A preliminary analysis of bedform evolution in the Waal during 2002-2003 flood event using Delft3D

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ABSTRACT: This work includes a brief analysis of measured data of bedform evolution process during flood event of 2002-2003 in the River Waal, and comparison with simulation result for some of the test cases. Simulation and data analysis were made for the entire Waal reach, which is divided into 63 characteristic morphological units. We briefly analyzed the nature of the measured data in order to elucidate the behavior of large scale dune evolution under varying flows at different morphological units. Furthermore, we carried out some test simulations by using a conventional approach of dune evolution including relaxation and advection, and in turn, coupling with Delft3D morphological model. Performance of different equilibrium bedform predictors together with the effect of relaxation and advection parameters on dune evolution feature was evaluated. It is evident that the dune evolution during flood event of 2002-03 can be predicted with a simple approach of bedform evolution coupling with an advanced morphological model. Proposed model appears to be rather efficient and handy for the real-world application, which is an important prerequisite for this work.

1 BACKGROUND
1.1 *Existing knowledge*

Many attempts have been made to improve both understanding and predictive capability of bedform evolution, transition and associated resistance under varying flow conditions (Simons & Richardson, 1961; Engelund, 1966; Kishi & Kuroki, 1973; Itakura et al., 1986; Tsujimoto et al., 1990; Wright & Parker, 2004, etc.). Most, if not all, of the approaches are empirical or semi-empirical. There is still no prediction method that can treat these phenomena in a coupled manner based on a first-principles physical formulation, i.e. a model which explicitly treats the physics of flow, morphodynamics of bedforms, non-equilibrium sediment transport, drag effect due to the pressure variation in the presence of bedforms and associated flow-field modification, thereby effects on water surface variation. The interactions among the flow-field, bed geometry and sediment transport are quite complex and difficult to capture in simple models. The bedforms are created and altered by the flow and, conversely, the flow is acted upon by the bedforms through the production of form drag and significant variation in local flow fields (Nelson et.al, 1993). Giri & Shimizu (2006) made a significant effort to numerically replicate the dune formation and evolution process. They proposed a morphodynamic model that successfully reproduces fluid and bedform dynamics in a coupled manner under arbitrary steady or unsteady flow condition.

Recently, their model has been extended in order to replicate the dune evolution, transition as well as variation of form drag produced by the temporal growth or decay of bedforms under unsteady flow conditions (Giri et al., 2007; Shimizu et al., submitted). The most important outcome of their analysis is that the dune evolution under varying flows seems to alter significantly depending on the flow intensity and the shape of the hydrograph. In their numerical experiments, they revealed different scenarios of dune evolution process with and without hysteresis characteristics. It implies that the hysteresis of dune evolution and thereby stage-discharge loop is not always the case even under the varying flow condition.

Number of studies was performed to observe the development of bedforms during different flood events in the Dutch Rhine branches (Wilbers & Brinke, 2003; Sieben, 2006). These studies have provided a valuable insight into the behavior of dune morphodynamics under varying flows in the Rhine branches. Wilbers & Brinke (2003) found different characteristics of dune growth and decay for the various sections during different flood events. They attributed those differences to grain size as well as the distribution of discharge over the main channel and the floodplain. They concluded that the growth and migration rate of dunes as well as bedload transport rates during the rising stage of a flood wave can

be predicted from the mobility of the bed material with simple power relations. Sieben (2006) observed a lag between the discharge and bedform amplitude during flood event of 1997 and 1998 in the Waal. Analyzing bedform evolution data for 1998 flood event, he found that bedform length becomes lowest during maximum flood discharge and subsequently increases during falling stage.

1.2 *Present study and objective*

The Rhine River is the most heavily navigated inland waterway in Western Europe. Due to its advantageous location in the Rhine delta, the inland waterways in the Netherlands form a natural access to the continent of Europe. The Waal branch of Rhine system is considered to be an important fairway. The flood events and extensive navigation cause significant morphological changes, which create difficulties for safe and efficient navigation particularly during low water period. Moreover, such a morphological behavior might lead to the formation of nautical bottlenecks in the Rhine branches.

At present, the fairway in the Waal is maintained by dredging operation at the shallow parts, but the same problem is repeated after the subsequent flood season (Sieben, 2006). Therefore, it is important to predict the bed level changes, particularly evolution of bedforms during the flood event including both the high water and low water periods as the navigable depth is determined based on bedform averaged levels.

Most models with the real-world application only consider the large scale and long-term morphological behavior. They are incapable to predict the evolution of geometric characteristics of micro-scale bedforms. On the other hand, the advanced models, which are capable to predict bedform characteristics in a physically based manner (Tjerry & Fredsoe, 2005; Giri & Shimizu, 2006) cannot be applied to resolve sophisticated real-world problems due to rather intensive computational efforts. Consequently, a pragmatic and efficient tool that can be applied in a real-world river system to replicate morphodynamic behavior of micro-scale bedforms considering their resistance to flows is of importance in the present context.

2 BRIEF ANALYSIS OF MEASURED DATA
2.1 *Measured quantities and approach*

The whole Waal is divided into 63 morphological units. The length of each morphological unit varies between 0.5 and 4 km. We can characterize three parts of the Waal, i.e. upper Waal, middle Waal and lower Waal respectively. The time-series data are available for 63 morphological units along the Waal

for the period of 1994 to 2003. These are multibeam data on a grid of 5x5 m. The bedform parameters are determined for a river bed sub-area (50m wide and 500m long). For every sounding, these parameters per sub area are available. In order to construct time series, all sub areas are assigned to the corresponding morphological unit (covering a bend or crossing). Only the parameters left and right are distinguished. The purpose was to derive morphological response relations for every morphological unit. The detailed data processing method has been described in Sieben (2004).

In this study, we used the data corresponding to the flood period of 2002-2003 including the low water period. . It is to be noted that the data set (particularly emphasis was given to the dune amplitude) for the period of 2002-2003 seems to be qualitatively more reliable, where we can clearly see the dune evolution feature during high water as well as low water period.

2.2 *Characteristics of dune evolution in different morphological units*

We attempted to analyze the dune evolution process (growth and decay of dune amplitude and length during flood event) for some of the characteristic morphological units of the Waal. As we mentioned, the Waal can be divided into three different parts, namely the upper part with a meandering reach followed by the middle part more or less straight, and the third part, where channel changes its direction abruptly and also comprises a bend part. It is to be noted that two bends, i.e., bends at Nijmegen (unit-16) and St. Andries (unit-49), have non-erodible layers.

In the first bend after Pannerdensch kop (starting from unit-6 up to unit-10, Fig. 1), the evolution of dune amplitude along the channel center appears to be regular (without any growth and decay) and of lower magnitude (less than 1.5m). Moreover, in this reach, dune amplitude appears to be higher along the right bank (i.e., along the outer bend). After morphological unit-10, where bend changes its direction (bend at Erlecom, Fig. 2), the dune amplitude along right bank appears to become similar to the values along the centerline (even less in some points), which is logical. Dune amplitude along the channel center in this bend seems to be somewhat higher than the preceding bend. Again, from the morphological unit-13 when new bend (bend at Haalderen- unit-13 and 14, Fig. 2) starts, the dune height at outer bend increases. Dune evolution is rather regular until unit-20 (Slijk-Ewijk) and of lower magnitude.

Specifically, dune growth and decay during flood event of 2002-2003 is rather clear for the middle Waal region starting from the morphological unit-21 up to 47 (i.e. from km 891 to km 921, see some typi-

cal examples in Fig. 3). Dune height decreases during falling stage in these units up to 0.5m. Dune amplitude near both banks and along the channel center appears to be of same magnitude.

In the lower Waal starting from the morphological unit-41 (after km 916), where river abruptly changes its direction, dune height does not seem to decrease much rapidly during falling stage of the flood event. The dune height in this region remains higher than preceding morphological units. Moreover, in and after the bend of St. Andries (unit-49 to 54, Fig. 2), the evolution of dune amplitude remains almost constant during the flood event with a considerable data scatter. This could be the effect of fixed layer in the outer bend. On the other hand, this indicates the dependency of the bedform adaptation on channel configuration/planform. In the lower part of the Waal, we can observe some trend of dune growth and decay in some morphological units, though most morphological units lack the detailed time-series data and thus the trend is not as clear as in the middle Waal.

Regarding the dune length evolution, the range of the data scatter is rather high and trend is not clear. The dune length appears to be somewhat constant during the whole period, and was severely underpredicted by existing predictors. It is to be noted that bedform parameters from measured quantities fit to a sinusoidal shape. Compared to a more realistic triangular shape, this tends to an overestimation of length mainly. On the other hand, if closely look at the dune length evolution for some morphological units (Fig. 4), one can observe the increment of dune length during falling stage, which is an expected behavior as observed in many previous investigations.

3 NUMERICAL COMPUTATION

3.1 Computational model

A depth-averaged version of morphological model Delft3D (Lesser et al., 2004) was used to compute hydrodynamic and morphological behavior of the Waal. The model incorporates all kinds of innovative, recently developed aspects, amongst which domain decomposition, consideration of floodplains including wet and dry processes, sediment transport over non-erodible layers and functionality for sediment management to assess dredging and dumping strategies (Yossef et al., submitted).

An advanced morphological model can be used to assess the long-term large-scale evolution of the Rhine system. As the model incorporates also complex time-dependent multi-dimensional phenomena, such as curvature-induced point bar and pool patterns in bends, assessment is also possible at the small and intermediate spatial scales.

A curvilinear grid was used for the computation. All relevant hydraulic and morphological features were projected on the grid. These include bed topography, bed composition (including fixed layers and spatial distribution of median sediment diameter, hydraulic roughness in the floodplains that was based on vegetation coverage, hydraulic structures including groynes, summer dikes and longitudinal dams).

We used a spatial distribution of median grain-size along the Waal reach based on observed data scatter (Fig. 5). The boundary conditions of the model were deduced from long term measurements. The hydraulic boundary conditions were imposed as follows, a time-dependent discharge at the upstream boundary (during 26th September 2002 to 31st December 2003, see Fig. 6), and a depth-discharge relation at the end of the Waal. The maximum peak discharge (6127 m^3/s) occurred on 6th January 2003 (i.e., on 103rd day of the simulation). There is one more peak (4383 m^3/s) on 15th November 2002 (i.e., on 51st day). Consequently, there were two flood waves; flood wave with the lower amplitude lasted about two months, whereas with the higher amplitude-about 35 days.

3.2 Evolution of bedform characteristics

As it was mentioned heretofore, it is extremely intensive in terms of computational effort to couple an advanced hydrodynamic model with bedform evolution process in a physically based manner. Consequently, a pragmatic approach was implemented in Delft3D (Van Vuren & Ottevanger, 2006).

An empirical model proposed by Allen (1976), extended to the two dimensional case, was implemented in order to describe the relaxation/advection processes of bedform geometry in both space and time (Sieben, 2006):

$$\frac{\partial H}{\partial t} + c_{Hx}\frac{\partial H}{\partial x} + c_{Hy}\frac{\partial H}{\partial y} = \frac{H_e - H}{T_H} \qquad (1)$$

where H = temporal dune height; H_e = equilibrium dune height; c_{Hx} and c_{Hy} = characteristic celerity in streamwise and transverse direction respectively; T_H = time scale for temporal adaptation of bedform height.

T_H can be defined as:

$$T_H = \frac{L_H}{c_H} \qquad (2)$$

where L_H = length scale for spatial adaptation of bedform height.

The length scale L_H is found to be proportional to the dune length, and the characteristic celerity is proportional to the celerity of bed disturbance as follows:
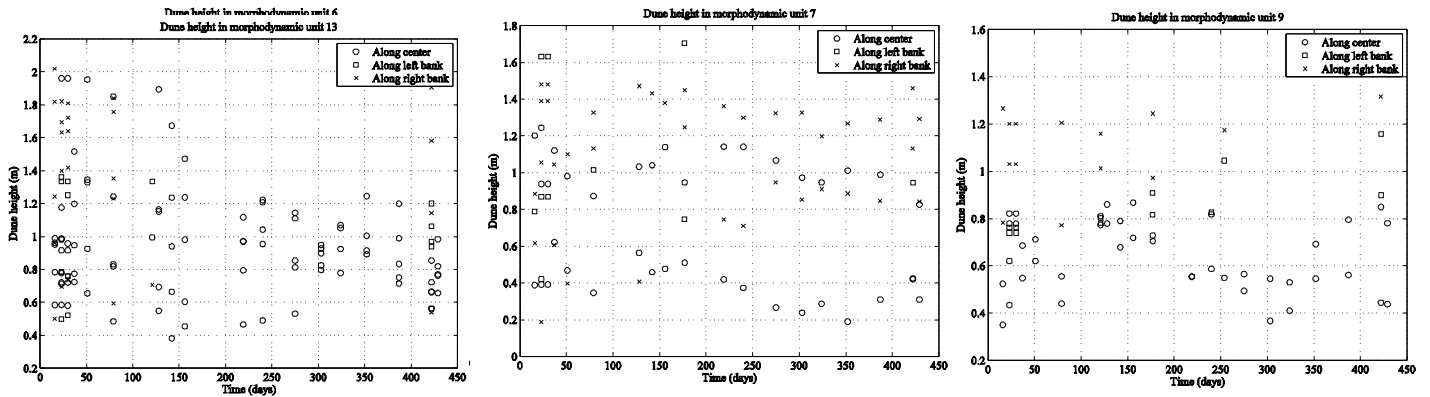
Figure 1. Typical feature of dune evolution in some selected morphological units of the upper Waal region.
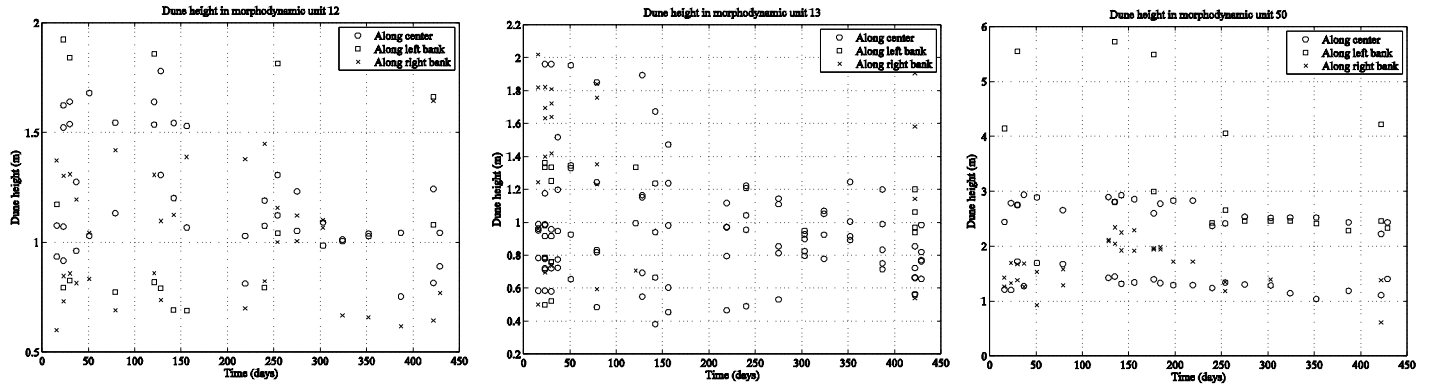

Figure 2. Dune evolution in the bend reach of Erlecom (unit-12), Haalderen (unit-13), and after St. Andries (unit-50).
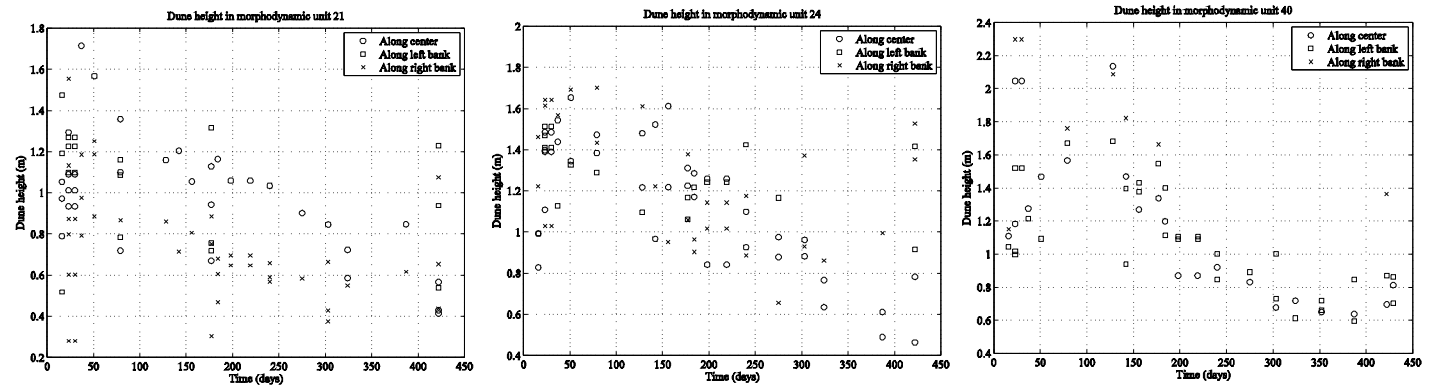

Figure 3. Typical feature of dune evolution in some selected morphological units of the middle Waal region.
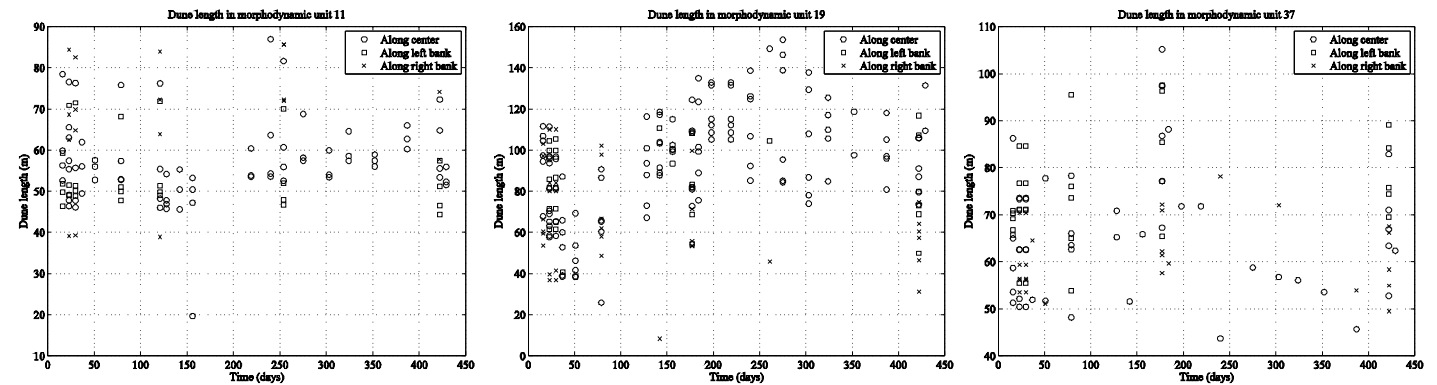

Figure 4. Evolution of dune length in some selected morphological units during flood event of 2002-2003.
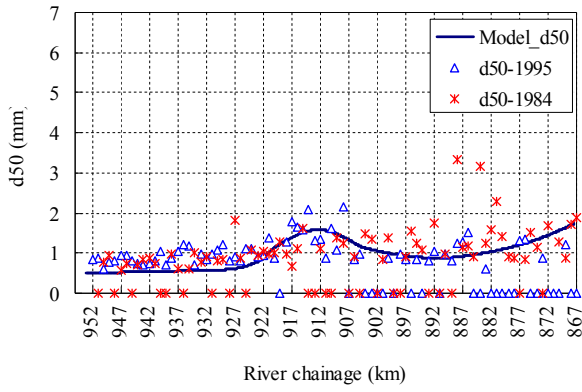
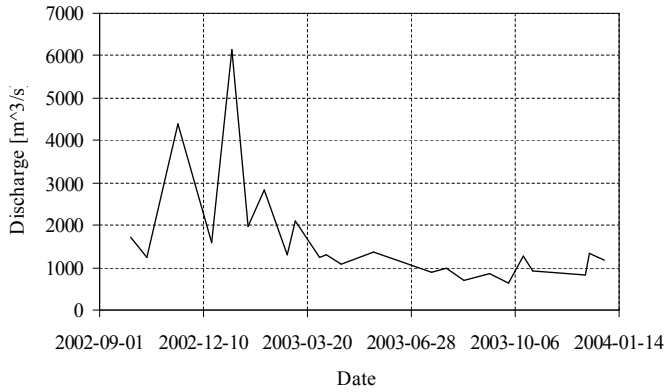Figure 5. Spatial distribution of mean diameter along the Waal

Figure 6. Discharge hydrograph in the Waal during 2002-2003, used for the simulation

$$L_H = \varphi L \tag{3}$$

$$c_H = \gamma c \tag{4}$$

where for the Waal φ = 1-3 and γ = 0.2-1.

The celerity of bed disturbance can be assigned based on field observation or calculated using empirical/semi-empirical approach.

Also, there is an option to use T_H and L_H as a constant parameter to check whether or not it provides an acceptable prediction in falling stage and low water period. Another approach is to relate the adaptation time scale to the sediment transport in order to make it variable for high and low water period for the better prediction of dune evolution particularly in low water period. However, this approach has not been implemented in present work.

3.3 Equilibrium dune height predictor

Since the relaxation model (Eq.1) includes the equilibrium dune height, we used a couple of predictors in order to calculate equilibrium dune height at each spatial and temporal step. One of the prediction methods proposed by Fredsoe (1982) reads as:

$$\frac{H_d}{h} = \frac{\varepsilon}{\dfrac{u}{f(u)}\dfrac{\partial f(u)}{\partial u} - \dfrac{h}{f(h)}\dfrac{\partial f(h)}{\partial h} - \dfrac{k}{f(k)}\dfrac{\partial f(k)}{\partial k}}$$
$$= \frac{\varepsilon}{b - l - m} \tag{6}$$

where b = nonlinearity in sediment transport due to flow velocity, i.e. $s = f(u)$; l = nonlinearity in sediment transport due to flow-depth, i.e. $s = f(h)$; m = nonlinearity in sediment transport due to flow velocity, i.e. $s = f(m)$; ε = a constant.

ε is found to be 0.5-1.5 for the Rhine branches (Sieben, 2004). In our simulation, ε =1 was used

Sieben (2004) analyzed the formula of Fredsoe using sediment transport formula of Engelund & Hansen that revealed b = 5, l = -0.5 and m = 0.5. This leads to a simple expression of Fredsoe's dune height predictor as follows:

$$\frac{H_d}{h} = \frac{\varepsilon}{5} \tag{7}$$

We as well analyzed the Fredsoe's predictor using van Rijn's sediment transport formula for the Waal and found that b-l-m is somewhat higher ($>$5).

For the sake of comparison, we also used the dune height and length predictors proposed by van Rijn (1984) and Julien & Klaassen (1995). However, in the preliminary offline calculation, the predictors of Fredsoe (in form of Eq. 7) and van Rijn (1984) appeared to be reliable, so we used only these two formulae for the further simulation. Particularly Equation 7 appears to be rather simple to be used in numerical computations.

3.4 Roughness predictor

Since we put emphasis on prediction of dune characteristics, the roughness prediction is not our primary concern herein. However, the model should provide a feedback effect of the form roughness induced by bedforms to the hydrodynamic module. As a consequence, the prediction of dune height for the subsequent time step is affected by the flow modification. This fact appears to be rather important particularly for the falling stage and low discharge period. Obviously, not only the different dune height formulations, but also the roughness height calculated by different predictors might lead to the altered interpretation of roughness. So, this fact is supposed to be investigated rigorously. However, the detailed study on roughness prediction is beyond the scope of this report.

For the calculation of roughness height, we used two different approaches, namely Van Rijn (1984) and Van Rijn (2007). In latter approach, Van Rijn distinguishes the roughness for different type of bed-

form, i.e. roughness induced by micro-scale ripples, mega ripples and dunes respectively. This approach appears to be useful, since during low water period the contribution of large scale dunes to roughness is almost negligible (though, they are rather important for the evaluation of navigation depth). The bed-forms with smaller size that are found to be superimposed on large dunes become dominant for roughness. We aim to include a methodology to predict the small scale bedforms as well. However, within the scope this study, we just used the approach of Van Rijn (2007).

4 PRELIMINARY RESULTS

Within the scope of this paper, we present numerical simulations for four different test cases as follows:

Case-1: Fredsoe (1982) for dune height and Van Rijn (1984) for roughness; Case-2: Fredsoe (1982) for dune height and Van Rijn (2007) for roughness, Case-3: Van Rijn (1984) for dune height and Van Rijn (1984) for roughness; Case-4: Van Rijn (1984) for dune height and Van Rijn (2007) for roughness.

Simulation results are depicted in Figure 7-9. Results were compared for the quantities derived along the center, the left part and the right part of the Waal, since we found some alteration in dune height evolution depending upon the transverse location at some morphological units, as mentioned in subsection 2.2. We attempted to evaluate the model performance for this behavior.

The result depicted in these figures clearly provides an overall impression about the model prediction capability. The different dune height predictors behave slightly distinctively in terms of magnitude, nonetheless, the trend appears to be similar. Van Rijn's predictor seems to provide somewhat underestimation, though in many cases it replicates the observed quantities reasonably well. On the whole, results leave an impression that Fredsoe's predictor gives better prediction, particularly if consider the low water period and also along the near-bank region. The implementation of a relaxation model in terms of adaptation time-scale parameter (T_H) appears to be useful for considering the lag behavior of bedform geometry, which is evident in some morphological units. We found that even a constant value of adaptation time scale (i.e., 20 days) seems to provide an acceptable result in many cases.

The roughness predictor shows some influence on the dune evolution feature, particularly during falling stage and low water period. Van Rijn (2007) formula predicts lower value of roughness; thereby dune height appears to be decreasing subsequently. As far as the prediction of roughness height is concern, Van Rijn (2007) formula gives the average value of nearly

0.2, whereas the van Rijn (1984) gives within the range of 0.3-0.4. The detailed quantitative analysis on this aspect is in progress.

5 PRELIMINARY CONCLUSION AND DISCUSSION

Some preliminary conclusions that can be drawn from this study are as follows:

1) In most morphological units of the Waal, dune evolution under varying flows is evident during 2002-2003 flood event. Measured quantity shows slow decrease in dune amplitude during falling stage in some morphological units. However, this is not the case for others, and those can simply be predicted with a constant relaxation parameter. The dune characteristic appears to depend on the channel planform as we found some alteration on bedform evolution feature associated with the location.

2) It is important to note that during low water season the amplitude of the dune might be affected by extensive navigation as well. However, effect of navigation on dunes was not considered herein.

3) The dune height predictor of Van Rijn underpredicts the dune amplitude. This problem can be resolved, to some extent (for some morphological units), by simply employing a multiplication factor to the dune height predictor.

4) Likewise, dune height predictor of Fredsoe as proposed by Sieben (Vuren & Ottevanger, 2006), with the value of ε =1, appears to be appropriate so as to predict dune height in most morphological unit, particularly during falling stage of flood event.

5) Julien & Klaassen's formula did not appear to provide any improvement (not shown herein).

6) Selection of dune height predictor does not appear to be a significant factor. Rather, adaptation time scale could be of more significance to smoothing the prediction and reproduce adaptation of dune evolution during falling stage of the flood event and the low flow period.

7) For most cases, measured data shows the adaptation time (T_H) to be 20-40 days, though for some cases it seems to be up to 100 days. A constant value of T_H, namely 20 days, was found to work well as shown by the numerical simulation.

8) The effect of advection on dune evolution was found to be negligible in present study. We, therefore, did not explore it extensively.

9) For some morphological units, prediction of dune evolution near the right and left bank is not that satisfactory as along the channel center. Though, simulations are found to be acceptable with Fredsoe's predictor for most cases.

10) So far as dune length is concerned, it seems to continue to grow for some time during the decay of

the flood. However, it appears that dune length does not explicitly depend on the flow depth. Since, in all predictor dune length is described as a function of the water depth, they are not able to reproduce dune length correctly (it is severely underestimated). It is to be noticed however that, in order to derive the bedform length from measured quantities, bedform parameters fit to a sinusoidal shape. Compared to a more realistic triangular shape, this tends to an over-estimation of bedform characteristic, particularly dune length. Results of a triangular routine for this purpose might correspond better to the predicted quantities. Also, a relaxation model can be used to replicate the dune length evolution with adaptation. More detailed analysis on dune length evolution is supposed to be done.

11) The dune evolution model with relaxation appears to be useful to replicate the adaptation of bedform characteristics; however it would be more reliable if we incorporate the sediment transport in a relaxation model. For instance, Tsujimoto et al. (1990) proposed a convolution-integral model to describe relaxation process of dune geometry and relate it with non-equilibrium sediment transport. They showed that the model can reproduce the hysteresis loop as well.

12) The model simulation is not physical in the sense that it does not reproduce the micro-scale bedform physically, but merely computes the bedform characteristics, and subsequently provides a feedback to hydrodynamic model by taking into account the bedform-induced roughness. Nonetheless, an advanced morphological model with many innovative features coupled with simple empirical/semi-empirical approaches seems to be rather effective to evaluate bedform characteristics and associated roughness.

On the whole, it can be inferred that the results are more or less within the acceptable range. They show at least qualitatively consistent trend with the measured quantities. The approach has been proved to be efficient and reliable to be used in engineering application. Next step is supposed to be proper reproduction of roughness induced by the predicted bedforms. In particular, it is of significance to develop a methodology to predict concurrently small scale bedforms, superimposed on large dunes, as they possess dominant impact on roughness during falling stage and low water period.

6 ACKNOWLEDGEMENT

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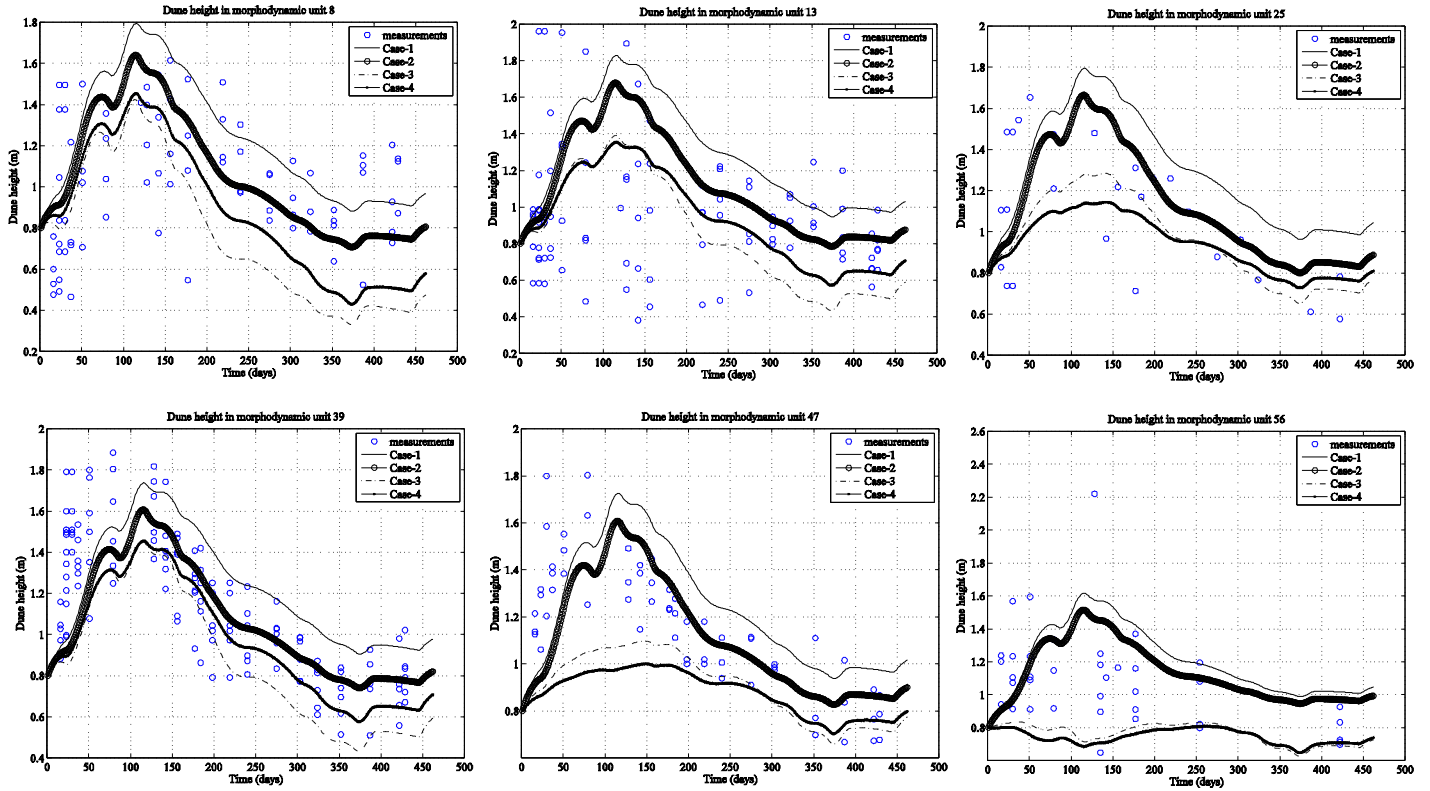
Figure 7. Model prediction on dune evolution with different cases for some selected morphological units.
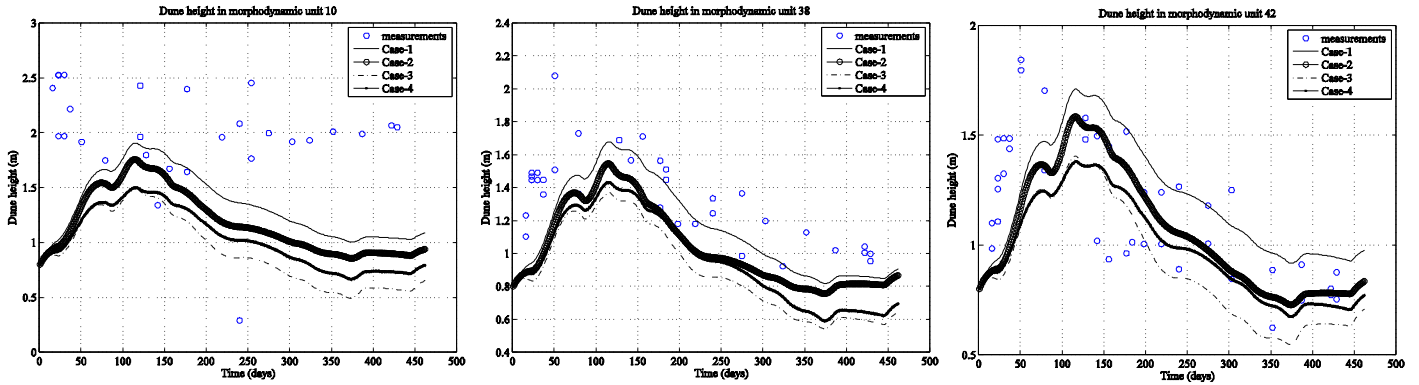

Figure 8. Model prediction on dune evolution with different cases along the right bank for some selected morphological units
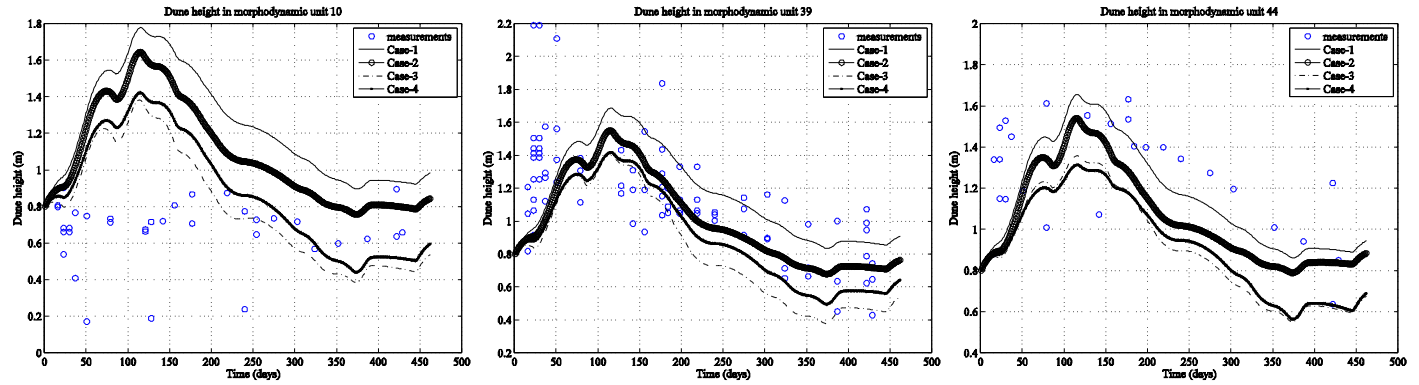

Figure 9. Model prediction on dune evolution with different cases along the left bank for some selected morphological units