E. W. Brascamp

Department of Animal Production, Section Animal Breeding, Agricultural University, Wageningen, the Netherlands

Model calculations concerning economic optimalization of AI-breeding with cattle

pudoc

Centre for Agricultural Publishing and Documentation

Wageningen – 1975

ISBN 90 220 0594 1

The author graduated on 12 December 1975 as Doctor in de Landbouwwetenschappen at the Agricultural University, Wageningen, the Netherlands, on a thesis with the same title.

Abstract

Brascamp, E. W. (1975) Model calculations concerning economic optimalization of AI-breeding with cattle. Agric. Res. Rep. (Versl. landbouwk. Onderz.) 846, ISBN 9022005941, (v) + 40 p., 30 refs. Eng. and Dutch summaries. This publication partly is a summary of work published in Z. Tierz. Züchtungsbiol. 90 (1973) 1-15; 126-140; 91 (1974) 176-187.

Also: Doctoral thesis, Wageningen.

The effect of costs on the optimum breeding plan for selection for milk traits and the profitability of performance-test selection according to meat production were studied, including the consequences of beef crossing. Returns from breeding schemes were calculated from the expression of genetic superiority of selected parents (paths) in subsequent generations of offspring. Measuring the contribution of separate paths to returns was based on 'discounted expressions per cow'. The relative contribution of paths to returns and to annual genetic improvement differed, especially for path sire to breed daughter, showing a higher relative contribution to returns than to genetic improvement. A breeding plan with highest net returns (returns minus costs) was designated as optimum. Two types of breeding plans for selection for milk traits were compared: a system with semen storage during the waiting period and including slaughtering of bulls after production of a predetermined number of doses, and a system without semen storage. The first system proved to be economically advantageous. Optimum proportion selected, after performance testing, was between 1 in 2 and 1 in 4. Optimum weighing of milk and meat traits – the product of actual economic values and discounted expressions per cow – differed by path, and increasing proportion of beef crossing resulted in a shift of emphasis to milk traits. The conclusions remained unaltered if returns per cow from the expression of genetic superiority were calculated in subsequent years instead of generations, even though the generation approach gave systematic errors in discounted expressions.

Descriptors: gene flow, net returns, milk yield, performance test, beef crossing.

Contents

List of symbols

For symbols used in Papers I, II and III see Appendices of respective papers. Symbols used in this text are summarized below.

Breeding schemes

DD	path dam to breed daughter
DS	path dam to breed son
k	proportion of beef crossing
L_j	generation interval for Path j
SD	path sire to breed daughter (proven bulls)
SS	path sire to breed son
y	proportion of dual-purpose inseminations with semen of young bulls
YB	path sire to breed daughter (young bulls)

Population structure

A	probability that a first insemination (including repeats) by a young bull results in a dairy replacement ($k = 0$)
B	the same probability for inseminations of proven bulls
C	average number of lactations per cow
ΔG	genetic improvement per year

The dimension of all following vectors is $m \times 1$, where m is given by the sum of the number of age classes in males and females.

$\mathbf{g}$	vector with fractions of lactating cows in age classes
$\mathbf{h}$	vector with relative phenotypic merit of lactating cows in age classes
$\mathbf{l}_j(t)$	vector with the genetic makeup for meat traits of slaughter animals of Age 1 in Year t. Transmission of genes to the first generation is via Path j only
$\mathbf{m}(t)$	vector with the genetic makeup by reproduction and ageing for milk traits of animals in Year t in relation to the initial situation $\mathbf{m}(0)$
$\mathbf{m}_j(t)$	the same vector as $\mathbf{m}(t)$ except that transmission of genes to first-generation offspring is via Path j only. The initial situation is given by $\mathbf{n}(0)$
$\mathbf{n}(t)$	vector with the genetic makeup by ageing for milk traits in Year t. The initial situation is $\mathbf{n}(0)$
$\mathbf{z}$	vector with the fraction of all calves born in a year kept for slaughter

The dimension of all following square matrices is given by the sum of the number of age classes in males and females. The matrices show reproduction by paths or ageing of breeding animals.

M	matrix showing reproduction for meat traits
$\mathbf{N}_j$	matrix showing reproduction for meat traits via Path j only
P	matrix showing reproduction for milk traits and ageing of breeding animals
Q	matrix showing ageing of breeding animals
$\mathbf{R}_j$	matrix showing reproduction for milk traits via Path j only
N	population size
s	proportion of purebred dual-purpose calves surviving to slaughter age
s_i	proportion of purebred dual-purpose calves from dams of Age Class i surviving to slaughter age
s'	proportion of crossbred calves for meat production surviving to slaughter age
s'_i	proportion of crossbred calves for meat production from dams of Age Class i surviving to slaughter age
S	proportion of calves surviving to slaughter age

Economic evaluation

δ_{jt}	discounted expression per cow for milk traits for Path j in an isolated Year t
ε_{jt}	discounted expression per cow for meat traits for Path j in an isolated Year t
r	interest rate

1 Introduction

Research on cattle breeding can be divided into three major areas: aim of breeding; assessment of breeding values; structure of breeding programmes.

It is recognized that the value of cattle for breeding depends on numerous traits. For an operational breeding objective an aggregate genotype can be defined by finding the relative (economic) weights of traits to be selected for. Niebel et al. (1972) showed for dual-purpose cattle that for the German situation milk yield (and components) together with growth rate or feed conversion ratio were by far the most important traits in the aggregate genotype.

An aspect of the second area of research is the analysis of (field) data to obtain reliable estimates of the breeding values as a basis for selection.

In the third area the breeding programme is the subject of research: what form should the breeding plan take to maximize the selection results for the population?

A powerful tool for genetic improvement of cattle is artificial insemination. It enables us to obtain reliable estimates of the breeding value of a bull for traits which can not be measured as the bull's own performance and the number of descendants of a (superior) bull can become very large.

This thesis deals with the third question: optimization of breeding plans using AI within a dual-purpose breed of cattle with respect to selection for milk and meat traits. The criterion for which a breeding scheme should be optimized has changed during the last decades. Skjervold & Langholtz (1964) studied genetic improvement resulting from a breeding scheme. It was realized, however, that schemes with maximum genetic improvement were probably too expensive and so, from an economic view point, not optimum. Skjervold (1966) suggested that schemes giving about 90% of maximum genetic improvement were near the economic optimum. Lindhé (1968) included cost calculations while monetary returns were calculated as a linear function of annual genetic improvement (ΔG). ΔG was calculated according to Rendel & Robertson (1950). Other methods to calculate monetary returns have been developed by Hinks (1971), Hill (1971) and McClintock & Cunningham (1972). Estimation of monetary returns was not based on ΔG but on the expression of genetic improvement in time according to the pattern by which genes of selected parents are passed on in the population. These methods, however, are not easily incorporated in a model calculation. Recently, Hill (1974) published a general method to evaluate by matrix procedures monetary returns from breeding schemes based on the pattern of passing on genes in a population.

The basis of this thesis is formed by three papers dealing with optimum breeding programmes for selection for milk traits and with the profitability of performance-test selection for meat traits. Some results are reconsidered in view of Hill's method.

In the first paper (summarized in Section 2.1) a method is developed to estimate monetary returns from selection for milk traits, which could be used in model calculations. Expression of genetic improvement in subsequent generations is the basis for this method.

The effect of costs on the optimum breeding plan to select for milk traits is studied in the second paper (summarized in Section 2.2).

Meat production can be improved genetically by selection within dual-purpose breed and by beef crossing. In the third paper (summarized in Section 2.3) the method to estimate returns is extended for meat traits and the profitability of performance testing in a dual-purpose breed according to meat production is examined. Consequences of crossing the dual-purpose breed with bulls of beef breeds, to produce slaughter animals, are studied.

Comparison of the method of Hill (1974) and that developed in the first paper is the subject of Chapter 3. Return calculations by Hill's method are based on the expression of genetic improvement in subsequent years whereas the calculations in the three papers are based on subsequent generations.

2 Summaries of papers

The papers which are part of this thesis are:

- Paper I, The economic value of genetic improvement in milk yield. Z. Tierz. Züchtungsbiol. 90 (1973) 1–15.
- Paper II, Effect of costs on the optimum breeding plan. Z. Tierz. Züchtungsbiol. 90 (1973) 126–140.
- Paper III, Profitability of performance testing in a dual-purpose breed according to meat production and the effect of beef crossing. Z. Tierz. Züchtungsbiol. 91 (1974) 176–187.

In the three papers deterministic models have been used. The most important parameters whose values were varied, are summarized below.

Parameter	*range*
population size	50000 – 1000000
proportion in milk recording	0.30 – 1.00
proportion of beef crossing	0.00 – 0.70 (approximately)
proportion of dual-purpose inseminations with young bulls	0.10 – 0.90
progeny group size	50 – 600
number of doses per bull	3000 – 220000
number of doses produced per bull per year	15000 – 35000

2.1 Paper I: The economic value of genetic improvement in milk yield

The approach to calculate the economic value in milk yield is illustrated in Fig. 1. The closed circles in Fig. 1 represent batches of offspring from groups of parents for four generations. For each batch the number of (female) offspring entering the dairy herd can be calculated. By inclusion of the genetic relationship between selected parents and animals in a batch, the total genetic superiority of parents expressed by the cows in the batch is found. The units of this superiority are for instance, kg (of milk) or money units. To obtain the economic value of the total genetic superiority expressed in a batch of offspring, the monetary value is discounted to a reference year. The birth of young bulls (Parents YB in Fig. 1) is taken as reference year (Year 0). Summing the discounted value of genetic superiority for all batches results in the economic value of genetic improvement for all four generations.

Another approach (Lindhé, 1968; Lindström, 1971) to estimate the economic value

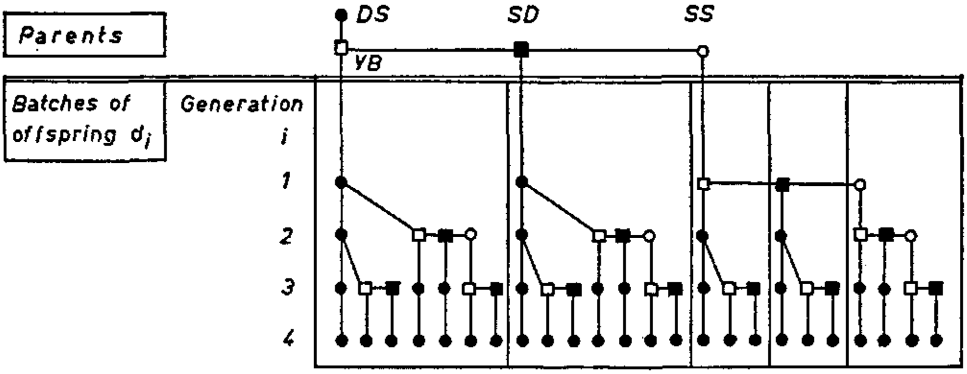

Fig. 1. The parents and the batches of offspring (d_i) when four generations of offspring are taken into consideration.
DS = dams to breed son; YB = young bulls of generation zero; SD = sire to breed daughter; SS = sire to breed son; ● = batch of offspring; □ = young bulls; ■ = proven bulls to breed daughter; ○ = sires to breed son.

of genetic improvement is based on the value of the annual genetic improvement (ΔG) estimated with the formula of Rendel & Robertson (1950). The economic value of genetic improvement is then estimated as a linear function of ΔG.

For comparison of both approaches a discount factor was introduced for the time lag between Year 0 and the expression of genetic improvement in females in the population. This discount factor was calculated as the ratio between the estimate of the economic value of genetic improvement over four generations and the estimate made with the linear function of ΔG.

Conclusions can be summarized as follows: the relative contribution of Path SS (sire to breed son) to the monetary returns is lower than to ΔG. For Paths SD (proven bulls) and DD (dam to breed daughter) the opposite is true. The relative contribution of Path DS (dam to breed sire) to both returns and ΔG is about equal.

The discount factor for the time lag is not a constant. Most important is the increase of the discount factor with increasing numbers of doses of sperm per bull. The discount factor based upon 10% interest rate ranges from 0.28 – 0.30 for 3000 doses per bull up to 0.35 – 0.40 for 80000 doses per bull.

Further the effect of the decrease of the population size has been studied, assuming a decrease during about 25 years with a constant rate q per year. The value of genetic improvement decreases roughly to $(1\text{-}q)^{16}$ times the value of genetic improvement when the population size is constant.

2.2 Paper II: Effect of costs on the optimum breeding plan

Gross returns were calculated with the method developed in Paper I. An interest rate of 10% was used to calculate gross returns. Costs based on cost factors summarized in Table 1 were calculated with an interest rate of 8%. The data in Table 1 are relative to Cost alternative 1, the assumed Norwegian situation.

The breeding plan with the largest difference between gross returns and costs (i.e. maximum net returns) was taken as the optimum. A suboptimum breeding plan was adopted to cover situations where one is not prepared to invest the amount of money in AI justified by the criterion maximum net returns. Such a suboptimum plan shows highest net returns given a certain cost level.

Two management systems were compared, the waiting-bull system B, and the system of storing deep-frozen semen and slaughtering the bulls as soon as a predetermined number of doses per bull has been stored: A.

For most cost alternatives, optimum and suboptimum breeding plans for System A were more profitable than those under System B. Under System A the number of doses per bull was high and the same for all these plans. Exceptions were cost alternatives with high costs for semen preparation and storage, and low costs for maintenance (Cost alternatives 4, 7 and 10). Then at low cost levels, suboptimum plans for System B were more profitable than those under System A, while for System A the suboptimum plans were characterized by a lower number of doses than the optimum plan.

Net returns of optimum and suboptimum plans increase with population size. In

Table 1a. Meaning of symbols and values of cost factors for Cost alternative 1.

Symbol	Cost factor	Value	
a1	milk price	0.25	Nkr per kg
a2	carcass value	10	Nkr per kg slaughter weight
a3	'first year'	2500	Nkr per bull
a4	maintenance	7	Nkr per bull per day
a5	dose preparation	0.17	Nkr per dose
a6	dose storage	0.033	Nkr per dose per year
a7	building	see Appendix 2, Paper II	
a8	labour	33000	Nkr per man-year

Table 1b. Summary of cost alternatives. Costs relative to cost factor a1 = 1 and to cost alternative 1 = 1. See Paper II.

Cost factor	Cost alternative											
	1	2	3	4	5	6	7	8	9	10	11	12
a1	1	1	1	1	1	1	1	1	1	1	1	1
a2	1	1	1	1	1	1	1	1	1	1	0.40	1.20
a3	1	5	1	1	1	1	1	1	0.5	1	2.0	6.0
a4	1	1	1	1	2	3	1	2	1.2	0.6	1.3	3.9
a5	1	1	2	3	1	1	5	0.5	2	3.7	0.13	0.4
a6	1	1	1	1	1	1	5	0.5	0.75	0.75	0.30	0.9
a7	1	1	1	1	1	1	1	1	1	1	0.35	1
a8	1	1	1	1	1	1	1	1	1	1	0.55	1.65

the model, however, cost factors were independent of population size. So no optimum population size could be determined.

2.3 Paper III: Profitability of performance testing in a dual-purpose breed according to meat production and the effect of beef crossing

The method developed in Paper I was extended for meat traits to estimate gross returns from performance-test selection for meat traits within a dual-purpose (milk/meat) breed.

The concept 'discounted expression per cow' was introduced to estimate monetary returns from performance-test selection and to determine optimum weighing of milk traits and meat traits in an aggregate genotype. The discounted expression for a trait and a path was defined as the discounted gross returns from *that* expression of a trait in offspring which results from parental genetic superiority of one money unit, divided by the population size. This concept is similar to the 'number of standard discounted expressions' of McClintock & Cunningham (1972).

The effect of beef crossing on the profitability of performance testing and on the weighing of milk and meat traits in an aggregate genotype was studied.

The major conclusions are summarized below:

Profitability of performance testing mainly depends on the selection intensity of bull dams, the relative economic value of milk and meat traits, costs and the fraction of beef crossing. The optimum proportion selected seems to be between 1 in 2 and 1 in 4. Generally the profitability of performance testing within the dual-purpose breed decreases when the fraction of beef crossing increases.

The optimum weighing of milk and meat traits in the aggregate genotype is the product of their actual economic value and their discounted expression per cow. This weighing is different for each path and the emphasis on milk and meat traits shifts to milk traits when the fraction of beef crossing increases.

3 Comparison of generation approach and year approach

In Papers I and III discounted expressions per cow have been calculated for four generations. With these discounted expressions returns from breeding schemes have been calculated in Papers II and III. In this chapter discounted expressions per cow are calculated for a certain number of years, instead of for a number of generations. These calculations are done with the method described by Hill (1974). In Section 3.1 that part of Hill's approach needed to calculate discounted expressions per cow is explained. This approach is illustrated with an example. Further some extensions are described. The notation of Hill (1974) is followed.

In Section 3.2 the methodology of calculating discounted expressions based upon generations is compared with the approach based upon years for the example situation. Discounted expressions as given in Papers I and III (four-generations approach) are compared with discounted expressions based upon years. Assumptions used in these calculations are consistent with those in Papers I and III and are given in appendices. Implications for the conclusions of the papers will be discussed.

3.1 Year approach: methods

The crucial question in Hill's approach is: which part of the genes (genetic superiority) of a certain group of animals (selected parents) is expressed in animals in subsequent years. Let us consider this in a simple unrealistic example, in which bulls produce female offspring when they are 2 years of age (untested young bulls, YB) and when they are 4 years old (proven bulls, SD). Bulls (SS) produce male offspring (young bulls) when they are 4 years of age. Females (DS and DD) survive up to 3 years of age and produce an equal number of offspring at 2 and 3 years old.

The genetic makeup of sexes and age classes starting from bulls of Age 1 in Year 0 is given in Table 2 for this example. In Year 0 only bulls of Age 1 contain 100% of their own genes. In Year 1 these bulls are one year older, so Age class 2 contains 100% of the genes of bulls of Age class 1 in Year 0. In Year 1 the bulls reach reproductive age. So in Year 2 the females of Age class 1 contain 10% of the genes of the bulls considered, as young bulls perform 20% of the inseminations and transmit 50% of their genes to an offspring. In Year 2 bulls of Age class 3 and females of Age class 1 contain genes of bulls considered but they are not of reproductive age. So from Year 2 to Year 3 the animals only grow one year older and have no offspring.

In Year 4, bulls of Age class 1 contain 50% of the genes of bulls considered transmitted by the bull fathers of Year 3. Further they contain $\frac{1}{2} \times 0.5 \times 10\% = 2.5\%$ of

Table 2. Genetic makeup of sexes and age classes in an example situation.

Year	Age class							Explanati on
	males				females			
	1	2	3	4	1	2	3	
0	1	0	0	0	0	0	0	
1	0	1	0	0	0	0	0	
2	0	0	1	0	0.10	0	0	($0.10 = \frac{1}{2} \times 0.2 \times 1$)
3	0	0	0	1	0	0.10	0	
4	0.525	0	0	0	0.425	0	0.10	($0.525 = \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 0.5 \times 0.10$)
								($0.425 = \frac{1}{2} \times 0.8 \times 1 + \frac{1}{2} \times 0.5 \times 0.10$)
5	0.025	0.525	0	0	0.025	0.425	0	($0.025 = \frac{1}{2} \times 0.5 \times 0.10$)
6	0.10625	0.025	0.525	0	0.15875	0.025	0.425	($0.10625 = \frac{1}{2} \times 0.5 \times 0.425$)
								($0.15875 = \frac{1}{2} \times 0.2 \times 0.525 + \frac{1}{2} \times 0.5 \times 0.425$)
7	0.1125	0.10625	0.025	0.525	0.115	0.15875	0.025	($0.1125 = \frac{1}{2} \times 0.5 \times 0.025 + \frac{1}{2} \times 0.5 \times 0.425$)
								($0.115 = \frac{1}{2} \times 0.2 \times 0.025 + \frac{1}{2} \times 0.5 \times 0.025 + \frac{1}{2} \times 0.5 \times 0.425$)

genes from cows of Age class 2. The remaining ¼ they get from cows of Age class 3, but the latter contain no genes of bulls considered. Females of Age class 1 contain 40% of genes transmitted by proven bulls and 2.5% transmitted by cows of Age class 3 ($\frac{1}{2} \times 0.8 \times 1 + \frac{1}{2} \times 0.5 \times 0.10 = 0.425$).

This process of ageing and reproduction can be formalized as follows. Define a Matrix **P** as

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0.25 & 0.25 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.4 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The blocks in **P** correspond to paths of gene transmission

$$\begin{bmatrix} \text{sire to breed son (SS)} & \text{dam to breed son (DS)} \\ \text{sire to breed daughter (YB and SD)} & \text{dam to breed daughter (DD)} \end{bmatrix}$$

The Matrix **P** describes reproduction and ageing for the example in Table 2. The actual makeup of **P** is given in Appendix 1.

Ageing alone can be described by a Matrix **Q**:

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

A column vector $\mathbf{m}(t)$ describes the genetic makeup of sexes and age classes in Year t, starting from the situation in Year 0, $\mathbf{m}(0)$. So

$$\mathbf{m}'(0) = (1 \quad 0 \quad 0 \quad 0 \mid 0 \quad 0 \quad 0) \text{ and}$$
$$\mathbf{m}'(5) = (0.025 \quad 0.525 \quad 0 \quad 0 \mid 0.025 \quad 0.425 \quad 0)$$

Now

$$\mathbf{m}(t) = \mathbf{P}\,\mathbf{m}(t\text{-}1) = \mathbf{P}^t\,\mathbf{m}(0) \qquad \text{(Hill, 1974)} \qquad (1)$$

The genetic makeup of sexes and age classes by reproduction alone is given by

$$\mathbf{m}(t) = (\mathbf{P}^t - \mathbf{Q}^t)\,\mathbf{m}(0) \qquad \text{(Hill, 1974)} \qquad (2)$$

Milk traits Returning to the example, let us consider what the previous reasoning means in terms of genetic improvement of milk traits. Suppose that the genetic superiority of young bulls (by selection of bull dams, see Fig. 1), is 1 kg of milk. Then the first returns are attained in Year 3 when the average superiority of cows in Age class 2 is 0.10 kg of milk. Per cow in Year 3 this is 0.05 kg because only half of the lactating cows in a year are of Age 2.

The discounted expression per cow (δ_3) in Year 3 can be calculated as $0.05 \times (\frac{1}{1+r})^3$.

Here r stands for the interest rate and discounting is done to the value in Year 0. (The actual monetary value of 1 kg = 1). This can be formalized as

$$\delta_t = \mathbf{m}'(t)\ \mathbf{h}\ (\frac{1}{1+r})^t \qquad (3)$$

In the example $\mathbf{h}' = (0 \quad 0 \quad 0 \quad 0 \quad | \quad 0 \quad 0.5 \quad 0.5)$, the proportion of lactating cows in different age classes. In reality, however, the proportion of lactating cows in different age classes will not be equal. Furthermore, the average level and standard deviation of production in different lactations will not be equal. These effects should be included in $\mathbf{h}$. The vector of fractions of lactating cows in different age classes will be noted here as $\mathbf{g}$. For the actual assumptions of $\mathbf{g}$ and $\mathbf{h}$ see Appendix 2. Now the (cumulative) discounted expression per cow up to Year t is obtained by adding all δ_i from Year $i = 0$ to $i = t$.

To be in line with Paper I and Paper III cumulative discounted expressions per cow will be calculated for each path separately. In the example, the female offspring of Path SS will first lactate in Year 7 (Table 2) containing a fraction $\frac{1}{2} \times 0.2 \times 0.5 = 0.05$ of the genetic superiority of SS. For Path SD the first lactation occurs in Year 5, cows of Age class 2 containing $\frac{1}{2} \times 0.8 \times 1 = 0.4$ of the SD genetic superiority. This splitting of selection response by paths can be formalized by

$$\mathbf{n}(t) = \mathbf{Q}^t\ \mathbf{n}(0) \qquad (4)$$

$$\mathbf{m}_j(t) = \mathbf{R}_j\ \mathbf{n}(t-1) + \mathbf{P}\ \mathbf{m}_j(t-1) \qquad (5)$$

$\mathbf{n}(0) = \mathbf{m}(0)$, and $\mathbf{m}_j(t)$ represents the genetic makeup of sexes and age classes in Year t for Path j. The vector $\mathbf{m}_j(0)$ contains zeroes only. So for an isolated Year t

$$\delta_{jt}\ \ (t = \mathbf{m})'_j\ \mathbf{h}\ (\frac{1}{1+r})^t \qquad (6)$$

where δ_{jt} is the discounted expression per cow for Path j in an isolated Year t.

In the example the $\mathbf{R}_j$ matrices for Path YB and SD are

$$\mathbf{R}_{YB} = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_{SD} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathbf{R}_{SS}$ contains only the SS reproduction part of matrix $\mathbf{P}$.

For the Paths SS, SD and YB the $\mathbf{n}(0)$ vector equals $\mathbf{n}'(0) = (1 \quad 0 \quad 0 \quad 0 \quad | \quad 0 \quad 0 \quad 0)$ in the example; or generally, $\mathbf{n}(0)$ contains all zeroes except males in Age class 1.

Equation (4) gives only ageing of the initial bulls. Note that $\mathbf{Q}^t\, \mathbf{n}(0) = \mathbf{0}$ when $t \geqslant$ (number of male age classes) (Hill, 1974). The part $\mathbf{R}_j\, \mathbf{n}(t-1)$ of Eqn 5 gives the genetic makeup of the offspring of the first generation, only via the path considered. The part $\mathbf{P}\, \mathbf{m}_j(t-1)$ gives reproduction of this first generation offspring and of later generations. This structure of separating reproduction by paths is seen also in Fig. 1.

The discounted expressions per cow for Path DS equal half those of Path YB, as follows from the position of DS in Fig. 1.

Meat traits Animals for breeding are produced via five paths: SS, SD, YB, DS and DD. All calves surviving to age of slaughter, except breeding animals and cows kept for milk production, are regarded as slaughter animals. So bullfathers (SS) and bulldams (DS) do not transmit genes directly to slaughter animals. Slaughter animals contain genes from dual-purpose breed parents YB, SD and DD, and possibly from bulls of beef breeds. Thus the genetic makeup of slaughter animals in Year t can be calculated as

$$\mathbf{l}_j(t) = \mathbf{N}_j\, \mathbf{n}(t-1) + \mathbf{M}\, \mathbf{m}_j(t-1) \tag{7}$$

It should be mentioned that $\mathbf{m}_j(t-1)$ in Eqn 7 is calculated from Eqn 5.

Matrix $\mathbf{M}$ contains reproduction of slaughter animals via Path YB, SD and DD. Matrix $\mathbf{N}_j$ contains reproduction of slaughter animals only via Path j. If all other elements of matrices $\mathbf{M}$ and $\mathbf{N}_j$ are put equal to zero, $\mathbf{l}_j(t)$ will contain zeroes except female Age class 1. This element represents the genetic makeup of all slaughter animals, irrespective of sex, at Age 1 in Year j.

In the example Matrix **M** will equal

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.4 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For Path YB $\mathbf{m}'_{YB}(5) = (0.025 \quad 0.525 \quad 0 \quad 0 \quad | \quad 0.025 \quad 0.425 \quad 0)$. Pre-multiplication of $\mathbf{m}_{YB}(5)$ by Matrix **M** gives as the only non-zero element of $\mathbf{l}_{YB}(6)$: $0.1 \times 0.525 + 0.25 \times 0.425 = 0.15875$, representing the fraction of young bulls' initial genetic superiority for meat inherited by all slaughter animals of Age 1 in Year 6.

N_{SS} contains zeroes only. N_{SD} contains the reproduction part for SD in Matrix **M** (in the example 0.4 and zeroes), and N_{YB} the reproductive part of bulls (in the example 0.10 and 0.40 and zeroes).

The actual makeup of Matrix **M** is more complicated if beef crossing is considered. In Appendix 3 the actual makeup of Matrix **M** is derived.

Discounted expressions per cow for meat traits for Path j in an isolated Year t (ε_{jt}) can be calculated as

$$\varepsilon_{jt} = \mathbf{l}'_j(t)\ \mathbf{z}\left(\frac{1}{1+r}\right)^t \qquad (8)$$

where **z** is a vector containing zeroes except the element female Age class 1. This element equals the proportion of all calves born in one year surviving to age of slaughter (S). Animals are taken to be slaughtered at one year of age, simply as a result of the definition of matrices. If animals are slaughtered at a different time, extra discounting is needed.

Remarks

– The DD part of Matrix **M** will only equal the reproduction part of DD, Matrix **P**, if slaughter animals inherit genes from dams in different age classes at the same frequency as the calves for female replacement. In practice this assumption will not hold. For example, a daughter of an older cow will be kept for replacement rather than a daughter of a heifer. In notation used here, the DD part of the **M** and **P** matrices mentioned will only be equal when reproduction via Path DD is given by $\frac{1}{2}\mathbf{g}$, vector **g** containing the fraction of cows in subsequent lactations.

– The reproduction part of matrices **M** and **P** for the paths males to females will only be equal to each other if daughters of young bulls and of proven bulls have an equal chance to become replacement heifers, and if no beef crossing is practised, as milking cows do not contain beef breed genes, but slaughter calves do.

3.2 Results of comparison

In this section two methods to estimate discounted expressions per cow will be compared with the example of the previous section (Table 2). In addition results from both approaches will be discussed. The two methods are based upon:
- generations (Papers I and III; Chapter 2)
- years (Section 3.1).

The discounted expression per cow for Trait i and Path j has been defined as the discounted gross returns from *that* expression of i in offspring which results from genetic superiority of parents (Path j) of one money unit, divided by the population size. The discounted expression for milk traits in two batches of offspring extracted from Fig. 1 will be calculated by both methods.

3.2.1 Discounted expression per cow: comparison of methodology

Case 1 Consider the batch of offspring from young bulls (in Generation 1) in Fig. 2. (This is a part of Fig. 1). Suppose that the population size is N. Since young bulls perform 20% of the first inseminations and the replacement rate is $\frac{1}{2}$, the number of offspring in Batch 1 entering the dairy herd will be $0.2 \times N \times \frac{1}{2}$. These offspring will have $2 \times 0.2 \times N \times \frac{1}{2}$ lactations. The genetic relationship between the young bulls and their offspring is $\frac{1}{2}$, so, if the genetic superiority of young bulls is one money unit, the total increase in milk production in Batch 1 is $\frac{1}{2} \times 2 \times 0.2 \times N \times \frac{1}{2} = \frac{1}{2} \times 0.2 \times N$ money units.

The discount factor for the time lag between birth of young bulls and average birth of the offspring in Batch 1 will be $(\frac{1}{1+0.10})^{L_{YB}}$, where L_{YB} is the generation interval for young bulls (2 years). Further discounting is necessary for the time interval between birth of the batch and actual expression of genetic superiority. In the example both lactations occur with the same probability ($\frac{1}{2}$) and at 2 and 3 years of age,

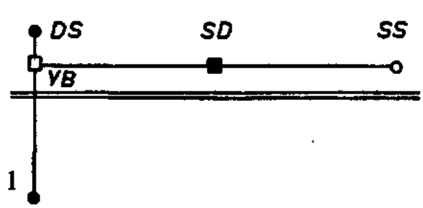

Fig. 2. A batch of first-generation offspring.

Table 3. Calculation of the discounted expression per cow for Path YB for Batch 1 (Fig. 2) only. Lactating cows: factor in italics.

Year	Age class							Discounted expression per cow
	males				females			
	1	2	3	4	1	2	3	
0	1	0	0	0	0	0	0	
1	0	1	0	0	0	0	0	
2					0.10	0	0	
3					0	*0.10*	0	$\frac{1}{2} \times 0.10 \times (\frac{1}{1+0.10})^4 = 0.0342$[a]
4					0	0	*0.10*	$\frac{1}{2} \times 0.10 \times (\frac{1}{1+0.10})^5 = 0.0310$
								Total 0.0652

a. Young bulls in Year 0 are of Age 1.

respectively. So the weighted discount factor equals

$$\tfrac{1}{2}\left(\frac{1}{1+0.10}\right)^{2} + \tfrac{1}{2}\left(\frac{1}{1+0.10}\right)^{3} = 0.7889.$$

Then the discounted expression per cow, for Batch 1 only, equals

$$\tfrac{1}{2} \times (0.2 \times \mathrm{N}) \times \left(\frac{1}{(1+0.10)}\right)^{L_{YB}} \times 0.7889/\mathrm{N} = 0.0652.$$

Monetary values are discounted to the year of birth of the batch of young bulls: the reference year chosen (Paper I; Chapter 2).

In Table 3 the discounted expressions per cow are derived based upon years. Fractions of genes (genetic superiority) in different age classes and years are given, according to Table 2. Fractions for lactating animals in Batch 1 are in italics.

In this case both approaches give identical results.

Case 2 Consider now the third-generation batch of offspring of proven bulls in Fig. 3. This is Batch 3. (It is also one of the batches of third-generation offspring of young bulls.) The number of offspring entering the dairy herd in Batch 1, Fig. 3, is $0.8 \times \mathrm{N} \times \frac{1}{2}$, as proven bulls perform 80% of the inseminations. Each cow produces on average one replacement heifer, if the population size is constant. So also in Batch 3 the number of offspring entering the dairy herd is $0.8 \times \mathrm{N} \times \frac{1}{2}$, with a total of $0.80 \times \mathrm{N}$ lactations. The genetic relationship between the offspring in Batch 3 and the proven bulls (or young bulls) is $\frac{1}{8}$. The time interval between birth of young bulls (reference year) and average birth of offspring in Batch 3 is $L_{SD} + 2L_{DD} = 4 + 2 \times 2\frac{1}{2} = 9$ years. L_{SD} and L_{DD} are generation lengths for Paths SD and DD, respectively. So the discounted expression per cow, for Batch 3 only, becomes

$$\tfrac{1}{8} \times (0.80 \times \mathrm{N}) \times \left(\frac{1}{1+0.10}\right)^{L_{SD} + 2L_{DD}} \times 0.7889/\mathrm{N} = 0.0335.$$

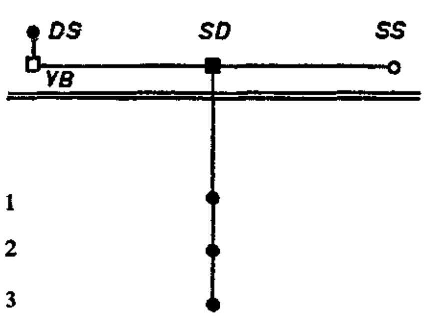

Fig. 3. A batch of third-generation offspring.

Table 4. Calculation of the discounted expression per cow for Path SD (or YB), for Batch 3 (Fig. 3) only. Lactating cows: factor in italics.

Year	Age classes							Discounted expression per cow	
	males				females				
	1	2	3	4	1	2	3		
3	0	0	0	1	0	0	0		
4					0.40	0	0		
5					0	0.40	0		
6					0.10	0	0.40		
7					0.10	0.10	0		
8					0.025	0.10	0.10		
9					0.05	*0.025*	0.10	$\frac{1}{2} \times 0.025 \times (\frac{1}{1+0.10})^{10}$	$= 0.0048$
10					0.025	*0.05*	*0.025*	$\frac{1}{2} \times (0.05 + 0.025) \times (\frac{1}{1+0.10})^{11}$	$= 0.0131$
11						*0.025*	*0.05*	$\frac{1}{2} \times (0.025 + 0.05) \times (\frac{1}{1+0.10})^{12}$	$= 0.0119$
12							*0.025*	$\frac{1}{2} \times 0.025 \times (\frac{1}{1+0.10})^{13}$	$= 0.0036$
								Total	0.0334

In Table 4 the discounted expressions per cow based upon years are derived. Table 4 is an extension of Table 2.

In Case 2 both approaches have a slightly different outcome. This is caused by the assumption in the generation approach that the lactations per cow are equally divided over Years 10 and 11. In 'reality' the cows lactate in Years 9, 10, 11 and 12. In the more realistic situation of many lactations (Appendix 2) the difference between the set of years in which the cows actually lactate and the years of lactation assumed in the generation approach will be larger, especially in later generations.

Two other differences between the generation approach and the year approach will be explained below.

Assume that four generations of offspring (Fig. 1) cover a period of 25 years. Then some animals included in the generation approach will not have completed all their lactations within 25 years. Other animals in later generations, on the contrary, will have started lactation before Year 25. These effects will balance each other to some extent.

The average generation interval varies by varying y. In other words, when y increases, the number of offspring in Fig. 1 resulting from young-bull inseminations will increase. This offspring is born earlier than offspring resulting from proven-bull inseminations. Thus when y increases, the number of years which covers four generations will decrease. This effect will cause a bias in the outcome of the generation approach compared with that of the year approach with a fixed number of years. This effect will be quantified below.

The average generation interval is between 5.1 and 5.9 years depending on y. (Paper I, Appendix 1). On average cows have $3\frac{1}{2}$ lactations. So four generations can be expected to cover a period of about 24 to 28 years. To compare results from the generation and year approach, 25 years are taken.

3.2.2 Discounted expressions per cow for milk traits (Paper I)

The reason for the differences in discounted expressions for milk traits calculated on a generation basis or on a year basis are shown in Table 5. As discussed the number of years which covers four generations decreases when the proportion of insemina-

Table 5. The year in which the cumulative discounted expressions per cow for milk traits calculated on a year basis equal those calculated on a four-generation basis. Interest rate = 10%.

Path	Proportion of dual-purpose inseminations with young bulls					
	0.10	0.20	0.30	0.50	0.70	0.90
SS	30	29	29	28	27	25
SD	27	27	26	26	25	25
DS and YB	28	27	27	26	24	23

Table 6. Discounted expressions per cow for milk traits based upon 4 generations (a) and upon 25 years (b), relative to Path SS, y = 0.90. Interest rate is 10%.

Path and method	Proportion of dual-purpose inseminations with young bulls					
	0.10	0.20	0.30	0.50	0.70	0.90
SS						
a	73	76	79	85	92	100
b	63	67	71	79	89	100
SD						
a	206	188	169	129	84	30
b	204	186	168	130	85	31
DS						
a	153	159	165	179	194	212
b	147	155	162	179	199	223

Table 7. Bias (%) in estimates of discounted expressions per cow for milk traits on a 4-generations basis relative to the (cumulative) discounted expressions per cow in Year 25.

Path	Proportion of dual-purpose inseminations with young bulls					
	0.10	0.20	0.30	0.50	0.70	0.90
SS	+18	+15	+13	+9	+5	+1
SD	+ 2	+ 2	+ 2	+1	+0	−0
DS and YB	+ 5	+ 4	+ 3	+1	−1	−3

tions with young bulls increases.

The discounted expression for Path SS with y = 0.90, equals 0.1036 on a four-generation basis and 0.1022 on a 25-year basis. In Table 6 discounted expressions are given relative to Path SS, y = 0.90, both based on four generations and on 25 years. When y decreases the discounted expressions for all paths on 25 years decrease compared to those based on four generations.

The size of this tendency and the size of difference between discounted expressions based on both methods is different for each path, which can also be seen clearly in Table 7. In this table the deviations in estimates of discounted expressions based upon four generations from those based on 25 years are given relative to the estimates based on 25 years. These effects, common to Tables 5 to 7, can be explained as follows. The tendency common to all paths was explained before: the average generation interval decreases when y increases because of the balance between the number of offspring from young-bull inseminations and the number of offspring from proven-bull inseminations. For Path SD the effect of this balance first occurs in the third generation (see Fig. 1). The female offspring in the first and second generation account for a

Table 8. Weighing of paths in ΔG, relative to Path SS, $y = 0.90$.

Path	Proportion of dual-purpose inseminations with young bulls					
	0.10	0.20	0.30	0.50	0.70	0.90
SS	86	88	89	93	96	100
SD	78	70	63	46	29	10
DS	86	88	89	93	96	100

large part of the (cumulative) discounted expression for Path SD. For the first and second generation the difference in discounted expressions calculated by both methods is small, as illustrated in the previous example.

For Path SS on the contrary, the first female offspring is obtained in the second generation. The years in which this offspring produces is influenced by y, and with that the number of years covering four generations. So the effect of y for Path SS will be larger than for Path SD.

As seen in Fig. 1 all batches of female offspring contribute to discounted expressions for Path YB and DS.

The first female offspring via these paths is first-generation progeny, both from young-bull inseminations and from proven-bull inseminations. So the effect of y will be larger for these paths, just as for SS.

The four generations offspring of young-bull inseminations of parent group YB in Fig. 1 cover the fewest years: the four generations offspring of parent group SS the most. This fact explains the differences in outcomes for different paths at a certain y given in Table 5 and related effects in Tables 6 and 7.

A conclusion in Paper I was that the discount factor for the time lag increases as the number of doses per bull increases. This conclusion has important consequences for the calculation of returns from breeding plans. The conclusion follows from the higher contribution of Path SD to returns than to genetic improvement while for Path SS the opposite is true. Table 8 shows the weighing of paths in ΔG. Tables 6 and 8 show that the increase in discount factor at increasing number of doses holds irrespective of calculation of discounted expressions per cow: based upon four generations or on 25 years.

3.2.3 Discounted expressions per cow for meat traits (Paper III)

Table 9 shows the number of years for which the (cumulative) discounted expressions based on years equal those based on four generations dependent on y, the proportion of dual-purpose inseminations with young bulls, and on k, the proportion of beef crossing. The difference between years in Table 9 and comparable data in Table 5 is about 5. This difference arises because discounted expressions for meat are calculated at the birth of calves (see Appendix 3) and because animals express their genetic superiority for milk at about five years of age in average.

Table 9. The year in which the cumulative discounted expressions per cow for meat traits calculated on a year basis equal those calculated on a four-generation basis. Interest rate = 10%. y is the proportion of dual-purpose inseminations with young bulls. k is the proportion of beef crossing.

y	Path SS			Path SD			Path YB (or DS)		
	k = 0.0	k = 0.1	k = 0.2	k = 0.0	k = 0.1	k = 0.2	k = 0.0	k = 0.1	k = 0.2
0.10	25	25	25	22	22	21	23	23	23
0.20	25	25	24	22	21	21	23	22	22
0.30	24	24	24	21	21	21	22	22	22
0.50	23	23	23	21	21	20	21	21	20
0.70	22	22	22	20	20	20	20	19	19
0.90	21	21	21	20	20	20	18	18	18

Table 10. The ratios between discounted expressions per cow for milk traits and for meat traits. Period considered 25 years, interest rate = 10%. y is the proportion of dual-purpose inseminations with young bulls. k is the proportion of beef crossing.

y	Path SS		Path SD		Path YB	
	k = 0.0	k = 0.2	k = 0.0	k = 0.2	k = 0.0	k = 0.2
0.10	0.847	1.082	0.960	1.213	0.917	1.160
0.50	0.841	1.072	1.052	1.331	0.904	1.143
0.90	0.868	1.106	1.184	1.493	0.926	1.170

The ratios between discounted expressions for milk traits and those for meat traits both based on 25 years are given in Table 10. The actual ratios will be slightly higher, dependent on slaughter age of beef animals.

The ratios based on 25 years given in Table 10 will be lower than on a four-generation basis (Paper III), as follows from Tables 5 and 9. The ratios for Paths SD, DS and YB based on 25 years are approximately 7% lower than when based on four generations. For Path SS this is 13%. The conclusions in Paper III (see Chapter 2) about tendencies in the ratio between discounted expressions per cow for milk and meat traits are not affected by the method of calculating discounted expressions.

3.2.4 Optimum breeding plans for milk traits (Paper II)

For management system A (deep-frozen semen storage) net returns for breeding plans are recalculated with discounted expressions based on 25 years. In Paper II the calculations were based on the four-generations approach. Cost alternatives 1 and 7 were studied (see Table 1). Main conclusions from Paper II are still valid. Some minor changes, however, warrant further discussion.

Optimum and suboptimum breeding schemes based upon four generations were characterized by a high number of doses stored per bull (e.g. for a population size of

400000 and a production of 25000 doses per bull per year the optimum number of doses stored per bull was 135000). Further these schemes are characterized by a progeny group size exceeding 100 with y between 0.20 and 0.30. For cost alternatives with relatively high costs of semen production and storage, suboptimum schemes were characterized by a low number of doses per bull (3000 – 10000). See Fig. 1, Paper II. For these schemes the proportion of inseminations with young bulls was high ($y = 0.90$).

Compared to the 25-year approach, discounted expressions based on four generations for all paths are overestimated when y is low and underestimated when y is high. The situation will be the same for returns. This explains why on a 25-year basis y is higher than on a four-generation basis for optimum and suboptimum schemes. The order of size of this change in optimum y is 0.10.

Also the difference in net returns between optimum schemes with low y (and a high number of doses stored), and schemes with high y (and a low number of doses stored) will be somewhat smaller when returns are calculated on a 25-year basis compared with the four-generations approach. For the alternatives studied this difference remained positive.

Suboptimum schemes for Cost alternative 7 (below a certain cost level) are characterized by low number of doses per bull, both when returns are calculated over four generations or over 25 years. When calculations of returns are based on 25 years, suboptimum schemes with high y and a low number of doses stored will be found at cost levels where the four-generations approach gives suboptimum schemes with low y and a high number of doses stored. Compare Fig. 1, Paper II.

4 General discussion

– Fig. 1 shows the structure of the evaluation of returns from a breeding scheme. The position of Path DS in this figure has been discussed in Paper I. The position of Path SS and, connected with that, the question which costs account for which returns, warrant further discussion.

In a breeding programme the first step is the purchase of a batch of young bulls. Returns are evaluated from genetic superiority expressed in progeny of this batch of young bulls. Path DS is represented by the dams of this batch, Path SD and SS are represented by bulls selected from the batch. Costs for the breeding scheme are costs associated with the batch of young bulls: selection of bull dams, purchase of bulls, sperm production, progeny testing etc. So proven bulls, including bull sires, can not be obtained without incurring all costs (except for bull dam selection) for the scheme. For the selection of bull dams, however, only milk recording is needed.

In this setup net returns (returns minus costs) from a breeding scheme (associated with one batch of young bulls) are the same whether the breeding scheme is in an initial stage or whether it has been in operation for a long time. However, if a breeding scheme has been in operation for many years, it may be tempting to consider the sires of a batch of young bulls as Path SS (i.e. the position of Path SS is then the same as the position of Path DS in Fig. 1). So return calculations will give very different results, as the discounted expressions per cow for Path SS then equal those for Path DS. However costs of an earlier batch of young bulls should also be considered and costs associated with one batch of young bulls should be allotted to Path SS or SD. This distribution of costs is unrealistic because selection of proven bulls including

Table 11. Time period considered and interest rate adopted.

Authors	Period	Interest rate (%)
Lindhé (1968)	∞	10
Hinks (1971)	2 generations	8
Hill (1971)	20 years	8.15; 20
McClintock & Cunningham (1972)	15 years[1]	8
Niebel (1974)	25 years	8
Peterson et al. (1974)	∞	10

1. These authors evaluate returns from 10 years of progeny of proven bulls. This period covers about 15 years of progeny of young bulls.

bull sires is part of one operation.

– In calculating discounted expressions per cow or monetary returns four generations of offspring of selected parents were included. In the previous sections calculations were based upon 25 years. An interest rate of 10% was used. In Table 11 the period considered and the interest rate adopted by different authors are given. The infinite number of years considered by Lindhé (1968) followed from his criterion to detect an optimum breeding scheme (see Paper I, discussion). Peterson et al. (1974) used the same criterion. An argument against taking a lot of years or generations is the uncertainty of future returns (McClintock & Cunningham, 1972). This uncertainty is also noted by Poutous & Vissac (1962). One way to cope with this uncertainty is by adopting a higher interest rate (Lindhé, 1968).

In the literature, and also in this thesis, the nominal interest rate (e.g. for mortgage loans) has always been chosen. Returns from breeding schemes are obtained over a long period. During this period, in many countries the net milk price will follow the rate of inflation. So interest rates might be adopted excluding inflation.[1] This real interest rate is about 2 to 3%.

When an interest rate for the evaluation of returns from breeding schemes is chosen, the following should be considered:

1. What is the real interest rate?
2. To what extent will the net value of products (e.g. milk) follow the inflation rate?
3. How can uncertainty in predictions of future returns be dealt with?

The real interest rate can be seen as a basic interest rate. The other two considerations will modify the basic interest rate.

Probably the net value of products will not quite follow the inflation rate. Or, if inflation is zero, the net value of one unit of product tends to decrease because more efficient production, e.g. by better organization or technical improvement, tends to result in smaller margins. Uncertainty, for instance caused by change in preference of the consumer, in predictions of future returns is smaller for returns early in time than for later returns. So this uncertainty can better be dealt with by choosing a higher interest rate than by adopting a constant loss factor.

The effects of choice of interest rate and time period are shown in Table 12 for discounted expressions per cow for milk traits. Returns calculated with an interest rate of 5%, including 25 years, are roughly twice as high as with an interest rate of 10%. A zero interest rate leads to values about four times as high (Table 12). The influence of time period considered on discounted expressions depends both on path and interest rate. For example, if we consider only 10 years, Path SS hardly contributes to returns, whereas via Paths SD, DS and YB 30 to 50% of returns via these paths are attained in this period compared with 25 years (interest rate 10%). Further examination of Table 12 shows substantial increase of discounted expressions from 25 to 50 years, especially for Path SS, even at the interest rate of 10%.

Returns from breeding schemes over 10, 15, 20 and 50 years in addition to those over

1. This argument was brought to my attention by Dr J. H. Renkema.

Table 12. Relative discounted expression per cow influenced by the number of years considered and interest rate r (%). Discounted expressions are given relative to classes with 100 (underlined).

Proportion of inseminations with young bulls is 10%

Years	Path SS r = 0	5	10	15	Path SD r = 0	5	10	15	Path DS or YB r = 0	5	10	15
10	~0	~0	~0	~0	99	61	39	26	81	52	34	22
15	67	33	17	9	231	131	77	47	192	109	65	40
20	360	153	69	32	316	166	92	53	319	161	87	50
25	653	247	<u>100</u>	43	392	190	<u>100</u>	56	439	200	<u>100</u>	54
50	2044	472	144	53	788	254	113	59	1034	296	119	59

Proportion of inseminations with young bulls is 90%

Years	Path SS r = 0	5	10	15	Path SD r = 0	5	10	15	Path DS or YB r = 0	5	10	15
10	3	2	1	1	93	58	37	24	99	68	49	36
15	166	86	46	26	221	125	73	44	191	117	75	50
20	358	164	79	40	320	166	91	52	282	153	90	57
25	553	226	<u>100</u>	48	408	193	<u>100</u>	55	372	182	<u>100</u>	60
50	1566	391	133	55	838	263	114	59	827	256	115	64

25 years have been calculated for the same alternatives as in Section 3.2 (interest rate 10%). Calculation of returns over 20 and 50 years gives a ranking of breeding schemes with respect to returns very similar to the ranking at 25 years. For 10 and 15 years the optimum cost level is lower than for 25 years, while for Cost alternative 7 with calculation of returns over 10 and 15 years, optimum schemes were characterized by low number of doses stored per bull. This finding is similar to the conclusion for sub-optimum schemes when 25 years are considered (Section 3.2).

- Uncertainty of costs for a breeding scheme can not be compensated for in the same way as that of returns. Most costs for a breeding plan are incurred before there are any returns and also the period over which returns are to be expected is much longer than that of investment. In Paper II, the effect of variation in cost factors on the optimum breeding plan was studied. This analysis covered the actual situations in different AI organizations or countries, but at the same time revealed the sensitivity of the model for changes in costs.
- Calculation of discounted expressions and of returns is based upon a fixed population structure. Dissemination of genes, however, may be influenced by the breeding value of individual bulls or may change for other reasons. Also the population size

may change with time (see Paper I). Further, predictions of genetic superiority and selection responses are expectations. Deviations by chance of individual bulls from the expectation may influence the intensity of their use and the dissemination of their genes. Also the average breeding value of a group of parents generally will differ from the expectation. Part of these effects can be met by applying Monte-Carlo methods (e.g. Simon, 1969).

Many replicates of each breeding programme should be run and the average result will probably not differ much from deterministic-model approaches. It gives, however, insight into the range of predictions, and further into the distribution of returns around the expectation.

A related problem is: how large a difference between net returns of two breeding programmes is a 'real' difference. To answer this question the standard deviation of predictions, obtained by Monte-Carlo methods, can be applied. On the other hand it may be argued that if one scheme has higher net returns than another, the probability of actually attaining higher net returns is over 50%; this makes the scheme with highest net returns the most worthwhile choice. However, distribution of returns possibly is not symmetric, and the range of predictions may be different from one breeding scheme to another.

Summary

The effect of costs for AI breeding on the optimum breeding plan for milk yield and the profitability of performance-test selection for meat traits within a dual-purpose breed of cattle were studied in three papers. Methods and results given in these papers are summarized in Chapter 2.

Returns from breeding schemes were calculated with a generation approach, i.e. the expression of genetic improvement in subsequent *generations* was the basis for the estimation of returns. An interest rate of 10% was adopted.

The method of Hill (1974) was used to calculate returns with a year approach in which the expression of genetic improvement in subsequent *years* was the basis. Both methods were compared by calculation of discounted expressions per cow, for different pathways, over 4 generations with the generation approach and over 25 years with the year approach. Major conclusions, summarized in Chapter 2, did not change though discounted expressions per cow, and returns too, calculated by either method differed systematically. For Paths SS and DS, but less for SD, discounted expressions per cow were found to be overestimated by the generation approach compared with the outcomes of the year approach when the proportion of inseminations with young bulls (y) was low, and underestimated when y was high (Tables 6, 7 and 9).

To study the effects of the interest rate and time period, discounted expressions per cow were calculated for interest rates of 0, 5 and 10%, with the year approach. The discounted expressions were calculated over 10, 15, 20, 25 and 50 years (Table 12). At 10% interest rate ranking of breeding schemes with respect to returns was similar if returns were calculated over 20, 25 or 50 years. Calculating returns over 10 or 15 years, however, resulted in optimum breeding schemes characterized by a lower cost level.

Samenvatting

Het onderzoekterrein van de rundveefokkerij kan opgesplitst gedacht worden in drie deelgebieden. Het eerste deelgebied betreft de definitie van een fokdoel. Er zijn vele kenmerken die van belang zijn, en het definiëren van een fokdoel of samengesteld genotype komt neer op het vinden van relatieve (economische) waarden van kenmerken waarop geselecteerd moet worden. Voor Duitse omstandigheden is dit probleem onderzocht door Niebel et al. (1972). Uit hun resultaten kan geconcludeerd worden dat melkproduktie (en -bestanddelen) benevens groei per dag, dan wel voederconversie, de belangrijkste elementen zijn in het fokdoel.

Het tweede gebied beschouwt de fokwaardeschatting. De analyse en correctie van (veld)gegevens valt hieronder; deze kunnen bijdragen tot een betrouwbare schatting van de fokwaarde van dieren voor kenmerken in het fokdoel.

Het derde gebied heeft als vraagstelling: hoe moet het fokprogramma georganiseerd worden opdat het selectieresultaat voor de populatie zo groot mogelijk wordt. Met runder-k.i. kan een hoog selectieresultaat behaald worden doordat hij een nauwkeurige fokwaardeschatting van stieren mogelijk maakt voor kenmerken die aan de stieren zelf niet te meten zijn. Verder kunnen goed vererende stieren op ruime schaal gebruikt worden.

Het onderwerp van dit proefschrift valt binnen het derde gebied: optimalisatie van fokprogramma's in een populatie met kunstmatige inseminatie. Met andere woorden, welke opzet van het fokprogramma levert een zo groot mogelijk selectieresultaat. De vraagstelling is beperkt tot melkproduktie (en -bestanddelen), terwijl voor vleesproduktiekenmerken een algemene benadering is gevolgd, met een uitwerking voor het kenmerk groei per dag voor Nederlandse kostenverhoudingen. Een fokprogramma heet optimaal wanneer de netto inkomsten uit het fokprogramma maximaal zijn. Het selectieresultaat wordt dus gemeten als netto inkomsten uit het fokprogramma. Netto inkomsten zijn inkomsten minus kosten voor het fokprogramma.

Het proefschrift bestaat uit drie artikelen, en een vergelijking van de daarin gehanteerde methodiek om de geldwaarde van het selectieresultaat te schatten met een methode beschreven door Hill (1974).

Het belangrijkste element in de schattingsmethode van de geldwaarde van het selectieresultaat, in genoemde artikelen, is het aantal nakomelingen van geselecteerde ouderdieren (selectiewegen) in vier opvolgende generaties. Deze nakomelingen erven een deel van de genetische superioriteit van ouderdieren en uiten deze in verbeterde produktie. Zie hiervoor fig. 1. Als selectiewegen worden beschouwd: SS, stieren om stieren te fokken; stieren om dochters te fokken (proefstieren YB and fokstieren SD) alsmede DS, koeien om stieren te fokken. De selectieweg: koeien om dochters te fokken (DD) is buiten beschouwing gelaten voor het berekenen van inkomsten uit fokprogramma's omdat de opzet van het fokprogramma de selectie via deze laatste selectieweg niet beïnvloedt.

Daar nakomelingen van geselecteerde ouders op zeer uiteenlopende momenten produceren, is het nodig van de toekomstige inkomsten de huidige (contante) waarde te berekenen. Als referentiejaar is gekozen het jaar van geboorte van een jaargang proefstieren. Er is een rentevoet van 10% gehanteerd.

De methode om inkomsten uit selectie op melkproduktie uit een fokprogramma te schatten is beschreven in het eerste artikel. De relatieve bijdrage van de selectiewegen SD en DD aan de inkomsten bleek hoger te zijn dan hun relatieve bijdrage aan de erfelijke vooruitgang per jaar (ΔG). Voor de selectieweg SS geldt het tegenovergestelde, terwijl voor selectieweg DS (en voor YB) de relatieve bijdrage aan inkomsten en aan ΔG vrijwel gelijk was. ΔG werd berekend met de formule van Rendel & Robertson (1950).

Om de relatie te leggen tussen inkomsten en ΔG, werd een contante-waardefactor (DF) gedefinieerd. Deze contante-waardefactor is de verhouding tussen de inkomsten uit een fokprogramma en de inkomsten berekend als een lineaire functie van ΔG. DF kan geïnterpreteerd worden als de contante-waardefactor voor de tijdsperiode die ligt tussen de geboorte van een jaargang proefstieren en het tijdstip waarop inkomsten tot stand komen voorvloeiend uit selectie van moeders van die proefstieren, en uit selectie van fokstieren en stiervaders uit de jaargang proefstieren. DF varieerde van 0,28 tot 0,40, wat betekent dat genoemde tijdsperiode ligt tussen circa 9 en 13 jaar. De belangrijkste invloedsfactor op DF bleek te zijn het aantal inseminaties dat verricht wordt per fokstier (d.i. het aantal doses dat per stier verzameld wordt). Voor 3000 doses per stier varieerde DF van 0,28 tot 0,30 en voor 80000 doses per stier van 0,38 tot 0,40. Andere invloedsfactoren (bij een vaste rentevoet) bleken het aandeel der eerste inseminaties dat verricht wordt met zaad van fokstieren, en de grootte der nakomelingengroepen. Gerekend werd met een populatiegrootte van 400000 koeien.

In het tweede artikel werd de invloed nagegaan van kostenfactoren op het optimale fokprogramma (voor selectie op melkproduktie). Er werden 12 kostenalternatieven gekozen (tabel 2) met uiteenlopende verhoudingen tussen kostenfactoren als kosten voor sperma-opslag en kosten voor voer.

Twee systemen zijn vergeleken, te weten het Proefstier-Wachtstier-Fokstier(PWF)-systeem, (B), zonder opslag van diepvriessperma gedurende de wachtperiode, en het systeem waarbij stieren worden geslacht zodra een vooraf bepaald aantal doses sperma per stier zijn opgeslagen, (A). Het invriezen van sperma onder Systeem A start meteen nadat proefinseminaties verricht zijn.

De populatiegrootte werd gevarieerd van 50000 tot 1 miljoen eerste inseminaties. Verder werd het aandeel der inseminaties verricht met zaad van proefstieren (y) gevarieerd van 0,10 tot 0,90, en de grootte der nakomelingengroepen van 50 tot 600.

Het aantal doses per stier werd gevarieerd van 3000 tot 220000 en de dosesproduktie per stier per jaar van 15000 tot 35000.

Voor alle beschouwde kostenalternatieven bleek Systeem A economisch aantrekkelijker dan Systeem B. Optimale fokprogramma's onder Systeem A werden gekarakteriseerd door opslag van een groot aantal doses per stier. (B.v. in een populatie van 400000 koeien en een spermaproduktie van 25000 doses per stier per jaar, bleek een opslag van 135000 doses per stier optimaal). Verder bleek de optimale waarde voor y te liggen tussen 0,20 en 0,30 en de optimale grootte van de nakomelingengroep boven de 100.

Een optimaal fokprogramma is een programma met het grootste verschil tussen inkomsten en kosten. Om echter situaties te dekken waarin men niet bereid is de bij het optimale fokprogramma behorende kosten te investeren, werden tevens suboptimale fokprogramma's gedefinieerd. Dit zijn fokprogramma's met de hoogste netto inkomsten bij een bepaald kostenniveau. Over het algemeen hadden suboptimale fokprogramma's dezelfde karakteristieke kenmerken als optimale. Uitzonderingen werden echter gevonden voor kostenalternatieven met hoge kosten voor spermaproduktie en opslag, en lage kosten voor voer (alternatieven 4, 7 en 10, tabel 2). In die gevallen bleek bij lage kostenniveau's Systeem B economisch aantrekkelijker dan Systeem A. Onder Systeem A werden dan suboptimale programma's gevonden bij een laag aantal doses per stier en een hoog aandeel der inseminaties met zaad van proefstieren.

Netto inkomsten uit optimale en suboptimale fokprogramma's stegen met de populatiegrootte. In het model was de grootte van kostenfactoren echter onafhankelijk van de populatiegrootte, zodat een optimale populatiegrootte niet kon worden vastgesteld.

Het derde artikel handelt over de rentabiliteit van eigen-prestatietoets op vleesproduktiekenmerken. Behalve de optimale selectiescherpte na eigen-prestatietoets, werd de optimale weging van melk- en vleesproduktiekenmerken in het samengesteld genotype bestudeerd. Daartoe werd het begrip 'discounted expression per cow', 'contante waarde per koe', geïntroduceerd. De contante waarde per koe voor kenmerk i en selectieweg j is gedefinieerd als de naar huidige waarde berekende inkomsten voortvloeiend uit verhoogde produktie voor kenmerk i van nakomelingen van ouders (selectieweg j) met een genetische superioriteit van één eenheid, gedeeld door de populatiegrootte. (Stel bijvoorbeeld dat voor het kenmerk melkproduktie voor fokstieren een contante waarde per koe geldt van 0,2; dit betekent dat inzet van fokstieren met een genetische superioriteit van f 100,— een totale opbrengst uit verhoogde melkproduktie van nakomelingen oplevert van f 20,—).

Verder is de invloed van gebruikskruising met vleesrassen op de rentabiliteit van eigen-prestatietoets en op de weging van melk en vlees in het samengesteld genotype onderzocht. De belangrijkste conclusies kunnen als volgt worden samengevat. De rentabiliteit van de eigen-prestatietoets hangt voornamelijk af van de selectiescherpte van stiermoeders, de relatieve economische waarde van melk- en vleesproduktiekenmerken, de kosten, en van het aandeel gebruikskruisingen. De optimale geselecteerde fractie na eigen-prestatietoets ligt over het algemeen tussen 1 op 2 en 1 op 4. Over het algemeen neemt de rentabiliteit van eigen-prestatietoets af bij toename van het aandeel gebruikskruising. De optimale weging van melk- en vleesproduktiekenmerken in het samengesteld genotype is het produkt van hun actuele economische waarde en de bijbehorende contante waarde per koe. Deze weging verschilt per selectieweg en de nadruk op de kenmerken verschuift naar melkproduktiekenmerken bij een toenemend aandeel gebruikskruising (tabel 10).

Contante waarden per koe werden tevens geschat met behulp van de methode van Hill (1974). In de drie tijdschriftartikelen berustten de berekeningen op het aantal nakomelingen van geselecteerde ouders in vier opvolgende generaties. Toepassing en uitbreiding van de door Hill (1974) beschreven methode maakt het mogelijk contante waarden per koe (en inkomsten uit fokprogramma's) te schatten op basis van het aantal nakomelingen van geselecteerde ouders in een opvolgend aantal jaren. De methodes zijn vergeleken door de inkomsten over 25 jaar te evalueren.

De conclusies van de artikelen bleken niet beïnvloed te worden door de keuze van methode: op basis van generaties of op basis van jaren. Wel vertoonden de twee methodes systematische verschillen in de contante waarden per koe. Speciaal voor de selectiewegen SS en DS, en in mindere mate voor SD, bleken de contante waarden per koe op basis van de generatiemethode overschat te zijn ten opzichte van de uitkomst van de jaarmethode, tenminste als het aandeel van de inseminaties met zaad van proefstieren laag was. Was dit aandeel hoog, dan bleken de contante waarden onderschat.

Met gebruikmaking van de jaarmethode is bestudeerd wat de invloed is van de keuze van de rentevoet en van de periode waarover men inkomsten evalueert op contante waarden per koe en op inkomsten. Bij een periode van 25 jaar zijn de contante waarden per koe bij een rentevoet van 0%, vergeleken met de uitkomst bij een rentevoet van 10%, globaal vier maal zo hoog. De invloed van de beschouwde periode op de contante waarde per koe verschilde per selectieweg. Bij een periode van 10 jaar bijvoorbeeld, bleek de selectieweg SS nauwelijks bij te dragen aan de inkomsten, terwijl de inkomsten via de selectiewegen SD, DS en YB voor elke weg 30 – 50% bedroegen van hetgeen in 25 jaar kon worden verkregen (rentevoet 10%, tabel 12). Keuze van een periode van 20, 25 of 50 jaar heeft nauwelijks invloed op de rangorde van fokprogramma's op basis van netto inkomsten, terwijl voor een kortere periode programma's met een lager kostenniveau optimaal zijn.

References[1]

Bech Andersen, B. & E. Ernst, 1972. Ergebnisse von Ultraschallmessungen an Jungbullen. Züchtungskunde 44: 81–90.

Bichard, M., 1971. Dissemination of genetic improvement through a livestock industry. Anim. Prod. 13: 209–218.

Boer, H. de, 1973. Working party on Assessment of carcass characteristics in cattle. 24th ann. meeting of EAAP, Vienna.

Brascamp, E. W. & D. Minkema, 1972. Economic aspects of selection for milk, fat-% and protein-% in a dairy cow AI population. Z. Tierz. Züchtungsbiol. 89: 99–108.

Cochran, W. G., 1951. Improvement by means of selection. Proc. 2nd Berkeley Symp. on Mathematics, Statistics and Probability. University of California Press, p. 449–470.

Cunningham, E. P. & A. E. McClintock, 1972. Selection in dual-purpose cattle populations: Effect of beef crossing and cow replacement rates. Submitted to Ann. Génét. Sél. Anim. (Ann. Génét. Sél. Anim. (1974) 6 : 227–239).

Gravert, H. O. & E. Rosenhahn, 1965. Welche Kriterien sind wirtschaftlich wichtig. Züchtungskunde 37: 244–250.

Haring, H. J. F., 1972. Die Zuchtplanung in der Rinderzucht am ökonomischer Sicht. Diss. Göttingen.

Hill, W. G., 1971. Investment appraisal for national breeding programmes. Anim. Prod. 13: 37–50.

Hill, W. G., 1974. Prediction and evaluation of response to selection with overlapping generations. Anim. Prod. 18: 117–140.

Hinks, C. J. M., 1970. The selection of dairy bulls for artificial insemination. Anim. Prod. 12: 569–576.

Hinks, C. J. M., 1971. The genetic and financial consequences of selection amongst dairy bulls in artificial insemination. Anim. Prod. 13: 209–218.

Knaack, J., H. Nehring & G. Lorenz, 1973. Neue Ergebnisse der experimentellen willkürlichen Geschlechtsbeeinflussung beim Rind. Tierzucht 27: 156–159.

Lindström, U., 1971. Some points of view on the estimation and importance of genetic change in populations of dairy cattle. Ann. Génét. Sél. Anim. 3: 161–168.

Lindhé, B., 1968. Model simulation of AI breeding within a dual purpose breed of cattle. Acta Agric. scand. 18: 33–41.

Mason, I. L., V. E. Vial & R. Thompson, 1972. Genetic parameters of beef characters and the genetic relationship between meat and milk production in British Friesian cattle. Anim. Prod. 14: 135–148.

McClintock, A. E. & E. P. Cunningham, 1972. Selection in dual purpose cattle populations: Defining the breeding objective. Submitted to Anim. Prod. (Anim. Prod. (1974) 18: 237–247).

Niebel, E., 1974. Methodik der Zuchtplanung für die Reinzucht beim Rind bei Optimierung nach Zuchtfortschritt und Züchtungsgewinn. Diss. Hohenheim.

Niebel, E. & D. Fewson, 1971. Methodik der Zuchtplanung für die Reinzucht beim Rind. LH Hohenheim; Inst. f. Tierzucht.

Niebel, E., A. Rittler & D. Fewson, 1972. Die Leistungsmerkmale beim Rind. Teil B: Selektionswürdigkeit der Leistungsmerkmale. Stuttgart, Ulmer.

1. Includes the references of the papers.

Oscarsson, G., 1968. Mjølk- og kjøttproduksjonen i 1970-åra. Produksjonsutvecklingen. Buskap og Avdrått 2: 175–179.

Oscarsson, G. & B. Lindhé, 1970. Spermakostnader i seminverksamheten. SHS Meddelande 36.

Petersen, P. H., L. G. Christensen, B. Bech Andersen & E. Ovesen, 1974. Economic optimisation of the breeding structure within a dual-purpose cattle population. Acta Agric. scand. 24: 247–259.

Poutous, M. & B. Vissac, 1962. Recherche théorique des conditions de rentabilité maximum de l'épreuve de descendance des taureaux d'insémination artificielle. Ann. Zootech. 11: 233–256.

Rendel, J. H. & A. Robertson, 1950–1952. Estimation of genetic gain in milk yield by selection in a closed herd of dairy cattle. J. Genet. 50: 1–8.

Simon, D. L., 1969. Erstellung und Einsatz einer simulierten Rinderpopulation als Instrument der Tierzüchtungslehre 1. Methodik, Programm und Auswertungsverfahren. Z. Tierz. Züchtungsbiol. 86: 101–126.

Skjervold, H., 1966. 9th International Congress of Animal Production, Edinburgh. Edinburgh, Oliver and Boyd, p. 250–261,

Skjervold, H. & H. J. Langholz, 1964. Factors affecting the optimum structure of Al breeding in dairy cattle. Z. Tierz. Züchtungsbiol. 80: 25–40.

Smith, C., 1969. Optimum selection procedures in animal breeding. Anim. Prod. 11: 433–442.

Syrstad, O., 1971. Selektion for proteininhalt i mjølk. Meld. Nor. Landbrukshøgsk. 50: 27.

Appendix 1 Multiplication matrix P (for milk traits)

Sire to breed son A generation interval of $6\frac{3}{4}$ years is assumed (Paper I, page 4). It is therefore appropriate to define 7 age classes for males. Reproduction is $\frac{1}{4}$ in Age Class 6 and $\frac{3}{4}$ in Age Class 7. So males in Age Class 1 in a certain year contain $\frac{1}{8}$ of the genes of males in Age Class 6 in the previous year and $\frac{3}{8}$ of the genes of males in Age Class 7 in the previous year.

Sire to breed daughter, Proven bulls (SD) A generation interval of $6\frac{3}{4}$ year is assumed. Reproduction is $\frac{1}{4}$ in Age Class 6 and $\frac{3}{4}$ in Age Class 7. With semen of proven bulls a fraction (1—y) of the inseminations is performed (Paper I, page 3). The probability that a first insemination results in a female replacement is 1/C. For proven bulls this probability is **B** (Paper I, page 3). Thus females in Age Class 1 in a certain year inherit $\frac{1}{8}$ BC $(1-y)$ of their genes from males in Age Class 6 in the previous year and $\frac{3}{8}$ BC $(1-y)$ from males in Age Class 7.

Sire to breed daughter, Young bulls (YB) A generation interval of $2\frac{3}{4}$ years is assumed. Young bulls perform a proportion y of the inseminations and the probability that a first insemination results in a female replacement is A. Females in Age Class 1 in a certain year inherit $\frac{1}{8}$ ACy of their genes from males of Age Class 2 of the previous year, $\frac{3}{8}$ ACy from males in Age Class 3.

Dam to breed son A generation interval of 6 years is assumed. Defining the proportion of sons (young bulls) reproduced by females in Age Class i in Matrix **P** as ds(i), the generation interval will be

$$L_{DS} = \sum_{i=1}^{i=n} i\,ds(i), \text{ where n is the number of female age classes.}$$

In Paper I, Table 1, 7 lactations (8 age classes) are considered. If the assumed generation interval is used unrealistic values of ds(i) will be found. Therefore in the year approach 13 age classes for females are assumed. The values for ds(i) are given below.

Age Class i	1	2	3	4	5	6	7	8	9	10	11	12	13
ds(i)	0	0	0.22	0.18	0.14	0.11	0.08	0.07	0.05	0.04	0.04	0.04	0.03

Males of Age Class 1 in a certain year receive a fraction $\frac{1}{2}$ ds(i) from females in Age Class i in the previous year.

Dam to breed daughter A generation interval of $4\frac{1}{2}$ year is assumed. The proportion of replacement daughters from females in Age Class i is defined as dd(i). Values chosen for dd(i) are given below.

Age Class i	1	2	3	4	5	6	7	8	9	10	11	12	13
dd(i)	0	0.25	0.22	0.15	0.10	0.08	0.06	0.05	0.03	0.02	0.02	0.01	0.01

Appendix 2 Vectors g and h

The proportion of cows in first lactation is $1/C$ ($= 0.30$). The fractions of cows in different age classes are given in the table below.

Vector element	Female age class i	g	Relative production, average = 1	Genetic correlation between first and later lactations	h
8	1	0			0
9	2	0.30	0.8319	1.0	0.2496
10	3	0.20	0.9812	0.8	0.1570
11	4	0.15	1.0807	0.8	0.1297
12	5	0.10	1.1145	0.8	0.0892
13	6	0.07	1.1225	0.8	0.0629
14	7	0.05	1.1225	0.8	0.0449
15	8	0.04	1.1225	0.8	0.0359
16	9	0.03	1.1225	0.8	0.0269
17	10	0.02	1.1225	0.8	0.0180
18	11	0.02	1.1225	0.8	0.0180
19	12	0.01	1.1225	0.8	0.0090
20	13	0.01	1.1225	0.8	0.0090

The phenotypic value of Age Class i equivalent to 1 unit genetic superiority in heifers depends on the genetic correlation between Lactation (i—1) and Lactation 1. Further it depends on age effects. Both are given in the above table. These values are based on the assumptions in Paper I, Table 1. Elements 1 to 7 of vectors **g** and **h** are zero.

In the approach here a lactation starts at the beginning of the year. For the calculation of the monetary value of lactation yield, discounting for a period x is needed. This period x is the time between start of lactation and the moment that half the lactation yield has been produced. To be in agreement with Paper I, Table 1, the following expression should hold:

$$\sum_{i=2}^{13} h(i) \left(\frac{1}{1 + 0.10}\right)^{i+x} = 0.5474$$

Therefore x is put at 0.4879 (years).

By the definition of Matrix **P** a cow calves first at two years of age and the calving interval is one year. Choice of elements in **P** gives realistic generation intervals, while the figure 0.4879 adjusts for the fact that a cow starts her first lactation at approximately 26 months on average and that half the lactation yield is produced about 4 months later.

Appendix 3 Multiplication matrix M (for meat traits)

'*Sire to breed daughter*' Calves contain a fraction $\frac{1}{2}$ of genes from sires. These sires can be proven bulls, young bulls, both of the dual-purpose (milk/meat) breed, or bulls of beef breeds. Below are tabulated groups of calves and the source of their genes. In this table k is the fraction of beef crossing; s is the proportion of dual-purpose calves surviving to productive age; s′ is the proportion of crossbred calves surviving to productive age.

Group of calves	Frequency	Genes from: young bulls	proven bulls	beef breed
All	1	$\frac{1}{2}y(1-k)$	$\frac{1}{2}(1-y)(1-k)$	$\frac{1}{2}k$
Dead before productive age				
purebred	$(1-k)(1-s)$	$\frac{1}{2}y$	$\frac{1}{2}(1-y)$	0
crossbred	$k(1-s')$	0	0	$\frac{1}{2}$
Replacement females	$1/C$	$\frac{1}{2}ACy$	$\frac{1}{2}BC(1-y)$	0
Calves for slaughter	$ks'+(1-k)s-1/C=S$	p_1	p_2	p_3

So for young bulls:

$$\tfrac{1}{2}y(1-k) = (1-k)(1-s) \times \tfrac{1}{2}y + k(1-s') \times 0 + 1/C \times \tfrac{1}{2}ACy + S \times p_1$$

$$p_1 = y((1-k)s - A)/2S$$

For proven bulls:

$$p_2 = (1-y)\ ((1-k)s-B)/2S$$

For bulls of beef breeds:

$$p_3 = ks'/2S$$

Calves surviving to slaughter age (with Age 1 in a certain year) inherit $\frac{1}{4}p_1$ of their genes from dual-purpose males of Age Class 2 in the previous year, $\frac{3}{4}p_1$ from those in Age Class 3, $\frac{1}{4}p_2$ from Age Class 6 and $\frac{3}{4}p_2$ from Age Class 7.

'Dam to breed daughter' Below are tabulated groups of calves and the source of their genes. In this table k_i is the proportion beef crossing performed with females in Age Class i; s_i is the proportion of purebred calves from dams of Age Class i surviving to slaughter age; s_i' is proportion of crossbred calves from dams of Age Class i surviving to slaughter age.

Group of calves	Frequency	Genes from dual-purpose females in Age Class i
All	1	$\frac{1}{2}g(i)$ (See Appendix 2)
Dead before productive age	$1-ks'-(1-k)s$	$\frac{1}{2}g(i)\,(1-k_is_i'-(1-k_i)s_i)\,/\,(1-ks'-(1-k)s)$
Female replacement	$1/C$	$\frac{1}{2}dd(i)$ (See Appendix 1)
Calves for slaughter	$ks'+(1-k)s-1/C = S$	p_i

So:

$$\tfrac{1}{2}g(i) = \tfrac{1}{2}g(i)\,(1-k_is_i'-(1-k_i)s_i) + 1/C \times \tfrac{1}{2}dd(i) + S \times p_i$$

$$p_i = (g(i)\,(k_is_i' + (1-k_i)s_i) - dd(i)\,/C)\,/2S$$

Assuming that $s_i = s$ and $s_i' = s'$ and (as in Paper III), $s = s'$, p_i reduces to

$$p_i = (g(i)\ s - dd(i)\,/C)/2S$$

where S reduces to $s - 1/C$.

For dual-purpose females the gene contribution to slaughter calves is given by p_i. In the calculations of discounted expressions for meat traits the reduced formula for p_i is used.

The discounted expression per cow at birth of calves for Path j and an isolated Year t can be calculated as

$$\varepsilon_{jt} = \mathbf{l}_j'(t+1)\ \mathbf{z}\left(\frac{1}{1+r}\right)^t$$

ε_{jt} is calculated with $\mathbf{l}_j\ (t+1)$, because this vector gives the gene makeup of slaughter calves of Age 1 in Year t+1, or birth date in Year t. So the discounting is done to the value at birth of calves as in Paper III.

Vector **z** contains zeroes, except element 8. This element equals S.