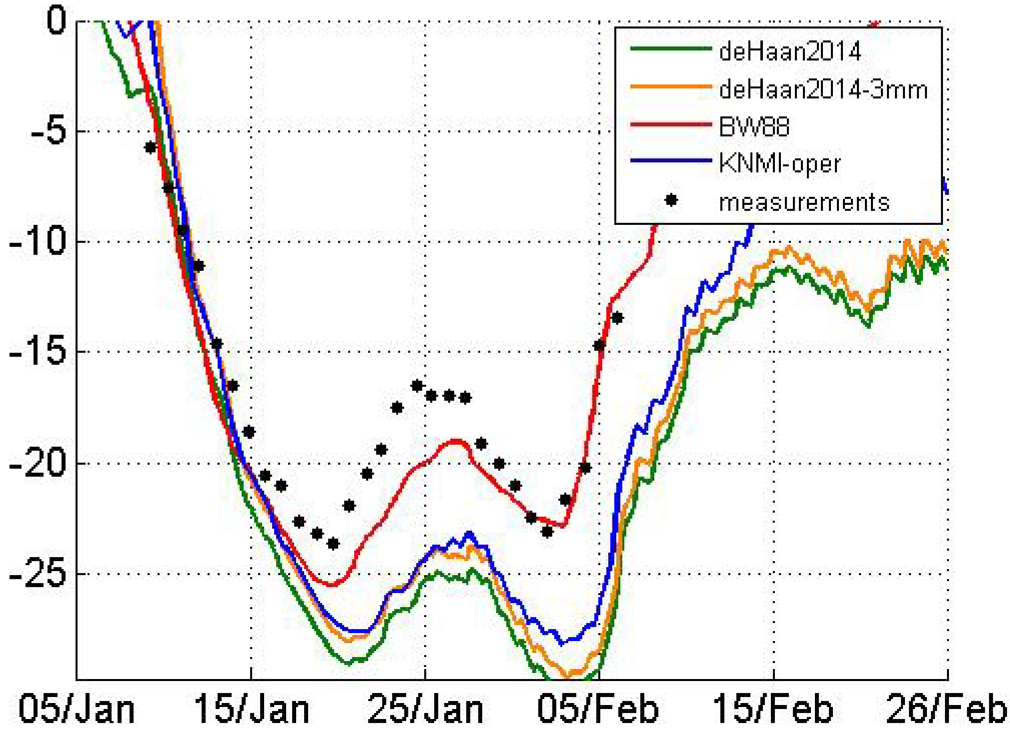

Figure 6: Ice thickness simulation during the winter of 1987 for Leeuwarden and measurements at Veenwouden. Green: the deHaan2014 model without ice removal, orange: the deHaan2014 model with ice removal of 3mm and less, red: the results of the BW88 model, blue the KNMI-oper model and black: the measurements.

The same analysis is done for the year 1987; the results are shown in the figure 6. The situation for 1987 is almost the same as for 1986 in the case that the KNMI-oper model and the deHaan2014 model show almost the same results and the BW88 model of shows lesser ice thicknesses. The case is snow free again and there is a rapid growth of the ice from the 5th of January till the 20th of January, then a reduction in the ice growth due to positive air temperatures, then a small frost period and after this period a fast temperature increase causes the final melting of the ice. When removing the ice of 3mm or less in the deHaan2014 model, the KNMI-oper model matches deHaan2014 except for the second growth and melting periods, it is unknown what primary factor causes this difference. De Bruin and Wessels (1988) also found a difference of 5cm for different input data of different stations during the melting phase; see page 171: sensitivity of the model for input parameters, further investigation on the melting period is therefore needed.
If we compare the BW88 results with the measurements there are differences up to 4cm. They connect this to the heat stored in the bottom and banks due to a sudden cooling after a period of relatively warm weather.

5.2 Snow

In the case of snow cover on the ice, the 3 models behave differently among each other, figures 7 and 8 illustrate this difference. The results of BW88 are the same as in the paper of de Bruin and Wessels (1988). The first frost period, from the 1st of January to the 1st of February 1985 is a period where snow plays a role. In figure 8 we study the impact of ice and snow removal for ice that is less than 3mm thick. The second frost period is completely snow free and runs from 7 February till the beginning of March 1985. We illustrate with the figures the differences in ice thickness with or without a removal of ice less than 3mm thick. This ice removal is not given in the original paper of de Bruin and Wessels (1988).
An important result from figure 7 is the higher ice growth for deHaan2014-3mm and KNMI-oper-3mm and a lower snow height. But even more important is the timing of the snow. When removing

ice lower than 3mm thick the snow is also removed, in this case the timing of a snow layer is about 1 day later. Due to this delay the ice can grow at a faster rate. Because the thermal conductivity of snow disappears completely due to this ice removal, the thermal heat released by fusion beneath the ice can now enter easier trough the ice to the atmosphere. Unfortunately reliable observations to test the model results were absent in this case; they are however plotted in the original results of BW88.

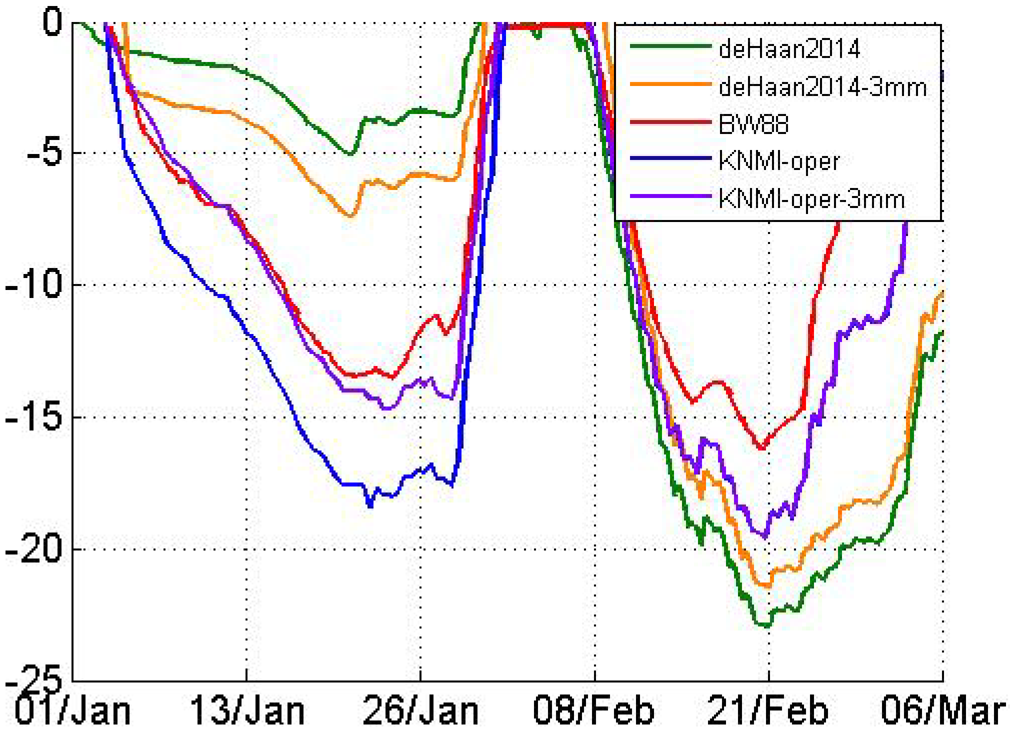

Figure 7: Ice thickness simulation during the winter of 1985 for Leeuwarden. Ice is influenced by snow in the first frost period. Green: the deHaan2014 model without ice removal, orange: the deHaan2014 model with ice removal of 3mm and less, red: the results of the BW88 model, blue the KNMI-oper model without ice removal and purple: the KNMI-oper model with ice removal of 3mm and less.

The modelled ice growth is very sensitive to the modelled snow and in particular the thermal conductivity of snow. This is an explanation for the difference in ice growth for period 1 (the first frost period where snow is present) between the deHaan2014 model and the KNMI-oper model. The piece of code that describes this thermal conductivity is different between the two models and probably also different in the original code from de Bruin and Wessels (1988). The way that this thermal conductivity is calculated in the KNMI-oper model is not totally clear, but does not meet their formula 21 of the original paper by de Bruin and Wessels (1988). It seems the KNMI-oper model uses a thermal conductivity of 2.0 W m^{-1} K^{-1} for snow, belonging to a standard density of fresh snow of 90 kg m^{-3}, and multiplies this by the rain intensity divided by the snow height:

$$\frac{\sum R_j}{h_s}, \qquad (26)$$

the reason for this unknown.

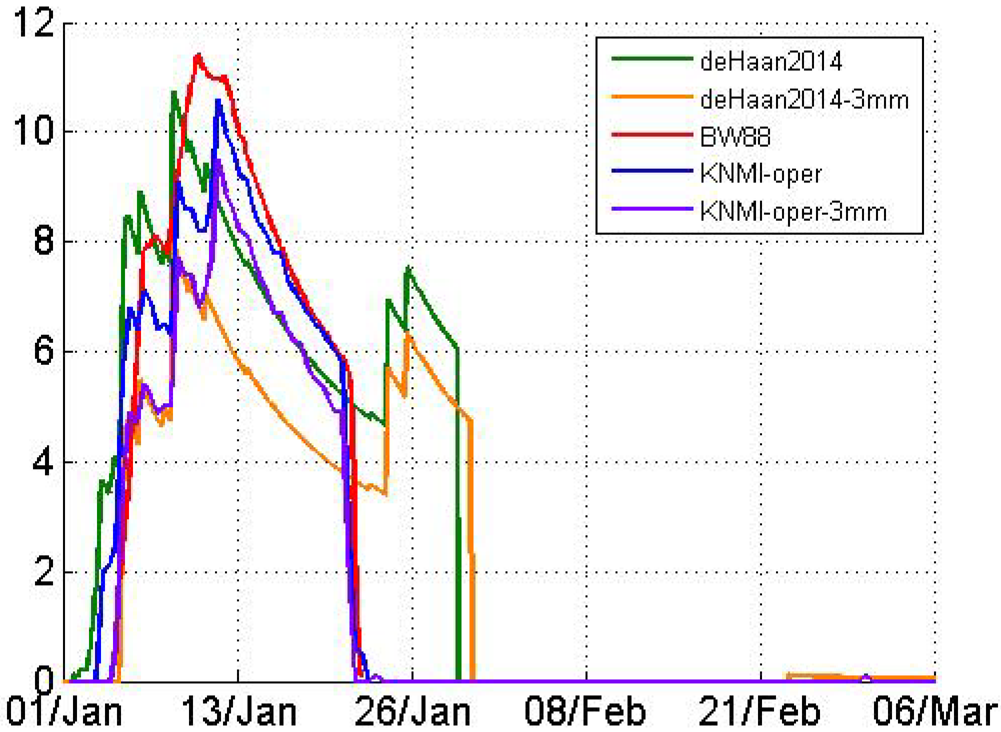

Figure 8: Snow height on the ice in centimetres for the winter of 1985. Green: the deHaan2014 model without ice removal, orange: the deHaan2014 model with ice removal of 3mm and less, red: the results of the BW88 model, blue the KNMI-oper model without ice removal and purple: the KNMI-oper model with ice removal of 3mm and less.

The model has also been tested for more recent years in the Netherlands. These data cover the winters of 2010-2011, 2011-2012 (in appendix) and 2012-2013 (in appendix). The ice and snow of these datasets are mostly measured in the province of Friesland. The city of IJlst has been selected because of the frequently measurements during the three periods done by the royal association of the "Friesche Elf Steden". Because there is no KNMI station in IJlst we compared data from the nearest stations; Leeuwarden, Stavoren and Eelde. Due to the difference in location of these stations compared with IJlst there can be a significant difference in snow height because of the spatial varying precipitation amounts. In the results and discussion section of de Bruin and Wessels (1988), a sensitivity of model input parameters is presented. They tested the sensitivity of the model for the station Leeuwarden and the station de Bilt at 160 km south-south-west from Leeuwarden. They found that differences of ice growth between these stations remained within 3cm. However this is only for two cases were no precipitation and therefore no snowfall is present. It is very important to mention that in a case with snow the ice growth can differ significantly; this will be illustrated with figure 9.
Figure 9 presents ice and snow measurements for the winter of 2010-2011. The measurements of ice are between 0 and 8cm and the snow measurements between 0 and 3cm. The figures show that the deHaan2014 model is closer to the observations than the KNMI-oper model for both 3 stations and both 3 winters. During the research this was present for all forcings with snow. The KNMI-oper model forecasts an ice layer that is 7cm too thick in the first frost period and 12 cm too thick in the second frost period in 2011. In 2012 the KNMI-oper model forecasts an ice thickness of 8cm too thick and in 2013 the maximum thickness is 4cm. It should be noted that in the deHaan2014 model the evaporation and melting of snow are not incorporated where the KNMI-oper model does include this. This is visible in figure 9B where for Leeuwarden in the period between 18 and 28 December the snow of the KNMI-oper model is about 1cm thinner. When snow melts or evaporates the snow layer on top of the ice will decrease. Therefore the ice thickness will increase because the ice can grow faster due to the higher thermal conductivity. The figures 9B, 9D and 9F with snow will explain this

difference, the ice thicknesses could be slightly higher because no snow melting is present for deHaan2014. In Eelde (figure 9F) the snow of KNMI-oper is also less than deHaan2014 because of melting and evaporation. Both models show a higher snow height than the observations, but keep in mind that it is a challenging task to draw snow height from precipitation amounts. Besides that, the precipitation has a high spatial variation and can differ for IJlst. The reason for the difference in ice thickness between the two models is explained in the previous sections. The difference is large because of the different formulation of the thermal conductivity of snow. The dehaan2014 model is very sensitive to this thermal conductivity and further investigation is needed in order to conclude if the KNMI-oper model has a wrong formulation of this thermal conductivity.

The timing of the snow event is of equal importance as the different formulation of thermal conductivity. If the timing is in the beginning of the ice growth phase, the ice will immediately stop growing fast because the thermal conductivity is lowered. This can be seen from figure 9 for the winter of 2010/2011 (figure 9 A, C and E around 2 December). In the winter of 2012/2013(figure 13 appendix) a snow event occurred later in the freezing period, therefore the ice can grow much thicker. This is not linked to model performance but it does explain that the deHaan2014 is very sensitive on the timing of the snow event.
In the winter of 2011/2012 the snow fell early during the ice event and it froze harder, the ice is influenced by the snow during a longer time than in the winter of 2012/2013. This is visible in the three ice growth figures of 2011/2012 from 3 February until the melting phase (figure 12 appendix). During this period the amount of snow does influence the ice growth. For Leeuwarden and Eelde the amount of snow in the model is about 2cm, whereas for Stavoren the amount is 5cm. This extra amount of 3cm snow will reduce the ice growth in Stavoren, compared with Leeuwarden and Eelde, with 2.5cm.
The timing of ice is also an important factor that can have an influence in the ice thickness. When the ice growth starts later but the precipitation of snow starts at the same time, the ice can grow less thick because the ice cannot grow at a fast rate because of this later timing.

Another important factor for the ice growth modelling is the melting of ice. When the ice melts down to 0cm, the snow on top of the ice will also melt and therefore be removed. It is thus very important for a second frost period if the ice is totally melted down or whether there is still some ice with snow on top of it left. In the winter of 2010/2011 an example of this is present for the run forced with weather observations of Stavoren (figure 9D). There the ice in the second frost period can grow very thick because all the snow melted during the breakdown of ice (figure 9C). Because for Leeuwarden and Eelde the ice did not disappear in their observational record during the first melting phase (figure 9 B and F), the snow remained on top of the ice and the ice in the second period could not grow as fast as it did for Stavoren. Because the KNMI-oper model uses a different formulation for the thermal conductivity of snow, this factor is less visible in their model output. For these results we did not incorporate the melting of snow due to higher air temperatures because we want to show what the impact was of melting of snow due to the melting/removal of the ice.

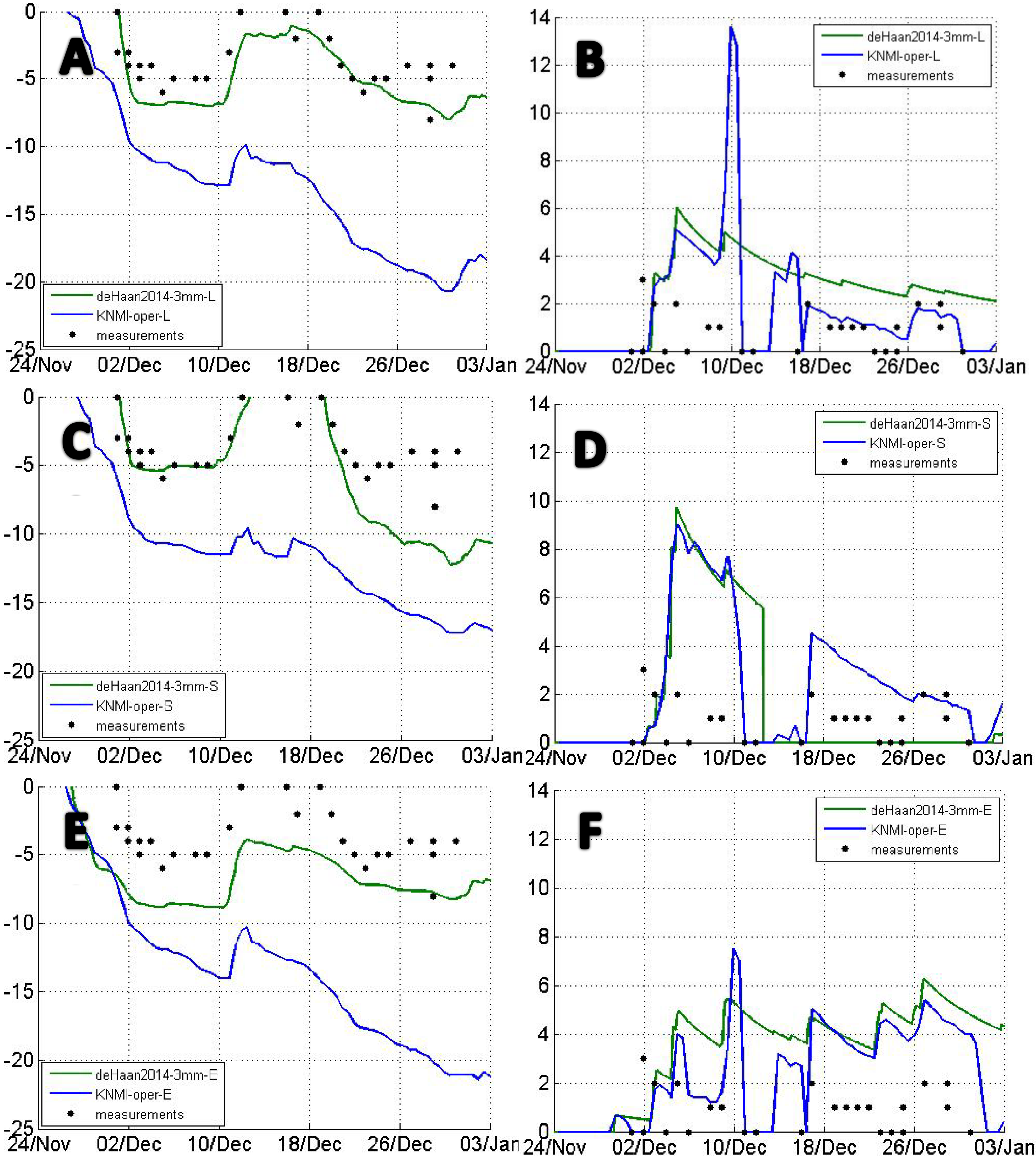

Figure 9: Ice and snow measurements for Leeuwarden, Eelde and Stavoren for the deHaan2014 model (green line), the KNMI-oper model (blue line) and the measurements in IJlst of the royal association of the "Friesche Elf Steden" (black dots). The model output and measurements are given for November and December 2010 and the last part is in January 2011. Both models include a removal of ice less than 3mm thick.

5.3 Spatial variability of ice thickness observations

Besides the variation between the models we also need to take into account the representativity of the measurements. To illustrate this aspect, measurements from Workum for 2010 are plotted in figure 10. It is visible that in the beginning of the ice growth period the differences between the ice thickness measurements are 2-4cm (from December 1 untill December 10).

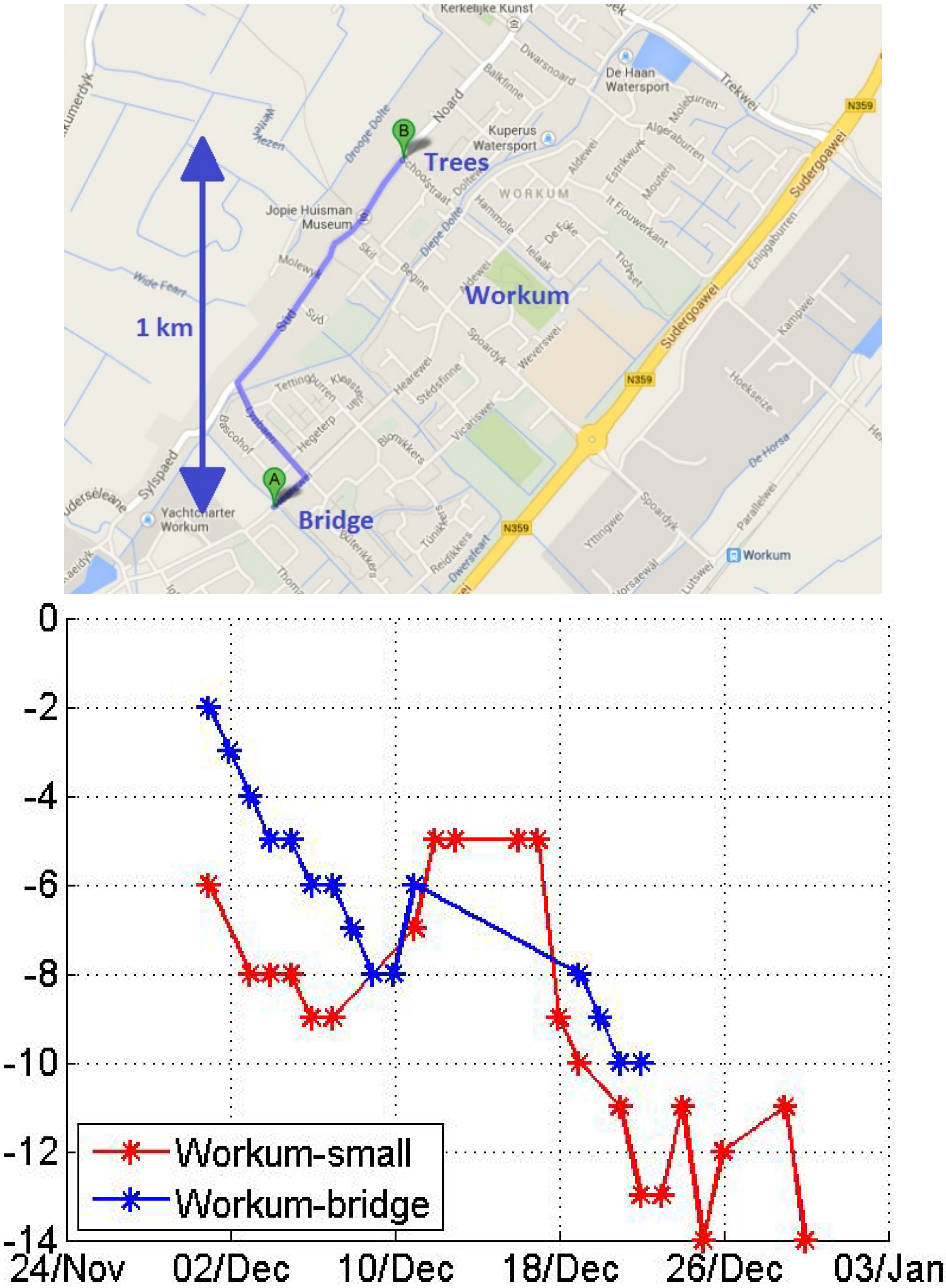

Figure 10: Spatial variation between measurements from Workum in 2010 for a small ditch surrounded by trees (red) and a significant larger ditch surrounded by a bridge (blue). Above the geographical location of these measurement locations are present. The distance between these locations is 1 kilometre. (Google maps, 2014)

It is therefore important to collect next to the ice and snow measurements the metadata of these measurements. This could be surrounding trees, a bridge nearby or a city that transfers warm air to the ice-water system by advection. When these data is collected we can say something about the deviation of the models from the measurements. The spatial variation between the measurements needs to be taken into account when analysing the model results.

5.4 Sensitivity analyses of input parameters

To give an indication which input variables are the most critical for our model and thus the ice thickness, a sensitivity study is done on the most important input variables.
The following variables are used as input for the model: the date (years, months days and hours); the wind speed (m s^{-1}); the air temperature (°C); the amount of precipitation (mm); the pressure (hPa); the total cloud fraction N (octas); the cloud cover of the first and second lower levels (octans) and the occurrence of rain and snow (0 or 1). In figure 11 the default run is the same run as in figure 5. The approach for the sensitivity analyses is based on formula 27:

$$i_n = i + (\max_i - \min_i) * \frac{P}{100} \qquad (27)$$

Where *i* is the variable at a certain time step, max_i is the maximum of the dataset of all *i's* and min_i is the minimum of that dataset, *P* is the increase in percentage of the variable and i_n is the new variable at a certain time step. In this case all variables will be increased by a percentage of their own range. The amount of snow is treated in a different section because the appearance of snow on the ice thickness does have a different impact. Therefore the amount of precipitation is not treated in figure 11. Because the rain and snow occurrence only influence the precipitation these variables are also not used in figure 11. The performed sensitivity analyses do not include a combined test with 2 variables. Only the separate variation of the input variables are analysed.

Table 4: minima and maxima of input variables of the model

	U (ms^{-1})	Ta (°C)	Td (°C)	N (-)	Nprime (-)
Min	0.5	-13.6	-14.5	0	0
Max	13.9	15.5	7.1	8	8

In table 4 the utilized ranges for the frost period in 1986 are presented. The air temperature has the largest range and the cloud cover the smallest. Figure 11 shows that the air temperature is by far the most governing parameter to the model. The air temperature range, maximum minus minimum, is the largest from the tested variables but it is also the variable that is implemented in many formulas of the model and it is the most dominance parameter. It is therefore expected to be the most critical, a 10% increase results in a 2.9°C increase in air temperature which results in a 5 – 15cm decrease in ice thickness. The dew point is only used in the formulation for the wet/ice bulb temperature but an increase of 10% in range results in this case in about 2.2°C. This 2.2°C increase results in a 3cm decrease in ice thickness. A 10% increase in wind speed results in a 1.3 m/s increase in wind speed and 2cm increase in ice thickness in the freezing phase but no increase in the melting phase. The opposite is visible for cloud cover, an increase of about 0.8 octas in cloud cover results in no in- or decrease in ice thickness in the freezing phase. However in the melting phase the ice is influenced by cloud cover, about 1-2cm decrease for Nprime and a 1-2cm increase for the total cloud cover. These values are all expected because an increase in wind speed will cause the ice to cool faster. When increasing total cloud cover, the short wave incoming radiation is decreased but the long wave radiation is increased, probably the decrease by short wave radiation is higher because an increase in total cloud cover results in a larger ice thickness. For the cloud cover of low and middle clouds only the long wave radiation is increased, this results therefore in a decrease in ice thickness. For a further sensitivity study, also the sensitivity of internal variables of the models could be tested. This gives a clear indication which formulas of the model or which parameters are critical. We showed different sensitivities for model input, for removal of ice and a difference in the thermal conductivity of snow. However a clear overview of sensitivities of all model variables could give a better representation of the sensitivity of the model and how the model interacts between different variables.

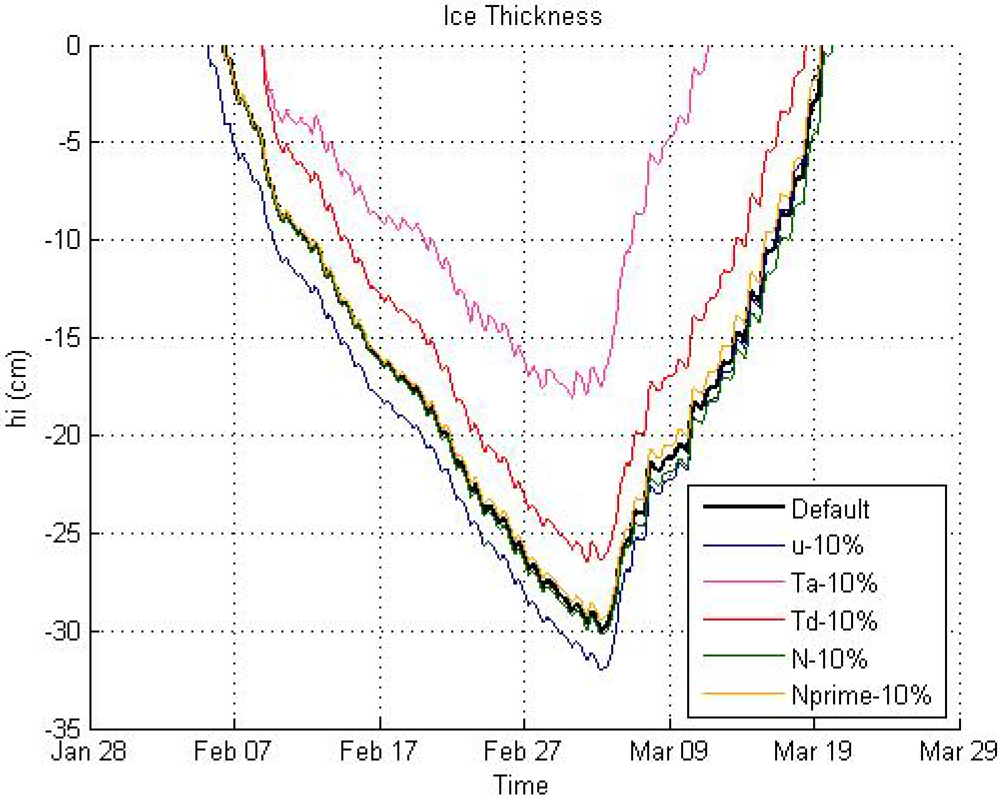

Figure 11: Sensitivity analyses for the winter of 1986 where 10% of the range of the variables is added to the variable. 10m wind (u-10% in blue), air temperature (Ta-10% in pink), dew point temperature (Td-10% in red), total cloud cover (N-10% in green), the cloud cover of low and middle clouds (Nprime-10% in orange) and the default run with no increase (Default in black).

5. Conclusions and recommendations

In this research we formulated the main question: What problems do we encounter when evaluating the results of the three different ice thickness models?

The three different models include:

1. The original created model for the formation and melting of ice on surface waters (BW88) by de Bruin and Wessels (1988).
2. The currently running operational model from the KNMI (KNMI-oper)
3. Our developed code from the results of the original paper (deHaan2014)

To answer the main question we divided this in 2 sub questions:

- Does the deHaan2014 and KNMI-oper model reproduce observations sufficiently well?
- What are the key differences/sensitivities between the results of BW88, KNMI-oper and deHaan2014

Actually the three models are supposed to be the same, because they are all based on the same paper by de Bruin and Wessels (1988). However due to undocumented modifications in time and different interpretations, the models are not the same.

We tried to reproduce BW88 but due to these undocumented changes the BW88 model results could not be reproduced. However, we found differences between the models that give an answer to the second research question. This answer starts with important differences of the removal of ice when the ice thickness is less than 3mm thick and a different interpretation of the thermal conductivity of snow. We tested the results of the 3 models against ice and snow observations of 1985, 1986, 1987, 2010, 2012 and 2013.

The deHaan2014 model and the KNMI-oper model do show reasonable results for cases without snow. Maximum deviations in the ice growing period remain within 2cm for the winter of 1986 and 1987. This indicates that both models do show the same results in a frost period that is free of snow. On the other hand, in the second snow free period of 1985 in the melting phase the two models differ from each other, probably due to different cloudiness conditions.

In case snow is present the above conclusions are different. The KNMI-oper model has a different formulation for the thermal conductivity of snow than the documented thermal conductivity from de Bruin and Wessels (1988). Therefore differences between the models occur in case when snow is present at the ice. When snow is present in the deHaan2014 model the ice growth is much more reduced than in the KNMI-oper model. This is because the thermal conductivity is formulated differently in both models. We suppose that in the KNMI-oper model they calculate the thermal conductivity by multiplying by the density of snow whereas de Bruin and Wessels (1988) and the deHaan2014 model multiply by the density of snow squared. But more research is needed to confirm the results.

A second important difference is the removal of snow and ice when the ice thickness is smaller than 3mm. This can lead to a thickness of 5 cm or more in case a second frost period without snow takes place. Then, the snow on top of the ice will be removed and the ice can grow much thicker. Due to this ice removal, the ice is also decreasing with a couple of centimetres because the ice will start growing later in time. Partly because of this, the timing of ice and/or snow is influenced. The timing of a snow event is important because snow dampens the ice growth. When the timing is a day later the ice can grow at the rate of a conductivity of only ice and therefore can grow much faster. Therefore, during ice growth when snow is present at the ice, the geographical location in latitude and longitude and the appropriate forcing are important. In the future, perfect conditions for a comparison study with observations should therefore be done with both model input and observations at the same location.

The measurements during the winter of 2011/2012 and 2012/2013 are sparse. It is thus a challenging way to draw conclusions based on these measurements. For 2011/2012 the timing of the measurements matches with both models for both snow and ice. The timing of the ice growth start is a bit too late for Stavoren. We conclude therefore that Leeuwarden and Eelde show the most reliable ice growths.

The measurements of 2012/2013 are even sparser and the timing of snow between the measurements and the both models do not correspond. Sometimes the measurements indicate that there is a snow height measured when no ice is present, this is not possible and creates doubts about the measurements.

Improvements for further study can therefore be based on reliable ice and snow thickness measurements. The timing of the events should be clearly documented and conditions of the ditch or lake need to be given. After a clear and extensive study of testing the model against observations the model can be improved by studying the thermal conductivity of snow. Improvements in model performance could be on the following additions: a layer model for the water, local sources like bridges which withhold the energy from going up or salt that is near the ditch, salinity of the water is important because it influences the freezing point of water. a sky view factor to include buildings from cities and trees, a more detailed snow parameter for the aging of snow, advection from water from somewhere else which can transport warm or cold water and a model for the thermal conductivity of the bottom underneath the ditch/pond which transports heat from the bottom of the ditch/pond to melt ice from below. Another recommendation we got from Hazeleger (2014) was more on the inside variables of the model and the optimising of the coefficients in the model with numerical methods. An example of the robustness of such an optimisation and the pros and cons are investigated by Franks et al (1997). Such a method could reduce the errors and optimize the coefficients of the model.

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Appendix

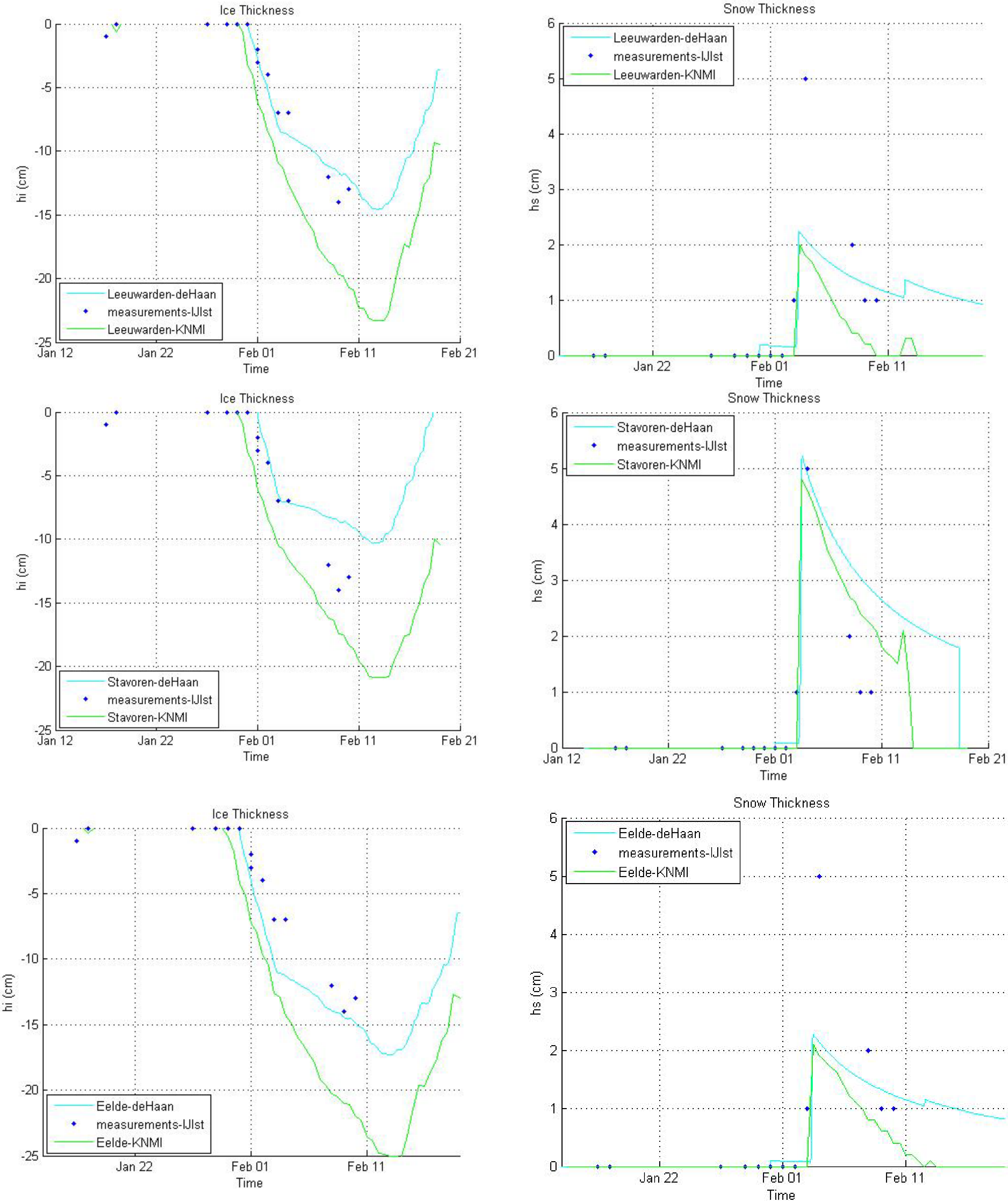
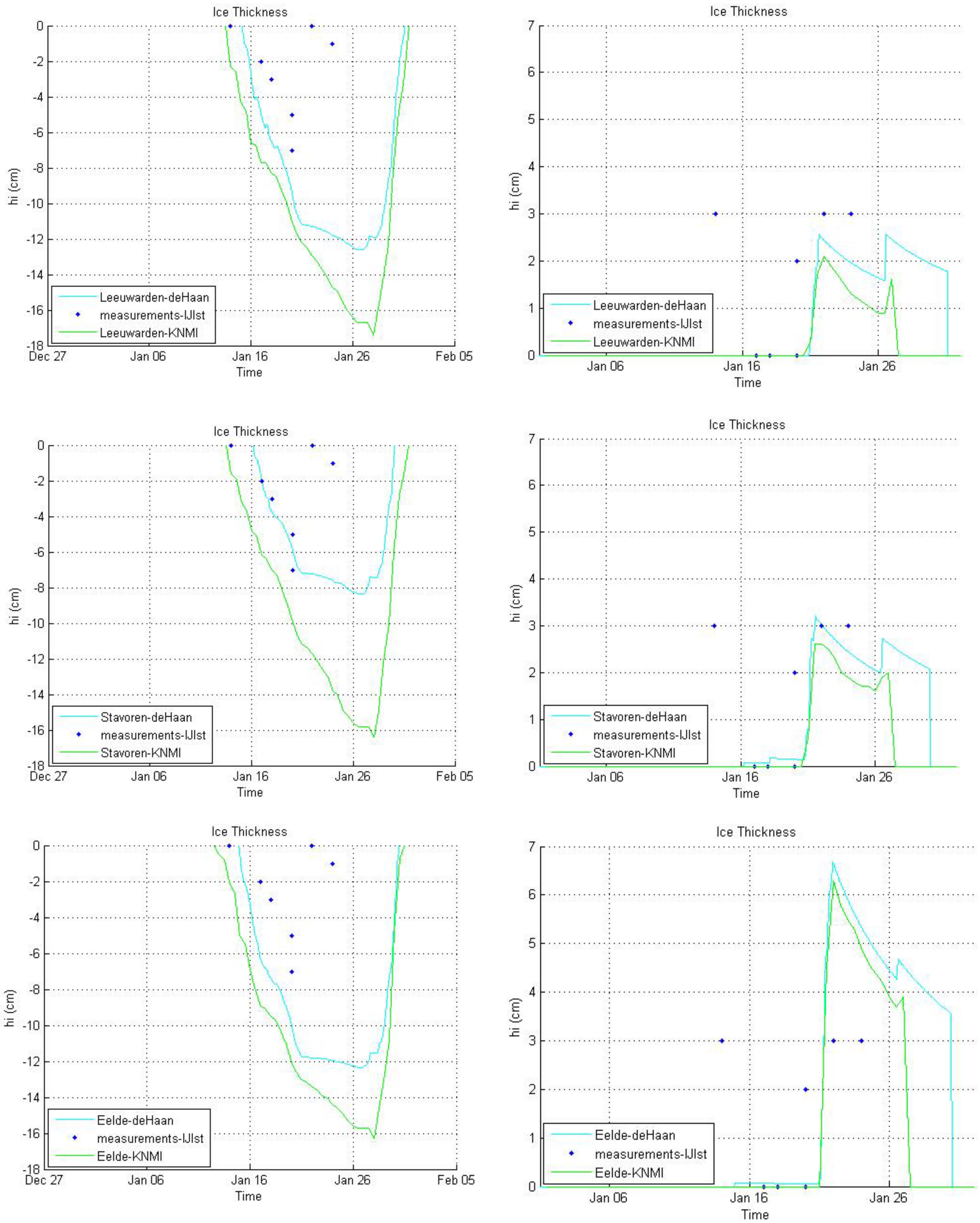


Figure 12: Ice and snow measurements for Leeuwarden, Eelde and Stavoren for the deHaan2014 model (cyan line), the KNMI-oper model (green line) and the measurements in IJlst of the royal association of the "Friesche Elf Steden" (blue dots). The model output and measurements are given for January and February 2012



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