

# On Recursive Estimation in Environmental Systems Analysis

J.D. Stigter<sup>a</sup>

<sup>a</sup>Systems and Control - Wageningen University and Research Centre  
Bornsesteeg 59, 6708 PD Wageningen (The Netherlands)  
hans.stigter@wur.nl

Recursive estimation is a well-established field in systems and control theory starting with the celebrated work of Rudolf Kalman in the early sixties that includes an *optimal* state reconstruction algorithm for linear systems. The extension to the non-linear problem of state-parameter reconstruction followed soon by Kob and Orford, presenting a *sub-optimal* solution to the non-linear state-parameter estimation problem, known as the extended Kalman filter. Beck and Young have been one of the first to apply this algorithm in an environmental setting.

The inherent problem in environmental systems analysis is that the model is usually ill-defined, the data are only sparsely available (both in time and space), and the mechanisms underlying the dynamical behavior of the observations are complex and not always well-understood [4]. Moreover, it is well known that the extended Kalman filter algorithm itself exhibits bad convergence properties if the noise characteristics of the measurement and system noise model are not well known. Clearly, in an environmental setting the problem of specifying the spectral density matrix of the system noise time series  $\{\xi(t), t > 0\}$  is virtually impossible. Hence, the need for a modified algorithm in which these requirements are less demanding is easily motivated.

Stigter [5] has developed on the basis of Ljung's analysis of the extended Kalman filter (Ljung's analysis of the EKF was the first one since its development) a continuous-discrete type of algorithm based on the so-called innovations format of the system model, including a parametrization of the steady-state Kalman gain matrix associated with a feedback of the observed mismatch between model and predicted observations. Some results generated with this algorithm for several environmental case studies will be presented.

In addition, the on-line estimates of the parametric output sensitivities that become available in the recursive framework make clear on which part of the data record a parameter is or is not identifiable and this opens the question of optimal input design, i.e. how should the input sequence be manipulated in order to have the maximal information content in the observations with respect to one or more parameters in the model structure. Although, admittedly, the issues raised in the presentation are well-known problems in the systems identification literature, they are still interesting subjects of study worth the attention, especially so in an environmental setting.

## References

- [1] Beck, M.B., and Young, P.C. (1976). Systematic Identification of DO-BOD Model Structure. Journal of the Environmental Engineering Division, 102, EE5:902–927.
- [2] Kopp, R.E., and Orford, R.J., (1963). Linear Regression Applied to System Identification and Adaptive Control Systems, AIAA Journal, 1(10):2300–2306.
- [3] Ljung, L., (1979). Asymptotic Behaviour of the Extended Kalman Filter as a Parameter Estimator for Linear Systems, IEEE Transactions on Automatic Control, AC24:36–50.
- [4] Young, P.C., (1978). General Theory of Modeling for Badly Defined Systems
- [5] Stigter, J.D., (1997), The Development and Application of a Continuous-Discrete Recursive Prediction Error Algorithm in Environmental Systems Analysis, PhD-Thesis, University of Georgia, Athens, GA, USA.