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On the computation of Negotiation Positions and Preferences in a Spatial Coalition Model

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On the computation of Negotiation Positions and Preferences in a Spatial Coalition Model

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Negotiations to form a coalition in politics appear in any parliamentary democracy. Many studies on literature deal with coalition formation games. Starting point of this paper is based on a model on political coalition formation. In this model, two different procedures of coalition formation between political parties are considered. In the first approach, a step-by-step procedure is used and new members are added one-by-one. In the second approach, a simultaneous procedure is applied in which members in a coalition decide and negotiate simultaneously. Furthermore, when the players are political parties, many different decision variables play a role in the game. A government is defined as consisting of a majority coalition and a policy supported by this coalition. Because of the different party positions on different topics, a multidimensional decision space is considered in which each party has an ideal position and the coalition policy is formed. When considering multidimensional space and a large number of parties, computational methods become an important tool to find which stable government(s) is(are) in equilibrium. We analyse and develop computational algorithms for both procedures. Different cases in political games are used to illustrate the methods and data is used to test hypotheses on coalition formation.

Key words:

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1. Introduction

The topic of coalition formation is widely studied in literature. This paper focuses on a model described in de Ridder and Rusinowska (2005) of multidimensional coalition formation in politics. In the model, a government consists of a majority coalition and a policy supported by this coalition. There are n parties trying to form a government. A formed government has a policy agreement represented in a m-multidimensional Euclidean policy space $\mathbb{R}^m$. The complexity increases with the number of parties n and the dimension of the policy spaces m. Given the number of parties n and policy dimension m, computational methods are necessary to compute all possible winning coalitions and preferences of parties over those (if many) coalitions. Furthermore, two ways of forming a government are considered: step-by-step and simultaneously. Each procedure requires a specific algorithm.

In de Ridder et al. (2007) the model is used to show that procedure plays an important role in reaching a coalition agreement and that political parties do not necessarily benefit from being a first-mover. Moreover, that study shows that a decrease in a party's flexibility can be beneficial in coalition negotiations. Hypotheses on power sharing tactics are also investigated. In the current paper, we develop methods to study the two dynamical aspects of coalition formation (procedure and policy flexibility) and report on the findings for testing hypotheses by analysing the formal model and deducing implications from this model based on real-life data. The computational aspects of both the model and the empirical test are discussed.

In Section 1.1, the model is embedded in literature on application of Game Theory in political coalition formation. In Section 1.2, theoretical backgrounds about the multidimensional spatial coalition formation model are outlined. In Section 1.3, the procedures to form a coalition and complexity of both are introduced. In Section 2, the algorithms for two different procedures of coalition formation are described. In Section 3, hypotheses are formulated and checked based on Dutch political data. Finally, conclusions are drawn in Section 4.

1.1. Game Theory in Political Coalition Formation

In multi-party democracies, political parties have to form coalitions to achieve majority governments. As a part of coalition negotiations, coalition members bargain and agree on a package of policy agreements, the coalition agreement (see Timmermans (2003) for an extensive discussion on coalition agreements). In 63% of the coalition formations in Western-Europe studied by Müller and Strøm (2003), such coalition agreements were reached (in e.g. Austria, Ireland, Belgium, and The Netherlands). In order to reach such a coalition agreement, parties in the coalition will have to make compromises as each party has its own ideal policy. Only by adjusting their policy positions, parties can reach the compromise needed for the coalition agreement.

An important subject is the procedure used to reach a coalition. Roughly speaking, two different ways of coalition formation can be discerned: a step-by-step or hierarchical procedure versus a simultaneous or non-hierarchical procedure (Laver and Schofield (1990)). The step-by-step approach sees coalition formation as a process in which the group incrementally forms: new members are added gradually. An alternative approach is to negotiate immediately with *all* the members of the coalition, as in a simultaneous procedure. In spite of these two different procedures which are recognized in the literature and which both occur in real life coalition formation, little attention has been paid to the consequences of these procedures for the result of coalition formation. Some earlier theoretical results show that procedure plays an important role in coalition formation and that, except for some special situation, different procedures lead to different results (de Ridder and Rusinowska (2005)). The special conditions require that the ideal positions of the players are really close, which is unrealistic in a political setting. The model introduced in de Ridder and Rusinowska (2005) is positioned among spatial coalition models (based on Downs (1957), see e.g. Grofman (1982); Laver and Shepsle (1996)) and has been applied to alliance formation between firms.

The field of research of formal coalition models is large and extensive, see, amongst others, Axelrod (1970), Vries (1999), Martin and Stevenson (2001), Grofman (1982), Laver and Shepsle (1996), van Deemen (1989), von Neumann and Morgenstern (1944), and Warwick (1998). So far, most of those studies have focused on why coalition form and, based on that, which parties will cooperate. Arguments for coalition formation were found in power, policy, or institutional arguments. However, the strategy and process of coalition formation have been ignored in the literature (Laver and Schofield (1990): how will coalitions be formed, and, what is the best strategy for a party during the process of coalition formation? Also, from a more formal theoretical point of view, several authors have pointed at this lack of dynamics in the models (Arnold and Schwalbe (2002); Tohmé and Sandholm (1999); van Deemen (1997)). It seems unnatural to analyse coalition formation with a static approach, since coalition formation is clearly dynamic in nature: for example, parties need a few weeks, sometimes months, to reach a coalition agreement, different procedures are used to form a coalition, and parties move their positions to be able to compromise. The suggestion that process plays a role in coalition formation - and should thus be included as an explanatory variable - is strengthened by earlier research (Austen-Smith and Banks (1988); Baron (1993); Bloch (1996); Brams et al. (2005); de Ridder and Rusinowska (2005)). This earlier research has not evolved towards a coherent and empirically verified stream of research, and, moreover, the role of procedure has been ignored.

1.2. The Model

We deal with the following model of spatial coalition formation, considered in de Ridder and Rusinowska (2005). There are n players, here political parties, which try to form a majority coalition S and to decide about a policy of the coalition x_S hereafter called the coalition position. This coalition position is the formal representation of the policy agreement of a coalition. Party $i \in N$, where N denotes the set of all parties, has a weight $w_i > 0$, which is based on the number of seats in parliament party i possesses.

Each party i may choose a policy position x_i from an m-multidimensional Euclidean policy space $\mathbb{R}^m$, $m \geq 1$. A distance between two positions $x_i = (x_{i1}, ..., x_{im})$ and $x_j = (x_{j1}, ..., x_{jm})$ is given by

$$d(x_i, x_j) = \sqrt{\sum_{k=1}^{m} (x_{ik} - x_{jk})^2}. \tag{1}$$

Parties have a certain amount of flexibility on the policy positions, i.e., they have their preferences defined in $\mathbb{R}^m$. Each player $i \in N$ is assumed to have an ideal position $x_i^* \in \mathbb{R}^m$, which is the most preferred position of party i, and a maneuvering space, an equivalent of the policy horizon by Warwick (2000), which consists of all positions acceptable to party i. The model assumes the maneuvering space to be a ball in $\mathbb{R}^m$. M_i denotes the maneuvering space of party i with middle point x_i^* and radius r_i, i.e.,

$$M_i = \{y \in \mathbb{R}^m \mid d(x_i^*, y) \leq r_i\}. \tag{2}$$

The maneuvering space of a party is then the set of policy positions with distances from the ideal position of the party not greater than the radius. Of course, some positions are more preferred to a party than others. Preferences of a party on positions are expressed by the following rule: the closer a position is to the ideal position of a party, the more preferred this position is to the party.

Given coalition $S \subseteq N$ and the ideal positions x_i^* for $i \in S$, all parties of the potential coalition S have to agree on a coalition position for S. We consider two alternative procedures for forming a coalition and choosing a coalition position for that coalition. Although the procedures differ from each other, there are two common assumptions for these procedures. First of all, it is assumed that no party will agree on a position which does not belong to its maneuvering space as these positions are unacceptable for a party. In other words, the necessary condition for a coalition S to be formed is a non-empty intersection of the maneuvering spaces of all members of S (we call this a feasible coalition), i.e.,

$$\bigcap_{i \in S} M_i \neq \emptyset,$$

and of course, the position x_S of the formed coalition S must belong to this intersection as there has to be commonality in positions, i.e.,

$$x_S \in \bigcap_{i \in S} M_i.$$

A similar assumption is adopted in the policy-horizon model: 'With horizons, there are definite limits to the willingness of parties to compromise on policy in order to participate in government; beyond those limits, parties would prefer to remain in opposition' (Warwick, 2000, 39).

An illustration of the model in a three-party, two dimensional example is given in Figure 1. Based on the preferences rule, the valuation (loss) of a party i when a winning coalition S is formed, denoted by Π_i^S, is defined as follows:

$$\Pi_i^S(x_S) = d(x_i^*, x_S) \tag{3}$$

In Section 1.3, both procedures are outlined.

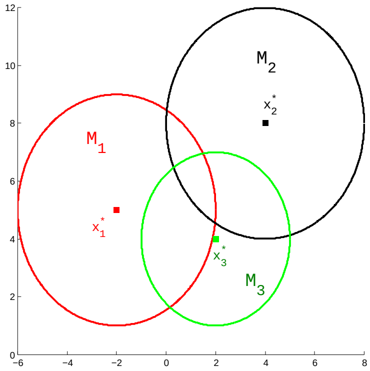

Figure 1 Illustration of the model.

1.3. Coalition formation process

Now, our approach takes a different course from the one adopted by Warwick. To find a solution to the basic coalition formation model, we consider and compare two procedures: a step-by-step procedure and a simultaneous procedure. These two procedures coincide with the distinction in political science literature between hierarchical and non-hierarchical coalition formation (Laver and Schofield (1990)). So far, spatial coalition theories have most often neglected the different procedure of forming a coalition (as in Grofman (1982) who studies one procedure, but see Brams et al. (2005), and Bloch (1996) who do consider the consequences of different procedures). In de Ridder and Rusinowska (2005), it has formally been proven that it matters which procedure is adopted, and also that there is no procedure which is always better.

The first kind of procedure, the hierarchical view, sees ‘ ... coalition building as a process in which actors with similar policy preferences first get together in some sort of provisional alliance and, only after this has been done ..., do they cast around for other coalition partners, adding these until the formation criterion is satisfied’ (Laver and Schofield (1990), p. 140). The proto-coalition model of Grofman (1982) is such a hierarchical model. In the model we present here, the step-by-step procedure is a hierarchical procedure. Although it is difficult to look behind the often closed doors of coalition negotiations, e.g. Ireland, Belgium, and Denmark have known instances of this step-by-step approach (Müller and Strøm (2003)).

In the step-by-step procedure, the first step is that two parties (e.g. party 1 and 2) negotiate. These two will reach an agreement if their maneuvering spaces overlap and hence a first coalition position $x_{\{1,2\}}$ is agreed on. This coalition position is determined by choosing a position in the intersection of their maneuvering spaces and taking the weights of the players into account. That is, a big party can pull the coalition position more towards its ideal. To be more precise, when determining $x_{\{1,2\}}$, first, parties 1 and 2 each choose a position (called the negotiation position) in the intersection of the maneuvering spaces such that the distance of that position to the ideal point of the party is minimal. These negotiation positions are denoted with $\widetilde{x}_1$ and $\widetilde{x}_2$. The coalition position $x_{\{1,2\}}$ is the gravity center (a weighted average) of the negotiation positions.

Now, a third party (3) joins the negotiations. Players 1 and 2 operate as proto-coalition {1,2}, and an agreement with 3 is only reached if the maneuvering spaces of 1, 2, and 3 overlap. If so, coalition $\{1,2,3\}$ with position $x_{\{\{1,2\},3\}}$ is formed, which is the gravity center of the negotiation positions of the proto-coalition $\{1,2\}$ and party 3. This process continues with adding new parties until a majority coalition S with position $x_{\overline{S}}$ has been reached, where $\overline{S}$ denotes an order, a set of parties, that indicates the sequence that leads to coalition S. In de Ridder and Rusinowska (2005), it has been proven that this step-by-step procedure leads to a unique and Pareto efficient solution. Hence, one coalition position is reached such that there is no other position in the intersection of the maneuvering spaces that is more preferred by all members of the coalition. An illustration of the step-by-step procedure of forming a three-party coalition is given in Figure 2.

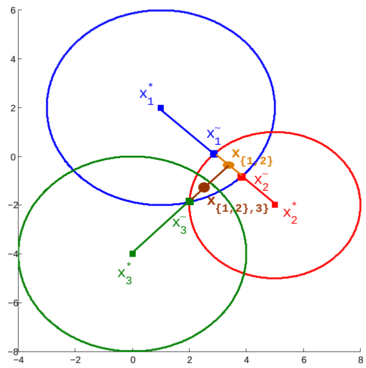

Figure 2 The step-by-step procedure.

Second, we also find a non-hierarchial approach which considers coalition formation as a one-step procedure. Laver and Shepsle (1996) generalize political coalition formation as a process in which one party proposes a particular cabinet, which can be vetoed by all its members. In such a case, there are no proto-coalitions which form intermediate steps before a definitive coalition is reached. Non-hierarchical coalition formation is a process in which all the parties of a coalition sit round the table to negotiate simultaneously. In the overview of coalition formation in Western-Europe, Müller and Strøm (2003) report many instances of such a way of bargaining.

In our model, the simultaneous procedure looks as follows. If parties $1, 2$, and 3 form coalition $\{1, 2, 3\}$, their coalition position is $x_{\{1,2,3\}}$. A coalition forms if maneuvering spaces of all three parties overlap. The coalition position will be in the intersection of their three maneuvering spaces and will depend on the weights of the players. The position $x_{\{1,2,3\}}$ is the gravity center of the negotiation positions of all parties in question. More general, the simultaneous procedure of forming a majority coalition S results in a position x_S of the coalition. Again, it has also been proven that this procedure leads to a unique and Pareto optimal solution (de Ridder and Rusinowska (2005)). An illustration of the simultaneous procedure of forming a three-party coalition is given in Figure 3.

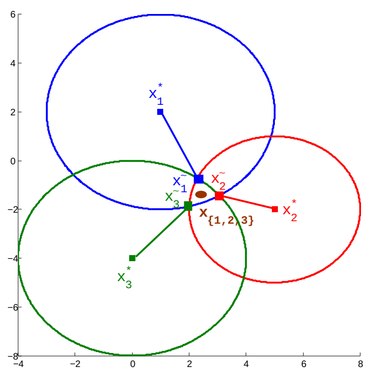

Figure 3 The simultaneous procedure.

Beware that although both the step-by-step procedure and the simultaneous procedure can study a coalition with for instance parties $1,2$, and 3, their respective outcomes are usually different [1]. According to the step-by-step procedure, coalition $\{1,2,3\}$ can form in three different ways: first a bilateral agreement with two parties and then the third party $1,2$ or 3 respectively joins. The simultaneous procedure predicts just one way of forming the coalition: all negotiate together. Hence, in spite of a cooperation between the same three parties, four different paths to form a coalition and four different coalition positions are discerned: $x_{\{\{1,2\},3\}}$, $x_{\{\{1,3\},2\}}$, $x_{\{\{2,3\},1\}}$, and $x_{\{1,2,3\}}$.

Calculations have shown that the number of different paths and coalition positions can increase dramatically. In a coalition game with ten parties, $2^{10}-11=1013$ different 10-party coalitions are possible. However, when taking different procedures into account, 4932045 different step-by-step coalitions can be discerned plus 1013 simultaneously formed coalitions. In sum, if ten parties play a coalition game, there are 4933058 different ways of forming a coalition. Figure 4 shows the different paths and coalitions to analyse in a case with only 3 parties.

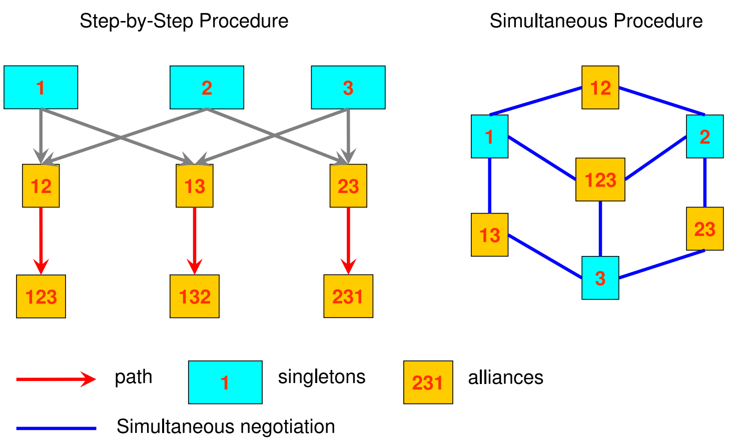

Figure 4 Number of paths and coalitions.

In this way there are $\frac{n!}{2}$ possible paths each represented by a permutation of n parties. The number of different paths of forming a coalition S is $\frac{|S|!}{2}$. The number L of coalitions in a step-by-step procedure can be calculated:

$$\begin{aligned} L &= C_n^2 + C_n^2 * (n-2) + \ldots + C_n^2 * (n-2) * \ldots * (n-(n-2)) * (n-(n-1)) \\ &= \frac{n!}{2} \times \sum_{k=2}^{n} \frac{1}{(n-k)!} \end{aligned} \tag{4}$$

Compared to the step-by-step procedure, the simultaneous procedure has only one path to be followed. It models the situation where $|S|$ parties are sitting together to come to an agreement. Besides the grand coalition, there are many possible partial-coalitions. The more parties, the more possible coalitions can be formed. The number K of possible coalitions is given by:

[1] When the ideal positions of two parties starting the coalition formation process belong to the intersection of the maneuvering spaces of the three parties, the step-by-step procedure with the given parties' order of forming a coalition, and the simultaneous procedure lead to the same position for the coalition.

$$K = C_n^2 + C_n^3 + \ldots + C_n^n = \sum_{k=2}^{n} C_n^k = 2^n - (n+1) \qquad (5)$$

Table 1 shows how complexity increases with the number of players (parties) in both procedures.

Table 1 Number of coalitions following different procedures.

	step-by-step		simultaneous
Number of parties	Possible paths	Number of Coalitions	Number of Coalitions
2	1	1	1
3	3	6	4
4	12	30	11
5	60	160	26
6	360	975	57
7	2520	6846	120
8	20160	54796	247
9	181440	493200	502
10	1814400	4932045	1013

Disregarding some special conditions, the two procedures usually lead to different positions for the coalition and consequently different appreciations by the coalition members. Given the distance between the ideal position of a party and the coalition position, parties will have a preference ranking over the different positions of the coalitions, over the different coalitions, and hence over the procedures to reach them. The closer a coalition agreement is to the ideal position of a party, the more this party will prefer this coalition agreement. In this way, we show that parties should not only form preferences over coalitions, but should also take the procedure into consideration. In conclusion, the procedure of coalition formation should be a strategic resource in coalition formation and should play a role in coalition negotiations similar to the composition of the coalition.

2. Algorithms for the different procedures

The coalition compromise differs for each different path in the step-by-step procedure. According to the all-coalition-path configuration, one can calculate the agreement points of all coalitions and corresponding valuations by following the procedure described in the next section. An index l is used to distinguish coalition. A coalition S in a step-by-step formation is an ordered subset of N. A coalition S in a simultaneous formation is a subset of N. Table 2 summarises the notation used.

Table 2 Notation.

N	Set of parties
i, j	Index of parties
L	Total number of coalitions
l	coalition index
x_i^*	Ideal position for party i
S_l	A coalition, $\|S_l\| \geq 2, S_l \subseteq N$
x_i^S	Negotiation position of party i when coalition S is formed
x_S	Compromise coalition position of S
$x_S^{S\cup\{i\}}$	Negotiation position of a coalition S when party i joins

2.1. Forming a Coalition Step-by-Step

Consider two parties, i and j, forming a coalition $S = \{i, j\}$. By proposition 3.1 in de Ridder and Rusinowska (2005), the negotiation positions for the two parties are calculated as follows. The negotiation position for party i is: if $r_j < d(x_i, x_j)$,

$$x_i^S = x_j^* + r_j \times \frac{x_i^* - x_j^*}{d(x_i^*, x_j^*)} \tag{6}$$

otherwise,

$$x_i^S = x_i^* \tag{7}$$

where the negotiation position of j is given by switching i and j in (6) and (7). Once the parties have defined their negotiation positions, the compromise position is calculated by

$$x_S = \frac{w_i \times x_i^S + w_j \times x_j^S}{w_i + w_j} \tag{8}$$

Let S be a coalition with p members, $p \geq 1$. The compromise position of the coalition is x_S. If party i joins the coalition, both, the coalition S and the party i have to choose new negotiation positions: $X_S^{S\cup\{i\}}$ and $x_i^{S\cup\{i\}}$ respectively. Next step is to agree on a compromise coalition position $X^{S\cup\{i\}}$. To choose the new negotiation positions, the problem to solve is:

$$x_i^{S\cup\{i\}} = \arg \min_{z \in \bigcap_{j \in S\cup\{i\}} M_j} d(x_i^*, z) \tag{9}$$

$$x_S^{S\cup\{i\}} = \arg \min_{z \in \bigcap_{j \in S\cup\{i\}} M_j} d(X^S, z) \tag{10}$$

The compromise position for the new coalition $S \cup \{i\}$ is calculated as follows:

$$x_{S\cup\{i\}} = \frac{w_i \times x_i^{S\cup\{i\}} + X_S^{S\cup\{i\}} \times \sum_{j\in S} w_j}{\sum_{j\in S} w_j + w_i} \tag{11}$$

Based on the model by de Ridder and Rusinowska (2005) we introduce a procedure to determine the compromise (agreement) points and valuations of all coalitions at each possible path. In this procedure, (see Algorithm 1) first the negotiation positions and compromise points for all the possible two-party coalitions are computed. Procedure 2 is used to compute the negotiation positions. For each two-party coalition S, the procedure builds up the coalition adding one-by-one new members. If the maneuvering spaces of the new member i and the members of S overlap, the negotiation positions (for the new member and the coalition) are computed (Procedure 3). If the new coalition $S \cup \{i\}$ is a winning coalition, then valuations for each member are calculated. For the computation of (9) and (10), Procedure 3 uses an external non-linear programming algorithm, *fmincon*. Additionally, a penalty approach is used to check whether or not an intersection (feasible area) exists between the maneuvering spaces of the negotiating parties. Given potential coalition S, we minimise over x the penalty function

$$F(x) = \max_{j\in S}(d(x_i^*, x) - r_i) \tag{12}$$

If the result is negative the intersection is nonempty.

Algorithm 1 Step-by-Step algorithm.

Funct Step-by-Step(Ideal positions of parties, X; Radius for each party, R; weights, voting power, W; quota q; number of parties, n and dimension, m)

$l := 0$
$L := n!/2 * \sum_{k=2}^{n} 1/(n-k)!$ ▷ *number of possible coalitions*
for each two-party coalition ▷ *Compute new positions and negotiation points for all the possible two-party coalitions*
  if $M_i \cap M_j \neq \emptyset$
    $l := l + 1$
    $S_l := \{i, j\}$
    $NS_l := N \backslash S_l$
    $[x_i^{\{i,j\}}, x_j^{\{i,j\}}] :=$ **Neg-Pos2**(x_i^*, x_j^*)
    $X_l^G = \frac{w_i \times x_i^{\{i,j\}} + w_j \times x_j^{\{i,j\}}}{w_i + w_j}$
    if $\sum_{i \in S_l} w_i > q$
      for $i \in S_l$
        $\Pi_{li} = d(x_i^*, X_l^G)$
for $k = 1$ to $L - n$
  while $j \in NS_k$ AND $\bigcap_{i \in S_l} M_i \cap M_j \neq \emptyset$
    $l := l + 1$
    $S_l := S_k \cup \{j\}$
    $NS_l := NS_k \backslash \{j\}$
    $[x_j^{S_l}, X_k^G] :=$ **Negotiation**(x_j^*, X_k^G)
    $X_l^G = \frac{w_j \times x_j^{S_l} + \sum_{i \in S_k} w_i \times X_k^G}{\sum_{i \in S_l} w_i}$
    if $\sum_{i \in S_l} w_i > q$
      for $i \in S_l$
        $\Pi_{li} = d(x_i^*, X_l^G)$
OUTPUT: {Coalitions, compromise positions and valuations of winning coalitions}

Algorithm 2 Computes negotiation positions for parties and coalition position.

Funct Neg-Pos2(Ideal positions for each of the two parties: x_i^*, x_j^*)

New position for party i
if $r_j < d(x_i, x_j)$
  $x_i^{\{i,j\}} = x_j^* + r_j \times \frac{x_i^* - x_j^*}{d(x_i^*, x_j^*)}$
else
  $x_i^{\{i,j\}} = x_i^*$
New position for party j
if $r_i < d(x_i, x_j)$
  $x_j^{\{i,j\}} = x_i^* + r_i \times \frac{x_j^* - x_i^*}{d(x_i^*, x_j^*)}$ **else**
  $x_j^{\{i,j\}} = x_j^*$
OUTPUT: {New positions for the parties and coalition position: $x_i^{\{i,j\}}, x_j^{\{i,j\}}$}

Algorithm 3 Computes new negotiation positions for coalition and new member (party).

Funct Negotiation(Ideal position for the new member i, x_i^*, of the new coalition $S \cup \{i\}$ and compromise position of coalition S, X^S)

objfun=objective function; objcon=objective constraints;x_0*=starting point*
if $M_i \cap \bigcap_{j \in S} M_j \neq \emptyset$
  $x_i^{S \cup \{i\}} =$ **Fmincon**$(objfun, objcon, x_0, x_i^*)$
  $X_S^{S \cup \{i\}} =$ **Fmincon**$(objfun, objcon, x_0, X^S)$
OUTPUT: {New negotiation positions for party i and coalition S}

2.2. Forming a Coalition Simultaneously

Let $S \subseteq N$ be a coalition and $M_i(x_i, r_i)$ for $i \in S$ be maneuvering spaces in $\mathbb{R}^m$ such that $\cap_{i \in S} M_i \neq \emptyset$.

1. Each party $i \in S$ chooses the negotiation position x_i^S:

$$x_i^S = \arg \min_{z \in \bigcap_{j \in S} M_j} d(x_i^*, z) \tag{13}$$

2. Coalition position x_S is chosen as gravity center of positions x_i^S with weights w_i:

$$x_S = \frac{\sum_{j \in S} w_j x_j^S}{\sum_{j \in S} w_j} \tag{14}$$

Algorithm 4 is the main procedure when coalitions are formed simultaneously. It computes the winning coalitions, its coalition positions and valuation-preferences for the parties. First, it generates all possible coalitions based on $0-1$ notation: 0 means the party is not a member; 1 means the party is a member of the coalition. If the coalition is winning and the maneuvering spaces of the members overlap, the algorithm calls a second procedure (Algorithm 5) to compute the negotiation positions of the members. Algorithm 5 uses an external non-linear programming algorithm to calculate the positions. Back in the main algorithm, the coalition position and valuations are computed in order to generate a preference order.

Algorithm 4 Simultaneous algorithm.

Funct Simult(Ideal positions of parties, x^*; Radius for each party, r; weights, voting power, w; quota q; number of parties, n and dimension, m)
1. *number of possible coalitions:* $L := 2^n - (n+1)$
2. **for** $k = 1$ to L ▷ *Compute new positions and negotiation points for all the feasible coalitions*
3. *Generate coalition* S_k
4. **if** $\sum_{i \in S_k} w_k > q$ and $\bigcap_{i \in S_k} M_i \neq \emptyset$
5. **for** $i \in S_k$
6. $x_i^{S_k} :=$ **Neg-Sim**(x^*)
7. $X_k^G = \frac{\sum_{i \in S_k} w_i \times x_i^{S_k}}{\sum_{i \in S_k} w_i}$
8. $\Pi_{ki} = d(x_i^*, X_k^G)$
9. OUTPUT: {Coalitions, new party and negotiation positions for the simultaneous procedure}

Algorithm 5 Procedure to compute new negotiation positions for members in coalition S.

Funct Neg-Sim(Ideal position for the party i, x_i^*)
1. *objfun=objective function; objcon=objective constraints;x0=starting point*
2. $x_i^S =$ **Fmincon**$(objfun, objcon, x0)$
3. OUTPUT: {New negotiation positions for party i}

2.3. Numerical Illustration

We provide an example here to illustrate how the algorithms and model work. This example uses the Dutch election result of 2003 (Klingemann et al. (2006)). As input for the model, we need ideal policy positions of Dutch parties, and a weight and a radius for each political party. The ideal policy positions are derived from a data set with policy positions of Dutch political parties on 56 dimensions from 1998 and 2003 (Klingemann et al. (2006)). Because the model is working with spherical maneuvering spaces based on distance calculations, the data is all scaled between 0 and 10. The weight of the parties is determined by the amount of seats each party had in parliament (total of 150 seats). The radii that model the flexibility of the parties is relatively arbitrary for illustrative purposes and leave a degree of freedom for our analysis. In reality, each party has its own radius which is dependent on the specific situation and which might be subject to change. In this case we have used similar radii for all parties. The names of the parties are the following:

CDA - Christian Democrats (Christen Democratisch Appel)
CU - Christian Union (Christen Unie)
D66 - Democrats 66 (Democraten '66)
GRL - Green Left (Groen Links)
LPF - List Pim Fortuyn (Lijst Pim Fortuyn)
PvdA - Labor Party (Partij van de Arbeid)
SP - Socialist Party (Socialistische Partij)
VVD - People's Party for Freedom and Democracy (Volkspartij voor Vrijheid en Democratie)

Note that the SGP (Political Reformed Party) is not included in this table, as it was not included in the dataset from Klingemann et al. (2006) (in Klingemann et al. (2006), Appendix IV, is explained that the election program for the collection of data was missing).

Table 3 Data for 2003.

	Parties							
	CDA	CU	D66	GRL	LPF	PvdA	SP	VVD
Radius	30	30	30	30	30	30	30	30
Weight	44	3	6	8	8	42	9	28

As output of the model, we only consider coalition positions of majority coalitions of parties that have an overlap of their maneuvering spaces given their ideal policy positions, i.e. of feasible winning coalitions. As said earlier, the biggest party gets the initiative for coalition formation in The Netherlands. In 2003, this was the CDA. The majority coalitions with overlapping maneuvering spaces containing CDA are included in Table 4. For each coalition reached with a certain procedure, the distance between the coalition position and the ideal position of the party are calculated. The {PvdA, CDA} coalition leads to the same coalition position with both procedures as no third party joins here. However, for a coalition between CDA, PvdA, and LPF (e.g. {{CDA, PvdA}, LPF} and {CDA, PvdA, LPF}) procedure plays a role as different procedures lead to different distances. More generally, we see in all the calculations done for this paper that procedure really makes a difference: different procedures lead to different results.

Table 4 Distances from ideal points for 2003 example.

Step-by-Step Procedure									
Coalition	Seats	Distance							
		CU	D66	GRL	PvdA	SP	VVD	LPF	CDA
$\{CDA, PvdA\}$	86	-	-	-	20.52	-	-	-	20.07
$\{\{CDA, PvdA\}, SP\}$	95	-	-	-	24.92	29.39	-	-	26.92
{{CDA,PvdA},LPF}	94	-	-	-	25.45	-	-	29.24	23.69
$\{\{CDA, SP\}, PvdA\}$	95	-	-	-	26.04	29.42	-	-	25.97
{{CDA,LPF},PvdA}	94	-	-	-	25.38	-	-	29.15	23.86
Simultaneous Procedure									
		CU	D66	GRL	PvdA	SP	VVD	LPF	CDA
$\{CDA, PvdA\}$	86	-	-	-	20.52	-	-	-	20.07
$\{CDA, PvdA, SP\}$	95	-	-	-	26.01	29.00	-	-	26.47
{CDA,PvdA,LPF}	94	-	-	-	25.31	-	-	28.62	24.59

Based on these distances, the preferences of the players can be calculated. The closer the coalition position to the ideal position of a party, the more the party will prefer this coalition and the procedure. Table 5 reports this. As an example, CDA's most favorite option is to cooperate with PvdA. If CDA would cooperate with PvdA and SP, then the best procedure for CDA would be to negotiate first with SP alone. The step-by-step procedure with SP joining as last is CDA's least preferred procedure for this coalition. Note that we do not consider preferences of the parties not participating in the coalition.

Table 5 Preference order for 2003.

Step-by-Step Procedure									
Coalition	Seats	Preference order							
		CU	D66	GRL	PvdA	SP	VVD	LPF	CDA
$\{CDA, PvdA\}$	86	-	-	-	1	-	-	-	**1**
$\{\{CDA, PvdA\}, SP\}$	95	-	-	-	2	2	-	-	**7**
$\{\{CDA, PvdA\}, LPF\}$	94	-	-	-	5	-	-	3	2
$\{\{CDA, SP\}, PvdA\}$	95	-	-	-	7	3	-	-	5
$\{\{CDA, LPF\}, PvdA\}$	94	-	-	-	4	-	-	2	3
Simultaneous Procedure									
		CU	D66	GRL	PvdA	SP	VVD	LPF	CDA
$\{CDA, PvdA\}$	86	-	-	-	1	-	-	-	1
$\{CDA, PvdA, SP\}$	95	-	-	-	6	1	-	-	6
$\{CDA, PvdA, LPF\}$	94	-	-	-	3	-	-	1	4

In reality, the coalition that formed was {CDA, VVD, D66}. Although it is not the aim of this paper to predict which coalitions have occurred, we can explain why this coalition did not appear in the results. According to the model and, in particular, the adopted input, this coalition would not be viable. That means that the adopted radii did not lead to an overlap of the parties' maneuvering spaces; the {CDA, VVD, D66} coalition is less acceptable than the coalitions that appear in the table.

3. Hypothesis Testing

The described model and introduced computational method was used in de Ridder et al. (2007) to do an extensive study to test hypotheses derived from intuition with the aid of Dutch data. The rest of this paper reports on the findings. First we formulate the hypotheses and the Dutch situation as a platform of analysis. After that we point wise discuss the results that can be found in de Ridder et al. (2007).

3.1. Procedure: Hypothesis on First Mover

Empirical observations of how coalitions form show that procedures are in some countries standard and formalized in laws (e.g. Belgium, Finland, Luxembourg, and The Netherlands, Müller and Strøm (2003)). That diminishes the opportunity for parties to use procedure as a strategic means during the coalition process. An important observation is that many multi-party democracies have the (unwritten) law that the party that came out of the elections as the largest gets the initiative (from a head of state) for forming a coalition. Examples of countries in which this (more or less frequently) happens are The Netherlands, Sweden, Finland, Austria, Belgium, and Luxembourg (Müller and Strøm (2003)). The idea behind this is that these initiative taking parties are supposed to lead the negotiations and to have an advantage in the bargaining situation. The earlier a party is involved in coalition negotiations, the more this party is able to pull the negotiations towards its own ideas. In this way, this party can determine and influence the negotiations more and can get advantage out of it. This brings us to the first hypothesis: *Being a first-mover in coalition negotiations is advantageous.*

3.2. Flexibility Hypothesis

The second innovation of our model, is the flexibility during negotiations we attribute parties via maneuvering spaces. In the literature of coalition formation models, it has most often been assumed that political parties have a fixed position in policy space (Grofman (1982); Vries (1999)). However, more scholars begin to acknowledge the importance of studying the dynamics of party competition (Laver (2005); Timmermans (2003), van der Brug (1999)): '...positions are not frozen or fixed; parties move in the policy space in different directions over time' (Timmermans (2003), p. 9). Here, we concentrate on dynamics of policy positions not with vote maximizing as goal (as e.g. Budge (1994); Enelow and Hinich (1984); Laver (2005)), but dynamics due to coalition formation.

The idea is that in order to form a coalition, political parties will move their policy position, but only *to a certain limit* (Warwick (2000)) as formalized by the maneuvering space. Coalition formation implies making a coalition agreement: a compromise between the members of a coalition on the ideological course of the coalition, consisting of a position for the coalition. As a consequence, parties participating in a coalition need to adjust their position in order to reach such an agreement (also see Martin and Vanberg (2004)). It is not likely that parties will cooperate with a party which has opposing policy ideals. We therefore assume parties will only be willing to compromise if they can stay within their maneuvering space of acceptable positions.

The question now rises what is mostly in a party's interest: a big or small maneuvering space? When forming a two-party coalition, the answer is straightforward: being less flexible is never disadvantageous. If a coalition consists of only two parties, the more flexible party of the two will be forced to move its position more than the other. One can speak of a zero-sum situation: what one wins, is lost by the other.

Nonetheless, when forming a k-party coalition, for $k \geq 3$, the answer is less easy. Intuitively, one would consider that staying closer to a party's ideal position is also better in multi-party coalitions. Hence, a decrease in flexibility would always be in a party's advantage. However, this is less easy to analyse due to the amount of players involved. Therefore, we use the data and theoretical results to study whether the following (second) hypothesis holds: *Being less flexible in coalition negotiations is more advantageous.*

3.3. Sharing Power: Hypothesis on Minimal Winning

As a final point, we study the role of sharing power. Coalition formation has long been considered as a combination of achieving power, and simultaneously sharing this power with coalition partners. Coalition formation is therefore a delicate balance between on the one hand getting this power by compromising into the coalition, and on the other hand, forming a coalition which gives a party relatively the best power. In this tradition, the minimal winning (von Neumann and Morgenstern (1944)) and minimum size theory (Riker (1962)) have been formulated. Minimal winning coalitions are coalitions that contain enough members to be winning, but are not oversized. Minimal winning coalitions cannot miss any member without becoming losing. Minimum size coalitions contain enough weight to be winning, but not more than that.

In line with this, one could reason that oversized coalitions imply sharing power with more partners and hence compromising with more partners than necessary. The chance is bigger that a coalition position will be reached which is farther from a party's ideal position. Less members in a coalition make it easier to reach an agreement which is closer to a party's ideal point. Hence hypothesis 3a: *Being in a smaller (winning) coalition is more advantageous than being in an oversized coalition.*

In a similar way, we can argue that forming a coalition with a stronger partner is not advantageous, since the stronger party may 'pull' the position of a formed coalition more towards it's own ideal position. Hence we propose hypothesis 3b: *Increase of a party's weight is disadvantageous for its coalition partners.*

3.4. Dutch Situation and Data

In The Netherlands, coalition governments are the standard, considering that the Dutch multi-party democracy only has had coalition governments since 1945 (Müller and Strøm (2003)). Also, The Netherlands has a tradition of majority coalitions. Furthermore, two of the issues we highlight - procedures and flexibility - are important. Concerning procedures, the process of coalition formation is by far the longest in Western Europe with an average of 70.6 days. This could denote an important role for procedures. The first mover issue is relevant as it is characteristic for the Dutch coalition practice that the biggest party gets the initiative to form a coalition. Concerning flexibility, coalition agreements play an important role in coalition negotiations: each cabinet agrees on such a document as the course of action during their period of government. Data however show different ideal policy positions of Dutch parties (e.g. Vries (1999); van der Brug (1999)) which implies compromises and hence flexibility of parties. As explained in Section 2, the radii that model the flexibility of the parties is relatively arbitrary for illustrative purposes and leave a degree of freedom for our analysis. Due to the lack of empirical data on this aspect, we have taken two different ways to determine the radius: a radius similar for each party and a radius different for each party, randomly generated. In the case (as in the case of Section 2) in which we have used similar radii for all parties, the radii have been determined by optimizing the case such that enough, but not too many, instances were found which could help us investigate the hypotheses.

To run the model with real-life multidimensional data, one needs computational algorithms. We have performed calculations with the model using data from Dutch politics, and, moreover, we present some theoretical results. Both the empirical and theoretical calculations provide some counter-intuitive situations which show that certain expectations do not always hold. Also, we illustrate that certain traditions in real-life coalition formation are not necessarily advantageous.

During the paper, we study which strategic moves are advantageous for a potential coalition member. *Advantageous* is defined in terms of preference of a party over a coalition and the path to reach this coalition. This is measured by taking the distance from the ideal position of the party to the position of the coalition compromise. The closer the coalition position, the better. The policy-distance effect on government composition, meaning that the incentive of a party to join a parliamentary coalition government decreases with the distance between the policy position and the position of the government, was elaborated and tested in particular by Warwick (1998).

3.5. Results on Procedure

Two different procedures of coalition formation, leading to different coalition positions, are under study. The research question is whether being a first-mover is always advantageous for a party in coalition negotiations, as in real-life the biggest party, after elections, is most often rewarded with the initiative for coalition negotiations.

In the 2003 case presented in Section 2, we indeed saw that for the LPF being the first mover was advantageous. When comparing the LPF's preference on the two step-by-step procedures it is involved in, it prefers {{CDA, LPF}, PvdA} over {{CDA, PvdA}, LPF}. So, it prefers being a first mover over being a late mover. A small counter example can be found due to the PvdA that in the same coalition prefers to step in later. The data of 1998 and Table 3.5 show a stronger counter example, as can be observed from Table 3.5.

Table 6 Weights and radius 45 for 1998 data

	Parties					
	GRL	SP	PvdA	D66	VVD	CDA
Radius	45	45	45	45	45	45
Seats	11	5	45	14	38	29

Table 7 Preference order for 1998 data

Coalition	Seats	GRL	SP	PvdA	D66	VVD	CDA
Step-by-Step Procedure							
		Preference order					
$\{\{PvdA, SP\}, CDA\}$	79	-	8	3	-	-	2
$\{\{PvdA, D66\}, CDA\}$	88	-	-	5	7	-	4
$\{\{PvdA, CDA\}, SP\}$	79	-	4	1	-	-	7
$\{\{PvdA, CDA\}, D66\}$	88	-	-	2	4	-	8
$\{\{\{PvdA, SP\}, D66\}, CDA\}$	93	-	5	10	9	-	6
$\{\{\{PvdA, SP\}, CDA\}, D66\}$	93	-	1	7	8	-	9
$\{\{\{PvdA, D66\}, SP\}, CDA\}$	93	-	9	11	5	-	6
$\{\{\{PvdA, D66\}, CDA\}, SP\}$	93	-	7	8	1	-	10
$\{\{\{PvdA, CDA\}, SP\}, D66\}$	93	-	2	6	8	-	9
$\{\{\{PvdA, CDA\}, D66\}, SP\}$	93	-	7	5	2	-	10
Simultaneous Procedure							
		GRL	SP	PvdA	D66	VVD	CDA
$\{PvdA, SP, CDA\}$	79	-	3	4	-	-	1
$\{PvdA, CDA, D66\}$	88	-	-	9	3	-	3
$\{PvdA, D66, SP, CDA\}$	93	-	6	12	6	-	5

In the 1998 case, PvdA was the biggest party and had to take the initiative in coalition negotiations. For the three party coalition {PvdA, SP, CDA}, two step-by-step and one simultaneous procedures were considered as PvdA always had to be a first mover. In the two step-by-step procedures, CDA would be better off being a late instead of a first mover. Let $\succ_i$ denote the preference relation of party i. For this coalition, the preference order of CDA is as follows: {PvdA, SP, CDA} $\succ_{CDA}$ {{PvdA, SP}, CDA} $\succ_{CDA}$ {{ PvdA, CDA}, SP}. This also holds for SP, which in case of step-by-step formation rather joins as last member in the negotiations.

We can therefore conclude that hypothesis 1 does not hold:

Result 1: *Being a first mover is not always advantageous.*

3.6. Results on Flexibility

As a second major point, focus is on policy flexibility of parties. One of the central assumptions of the model is that parties have maneuvering spaces which reflect their flexibility to deviate from their ideal positions. No party will accept a coalition position which lies outside its maneuvering space. This assumption is similar to the one made in a policy-horizon model by Warwick (2000), 2005a, 2005b). We study the hypothesis *Being less flexible in coalition negotiations is more advantageous.*

A search in the data did not provide a counter example to this hypothesis. It was found that a decrease in a party's flexibility always seems to be in the party's advantage. In other words, the intuition which was provided earlier holds. As seen more easy in two-party coalitions, less flexibility always leads to a more advantageous coalition agreement for a party. Although we did not find a counter-example in the Dutch data, we did come up with a one-dimensional theoretical example which shows that being less flexible can be a *disadvantage*.

Example 1 We consider a three-party example, in which parties 1 and 2 have the same weight, while the weight of party 3 is twice as big as the weight of party 1 and 2, i.e.

$$N = \{1, 2, 3\}, \quad w_1 = w_2, \quad w_3 = 2w_2$$

The situation is illustrated in Figure 5. Since this is a one-dimensional example, the ideal positions x_1^*, x_2^* and x_3^* are points (denoted in Figure 5 by squares) on a line, while the maneuvering spaces M_1, M_2 and M_3 are intervals (denoted in Figure 5 by two-headed arrows). We have

$$x_1^* = 0, \quad x_2^* = 4, \quad x_3^* = -2$$

All parties are assumed to be equally flexible and their radii are equal to

$$r_1 = r_2 = r_3 = 6$$

Hence, the maneuvering spaces are

$$M_1 = [-6,6], \quad M_2 = [-2,10], \quad M_3 = [-8,4]$$

and their intersections (also two-headed arrows)

$$M_1 \cap M_3 = [-6,4], \quad M_1 \cap M_2 = [-2,6]$$

$$M_2 \cap M_3 = M_1 \cap M_2 \cap M_3 = [-2,4] \neq \emptyset.$$

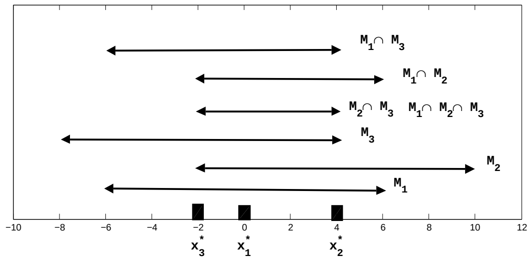

Figure 5 Counter-example "being less flexible can be a *disadvantage*". Ideal points (squares) and maneuvering spaces (two-headed arrows)

Since $M_1 \cap M_2 \cap M_3 \neq \emptyset$, the necessary condition for coalition $\{1,2,3\}$ to be formed is satisfied. Let us consider the step-by-step procedure of forming coalition $\{1,2,3\}$, in which first parties 1 and 2 form a coalition $\{1,2\}$, and then party 3 joins. The steps of the procedure are explained in Section 2. The negotiation positions $x_1^{\{1,2\}}$ and $x_2^{\{1,2\}}$ of parties 1 and 2 are equal to their ideal positions, because the ideal points lie in the intersection of the maneuvering spaces, i.e.

$$x_1^{\{1,2\}} = 0 = x_1^*, \quad x_2^{\{1,2\}} = 4 = x_2^*$$

Since the weights of parties 1 and 2 are the same and the coalition position is the gravity center of the negotiation positions, we get

$$x_{\{1,2\}} = 2 \in M_3$$

Next, party 3 joins proto-coalition $\{1,2\}$. Because x_3^* and $x_{\{1,2\}}$ lie in the intersection of the maneuvering spaces, the negotiation positions of party 3 and proto-coalition $\{1,2\}$ are equal to $x_3^* = -2$ and $x_{\{1,2\}} = 2$, respectively. Since the weight of party 3 is equal to the weight of $\{1,2\}$, we get

$$x_{\{\{1,2\},3\}} = 0 = x_1^*$$

Hence, the step-by-step procedure of forming $\{\{1,2\},3\}$, in which first parties 1 and 2 form a coalition, and then party 3 joins, leads to the coalition position $x_{\{\{1,2\},3\}}$ which is the best possible position for party 1.

Next, let us assume that party 1 becomes less flexible, that is, its new radius decreases to $r_1' = 3$. All remaining components of the example are unchanged. Then,

$$M_1' = [-3,3], \quad M_1' \cap M_2 = M_1' \cap M_2 \cap M_3 = [-2,3]$$

We consider the same step-by-step procedure of forming $\{1,2,3\}$ with the new radius $r_1' = 3$. The new negotiation position $y_1^{\{1,2\}}$ of party 1 is the same as before (equals $x_1^{\{1,2\}}$), since its ideal point lies in the intersection of the maneuvering spaces. However, the new negotiation position $y_2^{\{1,2\}}$ of party 2 is different, i.e.

$$y_1^{\{1,2\}} = x_1^* = 0, \quad y_2^{\{1,2\}} = 3.$$

The new position $y_{\{1,2\}}$, as the gravity center of $y_1^{\{1,2\}}$ and $y_2^{\{1,2\}}$ with equal weights $w_1 = w_2$, is now

$$y_{\{1,2\}} = \frac{3}{2} \in M_3$$

The new coalition position $y_{\{\{1,2\},3\}}$, as the gravity center of the negotiation positions $y_{\{1,2\}}$ and $x_3^* = -2$, with equal weights for $\{1,2\}$ and party 3, is now

$$y_{\{\{1,2\},3\}} = -\frac{1}{4}$$

Hence, the step-by-step procedure of forming $\{\{1,2\},3\}$, in which first parties 1 and 2 form a coalition, and then party 3 joins, results now in the coalition position $y_{\{\{1,2\},3\}}$ which is worse for party 1 than the coalition position $x_{\{\{1,2\},3\}}$, for the case where party 1 is more flexible, i.e.

$$x_{\{\{1,2\},3\}} \succ_1 y_{\{\{1,2\},3\}}$$

This means that becoming less flexible made party 1 worse off.

To conclude, although the data have shown that less flexibility always seems to be advantageous to a party, a theoretical counter example has illustrated how a decrease in flexibility can be a disadvantage for a party. Hence:
Result 2: *When forming a k-party coalition, for $k \geq 3$, being less flexible is usually advantageous, but can theoretically be a disadvantage.*

3.7. Results on Sharing Power

Additionally, we study a minor point: the role of sharing power. The question here is whether striving for a coalition in which a party gets the best relative power position is always advantageous. Earlier empirical results confirm the role of power-sharing motives of parties (Martin and Stevenson (2001)), but do not show that oversized can be an advantage for coalition members (cf. Volden and Carrubba (2004) who explain when oversized coalitions occur). Sub-issues here are the minimal winning argument (von Neumann and Morgenstern (1944)) and the influence of weight. The minimal winning argument states that only coalitions will form that have enough members to be winning, but not more than that. But is a minimal winning coalition necessarily advantageous for a party? Or, more general, is a smaller coalition necessarily more advantageous than an oversized coalition?

We have found many counter-examples in Dutch data which show that the hypothesis does not always hold. We consider Dutch data after the 1998 elections (see Table 3.5). Here, we change the radii for the parties and let the radius be different for different parties. We get an instance as shown in Table 3.7. Table 9 shows the preference order for this case. Note that under the step by step as well as simultaneous procedure, PvdA finds the non-minimal winning coalition formed by PvdA, VVD and D66 more attractive than the minimal winning coalition {PvdA, VVD}.

Concluding, we get the following result.

Result 3a: *Forming a minimal winning coalition is not always advantageous.*

Concerning weight, we like to consider the consequence the weight of a party (number of seats in parliament) has for its coalitional partners. The last research question is then: Does an increase of a party's weight imply a disadvantage for its coalition partners?

Table 8 Weights and different radii for 1998

	Parties					
	GRL	SP	PvdA	D66	VVD	CDA
Radius	45	55	25	65	85	45
Seats	11	5	45	14	38	29

Table 9 Preference order with different radii for 1998

Step-by-Step Procedure							
Coalition	Number of seats	Preference order					
		GRL	SP	PvdA	D66	VVD	CDA
$\{PvdA, VVD\}$	83	-	-	**3**	-	1	-
$\{\{PvdA, SP\}, VVD\}$	88	-	4	12	-	5	-
$\{\{PvdA, D66\}, VVD\}$	97	-	10	**1**	11	6	-
$\{\{PvdA, VVD\}, SP\}$	88	-	8	7	-	4	-
$\{\{PvdA, VVD\}, D66\}$	97	-	11	**2**	10	2	-
$\{\{\{PvdA, SP\}, D66\}, VVD\}$	102	-	1	8	1	12	-
$\{\{\{PvdA, SP\}, VVD\}, D66\}$	102	-	2	11	2	10	-
$\{\{\{PvdA, D66\}, SP\}, VVD\}$	102	-	6	5	5	12	-
$\{\{\{PvdA, D66\}, VVD\}, SP\}$	102	-	9	4	7	11	-
$\{\{\{PvdA, VVD\}, SP\}, D66\}$	102	-	5	7	4	9	-
$\{\{\{PvdA, VVD\}, D66\}, SP\}$	102	-	9	6	6	7	-
Simultaneous Procedure							
		GRL	SP	PvdA	D66	VVD	CDA
$\{PvdA, VVD\}$	83	-	-	**3**	-	1	-
$\{PvdA, SP, VVD\}$	88	-	7	10	-	3	-
$\{PvdA, D66, VVD\}$	97	-	11	**2**	8	2	-
$\{PvdA, SP, D66, VVD\}$	102	-	3	9	3	8	-

In a similar way, we can argue that forming a coalition with a stronger partner is not advantageous, since the stronger party may 'pull' the position of a formed coalition more towards it's own ideal position. Hence we propose hypothesis 3b: *Increase of a party's weight is disadvantageous for its coalition partners.*

One can show that forming a two-party coalition with a stronger party is never advantageous to the coalition partner. The intuition is that in such a 'zero-sum' situation, the larger party will always be able to pull the coalition position to its own position, further away from its partner. Nevertheless, it does not necessarily hold when forming a larger coalition. We can illustrate this with the following theoretical example.

Example 2 We consider the same situation as in Example 1 with party 1 being less flexible, i.e.,

$$N = \{1, 2, 3\}, \quad x_1^* = 0, \quad x_2^* = 4, \quad x_3^* = -2$$

$$r_1' = 3, \quad r_2 = r_3 = 6, \quad w_1 = w_2, \quad w_3 = 2w_2$$

$$M_1' = [-3, 3], \quad M_2 = [-2, 10], \quad M_3 = [-8, 4]$$

$$M_1' \cap M_2 = M_1' \cap M_2 \cap M_3 = [-2, 3]$$

As calculated in Example 1, the coalition position $y_{\{\{1,2\},3\}}$ results from the step-by-step procedure of forming $\{\{1,2\},3\}$, in which first parties 1 and 2 form a coalition, and then party 3 joins, is equal to $y_{\{\{1,2\},3\}} = -\frac{1}{4}$. Next, let us assume that the weight of party 1 increases: it is twice as big as the weight of party 2 and the same as the weight of party 3, i.e.,

$$w_1' = 2w_2 = w_3$$

The remaining components of the model remain unchanged. We consider the same step-by-step procedure of forming $\{\{1,2\},3\}$. The new negotiation positions $z_1^{\{1,2\}}$, $z_2^{\{1,2\}}$, and coalition positions $z_{\{1,2\}}$, $z_{\{\{1,2\},3\}}$ are now the following:

$$z_1^{\{1,2\}} = x_1^* = 0, \quad z_2^{\{1,2\}} = 3, \quad z_{\{1,2\}} = 1 \in M_3$$

$$z_{\{\{1,2\},3\}} = -\frac{1}{5}$$

Comparing the distance between coalition position $y_{\{\{1,2\},3\}}$ and the ideal point x_2^* of party 2 and the distance between the new coalition position $z_{\{\{1,2\},3\}}$ and x_2^*, one can conclude that

$$z_{\{\{1,2\},3\}} \succ_2 y_{\{\{1,2\},3\}}$$

It means that an increase of the weight of party 1 makes party 2 better off.

This gives the following result.

Result 3b: *When forming a k-party coalition, for $k \geq 3$, an increase of a party's weight may be an advantage for its coalition partner.*

In order to show a pure effect of an increase of a party's weight in Example 2, somewhat artificially we have increased the weight of party 1, keeping all remaining elements unchanged. This is of course not what happens in a parliament, since elections (usually) preceding coalition formation fix the weights of the parties. However, it can be used by parties defining a coalition formation strategy before elections. For example, in its campaign a party may be less negative with respect to another party whose bigger size might be beneficial. Nevertheless, although we believe that this result is mainly of a theoretical nature, we have also constructed an instance using the data. Consider the case of Table 3.7 that presents the 1998 data with varying flexibility for the parties taking the real number of seats. The distance of the ideal of D66 to the compromise of coalition $\{\{\{PvdA, SP\}, D66\}, VVD\}$ is 52.25. Let us now hypothetically assume that SP increases its weight by 30, while the other parties keep their original weights. Now the distance of the ideal of D66 to the coalition position becomes 51.53. This means its position improves due to an increase of another party.

4. Conclusions

In spite of the many unwritten laws and traditions during coalition formation in countries as Italy, Luxembourg, The Netherlands, Belgium, and Ireland, political parties should be aware of the important role of the process of coalition formation. In this paper, we have shown how several aspects of this coalition process play an important role for the result of the coalition negotiations. We use a formal model of coalition formation which considers political parties as players with ideal policy positions and maneuvering spaces denoting their flexibility to deviate from their ideal points. The output of the model is a set of feasible coalitions, which have a majority and whose members' maneuvering spaces overlap. The model describes which coalition position will be reached by the members given the procedure adopted. The complexity of the model increases with the number of players (parties) and policy dimension.

To generate coalitions from political data, algorithms have been presented. We have introduced computational algorithms for the different procedures. The algorithms compute all winning coalitions and preferences of parties over those coalitions. Furthermore, the algorithms are used to test different hypotheses.

The analysis in de Ridder et al. (2007) focused on three aspects of coalition formation and formulated hypotheses: procedure, flexibility, and power sharing. The following questions which political parties may (and should) take into account when forming a coalition were under study: Does procedure of coalition negotiations matter? Is it more advantageous to be a first-mover in the coalition process? Is it better to be more or less flexible in coalition formation? Should we invite more parties to join to a (minimal) winning coalition or is it better to stay with the existing one(s)? Is it better to form a coalition with a stronger party or rather with a smaller one? Applying the algorithms to Dutch data and using theoretical results, we have arrived at several (counter-)examples. These counter-examples have shown the importance of the process and give important implications for political parties involved in coalition formation. Also, these results have implications for future coalition research.

From the output of the applied methods the following can be observed. First, procedure matters. When forming a coalition, political parties should be aware of the important role procedure plays in determining the result of the coalition. The calculations have shown that procedure partly determines which coalition point is agreed on. However, earlier research has analyzed that there is not one procedure which is always best (de Ridder and Rusinowska (2005)).

Related to procedure, being a first mover is not necessarily advantageous. This result is also surprising in the sense that in many countries (e.g. The Netherlands, Belgium, Luxembourg, and Austria) the tradition is that the largest party can start the negotiations and determines who will negotiate first. Being involved early in the process is considered an advantage. However, from the model it appeared that this is not always the case. The rationale here is that, by studying coalition compromises the other coalition partners will reach without a party (assuming complete information), this party can estimate whether this compromise is close to its ideal position. If it is, it may pay to join later. If the compromise is not close, it may be better for the party to join earlier in the process.

With respect to flexibility, being less flexible is not necessarily advantageous. In the data, we have found that being less flexible results in a (pre-)coalition compromise which is closer to a party's position. So, being less flexible pays off. Nevertheless, we have presented a theoretical three-party counter-example in which being less flexible is a disadvantage. In this example, the first mover's ideal position was somewhere between the ideal position of the remaining two parties. Although being less flexible gave a better pre-coalition outcome, the final coalition position was worse for the party than the coalition position with the party being more flexible.

Related to power sharing theories (as minimal winning theory), computations show that forming a minimal winning coalition is not necessarily advantageous. Moreover, forming a coalition with a stronger party is not necessarily disadvantageous. So, it might pay off to share power with more and stronger parties than predicted by power sharing theory. To explain this counter-intuitive finding, for the minimal winning case it holds that new parties may determine a final coalition outcome closer to a party's ideal position, although this depends on the ideal positions of the new parties. For the stronger partner case, a stronger party joining usually moves the pre-coalition compromise further from a party's own ideal position. However, a strong party may determine a final coalition position which is closer to a party's position. In that case, a strong partner may be beneficial to cooperate with.

Game theoretic models like the coalition formation model allows analysis only for few player situations. The developed computational methods allow empirical testing of hypotheses using huge data sets with many players. We have provided theoretical examples and empirical cases which confirm the thesis that the coalition process matters. We aim to reach the agenda of coalition research with this message. Due to the focus on making and illustrating this message, we have neglected other aspects of the research. We suggest for future research to investigate how to empirically determine a party's flexibility, development of more dynamic coalition models, and empirical analyses of more countries.

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