Price setting and vertical coordination in food chains
A game theoretic approach

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The paper compares the outcomes of three alternative price mechanisms in vertically related chains: non-cooperative price setting, franchising and bargaining. The alternatives are compared in a game-theoretic framework encompassing consumer and producer behaviour. The paper repeats the familiar result that franchising lowers consumer prices and increases chain output compared to non-cooperative price setting. However, chain profits are only increased if the intersector elasticity of substitution is sufficiently high, i.e. if final demand is sufficiently price elastic. Both non-cooperative price setting and franchising are well documented in the literature. Bargaining, however, is not, although it occurs frequently at intermediate stages of production, especially in food chains. For this reason, bargaining is analysed as well. The price solution under bargaining turns out to be more general: the solutions under non-cooperative price setting and franchising are special cases of the bargaining solution.

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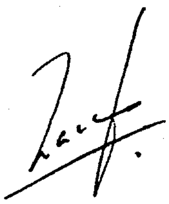
Preface

Vertical coordination and price institutions have a profound impact on the prices at intermediate and final levels in product chains. These prices determine both the level and the division of chain profits. This paper analyses the impact of three price institutions on chain prices, output and profits. The game theoretic analysis provides a thorough basis for theoretical evaluations of price mechanisms. The results also constitute a building stone for empirical estimates of behavioural relations and for price determination in empirical microeconomically founded simulation models.

Frank Bunte and Frank van Tongeren performed the analysis on which this paper is based. They are grateful to many others for comments and discussion.

The managing director,



Prof. Dr. L.C. Zachariasse

Summary

As vertical coordination becomes more important, it is worthwhile to get more insight into the economic motivations for vertical coordination, the types of coordination pursued and the implications of coordination for economic performance. This paper provides some insight by a model relating the economic behaviour and performance of actors from all links. The behavioural postulates employed in the model are derived using microeconomic and game theory. Examples of issues addressed by this model are: what types of coordination are pursued; what types of coordination are favourable for economic (chain) performance; what implications do changes in the environment, such as policy changes, have for coordination and performance.

The theoretical framework yields several empirical implications that can be submitted to econometric tests. The following conclusions are derived analytically:

1. prices set either upstream (primary production) or downstream (retail) fall with the elasticities of substitution and the number of varieties available;
2. prices set upstream rise with downstream factor costs. A rise in downstream costs causes a fall in the price elasticity of derived demand;
3. prices set either upstream or downstream rise with the number of varieties per firm. This result is due to the positive externality a price rise has on the demand of the other varieties sold by the multiproduct firm;
4. in the symmetric Stackelberg equilibrium with upstream firms as first movers, downstream seller concentration does not influence upstream pricing: Upstream prices depend on the number of varieties per firm upstream, but not on the number of varieties per firm downstream;
5. the paper shows that vertical coordination between firms leads to a fall in the consumer price and a rise in output. Aggregate profits also rise if the intersector elasticity of substitution is sufficiently high;
6. the wholesale price set in the bargaining model depends on the bargaining power of the upstream firm relative to the bargaining power of the downstream firm. When all bargaining power is upstream, the solution equals the Stackelberg solution. When all bargaining power is downstream, the wholesale price equals marginal cost, i.e. the franchise solution.

The relations laid down in these conclusions may be used in applied research in the following two ways. First, the relations derived allow one to make theoretical predictions on the relation between prices, output and profits on one hand and demand and supply characteristics and price institutions on the other hand. Second, these theoretical relations constitute a profound basis for empirical applications. In particular, the theoretical analysis yields precise hypotheses which can be subjected to econometric testing.

Conclusion 2 implies a positive relation between downstream (primary production) costs and upstream (retail) prices. This information may be used to indicate the implications of changes in downstream factor costs for both upstream and downstream prices and chain

output and profits. Empirical tests based on these relations may be used to obtain estimations of the quantitative impact of changes in downstream factor prices. Conclusion 4 states that - under certain conditions - downstream seller concentration does not influence upstream pricing. This conclusion has strong implications for the relation between seller concentration and chain prices, output and profits, and hence bears implications for competition policy. Again, this conclusion may be tested empirically. Conclusion 5 indicates under what conditions franchising increases chain profits. Whether these conditions are fulfilled for specific agro-food chains may be investigated using econometric tests. In addition, we obtain rather clear criteria for the assessment of contract design among vertically related partners in agro-food chains.

The paper may thus be used to give theoretical predictions and as a basis for econometric tests. The price equations derived will be estimated in future research in order to construct a microsimulation model relating chain prices, output and profits on the one hand to demand and supply characteristics and coordination and price mechanisms on the other hand.

1. Introduction

Vertical coordination is of increasing importance in food chains, both in the Netherlands and abroad. In the US, for instance, vertical coordination rose from 45% to 65% in fresh vegetables and from 75% to 95% in processed vegetables (Hennessy, 1996). In the Netherlands, contract production accounted for more than 80% of total production for vegetables like string beans, green peas, spinach and carrots (Van Scheppingen, 1985). The contracts between the food processing industry and cultivators are very detailed: They contain stipulations on the seed-date, the amount of seed, seed-race, the harvest-date, cutting, threshing, loading, transport, *et cetera*. Recently, retail trade also becomes more involved in vertical contracting, both with the food processing industry and cultivators. This trend is attended with marketing concepts such as Efficient Consumer Response and Category Management and logistic systems such as Just-In-Time Delivery and Electronic Data Interchange.

Since vertical coordination becomes more important, it is worthwhile to get more insight in the economic motivations for vertical coordination, the types of coordination pursued and the implications of coordination for economic performance. This paper provides some insight by a model relating the economic behaviour and performance of actors from all links. The behavioural postulates employed in the model are derived using microeconomic theory. Examples of issues addressed by this model are: what types of coordination are pursued; what types of coordination are favourable for economic (chain) performance; what implications do changes in the environment, such as policy changes, have for coordination and performance. Individual firms are taken as starting point for the analysis. However, since our ultimate interest refers to the product chain level, the aggregation in links and the coordination between links receive thorough attention.

The paper delivers a conceptual framework and a game theoretic analysis of various price setting regimes. The specification of the model is chosen in such a way that it can be implemented empirically in a numerical simulation model.

The paper is structured as follows. Chapter 2 addresses the double marginalisation problem briefly. The double marginalisation argument is one of the key theoretical insights of the Industrial Organisation literature on vertical coordination. Since double marginalisation refers to price behaviour, this insight will be elaborated in this chapter. In chapter 3 consumer demand and producer factor demand and costs will be derived. Chapter 2 and 3 provide the tools for analysing price behaviour. This will be done in chapter 4. Section 4.2 analyses price decisions under non-cooperation; section 4.3 those under cooperation (franchising). In these two sections, we distinguish single product firms only. In section 4.4 we complicate the analysis somewhat by analysing the price decisions of multiproduct firms. In section 4.2-4.4, upstream firms set wholesale prices just like downstream firms set consumer prices. In section 4.5, however, wholesale prices are determined as the outcome of a bargaining process between upstream and downstream firms.

2. Theoretical background

2.1 Introduction

New Industrial Organisation theory (Tirole 1988) considers externalities between vertically related firms as the prime ground for vertical coordination. Externalities cause a divergence between joint profits under vertical integration and under non-cooperation. Joint profits may be raised by internalising the externalities through vertical coordination. Four major externalities are identified:

1. *double marginalisation*
   A price rise in one link of the chain reduces the output of other links as well (Spengler 1950). Under non-cooperation the resulting loss of gross profits suffered by the other links is not taken into account in the pricing decisions. As a result, the price rise is too large from the chain's point of view;
2. *input substitution*
   When an upstream firm charges a monopoly price, his[1] product will be substituted for another product downstream. As a result, his output is contracted. The upstream firm may prevent input substitution and profit erosion by vertical coordination;
3. *moral hazard*
   Promotional activities do not only raise output in one link, but also in other links. These activities are again too low from the chain's perspective, when the positive externality is not taken into account. Moral hazard comes in when e.g. manufacturers can not monitor retailer promotion efforts and thus can not prescribe the level of activities;
4. *free riding behaviour*
   Free riding occurs when a company's service and quality efforts favour rival firms as well (Telser 1960). Rival demand is boosted by the positive externalities while rival costs are lower. This enables rival firms to set more competitive prices and drive the company performing service and quality efforts out of the market. As a result, the service and quality levels are too low, not only from the link's point of view, but also from the chains perspective.

There are two additional aspects which play a crucial role in concluding vertical coordination contracts: uncertainty and strategic interactions.

5. *uncertainty*
   Under uncertainty, risk aversion and insurance play a decisive role on the vertical arrangement to be concluded. Risk averse companies are willing to accept lower margins if income volatility is reduced;

[1] In this study, the upstream firm is referred to with the pronouns 'he' and 'his'; the downstream firm with the pronouns 'she' and 'her'.

6. *strategic considerations*
Firms may foreclose entry by vertical coordination in order to raise market power. This issue is still highly debated in Industrial Organisation and offers a fruitful area for future research. This area has large antitrust implications.

The model outlined in this study concentrates on the double-marginalisation problem. The double marginalisation problem is closely related to the price decisions in vertical relations and for this reason put at the heart of the analysis of this study. Since input substitution refers to price analysis as well, it will be relatively simple to incorporate input substitution in future research. The moral hazard and free-riding problems related to service and quality efforts are not addressed here. The incorporation of uncertainty, risk aversion and strategic considerations are intricate complications left for future research.

2.2 Double marginalisation

The double marginalisation argument can be best illustrated using a simple two-links-chain model (figure 2.1). In this chain, the product flows from the suppliers of raw materials via the upstream and the downstream firms to the consumers. The upstream firms pay P_v^u per product, the downstream firms P_v^d and the consumers P_v^c.[1] All upstream and downstream firms sell different varieties: Products are heterogeneous. This implies that each firm has a 'local' monopoly: Demand is downward-sloping.

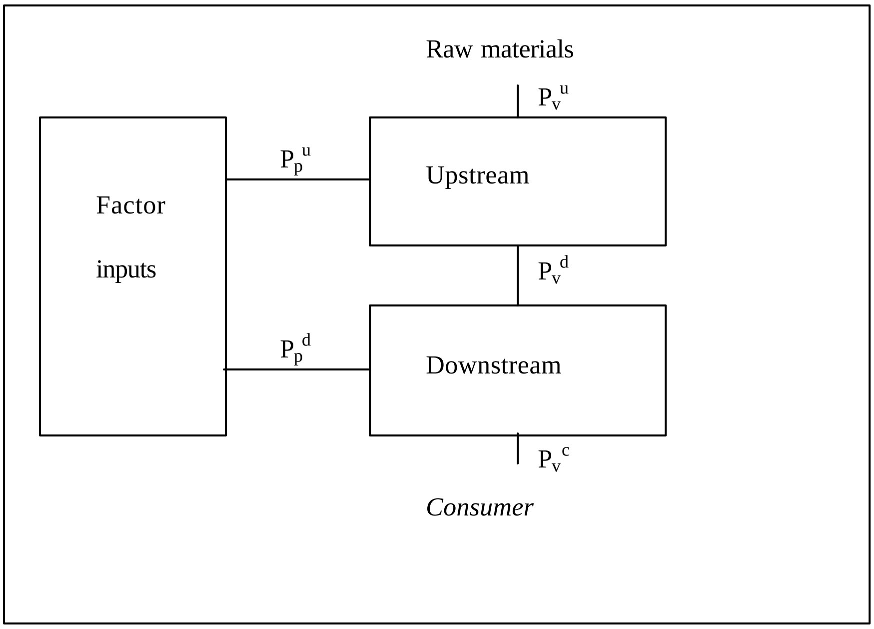


Figure 2.1 The chain

[1] P denotes price, while subscript v refers to a particular variety. Superscripts u, d, and c refer to the paying actor: the upstream firm, the downstream firm and the consumer respectively.

Double marginalisation (Spengler 1950) occurs when both firms charge a price above marginal cost. When the downstream firm raises her price above marginal cost, both her own and the upstream firm's output is reduced. The downstream firm's price rise hurts the upstream firm's sales (and *vice versa*). However, since this effect does not affect the non-cooperative downstream firm herself, the latter does not take this external effect into account. As a result, the downstream firm raises the consumer price rise too much, since the joint cost of the price rise - the margin lost by both the upstream and the downstream firm due to the output reduction - is higher than the downstream firm's own cost - the margin lost by the downstream firm. The downstream firm's gain is the margin increase due to the price rise.

Vertical coordination internalises the upstream firm's cost and leads to a price drop and an increase in output. As a result consumers benefit as well. Vertical coordination is not necessarily as detrimental as horizontal coordination.[1] Note that when one of the two links is competitive, the problem does not arise, since the margin lost on the output reduction vanishes for the competitive link: There is no external effect. Vertical coordination may be obtained through a franchise-fee-system, resale price maintenance, quantity fixing or even vertical integration.

In a franchise-fee-system the upstream firm charges a wholesale price as well as a franchise fee which is fixed, i.e. non-related to the output sold. The downstream firm now maximises joint profits, since its marginal cost, P_v^d, equals the chain's marginal cost, P_v^u. The only problem the upstream firm now faces is the fact that the downstream firm earns all profits. However, the franchise fee enables him to capture (a part of) the downstream firm's profits. The division of profits depends on the firms' bargaining power.

Resale price maintenance also enables the upstream firm to capture monopoly profits. He merely sets the consumer price at the joint optimum level and leaves the downstream firm a margin equal to downstream marginal factor costs. However, resale price maintenance is *per se* illegal in both the US and the EU. The upstream firm may also force the downstream firm to sell the optimum quantity (at the optimum price) through quantity forcing. Shelf arrangements and minimum purchase requirements are equivalent to quantity forcing. Vertical integration is, of course, also a solution to the double marginalisation problem, provided it solves the agency problem internally.

Abiru (1988) shows that the above argument also applies to oligopolistic market structures. The consumer price falls in the so-called pure case of vertical integration.[2] In Abiru, there is Cournot competition in the upstream as well as the downstream market. However, the consumer price may not fall when vertical integration is attended with horizontal concentration. This may occur when the number of upstream firms is not equal to the number of downstream firms, more in particular when the former number is smaller than the latter.

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[1] This result explains the Chicago School's hypothesis that a firm with monopoly power in one link cannot extend this power by vertical integration into competitive links. If monopoly power cannot be enhanced and efficiency may be increased by vertical integration, there is no need for competition policy against vertical arrangments (Scherer and Ross 1990).

[2] In the pure case, the number of upstream oligopolists equals the number of downstream oligopolists.

3. The model

3.1 Introduction

The behavioural postulates employed in this study are derived using microeconomic theory. Consumer demand is derived from a utility function and subjected to a budget constraint. As a result, the system of demand equations derived below satisfies both the preferences laid down in the utility function and the budget restriction in a consistent way. Factor demand and costs are derived from cost minimisation subject to a production constraint. The system of factor demand equations found in this way minimises producer costs and corresponds with the underlying relation between factor inputs and production.

3.2 Consumer demand

The utility function employed in this study has a CES-CES nature and reads as follows

$$U = (\Sigma_{j=1}^{J} X_j^{\alpha} + (\Sigma_{v=1}^{N} X_v^{\tau})^{\alpha/\tau})^{1/\alpha} \qquad (1)$$

where α and τ are parameters reflecting consumer utility ($0 < \alpha < \tau < 1$). Utility depends on the consumption of two types of products: heterogeneous varieties of food X_v and non-food products X_j. Total food consumption is represented by $X_i = (\Sigma\ X_v^{\tau})^{1/\tau}$. X_i equals both the quantity index for food and the subutility function for food. There are N food varieties and J non-food products. The elasticity of substitution between any combination of two varieties σ: $\sigma = 1/(1-\tau)$ is constant and identical which is characteristic of the CES (Constant Elasticity of Substitution) function. Likewise, the elasticity of substitution between any combination of two products ς: $\varsigma = 1/(1-\alpha)$ is constant and identical. Both the subutility function for food and the aggregate utility function exhibit 'love-of-variety': Consumers prefer to divide their consumption over as many food varieties and as many products as possible (at given prices). The elasticity of substitution between the food varieties σ - the intrasector elasticity - is larger than the elasticty of substitution between food and other commodities ς - the intersector elasticity - since by assumption $\alpha < \tau$. The latter assumption is plausible since one may expect two varieties of the same product - Gouda cheese and Brie - to be closer substitutes than two products - flowers and potatoes (or televisions).

Consumer demand is derived by maximising utility with respect to all products subject to the budget constraint. The maximisation problem is the following

$$\max\ L = (\Sigma_{j=1}^{J} X_j^{\alpha} + (\Sigma_{v=1}^{N} X_v^{\tau})^{\alpha/\tau})^{1/\alpha} \quad s.t. \ \Sigma_{j=1}^{J} P_j X_j + \Sigma_{v=1}^{N} P_v^{c} X_v = I \qquad (2)$$

where P_j is the consumer price of product j, P_v^c is the consumer price of variety v and I is the overall budget. Demand for each variety v can be shown to be (Appendix)

$$X_v = \frac{I}{(P_v^c)^{\sigma}\,(P_i^c)^{\varsigma-\sigma}\,P^{1-\varsigma}} \tag{3}$$

where P_i^c is the price index of the food varieties defined by

$$P_i^c = \left(\Sigma_{v=1}^{N}\,(P_v^c)^{1-\sigma}\right)^{1/(1-\sigma)} \tag{4}$$

and P is the overall consumer price index defined by

$$P = \left(\Sigma_{j=1}^{J}\,(P_j^c)^{1-\varsigma} + (P_i^c)^{1-\varsigma}\right)^{1/(1-\varsigma)} \tag{5}$$

3.3 Factor demand and producer costs

Producer behaviour with respect to factor demand depends on the relation between factor inputs and production. This relation is referred to as the production function. Producer behaviour with respect to factor demand is derived from cost minimisation. The production function employed in this study is of the Leontief-CES-type. We identify two types of inputs: intermediate input Q_v and primary factors input Q_p. The latter is decomposed into capital input Q_c, labour input Q_h and land input Q_l. Production is modelled as follows

$$X_v = \min(Q_v, Q_p) \tag{6}$$

$$Q_v = Q_v^* \tag{7}$$

$$Q_p = \left((A_c Q_c)^{-\rho} + (A_h Q_h)^{-\rho} + (A_l Q_l)^{-\rho}\right)^{-1/\rho} \tag{8}$$

where A_c, A_h and A_l and ρ are parameters and Q_v^* represents a certain level of food input. The elasticity of substitution between any combination of two primary production factors (-1/(ρ+1)) is constant which is, again, characteristic of the CES function. The Leontief-nature of the production function reflects the idea that the supply of food requires a minimum of both intermediate inputs and primary factor inputs. Within the set of primary factors some substitution is possible. The latter is reflected in Q_p, the CES part of the production function.

Cost minimisation is a necessary condition for profit maximisation. Factor demand is derived on basis of cost minimisation. Given production level X_v^*, costs are minimised when material inputs equal $Q_v = X_v^*$ and primary factor inputs $Q_p = X_v^*$. The only decision left for the producer is the choice of primary factor inputs minimising costs while meeting the constraint $Q_p = Q_p^* = X_v^*$. This problem may be modelled as follows:

$$\min\ C_p = P_c Q_c + P_h Q_h + P_l Q_l \quad s.t.\ Q_p = Q_p^* \tag{9}$$

where C_p denotes production factor costs, P_c the price of capital, P_h the price of labour (wage) and P_l the price of land (rent). Taking first order derivatives with respect to Q_c, Q_h and Q_l and the production constraint gives the following demand for factor inputs

$$A_j Q_j = X_v \left(\frac{P_p}{P_j / A_j} \right)^{1/(1+r)} \tag{10}$$

where j 0 {c,h,l} and P_p is the price index for the primary production factors defined by

$$P_p = \left(\Sigma_{j=1}^{3} (P_j / A_j)^{r/(1+r)} \right)^{(1+r)/r} \tag{11}$$

Substituting the demand for primary factors into the cost function gives the reduced cost equation

$$C = (P_p + P_v)\, X_v \tag{12}$$

where C denotes total costs. This cost function exhibits constant returns to scale: costs are a linear function of output. This characteristic follows from the choices made with respect to the production function.[1] Constant returns to scale facilitate price analysis enormously (chapter 4). We could, of course, have postulated constant returns to scale straight away, as is quite common in the literature on Industrial Organisation. However, since the cost function is explicitly based on a production structure, one may relate chain and firm performance to factor performance in a later stage.

[1] Increasing and decreasing returns to scale may be modelled by incorporating a constant in the CES function: $Q_p = (A_0 + \Sigma (A_j Q_j)^{-\rho})^{-1/\rho}$.

4. Price decisions

4.1 Introduction

In this chapter, pricing behaviour is analysed for four different regimes. Section 4.2 derives price decisions under non-cooperation; section 4.3 those under cooperation (franchising). In these sections, we distinguish single product firms only. In section 4.4, we complicate the analysis somewhat by analysing the price decisions of multiproduct firms. In the previous three sections, upstream firms set wholesale prices just like downstream firms set consumer prices. In section 4.5, however, wholesale prices are determined as the outcome of a bargaining process between upstream and downstream firms.

In the first three regimes, price setting is possible since firms have some monopoly power due to product heterogeneity. Consider figure 2.1 once again. Upstream firms set P_v^d, the varieties' prices to be paid by downstream firms. The prices of raw materials, P_v^u, and the prices of primary factors, P_p^u, are taken as given by the upstream firms. Downstream demand, however, is not taken as given. Downstream demand depends on consumer demand which in turn is a function of downstream marginal costs a part of which is made up of the varieties' prices set upstream, P_v^d. Downstream firms set P_v^c, the varieties' prices to be paid by consumers. The varieties' prices charged by the upstream firms, P_v^d, and the prices of primary factor inputs, P_p^d, are taken as given. Consumer demand is given for downstream firms. Hence, firms exhibit price taking behaviour on their input markets and price setting behaviour on their output markets. In the fourth regime, however, upstream firms do not set prices, but bargain over wholesale prices with downstream firms.

In all four regimes, the model is analysed as a two-stage game. Wholesale prices are determined before consumer prices are. The model is solved backwardly by first deriving optimal downstream prices as a function of upstream prices. The reason for this order is the following. The downstream firms take all input prices and consumer demand as given. The upstream firms, however, know that they may influence downstream price decisions by their own price decisions. (This also holds for both upstream and downstream firms in the bargaining-regime.) By analysing downstream equilibrium first, this influence is taken into account. In the second step, optimal upstream prices are derived, which are thus based, among other things, on the relation between upstream prices and downstream equilibrium prices. The game-theoretic structure makes the upstream firms Stackelberg leaders and the downstream firms Stackelberg followers, when prices are decided upon non-cooperatively.

The price solutions derived below constitute subgame perfect Nash equilibria. A price solution constitutes a Nash equilibrium, if every firm's price is the best possible reaction to the prices set by all other firms. Since the price solution meets the main conditions for profit maximisation - the first order conditions - for all firms, the solution is a Nash equilibrium. The solution is subgame perfect since the price solution is a Nash equilibrium in every subgame. Downstream prices constitute a Nash equilibrium of the downstream price game, the subgame of the overall game. Upstream prices constitute a Nash equilibrium, since the upstream firms take account of each others' price reactions and downstream equilibrium.

4.2 Non-cooperative price decisions

In this section, non-cooperative price decisions are analysed. The price decisions are non-cooperative, since both upstream and downstream firms maximise individual rather than joint profits. Upstream and downstream firms set wholesale and consumer prices respectively and do not use coordination devices such as franchise-fee-systems. In order to keep the argument simple for the moment, we assume that there are N one-product firms in both links. Each upstream firm sells its product to one downstream firm. This implies that all downstream firms buy from one upstream firm. The complications brought about by the multiproduct nature of firms is dealt with in section 4.4.

All N downstream firms maximise the following profit equation

$$\max \ \Pi_v^d (P_v^c) = (P_v^c - P_v^d - P_p^d)\, X_v\, (\mathbf{P^c}) \qquad (13)$$

where again P_v^c is the consumer price of variety v, P_v^d is its wholesale price set by the upstream firm and P_p^d is the factor input price (figure 1). X_v is consumer demand which depends on $\mathbf{P^c}$, the price vector of all consumer prices: $\mathbf{P^c} = [P_v^1, ..., P_v^N]$. Equating marginal revenues and marginal costs gives (Appendix)[1]

$$P_v^c = (P_v^d + P_p^d)\left(\frac{(s\ -1)(N-1) + (V-1)(1-i_{nc}^*) + N}{(s\ -1)(N-1) + (V-1)(1-i_{nc}^*)}\right) \qquad (14)$$

where $\iota_{nc}^* = (P/P_i^c)^{\varsigma-1}$ represents the equilibrium propensity to consume. The consumer price depends on downstream marginal cost ($P_v^d + P_p^d$) multiplied by a mark-up factor. One may show that the consumer price depends negatively on the elasticities of substitution $\sigma: \partial P_v^c/\partial\sigma < 0$ and $\varsigma: \partial P_v^c/\partial\varsigma < 0$, and on the number of varieties N: $\partial P_v^c/\partial N < 0$.These findings are intuitively plausible. When the elasticity of substitution and thus the price elasticity of demand increase, the opportunity cost of a price increase - a fall in demand - rises. As a result, the mark-up over marginal costs falls. The elasticity of demand also rises with the number of varieties available. Again, the mark-up falls. On the other hand, price is positively related to the propensity to consume food: $\partial P_v^c/\partial\iota > 0$. The propensity to consume food rises when the ratio between the overall price index and the food price index increases.

All N upstream firms maximise the following profit equation

$$\max \ \Pi_v^u (P_v^d) = (P_v^d - P_v^u - P_p^u)\, X_v (P^c) \qquad (15)$$

[1] In this chapter, downstream and upstream firms are treated symmetrically, i.e. they are identical. The solutions presented in this chapter hereby provide a benchmark for various types of asymmetries introduced in a later stage.

where P_v^d is the price of variety v set by upstream firm v, P_v^u is the wholesale price paid by the upstream firm and P_p^u its factor input price (figure 2.1). X_v represents consumer demand which depends on $\mathbf{P^c}$, the vector of all consumer prices, and indirectly on the wholesale price the upstream firm sets, P_v^d. The upstream firms take downstream equilibrium into account via the demand term: $X_v(\mathbf{P^c})$. Equating marginal revenues and marginal costs gives (Appendix)

$$P_v^d = \left(\frac{(P_v^u + P_p^u)\left[(\sigma - 1)(N-1) + (\varsigma - 1)(1 - \iota_{nc}^*) + N\right] + N \cdot P_p^d}{(\sigma - 1)(N-1) + (\varsigma - 1)(1 - \iota_{nc}^*)} \right) \tag{16}$$

The wholesale price depends on two terms. The first term is analogous to the one found above: the upstream firm's marginal cost multiplied by a mark-up factor. The second term indicates the influence of the downstream price decision on the upstream price decision. The price set by the upstream firm rises with factor costs downstream. This finding is due to the following mechanism. A rise in primary factor costs downstream causes an increase in the consumer price. As a result, the elasticity of downstream demand as perceived upstream increases. This causes the upstream firm to charge an extra mark-up. Again, the price set corresponds negatively with both the elasticities of substitution σ: $\partial P_v^d/\partial\sigma < 0$ and ς: $\partial P_v^d/\partial\varsigma < 0$, and the number of varieties N: $\partial P_v^d/\partial N < 0$. The price set is again positively related to the propensity to consume food: $\partial P_v^d/\partial\iota > 0$. The same arguments as above apply.

Now both upstream and downstream prices are derived, we may summarise our findings. The wholesale price P_v^d may be used to rewrite the consumer price P_v^c. When one substitutes for P_v^d, the following consumer price is found

$$P_v^c = \nu^2 (P_v^u + P_p^u + P_p^d) \tag{17}$$

where $\nu = [((\sigma-1)(N-1)+(\varsigma-1)(1-\iota_{nc}^*)+N)/((\sigma-1)(N-1)+(\varsigma-1)(1-\iota_{nc}^*))]$. Consumer demand equals

$$X_v = \frac{\iota_{nc}^* I}{\nu^2 (P_v^u + P_p^u + P_p^d) N} \tag{18}$$

Downstream profits per variety are

$$\Pi_v^d = \frac{(\nu^2 - \nu)\iota_{nc}^* I}{\nu^2 N} \tag{19}$$

Upstream profits per variety are

$$\Pi_v^u = \frac{(\nu - 1)\iota_{nc}^* I}{\nu^2 N} \tag{20}$$

Aggregate profits per variety are

$$\Pi_{v} = \frac{(n^{2} - 1)\ i^{*}_{nc}\ I}{n^{2}\ N} \qquad (21)$$

The prices set, the quantities sold and the profits received are all endogenous to our model. We summarised the endogenous variables in order to compare the results under non-cooperation with those obtained under cooperation, in particular franchising (section 4.3).

4.3 Franchising as coordination device

In section 2.2, we argued that the double marginalisation problem arising under non-cooperation may be solved by franchising. Under franchising the upstream firm sells at marginal cost. As a result, it does not earn any net profits, but receives a franchise fee in return. The franchise fee depends on its bargaining power relative to the downstream firm. Recall that a franchise fee system maximises aggregate net profits, since the downstream firm is the only residual claimant. The downstream firm maximises aggregate profits, since her marginal costs coincide with the chain's marginal costs. Her marginal costs do not include the mark-up set by the upstream firm under non-cooperation (section 4.2).

The wholesale price P_v^d under franchising equals the upstream firm's marginal costs

$$P_v^d = P_v^u + P_p^u \qquad (22)$$

The downstream price is again given by equation (14). The downstream firms' price decisions are not affected by the franchise fee system, since this is a lump sum transfer. The consumer price may be derived by substituting equation (22) into equation (14). The consumer price equals

$$P_v^c = n\,(P_v^u + P_p^u + P_p^d) \qquad (23)$$

The consumer price under franchising is smaller than under non-cooperation, because $\nu^2 > \nu > 1$. Output now equals

$$X_v = \frac{i_f^{*}\ I}{n\,(P_v^u + P_p^u + P_p^d)\ N} \qquad (24)$$

Output under franchising exceeds output under non-cooperation because $\nu^2 > \nu > 1$ and $\iota_f^* > \iota_{nc}^*$ since $\partial\iota/\partial P_v^c < 0$. Aggregate profits per variety now are

$$\Pi_v = \frac{(n - 1)\,i_f^{*}\ I}{n\ N} \qquad (25)$$

Aggregate profits are divided as follows. Upstream profits equal the franchise fee F paid by the downstream firm: $\Pi_v^u = F$. Downstream profits equal aggregate profits minus the franchise fee: $\Pi_v^d = \Pi_v - F$. Aggregate profits under franchising exceed those under non-cooperation if

$$\frac{(\nu(\iota_f^*)-1)\iota_f^*}{\nu(\iota_f^*)} > \frac{\left(\nu(\iota_{nc}^*)^2-1\right)\iota_{nc}^*}{\nu(\iota_{nc}^*)^2} \tag{26}$$

Since this condition can not be simplified analytically, (ι_f^*/ι_{nc}^*) and ν are approximated by taking limits. The approximation of (ι_f/ι_{nc}) is valid when the chain is a small part of the economy ($J\rightarrow\infty$ in Eq. (5)). The approximation of ν is valid when both links are competitive ($N\rightarrow\infty$). When we use the fact that $\lim_{N\rightarrow\infty} \nu(\iota_f^*) \approx \lim_{N\rightarrow\infty} \nu(\iota_{nc}^*) \approx \sigma/(\sigma-1)$ and $\lim_{J\rightarrow\infty} (\iota_f^*/\iota_{nc}^*) \approx \nu^{\varsigma-1} \approx (\sigma/(\sigma-1))^{\varsigma-1}$, the above inequality may be reduced to

$$\left(\frac{\sigma}{\sigma-1}\right)^{\nu} > \frac{\sigma}{\sigma-1}+1 \tag{27}$$

Franchising raises aggregate profits, if the intersector elasticity of substitution ς is high, i.e. close to σ. When the intersector elasticity of substitution is high, food expenditures rise when the food price index falls.

There are other mechanisms than the franchise fee system which attain the cooperative outcome, notably resale price maintenance (RPM). Since these mechanisms are equivalent in the simple framework set-out so far, we ignore these mechanisms for the moment. The price mechanisms are no longer equivalent when complications such as uncertainty and asymmetric information are allowed for.

4.4 Multiproduct firms

This section complicates the above analysis by allowing firms to have more than one product. In particular, we allow the upstream firms to sell m varieties each and the downstream firms to sell n varieties each. Since there are N varieties, there are (N/m) upstream firms and (N/n) downstream firms. Because the subject of our study, vertical contracts, refers to specific firms and specific products, we assume that each upstream firm sells each of his m varieties to one firm; however, he may sell more than one variety to the same firm. For the same reason, we assume that each downstream firm buys each of her n varieties from one firm, although she may buy more than one variety from the same firm. This assumption is particularly suited for analysing the effects of vertical coordination on aggregated profits per variety. However, it does not allow the possibility that one variety is sold to or bought from more than one firm. In the first part of this section, we analyse non-cooperative behaviour; in the second part, we deal with franchising.

4.4.1 Non-cooperative price decisions

Multiproduct downstream firms maximise

$$\max \Pi^d (P_1^c,...,P_n^c) = \Sigma_{v=1}^{n} (P_v^c - P_v^d - P_p^d)\, X_v(\mathbf{P^c}) \qquad (28)$$

over n varieties. As before, P_v^c is the consumer price of variety v, P_v^d the wholesale price and P_p^d the factor input price; X_v represents consumer demand (figure 2.1). Equating marginal revenues and marginal costs gives

$$P_v^c = (P_v^d + P_p^d)\left(\frac{(\sigma - 1)(N-n) + n(\varsigma - 1)(1 - \iota_{nc}^*) + N}{(\sigma - 1)(N-n) + n(\varsigma - 1)(1 - \iota_{nc}^*)} \right) \qquad (29)$$

The consumer price for the n-product downstream firm is similar to the consumer price for the one-product downstream firm (equation (14)). The latter is a special case of the n product case (n = 1). The consumer price depends on downstream marginal cost (P_v^d + P_p^d) multiplied by a mark-up factor. One may show that the consumer price depends negatively on the elasticities of substitution σ: $\partial P_v^c/\partial\sigma < 0$ and ς: $\partial P_v^c/\partial\varsigma < 0$, and on the number of varieties N: $\partial P_v^c/\partial N < 0$. The argument is the same as above. On the other hand, price is positively related to the number of varieties per firm n: $\partial P_v^c/\partial n > 0$. When single product firms decide on price, they weight an increase in a variety's price-cost margin against its decrease in demand. Multiproduct firms also take the demand externality into account with respect to the other products sold. A rise in variety 1's price coincides with an increase in demand for varieties 2 to n. This dampens the opportunity cost of a rise in variety 1's price, the fall in variety 1's demand. As a result, prices are higher, the higher the number of varieties per firm is. Price is also positively related to the propensity to consume food: $\partial P_v^c/\partial\iota < 0$. The propensity to consume food rises when the ratio between the overall price index and the food price index increases.

Multiproduct upstream firms maximise

$$\max \Pi^u (P_1^d,...,P_m^d) = \Sigma_{v=1}^{m} (P_v^d - P_v^u - P_p^u)\, X_v(\mathbf{P^c}) \qquad (30)$$

over m varieties. P_v^d is the wholesale price of variety v as set by the upstream firm, P_v^u the wholesale price as paid by the upstream firm and P_p^u the factor input price; X_v represents consumer demand (figure 2.1). The upstream firms take downstream equilibrium into account via the demand term: $X_v(\mathbf{P^c})$. Equating marginal revenues and costs gives the following price solution

$$P_v^d = \left(\frac{(P_v^u + P_p^u)\left[(\sigma - 1)(N-m) + m(\varsigma - 1)(1 - \iota_{nc}^*) + N\right] + N.P_p^d}{(\sigma - 1)(N-m) + m(\varsigma - 1)(1 - \iota_{nc}^*)} \right) \qquad (31)$$

The consumer price for the m product upstream firm is similar to the consumer price for the single product upstream firm. The latter is a special case of the m product case (m = 1).

The consumer price equals upstream marginal cost (P_v^u + P_p^u) times a mark-up factor. Again, the wholesale price corresponds negatively with both the elasticities of substitution σ: $\partial P_v^d/\partial\sigma < 0$ and ς: $\partial P_v^d/\partial\varsigma < 0$, and the number of varieties N: $\partial P_v^d/\partial N < 0$. The wholesale price is also positively related to the number of varieties per firm m: $\partial P_v^d/\partial m > 0$, and the propensity to consume ι: $\partial P_v^d/\partial\iota < 0$. The arguments are the same as the ones used for the consumer price.

We now present the reduced form equations for the consumer price, output and profits. We do so briefly, since the results are similar to the results of section 4.2. The consumer price equals

$$P_v^c = mn\,(P_v^u + P_p^u + P_p^d) \tag{32}$$

where μ = [((σ-1)(N-m)+m(ς-1)(1-ι)+N)/((σ-1)(N-m)+m(ς-1)(1-ι))] and ν = [((σ-1)(N-n)+n(ς-1)(1-ι)+N)/((σ-1)(N-n)+n(ς-1)(1-ι))]. Output is

$$X_v = \frac{i^*_{nc}\,I}{mn\,(P_v^u + P_p^u + P_p^d)\,N} \tag{33}$$

Downstream profits per variety are

$$\Pi_v^d = \frac{m(n-1)\,i^*_{nc}\,I}{mn\,N} \tag{34}$$

Upstream profits per variety are

$$\Pi_v^u = \frac{(m-1)\,i^*_{nc}\,I}{mn\,N} \tag{35}$$

Aggregate profits per variety are

$$\Pi_v = \frac{(mn-1)\,i^*_{nc}\,I}{mn\,N} \tag{36}$$

These results refer to non-cooperation.

4.4.2 Franchising as coordination device

Under franchising, the following results are obtained. The wholesale price is

$$P_v^d = P_v^u + P_p^u \tag{37}$$

The downstream price is again given by equation (29). The consumer price equals

$$P_v^c = n(P_v^u + P_p^u + P_p^d) \tag{38}$$

The consumer price under franchising is smaller than under non-cooperation since μ > 1. Output now equals

$$X_v = \frac{i_f^* I}{n(P_v^u + P_p^u + P_p^d) N} \tag{39}$$

Output under franchising exceeds output under non-cooperation because μ > 1 and $\partial\iota/\partial P_v^c < 0$. Aggregate profits per variety now are

$$\Pi_v = \frac{(n-1) i_f^* I}{n N} \tag{40}$$

Aggregate profits are divided as follows. Upstream profits equal the franchise fee F paid by the downstream firm: $\Pi_v^u = F$. Downstream profits equal aggregate profits minus the franchise fee: $\Pi_v^d = \Pi_v - F$. Aggregate profits under franchising exceed those under non-cooperation if

$$\frac{(n(i_f^*)-1) i_f^*}{n(i_f^*)} > \frac{\left(m(i_{nc}^*) n(i_{nc}^*)-1\right) i_{nc}^*}{m(i_{nc}^*) n(i_{nc}^*)} \tag{41}$$

Since this condition can not be simplified analytically, (ι_f^*/ι_{nc}^*), μ and ν are approximated by taking limits. Using the same approximations as in section 4.3, the above inequality may be reduced to

$$\left(\frac{s}{s-1}\right)^{v} > \frac{s}{s-1} + 1 \tag{42}$$

Again, franchising raises aggregate profits, if the intersector elasticity of substitution ς is high, i.e. close to σ. When the intersector elasticity of substitution is high, food expenditures rise when the food price index falls.

In principle, other approximations are also possible. The situation in which one or both links are monopolised may be studied as well.[1] This implies that at least three alternative conditions may be derived. (1) The situation in which the downstream link is monopolised and the upstream link is competitive. (2) The situation in which the upstream link is monopolised

[1] When the downstream link is monopolised (N = n), ν reduces to ν = ((ς-1)(1-ι)+1)/((ς-1)(1-ι))). When the upstream link is monopolised (N = m), μ also reduces to μ = ((ς-1)(1-ι)+1)/((ς-1)(1-ι))).

and the downstream link is competitive. (3) The situation in which both links are monopolised. The reader may use footnote 7 to derive similar conditions as equation (42).

4.5 Bargaining

In the above analysis, the upstream firms are able to act as Stackelberg leaders. This ability is due to the order in which the products flow through the chain in combination with the principles of subgame perfection and backwards induction. The ability to act as Stackelberg leaders gives the upstream firms 'bargaining power' over the downstream firms. With respect to the relation between food processing and retail industries, the Stackelberg assumption is often valid. There is substantial bargaining power with food processing companies, especially those selling A-brands. However, with respect to the relation between farmers on one hand and either the food processing or the retail industry on the other hand, this assumption is no longer valid. Market power lies downstream with food processing companies or horizontal retail chains. For this reason, the model outlined above is adapted slightly.

As before, the downstream firms set prices after the wholesale prices have been determined (figure 1). The wholesale prices are determined as a solution to a bargaining process between the upstream and the downstream firms. The bargaining solution depends on relative bargaining power. The upstream firms no longer set prices unilaterally, as they did in the previous sections, unless all bargaining power is upstream.

Downstream prices are determined as before, since they are set after the wholesale prices have been determined. Consumer prices thus are given by Eq. (14). The wholesale price is derived as an asymmetric Nash bargaining solution (Gravelle and Rees 1992). The bargaining process may be modelled by the following maximisation problem

$$B = (\Pi_v^d(P_v^d))^{\phi}\,(\Pi_v^u(P_v^d))^{1-\phi} \tag{43}$$

where $0 < \phi < 1$. The upstream and downstream firm maximise the product of downstream and upstream profits weighted by relative bargaining power. The downstream firm's bargaining power is given by ϕ; the upstream firm's bargaining power by $1-\phi$. Maximising the above function with respect to the wholesale price P_v^d gives the following solution after some substitution

$$P_v^d = \left(\frac{(P_v^u + P_p^u)\left[(N-m)(s-1) + m(V-1)(1-i_b^*) + (1-\phi)N\right] + (1-\phi)N P_p^d}{(N-m)(s-1) + m(V-1)(1-i_b^*)} \right) \tag{44}$$

The bargaining solution is a more general solution than the solution presented by Eq. (16). The latter equation accords with the case where all bargaining power is with the upstream firm ($\phi = 0$). The other extreme solution has also been derived above. When all bargaining power is with the downstream firm ($\phi = 1$), the upstream firm is forced to sell at marginal cost. The latter situation probably reflects the actual relation between farmers and their contracting parties in either the food processing or the retail industry quite well. The wholesale price is forced down to marginal cost.

4.6 Conclusion

In this paper a theoretical framework has been outlined which has a number of empirical implications that can be submitted to econometric tests. Consumer demand and producer factor demand, costs and prices are all based on microeconomic behavioural postulates. Moreover, consumer demand is modelled with a utility function exhibiting 'love of variety', i.e. the desire for product differentiation. The model offers a framework for analysing firm, industry, chain and factor performance, since both input and output prices and quantities are accounted for and even determined endogenously (except for the prices of raw materials). The following conclusions are derived analytically:

1. prices set either upstream or downstream fall with the elasticities of substitution and the number of varieties available;
2. prices set upstream rise with downstream factor costs. A rise in downstream factor costs causes a fall in the price elasticity of derived demand;
3. prices set either upstream or downstream rise with the number of varieties per firm. This result is due to the positive externality a price rise has on the demand of the other varieties sold by the multiproduct firm;
4. in the symmetric Stackelberg equilibrium with upstream firms as first movers, downstream seller concentration does not influence upstream pricing: Upstream prices depend on the number of varieties per firm upstream, but not on the number of varieties per firm downstream;
5. the paper shows that vertical coordination between firms leads to a fall in the consumer price and a rise in output. Aggregate profits also rise if the intersector elasticity of substitution is sufficiently high;
6. the wholesale price set in the bargaining model depends on the bargaining power of the upstream firm relative to the bargaining power of the downstream firm. When all bargaining power is upstream, the solution equals the Stackelberg solution. When all bargaining power is downstream, the wholesale price equals marginal cost, i.e. the franchise solution.

An extension of the current model to more than two links does not invoke major problems, as long as the other assumptions are uphold. More involved extensions include the derivation of long run equilibria which enable one to determine the equilibrium number of firms and varieties endogenously; the introduction of asymmetries between firms; empirical tests of the hypotheses derived above; and the construction of a microsimulation model.

The model describes the Dutch vegetables chain very well. The vegetable processing industry indeed 'bargains' with cultivators over the wholesale price. The industry uses its bargaining power to enforce marginal cost pricing. On the other hand, the industry provides cutting, treshing, loading and transport services to the contracted cultivators upstream. If the services would not be provided, fixed outlays would be necessary upstream. This would inhibit marginal cost pricing for the wholesale price. The services are equivalent to a franchise fee payment from the vegetables processing industry to the cultivators.

5. Conclusion

In the previous chapter, several theoretical relations have been derived which are summarized in its conclusion. These relations may be used in applied research in the following two ways. First, the relations derived allow one to make theoretical predictions on the relation between prices, output and profits on one hand and demand and supply characteristics and price institutions on the other hand. Second, these theoretical relations constitute a profound basis for empirical applications. In particular, the theoretical analysis yields precise hypotheses which can be subjected to econometric testing.

Conclusion 2 of section 4.6 implies a positive relation between downstream (primary production) costs and upstream (retail) prices. This information may be used to indicate the implications of changes in downstream factor costs for both upstream and downstream prices and chain output and profits. Empirical tests based on these relations may be used to obtain estimations of the quantitative impact of changes in downstream factor prices. Conclusion 4 states that - under certain conditions - downstream seller concentration does not influence upstream pricing. This conclusion has strong implications for the relation between seller concentration and chain prices, output and profits, and hence bears implications for competition policy. Again, this conclusion may be tested empirically. Conclusion 5 indicates under what conditions franchising increases chain profits. Whether these conditions are fulfilled for specific agro-food chains may be investigated using econometric tests. In addition, we obtain rather clear criteria for the assessment of contract design among vertically related partners in agro-food chains.

The paper may thus be used to give theoretical predictions and as a basis for econometric tests. The price equations derived will be estimated in future research in order to construct a microsimulation model relating chain prices, output and profits on the one hand to demand and supply characteristics and coordination and price mechanisms on the other hand.

References

Abiru, M., 'Vertical integration, variable proportions and successive oligopolies'. In: *Journal of Industrial Economics* (1988) 36, pp. 315-325.

Gravelle, H. and R. Rees., *Microeconomics*. Second edition. Longman, London, 1992.

Hennessy, D.A., 'Information asymmetry as a reason for food industry vertical integration'. In: *American Journal of Agricultural Economics* (1996) 78, pp. 1034-1043.

Rey, P. and J.E. Stiglitz., 'The role of exclusive territories in producers competition'. NBER WP: 4618, 1994.

Scheppingen, A.J.T. van., *Het belang van de conservencontractteelt voor de Nederlandse landbouw*. Rabobank, 1985.

Scherer, F.M. and D. Ross., *Industrial market structure and industry performance.* Third edition. Houghton Mifflin, Boston, 1990.

Spengler, J.J., 'Vertical integration and antitrust policy'. In: *Journal of Political Economy* (1950) 58, pp. 347-352.

Telser L.G., 'Why do manufacturers want fair trade'? In: *Journal of Law and Economics* (1960) 3, pp. 86-105.

Tirole, J., *The theory of industrial organization*. MIT Press, Cambridge MA 1988.

Appendix

Consumer demand

Utility is given by

$$U = (\Sigma_{j=1}^{J} X_j^{\alpha} + (\Sigma_{v=1}^{N} X_v^{\tau})^{\alpha/\tau})^{1/\alpha} \qquad (1)$$

where $0 < \alpha < \tau < 1$. Since the above utility function is (weakly) separable, a two-stage maximisation procedure may be adopted. First, the demand X_v for each variety v is derived given the expenditure on all the varieties of the commodity under consideration. Second, the demand for the commodity under consideration, the bundle of v's, is determined. The first maximisation problem is the following

$$\max L = (\Sigma_{v=1}^{N} X_v^{\tau})^{1/\tau} - \lambda(\Sigma_{v=1}^{N} P_v^c X_v - I) \qquad (2)$$

where P_v^c is the consumer price of variety v and I income spent on the varieties under consideration. The first part of the Lagrangean function L is the sub-utility function V. The second part is the budget constraint. The first order derivatives towards X_v and λ are

$$\frac{\partial L}{\partial X_v} = (V/X_v)^{1/\sigma} - \lambda P_v^c = 0 \qquad (3)$$

$$\frac{\partial L}{\partial \lambda} = \Sigma_{v=1}^{N} P_v^c X_v = I \qquad (4)$$

where $\sigma = 1/(1-\tau)$ is the elasticity of substitution between any two varieties. Demand X_v for may be derived by solving for X_v using equation (3); substituting X_v into equation (4); solving for λ using equation (4) and substituting λ into equation (3). This gives

$$X_v = \frac{I}{(P_v^c)^{\sigma} (P_i^c)^{1-\sigma}} \qquad (5)$$

where P_i^c is the price index corresponding to V and defined by

$$P_i^c = \left(\Sigma_{v=1}^{N} (P_v^c)^{1-\sigma}\right)^{1/(1-\sigma)} \qquad (6)$$

Now the demand for each variety v has been determined given the expenditure on all v's, we may now turn to the demand decision with respect to the bundle of v's. For this reason, the analysis is repeated using P_i^c as the price index for the bundle of v's and $X_i = (\Sigma X_v^{\tau})^{1/\tau}$ as the quantity index. Since both the aggregate and the sub-utility function are CES-functions, the analysis is exactly the same. For this reason, we skip the analysis and merely present the results. The demand for X_i equals

$$X_i = \frac{Y}{(P_i^c)^{V} \cdot P^{1-V}} \tag{7}$$

where $\varsigma = 1/(1-\alpha)$ is the elasticity of substitution between any two bundles, Y is consumer income and P is the consumer price index corresponding to U and defined by

$$P = \left(\Sigma_{j=1}^{J} (P_j^c)^{1-V} + (P_i^c)^{1-V}\right)^{1/(1-V)} \tag{8}$$

Using the fact that $I = P_i^c X_i$, the demand for each individual variety X_v becomes

$$X_v = \frac{Y}{(P_v^c)^{s} \cdot (P_i^c)^{1-s} \cdot (P_i^c)^{V-1} \cdot P^{1-V}} \text{ or} \tag{9}$$

$$X_v = \frac{i\,Y}{(P_v^c)^{s} \cdot (P_i^c)^{1-s}} \tag{10}$$

where $\iota = (P/P_i^c)^{\varsigma-1}$. The price elasticity of X_v with respect to P_v^c is

$$e_d^d = \frac{\partial X_v}{\partial P_v^c}\frac{P_v^c}{X_v} = -s + (s - V)\left(\frac{P_i^c}{P_v^c}\right)^{s-1} + (V-1)\left(\frac{P_i^c}{P_v^c}\right)^{s-1}\left(\frac{P}{P_i^c}\right)^{V-1} \tag{11}$$

In case of symmetry, $(P_i^c/P_v^c)^{\sigma-1}$ may be simplified to 1/N; $\iota(P_i^c) = (P/P_i^c)^{\varsigma-1}$ equals the propensity to consume. Below, we will merely replace $(P/P_i^c)^{\varsigma-1}$ with $\iota(P_i^c)$ if appropriate, but, of course, this does not make $\iota(P_i^c)$ independent from P_i^c.

Single product firms

Profits are maximized when marginal revenues MR equal marginal costs MC: MR = MC. Marginal revenues may be written as a function of price P_v and the price elasticity of demand ε: $MR = P_v[1+1/\varepsilon]$. Since we already derived the price elasticity of demand above [equation (11)], we may now determine the profit maximizing prices. Downstream profits are maximized when

$$P_v^c\left(1 + \frac{1}{e_d^d}\right) = P_v^d + P_p^d \tag{12}$$

where ε_d^d is given by equation (11). When we impose symmetry, $(P_i^c/P_v^c)^{\sigma-1}$ may be simplified to (1/N). $(P/P_i^c)^{\varsigma-1}$ is replaced by ι_{nc}^*, the equilibrium propensity to consume under non-cooperation. This gives the following result

$$P_v^c = (P_v^d + P_p^d)\left(\frac{(\sigma-1)(N-1) + (\varsigma-1)(1-\iota_{nc}^*) + N}{(\sigma-1)(N-1) + (\varsigma-1)(1-\iota_{nc}^*)}\right) \tag{13}$$

The upstream price elasticity of derived demand equals

$$\varepsilon_d^u = \frac{\partial X_v}{\partial P_v^c}\frac{\partial P_v^c}{\partial P_v^d}\frac{P_v^d}{X_v}\frac{P_v^c}{P_v^c} = \varepsilon_d^d \varepsilon_p \tag{14}$$

where ε_p represents the price elasticity of the consumer price with respect to the wholesale price. This elasticity equals

$$\varepsilon_p = \nu\frac{P_v^d}{P_v^c} = \frac{P_v^d}{P_v^d + P_p^d} \tag{15}$$

where $\nu = [(\sigma-1)(N-1)+(\varsigma-1)(1-\iota_{nc}^*)+N]/[(\sigma-1)(N-1)+(\varsigma-1)(1-\iota_{nc}^*)]$. The upstream's firm price are maximized if marginal revenues equal marginal costs or

$$P_v^d\left(1 + \frac{P_v^d + P_p^d}{P_v^d \varepsilon_d^d}\right) = P_v^u + P_p^u \tag{16}$$

$$P_v^d = \nu(P_v^u + P_p^u) + (\nu-1)P_p^d \tag{17}$$

Simplifying $(P_i^c/P_v^c)^{\sigma-1}$ to (1/N) and substituting $(P/P_i^c)^{\varsigma-1}$ for ι_{nc}^*, gives the following result after some substitution

Resume - Oligopoly

Substituting equation (17) into equation (13) gives the consumer price as a function of the exogenous variables

$$P_v^c = \nu^2(P_v^u + P_p^u + P_p^d) \tag{18}$$

Consumer demand can be found by substituting equation (18) into equation (10)

$$X_v = \frac{i^*_{nc}\, Y}{n^2 (P^u_v + P^u_p + P^d_p)\, N} \qquad (19)$$

Note that $(P_i^c)^{1-\sigma} = N(P_v^c)^{1-\sigma}$ because of the symmetry assumption. Downstream profits per variety $\Pi_v^d = (P_v^c - P_v^d - P_p^d)X_v$ are found using equations (17), (18) and (19)

$$\Pi^d_v = \frac{(n^2 - n)\, i^*_{nc}\, Y}{n^2 N} \qquad (20)$$

Upstream profits per variety $\Pi_v^u = (P_v^d - P_v^u - P_p^u)X_v$ are found using equations (17) and (19)

$$\Pi^u_v = \frac{(n - 1)\, i^*_{nc}\, Y}{n^2 N} \qquad (21)$$

Aggregate profits per variety $\Pi_v = \Pi_v^d + \Pi_v^u = (P_v^c - P_v^u - P_p^d - P_p^u)X_v$ are found using equations (20) and (21)

$$\Pi_v = \frac{(n^2 - 1)\, i^*_{nc}\, Y}{n^2 N} \qquad (22)$$

Franchising - Oligopoly

The wholesale price P_v^d under franchising equals

$$P^d_v = P^u_v + P^u_p \qquad (23)$$

The consumer price is found by substituting equation (23) into equation (13)

$$P^c_v = n\,(P^u_v + P^u_p + P^d_p) \qquad (24)$$

$$X_v = \frac{i^*_f\, Y}{n\,(P^u_v + P^u_p + P^d_p)\, N} \qquad (25)$$

Equilibrium demand is found by substituting equation (24) into equation (10)

Output under franchising exceeds output under non-cooperation since $\nu^2 > \nu$, since $\nu > 1$. Downstream (gross) profits per variety now are

$$\Pi_v^d = \frac{(n-1)\,i_f^*\,Y}{n\,N} \tag{26}$$

Upstream (gross) profits per variety are zero. Aggregate profits per variety thus are

$$\Pi_v = \frac{(n-1)\,i_f^*\,Y}{n\,N} \tag{27}$$

Aggregate profits under franchising exceed those under non-cooperation if

$$\frac{(n(i_f^*)-1)\,i_f^*}{n(i_f^*)} > \frac{(n(i_{nc}^*)^2-1)\,i_{nc}^*}{n(i_{nc}^*)^2} \tag{28}$$

There is no clear-cut analytical solution to this inequality.

Multiproduct firms

A n product firm maximizes its profits over all the n markets in which it sells. Its profits are

$$\Pi = \Sigma_{v=1}^{n} (P_v - c_v)\, X_v \tag{29}$$

where c_v denotes variety v's per unit costs. Profits are maximized with respect to P_v if

$$\frac{\partial \Pi}{\partial P_v} = X_v + (P_v - c_v)\frac{\partial X_v}{\partial P_v} + \Sigma_{w=1}^{n-1} (P_w - c_w)\frac{\partial X_w}{\partial P_v} = 0 \tag{30}$$

Subscript w refers to the other varieties sold by the multiproduct firm. The equation should hold for all varieties v sold by the multiproduct firm. The equation may be simplified to

$$P_v + (P_v - c_v)\,e_v + \Sigma_{w=1}^{n-1} (P_w - c_w)\,e_w \frac{X_w}{X_v} = 0 \tag{31}$$

where ε_v denotes the price elasticity of X_v with respect to P_v and ε_w the cross price elasticity of X_v with respect to P_w. For the downstream firm we then have

$$P_v^c + (P_v^c - P_v^d - P_p^d)\,e_v^d + \Sigma_{w=1}^{n-1} (P_w^c - P_w^d - P_p^d)\,e_w^d \frac{X_w}{X_v} = 0 \tag{32}$$

where

$$\varepsilon_v^d = -\sigma + (\sigma - \varsigma)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1} + (\varsigma - 1)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1}\left(\frac{P}{P_i^c}\right)^{\varsigma-1} \tag{33}$$

$$\varepsilon_w^d = (\sigma - \varsigma)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1} + (\varsigma - 1)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1}\left(\frac{P}{P_i^c}\right)^{\varsigma-1} \tag{34}$$

In case of symmetry, $(P_i^c/P_v^c)^{\sigma-1} = 1/N$, $X_v = X_w$, $P_v^c = P_w^c$ and $P_v^d = P_w^d$. Imposing symmetry and substituting ι_{nc}^* for $(P/P_i^c)^{\varsigma-1}$ reduces equation (32) to

$$P_v^c = (P_v^d + P_p^d)\left(\frac{(\sigma-1)(N-n) + n(\varsigma-1)(1-\iota_{nc}^*) + N}{(\sigma-1)(N-n) + n(\varsigma-1)(1-\iota_{nc}^*)}\right) \tag{35}$$

The own and cross price elasticities of derived demand as faced upstream are

$$\varepsilon_v^u = \frac{P_v^d}{P_v^d + P_p^d}\left(-\sigma + (\sigma - \varsigma)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1} + (\varsigma - 1)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1}\left(\frac{P}{P_i^c}\right)^{\varsigma-1}\right) \tag{36}$$

$$\varepsilon_w^u = \frac{P_v^d}{P_v^d + P_p^d}\left((\sigma - \varsigma)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1} + (\varsigma - 1)\left(\frac{P_i^c}{P_v^c}\right)^{\sigma-1}\left(\frac{P}{P_i^c}\right)^{\varsigma-1}\right) \tag{37}$$

where $\varepsilon_v^u = \varepsilon_v^d \varepsilon_p$ and $\varepsilon_w^u = \varepsilon_w^d \varepsilon_p$. Substitution into the first order condition [equation (31)] gives

$$P_v^d + (P_v^d - P_v^u - P_p^u)\varepsilon_v^u + \Sigma_{w=1}^{m-1}(P_w^d - P_w^u - P_p^u)\varepsilon_w^u \frac{X_w}{X_v} = 0 \tag{38}$$

and after imposing symmetry

$$P_v^d = \mu(P_v^u + P_p^u) + (\mu - 1)\, P_p^d \tag{39}$$

where $\mu = [(\sigma-1)(N-m)+m(\varsigma-1)(1-\iota_{nc}^*)+N]/[(\sigma-1)(N-m)+m(\varsigma-1)(1-\iota_{nc}^*)]$.

Resume - Oligopoly

Substituting equation (39) into equation (35) gives the consumer price as a function of the exogenous variables

$$P_v^c = \mu\nu\,(P_v^u + P_p^u + P_p^d) \tag{40}$$

where $\mu = [(\sigma-1)(N-m)+m(\varsigma-1)(1-\iota_{nc}^*)+N]/[(\sigma-1)(N-m)+m(\varsigma-1)(1-\iota_{nc}^*)]$ and $\nu = [(\sigma-1)(N-$

n)+n(ς-1)(1-ι_{nc}^*)+N]/[(σ-1)(N-n)+n(ς-1)(1-ι_{nc}^*)]. Consumer demand can be found by substituting equation (40) into equation (10)

$$X_v = \frac{i_{nc}^* Y}{mn(P_v^u + P_p^u + P_p^d) N} \tag{41}$$

Downstream firm profits per variety are [equation (39), (40) and (41)]

$$\Pi_v^d = \frac{m(n-1) i_{nc}^* Y}{mn N} \tag{42}$$

Upstream firm profits per variety are [equation (39) and (41)]

$$\Pi_v^u = \frac{(m-1) i_{nc}^* Y}{mn N} \tag{43}$$

Aggregate profits per variety are [equation (42) and (43)]

$$\Pi_v = \frac{(mn-1) i_{nc}^* Y}{mn N} \tag{44}$$

Franchising - Oligopoly

The wholesale price under franchising equals

$$P_v^d = P_v^u + P_p^u \tag{48}$$

The consumer price is derived by substituting equation (45) into equation (35)

$$P_v^c = n(P_v^u + P_p^u + P_p^d) \tag{49}$$

Consumer demand and output are found by substituting equation (46) into equation (10)

$$X_v = \frac{Y}{n(P_v^u + P_p^u + P_p^d) N} \tag{50}$$

Output under franchising exceeds output under non-cooperation because μν > ν - since μ > 1 - and $\iota_f > \iota_{nc}$ since Mι/MP < 0. Aggregate profits follow from equation (46) and (47)

$$\Pi_v = \frac{(n-1) i_f^* Y}{n N} \tag{51}$$

Aggregate profits under franchising exceed those under non-cooperation if

$$\frac{(n(i_{\mathrm{f}}^{*})-1)\,i_{\mathrm{f}}^{*}}{n(i_{\mathrm{f}}^{*})} > \frac{(n(i_{\mathrm{nc}}^{*})^{2}-1)\,i_{\mathrm{nc}}^{*}}{n(i_{\mathrm{nc}}^{*})^{2}} \tag{52}$$

There is no clear-cut analytical solution to this inequality.