WORKING PAPER

Risk and Uncertainty in OR models which can be used in agriculture.

This paper serves as a discussion paper and a starting point for further research on the subject of risk aspects in optimisation models that can be used in agriculture.

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I. Concept of risk and uncertainty.

Despite the many papers written on this subject, the discussions and the conferences, there is no definition of risk everybody agrees upon. Uncertainty is a very important aspect of life. We will not mention dull examples here.
In optimisation models this uncertainty is often built in as probabilities in relation to possible states of nature. So here uncertainty is just caused by nature. However, not all uncertain factors are treated this way.

Another viewpoint is that uncertainty exists due to lack of knowledge. Indeed if we knew everything in advance, there would not have been any uncertainty.
To apply this to the field of optimisation, let us consider three uncertain factors which occur in the formulation of a certain optimisation model:

- We are not sure what will be the right hand side of certain inequality constraints e.g. the budget.
- What is exactly the average evaporation of the soil. A technical factor influencing the relations in the model.
- What will be the weather in the planning period of the model.

All three factors are indeed related to lack of knowledge, though their character is different. The factor of the budget depends on behaviour of other people and after some time we really know what the budget will be. We apply sensitivity analysis and study the behaviour of the shadow price. The behaviour of other people is in fields as game theory, economics and in general social sciences also described with probabilities.

The technical factor of evaporation is not known exactly and it will not be known, unless some study is done on the technical details of the soil under consideration. Also here sensitivity analysis can be used to study the sensitivity of the model for this parameter. In this sense an impression is created of the relevance of an additional study on this technical factor. To get this insight can be an additional objective of the use of the optimisation model.

Whether it is going to rain tomorrow can be described with a probability. With this kind of uncertainty it appears to be not worth the trouble to make an exact prediction, but stochastic variables are introduced to describe reality. Apparently stochastic variables satisfy very well in the description of reality in optimisation models, in particular in discrete simulation models.

Still risk is not defined. Risk includes the attitude of people towards this uncertainty and this brings in the subjective aspect. Risk is defined by Cooper and Chapman [6] as exposure to the possibility of economic or financial loss or gain, physical damage or injury, as a consequence of the uncertainty associated with pursuing a particular course of action. In general we will call risk simply the possibility of bad outcomes caused by uncertain events. Much is done by economists and psychologists to measure the attitude towards uncertainty. In this attempt the terms risk perception, how is the risk experienced, and risk preference are distinguished.
It is also hard to see this attitude disconnected from subjective probabilities. See e.g. the interesting study of B. Verplanken on attitudes and subjective probabilities towards the use of nuclear energy

before and after Chernobyl in the Netherlands [35].

In our study we don't want to consider this subjective aspect of probabilities but describe the stochastic nature of the outcomes and link this with various risk criteria which appear to represent different descriptions of attitudes towards uncertainty, various risk perceptions. In practice various criteria and formulations have been applied and compared e.g. by Pintér et. al. in the field of water quality control. See Somlyódy and Pintér [31].
We will discuss risk criteria on a conceptual base and see which criteria are used as a tool in applications.

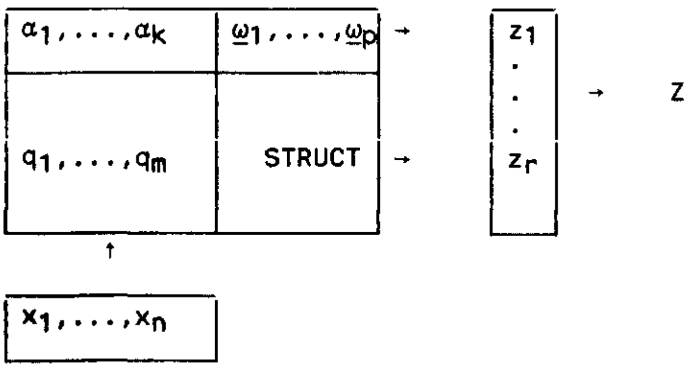
II. Risk and optimisation. A general model.

In this section we will try to come to a general description of optimisation models regardles the technique that is used to find the optimal decision. It should be kept in mind that focus is on this model in a stage just before the optimisation process. The optimisation is namely only one stage in the decision support process. In the formulation of such a model already decisions are made on which objectives are hidden in the constraints, which are left out and which appear in the objective function(s).

Another formulation decision that has been made is which uncertain factors are expressed in stochastic variables and which factors are taken deterministic and only taken in consideration as uncertain elements later on, like the technical factor we mentioned in section I. The optimisation can also be seen as a stage in a hierarchical decision process in which f.i. the budget we mentioned earlier can be regarded as a decision on a higher level.

The various items of an optimisation model are represented in nearly every handbook of Management Science/OR. See e.g. Dannenbring & Starr [7]. The model consists at least of a more or less detailed description of reality. The entrance of the model is formed by the decision variables, which we will call $x_1,\ldots,x_n$. We can think of continuous variable or integer variables like in combinatorial problems or there exist perhaps a finite number of possible settings of $x_1,\ldots,x_n$. To describe the system some internal dependent variables are necessary which can only be changed indirectly say $q_1,\ldots,q_m$. We can think e.g. of inventory in production-inventory models where decision variables consists of the production scheme over time and inventory is developing via the balance equation:

$$It = I_{t-1} + Prod_t - Demand_t.$$



Next to this, as already mentioned, an important tool to describe reality is found in including stochastic exogenous events, $\underline{\omega}_1,\ldots,\underline{\omega}_p$, in the model. In inventory models we can think of demand. In agriculture the main uncertain factors that are described are formed by the uncertainty in the weather, in occurance of pests or in more higher aggregated and economic models by the uncertainty in prices of inputs and outputs which is also related to the stochastic nature of yield. See e.g. Hazell 1979 [10]. Another important factor in the description of reality is formed by the parameters, here called $\alpha_1,\ldots,\alpha_k$. In linear programming we think of the right hand sides, the

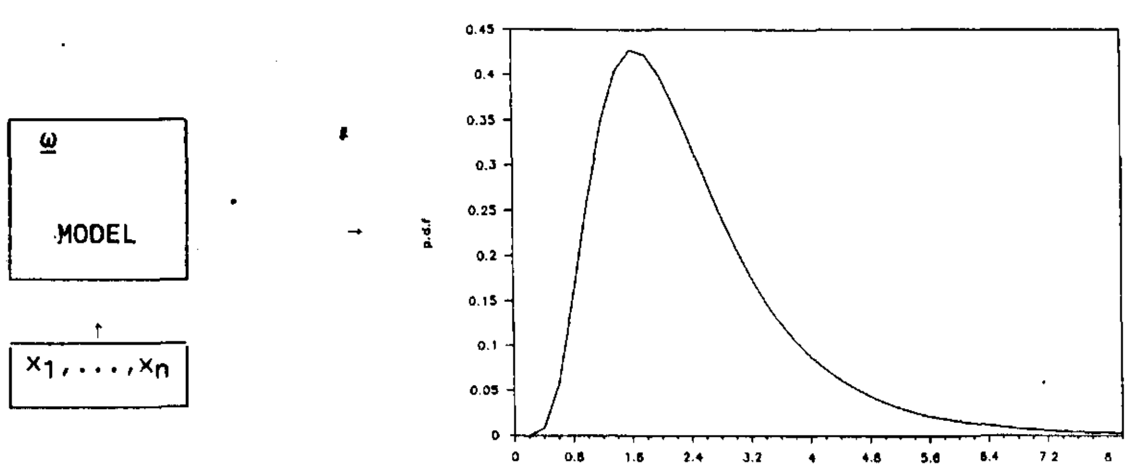
technological coefficients and the cost coefficients.
We can also think of more technical parameters like the evaporation character of the soil we already mentioned. On the moment of optimisation they are taken deterministic.
Often some parameters are set by decisions on a higher, more strategic, level. In the long run those are variable but for the optimisation they are seen as exogenous.

In order to distinguish ω from the model parameters α and the variables x and q, we will make use of the terms stochastic factors, events, random elements etc. and not of the terms stochastic parameters and stochastic variables.

Finally the model delivers certain outcomes $z_1,\dots,z_r$ which with the aid of multicriteria tools, are turned to one objective Z.
Optimisation is nothing else than calculation of the set of which $x_1,\dots,x_n$ has to be an element so that Z is optimal.
This optimisation can be done by various techniques. We will consider simulation in this context as the technique of calculating the consequence in terms of z_i for a certain setting of the decision variables $x_1,\dots,x_n$. Simulation in this sense is regarded as an alternative to find the optimal decision. Usually also varying the parameters in order to do sensitivity analysis is seen as simulation and it is not always clear what are decision variables and what are the parameters. We act now as if they can be distinguished very well.

Other techniques are linear programming and non-linear programming. In this the best $x_1,\dots,x_n$ is calculated according to the criterion of Z. In deterministic dynamic programming an optimal path is calculated for a number of decision stages. However if uncertainty is brought in by stochastic dynamic programming only the decision for one stage can be calculated and decision rules for the other stages are derived. In each stage after a stochastic factor has been realised, this rule can be used to determine the optimal $X = x_1, \dots, x_n$. In this sense we should speak of Z_t and X_t and the former figure only holds for the current stage.

We consider further the case in which the outcome is not one Z, but a distribution function of the objective $\underline{z}$. Represented in the figure:



The strong assumption is that all endogeneous stochastic factors are brought into the objective function. Klein Haneveld [16] distinguishes stochastic feasibility and stochastic optimality. We act as if all stochastic coefficients, $\underline{\omega}_i$, in the description of the feasible space are, one way or the other, transferred to the stochastic objective $\underline{z}$.

Due to the stochastic nature of yield many agricultural models can be regarded in this way. First of all discrete simulation models have got the property that incoming probability distribution functions of $\underline{\omega}_1,\ldots,\underline{\omega}_p$ are transformed to one or more outgoing distributions. For special cases mathematicians have been working on the analytical derivation of composed distribution functions. For convolutions this in general can be done easier than for cases in which stochastic variables are multiplied or divided, but this depends completely on the distributions that are chosen.

In the case of linear programming e.g., if the coefficients of the objective function, prices or margins, are stochastic with a known probability distribution and there is enough information on covariances etc., usually* the composed probability distribution of $\underline{z}$ can be derived for every $x_1,\ldots,x_n$.
The case becomes more complex in LP if also the RHS are stochastic and/or the technological coefficients. See Vajda 1972 [34]. The field of probabilistic programming is still attractive for research. See the more recent publication of M.A. Grove [8]. A kind of goal programming approach is suggested to solve the case of a stochastic RHS with known pdf by him.

Besides simulation and probabilistic programming another idea to describe problems in an uncertain environment is related to the technique of Stochastic Dynamic Programming, in which in every period or stage a decision has to be made. This approach can be very useful in agricultural problems, describing the actions in time a farmer has to consider as to get the desired yield, or in general total return. A nice example is that of pest control in which a farmer has to decide on the timing of use of pesticides. Every week or day in the growing season he has to decide on application of pesticides yes or no. See on this Carlson [4] and Rossing [26]. Pest damage is described for every growth stage of the plant and the occurance of the pest is directly or indirectly stochastic.

Another example can be found in the study of Schouwenaars [28] in which the farmer has to decide during the year on sowing or not. The stochastic rainfal gives whether the sowing strategy is succesful or not. However, Schouwenaars doesn't use dynamic programming but only takes a finite number of possible strategies into account. In this kind of dynamic models the actions which can be undertaken are more or less on the same level. Usually however the farmer first has to make more strategic decisions, on a higher level, and after this he has to decide on the timing of certain actions which can be described by the dynamic model.

It depends on the structure of the model whether the final probability distribution of $\underline{z}$, which is necessary to calculate the value functions to solve the DP problem, can be calculated analytically or has to be found in simulation.

It is useful now to use a small example which can serve for illustrative purposes later on. We take this example from Schweigman

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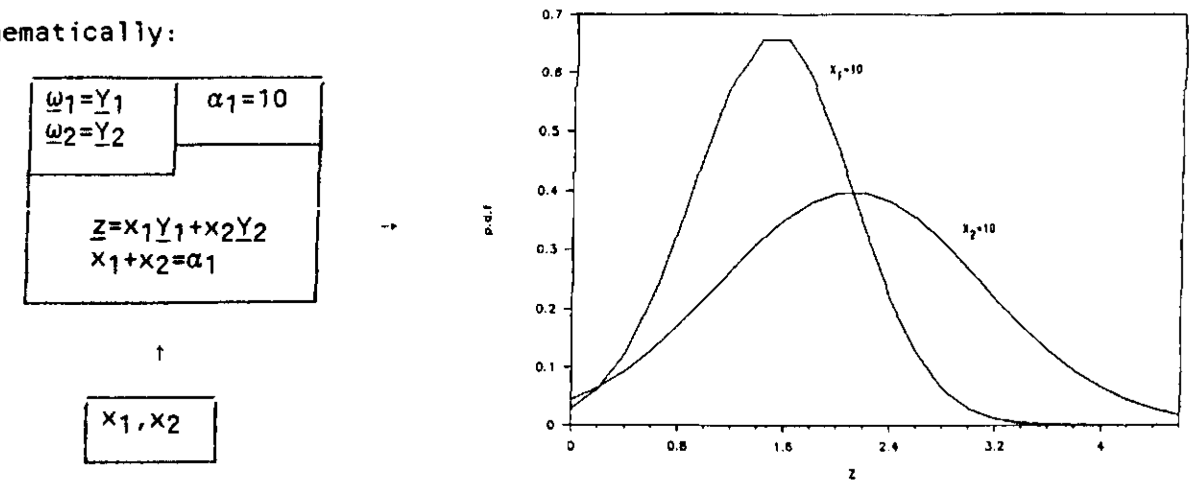
* at least this can be done for normally distributed coefficients.

and Klein Haneveld [29] 1985. Let us consider a farmer who has to *decide on the use of his fields. He has to make a more stategic decision* on which variety to use of maize on his 10 acres of land. Distributions are given of the yield per acre of two different varieties $\underline{Y}_1$ and $\underline{Y}_2$.
x_1 acres will be planted with variety 1 and x_2 acres with variety 2. The stochastic harvest of maize is then $\underline{z} = x_1\underline{Y}_1 + x_2\underline{Y}_2$ under $x_1 + x_2 = 10$ and $x_1, x_2 \geq 0$.

Schematically:



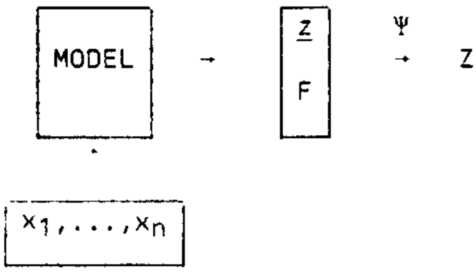
Two probability distributions of $\underline{Y}_1$ and $\underline{Y}_2$ go into the system. The probability distribution of $\underline{z}$ comes out of it and its shape depends on the decisions x_1 and x_2.
We have seen until now that the outcome of a stochastic model gives a distribution function of $\underline{z}$. Let this probability density function of $\underline{z}$ be called f and the cumulative distribution function F.
What is optimised if the outcome consists of the cdf F and not of the *scalar Z? In general, optimisation takes place by taking simply only* the expected value $E(\underline{z})$ into account. However how can risk, defined as anxiety for negative outcomes, be dealt with now?

The answer is very simple in principle, but in practice very complex. Just give an ordering of which cdf's, F, are preferred above others. Give for every pair of cdf's F, G whether $F \succ G$, or $G \prec F$ or $G \sim F$. *To state it in another way: Label all cdf's, attach a number to them which represents the grade of preference. Mathematically seen* there should exist a functional $\Psi[F]$ which forms a function from the space of cdf's to the R.

Now the subjective aspect enters our discussion again, because people experience uncertainty differently, not only in the sense that they are less or more risk averse (risk preference), but also in different atti*tudes towards regret, bad outcomes, reference points, etc. (risk* perception).

The use of different functional forms Ψ and the derivation of properties has been done already in the sixties by Schneeweiss in his book 'Entscheidungskriteriën bei Risiko'. See Schneeweiss [27]. In the next paragraphs we will summarize the ideas of various risk criteria *and discuss their consequences and properties on the basis of this* book, but also on the work of Anderson 1979 [2], Boussard 1979 [3], and Montazani, Wright 1982 [23].

The general model can now be represented in the following way:



The outcome of the model consists of a distribution F of $\underline{z}$ which is determined by the decision $x_1,\dots,x_n$. The functional ψ turns the distribution F into a preference value Z. In this way the stochastic optimisation model is transformed into a deterministic optimisation model; *determine X (or a set of which X has to be a member)* so that Z is at its maximum. This transformation question is the main subject of stochastic programming in general. Problem with the model formulated above is, that the relation $Z = g(X)$ most of the time has to be found by simulation.

In further studies the purpose will be to trace the consequence of the use of different functionals Ψ for the optimum values $x_1^*,\dots,x_n^*$. A basic assumption in the following sections will be that, no matter which risk criterion is used, only risk aversion is taken into account. In general in the agricultural sector this is the case. We have to admit that studies have been done which gave the opposite result. In this context we mention the study of Kunreuther and Wright [18] who found for very small farmers strange enough a risk seeking attitude.

III. Expected Utility as a possible approach

One way to solve stochastic optimisation models is to look at the expected values of z only and treat it as a deterministic problem further. In this case $\Psi[F] = \int_{-\infty}^{\infty} z dF(z)$. The set of distribution functions with the same Z-value are those with the same expectation.

One way to model the aversion towards bad outcomes is to formulate a so called utility function on the outcome space of $\underline{z}$, u(z), or a penalty function on a part of this space, expressing the aversion towards these outcomes.

The expected utility criterion (EU), $\int_{-\infty}^{\infty} u(z)dF(z)$ can be seen as a functional $\Psi[F]$. The concavity of the utility functions represents very often in this idea risk aversion.

Difficulty with application of the EU idea in optimisation is that we do not know a general applicable form of this function. However the idea is very useful to evaluate other risk criteria as is done by Schneeweiss who uses the concept of Bernoulli rationality of the criteria. Unfortunately the term rationality suggests that criteria not matching this concept are not rational or not good. Our interpretation of Bernoulli rationality is that criteria can be translated in the EU form.

Stated in an other way: Does there exist a utility function so that $\Psi[F] = \int_{-\infty}^{\infty} u(z)dF(z)$ or not. We will come back on this idea in the next section.

The idea of EU has also been used in mathematics to define a metric of probability functions with a distance function based on the functional $\int_{-\infty}^{\infty} u(z)dF(z)$. In this topology of probability distribution functions (cdf's), two distributions are called close if their expected utility values are close. In this sense for any u-function a metric is defined. Regoli [25] has been working on the specific conditions for these functions in order to have a proper definition of such a metric. The idea of convergence $F_n \xrightarrow{u} F$ is used in her work to come to special sets of u-functions. In such a set the convergence of cdf's have to be closely connected.

In reality we do not know what the utility function is, so that the idea of a complete set of u-functions leading to the same ordering of cdf's would be very useful. If one particular feasible cdf, F, is preferred above the other feasible cdf's, it would be nice if its expected utility is higher for a complete set of utility functions. In general however such a wider definiton of convergence or distance function will only lead to a partial ordering of the cdf's. We will see such a partial ordering also later on in the discussion of the idea of stochastic dominance.

Not only scientists have studied the idea of a utility function as such, whether it exists or not and whether there is a general form, but early research has also found that the idea of EU does not completely satisfy. To state it in our terms: The EU idea cannot explain completely the preference relations of distributions revealed by people in reality. People violate the axioms intuitively. A recent study which gives a good summary of the problems with the EU concept is due to Weber and Camerer [36].

In 1952 Allais showed with a small experiment this violation already and this example is still used to try to come to other theories in which behaviour of people in choice between cdf's (in these theories the term lotteries is used). Another name in this field is Machina who postulated a generalised EU utility model in asking for an additional function on the probabilities say q(F) so that $\Psi[F] = \int u(z)dq(F)$ gives the preference functional. However in this kind of experiments only lotteries are considered with a finite number of outcomes, usually two, and graduate students appear to be the aselect representation of mankind to experiment on choices between lotteries.

The disability of EU theory to explain peoples behaviour formed an *important subject of conversation on the Fourth International* Conference on Risk, Utility and Decision theory held in Budapest 1988. Important flaws of the EU idea can be found in the fact that e.g. regret plays an important role in peoples behaviour and that people need a certain reference point to evaluate the utility of the outcomes which changes quickly over time.

If we link these ideas to the behaviour of a farmer during the growth season, it is important to see that the attitude of the decision-maker, the farmer, towards the outcomes, return, of his actions can change as time proceeds. Let us consider the case of DP in pestcontrol. The attitude towards a loss of 20% due to pests at the beginning of the season will be different from that of the same loss of the end of the season, whereas in the optimisation model this loss has the same influence on the objective function, if the same utility function is taken in every stage. Consequently in every decision stage another pre*ference relation translated in an utility function or risk criterion* should be used.

Chikán [5] uses the idea of regret explicity in his definition of risk. He considers risk as the difference between the expected result of the actions undertaken, which were optimal before something happened *and uncertain events realised , and the result of the optimal actions* $x_1^*,\dots,x_n^*$ after some events are *realised*.

In this sense risk is defined as a kind of opportunity losses. See Dannenbring & Starr [7].

Minimisation of risk in this sense will lead to minimisation of regret.

To come back to the idea of the functional Ψ, one has to realise that, in general, a pdf contains an infinite number of information pieces. If we consider the moment generating function of a density function it appears that, if this moment generating function exists for an open interval around 0, the pdf can be determined completely by its moments. Of course the function can also be seen determined by its factors in the Taylor expansion and the cdf can be considered determined by an infinite number of quantiles. To describe the full set of all possible pdf's one needs an infinite series of information pieces, whereas Ψ turns one pdf into one single number namely the preference value.

Fortunately the outcome F of a model will only be a member of a subset of the set of all possible density functions. Consider e.g. the classical Markowitz model in which the density functions are only

described by two moments.
In this sense it is not necessary to look for a generally applicable criterion, functional Ψ, which covers all density functions.

IV. Risk Criteria

Nearly all objectives in optimisation models contain the expected value of $\underline{z}$, $E(\underline{z})$. Combined with some measure for risk it forms the functional Ψ, which gives the preference in the feasible pdf's. We will mention many of these criteria here and also refer to some applications in which these were used.

In the minimax criterion only the return of the worst possible case is taken into account. Note that the formulation of the model should be given in the sense that there exists a worst case. This is f.i. the case if a finite number of states of nature are taken into account. In LP this can be implemented by using together with the other restrictions the formulation:

$$\begin{aligned} &\max M \\ &\text{s.t.} \\ &\qquad \sum_j C_{ij}x_j \geq M \qquad\qquad i = 1, \ldots, m \end{aligned}$$

in which C_{ij} is e.g. the margin of activity j in weather i. Notice that no probabilities are needed because the criterion is not weighted. Combining this criterion with the expected return, E, (for which probabilities are needed) would imply the E-M frontier. Such a frontier will indicate the highest minimum M possible, given an expected return, and the highest expected return possible given a certain value of M. Heyer used this formulation to explain the behaviour of the Masii in Kenya. See Heyer [13]. We will see other criteria which do not use the expected value later on.

Let us consider now functionals which take into account $E(\underline{z})$, but also other moments of F; by Schneeweiss called classical criteria. If K moments are taken into account, Ψ reduces to $\Psi(F) = \theta(\mu_1, \mu_2, \ldots, \mu_k)$. How many moments should be taken into account and what should be the function which relates the moments? This depends on the possible pdf's that can come out of the model.

If only two parametric families of distributions like normal, Cauchy, lognormal and gamma can come out, it is sufficient to look only at the first two moments, provided that the moments exist. It will be very likely that these moments will exist; who builds a model that gives a Cauchy distributed outcome?
For the more general case however, if risk aversion is anxiety for the left tail of the pdf, this leads to preference of right skewed pdf's and therefore the third moment should also be taken into account.
The two parameter-case with $\theta(\mu,\sigma^2)$ is used very much.
Markowitz designed the portfolio model in the fifties, which could be solved by quadratic programming. Let x_i be the money invested in asset i with expected return e_i and variance and covariances with the other assets represented in a variance covariance matrix Q.
The expected value of portfolio $x_1,\ldots,x_n$ is then $\mu_x = x'e$ and the variance is given by $\sigma_x^2 = x'Qx$. Let $\theta(\mu,\sigma^2) = \mu - \lambda\sigma^2$. Varying λ and choosing the highest $\psi = \theta[\mu(x),\sigma^2(x)]$ gives the so called efficient line
A more recent study on this same idea can be found in Kriens and Van Lieshout 1986 [18]. This basic idea of quadratic programming was recently implemented in a farm planning model in Greece by Manos and Kitsopandis, 1986 [21].
They also derive this efficient line and claim that the farmers in

Macedonia were very satisfied with the result compared with LP plan outcomes. Huisman [14] states that in economy, in which the attempt is to explain people's behaviour, this behaviour is always better explained if a certain form of risk is taken into account than if only the expected value is maximized. Note that it is necessary to know all covariances with this approach.

The idea to insert the third moment in Ψ, has also been tried relatively early. By Marchack [22] in social science in 1955 and by Albach [1] in the financial environment in 1959. It appeared that the chosen pdf's and with that the decision $x_1,\ldots,x_n$, didn't differ very much compared with the criterion based on two moments. Again, this depends on the possible pdf's.

Jean [15] used recently again even higher moments in the portfolio model. In his study the starting point is an unknown utility function which is seen in the light of its Taylor expansion. The strong assumption is that all comoments of the securities are known. The comoment between the return of asset i, $\underline{y}_i$, and the return of asset j, $\underline{y}_j$, is defined as $\mu_{pq} = E(\underline{y}_i^p\ \underline{y}_j^q)$. Some necessary first order conditions for the optimal portfolio, expressed in terms of comoments, are derived. Also Sinn [30] applied among others various risk perception ideas on the portfolio theory and the theory of currency speculation.

The practical use of such portfolio models as a tool to describe the portfolio choice of people can be doubted, because this choice depends on the subjective probability distribution of the 'economic agent'. In this sense it is the question how the subjectivity of the utility function can be distinguished from the subjectivity in the pdf which serves as a base for this decision.

It is easy to show the relation between a linear Ψ in terms of the central moment, like in the Markowitz case, and polynomial utility functions.

For two moments:

Let $u(z) = az + bz^2$ then

$$Eu = E[a\underline{z} + b\underline{z}^2] = aE(\underline{z}) + bE(\underline{z}^2) = a\mu_1 + b\mu_2$$

For three moments:

Let $u(z) = az + bz^2 + cz^3$:

$$Eu = E[a\underline{z}+b\underline{z}^2+c\underline{z}^3] = aE(\underline{z})+bE(\underline{z}^2)+cE(\underline{z}^3) = a\mu_1+b\mu_2+c\mu_3$$

A functional which is linear in the moments can be translated to an EU model with a polynomial utility function. This translatability is called Bernoulli rationality by Schneeweiss [26]. Most criteria are evaluated by him in this sense.

Schweigman and Klein Haneveld [29] show that λ in the μ,σ^2 criterion $\phi(\mu,\sigma^2) = \mu - \lambda\sigma^2$ is sensitive to scale while in $\phi(\mu,\sigma^2) = \mu - \lambda\sigma$ this is not the case.

This criterion with the standard deviation appears not to be Bernoulli rational so that there does not exist a u-function which gives $E[u(\underline{z})] = \mu - \lambda\sigma$ for all two parametric distributions. The opposite is the case with the criterion μ/σ.

A similar criterion as the μ,σ criterion is that of the mean absolute deviation MAD = $E(|\underline{z}-\mu|)$ but it is not the same. In this case the matching utility function is $u(z) = |z-\mu|$. In this sense the deviations at the right side are also taken in the risk measurement, while we do not mind outcomes that are better than average. It looks more appropriate to take only the middle under deviation $\int_{-\infty}^{\mu} z dF$ as a measurement for risk. What is left is the expectation of the left side of the pdf, which is of course more negative the more F is skewed to the left or stated in another way, the longer and thicker the left tail is. However this left wing expectation in principle does not differ very much from the criterion of the absolute deviation expectation, as the right wing deviation equals the left wing deviation:

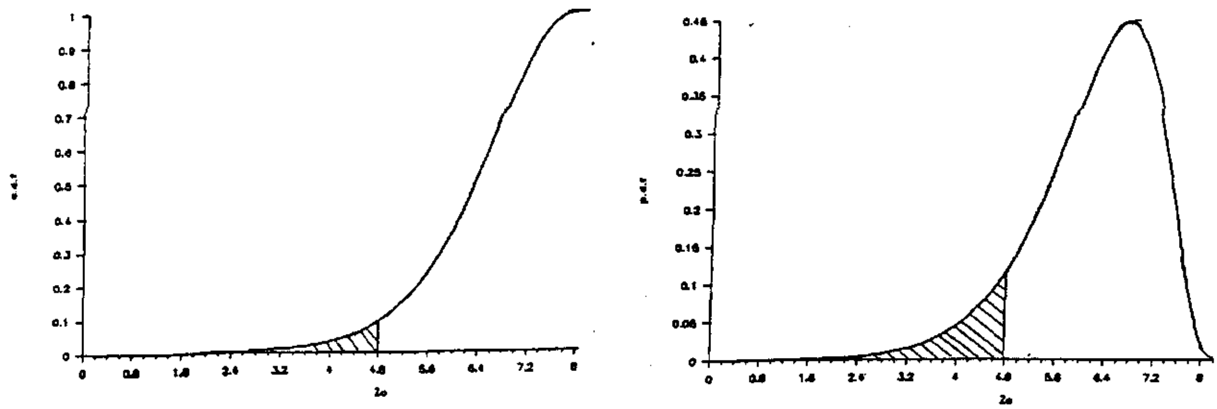
$$\int_{-\infty}^{\infty} |z-\mu| dF = \int_{\mu}^{\infty}(z-\mu)dF - \int_{-\infty}^{\mu}(z-\mu)dF = -2\int_{-\infty}^{\mu}(z-\mu)dF = -2\int_{-\infty}^{\mu} zdF + 2\mu F(\mu).$$

If $\mu = 0$ the two criteria coincide.

To accentuate this left tail and to see an analogon with the variance we can also think of a quadratic penalty in the left tail: $\int_{-\infty}^{\mu} (z-\mu)^2 dF$ which is called the semivariance. In 1970 Markowitz changed his model into this direction

The above mentioned criteria take the total outcome space into account. Very often however outcomes are experienced as bad if they come below a certain level z_0.

One criterion used very much, takes the expected value of the left tail below the threshold value z_0 as a meaurement for risk: $\int_{-\infty}^{z_0} (z_0-z)dF$.

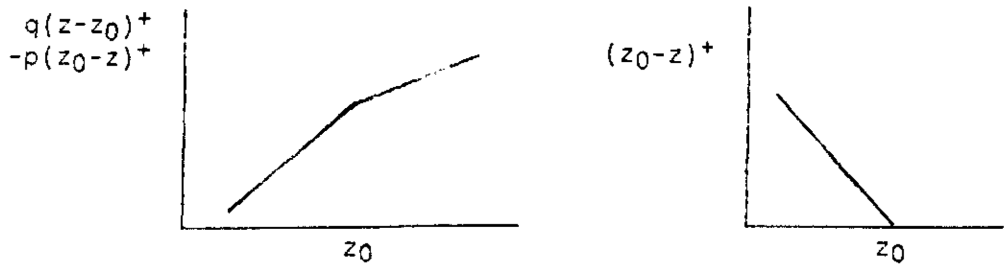


The measure is shown here in a positive sense and is interpreted by Schweigman [29] as expected shortage. Consider a situation of a farmer in a developing country, who is confronted with $\underline{z}$ as the yield of food. To feed his family he needs the minimum amount z_0. If the return falls below z_0 he has to buy food at a high price p and all which is left over can be sold on the market for a lower price q.

In this case his return is given by

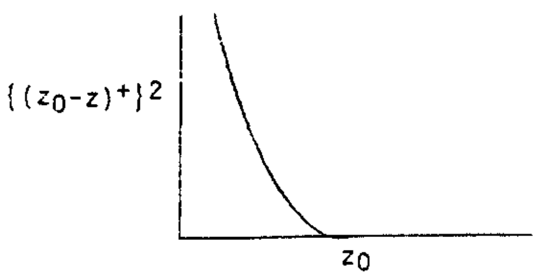
$$-p\int_{-\infty}^{z_0}(z-z_0)dF + q\int_{z_0}^{\infty}(z-z_0)dF.$$

Notice that subjectivity has disappeared in this interpretation. In this sense he is confronted with a kind of utility function on the total outcome space resp. penalty function on the space of "bad" outcomes $(-\infty, z_0]$ in the following sense:



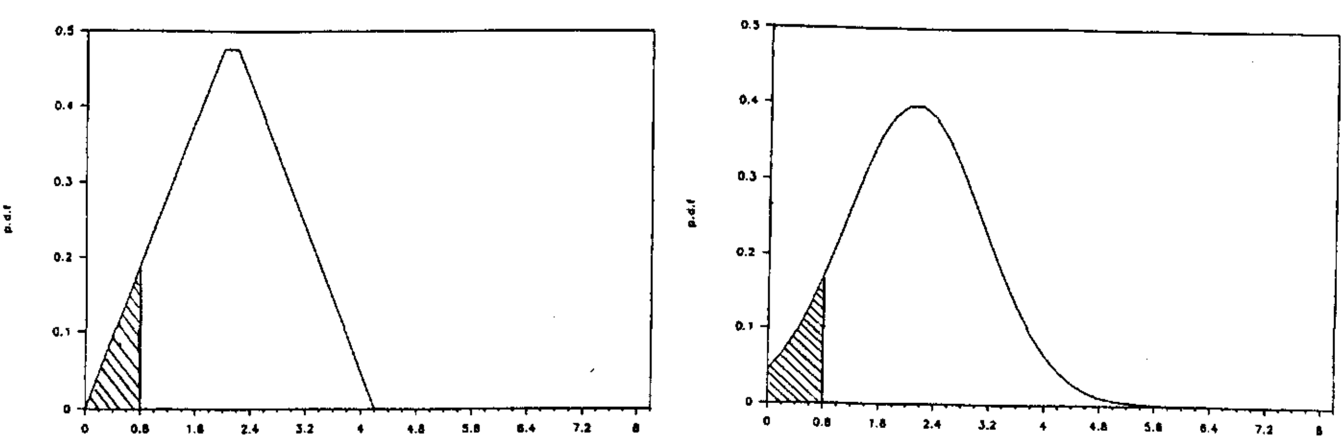
The expected shortage criterion can be introduced in this linear sense, max $E(\underline{z}) - \lambda E(z_0-\underline{z})^+$, but also in a form which uses a restriction like: max $E(\underline{z})$ under $E(\underline{z}-z_0)^+ \leq s_0$. What happens in the idea of Schweigman if there is hardly any alternative food available like in many regions in Mozambique?

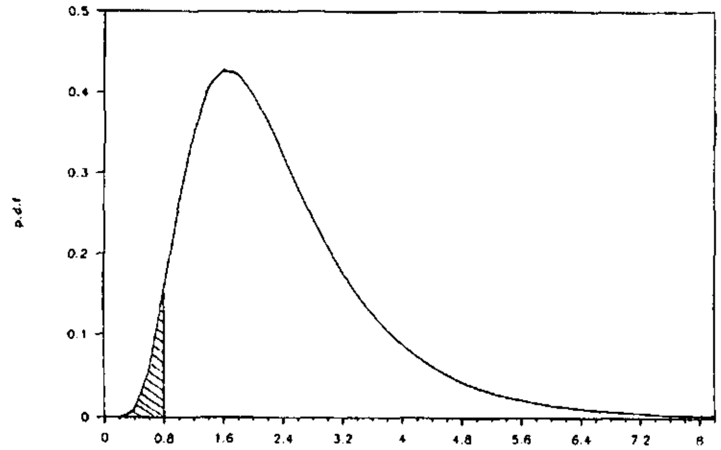
To express the fact that it becomes more and more difficult to find alternatives the penalty on coming below the threshold value can be made strict convex by using e.g. a quadratic penalty:

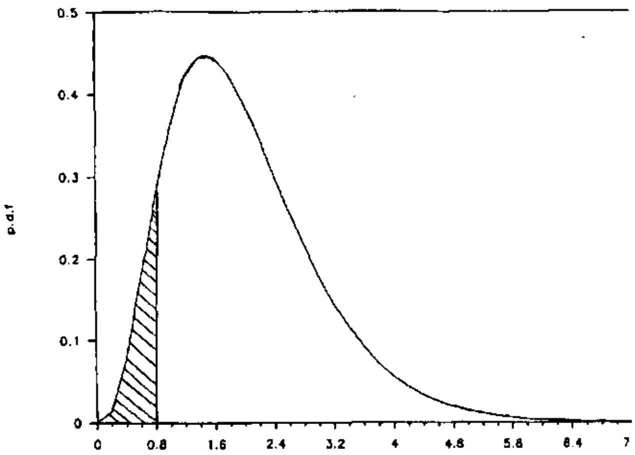


This corresponds to the idea of the semivariance. The convexity can also be carried on to linear models in creating a piecewise linear penalty function. Let us define semimoments $\nu^k(z_0)$ as $\nu^k(z_0) = \int_{-\infty}^{z_0} |z_0-z|^k dF(z)$.

In fact the quadratic penalty can be generalised, because it makes use of the two semimoments $\nu^1(z_0)$ and $\nu^2(z_0)$ generalized quadratic penalty, $p(z) = a\{(z_0-z)^+\}^2 + b(z_0-z)^+ + c$, uses more information on the shape of F in the left tail, because it uses more parameters.







These figures give four distributions with each a very different left tail. The parameters of $\theta(\mu_1, \nu_1, \nu_2)$ will determine which pdf is preferred. The expectation and the standard deviation is the same for all depicted distributions.

One of the most used ideas is not to make use of the left tail information of F, but to declare any outcome under the threshold value a disaster. The measure of risk is given by $P_0 = P(\underline{z} \leqslant z_0) = F(z_0)$. We want the probability of falling under z_0 as low as possible. Minimize the probability of bancruptcy. The penalty or utility translation is given by

$$\text{penalty}(z) = M, \qquad z \in (-\infty, z_0]$$

Usually the idea is implemented in a restriction $P(\underline{z} \leqslant z_0) \leqslant \alpha$. In stochastic dynamic programming after some periods it can occur that this probability is always bigger than α for the outcome z_t. See e.g. Rossing [26]. An approach which will give the same optimal decisions is to use the penalty equivalent with a very big M instead of the restriction formulation. In this chance constraint formulation it is also possible to state more chance constraints on $\underline{z}$: $P(\underline{z} \leqslant z_{0k}) \leqslant \alpha_k$ k = 1, 2, ... This defines more of the desired distribution.

A variation on the idea which is very simular to the minimax criterion, is to formulate the objective the other way around as max z_0 in $F(z_0) \leqslant \alpha$; shift the α-quantile as far as possible to the right. Schneeweiss shows [27] that there does not exist a utility equivalent formulation of this criterion. This is the case for all ordinal measures like quantile, median and interquantile distance.
It will not always be easy to solve such a quantile formulation analytically. Severe assumptions have to be made about the underlying distributions as to make an analytical form of the restriction $P_0 \leqslant \alpha$. It will be easier to check whether for a given strategy $P_0 \leqslant \alpha$.

More on this idea can be found in Klein-Haneveld [16]. He discussed the penalty idea and P_0 idea for individual restrictions. The threshold value can also be encountered in inventory models. See e.g. Hadley & Whitin [9].

z_0 in this case is the zero level of inventory. The probability of coming below this level, P_0, has to be as small as possible. In these models the term service degree is used. Also the expected shortage idea is used with costs for negative inventory in terms of lost sales costs and back-order costs. Here we see that the ideas discussed above are used in parts of models. As we mentioned before, we want to focus further on the use of criteria on the final outcome $\underline{z}$.

A LP implementation of the idea that $\underline{z}$ should exceed the threshold value can be found in Montazemi and Wright [23]. Next to the usual constraints, the following formulation is implemented:

$$\max \quad \Sigma\, e_j x_j$$

$$\text{under } \Sigma\, C_{ij} x_j \geqslant z_0 \quad \forall i$$

Low [20] uses this approach to describe farmers behaviour in Ghana and to come to e.g. minimum farm sizes in which special techniques can be used without too much risk of a disaster. In general the formulations that contain restrictions form a lexicographic preference ordering in terms of ψ. First the feasible pdf's which fulfill the restriction are looked after and then within this group the other objective is optimized. This lexicography doesn't match the utility theory very much. This doesn't mean that such an ordering is inferior. See Klein Haneveld [16].

The P_0 approach, not to look below the threshold value z_0, can be interpreted as a certain attitude towards uncertainty. Consider a farmer whose yield falls below the z_0 value. It is possible that it is not worth the trouble to elaborate the field any further, because it costs more than the benefit of the harvest to take further actions. In this sense it does not matter any more how far z falls below z_0. Huisman [14] used this concept in explaining the behaviour of farmers in a Philippine village.

Let us state again that this last criterion does not take any form of the left tail of the cdf F into account. The expected shortage idea of Schweigman and the quadratic expectation of the left tail on the contrary do. The P_0 criterion gives $\int_{-\infty}^{z_0} f(z)dz = F(z_0)$ as a measure for risk, so that the shape of F below z_0 is not taken into account.

The expected shortage criterion $E(z_0-z)^+ = \int_{-\infty}^{z_0} (z_0-z)f(z)dz$ by partial integration leads to $\int_{-\infty}^{z_0} F(z)dz$ which is exactly the area under F left from z_0, so that the shape does matter.

If we proceed with partial integration, it is easy to see that the quadratic penalty $\int_{-\infty}^{z_0} (z-z_0)^2 f(z)dz$ is equal to $\int_{-\infty}^{z_0}\int_{-\infty}^{q} F(z)dzdq$. By additional integration more and more of the left tail is weighted. This gives the link to stochastic dominance and partial stochastic dominance. We will work out this idea in the next section.
Klein Haneveld [16] compares mathematically the chance constraint formulation, $P_0 < \alpha$, the expected shortage criterion $E(z_0 - \underline{z}) \leqslant \beta$, the conditional expected shortage criterion of Prekopa, $E(z_0 - \underline{z}) \leqslant \gamma P_0$ and

a formulation with $E(z_0 - \underline{z})/E(\underline{z}) \leq \delta$. An important property of chance constraint programming is that the feasible region isn't necessarily convex.

Let us first go back to the optimization models. Returning to the implementation of risk expression in models, a lot appears to be elaborated on the idea of absolute deviations. Hazell introduced in 1971 his MOTAD model [11]. MOTAD stands for Minimum of Total Absolute Deviation. The risk expressed in absolute deviations is minimized. This corresponds to the criterion of expected absolute deviation we discussed earlier.

The LP implementation contains the following items. Let i be a state of nature, e_j be the average coefficient and C_{ij} be the coefficient of action x_j under state i. So e_j is the expected profit determined by $\sum_i C_{ij} = e_j$. In LP terms $D_i = D_i^+ - D_i^-$ will give the deviation under state i via

$$\sum_j (C_{ij}-e_j)x_j - D_i = 0$$

Now there exists a trade-off between

$$\sum_i D_i^+ + D_i^- \qquad : \text{risk expressed in absolute deviation}$$

$$\sum_j e_j x_j = E(\underline{z}) \qquad : \text{the expected profit.}$$

This leads to a trade off line as in the Markowitz model, the so called A-E frontier. Tan and Fong [32] applied this model for the optimal crop mix in the rubber and oil plantation Malaysia. They also give the A-E frontier. Again there is the question: What is wrong with deviations if they are positive? We are only afraid of negative outcomes.

As we mentioned before in the discussion of the expected absolute deviation criterion, the sum of positive deviations is exactly the same as the sum of negative deviations:

$$\sum_i D_i^+ - \sum_i D_i^- = \sum_i D_i =$$

$$\sum_i \sum_j (C_{ij}-e_j)x_j = \sum_j \sum_i (C_{ij}-e_j)x_j = \sum_j 0.x_j = 0.$$

This means that in the MOTAD application less variables and restrictions are necessary in the reformulated version:

minimize the expected absolute deviation: $2 \sum_i D_i^-$

$$\sum_j (C_{ij}-e_j)x_j + D_i^- \geq 0 \qquad i = 1, 2, \ldots, m.$$

This formulation gives exactly the same results. This idea can also be found in the later work of Hazell, see Hazell 1986 [12]. In this work it also becomes clear why no probabilities are attached to the possible states of nature. Hazell uses time series data for the estimation of the coëfficients, so that state i corresponds to a certain year or time period t. This means again that the time dependent data are not weighted.

The application of deviations in restrictions in LP has also been used in multicriteria environment, in which it gives the deviation from the ideal point, but it also has been applied to express the uncertainty involved in the restriction. This idea has been used in applications of the fuzzy set theory. In the fuzzy set theory the idea is that there exists a so called membership function which represents the grade of the membership of a certain element of a certain set.

On the FUR conference we saw two applications.
on Nachtnebel [24] gives a trade-off between economic goals and environmenta goals in water resource management for a special case, which includes uncertain discharge of the river. Korhonen [17] uses the membership value to express that certain RHS of some restrictions are not known for sure but that they lei between two values. The slacks of the restrictions are put together in a composed way in the objective so that also here a efficient frontier can be derived. In principle this approach does not differ from the multicriteria and soft constraint methods that have been used for many applications. In the two studies mentioned here, linear membership values where applied, which is the same as putting the slacks indirectly in the objective function. From these applications we got the impression that fuzzy set theory is nothing else than, or an economic interpretation of, the soft constraint methods which have been applied satisfactorily for a long time.

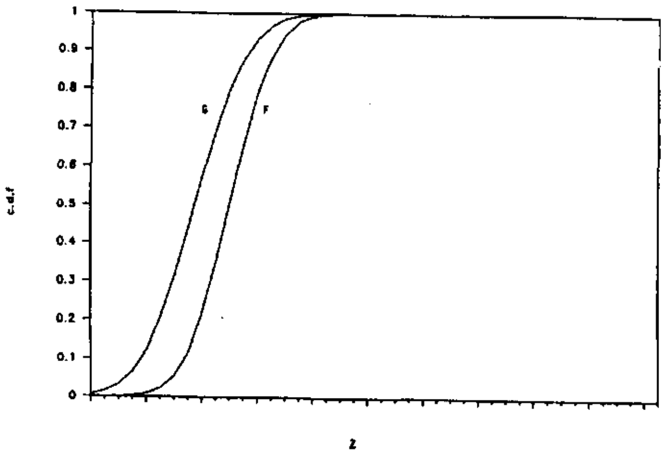
In LP-like models, like in the MOTAD case, the uncertainty brought in is translated directly in the objective without expressing more information on the distribution of $\underline{z}$. However it is good to keep in mind that a model consists of the various aspects as presented in section 2.
What in the model is due to ψ; what is soft constraint or fuzzy, what are the main restrictions; what is technical, what is behavioural etc.

Before this paper is finished we will shortly discuss the idea of stochastic dominance.

V. Stochastic Dominance

We focus briefly on the concept of stochastic dominance and its relation with other functional ideas namely those based on moments and those based on utility functions. We should refer to the earliest work in this field but our information depends heavily on the publications of Anderson [2]. In the direction of applications we found his ideas completely back, repeated, in the PhD research of Thornton [33].
Thornton studied a special case of pest control and applied explicitly a utility function to express the preferences of the farmers.

In nearly all books that contain the subject of decision under risk, the idea of stochastic dominance is at least presented for a pair of CDFs as $F(z) < G(z)$ for all z.

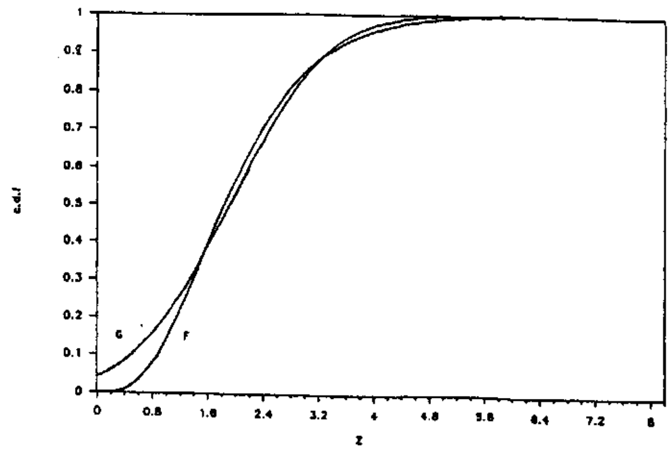


This means that the CDF F lies at the right of G so that F is always preferred for all people. This gives a kind of Pareto idea. There is no trade-off between risk and the expected value.
This stochastic dominance rule and the Pareto rule have in common that they are not very powerful. In terms of the functional ψ, hardly any discrimination between CDFs can be made. On the other hand all people prefer F above G, if F dominates G.

For people with a concave utility function it appears that this rule can be made weaker. This corresponds with people who are risk averse.
Second degree stochastic dominance is defined as $F \succ G$ if and only if

$$\int_{-\infty}^{z} F(q)dq < \int_{-\infty}^{z} G(q)dq \qquad \text{for all } z.$$

$F \succ G$ only if the expectation of F is bigger and the left tail of F lies at the right of G.
This means that the CDFs are allowed to intercept at some points as long as the surface under F from the left tail to z is smaller than the same surface under G.



In this same sense third degree stochastic dominance can be defined as

$$F \succ G \text{ if } \int_{-\infty}^{z}\int_{-\infty}^{p} F(q)dqdp < \int_{-\infty}^{z}\int_{-\infty}^{p} G(q)dqdp \qquad \forall z$$

and the expectation of F is bigger than that of G.

This rule is weaker and applies for people with a concave utility function which has a positive third derivative. This rule also considers skewness to the right. As we were focusing on risk averse decision makers, this rule will be sufficient for all people under consideration, if we take again risk as anxiety for left tail outcomes. Higher order stochastic dominance of course determines more the shape of the utility function.

Partial stochastic dominance is defined as stochastic dominance on a convex subset of the outcome space of z. It is suggested by Anderson not to take e.g. the extreme tail behaviour of the distribution function into account. It is more important that F dominates G on the rest of the outcome space. This is strange because we have already seen some criteria which were based on this partial stochastic dominance ideas, namely, the P_0 criterion gives the first order dominance on the point z_0 as a measure of risk; $F(z_0)$. According to the safety first rule F is preferred above G if $F(z_0) < G(z_0)$.

In this same way there is a connection between the partial SD idea on the $\{z_0\}$ subspace and the other criteria. As we already saw, the expected shortage criterion gave by partial integration

$$E(z_0-z)^+ = \int_{-\infty}^{z_0} F(z)dz$$

so that F is preferred above G if

$$\int_{-\infty}^{z_0} F(z)dz < \int_{-\infty}^{z_0} G(z)dz.$$

In the way this criterion corresponds to a special case of partial SD of the second order. To go on in this way, the quadratic loss function which corresponds with the idea of semi-variance would prefer F above G, if

$$\int_{-\infty}^{z_0}\int_{-\infty}^{q} F(z)dzdq < \int_{-\infty}^{z_0}\int_{-\infty}^{q} G(z)dzdq$$

which corresponds to the concept of third degree partial SD.
With these cases of partial dominance only the dominance in the point z_0 was taken into account. It appeared that the number of semimoments on $(-\infty, z_0)$ corresponds to partial dominance on $\{z_0\}$ only of one degree higher. However, stated the other way around: Stochastic dominance of order n corresponds to the loss criterion

$$\int_{-\infty}^{z_0} (z-z_0)^{n-1}dF \qquad \text{for all } z_0.$$

For one value of z_0 at least the expression $\int^{z_0} (z-z_0)^{n-1}dF$ can be used as a functional ψ; it gives a value. The stochastic dominance rules indeed give a preference relation but do not give one value. Dominance has to be checked for all values in the outcome space of z. It only forms a partial ordering of the set of pdf's.

VI. Conclusion and Future Research

In operations research quantitative models are built with the purpose to serve as a support in decision processes. Already the description of the objectives and variables makes clear what is decided upon and which are the instruments and main restrictions. With the mathematical models we had in mind a description of an agricultural system in this paper, in which the main source of uncertainty was not caused by some lack of knowledge, but more exogenous (naturally stochastic) like the weather outcomes.

The quantitative description of the input-output system from weather to yield gives insight in what the main problems are. In optimisation or simulation models the main factors are evaluated on the base of the objectives. Later on the other parameters α_i are checked in the direction of the objective to trace what does really matter and what does not. The risk criteria constitute a bridge between uncertain outcomes and the objective.

In section II we gave a schematic model in order to discuss the status of such criteria in the context of the model. Sometimes short-cuts can be made, like in the Markowitz case, in which it is not necessary to express the distribution function explicitly. In the Markowitz model the criterion is formulated in moments and the ingoing and other outgoing information is in terms of moments only.

In this paper an enumeration of the criteria was given and a link was lead to applications on the one hand and more theoretical framework on the base of utility functions and stochastic dominance on the other hand.
Interpretation of criteria, what do they mean in relation to preferences of people is also very important. We have to admit that we did not want to go too deep into the subjectivity of attitudes towards risk.
It is more a task for a social scientist than for a mathematician to describe peoples behaviour. Though many applications we refered to form in reality a description of human behaviour from the assumption that people behave rational and optimal.
In this direction including a description of uncertainty and risk appears to be a succes.

What should be kept in mind by the mathematician is that every criterion has got its properties which, depending on the structure of the model may lead to very different outcomes. For instance the P_0 criterion which is used very often may not fully describe the decision maker's attitude towards uncertainty. In future research it may be interesting to find out for applied models, whether and how the optimal set changes if other criteria are used or if other formulations are tried.

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Notes:

The FUR conference refered to, is the Fourth international conference on Risk, Uncertainty and Decision Theory held in Budapest, June 1988.

The book 'Risk, Uncertainty and Agricultural Development' exists due to a seminar held on this item in 1976 and was published by the Southeast Asian Regional Center for Graduate Study and Research in Agriculture (SEARCA) on the Philippines in cooperation with the Agricultural Development Council (ADC) in New York.

EJOR is the European Journal for Operations Research.

List of the technical notes of the Department of Mathematics
(Dreijenlaan 4, 6703 HA Wageningen, The Netherlands)

77-01	P. van Beek	Explicit expressions for optimal reorder point and lot size by simultaneous optimization.
77-02	T.A. Buishand	Kansmodellen in de populatiedynamica.
77-03	P. van der Laan and J. Weima	Experimental determination of the power function of the two-sample rank test of Wilcoxon for logistic parent distributions, and comparison with the power of Student's t-test.
77-04	T.A. Buishand	Monte Carlo Technieken.
77-05	W. Meijles	A Stochastic model for chemical equilibrium.
78-01	T.A. Buishand	The binary DARMA (1,1) process as a model for wet-dry sequences.
78-02	A.C. van Eijnsbergen	Estimation of mean and variance of a variable in a random sample from a finite population.
79-01	L.R. Verdooren	Aspects of variance component estimation.
79-02	Th.H.B. Hendriks and L. Szelényi	Multiparametric programming for generalized transportation problems.
79-03	W.G. van Hoorn	The direct sum for semirearrings.
79-04	P. van Beek und M.H. Wientjes-van Rij	Eine Anwendung der Bendersschen Partitionsmethode in einem Modell für die Flurbereinigung (extended Abstract).
80-01	M.A.J. van Montfort	The expected gev order statistics and their derivatives, both at $\theta = 0$.
80-02	L.R. Verdooren	How large is the probability for the estimate of a variance component to be negative?
80-03	E.W. van Ammers	Correctness proofs by formalized refinement steps.
80-04	P. van Beek	Modelling and analysis of a multi-echelon inventory system, a case study.
81-01	M.A.J. van Montfort	Comparison of distributions.
82-01	P. van Beek and P. van Schuylenburg	An application of aggregation and decomposition to the N-product, 1-machine problem.
82-02	M.A.J. van Montfort	Modellen voor maxima en minima, schattingen en betrouwbaarheidsintervallen, keuze tussen modellen.
83-01	P.van Beek,A. Bremer and C. van Putten	Design and optimization of multi-echelon assembly networks: savings and potentialities.

83-02	B.R. Damsté	Procedures voor de numerieke berekening van meervoudige integralen.
83-03	B.R. Damsté	De gepreconditoneerde geconjugeerde-gradiënt-methode voor het oplossen van stelsels lineaire vergelijkingen met een grote ijle symmetrische positief-definiete matrix.
83-04	B.R. Damsté	Programma's voor de berekening van grondwaterstanden in een gebied met beekafvoer, onder gebruikmaking van de eindige elementen methode.
84-01	B. van Rootselaar	Output formulas for constant linear systems.
85-01	B. van Rootselaar	Some uses of a simple form for the solutions of X' = AX.
85-02	B. van Rootselaar	On Postma's third degree predator-prey model.
86-01	B.R. Damsté	Straling van een vlakke plaat naar een bol.
86-02	P. van der Laan and L.R. Verdooren	Classical analysis of variance methods and nonparametric counterparts.
86-03	P. van der Laan and B. van Putten	Exact critical values of the nonparamatric two-sample test of Halperin for censored samples with sample sizes up to and including 15, and comparison with the Normal approximation.
87-01	M. Funk	Calculation of ground water levels with use of the finite element method.
87-02	M. Funk	Growing, a program package for the calculation of ground water levels in natural geometries.
87-03	P. van der Laan	Selection from logistic populations.
87-04	S. Driessen, P. van der Laan and B. van Putten	Selection from normal populations with unequal variances and its robustness.
87-05	S. Driessen	A note on the probability of correct selection.
88-01	P. van der Laan and L.R. Verdooren	Selection of populations: An overview and some recent results.
88-02	H. Almekinders, P. van Beek and P. van Schuilenburg	The 1 Machine N Products Problem.
88-03	M. de Gee	Step-change techniques for extrapolation methods for delay-differential equations.
88-04	S. Driessen	Evaluation of a multiple comparison technique as a subset selection procedure.
88-05	J.M. ten Vregelaar	Note on the convergence to normality of quadratic forms in independent variables.

88-06	M.A.J. van Montfort, J.V. Witter	Statistische analyse met betrekking tot maxima.
88-07	L.R. Verdooren	Use of variance components in the analysis of complete block designs.
88-08	P. van der Laan	Selection from designed experiments.
89-01	E. Hendrix	Risk and Uncertainty in OR models which can be used in agriculture.