

Comparison of vegetation roughness descriptions

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ABSTRACT

Vegetation roughness is an important parameter in describing flow through river systems. Vegetation impedes the flow, which affects the stage-discharge curve and may increase flood risks. Roughness is often used as a calibration parameter in river models, however when vegetation is allowed to develop naturally in river restoration projects, it is important to have suitable predictions of the increased resistance caused by the vegetation. There are many formulas available for describing vegetation roughness in one or two dimensional flow, ranging from simple wall roughness approximations to (semi-) empirical or theoretically derived roughness descriptions that are a function of flow and plant characteristics. In this paper a number of roughness descriptions are compared. All descriptions give a reasonable fit to experimental flume data. However, the models show significant deviation when extrapolating to large water depths typical for extreme discharge conditions. Therefore, to identify the suitability of existing methods for flood modeling, more data at large water depths are necessary. At large submergence ratios vegetation roughness can be approximated by a constant Manning coefficient.

Keywords: river models, vegetation roughness, uncertainty

1 INTRODUCTION

River models are an important tool in designing safety measures along rivers. For a given discharge (design discharge) water levels are calculated to determine the threat for the surrounding environment. Measures like dikes, widening or deepening the river bed etc., can be incorporated to evaluate the effect on water levels and hence the changes in threats. For these types of studies 1 or 2 dimensional river flow models are commonly used. Examples of such models are HEC, MIKE11, SOBEK and WAQUA. These models contain different types of uncertainties caused by model assumptions,

spatial discretization and parameter estimation. The uncertainty in the outcome of models can be reduced by calibration and validation of the model with field data. In this process roughness is often used as a calibration parameter because the roughness parameter is the most difficult to quantify independently. However, in areas with high safety levels design discharges are much higher than the measured discharges that are used for calibration. This means that the applicability of the calibrated parameters becomes questionable at extreme discharge conditions. To warrant realistic model outcomes in extrapolated (extreme) conditions, it is therefore important that model components

are well understood and that remaining calibration parameters remain fairly constant with changing flow conditions. That way, the uncertainty of model outcomes for extrapolated conditions may be reduced.

In river systems with extensive floodplains the hydraulic resistance caused by vegetation can have a major impact on water levels. There are many different models that can be used to estimate the vegetation roughness. Commonly used approaches are based on equivalent Manning or Chézy roughness values. However, these equations were not derived to describe the complex interactions of vegetation with flow. Several models have been developed to specifically describe the resistance caused by emergent or submerged vegetation. So far, only few of these models have been applied in general river models. In this paper some of the existing equations will be compared to each other and to the more conventional approaches. The focus here will be on submerged vegetation.

2 VEGETATION ROUGHNESS DESCRIPTIONS

2.1 Conventional approaches

The most common way to describe hydraulic roughness is by the Manning equation:

$$U = \frac{1}{n} h^{2/3} \sqrt{i} \quad (1)$$

where U is the average velocity over the cross section, n the Manning coefficient, h the water depth and i the energy slope. This equation applies to uniform and steady flow conditions, but is commonly applied beyond these conditions. In Eq. (1) and in the following it is assumed that the considered system is much wider than the water depth, such that the hydraulic radius R can be replaced by water depth h . A variation on Eq. (1) is the Strickler equation where $n = k_s^{1/6}/25$, in which k_s is Strickler's equivalent roughness height.

Another commonly used equation is the Chézy equation:

$$U = C\sqrt{hi} \quad (2)$$

where C is the Chézy coefficient. Equivalent to the Chézy equation is the Darcy-Weisbach equation where $C = (8g/f)^{1/2}$ with g the acceleration due to gravity and f the Darcy-Weisbach coefficient. The Chézy and Manning coefficient are related by the following relation:

$$C = h^{1/6} / n \quad (3)$$

Note that Eq. (1) and (2) show a different dependency of the velocity or discharge on h , which makes the choice for one of the two equations a fundamental choice. Manning's equation was developed for fully turbulent flow in open channels; Chézy is often used for lower Reynolds numbers (Kadlec, 1990).

Although the Manning and Chézy equation are originally derived to describe wall roughness, they are also used for vegetation resistance. It is common practice to use a constant value for n determined from look-up tables (Chow, 1959, Arcement and Schneider, 1989). Values for the Chézy coefficient are often calculated with Keulegan's equation for rough channel flow (Keulegan 1938), which can be derived from the mixing length theory:

$$C = 18 \log \left(\frac{12h}{k_N} \right) \quad (4)$$

where k_N is the Nikuradse equivalent roughness height.

The vegetation resistance experienced by the flow depends on water depth, Reynolds number, and vegetation characteristics such as height, density, stem diameter, and flexibility (Yen, 2002). Several (semi) theoretical and empirical relations incorporate some of these factors in the roughness coefficient. However, many of these relations have not made their way to commonly used river models. Basic types that can be used in commercial software are:

$$n = a \ln(UR)^b \quad (5)$$

$$n = ah^b \quad (6)$$

where a and b are empirical constants. A specific form of Eq. (6) is the relation given by De Bos and Bijkerk (1963):

$$n = h^{1/3} / \gamma \quad (7)$$

with γ the De Bos and Bijkerk coefficient. Empirical relations as in Eqs. (5) and (6) are easy to incorporate in river flow models, however, since they lack a theoretical basis, their range of applicability is limited to the flow range in which the empirical constants are calibrated.

2.2 New approaches

More recently a number of equations have been developed to describe vegetation roughness based on parameters that reflect vegetation characteristics such as vegetation height k , density m , diameter of plant stems D , and the drag coefficient C_D . In this paper we will compare three such methods. For simplicity the bottom roughness is ignored but may become relevant for sparse vegetation. For flow through vegetation all three methods reduce to the same equation which is equivalent to the relation proposed by Petryk and Bosmajian (1975). Here we will only discuss submerged conditions.

Method 1 Van Velzen et al. (2003)

The first method by Van Velzen et al. (2003) is incorporated in the two-dimensional WAQUA model which is the standard software in the Netherlands used for legal purposes. This method is based on the approach by Klopstra et al. (1997) who derived an analytical solution for the depth averaged velocity U based on the momentum equation for the vegetation layer and the Prandtl mixing length concept (logarithmic velocity profile) for the surface layer overflowing the vegetation. The result is a rather lengthy expression:

$$U = \frac{k}{h} U_v + \frac{h-k}{h} U_s \quad (8)$$

where U_v is the depth-averaged velocity in the vegetation layer:

$$U_v = \frac{2\ell}{k} \left(\sqrt{Ke^{k/\ell} + u_s^2} - \sqrt{K + u_s^2} \right) + \frac{u_s \ell}{k} \ln \left(\frac{\left(\sqrt{Ke^{k/\ell} + u_s^2} - u_s \right) \left(\sqrt{K + u_s^2} + u_s \right)}{\left(\sqrt{Ke^{k/\ell} + u_s^2} + u_s \right) \left(\sqrt{K + u_s^2} - u_s \right)} \right) \quad (9)$$

and U_s the depth-averaged velocity in the surface layer:

$$U_s = \frac{u_*}{\kappa(h-k)} \left((h - (k - h_s)) \ln \left(\frac{h - (k - h_s)}{z_0} \right) - h_s \ln \left(\frac{h_s}{z_0} \right) - h + k \right) \quad (10)$$

u_s is the flow velocity through the vegetation for non-submerged conditions ($h < k$) and is equivalent to the expression derived by Petryk and Bosmajian (1975):

$$u_s = \sqrt{\frac{2gi}{C_D m D}} \quad (11)$$

ℓ is a scaling length defined as:

$$\ell = \sqrt{\frac{\alpha}{C_D m D}} \quad (12)$$

where α is a closure parameter derived from experimental data:

$$\alpha = 0.0227k^{0.7} \quad (13)$$

Note that α has unit of length and that Eq. (13) is therefore not dimensionally correct. Other relations have been proposed for this closure parameter as well (Klopstra et al., 1997, Huthoff, 2007).

K is a help variable equal to:

$$K = \frac{gi(h-k)\ell}{\alpha \cosh(k/\ell)} \quad (14)$$

The roughness height of the surface layer z_0 is defined as:

$$z_0 = h_s e^{-M} \quad (15)$$

where h_s is the distance between the top of the vegetation and the virtual bed of the surface layer, defined as:

$$h_s = \frac{1 + \sqrt{1 + 4L^2\kappa^2(h-k)/gi}}{2L^2\kappa^2/gi} \quad (16)$$

and

$$L = \frac{K \cosh(k/\ell)}{\ell \sqrt{2K \sinh(k/\ell) + u_s^2}} \quad (17)$$

$$M = \frac{\kappa \sqrt{2K \sinh(k/\ell) + u_s^2}}{u_*} \quad (18)$$

with κ the Von Kármán constant taken as 0.4 and u_* the shear velocity for the surface layer:

$$u_* = \sqrt{gi(h - (k - h_s))} \quad (19)$$

Method 2 Baptist et al. (2007)

Baptist et al. (2007) derived an expression for the Chézy coefficient by applying dimensionally aware genetic programming to a set of 990 results of a 1DV turbulence model developed by Uittenbogaard (2003). This model solves a simplification of the 3D Navier-Stokes equation for horizontal flow conditions. The 990 cases represent a wide variety of water depths and vegetation properties, representing vegetation as rigid cylinders. Substituting the expression for the Chézy coefficient of Baptist et al. into Eq. (2) yields the following flow equation:

$$U = \left(\sqrt{\frac{2g}{C_D m D k}} + \frac{\sqrt{g}}{\kappa} \ln\left(\frac{h}{k}\right) \right) \sqrt{hi} \quad (20)$$

At $h = k$ the equation reduces to u_s (Eq. 11). The second term in parentheses is equivalent to the Keulegan equation (Eq. 4), where $k_N = 12k$. In this relation the velocity, and hence discharge, shows a clear direct relationship to vegetation parameters: U decreases with increasing C_D , m , D , and k .

Method 3 Huthoff et al. (2007)

Huthoff et al. (2007) derived an analytical expression for bulk flow through and over vegetation using scaling assumptions. The

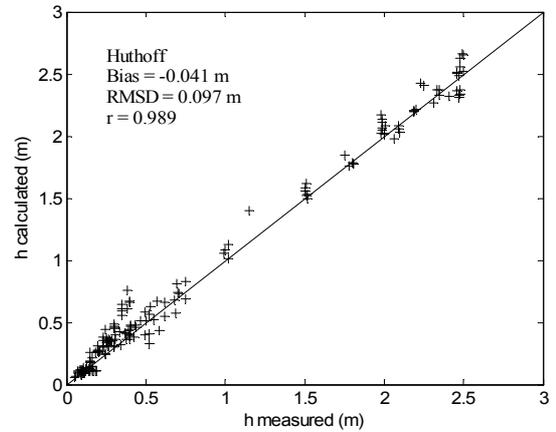
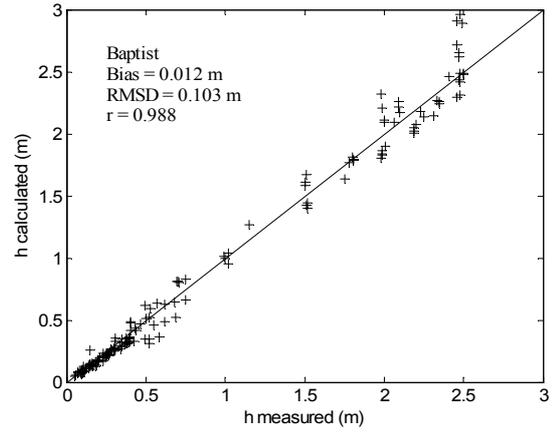
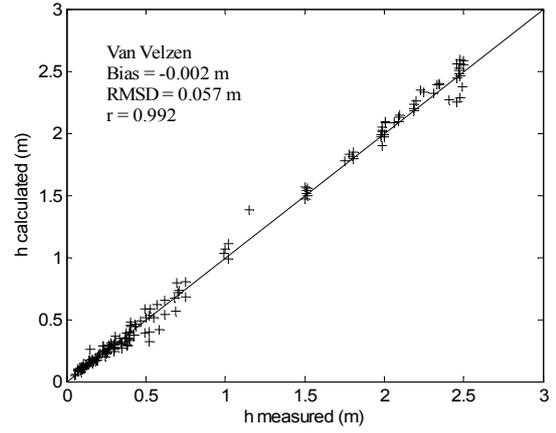


Figure 1. Performance of the three vegetation roughness descriptions in predicted water depth compared to experimental data.

resulting expression for the depth-averaged velocity is:

$$U = \sqrt{\frac{2gi}{C_D m D} \left(\sqrt{\frac{k}{h}} + \frac{h-k}{h} \left(\frac{h-k}{s} \right)^{\frac{2}{3}} \left(1 - \left(\frac{h}{k} \right)^{-5} \right) \right)} \quad (21)$$

with $s = 1/m^{1/2} - D$, the average spacing between vegetation. The first term on the right hand side is again the flow velocity through the

Table 1. Overview of flume experiments with (artificial) vegetation characteristics and fitted parameter values for conventional roughness descriptions. F = flexible, R = rigid, N = number of experiments.

| Reference | N | k (m) | D (m) | m (m ⁻¹) | C_D | n (s/m ^{1/3}) | C (m ^{1/2} /s) | γ (s) | k_N (m) | |
|-------------------------------|---|------------|------------|---------------------------|-------|------------------------------|------------------------------|-----------------|--------------|------|
| Ikeda and Kanazawa (1996) | F | 7 | 0.04 | 0.00024 | 20000 | 1 | 0.04 | 17.6 | 0.02 | 0.21 |
| Tsujimoto and Kitamura (1990) | R | 8 | 0.0459 | 0.0015 | 2500 | 1.46 | 0.05 | 12.1 | 0.02 | 0.20 |
| Murota et al. (1984) | F | 8 | 0.058 | 0.00024 | 4000 | 2.75 | 0.05 | 12.7 | 0.03 | 0.24 |
| Tsujimoto et al. (1993) | F | 12 | 0.065 | 0.00062 | 10000 | 2 | 0.08 | 8.4 | 0.04 | 0.53 |
| Kouwen et al. (1969) | F | 27 | 0.1 | 0.005 | 5000 | 3 | 0.09 | 9.0 | 0.06 | 1.06 |
| López and García (2001) | R | 5 | 0.12 | 0.0064 | 170 | 1.13 | 0.05 | 15.0 | 0.03 | 0.44 |
| Ree and Crow (1977) | F | 14 | 0.2032 | 0.005 | 1464 | 1 | 0.10 | 8.4 | 0.07 | 1.68 |
| Järvelä (2003) | F | 9 | 0.205 | 0.0028 | 12000 | 1 | 0.09 | 9.4 | 0.07 | 1.83 |
| | F | 3 | 0.295 | 0.003 | 512 | 1 | 0.08 | 10.4 | 0.06 | 1.27 |
| Ree and Crow (1977) | F | 16 | 0.3048 | 0.005 | 1076 | 1 | 0.14 | 6.4 | 0.12 | 3.05 |
| Meijer (1998b) | R | 8 | 0.45 | 0.008 | 64 | 0.97 | 0.05 | 20.5 | 0.07 | 1.59 |
| | R | 8 | 0.45 | 0.008 | 256 | 0.98 | 0.06 | 17.0 | 0.08 | 2.50 |
| | R | 8 | 0.9 | 0.008 | 64 | 0.97 | 0.09 | 12.9 | 0.11 | 4.60 |
| | R | 8 | 0.9 | 0.008 | 256 | 0.99 | 0.11 | 10.1 | 0.14 | 6.58 |
| | R | 8 | 1.5 | 0.008 | 64 | 0.96 | 0.15 | 7.5 | 0.20 | 10.4 |
| | R | 8 | 1.5 | 0.008 | 256 | 0.99 | 0.22 | 5.1 | 0.29 | 14.0 |
| Meijer (1998a) | F | 7 | 1.64 | 0.0057 | 254 | 1.805 | 0.28 | 4.1 | 0.36 | 15.5 |

vegetation for non-submerged conditions (u_s). The relation of U with k , m , and D here is less straightforward but in general U decreases with increasing values of the vegetation parameters. When h becomes large Eq. (21) approaches:

$$U = \sqrt{\frac{2g}{C_D m D s^{4/3}}} h^{2/3} \sqrt{i} \quad (22)$$

Comparison to the Manning equation (Eq. 1) provides an estimate for the Manning coefficient based on vegetation parameters.

3 COMPARISON OF ROUGHNESS DESCRIPTIONS

3.1 Comparison to experimental flume data

To compare the different roughness descriptions a data set compiled by Baptist (2005) was used. Table 1 summarizes the data. The data set includes rigid and flexible, artificial and natural vegetation types. For k the average deflected height is taken. Bed roughness was assumed negligible in the experiments. Values for the drag coefficient

were taken from the respective studies, or when no drag coefficient was given a value of 1 was assumed. It should be noted that the drag coefficient is determined in different ways in this data set. The values vary over a factor three and this has a large impact on the outcome of the models. It should also be noted that the data of Meijer (1998b) were used by Huthoff et al. (2007) to derive Eq. (21), and that part of the data set was used to derive the closure relation (Eq. 13) for the method by Van Velzen et al. (2003).

The three models can be compared in different ways. Since the aim of river models is to predict water levels, Figure 1 compares the measured and predicted water depths for the three vegetation roughness descriptions presented in section 2.2. All methods show high correlation coefficients (r), small biases and root mean square differences (RMSD) indicating good performance.

In Figure 2 fits of the models to three individual data sets are shown. The data sets are chosen to represent three distinctly different vegetation heights. Predicted water depths normalized to the vegetation height are plotted as a function of $U/i^{1/2}$. On the x-axis $U/i^{1/2}$ is used because the energy slope i is different for each experiment (data point). The fits of the

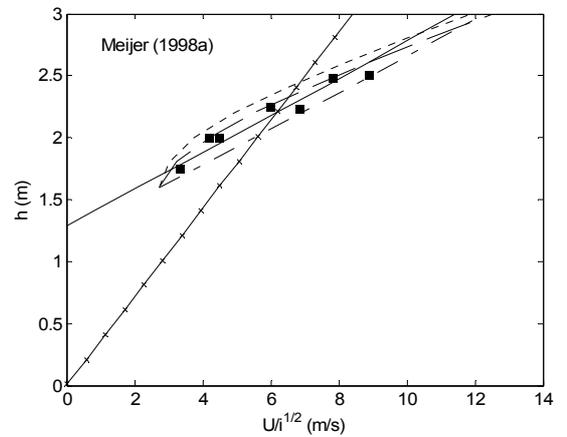
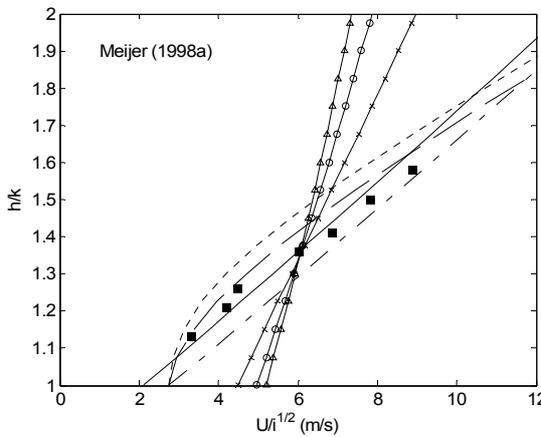
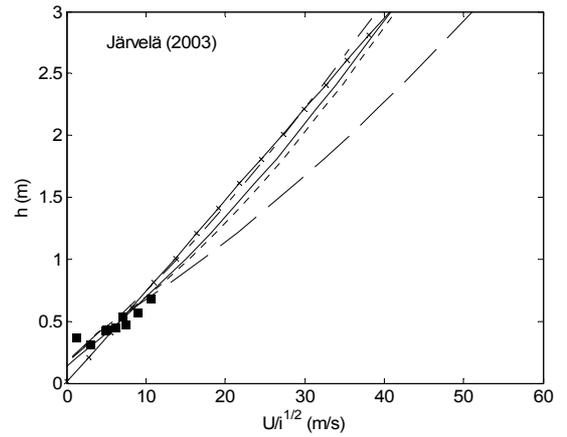
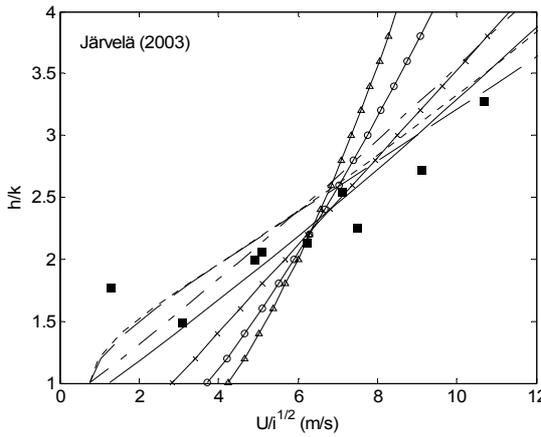
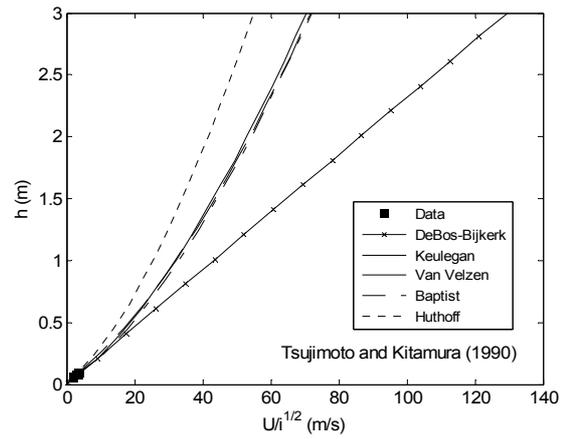
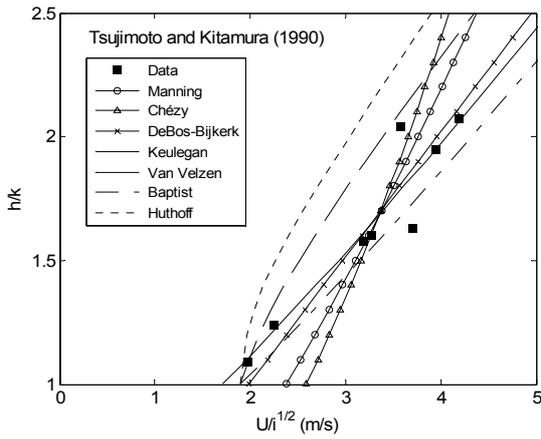


Figure 2. Three data sets with estimated lines for different models. Legend in the upper figure holds for all figures.

Figure 3. Same data sets as in Figure 2 with predictions extended to a water depth of 3 m.

three vegetation roughness descriptions are independent predictions based on the values for k , D , m and C_D given in Table 1. All these methods converge to the same value at $h = k$. The data were also used to fit the Manning, Chézy, Keulegan and De Bos and Bijkerk equations by minimizing the root mean square differences. The optimized parameters for these models are given in Table 1. Disadvantage of these methods is that they are based on empirical roughness coefficients that have to be

fit for each new data set, making them dependent on the data set for which they are calibrated. Most models provide a reasonable fit, only constant Manning and Chézy coefficients are in general not accurate as estimators for vegetation resistance. Although Van Velzen et al. (2003) suggest that the Nikuradse roughness height k_N is also a function of water depth, a fitted constant performs quite well. Moreover, on average the

Keulegan formula shows the best agreement with experimental data, although for some data sets other models may perform better. It can also be seen in Figure 2 that the Keulegan equation runs more or less parallel to the method of Baptist et al. (2007). As pointed out before, these equations are closely related. The experimental data show a more or less linear relation between h and $U/i^{1/2}$. De Bos and Bijkerk method (Eq. 7) describes a linear relation, only the intercept for this equation is forced through zero. With a similar base as the three roughness descriptions this relation would perform the best.

So far it can be concluded that the three roughness models perform equally well as do Keulegan and De Bos and Bijkerk. A disadvantage of the latter two is that they contain an empirical parameter that needs to be calibrated. However, the same can be said about the three more advanced methods. For the experimental data the vegetation parameters m , D , k and C_D are known, however in the field vegetation is quite heterogeneous and these parameters are not easily determined. Some indication values are possible, but they remain estimates, similar to the way other empirical parameters such as the Manning or Chézy coefficient have to be estimated or calibrated.

3.2 Comparison at extreme conditions

The water depths in the experimental data vary from a few centimeters to over 2 meter. In Dutch floodplains water depths can easily reach several meters at high discharges and certainly for the design discharge. The question then rises: how do these methods perform at such high discharges? To evaluate this the graphs from Figure 2 are extrapolated to discharges that generate water depths up to three meters (Figure 3). For some cases, the methods show considerable deviations in predicted water levels for a given discharge, especially the De Bos and Bijkerk model. To illustrate this in another way, Figure 4 shows the calculated water depths for the different methods for a given discharge ($U/i^{1/2} = 20$ m/s). The parameters used to calculate the water depths are taken from Table 1. The De Bos and Bijkerk model is left out here. Excluding the outlier (predicted by Huthoff's model for the

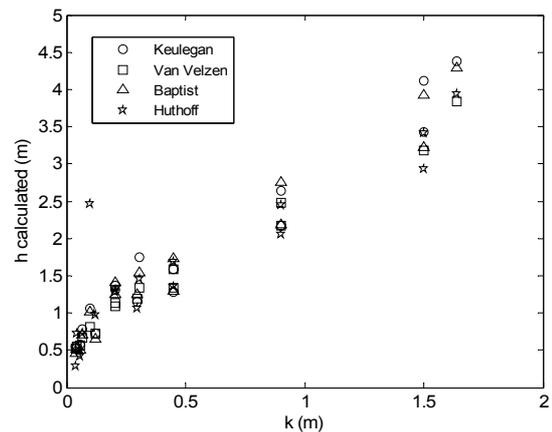


Figure 4. Water depths calculated by different models for the experiments in Table 1 and $U/i^{1/2} = 20$ m/s.

data of Kouwen et al.), the maximum difference between predicted water depths ranges from 0.08 to 0.71 m. The differences increase even more when discharge increases (see Figure 3). Figure 4 shows a clear relation between predicted water depth and the height of the vegetation. Other relations of plant characteristics with water depth were also examined, but none gave such a good correlation as k .

Based on the different predictions of vegetation roughness descriptors, there remains a large uncertainty in flow response to the presence of vegetation at high discharges. In fact, the uncertainty can be considered too large for designing safety measures. In conclusion, more data is required, especially for large submergence ratios to establish which description performs best under a wide range of flow conditions.

3.3 Comparison of Manning coefficients

In Figure 5 data are compared by calculating the Manning coefficient as function of submergence ratio h/k (Figure 5). It can be seen that for all methods the Manning coefficient decreases with increasing submergence ratio and eventually levels off. The rate by which they level off is different for different model parameters, but in general it can be said that for $h/k > 5$ the roughness coefficient can be approached by a constant Manning coefficient. Therefore, at high discharges and associated high submergence ratios calibration on constant Manning coefficient is acceptable.

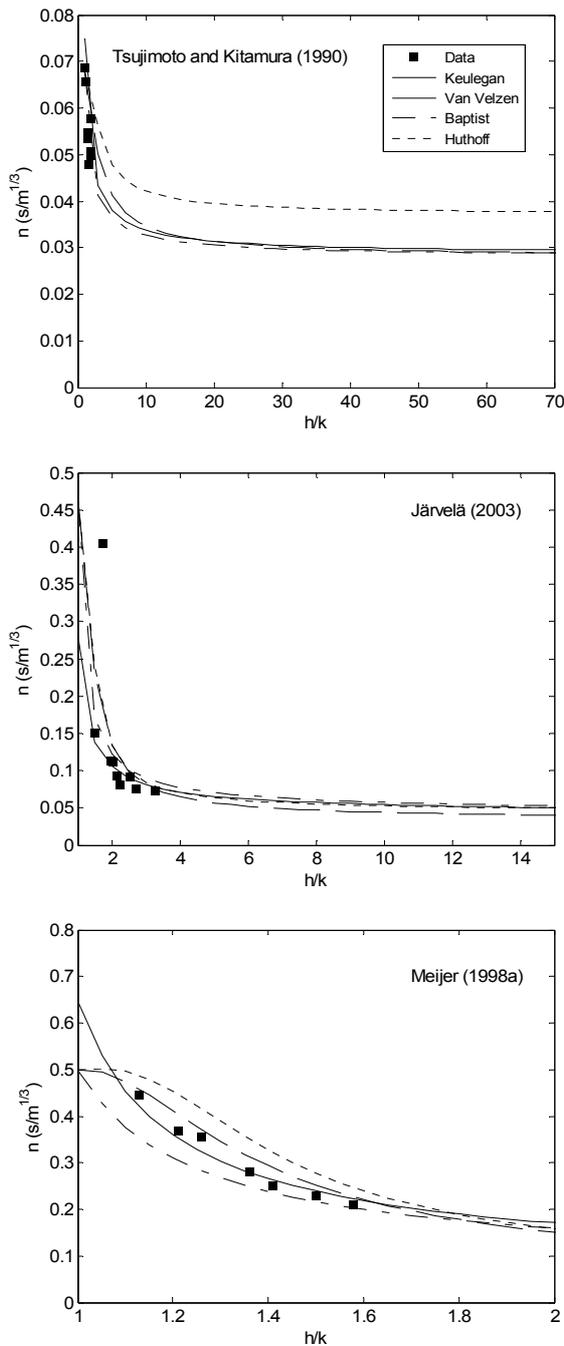


Figure 5. Comparison of calculated and measured Manning coefficients for three data sets. The highest submergence ratio (h/k) corresponds approximately to a water depth of 3 m.

4 DISCUSSION AND CONCLUSIONS

There are many ways to describe roughness, some of which are empirical and some theoretically derived. All descriptions can be fitted to data to yield reasonable agreement. However, when extrapolating data to larger water depths methods can deviate significantly from each other in predicted water levels. This is a worrying conclusion, since river models

are used to set safety standards and significant uncertainties herein are not wanted. Therefore, to determine which model performs best, data for higher water levels are required. Besides the limited range of submergence ratios, a difficulty with the used data base is the differences by which the drag coefficient C_D is determined, e.g. by force measurement, calibration or estimation. It is desired to agree on a standard way to determine the drag coefficient to make values comparable. A similar statement can be made about the vegetation height k . When water flows over flexible vegetation, the vegetation bends over decreasing the effective height, and hence the drag. However, the bended vegetation may be considered as an increase of the effective density or plant diameter near the top of the vegetation. In addition, waving of flexible plants in the flow causes turbulent energy losses. The net effect is not clear and will depend on individual species (Green, 2005). For practical purposes it is proposed to use the average vegetation height not affected by flow. At high submergence ratios the Manning equation provides a good approximation as long as the value is calibrated on data at submergence ratios $h/k > 5$. Besides experimental data from flume experiments it is also desired to collect field data at high discharges to derive bulk roughness estimates for different vegetation types.

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