LAMINAR FLOW HEAT TRANSFER
IN THE THERMAL ENTRANCE REGION
OF FLAT AND PROFILED DUCTS

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**NOMENCLATURE**

\[ a \quad \text{thermal diffusivity} \quad (\text{m}^2/\text{sec}) \]

\[ a_1 \quad \text{constant, eq. (52)} \]

\[ A_n \quad \text{constant, eq. (15)} \]

\[ b \quad \text{constant, eq. (52)} \]

\[ B \quad \text{width of duct cross section} \quad (\text{m}) \]

\[ Br \quad \text{Brinkman number} \]

\[ c_p \quad \text{specific heat capacity} \quad (\text{J/kg °C}) \]

\[ C_1 \quad \text{constant, eq. (4)} \]

\[ C_2 \quad \text{constant, eq. (7)} \]

\[ C_3 \quad \text{constant, eq. (19)} \]

\[ C_4 \quad \text{constant, eq. (29)} \]

\[ C_5 \quad \text{constant, eq. (48)} \]

\[ d \quad \text{thickness of thermal resistance layer} \quad (\text{m}) \]

\[ D_e \quad \text{equivalent or hydraulic diameter} \quad (\text{m}) \]

\[ f \quad \text{Blasius friction factor} \]

\[ g \quad \text{gravitational acceleration} \]

\[ Gr \quad \text{Grashof number} \]

\[ Gz \quad \text{Graetz number} \]

\[ Gz^+ \quad \text{reciprocal Graetz number} \]

\[ Gz^+_{\text{mod}} \quad \text{modified reciprocal Graetz number, duct VI} \]

\[ H \quad \text{height of duct cross section} \quad (\text{m}) \]

\[ H_i \quad \text{height of central core in duct VI, see fig. (3-25)} \]

\[ K \quad \text{constant, eq. (37)} \quad (\text{m}^2 \text{ °C/W}) \]

\[ L \quad \text{length of duct} \quad (\text{m}) \]

\[ M \quad \text{flowmodulus} \]

\[ Nu \quad \text{Nusselt number...} \]

\[ p \quad \text{pressure} \quad (\text{N/m}^2) \]

\[ \Delta p \quad \text{pressure drop} \quad (\text{N/m}^2) \]

\[ \text{Pé} \quad \text{Péclet number} \]

\[ Pr \quad \text{Prandtl number} \]

\[ Q_v \quad \text{volumetric flowrate} \quad (\text{m}^3/\text{sec}) \]

---

1 The following definitions of the (local) Nusselt number will be used:

\[ Nu = \alpha H/\lambda \quad \text{theoretically derived Nu-number for parallel plates and constant physical properties (Ch. 2) or experimentally obtained Nu-number (Ch. 3).} \]

\[ Nu_0 \quad \text{theoretical reference Nu-number for a certain Gz-value assuming fluid properties at entrance temperatures.} \]

\[ Nu_{\text{corr}} \quad \text{experimental Nu-number corrected for radial viscosity variations.} \]

\[ Nu_{\text{mod}} \quad \text{modified Nu-number for duct VI.} \]

\[ Nu_{\infty} \quad \text{limiting Nu-number for parallel plates and temperature independent fluid properties.} \]

\[ Nu_{\infty, \text{loc}} \quad \text{local limiting Nu-number on the perimeter of a rectangular duct.} \]

\[ Nu_{\infty, \text{av}} \quad \text{limiting Nu-number of a rectangular duct. It contains a heat transfer coefficient, which has been obtained by averaging the local wall temperatures over the duct perimeter.} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>Ra</td>
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</tr>
<tr>
<td>Ra'</td>
<td>Rayleigh number, eq. (34)</td>
</tr>
<tr>
<td>Ra''</td>
<td>Rayleigh number, eq. (36)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>s</td>
<td>velocity gradient at the wall</td>
</tr>
<tr>
<td>T</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>y</td>
<td>distance to the duct centre, see fig. (2-1) (m)</td>
</tr>
<tr>
<td>Y_n</td>
<td>Eigenfunction</td>
</tr>
<tr>
<td>z</td>
<td>distance to the duct entrance, see fig. (2-1) (m)</td>
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<td>z^+</td>
<td>hydrodynamic entrance length, eq. (7) (m)</td>
</tr>
<tr>
<td>α</td>
<td>heat transfer coefficient (W/m² °C)</td>
</tr>
<tr>
<td>β</td>
<td>thermal expansion coefficient (°C⁻¹)</td>
</tr>
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<td>β_n</td>
<td>Eigenvalue</td>
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<td>ε</td>
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<tr>
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<td>ξ</td>
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<tr>
<td>Φ_w</td>
<td>heat flux intensity (W/m²)</td>
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<td>ψ</td>
<td>dimensionless number, eq. (47) (—)</td>
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**indices**

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<thead>
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<tr>
<td>a</td>
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<td>c.c.</td>
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<tr>
<td>e</td>
<td>conditions at duct outlet</td>
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<td>m</td>
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<td>o</td>
<td>conditions at duct entrance or</td>
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<tr>
<td>r</td>
<td>radial direction</td>
</tr>
<tr>
<td>ref</td>
<td>reference value</td>
</tr>
<tr>
<td>w</td>
<td>conditions at duct wall</td>
</tr>
<tr>
<td>y</td>
<td>local conditions at distance (y) from duct centre</td>
</tr>
<tr>
<td>z</td>
<td>local conditions at distance (z) from duct entrance</td>
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<td>&lt; &gt;</td>
<td>flow average</td>
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1. INTRODUCTION

Heat transfer has been the subject of investigation for over eighty years now. Both analytically and experimentally a tremendous lot of research has been devoted to all sorts of problems connected with this kind of energy transmission. A general survey may be obtained from the classical works by McADAMS (1954), Gröber c.s. (1963), Jakob (1958) and Knudsen c.s. (1958).

The initial aim of the present work was to study the influence of the design of so-called plate heat exchangers on their heat transfer performance.

For various reasons plate heat exchangers are very popular in the food industry and they are also being used in the plastics industry. They are usually built from profiled plates in such a way that several ducts with profiled walls are formed. These ducts have rectangular cross sections of e.g. a few mm by several hundreds of millimeters. Fluids of low to rather high viscosities flowing through these ducts are heated or cooled from both sides.

It is claimed that the profilations of the plates not only have the advantage of increasing the strength and the heat exchanging area, but also of improving the heat transfer. This could be true in the case of laminar flow and then the type of profilation could be important. In fact numerous types are in use.

However, there is very little evidence in literature that the profilation has indeed a favourable influence on the heat transfer and the available data certainly are insufficient to allow a comparison of the various designs. In view of the importance of the plate heat exchangers it seemed desirable to amplify the information and to improve the understanding of their action.

The idea arose to determine heat transfer coefficients at variously profiled plates and to compare the results with those obtained in flat plates. However, a search in literature revealed a lack of information on heat transfer in simple flat ducts at least under conditions normal in plate heat exchangers. This may have been caused first of all by the fact that the problem of heat transfer to laminar flows in flat ducts, which at first sight looks so simple, is in fact theoretically extremely complicated. For immediately behind the entrances of such ducts the conditions are unestablished with respect to flow as well as to temperature distribution.

Further reasons for the afore mentioned lack in knowledge on these problems can be summarised as follows:
1. until the 'plastics age' the larger process industries were seldom interested in heat transfer to these flows;
2. the techniques, by which temperature distributions, cupmixed mean temperatures and heat fluxes have to be measured, are not easy to develop. They became available only after very thin thermocouples and heat flux measuring devices (thermocouple piles) became common practice;
3. in laminar flows the influence of the temperature dependence of the fluid properties on the heat transfer rate and on the pressure drop is enormous. The most common first approximation of constant fluid properties is a poor one.
Free convection and a radial gradient in the viscosity are so important that the large amount of theoretical information on these approximations is of restricted practical use, and 
4. most laminar flows in the food and plastics industry exhibit a non-newtonian behaviour.

There being a lack of data upon which to base the study of heat transfer in profiled ducts, it was decided to start the investigation with the simpler flat ducts.

Thus the first part of the present work deals with heat transfer to laminar Newtonian flows in flat ducts, in particular in the thermal and hydrodynamic entrance regions of such ducts. As the heat transfer coefficients vary strongly over the entrance length of the ducts it will be necessary to study the local coefficients. To measure them, heating by a uniform and known heat flux has been chosen. In that case the unknown physical quantity is the wall temperature, but this can be measured more simply than local cupmixed mean fluid temperatures that must be determined when a uniform wall temperature is used.

Finally for two reasons it was decided to confine the investigations to heat transfer to hydrodynamically developed flows. The idea was to study, in the second part of the investigation, the influence of the disturbance of the velocity distribution by application of profiled walls and for that purpose the flow had to be developed before heating started. Moreover the effects of the temperature dependent fluid properties on the development of the temperature distribution were so interesting, that it was preferred not to obscure these effects by the hydrodynamic development of the flow.

The purpose of the first part of the work can now be defined as:
1. the determination of local heat transfer coefficients over the entire length of the duct for a laminar newtonian flow,
2. to find the influence of a temperature dependent fluid viscosity on the heat transfer rate, and
3. to study the influence of free convection on local heat transfer.

The results of the first part of the investigation were used as a basis for the second part, which consisted of studying the influence of profilation of the duct walls. To that end a number of types of transversally ribbed walls were used. All but one of the ducts had constant cross sections. As an extreme case a regular variation of the cross sectional area in the flow direction was studied with the idea to disturb the velocity distribution strongly.

The report of the work is divided into two main parts, a theoretical (ch. 2) and an experimental (ch. 3). The subdivision of these chapters will be elucidated in the introductions.

The theoretical considerations are strictly confined to those problems, that are directly related to the experiments and to the discussion of the experimental results. They were obtained while using Newtonian fluids only, since the few, mainly theoretical, publications on the effect of the temperature dependence of viscosity and density are all concerned with Newtonian flows. 

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2. THEORETICAL CONSIDERATIONS

For a clear comprehension of the following discussion a brief description of the main part of the experimental apparatus and the interesting variables will be necessary. More information is given in Ch. 3.

In the first part of the investigation three rectangular ducts with flat walls were studied; their cross sections were $100 \times 10$, $100 \times 7.9$ and $100 \times 3.5$ mm resp. The length of the ducts was 1000 mm. Two oils of different viscosities were used (appr. 30 and 100 c S respectively at 20°C). The oil was pumped through the ducts and heated from both long sides. Re-numbers were in the order of 2–70. To be sure that the flow was developed hydrodynamically before heating started, an inlet section was fitted. Heating took place by radiation from electrical heaters; measures were taken to ensure a uniform heat flux over the length of the ducts.

The following factors were varied: type of ducts, viscosity of the oil, rate of flow and heat flux. With each experiment, the fluid temperatures were measured at the inlet and outlet of the duct, while the wall temperatures were determined at various distances from the inlet. From these observations local Nu-numbers were calculated. Some attention was also paid to studying pressure drops.

In the second part of the investigation various other types of ducts were studied. In principle the experiments were carried out in the same way.

An extension to the first part was formed by the third part. Here the above mentioned variables were studied using a duct of which the cross section consisted of two parallel walls bounded by two half circles.

From the above it follows that attention must be paid to:

a. velocity profiles and pressure drops with isothermal laminar flow and the length of the hydrodynamic entrance region (Ch. 2.1.1. and 2.1.2);

b. heat transfer to laminar flows, either fully established or hydrodynamically and/or thermally developing flows (Ch. 2.2.1 and 2.2.2); to simplify the complex problems in these chapters temperature independent fluid properties have been assumed;

c. the influence of temperature dependent physical properties on the heat transfer (Ch. 2.3); and

d. the possible influences of disturbances of the velocity distributions by the use of profiled walls (Ch. 2.4).

2.1 ISOTHERMAL LAMINAR FLOW

2.1.1 Velocity distribution and pressure drop (developed flows)

This chapter deals with the energy losses in flat ducts and with the variations in the local fluid velocities over duct cross sections. The discussion is restricted to isothermal conditions and theoretical data that are directly applicable to the experimental investigation.

In view of the ultimate goal of this work, the determination of local heat...
transfer coefficients and the interpretation of the values obtained, it will be necessary to discuss the velocity gradient at the wall. Moreover it must be examined under which conditions laminar flow exists. The final part of this chapter will then comment rather briefly upon the way in which the pressure drops may be best represented.

Isothermal laminar developed flow in closed conduits such as rectangular ducts, has been studied extensively theoretically as well as experimentally.

Holmes (1967) recently surveyed the present state of affairs concerning velocity distribution and pressure drop as functions of duct configuration. He also provided experimental confirmation of the existing exact and numerical solutions. The values calculated and measured by Holmes are in excellent agreement. The disadvantage of the theoretical solutions, however, is their length, which implies a good deal of computational work when one wishes to calculate the velocity gradient at the wall. To cut this down as much as possible, in the present investigation it was assumed that the influence of the side walls would be negligible when rectangular ducts are considered such as were used in the experiments (aspect ratio H/B < 0.1). This assumption did not seem unrealistic and was justified by the pressure drop measurements to be reported later. The consequence of this supposition is the existence of a parabolic velocity distribution similar to that existing between two parallel flat plates.

Convective heat transfer is mainly determined by the velocity gradient at the wall. It will therefore be necessary to know the velocity gradient at the place where the heat transfer coefficients will be measured; that is at \((x = 0; y = H/2)\) according to fig. (2-1).

![Fig. 2-1. Coordinate system of an arbitrary duct](image)

\[ \text{HEAT SUPPLY} \]

\[ \text{FLUID} \]
For laminar flow of a Newtonian liquid between two parallel plates, the velocity gradient at the wall is:

\[ s = (\frac{dv_z}{dy})_{y=H/2} = -6 \frac{\langle v_z \rangle}{H} \]  

(1)

In Ch. 2.2.2.1 this expression can be used to adapt LÉVÈQUE's solution, eq. (18), to the experimental situation.

If the effect of the side walls is indeed negligible, then the numerical values of the velocity gradient following from the eq. (1) must be identical with those obtained by using the RABINOWITCH equation for narrow gaps:

\[- (\frac{dv_z}{dy})_{y=b/2} = \frac{2Re}{B/H^2} \left\{ 2Qv + \Delta P \frac{dQ}{d(\Delta p)} \right\} \]  

(2)

It could be mentioned here that the eq. (2) may also be used under non-isothermal conditions, to determine the average velocity gradients at the wall from measurements if there is no free convection. To achieve this, the last term in the right member of eq. (2) can be determined to a sufficiently accurate degree by making small variations in \(Q_v\).

In an analysis of the onset of turbulence in rectangular ducts of various aspect ratios, HANKS (1966) predicted the existence of laminar flow up to \(Re = 2800\) for \(H/B = 0.1\). Their analytical results compare well with previous experimental data. However, the nature of the flow is insufficiently described by the Re-number alone, if heat transfer occurs simultaneously. In that case the velocity distribution may change completely, although the flow can still be characterized as laminar parabolic according to the Re-number.

There are several ways to demonstrate this phenomenon. The most illustrative, viz. flow visualization, is not well feasible with the experimental equipment used to perform the heat transfer experiments. And as this kind of investigation is beyond the scope of the actual subject, no special provisions have been made to visualize the flow patterns during the determination of the heat transfer coefficients.

An alternative procedure might be the use of the RABINOWITCH equation, eq. (2), as demonstrated earlier. Here again, difficulties arise. In the first place from the fact that free convection effects at the end of the duct are influencing the total pressure drop, and secondly because the average velocity gradient obtained over a certain length of pipe does not provide information on the local situation, which may vary considerably over that length.

After a thorough survey of the existing literature, it has to be concluded that, at the moment, there is no proper solution of this problem available. The matter will be discussed further in Ch. 2.3.

Now the question arises which of two possible methods is to be used in representing the kinetic energy losses.

The first method is almost classical and commonly accepted. It was originally designed for turbulent flows, and consists of the computation of the BLASIUS'
friction factor (four times the FANNING friction factor) from experimental data. This quantity is defined by equation (3)²:

\[ f = \frac{\Delta p D_e}{2 \rho} \left( \frac{v_z}{P} \right)^2 \text{L} \quad (3) \]

For laminar flows, this friction factor is related to the Re-number as follows:

\[ f \cdot \text{Re} = C_1 \quad (4) \]

The second method is provided by the flow modulus M (see LAHTI (1963), BEER (1965) and HOLMES (1967): being the ratio of the flow rate per unit width of a duct with aspect ratio \( H/B \ll 1 \), to that between parallel plates a distance \( H \) apart:

\[ M = 12 \Omega \mu \frac{L}{APBH^3} \quad (5) \]

Use of the second method meets objections that might be raised against the use of the friction factor. One of the objections could be that the friction factor is defined by the inertia term, which is negligibly small as compared to the viscous forces for laminar flows. Besides, the second method also offers two additional advantages: it is physically readily comprehensible, and it is also a simple function of the aspect ratio. Therefore the author rather prefers the flow modulus.

The observations of the pressure drops allowed a comparison of the experimental results with the theoretical predictions of the flow modulus by LAHTI (1963) and for the product \((f \cdot \text{Re})\) by CORNISH (1928). The solution by CORNISH is given in eq. (6):

\[ f \left( \frac{H}{D_e} \right)^2 \left[ 1 - \frac{192}{\pi^5 (B/H)} \left( \tanh \left( \frac{\pi B}{H} \right) - \frac{1}{3^5} \tanh 3 \left( \frac{\pi B}{H} \right) + \ldots \right) \right] = \frac{6.0}{\text{Re}} \quad (6) \]

The only reason for including the measurements of isothermal pressure drops in the investigation programme was, to supplement the experimental confirmation on M-factors and \((f \cdot \text{Re})\) products, that had been lacking for the configurations used.

In the foregoing chapter some aspects of laminar flows in flat ducts have been introduced. When the flow is developed and isothermal, there will be no changes in the velocity distribution. This situation differs from that existing in the entrance region of flat ducts as discussed in Ch. 2.1.2. The conclusions arrived at in this chapter, may also be applied to profiled ducts because each change in direction of flow may disturb the velocity profile. In that case entrance region conditions are being created in between two subsequent disturbances.

2.1.2 The hydrodynamic entrance region

In the introduction (Ch. 1) it has been mentioned that an investigation will be

2 The equivalent or hydraulic diameter introduced in eq. (3) is often used as the characteristic length in the definitions of the Reynolds and Nusselt number. It is defined as follows: \( D_e = 4 \) (surface area of duct cross section/perimeter of duct cross section).
made into the heat transfer to a fluid stream, which enters hydrodynamically established into a heated duct.

The established situation will exist some distance downstream from the entrance, or may be achieved by introducing a certain length of duct in which the velocity distribution becomes developed before entering the heating section. The second possibility has been preferred and the following discussion considers the theoretical data on hydrodynamic entrance lengths.

As has been done in Ch. 2.1.1, the experimental ducts are again considered as parallel plates. Several authors have studied the development of the velocity distribution in the entrance region, paying special attention to round tubes. A useful reviewing study for this type of ducts has been published by Christianesen (1962).

Until recently, solutions for both circular ducts and gaps have been obtained merely by making boundary layer assumptions to the equations of momentum and continuity. However, the boundary layer theory produces erroneous results in the vicinity of the leading edge, as pressure gradients in the radial direction will not be negligibly small. This has lately been recognized by Wang (1964), who also objected to the model of a uniform velocity profile at the entrance. The opinion that this is not right physically, is shared by Dealy (1965). Wang using a numerical integration of the complete momentum and continuity equations, showed the existence of a concavity immediately at the inlet of the duct. Apparently the radial momentum transport involved, influenced the pressure drops causing higher losses than those predicted by assuming a uniform velocity profile at the entrance.

However the different approaches lead to an almost identical value of the entrance length, which is the only important quantity with respect to the future investigation. Table (1) shows some of the results as values of the constant \( c_2 \) appearing in eq. (7):

\[
z^+ = C_2 H \text{Re}
\]  

(7)

Here \( (z^+) \) means the distance to the duct entrance needed by the fluid stream of a certain \( \text{Re} \)-number, to let the centre line velocity reach 98 or 99% of its maximum value, i.e. to let the flow become hydrodynamically established.

In an unpublished paper, Smith (1968) reported that eq. (7) does not correctly fit the experimental results, and has to be modified into eq. (8):

\[
z^+ = C_2 H \text{Re} + 2 H
\]  

(8)

If eq. (8) is right, this would imply that the entrance length reaches a limiting value equal to \( (2H) \) for \( \text{Re} \)-numbers tending to zero. According to this equation, the developed situation comes into being 50% further downstream than predicted by eq. (7) for \( \text{Re} = 100 \) \( (H = 1.0 \text{ cm}) \), which will be the upper limit of the future experimental programme. The causes of this divergence are not understood at the moment, but it is recommended to use broad safety margins in the application of eq. (7).

Next to calculating the entrance length to design a proper inlet section,
### Table 1. Hydrodynamic entrance lengths expressed as values of the constant \( C_2 \) of eq. (7).

<table>
<thead>
<tr>
<th>Method of Solution</th>
<th>Author</th>
<th>( C_2(z^+] / H \cdot Re )</th>
<th>( v_{x,y} = \partial \sqrt{v_{x, \text{max}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parallel Plates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appr. Sol. of the</td>
<td>SCHLICHTING (1958)</td>
<td>0.04</td>
<td>0.99</td>
</tr>
<tr>
<td>Bound. Layer Eq.</td>
<td>STEPHAN (1959)</td>
<td>0.045</td>
<td>0.99</td>
</tr>
<tr>
<td>Exact Sol. of the</td>
<td>BODOIA (1961)</td>
<td>0.044</td>
<td>0.99</td>
</tr>
<tr>
<td>Bound. Layer Eq.</td>
<td></td>
<td>0.034</td>
<td>0.98</td>
</tr>
<tr>
<td>Num. Sol. of the</td>
<td>WANG (1964)</td>
<td>0.034</td>
<td>0.98</td>
</tr>
<tr>
<td>Complete Eq. of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Round Tubes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appr. Sol. of the</td>
<td>BOUSSINESQ (1891)</td>
<td>0.065</td>
<td>0.99</td>
</tr>
<tr>
<td>Bound. Layer Eq.</td>
<td>LANGHAAR (1942)</td>
<td>0.0575</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>SCHILLER (1922)</td>
<td>0.0575</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>STEPHAN (1959)</td>
<td>0.0575</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>CAMPBELL (1963)</td>
<td>0.0588</td>
<td>0.99</td>
</tr>
<tr>
<td>Exact Sol. of the</td>
<td>HORNBECK (1963)</td>
<td>0.0552</td>
<td>0.99</td>
</tr>
<tr>
<td>Bound. Layer Eq.</td>
<td>SMITH (1968)</td>
<td>0.060</td>
<td>0.99</td>
</tr>
</tbody>
</table>

eq. (7) may also be used for the following purpose. If possible, it is desirable to operate practical heat exchangers under non-developed conditions. The methods used to achieve this, will be further discussed in Ch. 2.4. These involve the use of the profiled walls mentioned before. By means of eq. (7) it is then possible to estimate the maximum distance between two consecutive hindrances which prevent the flow becoming completely developed.

2.2 Heat transfer to laminar flows

The subject will be subdivided into three parts. The presentation of theoretical data starts with the existing solutions for hydrodynamically and thermally established flows (Ch. 2.2.1). In Ch. 2.2.2.1, the latter assumption is discarded to study the heat transfer coefficient as a function of the developing temperature distribution. Ch. 2.2.2.2 considers heat transfer to both hydrodynamically and thermally unestablished flows.

This sequence has been chosen because the problems thus become gradually more complex, although it is recognized that the solutions provided in Ch. 2.2.1 actually constitute one of the two asymptotes belonging to the general problem of heat transfer in the entrance region, as discussed in Ch. 2.2.2.

2.2.1 Fully established flows

This chapter deals with uniform heat flux and fully developed conditions which in practice occur far downstream the heated duct, and includes:

- the limiting Nusselt number (\( \text{Nu}_\infty \)) and its characteristic reference length;
### Table 2. Theoretical values of the limiting Nusselt number.

<table>
<thead>
<tr>
<th>Method of Solution</th>
<th>Uniform Heat Flux</th>
<th>Uniform Wall Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Author</td>
<td>$N_u_{\infty}$</td>
</tr>
<tr>
<td><strong>Parallel Plates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separation of Variables</td>
<td>SELLARS (1956)</td>
<td>4.117</td>
</tr>
<tr>
<td>Direct Numerical Integration</td>
<td>CESS (1959)</td>
<td>4.117</td>
</tr>
<tr>
<td>Numerical Relaxation</td>
<td>NORRIS (1940)</td>
<td>4.120</td>
</tr>
<tr>
<td>Analytical</td>
<td>CLARK (1953)</td>
<td>4.117</td>
</tr>
<tr>
<td>Thin Plate Theory</td>
<td>JANSSEN (1952)</td>
<td>4.117</td>
</tr>
<tr>
<td>Variational</td>
<td>MARCO (1955)</td>
<td>4.105</td>
</tr>
<tr>
<td>SPARROW (1960)</td>
<td>4.117</td>
<td></td>
</tr>
<tr>
<td><strong>Round Tubes</strong></td>
<td>Separation of Variables</td>
<td>SELLARS (1956)</td>
</tr>
</tbody>
</table>

*The influence of the aspect ratio ($H/B$) of the cross section of the duct on the $N_u_{\infty}$ number, averaged along the circumference of the conduit; and the variation of the local heat transfer coefficient along this circumference.*

Starting point is the assumption that a fluid of constant physical properties passes through the duct of fig. (2-1) with a parabolic velocity profile. It has a fully developed temperature distribution, which implies

$$\delta T(x, y, z)/\delta z = \text{constant} \quad (9)$$

The eq. (9) shows that the temperature gradient at the wall is independent of $z$. The same goes for the difference $(T_w - <T_z>)$ between the wall temperature ($T_w$) and the local cup mixed mean fluid temperature ($<T_z>$), if it is also assumed that there is an axially constant uniform heat input.

The cases of uniform wall temperatures and of a uniform heat flux to round tubes are not considered in detail. In table II only a few theoretical results for these situations are given in comparison with solutions for a uniform heat flux to parallel plates. The latter solutions can be used in the interpretation of future experimental results. This is justified since measurements are performed with ducts of aspect ratios ($H/B \ll 0.1$). Table II also mentions the various techniques used in the derivations of the limiting Nusselt number.

The definition of the Nusselt number:

$$Nu = \frac{H(\delta T/\delta y)_{y=H/z}}{T_{w,z} - <T_z>} \quad (10)$$

and the consequences of eq. (9) lead to the deduction that, under developed conditions, this dimensionless heat transfer coefficient becomes a constant. It will be called the limiting Nu number: $N_u_{\infty}$.

In order to calculate the local Nu number, which in the case under discussion is the same as $N_u_{\infty}$, the energy equation has to be solved in order to get the
local temperature gradients at the wall, and the local cup mixed mean temperature difference. To achieve this, the following assumptions are introduced:

a. two dimensional flow;
b. parabolic velocity and temperature distribution;
c. constant physical properties;
d. constant axially uniform heat flux;
e. negligible axial heat conduction;
f. negligible heat dissipation.

The rigidity of assumption c. is readily understood. The consequences of allowing for temperature dependent physical properties to the temperature and velocity distribution, will be discussed in Ch. 2.3. This leaves the assumptions e. and f. to be commented upon.

A good measure to weigh convective against conductive heat transfer, is provided by the Péclet-number \( \langle \nu \rangle H/a = \rho c_p <\nu>H/\lambda \), which is actually the ratio of both. SINGH (1958) has shown that axial heat conduction is negligibly small if \( \text{Pe} > 100 \). A simple calculation with the help of experimental data will demonstrate that this requirement is completely fulfilled during the experiments, and that assumption d) will generally be justified in theoretical solutions.

BRINKMAN (1951) has been the first to analyse the influence of viscous dissipation for round tubes of uniform wall temperatures. Named after him and called the Brinkman number, it describes the ratio of the temperature gradient needed to convey the dissipated heat against the temperature gradient caused by the imposed heat flux. For a uniform heat flux, it becomes: \( Br = (\Delta p \langle \nu \rangle > R/ \Phi_\text{a} \Phi_L) \). BEEK (1962), FAN (1964), and HWANG (1965) have shown that, as long as \( Br < 1 \), the dissipated heat is negligible as compared to the heat supplied. It is easily verified that by means of this criterion, assumption f) is valid in the present study.

The energy equation can now be reduced to:

\[
v_{x,y} (\delta T/\delta z) = a (\delta^2 T/\delta y^2)
\]  

The solution of eq. (11), however, is quite laborious in spite of the many simplifications. Jansen (1952) has arrived at his limiting Nu-number analytically by stating that integration of eq. (11) leads to expressions of the form:

\[
T(z, y) = T_1(y) + T_2(z)
\]  

This follows directly from eq. (9), assuming a uniform heat flux. His result compares favourably with the values obtained by other authors for the Nu-number. See table (2).

MARCO (1955) has solved eq. (11) by using the analogy to existing solutions for the deflection of thin plates under uniform load and supported along all sides. This lead them to expressions in simple Fourier series. SPARROW (1960) tackled the problem by means of a variational method. As might be expected,
the simplified energy equation has also been handled numerically. This was done by Clark (1953), who used a relaxation method.

The other limiting Nu-numbers reported in table (2) are the asymptotic values for flows with developing temperature distributions. The reader is referred to Ch. 2.2.2 to obtain some information on the techniques employed in solving this problem.

The data presented in table 2, together with figures (2-2) and (2-4) (to be discussed below), demonstrate that it is necessary to know the boundary conditions and the characteristic length in the definition of the Nu-number, before a good comparison of heat transfer results can be made. The necessity of this becomes even more obvious when it is realized that Nu-numbers are usually being reported in the literature with variously defined heat transfer coefficients—not to mention the causes for misinterpretation stated earlier.

From now on, the local heat transfer coefficient will be characterised by eq. (13):

\[ \alpha = \Phi_w'/(T_w,z - <T_z>) \]  

The next step will be to choose a proper reference length. The parallel plates model frequently forms a good idealization for heat exchangers with rectangular cross sections, which would imply the use of the distance (H) between the plates. However, many types of plate heat exchangers have aspect ratios that differ too much from the one with a gap for which \( H/B = 0 \). This leads to the question whether the equivalent diameter (\( D_e \)), or the dimension of the short side of the cross section (H), must be used. In view of this, special information is needed concerning the relation between the Nu∞-number and the aspect ratio.

This problem has been studied by Clark (1953), Marco (1955) and Sparrow (1960) for ducts of rectangular cross section and newtonian flow. The different techniques they used (see below) have produced remarkably corresponding results. They are collected in fig. (2-2), where Nu-numbers (\( \alpha D_e/\lambda \)) averaged around the perimeter of the ducts by taking the mean wall tempera-

\[ \text{FIG. 2-2. Limiting Nusselt number for Newtonian flow in rectangular ducts as a function of aspect ratio} \]
ture, are presented as a function of the aspect ratio (drawn line). In the same fig. (dotted curve) values for \((\alpha H/\lambda)\) calculated by the present author are given.

It can be seen that the curve for which the equivalent (or hydraulic) diameter \(D_e\) is being used as a characteristic length, gives less insight in the heat transfer process than when using \((H)\). This can be motivated as follows.

Considering a narrow gap with almost zero aspect ratio, it will be clear that the short side walls will have a disadvantageous effect on the average heat transfer coefficient. For the flow in the corners of the gap and therefore also at these side walls may be put equal to zero by action of viscous forces.

The effect of the side walls on the average heat transfer coefficient is at first practically negligible but increases steadily for larger \(H/B\) ratios. This may be noticed from a decrease of the average Nusselt number, which is later followed by an improvement. The amelioration is observed as soon as normal forced heat convection occurs over part of the side walls.

As can be seen from fig. (2–2) the initial decrease of the Nu-number \((\alpha D_e/\lambda)\) is caused predominantly by the reduction of the equivalent diameter; an increase does not arise.

For the above reasons the Nu-number defined as \((\alpha H/\lambda)\) is prefered.

The variation of the local \(\text{Nu}_{\text{loc}}\)-number around the perimeter of the duct, due to the existing differences in local flow conditions, is given in fig. (2–3) for three aspect ratios. They have been computed from the solution of Han (1955) by the present author. The presence of hot spots in the corners of the ducts is clearly demonstrated, which indicates the unfavourable local flow situation mentioned earlier.

![Fig. 2-3. Variation of local to average limiting Nusselt number around the perimeter of rectangular ducts for Newtonian flow](image)

The existence of hot spots in the corner limits the applicability of figs. (2–2) and (2–3) to situations in which small heat fluxes are imposed on the fluid, as otherwise the assumption of constant fluid properties would be violated too much. This suggests that in practice the heat transfer will be considerably improved by viscosity effects at the long sides and by natural convection in the corners.

Figures (2–2) and (2–3) may be used as follows to calculate the local Nusselt...
number in the center \( x = 0 \) of the two largest heat exchanging walls of a duct with aspect ratio \( H/B = 0.1 \). The average Nu-number is found from fig. (2-2) to be \( \alpha H/\lambda = 3.8 \). Fig. (2-3) shows that for \( x = 0 \) the ratio of the local to average Nu-number becomes 1.25. Combination of these two data leads to the result \( \text{Nu}_{\text{loc}} = 4.75 \).

This value seems to be unreliable since it is unlikely that the heat transfer coefficient at \( x = 0 \) will exceed the minimum value for parallel plates while assuming a parabolic velocity distribution in both cases.

This observation in connection with the above mentioned consequences of the neglected variation of fluid properties, makes it clear that the solutions of Clark, Han and Sparrow have to be used with some reserve in practical applications.

2.2.2 The entrance region

2.2.2.1 Hydrodynamically established flow with developing temperature distribution

The situation of a fully developed temperature distribution exists only downstream from the entrance of the heated section. This is the ‘time-distance’ needed by the heat to penetrate to the centre of the fluid. The quantities that determine this process are grouped together in the well-known Graetz number \( \text{Gz:} = \langle v_z \rangle H^2/az \). In the present investigation its reciprocal value called \( \text{Gz}^+ \) will be used. They both provide in the same way a means to bound the thermal entrance region, since the Nu-number is a function of the \( \text{Gz} \) or \( \text{Gz}^+ \)-number, as long as the temperature profile has not obtained its final shape.

The next pages of this chapter will deal with the solution of the energy equation, if the temperature distribution has to change, by force of an imposed uniform heat flux, from uniform at the entrance to parabolic somewhere downstream. While becoming thermally established, the Nusselt number drops from infinity immediately at the entrance, to its limiting value \( \text{Nu}_{\text{lim}} \). This evolution is generally regarded as completed, as soon as the Nu-number has reached its minimum value within 5%.

After dropping the assumption of a parabolic temperature distribution, the remaining set of conditions as given on page (10) may be used to study the development of the temperature profile in the entrance region. Introducing further the dimensionless variables \( \text{Gz}^+ = az/\langle v_z \rangle H^2 \), \( \xi = 2y/H \), \( \theta = (T_x - T_0)/(T_w - T_0) \), the energy equation in rectangular coordinates now becomes (see a.o. Brown, 1960):

\[
\frac{3}{8} \left( 1 - \xi^2 \right) \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi^2} \tag{14}
\]

The general procedure in the solution of eq. (14) is similar to the one adopted by Graetz (1885) for round tubes and uniform wall temperatures. It consists in
the first place of a separation of the variables in the \((z)\) direction, which leads to the solution:

\[
\theta = \sum_{n=0}^{\infty} A_n \ Y_n(\xi) \exp\left(-\frac{8}{3} \beta_n^2 Gz^+\right)
\]

(15)

The next step will be to find the constants \((A_n)\), eigenfunctions \((Y_n)\) and eigenvalues \((\beta_n)\), that satisfy eqs. (16) and (17), composing a Sturm-Liouville type problem.

\[
Y''_n + \beta_n^2 (1 - \xi^2) Y = 0
\]

(16)

\[
Y(0) = Y'(1) = 0
\]

(17)

Once these equations have been solved, the final solution to the heat transfer problem is found to be:

\[
\text{Nu} = \frac{2}{(17/35) + \sum_{n=0}^{\infty} A_n Y_n(1) \exp\left(-\frac{8}{3} \beta_n^2 Gz^+\right)}
\]

(18)

Most authors omit a graphical presentation or tabulation of local Nu-numbers against \(Gz\)-numbers. In this way the results become less accessible to the reader, whose interest is primarily directed at the practical application or physical meaning. This is the more regrettable as the computation of the Nu-number is quite laborious, even if the \(A_n\), \(Y_n\) and \(\beta_n\) values are known.

The experimental investigation only demands consideration of the physical consequences of the various solutions. Therefore the mathematical difficulties, and the ways in which these have been handled, will not be considered in detail. The merits of the various available data will be briefly discussed below. From the solutions of Sellars (1956) and Cess (1959) local Nu-numbers have been computed in order to illustrate this discussion by means of a graphical presentation by fig. (2-4) of the Graetz Nusselt relation.

Table (2) lists the different techniques employed while table (3) gives the maximal value of \((n)\) in eq. 15 as used by the various authors in their derivation of \(A_n\), \(Y_n\) and \(\beta_n\) values.

Eq. (18) shows that the solution can be divided into two parts. For large values of \((Gz^+)\), there is the fully developed situation which extends into the entrance region (decreasing \(Gz^+)\) by means of the additional term in the denominator. It is obvious that no problems are encountered in the established region: the series (eq. 15) converges rapidly, and sufficiently accurate results are obtained for \(n = 3\). If heat transfer occurs closer to the thermal entrance, i.e. \(Gz^+ < 10^{-2}\), more terms are needed, and it will be necessary to develop approximations for \(A_n\), \(Y_n\) and \(\beta_n\).

If the evaluation is not carried on beyond \(n = 3\), the result will be a lag in increase of the Nu-number and the heat transfer coefficient even seems to
TABLE 3. Solution of eq. (15) for the thermal entrance region for parallel plates.

<table>
<thead>
<tr>
<th>Author</th>
<th>Number of Eigenvalues, -functions and Constants determined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sellars (1956)</td>
<td>n = 3 (asympt. sol. for n &gt; 3)</td>
</tr>
<tr>
<td>Cess (1959)</td>
<td>n = 3 (corrected sol. of Sellars for n &gt; 3)</td>
</tr>
<tr>
<td>Prins (1951)</td>
<td>n = 3</td>
</tr>
<tr>
<td>Siegel (1958)</td>
<td>n = 7</td>
</tr>
<tr>
<td>Hatton (1962)</td>
<td>n = 10</td>
</tr>
<tr>
<td>Brown (1960)</td>
<td>n = 10</td>
</tr>
</tbody>
</table>

become constant. It follows directly from eq. (18), that this is physically impossible.

Prins (1951) has assumed a constant wall temperature and the result of this illustrates the foregoing statement, see fig. (2–4). Schenk (1955) has extended the problem to heat transfer between parallel plates with unequal thermal resistances; here also the evaluation has not been carried on for a sufficiently high value of (n). According to Dennis (1955), more terms can be obtained by means of the Rayleigh estimate, as proposed by Carslaw and Jaeger. It leads to an improvement of the Nu-Gz correlation, but this is not included in fig. (2–4) as it could obscure the figure.

![Fig. 2-4. Complete theoretical solution of the thermal entrance problem for hydrodynamically developed laminar Newtonian flows](image-url)
This solution is to be preferred to the solutions offered by Sparrow (1955) and Stephan (1959), as it does not make any boundary layer assumptions. Sparrow (1955) applied the Karman-Pohlhausen method to the thermal and velocity boundary layers for ducts of rectangular cross sections and uniform wall temperatures. When this method could no longer be applied, the results were smoothly combined with those given by Norris (1940) for parabolic velocity profiles. Siegel (1959) extended Sparrow's investigation to the case of uniform heat flux. Both authors have been shown that the problem can be reduced to the one discussed in the foregoing chapter, if \( \text{Pr} \gg 1 \). Interesting also is their observation that the hydrodynamic entrance region forms only a very small fraction of the thermal entry length for \( \text{Pr} \gg 1 \).

The same conclusions were reached by Stephan, who combined the Pohlhausen solution for heat transfer near the leading edge of a flat plate with the general Graetz-Nusselt solution for large values of \( (Gz)^+ \).

### 2.3 Influence of Temperature Dependent Physical Properties

So far analytical investigations have neglected changes in physical properties induced by the existence of radial temperature gradients. Only recently has it been recognized that this neglect may cause considerable deviations from actual industrial situations.

The properties that may exert influence on the heat transfer rates are:
viscosity (μ); density (ρ); specific heat conductivity (λ); and specific heat capacity (cp).

As will be seen later from table (4) it is justified to restrict the discussion in this chapter to the influence of the viscosity and density variations in the radial direction. Regarding these factors, it may be expected that free convection effects are negligible immediately behind the entrance, but that radial variations in viscosity will influence the heat transfer. Beyond a certain distance from the entrance, which is a function of the heat flux imposed, differences in density begin to contribute to the distortion of the velocity profile. This contribution will initially be small and undistinguishable from the viscosity influence. The buoyancy forces increase very rapidly and they soon prevail strongly over the viscosity effects.

The discussion on the influence of the latter influence is opened by stating the temperature dependence of the viscosity of the fluids used in the experimental part (fig. (2–6)). For information on the meaning of the dotted lines, the reader is referred to Ch. 3.1.

![Figure 2-6. The viscosity's temperature dependence of the fluids used in the experiments](image)

Several relations have been proposed to describe the viscosity variations as a function of temperature. The most simple is a linear behaviour according to eq. (21) in which the coefficient (ε) is negative and the temperatures belonging to the respective values of the viscosity are presented by (T₁) and (T₂).

\[
(μ_{T₁}/μ_{T₂}) = 1 + ε (T₁ - T₂)
\]  

(21)

In this way, the temperature dependence is approximated correctly, but only within a limited range of temperatures. For instance, the oils used in the experiments show this linearity from 20–40°C as can be seen in fig. (2–7). Pigford (1955) has stated that eq. (21) fits well up to a threefold range of viscosity, which seems slightly optimistic.
A better approximation to cover the entire temperature range is given by eq. (22), see fig. (2-6):

\[ \left( \frac{\mu_{T1}}{\mu_{T2}} \right) = \exp \left\{ - \varepsilon (T_1 - T_2) \right\} \]  

(22)

Qualitatively, the following may be expected to occur if the radial viscosity variations are taken into account for the case of an upward flow with heating. Starting from the centre line of the duct, the layers closer to the duct walls have a gradually lower viscosity. Nearer to the duct wall, the decrease becomes steeper in accordance with the temperature profile. As a consequence, the velocity distribution changes as indicated in fig. (2-8). The resulting increase of the velocity gradient at the wall will cause an improved heat transfer coefficient.

However, it does not necessarily have to be a radial temperature gradient that induces these deviations. The same result can be obtained isothermally, if the fluid behaves non-Newtonian. Assuming fully developed conditions, VALSTAR (1963) has computed the limiting Nu-number as a function of the ratio \( \frac{v_{\text{max}}}{v_{\text{med}}} \).
describing the velocity distributions of fluids of the Bingham, Eyring and Ostwald de Waele type. His results, presented in fig. (2-9), clearly demonstrate that the heat transfer increases when the velocity profile flattens, or when the velocity gradient at the wall increases (which is essentially the same).

It might be expected for \( \frac{v_{\text{max}}}{\langle v_z \rangle} = 1 \), that the \( \text{Nu}_a \)-number for rodlike flow (uniform velocity distribution) would be reached; this is indeed what happens.

Fig. (2-9) describes only the developed region. A complete solution, including the entrance region, has been given by CHRISTIANSEN (1962) for pseudo-plastic fluids. He also demonstrated that the upper limit of the \( \text{Nu} \)-numbers is formed by the solution for constant property plug flow. The curves for plug flow (\( \text{Pr} = 0 \)) and parabolic flow (\( \text{Pr} = \infty \)) are given in fig. (2-5), to show the boundaries within which the \( \text{Nu} \)-numbers may vary.

A further restriction to the use of fig. (2-9) is the fact that it does not contain a correlation between the ratio \( \frac{v_{\text{max}}}{\langle v_z \rangle} \) and, for instance, a characteristic ratio \( \frac{\mu_m}{\mu_w} \) of viscosities at mean bulk and wall temperature resp., which might describe the change in the velocity profile resulting from locally varying radial viscosity differences. The ideal situation would be an expression covering both the entrance and the developed region by means of a simple viscosity parameter.

One of the first attempts in that direction has been made by SIEDER (1936), who used experimental data. They proposed eq. (23), which was later widely accepted, not only for practical purposes but also in theoretical analyses (see PIGFORD).

\[
\text{Nu} = \text{Nu}_o \left( \frac{\mu_m}{\mu_w} \right)^{0.14}
\]

The \( \text{Nu} \)-numbers reported by SIEDER represent average values, which are
defined by an arithmetic mean bulk temperature: \( \frac{1}{2} (T_e + T_w) \). For constant wall temperatures, this could lead to incorrect predictions by eq. (23), as the rise of the fluid temperature in the thermal entrance is not linear. Sieder included this region in his test section, and averaged the Nu-numbers. Although the influence of the presence of the thermal entrance region upon the correlation between the averaged Nu-numbers and the average value of the viscosity parameter cannot be predicted precisely, it could be possible that a plot of local Nusselt numbers versus a local bulk to wall viscosity ratio would result in a different exponent in eq. (23). In other words: linearisation of the bulk fluid temperature development, wall temperature development, or viscosity temperature dependence, may lead to deviations between theory and experiment.

In another attempt, Beeck (1965), inspired by Hausenblas' (1950) analysis, arrived at eq. (24) for entrance region conditions:

\[
Nu = Nu_0 \left( \frac{1}{1 - 0.7 \left( \frac{\mu_w}{\mu_o} \right)} \right)^{1/3} \tag{24}
\]

In spite of the fact that the viscosity ratios in eq. (23) and (24) contain differently defined bulk viscosities, they may be regarded identical as the bulk temperature of the fluid in the entrance region differs only slightly from the entrance temperature.

With the exception of the solutions by Yang (1962) and Beeck (1965), all data apply to round tubes of uniform temperature. Yang has used an improved integral procedure for the entrance region, but his final expression is of little practical interest, in view of the serious difficulties that arise when it is adapted to experimental data.

Rosenberg (1965) has chosen a numerical finite difference procedure. His solution concerns the simultaneous development of velocity and temperature distribution. One of the conclusions is worth noting. It states that flow development effects are often of the same order of magnitude as viscosity variations effects. This observation is in accordance with the earlier statement, that dynamically established flows entering s heated tube, may have serious distortions of the velocity profiles very near the entrance. As a consequence, even the Léveque solution will have to be corrected.

Attention will now be paid to the occurrence of buoyancy forces causing natural (or free) convection in the regions further downstream. In principle the mechanism of free convection is fairly well understood. Quantitatively, however, the problem has been poorly analysed as a result of the complexity of the equations and boundary conditions involved. The momentum and energy equations are no longer uncoupled and give rise by combination to a fourth order differential equation that is of a complicated nature. Once again, the
mathematical aspects will not be discussed, and attention is being paid only to those results that have a direct bearing upon the future investigation in which combined free and forced convection occurs. \textsc{Hallman} (1959) has studied this case for round tubes and a uniform heat flux, while \textsc{Hanratty} (1958) coupled flow visualization experiments with a similar theoretical investigation including parallel plates. It could be clearly demonstrated that, subject to the temperature gradients, the flow patterns agreed with those also predicted by \textsc{Tao} (1960), for fully developed conditions.

The ultimate velocity distribution is a function of the dimensionless variable \((Gr/Re)\). This can be deduced from the momentum equation by including the buoyancy term. Expressed in physical terms the variable simply represents the ratio of buoyancy forces to viscous forces.

The Pr-number arises in the energy equation and so the heat transfer by free convection is generally described as a function of the Grashof and Prandtl number. For high Pr-numbers (as are encountered in viscous flows) the product \((Gr \times Pr)\), which is called the Rayleigh number, is used. In doing so it must be regarded as a drawback that the correlation between the heat transfer and the local velocity distribution is lost sight of. In fig. (2-10) the ultimate velocity profiles computed from a solution by \textsc{Tao} (1960) are therefore presented as a function of \((Gr/Re)\), while fig. (2-11) correlates the Nusselt and Rayleigh numbers for this case.

\[ \frac{Gr}{Re} = 32 \times 10^6 \]

\[ \frac{Gr}{Re} = 8 \times 10^6 \]

\[ \frac{Gr}{Re} = 4 \times 10^6 \]

\[ \frac{Gr}{Re} = 8 \times 10^3 \]

\[ \frac{Gr}{Re} = 3 \times 10^3 \]

\[ \frac{Gr}{Re} = 5 \times 10^2 \]

\[ \frac{Gr}{Re} = 32 \]

\textit{Fig. 2-10. Velocity distribution as a function of increasing buoyancy forces for combined free and forced convection between parallel plates}
The Ra-number can be easily derived from the (Grashof-Reynolds) ratio. For that purpose the general definition of the Gr-number has to be transformed first of all according to eq. (25):

$$\text{Gr} = \frac{g \beta H^4 (\delta T/\delta y)}{v^2} = \frac{g \beta H^4 \Phi''_w}{v^2 \lambda}$$

(25)

Introduction of the heat balance, eq. (26), into eq. (25), leads to the new expression for the Grashof number as given by eq. (27).

$$\Phi''_w = \frac{1}{2} \rho c_p <v_x> H (\delta T/\delta z)$$

(26)

$$\text{Gr} = \frac{1}{2} \frac{g \beta H^4 (dT/dz) \cdot H \langle v_x \rangle}{v^2 a}$$

(27)

The Grashof number is now divided by the Reynolds number to produce the desired result, eq. (28)

$$\frac{\text{Gr}}{\text{Re}} = \frac{1}{2} \frac{g \beta H^4 (dT/dz)}{a v} = \frac{1}{2} \text{Ra}$$

(28)

When returning to fig. (2–10) it is seen that, to begin with the 'constant property' parabolic velocity profile (which is almost equal to the curve drawn for Gr/Re = 32), for increasing (Gr/Re) values a change in the ultimate velocity distribution will occur. This change is at first similar to that caused by the influence of the viscosity parameter. As the (Gr/Re) ratio increases further, the plugflow distribution is exceeded and still larger values will cause a reversal of the flow in the core of the duct. This situation will gradually lead to instability and finally result in turbulence.

Fig. 2–11. Minimum Nusselt number for combined free and forced convection in flat ducts of various aspect ratios

Meded. Landbouwhogeschool Wageningen 68-18 (1968)
Fig. (2-11) shows that the buoyancy forces become important for \( Ra > 350 \). The Nu-number for plugflow is reached at \( Ra = 2000 \). The moment, that the flow becomes turbulent, cannot be indicated.

Fig. (2-11) also gives the same curves for the aspect ratios \( H/B = 0.035 \) and \( H/B = 0.1 \). They have been computed from the solution reached by HAN (1955) and not from those obtained by TAO (1960) or by AGRAWAL (1962), as these involved too much computational work. It is remarkable that the Nu-values differ from the limiting value for parallel plates if the Ra-number tends to zero. This is contrary to the expectation, as all other fluid properties with the exception of density, were supposed to be constant. It has been impossible to find an explanation for this phenomenon.

The ultimate velocity distribution, however, takes a certain distance to develop. A developing free convection boundary layer flow will be superimposed on the parabolic velocity distribution at the entrance. At a certain distance downstream, the 'boundary layer thickness' will become of the order of magnitude of the tube dimensions. In the case of uniform heat flux the velocity distribution will not change any more from that point on. It means that the ultimate velocity distribution, as introduced above, has been reached.

The heat transfer problem for the region in which the free convection boundary layer in a forced convection flow is still developing, has never been analysed. The problem is even more complex since the free convection boundary layer is already present in the thermal entrance region. Consequently for this kind of flow the Nu-number will be not only a function of the Gz-number, but also of a Ra-number in which the distance to the entrance occurs as the characteristic length. Because analyses of this problem are lacking, a further discussion of heat transfer in this region is postponed till chapter 3.3, when the experimental data will be presented.

Finally before commenting briefly on non-isothermal pressure drops, the following investigations are worth mentioning. EMERY (1965) and DROPKIN (1965) performed experiments on pure free convection between parallel plates, whereas BODOIA (1962) analysed the development of the velocity distribution for this case. The three references together provide a comprehensive review of the present state of research in this field. It seems that BROWN (1965) has been the only one to carry out an experimental investigation on combined free and forced convection in tubes of uniform temperature. The average Nu-numbers he measured were masked by entrance region effects, but they showed that the flow remained laminar until quite far into the range of Gr-numbers, where pure free convection would have become turbulent. This clearly indicates that the flow in combined free and forced convection is of a special nature about which little is known.

To conclude this chapter, a few remarks will be made on the pressure drops that are to be expected with the assumption of non constant physical properties.

Meded. Landbouwhogeschool Wageningen 68-18 (1968)
In view of the serious distortions occurring in the velocity distributions, it is no longer justified to neglect the radial momentum components. However, the radial momentum components cause analytical problems in the basic equations, which aggravate the solution to such an extent that as yet no theoretical results are available as regards the quantitative interpretation of the phenomena. It is hoped that future investigations will present data for engineering purposes in a form that is readily accessible. A means might be found in numerical techniques. In qualitative respect it may be said that the predicted pressure losses, based on a viscosity decrease, will probably have been underestimated. This has partly been caused by the radial pressure gradients, partly by the flow instabilities arising at high Ra-numbers.

2.4 Improvement of Heat Transfer by Artificially Disturbing the Velocity Distribution

So far the discussion has been restricted to hydrodynamically established flows, or flows entering with a disturbed velocity distribution in ducts with flat walls. It has been demonstrated that the conditions near the entrance of the heated section are favourable to the heat transfer process. Figs. (2-4) and (2-5) show that the heat transfer coefficients beyond the entrance soon become quite low. This is a serious drawback in practical situations where viscous flows cannot be made turbulent if this would demand too high fluid velocities (it will need no further comment that in heat transfer processes turbulent flows are preferred to developed laminar flows). In these cases other ways have to be found to improve the heat transfer coefficients.

The solution to this problem may be found in a disturbance of the flow. Such a disturbance can be achieved, for instance, by narrowing or widening the cross sectional area of the duct in a regular sequence, or by forcing the flow to change its direction continually.

Even if such provisions do not provoke disturbances of such intensity that one could speak of creating turbulence, they might still give rise to an improved heat transfer, as they will influence the velocity distribution to some degree at least. This statement follows directly from the consideration that all laminar flows are in fact developing. In this case, therefore, the situation is as illustrated in the left part of figs. (2-4) and (2-5). However, it cannot be exactly indicated with which part of the entrance region the disturbed flows may actually be compared. Therefore it is in fact not feasible to estimate the improvement that may be attained. At the moment no reference material reporting theoretical or experimental work on this subject is available.

A few additional remarks will be made on the influence of wall profilations on the velocity distribution. A very simple type of profilation has been chosen for this discussion, see fig. (2-12).

One can assume that the degree of disturbance of the velocity distribution depends on the Re-number and on the change in the direction of flow, that is the angle (\(\gamma\)). The lower the Re-number the more difficult it will be to bring about
disturbances, which improve the heat transfer appreciably.

When disturbances have been induced, the flow re-establishes hydrodynamically along the distance \( l_2 \) between two successive angles. Depending on the degree of disturbance and the duct length between the two changes in the direction of flow, the duct contains either an entrance region only or an entrance as well as a developed region. It would be of practical use to calculate the maximum distance between two successive angles that allows only entrance region conditions to be present in that part of the duct. For flows, that enter a duct with a uniform velocity profile, the establishing length to obtain a parabolic velocity distribution can be found by means of eq. (7) on p. (7). The velocity profile of disturbed flows is actually never flat, but it may be approximated by increasing the disturbance.

In general, disturbed flows may be expected to establish over a shorter length than flows with a uniform velocity profile. So if eq. (7) is used, the establishing length will be progressively over-estimated for less disturbed flows. Consequently the average heat transfer will be less favourable, as the part of the duct under consideration will also contain a region where the flow is dynamically fully established and thermally partly developed.

The simple model of a profiled duct has been chosen on purpose since it may be considered representative for plate heat exchangers. These have been introduced sufficiently on p. (1) of the introduction. A good review on plate heat exchangers is given by Böhm (1955), Mckillop (1960) and Leniger (1966). The claim that heat transfer is improved by the profilations has been doubted but only as regards viscous flows of low Re-numbers for which the plate heat exchanger is not often used. There is a tendency at the moment to extend the applicability range of this kind of apparatus to higher viscosities. For these cases it might be interesting to know whether the heat transfer is favourably influenced by the profilations.

The following calculation will give an example of the range of Re-numbers with which more viscous fluids will be pumped through plate heat exchangers.

Meded. Landbouwhogeschool Wageningen 68-18 (1968)
The plate distance generally amounts to approximately 0.5 cm. Assuming a viscosity of 1 Stokes, which is still not extremely high, a fluid velocity of 1 meter/second is needed to reach \( Re = 100 \) according to the definition \( Re = \frac{<v_z>D_e}{\nu} \). This fluid velocity is already rather high for practical situations. Therefore Re-numbers of at most a few hundreds may be expected to occur in plate heat exchangers for more viscous fluids, such as e.g. concentrated sugared milk.

For less viscous materials, e.g. beer or milk, the assumption of strongly disturbed flows may be correct. If the desired turbulence is not reached, it can easily be realised by increasing the fluid velocity. In the latter case this does not imply such a raise of the work pressure that the limit of tolerance, which is rather low for plate heat exchangers, would be exceeded.

An experimental investigation into the flow behaviour in plate heat exchangers has recently been performed by Rybinova (1964). In three kinds of plates she observed a transition from laminar to turbulent for \( 300 < Re < 800 \). This indicates that a substantial improvement of the heat transfer may be expected for the plates investigated.

Maslov (1963) studied the heat exchange performance for \( 1000 < Re < 20000 \). This leads to the conclusion that his interest was not directed towards the value of the critical Re-number. He was able to correlate the results in the classical form:

\[
Nu = C_4 (Re)^m (Pr)^n
\]  

Here the constant \((C_4)\) and the coefficients \((m)\) and \((n)\) have a characteristic value for each kind of plate used. However, the wide variety of the commercial plate designs limit the practical applicability of the provided solutions. Another interesting observation was made by Maslov, who claims that the geometry of the profilations is of minor interest in comparison with the Re-number and the total number of changes in axial direction of flow, i.e. the wavelength \((l_1)\) in fig. (2–12). This has most probably been caused by the fact that all the experiments were performed near or in the turbulent region.

The latter two references were the only that had something in common with the present experimental investigation. But the models used in the studies were far too complicated to provide basic information on the factors that determine the influence of the wall profilations on the flow patterns. So this problem actually remains unsolved. The following methods might be used in another attempt to solve this problem:

1. Flow visualization by observing coloured streaklines in transparent models;
2. Measurement of velocity and residence time distribution over the duct cross section;
3. Measurement of pressure drops; and

The combined problem may best be solved by simultaneous measurements of pressure drops and heat transfer coefficients. The advantage of the determination of the pressure drops is that it does not influence the velocity profiles, while
it also provides data that are qualitatively easily to interpret: Deviations from the values for straight unprofiled ducts under otherwise identical conditions, immediately show that the profilations exert some influence. Next to this the knowledge of pressure drops is also of economic importance.

One of the disadvantages of using pressure drops is the fact that a direct quantitative correlation with heat transfer coefficients is as yet not available for the problem under consideration. Moreover the quantitative interpretation of pressure drops obtained under non-isothermal conditions is very difficult as will be shown later in Ch. 3.3. However, these drawbacks are negligible if the pressure drops are only used as qualitative information.

The influence of wall profilations on the flow behaviour in ducts should actually be investigated over a range of Re-numbers up to approximately Re = 2000. As has been said before, the problem becomes most interesting for fluids of high viscosity. Therefore it seems logical to initiate investigations with the extreme range of 0 < Re < 100. The presentation of the experimental equipment, Ch. 3.1, and of the experiments carried out, Ch. 3.2, will show that this approach has been followed.

2.5 Conclusions

The preceding discussion on the theoretical data available has shown that the flow development and heat transfer in the entrance region of flat ducts has still been studied incompletely. Most investigations were purely theoretical and experimental research was found to be almost negligible.

The hydrodynamical entrance length as predicted theoretically, has not yet been confirmed by experiments. Furthermore no attempts have been made to investigate this problem for rectangular ducts of arbitrary cross sectional dimensions. In this case a solution may probably be obtained only by measurement.

Developed laminar flows are supposed to be stable, but little is known about the effort necessary to disturb the established velocity distribution. Here also more research is desired.

Under isothermal conditions pressure drops can be predicted accurately for a wide variety of duct configurations. However, the influence of temperature gradients within the fluid is quantitatively poorly understood with regard to these energy losses.

A comparison of thermal and hydrodynamical entrance lengths reveals that the first are strongly predominant when Pr >> 1.

The development of the temperature distributions has been analysed in detail with the aid of nearly all the available techniques. But experimental verification will also be necessary. Some fields have barely been touched upon, for example: The local Nu-number variation around the perimeter of rotational asymmetric ducts, and the study of free convection effects in the thermal entrance region of passages with so-called 'dead corners', such as may be present in square ducts.

Generally spoken, the influence of physical properties in the non-isothermal
situation, deserves more attention than it is being paid at the moment. It is now recognized that considerable deviations from the theoretical predictions assuming temperature independent properties may occur under practical conditions. A few initial investigations have been performed in this direction, to allow for variable fluid properties. Numerical techniques may be the only way to solve the complex analytical difficulties, while many problems will have to be encountered experimentally.

Finally, the absence of practical correlations is keenly felt and a re-examination of the present solutions would be desirable for engineering purposes.
3. EXPERIMENTAL PART

Chapter 3.1 discusses first of all the experimental apparatus, materials and methods used in the investigation. Thereupon Ch. 3.2 deals with the experimental programme and the way in which the collected data have been processed. This is followed by a presentation and discussion of the results (Ch. 3.3). That chapter also states the reasons why presentation and discussion are combined. The most interesting conclusions are then presented in Ch. 3.4.

3.1 EQUIPMENT, MATERIALS AND METHODS

This chapter includes the flowsheet (Ch. 3.1.1), the construction of the heat exchangers (Ch. 3.1.2) and the test to check the uniformity of the heat flux (Ch. 3.1.3); furthermore the provisions to measure the temperatures (sect. 3.1.4) and the pressure drops (sect. 3.1.5). Finally the physical properties of the fluids used in the experiments are given in section 3.1.6, while some miscellaneous subjects will be briefly considered in sect. 3.1.7.

3.1.1 The flowsheet

The course of the fluidstream can be followed from fig. (3-1). The fluid (oil I or II) is kept in a thermostated storage tank (a). With the aid of a Moyno pump (b) this fluid of uniform temperature is sucked out of the tank and pumped into the inlet section (c), to let it calm down prior to its entering the heated section (d). When the fluid passes the heat exchanger, the temperatures at the wall are measured by means of stick-on thermocouples attached to the outside of the duct. Meanwhile pressure drops are determined with aid of pressure taps fitted to one of the short sides of the duct cross section. The manometer (m) controls the total pressure in the system.

On leaving the heat exchanger, the fluid enters an outlet section (e), which is identical to the inlet section (c). It serves to ensure that no retro action is to be expected of any disturbances of the flow caused by the mixing chamber (g). Here the temperature distribution within the fluid stream is levelled in order to enable the measurement of the cup mixed mean temperature. The fluid finally passes the flowrator (h) before it returns to the tank (a). The flowrate can be regulated by means of the by pass (i) and valves (k, l). The total efflux is determined by weighing.

In this way the flowrator serves only as a means to establish the comparable volume rates of flow for duplicate runs, or for runs with the heat flux as the only variable.

3.1.2 The heat exchanger

Special attention will now be paid to the heated section and the way in which the heat flux to the walls is being created. The heat exchanger (d) is interchange-
With the exception of duct VIII, all ducts had rectangular cross sections, see fig. 3-2. The respective dimensions of height and width of the various ducts are given in this figure. The length can be read directly from the heated surface that is given in table (4).

The flat ducts carried supporting ribs transverse to the axis at 15 cm intervals, to prevent the walls from bending outwards. The ribs were supposed not to influence the uniformity of the heat flux.

The ducts themselves had been constructed from brass plates. The heat exchanging walls of the first three ducts that had been constructed, (duct IV,  

<table>
<thead>
<tr>
<th>Duct Nr.</th>
<th>Heated Surface Area (m²) (2BL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I, II, III</td>
<td>$1.50 \times 10^{-3}$</td>
</tr>
<tr>
<td>IV</td>
<td>$1.52 \times 10^{-3}$</td>
</tr>
<tr>
<td>V</td>
<td>$2.06 \times 10^{-3}$</td>
</tr>
<tr>
<td>VI</td>
<td>$1.98 \times 10^{-3}$</td>
</tr>
<tr>
<td>VII</td>
<td>$2.03 \times 10^{-3}$</td>
</tr>
<tr>
<td>VIII</td>
<td>$0.66 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
FIG. 3-2. Configurations of the experimental ducts

V and VI) were sandblasted in order to improve the absorption coefficient. For it was supposed that otherwise most of the heat emitted by the heating elements would be reflected. The heat flux transmitted by the wall would then become perhaps too small to investigate the variables as mentioned in Ch. 3.2.

Some trial measurements on these ducts showed however that the sandblasting was unnecessary since the measured intensity of the heat flux indicated that, even with walls of lower absorption coefficient, the transmitted heat would be sufficient to carry out the experimental programme. The heat exchanging walls of ducts I, II, III and VII have therefore not been sandblasted.

Since the ductwalls of the various ducts were all of different quality, the calibration tests on the measurement of the wall temperatures had to be performed for samples of each of the ductwalls. This special test and the apparatus used in it, will be described later in section 3.1.4.

A special case is tube VIII, which was included in the experimental programme after the experiments with other ducts had been completed. Duct VIII was constructed of steel that had been evenly blackened in order to obtain the highest possible heat fluxes. The ovaloid configuration was chosen, as this provided a means to extend the applicability of the results, obtained with the simple rectangular models, to wider range of practical heat exchange situations.

It should finally be mentioned that the surfaces of each single duct were
homogeneous in colour and structure, so that the condition of a uniform heat flux could be realised. However, for each duct the intensity of the heat flux varied because of the afore mentioned reasons, and also because of the fact that the un-sandblasted ducts could not be polished to exactly the same degree.

The ducts were heated equally from two sides by two electrical heaters (n), see fig. (3-1), each consisting of eight spiral elements. The space between the elements and the duct was bounded by reflecting stainless steel plates, which were connected with the reflecting curved wall behind the elements. This is shown in fig. (3-1). Initially the heaters were insulated all around, but a test with the heat exchanger outlined in fig. (3-3), which will be dealt with later on, demonstrated that this made the temperature of the air in the upper parts of the heaters rise to such a degree, that it induced an additional free convective heat flux to the duct. The test also showed that the initial distance to the heat exchanger was too small.

Both shortcomings were corrected by placing the spiral elements at 60 cm from the duct wall instead of the original 25 cm, and by removing the insulation. By leaving open the upper and lower ends of the heaters, it was ensured that the free convection effects of the irradiated sides were negligible. This sufficiently reduced the temperature gradients from the air to the wall.

The equality of both heat fluxes was proved by means of an Erich Marek Watt voltage current meter. It indicated an maximal emittance of 1.080 kW/h and 1.082 kW/h for the respective heaters at 220 V. The heat fluxes could be adjusted to the desired level, with a Variac had been included in the electrical circuit of each heater (see photo p. 46). Of course an equal change in voltage with regard to the original level of 220 V does not influence the equality of the heat fluxes. The functioning of both heaters could be checked by two current meters.

3.1.3 The check on the uniformity of the heat flux

The method to test the uniformity of the heat flux as given in fig. (3-3) is very simple and originates from the following consideration.

Axially uniform heat input means that the rise of the fluid temperature in the duct is a linear function of the distance (z) to the entrance. As a consequence the temperature differences between the entrances and exits of the compartments, formed by subdividing the duct into a number of equal parts, must also be equal. Inspection of fig. (3-3) reveals that such a subdivision has been chosen to confirm the assumption of a uniform heat flux.

Water was sucked from the storage tank, pumped through the successive compartments and carried back into the tank. The flowrates were regulated in such a way, that the flow in the connecting parts between the compartments was turbulent. In doing so, no special provisions were necessary to obtain the cup-mixed mean temperatures. Table (5) shows that the uniformity is well approximated, as the respective temperature differences for the successive compartments are equal for the same flow rate.

However, the small temperature differences corresponding with high flowrates made it inevitable that the test became less accurate than desired. This was
TABLE 5. Verification of assumed uniform heat flux.

<table>
<thead>
<tr>
<th>Thermo-couple</th>
<th>Temp. Diff. with Reference Thermocouple (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v_0 = 29$ (cm/sec)</td>
</tr>
<tr>
<td></td>
<td>$v_0 = 27$ (cm/sec)</td>
</tr>
<tr>
<td></td>
<td>$v_0 = 19$ (cm/sec)</td>
</tr>
<tr>
<td></td>
<td>$v_0 = 15.6$ (cm/sec)</td>
</tr>
<tr>
<td></td>
<td>$v_0 = 5.8$ (cm/sec)</td>
</tr>
<tr>
<td></td>
<td>$\mu V$</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>mean value</td>
<td>13</td>
</tr>
</tbody>
</table>

observed later during the determination of the correction on the measured wall temperatures, when the heat flux was found to be insufficiently uniform as contrasted with the conclusion obtained in the first experiment and set out in the above paragraph. Therefore the uniformity of the heat flux was further improved by the measures described in the following section. But before doing so, it will be demonstrated that axial heat conduction through the wall is negligible.

In the thermal entrance region the gradient of the wall temperature in the $(z)$-direction changes continuously from the theoretical infinite value at the
entrance to the constant value for established conditions. Under established conditions the wall temperature develops parallel to the rise of the cupmixed mean fluid temperature (assuming constant fluid properties). This means that the uniformity of the heat flux can only be influenced in the entrance region; in particular in that part, which is situated very close to the duct entrance. To estimate the importance of the 'unwanted' heat flux through the wall to the entrance, the following rough calculation has been made. It assumes that entrance region conditions are present over the whole duct length, by placing the duct outlet at \( Gz^+ = 7.6 \times 10^{-3} \). Assuming furthermore a heat flux of 5.0 kW/m\(^2\), the variation of the wall temperature downstream from the entrance can be calculated from the theoretical values of the Nu-number (fig. 2-4). The temperature gradient at, for instance \( z = 3 \), may then be obtained graphically. The heat flux through the cross section of the wall at \( z = 3 \), resulting from this gradient, appears to be less than 3.4 Watt as against a heat flux absorbed by the fluid of 353 Watts. This justifies the assumption of negligible axial heat conduction. The conclusion seems the more justified since the deviations from a uniform heat flux are actually determined by \( \frac{\partial T^2_w}{\partial z^2} \). The first derivative is used in the estimation which, in spite of this, still values under 1%-regardless of the fact that the conditions selected (theoretical Nu-number and maximal heat flux) were also unfavourable.

3.1.4 The method of temperature measurement

The subjects to be discussed in this section are presented as follows.

The method to determine the local wall temperatures will first be given in detail. This leads to a discussion of the corrections, that must be made on the obtained data. Therupon the factors that constitute these corrections are treated separately, including a description of the techniques and apparatus used to determine each factor quantitatively. Special attention is being paid to one of the approaches, which could also be used quite well to test the uniformity of the heat flux.

Finally the measurement of the cupmixed mean temperature is considered.

The wall temperatures were measured by means of Philips Stick-on Copper-Constantan Thermocouples (PR 6452 A/00). For the flat rectangular duct the location of the thermocouples has been indicated in fig. (3-4). For all ducts the distances of the thermocouples to the entrance is given by table (6). A similar grouping was used for the other ducts. With this arrangement the interesting parts of the duct got the attention they deserved. The localisation of the thermocouples is at \((x = 0, y = H/2)\) according to fig. (2-1).

The thermocouples and their references (at 0°C) were connected with the Philips Indicator PR 4015, as shown schematically by the wiring diagram in fig. (3-4).

The maximum accuracy attainable in the readings was about 0.2°C. The absolute value, which was reliable up to 0.5°C, is of no interest in the calcu-
TABLE 6. The localisation of the thermocouples

<table>
<thead>
<tr>
<th>Thermocouple</th>
<th>Duct Nr.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>4.0</td>
<td>3.8</td>
<td>4.2</td>
<td>18.5</td>
<td>4.9</td>
<td>15.5</td>
<td>13.2</td>
<td>7.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>14.0</td>
<td>23.0</td>
<td>12.5</td>
<td>19.0</td>
<td>5.5</td>
<td>40.0</td>
<td>13.5</td>
<td>15.2</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>29.0</td>
<td>43.5</td>
<td>20.5</td>
<td>34.5</td>
<td>29.0</td>
<td>40.0</td>
<td>17.0</td>
<td>25.5</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>43.5</td>
<td>63.5</td>
<td>33.5</td>
<td>35.5</td>
<td>39.0</td>
<td>63.5</td>
<td>47.5</td>
<td>35.2</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>58.5</td>
<td>73.5</td>
<td>40.5</td>
<td>53.5</td>
<td>46.5</td>
<td>87.0</td>
<td>49.5</td>
<td>50.2</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>71.5</td>
<td>59.0</td>
<td>54.0</td>
<td>65.0</td>
<td>87.0</td>
<td>52.5</td>
<td>65.3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td>71.0</td>
<td>71.0</td>
<td></td>
<td></td>
<td></td>
<td>82.0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>84.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>86.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 3-4. Wiring diagram of the thermocouples

lation of the local heat transfer coefficient. This is easily understood from an inspection of eq. (13):

\[ \alpha = \Phi^w_w / (T_w, z - \langle T_z \rangle) \]  (13)

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The measured wall temperatures are not the real fluid wall temperatures. Fig. (3-5) shows the actual situation.

The heat has to flow through the layers \((d_3, d_2, \text{ and } d_1)\) before it reaches the fluid. According to Fourier's law:

\[
\Phi''_w = \lambda \Delta T_i / d
\]

this will result in a temperature difference \((\Delta T_i)\) between the measured and the actual fluid wall temperatures. It was therefore necessary to correct for this temperature difference. The problem was handled as follows.

For a given heat flux the correction \((\Delta T_i)\) is determined by the total thermal resistance:

\[
1/k = \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3}
\]

Since the first term could be neglected, only the specific heat conductivity and the thicknesses of the paper and the cement layer were to be determined.

By means of a micrometer it was found that the thickness of the paper varied for all thermocouples between 0.7 and 0.8 millimeters.

Once the thermocouple was attached to the duct wall, the micrometre could no longer be used to determine the thickness of the cement layer.

The thermocouple and the duct wall together were now considered as a condensator, the capacity of which had to be determined. Having evaluated this quantity, it was easy to find the distance between the thermocouple and the duct wall, by comparing it with a condensator of known capacity and dimensions.

In this case the reference consisted of a dummy plate \((5 \times 5 \text{ cm})\), taken from the same material as that used in the construction of the duct walls, and a thermocouple was cemented on it as shown in fig. (3-6). In this way a second reference was also constructed.
1. Dummy plate
2. Paper + Cement layer
3. Thermocouple
4. Resistance 5000 Ω
5. Resistance 120 Ω
6. Measuring apparatus

Fig. 3-6. Wiring diagram to determine the distance from the thermocouple to the wall by means of a capacitive measurement.

With the aid of a micrometre, the distances \((d_2 + d_3)\) of both references were determined after which the capacities of the 'reference condensators' had to be measured. To this end a Peekel Strain Gauge Apparatus Type 540 DNH was adjusted to capacity measurements and placed in the measuring circuit of fig. (3-6), together with one of the 'condensators'. The method proved to be reliable as the micrometer indicated distances of 0.10 and 0.14 millimetres resp. for the references, while the other method produced values of 0.12 and 0.14 millimetres respectively.

Using the same measuring circuit, the capacities of the various thermocouples cemented on the ducts were determined. These could be converted into values of \((d_2 + d_3)\) in millimeters with the aid of the results of the reference thermocouples. The obtained data have been collected in table (7), which also contains the number of thermocouples on each duct as given in table (6). The distance to the duct wall appears to be almost equal for all thermocouples. The values of \((d_2 + d_3)\) being known, only the respective heat conductivities had to be determined to obtain the corrections on the measurements of the wall temperatures (see eq. 30). However the first trial to obtain the value of \((d_2 + d_3)\) experimentally, did not produce the required result. Therefore an approach was later developed, which was completely different from the one to be discussed now.

**TABLE 7. Measured distances from thermocouple to wall.**

<table>
<thead>
<tr>
<th>Duct Nr</th>
<th>(d_2 + d_3) (mm)</th>
<th>(d_2 + d_3), av. (mm)</th>
<th>number of thermocouples</th>
</tr>
</thead>
<tbody>
<tr>
<td>I, II, III</td>
<td>0.08-0.105</td>
<td>0.09</td>
<td>17</td>
</tr>
<tr>
<td>IV</td>
<td>0.11-0.16</td>
<td>0.12</td>
<td>7</td>
</tr>
<tr>
<td>V</td>
<td>0.10-0.15</td>
<td>0.13</td>
<td>6</td>
</tr>
<tr>
<td>VI</td>
<td>0.10-0.16</td>
<td>0.13</td>
<td>7</td>
</tr>
<tr>
<td>VII</td>
<td>0.09-0.14</td>
<td>0.11</td>
<td>9</td>
</tr>
</tbody>
</table>

*Meded. Landbouwhogeschool Wageningen 68-18 (1968)*
Three series of three thermocouples were attached to the heated wall of duct I. The series were located at \( z = 10, 50 \) and \( 70 \) cms, while the thicknesses of the cement layers varied within each series of three thermocouples. The thermocouples closest to the wall were regarded as references and the occurring temperature differences were read from the micro-voltmeter \((p)\), see fig. (3-4). It appeared that the results of the three series did not compare with each other. This could not have been caused by the existence of different flow conditions across the width of the duct, since the thermocouples had been placed at 0.5 cm intervals with the references in the centre at \((x = 0, y = H/2)\). A possible cause could have been that the assumption of uniform heat input had not been satisfied sufficiently in spite of the earlier experiments. A new experimental approach to this matter was therefore initiated.

The object was to measure directly the corresponding differences between the measured and the actual wall temperatures for various heat fluxes. Next to this, the entire irradiated surface area had to be investigated on its uniformity. The two goals were achieved with the aid of the apparatus in fig. (3-7).

![Fig. 3-7a. Front view of measuring section according to "P"
Fig. 3-7b. Wiring diagram belonging to the measuring section](image)

One side of the dummy plate was air-cooled, and a constant temperature was maintained by regulating the air flow. Various heat fluxes were imposed on the other side. Once again the micro-voltmeter was used to measure the resulting temperature differences of the thermocouples on both sides, that is \( C_1 - D_1, C_2 - D_3 \) and \( C_3 - D_5 \) resp., see fig. (3-7b).

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To find out if the stick-on thermocouples influenced the local heat flux, the couples $D_2$ and $D_5$ were soldered to the wall in between the other couples $D$.

Fig. (3–7b) shows that these couples $D_2$ and $D_4$ were not shielded by the Philips stick-on couples as distinct from the thermocouples $D_1, D_3$ and $D_5$. The arrangement as shown in this figure made it possible to measure the difference between the thermocouples at the heated and cooled side of the wall by various combinations of the cemented thermocouples and both sorts of soldered thermocouples (shielded or unshielded). No differences could be observed between the thermocouples $(D_1, D_3, D_5)$ and $(D_2, D_4)$. This confirms the assumption that it is allowed to measure the wall temperatures with the aid of stick-on thermocouples that are irradiated directly.

The uniformity of the heat flux over the duct length could now be tested by moving the test section up or down the bars in fig. (3–7). The uniformity (see section 3.1.3) appeared to be inaccurate for the upper and lower parts. The temperature differences between the two sides of the dummy plate were smaller and larger respectively over the first fifteen and the last ten centimeters of the irradiated area. The corresponding parts of the ducts were therefore insulated. The resulting heated surface areas of the flat, profiled and ovaloid ducts have been tabulated in table (4).

As remarked on page (33) the tests were performed with dummies taken from the various plates used in the construction of the ducts. As an example the profile of duct V is shown in fig. (3–7).

An illustration of one of the calibration results is given in fig. (3–8), which reads as follows: Suppose experiments to be performed on duct I. Under the experimental conditions, i.e. for the prevailing flow rate and the chosen voltage over the heating spirals, a heat flux of $\Phi_w = 2.0$ kW/m$^2$ is found by measuring the fluid temperature and the total fluid efflux. The wall temperatures obtained with thermocouples 1–6, see fig. (3–4), must then be corrected by substraction of 4.2°C according to fig. (3–7c).

The heat fluxes presented in fig. (3–7c) were borrowed from the experiments with duct I. This could be done as they were obtained for the same positions of the Variacs that were used in the determination of the temperature corrections. The same procedure was also adopted for the other cases.
Another problem to be solved was the determination of the cupmixed mean temperature of the fluid.

The fluid had been mixed thoroughly, both in the storage tank and in the pump. So the cupmixed mean fluid temperature was easily determined at the entrance by means of inserting thermocouples in the fluid stream. The thermocouples $A$, $B_1$, $B_2$ and $B_3$, see fig. (3-1), were insulated electrically by using heat shrinkable neoprene tubing, type Rayclad Tubes Inc.

Another situation existed at the outlet of the ducts. When leaving the heat exchangers, the fluid had a certain temperature profile that was almost parabolic. Several types of mixing chambers were tried to smooth out the existing temperature variations within the fluid. Without going into details it has to be remarked here that this problem was not easily solved. The result of the trials was a design that applied the simultaneous action of: conduction (by prolonged residence time), axial, tangential and radial mixing. These effects are illustrated in fig. (3–8).

![Fig. 3-8. Principle of the mixing chamber to obtain the cupmixed mean temperature of the fluid leaving the heat exchanger](image)

Three thermocouples ($B_1$, $B_2$ and $B_3$), see fig. (3–1), inserted to various depths in the stream leaving the mixing chamber (g), showed mutual differences up to about $4\mu V \sim 0.1 \degree C$. This is also the maximal attainable accuracy of the microvoltmeter (p) used in measuring the temperature differences. Therefore the mixing quality of the design was considered to be satisfactory.

3.1.5 The measurement of pressure drops

Small pressure drops of up to about 100 (N/m$^2$) were determined by means of the adjustable manometer, see the measuring circuit of fig. (3–9). For the range of 100–1000 (N/m$^2$), a membrane differential manometer (type De Wit Nr. 4–44) was used, which had been calibrated previously.
A Piezo-ring was mounted on the outlet section, in view of the fact that the pressure drops over ducts VI and VII could not be measured by means of the lateral pressure taps. This is easily recognised by an inspection of the profilation and the axial cross section of the duct in question. After completing the experiments on the flat rectangular ducts, the Piezo-ring was used for all ducts with profiled walls, which reduced the number of readings.

The measurements have been carried out both isothermally and non-isothermally. The results were compared with pressure drops that could theoretically occur in an isothermal flow between parallel plates of the same traversed length. These reference pressure drops ($\Delta p_0$) were considered to be the sum of two components ($\Delta p_1$) and ($\Delta p_2$), as is shown schematically in fig. (3-10). The former of them was calculated for entrance fluid temperatures by using eq. (5), while the latter was obtained assuming the same fluid temperatures as the fluid actually had at the outlet of the tube. In this way some sort of 'isothermal' situation was created, that made it possible to investigate the influence of the heating, and this resulted in a temperature development of the fluid as given by line (A) in fig. (3-10).

For various reasons it was not feasible to completely re-design the experimental equipment by which this cumbersome method of calculation, summariz-
ed by eq. (32), would have become unnecessary
\[ \Delta p_o = \Delta p_1 \text{ (at } T_o) + \Delta p_2 \text{ (at } T_e) \]  

Earlier experimental work, performed on ducts that differed from the present aspect ratios, had proved the reliability of eq. (6) for flat rectangular ducts. Since orientating measurements on duct I were in excellent agreement with the values predicted by eq. (6), it was decided that the equipment to determine the pressure drops functioned correctly.

The lateral pressure taps were fitted to the ducts with the intention to obtain the pressure drop as a function of the distance to the entrance for varying heat fluxes and entrance velocities. In this way, information might be obtained on the influence of the cup mixed mean temperature in the axial direction, as well as on the changes that could occur in the velocity distribution, which would indicate the existence of a radial pressure gradient. The already very complex matter for developed conditions is aggravated even more by the presence of the thermal entrance region. A complete quantitative analysis of the experimental results is actually prevented by the combined influence of the viscosity variations (axially and radially), the free convection and the entrance-region effects. However, the data obtained illustrate quite well the qualitative changes in the pressure drop and therefore in the flow patterns in the axial direction. The primary objective of these experiments, i.e. the calculation of velocity gradients, could not be realized. As said before in Ch. 2.1.1, this could be achieved in principle by using the Rabinowitch equation. But this would make the experimental programme too extensive, as it would involve very small variations of the flow rate in order to obtain correct approximations for the term \((\partial Q_v/\partial (\Delta p))\). These measurements have therefore not been carried out. During the discussion of the results it will be seen that part of the experimental data, that does not show too pronounced free convection influences, can be worked out quantitatively.
3.1.6 *The physical properties of the fluids to be heated*

The data of table (8) shown below were provided by Shell Research Laboratories. It can be seen that the components of the thermal diffusivity \( (a = \frac{\lambda}{\rho C_p} \text{ m}^2 \text{ sec}^{-1}) \) are only moderately temperature dependent. The thermal diffusivity and its temperature dependence are almost the same for both oils. This reduces the number of variables that could influence the experimental data.

| Table 8. Physical properties of the fluids used in the experiments. |
|------------------|---------------|------------------|---------------|---------------|---------------|---------------|
|                  | 20°C          | 35°C            | 50°C          |
|                  | Oil I | Oil II | Oil I | Oil II | Oil I | Oil II |
| \( C_p \) (kcal/kg°C) | 0.446     | 0.448     | 0.462     | 0.465     | 0.475     | 0.478     |
| \( \lambda \) (kcal/mh°C) | 0.112     | 0.114     | 0.111     | 0.113     | 0.110     | 0.112     |
| \( \rho \) (kg/m\(^3\)) \* | 882 | 872     | 609 | 225     | 384 | 137     |
| \( Pr \)          | 1360 | 405 | 609 | 225 | 384 | 137     |

\* Values determined at 15°C

A value of \( \beta = 7.10^{-4} \text{ (°C}^{-1}) \) for the thermal expansion coefficient will be used in the processing of the experimental results. This value has been taken from E. Schmidt (1963) and it seems to be a good approximation for oils with viscosities ranging from 0.1-100 Stokes.

The temperature dependence of the viscosity of both oils has already been discussed in Ch. 2.3 from fig. (2-6) and (2-7) on pages (23) and (25).

3.1.7 *Miscellaneous*

One of the experimental conditions assumed that the flow was hydrodynamically established when entering the heat exchanger. In order to satisfy this condition the inlet section (c) had been dimensioned according to eq. (7). It measured 25 cm in length, so a laminar flow could be achieved up to \( Re = 400 \).

A possible unsmooth transition from inlet to test section may be safely supposed not to have any influence on the heat transfer results in view of the Pr-numbers of the oils. This statement follows directly from the discussion in sect. 2.2.2.2.

The normal procedure in changing from the one oil to the other was to drain the whole system, after which the tank containing the second oil was installed. Owing to the fact that it was impossible to remove all remnants of the one oil before introducing the other, the viscosity of the oils underwent a gradual change in the course of the experiments. Before each new run, therefore, the viscosity was determined by means of a Haake Rotovisco. This viscosity meter provided data that might be regarded as sufficiently accurate for the present purpose.

Fig. (2-6) shows the changes in the viscosity of the two oils from the start of the experiments (represented by the solid lines) up till the end. A pessimistic estimation yields a fault of approximately 5% in the values of the viscosity used.
PLATE 1. General survey of the experimental equipment
PLATE 2. The measuring section of the experimental equipment.

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in the calculation of the Re-, Gr- and Ra-numbers in the processing of the results.

During the experiments with ducts V and VI, it was considered necessary to have some information about the flow phenomena inside the ducts. To this end a simple design was used. It is presented in fig. (3–11) and consists of a perspex copy of the real duct, mounted on a flat rectangular section also constructed of perspex. The latter is preceded by the inlet section that is part of the equipment for the heat transfer experiments.

A glycerine-water mixture (2:1) of viscosity \( \mu_{20^\circ C} = 14 \text{ cP} \) was sucked from a small container, pumped through the transparent model, and fed back into the tank. At the entrance of the flat rectangular section, five small tubes (injection needles of \( \varnothing = 1.0 \text{ mm} \)) were placed in the fluid stream as shown in fig. (3–11). The tubes were connected with a vessel containing \( \text{KMnO}_4 \) and when the fluid passed, it dragged some dye from the tubes, thus creating streaklines. The flow patterns obtained were photographed for various Re-numbers.

3.2 EXPERIMENTS AND CALCULATIONS

The investigations were carried out to obtain experimental information and confirmation on:

Fig. 3–11. Lay-out of the flow visualization experiments
a. the existing solutions for the heat transfer in the thermal entrance and
developed regions of flat ducts;
b. the influence of aspect ratio and duct geometry;
c. the deviations from theory due to the temperature dependence of both vis-
cosity and density;
d. the possibilities to improve the heat transfer at small Re-numbers;
e. the pressure drop for isothermal and non isothermal conditions.

Regarding c) it should be noted that this aim could be realised by using oils I
and II as their viscosities compose a range of 15–100c.S. (see fig. 2–7). The
variation of the viscosity enables also the study of the free convection effects
since it enlarges the range of Ra-numbers; this is necessary because the thermal
expansion coefficient of the oils is rather small. The fact that the temperature
dependence of the viscosity of both oils is the same implies that the data ob­
tained for oil I may be regarded to be duplicates of those obtained for oil II as
long as natural convection effects are negligible for both.

To achieve the contemplated objectives, the experimental programme has
been divided into three parts, which are shown schematically below.

<table>
<thead>
<tr>
<th>Scheme of experiments</th>
<th>Duct Nr.*</th>
<th>Heat flux (kW/m²)</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part I</td>
<td>I</td>
<td>0.49</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.53</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.60</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>0.86</td>
<td>3.88</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>0.74</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>0.75</td>
<td>2.83</td>
</tr>
<tr>
<td></td>
<td>VII</td>
<td>0.45</td>
<td>1.78</td>
</tr>
<tr>
<td>Part II</td>
<td>VIII</td>
<td>from 0.82 to 4.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part III</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* see fig. (3–2)

After part I, the study of the heat transfer in flat rectangular ducts, had been
completed, the ensuing programmes could gradually be simplified.

In part II the number of the variations in the heat flux was reduced. Only two
intensities of heat flux were imposed on the ducts, in order to obtain special
confirmation on the reliability of previous results. In part II both oils were used
again, although the influence of the physical properties had been studied suffi­
ciently in part I. It was hoped that one or more of the profiled duct walls used in
part II, might disturb the flow pattern in the duct if the less viscous oil II was
used.

Part III was initially included in the experimental programme to connect the
previous findings with a duct geometry, that is also closely related to a design

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of heat exchangers occurring in practice. At first sight, it seemed that this could be realized by experiments on duct VIII using only one oil and one level of heat flux. However, when the measurements were taken, the chaotic results necessitated a variation of the heat flux by small degrees. In this way supplementary data were collected on the appearance of free convection, which was obviously the cause of the unexpected and irregular behaviour of the Nu-number.

The experiments were carried out as follows. After starting the pump and selecting a certain flow rate, the heating elements were turned on. Depending on the intensity of the heat flux chosen by means of the Variacs, it took 1–2 hours before the process became stationary. This could be observed by means of the Micro-voltmeter, see fig. (3–5). Temperature readings were taken in triplicate as soon as the temperature difference indicated by the Micro-voltmeter had been constant during fifteen minutes.

A series of measurements in which the heat flux was kept constant with varying flow rates will be called an experiment. Each experiment consisted of 8 to 12 runs corresponding with the number of steps by which the flow rate was varied from approximately 0.04 to 0.5 m²/hr. Since the thermocouples were situated at varying distances from the entrance, and the velocities varied, the heat transfer for a certain heat flux could be studied over the range $2 \times 10^{-4} < Gz^+ < 10^{-1}$. Each change of flow rate was followed by a 15 to 30 minute interval, to let the situation become stationary again before new measurements were taken.

After completion of one experiment the intensity of the heat flux was changed and the same runs were carried out again. For each duct all the experiments as summarized on page (49) were thus performed before the second oil was introduced for an identical investigation.

The above mentioned upper and lower $Gz^+$-values could not always be reached. For very large volume rates of flow the difference between the inlet and the outlet temperatures of the fluid will be so small, that the errors involved in the measurement of the temperatures become inaceptable. This is easily recognized in view of the aforementioned accuracy of the Philips Indicator (0.3 °C) and the Micro-voltmeter (0.1 °C). Therefore the experiments were performed for increasing flow rates as long as $(T_o - T_e)$ exceeded 0.8 °C. It will be clear that this limit is reached sooner if small heat fluxes are applied, while still $Gz^+ >> 2.10^{-4}$.

On the other hand, very small volume rates accompanied by large heat fluxes caused wall temperatures that were too high to neglect free convection in the air at the irradiated sides. The observed effect was a decrease in the measured heat flux since part of the heat would be transferred to the air. At this point further experiments were scratched from the programme, as it was no longer reasonable to assume uniformity of heat flux.

To check the reproducibility of the results some experiments of part I were performed twice. It was found that, under identical conditions, the values obtained for each thermocouple deviated by 3% from the local Nu-number.
averaged over three series, if the same heat flux was applied and the $Gz$-number contained the same mean velocity. This reproducibility was considered satisfactory.

The Nusselt numbers were calculated from the obtained data with the aid of the following equations, which do not need any further explanation.

$$\Phi^*_w = \rho c_p Q_v (T_e - T_0)/2 B L$$
$$T_w, z = T_{(measured)} - \Delta T_r$$
$$<T_z> = T_0 + (T_e - T_w) z/L$$
$$Nu = \alpha H/\lambda = \{\Phi^*_w/(T_w, z - <T_z>)\}(H/\lambda)$$

The calculation of the $Gz^+$ number follows directly from its definition:

$$Gz^+ = az/<v_z> H^2$$

To illustrate the experimental data a form used in recording them is represented below. It contains the results of one run.

<table>
<thead>
<tr>
<th>Thermocouple*</th>
<th>T measured</th>
<th>$\Delta T_r$</th>
<th>$T_w$</th>
<th>$&lt;T_z&gt;$</th>
<th>$T_w - &lt;T_z&gt;$</th>
<th>$\alpha$</th>
<th>Nu</th>
<th>$Gz^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 8/8/67</td>
<td>1</td>
<td>24.2</td>
<td>1.1</td>
<td>23.1</td>
<td>20.3</td>
<td>2.8</td>
<td>153</td>
<td>13.4</td>
</tr>
<tr>
<td>Duct: I</td>
<td>2</td>
<td>25.6</td>
<td>1.1</td>
<td>24.5</td>
<td>20.5</td>
<td>4.0</td>
<td>108</td>
<td>9.5</td>
</tr>
<tr>
<td>Fluid: I</td>
<td>3</td>
<td>26.1</td>
<td>1.1</td>
<td>25.0</td>
<td>20.6</td>
<td>4.4</td>
<td>98</td>
<td>8.6</td>
</tr>
<tr>
<td>Flowrate: 45</td>
<td>4</td>
<td>26.8</td>
<td>1.1</td>
<td>25.7</td>
<td>20.8</td>
<td>4.9</td>
<td>88</td>
<td>7.8</td>
</tr>
<tr>
<td>Flow rate: 4.82 $\times 10^{-2}$ kg/sec</td>
<td>5</td>
<td>27.1</td>
<td>1.1</td>
<td>26.0</td>
<td>20.9</td>
<td>5.1</td>
<td>85</td>
<td>7.5</td>
</tr>
<tr>
<td>$&lt;v_z&gt;$</td>
<td>6</td>
<td>27.3</td>
<td>1.1</td>
<td>26.2</td>
<td>21.1</td>
<td>5.1</td>
<td>85</td>
<td>7.5</td>
</tr>
<tr>
<td>$\Phi^*_w$</td>
<td>0.50 kW/m²</td>
<td>20.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(T_0)$</td>
<td>26.0</td>
<td>B-A: $\mu V = 34$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temp. diff.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$^\circ C = 0.84$</td>
</tr>
<tr>
<td>* see fig. (3-1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.3 RESULTS AND DISCUSSION

Before entering into a more detailed interpretation of the experimental data, the subjects to be treated are summed up for guidance.

The discussion opens with a qualitative description of the effects observed in the rectangular ducts. Then the consequences of letting the heat flux approach zero are considered to establish a connection with the theoretical predictions. It will no doubt be remembered from the foregoing pages, that this means a discussion of the influence of the viscosity in the thermal entrance region, where free convection is negligable. This is followed by a closer inspection of the free convection forces in regions further downstream near or in the developed region.

The results of the profiled ducts will then be compared with the previous data. In the course of this part the side issue of flow visualization experiments will
also be discussed. Finally an attempt will be made to work out the pressure drops in such a way that they may serve as a special confirmation of the interpretation given to the heat transfer results.

To indicate the huge amount of data obtained in the experiments, it should be mentioned that 1500 local Nu-numbers were obtained in the first part, and 1300 and 600 in the second and third part respectively.

A direct presentation of these Nu-numbers as a function of Gz+-numbers would result in chaotic graphs, as the results would be simultaneously influenced by several factors. Such graphs do not give any useful information on the transport phenomena involved in the heat transfer process. This is instructively illustrated by fig. (3–21), page (61), which presents part of the original Nu-numbers as a function of the Gz+-number without processing them at first. Therefore the results are presented in an exceptional way as merely a selection of data will be given to illustrate the various variables. It is emphasized here that the discussion of the figures and the conclusions is based on the total amount of data.

The local heat transfer coefficients calculated from the measurements on the flat rectangular ducts (Part I) were first grouped for each duct according to Gz+ value and heat flux. Then the average Nu-number was determined for all data belonging to appr. the same Gz+-value and heat flux. This means that the local heat transfer coefficients were supposed to be independent of the way in which the Gz+-number had obtained its value (e.g. by variation of \( <v_z> \) and \( z \) or both of them). The variation of the Nu-numbers was temporarily ascribed to inaccuracies of the measurements and not to any physical influence. In other words constant fluid properties (isothermal conditions) were assumed.

The curves resulting from the lowest and highest intensities of the applied heat fluxes are shown in fig. (3-12), (3-13) and (3-14) for duct I, II and III respectively.

The other two heat fluxes produced similar curves, which were in between those represented. They are omitted in order to keep the figure clear.

As may be seen in fig. (3-12), the results obtained for duct I showed a marked influence of the heat flux; this effect is less striking for duct II and still more unpronounced for duct III, see fig. (3-13), (3-14). If the heat flux appears to be a parameter, this would mean that the temperature gradients affect the velocity distributions. In other words, the temperature dependence of the physical properties cannot be neglected. Therefore the proposed representation of the results is incorrect. It was observed that the variation of the local Nu-number within one Graetz number by varying \( z \) or \( (<v_z>) \), was less significant than the variation caused by changing the heat flux over big intervals.

All data will later be worked out in a different way, in order to evaluate the effects of variable fluid properties by the existence of temperature gradients at the wall.

Before discussing this matter in detail, an inspection is made of the validity of the asymptotic solutions for the region close to the entrance and for thermally developed flows.
Fig. 3-12. Local Nu-numbers as a function of Gz⁺-values for two different heat fluxes (duct I)

Fig. 3-13. Local Nu-numbers as a function of Gz⁺-values for two different heat fluxes (duct II)

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Lévéque's solution appears to be an asymptote indeed. The theoretical limiting Nu-number (\( \text{Nu} = 4.12 \) for parallel plates) is only reached for extreme conditions, which in practice seldom occur. So it may be concluded from figs. (3-12), (3-13) and (3-14) that for \( H = 3.5 \times 10^{-3} \) m the heat flux has to be decreased to less than \( \Phi^* \) \( w = 0.4 \) (kW/m\(^2\)) or, alternatively, the plate distance must be diminished, in order to approximate 'isothermal' conditions.

The deviations from theoretical results are even more striking when larger heat fluxes or wider ducts are concerned. The Lévéque solution is approached for smaller \( \text{Gr}^* \)-values, while the limiting Nu-number becomes a phantasm. Both phenomena are shown most clearly in figs. (3-12) and (3-13). It should be observed here that the accuracy of the experimental equipment did not allow a further reduction of the minimal imposed heat flux. However, the mere presence of a temperature gradient in radial direction makes it unlikely that the limiting Nu-number will ever be reached in practice, when fluids with temperature dependent viscosity are used.

In the case of constant properties, the deviations from theoretical results may be ascribed to the following causes. Immediately behind the entrance, the temperature gradients will be such that they can induce only radial viscosity variations; they cannot provoke free convection. When the fluid streams down the tube, the buoyancy forces will gradually increase, then equal the viscosity forces in magnitude, and eventually dominating them. This process is a function...
of the mean fluid velocity and heat flux. To study these phenomena the results obtained with duct I were chosen for reasons of accuracy. The viscosity influence will be considered first.

Assuming that the viscosity has a linear temperature dependence, its variation over the duct cross section may be described by the mean-bulk to wall viscosity ratio \( \frac{\mu_m}{\mu_w} \). The assumption of linearity is justified by fig. (3-15), in the composition of which all the original experimental data have been used. For \( \frac{T_w}{T_m} > 2.0 \) the deviations clearly become unacceptable.

![Fig. 3-15. Dimensionless viscosity as a function of the reduced temperature](image)

The experimental results were regrouped as follows, see fig. (3-16). Taking the Graetz-number as parameter, all the measured local Nu-numbers were plotted against the local viscosity ratio \( \frac{\mu_m}{\mu_w} \). The points represent mean values for the same viscosity gradient at different \( \varepsilon \). The scatter of the mean values around the lines in this figure is up to 5%.

From fig. (3-16) the constant property Nu-number can be read by taking it equivalent to the limiting value for \( \frac{\mu_m}{\mu_w} = 1 \). The obtained ‘constant property’ Nu-numbers ought to compare well with the theoretical solutions, if the applied approach is right. Fig. (3-17) shows this to be the case indeed.

The straight line, that was found to fit best the experimental data represented in fig. (3-16), has been compared with the theoretical corrections of Sieder and Tate, eq. (23), and of Hausenblas and Beek, eq. (24). After inserting a few values of the viscosity ratio \( \frac{\mu_m}{\mu_w} \) into these equations, the results were collected together with the experimental correlation in fig. (3-18).

It had been expected in Ch. 2.3 that the Sieder and Tate correction factor would lead to an underestimation of the Nu-number for variable viscosity. Fig. (3-18) shows that the experimental correlation led to the same correction as proposed by Beek and Hausenblas. Both appear to result in values, which
Fig. 3-16. Local Nu-numbers as a function of the bulk to wall viscosity ratio

\[ \text{Nu} = \frac{\alpha H}{\lambda} \]

\[ Gz^+ = \frac{az}{<v_z>H^2} \]

Fig. 3-17. Comparison of experimental to theoretical 'isothermal' Nu-numbers

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confirm the foregoing statement.

For the given range of viscosity ratios, \(1 < \left(\frac{\mu_m}{\mu_w}\right) < 2\), the Nu-numbers may be predicted by eq. (33)

\[
Nu = Nu_0 \left(\frac{\mu_m}{\mu_w}\right)^{0.21}
\]  

(33)

Up till now no reference has been traced indicating the range within which the correction factors are valid. Apart from the possibility that large temperature gradients may be accompanied by natural convection effects, it is to be expected that deviations will occur as soon as the supposed linear temperature dependence of the viscosity is violated too much. If this condition is not satisfied before free convection occurs, it may lead to erroneous results. More research in this field is desirable.

While composing fig. (3-16) the following observation was made. For high values of \(z\) or low values of \(\langle v_z \rangle\), given by \(Gz^+ > 7.10^{-3}\), the experimental values did no longer fit the correlation of Nu-versus \(\left(\frac{\mu_m}{\mu_w}\right)\), which is represented by the straight lines. It appeared that the Nu-number was influenced under these conditions by free convection if also simultaneously \(\left(\frac{T_w}{T_m}\right) > 2.0\). This was proved by the fact that the Rayleigh-numbers, as given by Tao (1960) and Han (1959), exceeded the upper limit of \(Ra = 75\) for which natural convection effects may be considered negligible, see fig. (2-11).

The presented Nu-values in fig. (3-16) for \(Gz^+ > 7.10^{-3}\) belong to \(Gz^+\)-numbers, which obtain their value for \(z < 0.5\ m\), since in that part of the duct the aforementioned combination of conditions could not be reached with the heat fluxes imposed. For \(Gz^+ > 1.43 \times 10^{-2}\) the series in fig. (3-16) was not continued for lack of sufficient data that were not influenced by free convection.

In the foregoing it was seen that natural convection could occur near or in the established region. Fig. (3-12) and (3-13) show that for non-isothermal flow it is rather difficult to determine the value of \(Gz^+\) beyond which fully developed conditions may be assumed. Such a situation will actually never exist, as the local fluid properties keep changing in the flow direction. The consequences of this with respect to the velocity distribution have been discussed in Ch. 2.3.

Developed conditions will be assumed above those values of \(Gz^+\) for which the Nu-number reaches a minimum.

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The problem of fixing the length of the entrance region arose, when an attempt was made to compare the experimental results with the solutions of Han (1959) and Tao (1960) for combined free and forced convection under fully developed conditions. An inspection of fig. (3-12) indicates that it seems reasonable to bound the established region at \( Gz^+ = 3.0 \times 10^{-2} \), taking the minimum Nu-number as the characteristic quantity. However, it turned out that in this way the Nu-numbers were not only a function of the Ra-number defined by Tao as \( Ra = g\beta(dT/dz)/av \) but also of the distance to the entrance \((z)\). This means that in the main part of the experiments the free convection boundary layer was still developing as suggested in Ch. 2.3.

Therefore an investigation was made to find out if the assumption of complete development of both the thermal and the velocity distribution would be met better when only those experimental data were regarded, which were obtained by means of the two thermocouples that were farthest from the entrance. Fig. (3-19), using the curves of fig. (2-11), shows that for \( Ra < 10^3 \) the data taken from these thermocouples confirm Han’s prediction very well. This is not so surprising in view of the fact that the corresponding velocity distributions of fig. (2-10) demonstrate that the free convection effects are not too pronounced within the range of \( 10^2 < Ra < 10^3 \). Beyond \( Ra = 1.5 \times 10^3 \) the Nu-numbers diverged from the theoretical predictions. Although the amount of data is limited, the tendency may be clearly observed.

![Fig. 3-19. Influence of free convection on local \( Nu_\infty \)-numbers (duct I) (1968)](image-url)
The theoretical lower limit of the Nu-number for \((Ra \to 0)\) could not be explained, as pointed out earlier in Ch. 2.3, but it is surprising that the value corresponds perfectly with the measured values. The experimental results are closely approximated when the viscosity variations are taken into account in correcting the theoretical value \(Nu = 4.12\) (see dashed line in fig. 3-16). However, if a comparison is made with the theoretical solutions that neglect viscosity influences, the good agreement for small heat fluxes may be considered to be incidental.

Inspection of figs. (2-11) and (3-19) creates the impression that the differences in theoretical Nu-values for the various aspect ratios for \((Ra \to 0)\) are to be ascribed simply to the inclusion of the aspect ratios in the derivation of the (Nu-Ra)-correlation. They seem to bear no physical meaning. It is therefore to be expected that the experimental and theoretical values will disagree increasingly if the aspect ratios tend to \(H/B = 1.0\) (square duct). With regard to the above mentioned considerations, it may be concluded that Han's and Tao's theoretical solutions have incidentally been confirmed. It is doubtful whether they will be supported further by measurements more downstream, if the experiments are performed on longer ducts. Anyhow, the solutions are proved to hold for \(Gz^+ > 3.0 \times 10^{-2}\) and \(55 < z/H < 75\), provided \(10^2 < Ra < 10^3\).

The situation of a free convection boundary layer, that is still developing will now be considered. It must be possible to describe the gradual increase of the buoyancy forces starting at the thermal entrance and continuing up to and including the developed region. To meet this problem it was decided that the ratio \((Nu/Nu_0)\) of the measured local Nu-number to the theoretical 'isothermal' Nu-number \((Nu_0)\) would be used. The latter values were obtained from the solution of Cess, see fig. (2-4).

Attention must then be paid to the proper definition of the Rayleigh number for this case, which would suitably describe the variation of the ratio \((Nu/Nu_0)\) for varying \(\langle v_z \rangle\), \(z\) and \(\Phi''w\). First, the Ra-number used by Han (1959) could be easily converted to the following expression with the aid of eq. (26) on p. (24):

\[
Ra' = g\beta H^3\Phi''w/\langle v_z \rangle \lambda \nu
\]

(34)

The advantage of this form is that it simplifies the computational work. The variation of the Ra-number over the duct length is found by inserting the local viscosity values once the experimental conditions \(\Phi''w\) and \(\langle v_z \rangle\) have been chosen. The Ra-numbers for duct I were initially computed in this way to obtain some information on the influence of \(z\). The curves for varying \(z\) appeared to differ indeed by a factor \((z/H)^3\), which proves that in this region the downstream distance \(z\) and not the plate distance \(H\) is the appropriate characteristic length in the Ra-number. Taking this factor into account, eq. (34) becomes:

\[
Ra'' = g\beta z^3\Phi''w/\langle v_z \rangle \lambda \nu
\]

(35)

If this is the desired expression, then the results of ducts II and III also had to
fit the curve belonging to duct I. This is indeed what happens, as shown in fig. (3–20).

For $Ra''<10^6$ the viscosity and buoyancy effects appeared to become of the same magnitude. The results indicated that $(Nu/Nu_a)$ is best described by the viscosity parameter $(\mu_m/\mu_w)$ for $Ra''<10^5$.

![Diagram](image)

**FIG. 3–20.** Experimental results of four different ducts on the influence of free convection on local Nu-numbers

The correction on the viscosity influence and the $(Nu-Ra'')$-correlation could be used to explain the chaotic results obtained for duct VIII. Although the measurements on this conduit were the last to be performed, the resulting data essentially belonged to the first part of the experiments.

To demonstrate the effects of radial viscosity and density variations the actual measured Nu-numbers have been plotted against the $Gz^+$-number in fig. (3–21). This figure contains the unprocessed Nu-numbers of a number of runs.

The deviations from theory were much more pronounced than those observed for ducts I, II and III. This was caused by an increase of the duct height ($H$), see fig. (3–2), and an extension of the range of heat fluxes made possible by blackening the duct surface. It will be clear to the reader that the theoretical solution (represented by the asymptotes in fig. (3–21)) is of little use in the prediction of the heat transfer coefficients under these circumstances.

The viscosity correction factor of eq. (33) was used up to the limiting value $(\mu_m/\mu_w) = 2.0$ to correct for radial viscosity variations. This manipulation produced the results collected in fig. (3–22).

For the other Nu-numbers the corresponding $Ra''$-values were computed. Fig. (3–20) demonstrates that the Nu-numbers, which may seem unreliable at first sight, agree very well with the results obtained previously. It is interesting to notice that for $Ra'' > 7.5 \times 10^8$ the ratio of experimental to theoretical Nu-numbers is a function of $(Ra'')^{1/4}$. Pure free convection shows the same dependence. So if the flow behaviour of the stream in the duct is comparable with that of free convection, this would imply that a reversal of the flow occurs in the core of the duct. The velocity distributions of fig. (2–10) would then no longer be
Fig. 3-21. Local Nu-numbers as a function of Gz-values for varying heat fluxes (duct VIII)

Fig. 3-22. Local Nu-numbers of duct VIII after correction for radial viscosity variations by means of eq. (33)

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valid. This might lead to instabilities that could be the cause of the deviation in the results from predicted values for $Ra > 1500$, as shown in fig. (3-19). The quantitative agreement between fig. (3-19) and (3-20) follows immediately from eq. (36) below, and from the fact that the Ra-numbers had been computed for $z/H = 71.5$ and $z/H = 58.5$ (see table 6).

\[
Ra'' = \frac{1}{2} Ra \left( \frac{z}{H} \right)^3
\]  

(36)

The discussion is continued with the heat transfer results of ducts IV, V and VI. Fig. (3-23) shows that under the imposed experimental conditions, the ducts IV and V produced data that did not differ from those obtained with the flat ducts. This is easily recognised by comparing fig. (3-23) to fig. (3-12).

![Fig. 3-23. Local Nu-numbers as a function of $G^+$-values (ducts IV and V)](image)

Evidently the flow was not disturbed sufficiently by the profilations of the ducts under consideration. It is very likely that the maximal Re-number was not high enough to provoke the desired disturbances. Afterwards more attention to this matter will be paid on pp. 64-66. Addition of more material is not necessary here as neither of the two heat fluxes showed any deviation from the data discussed earlier.

The heat transfer results of duct VI, see fig. (3-24) showed a different picture. The profiled walls have an unfavourable influence, which might be explained as
follows. Part of the fluid stream passes the duct unhampered and it is bounded by static and slowly rotating whirls, as is illustrated by fig. (3-25). These 'whirls' form a special outward thermal resistance to the core which may be considered constant over the duct length in view of the boundary condition of uniform heat flux, and assuming that the same flow situation exists in each of the rectangular ribs. To give an idea of this special thermal resistance it is assumed that it is caused by two 'immobile' fluid layers at either side of the central core. This might suggest that, to reach the core, the heat must be conducted from the wall through these layers. However, this cannot be correct, as it has been observed that the 'whirls' are slowly rotating. Therefore the thermal resistance will not be called \((d/\lambda)\) but \((K)\). Once the heat penetrates the 'boundary layer' of the central core, a normal thermal entrance problem is created for a hypothetical duct of height \(H_i\), see fig. (3-25). This immediately leads to the question whether the flow in the central core is to be considered as a plug flow or as a parabolic flow. In view of the fact that the viscosity forces at the boundaries cannot be completely neglected, the flow is assumed to be parabolic. An attempt will be made to adapt the measured results to the sketched model, although its imperfection is fully recognized.

If the assumptions are true, the Nu-numbers for the central core \((z_iH_i/\lambda)\) ought to fall within the asymptotic solutions for the thermal entrance region, after modifying the Graetz-number to: \(Gz_{\text{mod}}^+ = az/<v_z>H_i^2\).
The measured total thermal resistance \(1/\alpha_{\text{exp}}\) may be expressed by eq. (37):

\[
1/\alpha_{\text{exp}} = K + 1/\alpha_i
\]

(37)

For those \(Gz^+\)-values for which the Lévéque solution holds, the heat transfer coefficients at the boundaries of the central core are given by eq. (38):

\[
\left(\frac{\lambda}{\alpha_i} H\right) = \frac{H_i}{H} (Gz_{\text{mod}})^{1/3} / C_3
\]

(38)

with \(C_3 = 0.650\)

Eq. (38) can be rewritten to eq. (39) by using eq. (37):

\[
1/Nu_{\text{exp}} = \frac{H_i}{H} (Gz_{\text{mod}})^{1/3} / C_3 + \left(K\lambda/H\right)
\]

(39)

The constant in the right member of eq. (39) is determined by means of fig. (3–26). It is found to be \(K\lambda/H = 3.1 \times 10^{-2}\) or \(K = 3.2 \times 10^{-6}\) (kW/m²°C). The modified Nu-numbers \((Nu_{\text{mod}} = \alpha_i H_i/\lambda)\) can now be computed from the experimental data by means of eq. (37). The values obtained are given in fig. (3–26) as a function of the modified \(Gz^+\)-number.

The proposed model to explain the influence of the ribs on the walls appears to be justified by figs. (3–26) and (3–27) despite its rigid physical simplifications. The model shows that the main temperature gradients within the fluid occur in the corners of the walls at both sides and hardly influence the central core. Fig. (3–27) shows that the Lévéque region extends as compared with the results of duct II \((H = 0.79\ \text{cm})\), see fig. (3–13), in accordance with the smaller temperature gradients at the boundaries of the central core. In addition the limiting Nu-number is more closely approximated and free convection is suppressed to \(Gz_{\text{mod}}^+ = 5.0 \times 10^{-2}\).

The results of ducts IV, V and VI have been confirmed by an experimental investigation of the flow behaviour. Chapter 3.1. discussed the equipment that was used to perform the Reynolds flow visualization experiments.

In ducts IV and V no change in the velocity distribution could be observed for \(Re < 70\) as compared with the flat rectangular duct. In duct VI the un-
\[
\frac{1}{Nu} = \left( \frac{\lambda}{\alpha_{exp} H} \right)
\]

<table>
<thead>
<tr>
<th>(\Phi_w (kW/m^2))</th>
<th>(\Delta)</th>
<th>(\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.75</td>
<td>2.83</td>
</tr>
</tbody>
</table>

**Fig. 3-26.** Determination of the special thermal resistance caused by the cavities in duct VI

**Fig. 3-27.** Measured Nu-numbers of duct VI after correction for the special thermal resistance, versus modified \(G^+\) -values

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hampered core flow did exist, as assumed, up to \( \text{Re} = 125 \). This substantiated the theory explaining the experimental heat transfer data, as the experiments on heat transfer had not been extended beyond \( \text{Re} = 70 \).

In duct V complete turbulence was observed for \( \text{Re} > 180 \). Small deviations from the parabolic velocity distribution already occurred at \( \text{Re} \approx 90 \). The recurring changes in the flow direction resulted every time in an asymmetric velocity profile, because the flow was accelerated at one wall and retarded at the other.

With increasing Re-numbers this was followed by a back flow along the ‘retarded’ side, while the streaklines at the other side were still undisturbed. The latter were closer to the wall, indicating an increased velocity. The streaklines eventually also became unstable and turbulence resulted. According to the asymmetric velocity distribution the heat transfer will improve at one side and be impaired at the opposite side. The two effects may cancel each other so that a positive influence of the profiled walls can only be predicted for \( \text{Re} > 180 \). This value compares well with those obtained by Rybinova (1964).

The experiments with flow visualization demonstrated that the upper limit of the Re-numbers investigated in the study of the heat transfer, had been chosen too low to determine in profiled ducts the transition from laminar to turbulent flow. To include such a fundamental investigation in the experimental programme, the consequence would have been that next to an extension of the Re-number range, a wider variety of wall profilations would have to be used. For only in this way it could be possible to develop a general expression correlating the critical Re-number to the factors that determine the transition (see ch. 2.4). This, however, was not the purpose of the present work, which was intended mainly to give a decisive answer to the question whether the heat transfer for \( \text{Re} < 100 \) could be improved by continually changing the direction of the flow.

It was found by means of the heat transfer experiments that this could not be
realised; the flow visualization experiments confirmed this conclusion and also provided the indication that a reduction of the critical Re-number may indeed result from the application of wall profiling. The initial aim being reached, the experiments were not carried on for higher Re-numbers.

The flow patterns discussed above, will now be used to explain the results of duct VII. The configuration of this duct did not allow a direct comparison with the flat rectangular duct. The local heat transfer coefficients were measured for three different situations, as may be seen in fig. (3-28), which shows the localisation of the three series of three thermocouples that were cemented to the duct walls.

The measured local heat transfer coefficients were compared with the values ($\alpha_0$), that could have been expected to be found in flat ducts for the same Gz$^+$-value and experimental conditions as regards the heat flux. This Gz$^+$-value had been computed from the mean entrance velocity and the distance ($z$) to the entrance measured over the duct wall. The various ratios of ($\alpha_{\text{exp}}/\alpha_0$) thus obtained for a certain mean entrance velocity were averaged over the duct length simply by dividing the sum of the ratios by the number of thermocouples. Thereupon the arithmetic mean ratios were plotted against the entrance Re-number to give fig. (3-29).

It was observed that $(\alpha_3 H_1/\lambda) < (\alpha_2 H_2/\lambda) < (\alpha_1 H_2/\lambda)$. This agrees well with the velocity distributions in fig. (3-28). The sketched flow pattern has been composed of previous findings obtained for duct V by means of the flow visualization experiments.

![Graph](image1)

![Graph](image2)

Fig. 3-29. Local heat transfer coefficients and pressure drops in duct VII compared to theoretical values for parallel plates under the same experimental conditions

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It is to be noted that the given picture is only valid up to a certain Re-number, which will appear to be Re $\sim 20$. This value is based on the fact that a positive effect of the wall configuration on the mean Nu-number could only be observed beyond this limit. The pressure drop ratio ($\Delta p/\Delta p_0$) also remained independent of Re over this range, see fig. (3–29).

Fig. (3–29) shows that favourable and unfavourable influences evidently neutralized each other for low entrance velocities. The mean heat transfer coefficient increased as soon as the critical value Re = 20 was exceeded. Comparison with a correlation for heat transfer to turbulent flows will demonstrate that this improvement was caused by disturbances in the laminar flow, which was dynamically established when it first entered the heated section. Again as in Ch. 2.4, the concept of turbulence is intentionally not used for this disturbed flow, since it stays laminar in nature. Each disturbed laminar flow actually develops dynamically between two consecutive disturbances. As a consequence the improvement in the heat transfer coefficient has to be accredited largely to entrance region effects, as long as the flow has been disturbed only moderately. For increasing entrance velocities, the flow patterns in the ‘widenings’ will gradually and more and more resemble turbulence. The number of thermocouples attached to the walls of the ‘widenings’ is twice that of the thermocouples in the ‘narrowings’. Therefore the average heat transfer coefficient will also show a gradual resemblance with heat transfer under turbulent conditions.

The heat transfer to turbulent flows is described by eq. (40) in a simplified form. For laminar conditions eq. (41) is valid.

$$\alpha_{exp} \sim Re^{0.8}$$  \hspace{1cm} (40)

$$\alpha_0 \sim Re^{0.3}$$  \hspace{1cm} (41)

By combining these two correlations, eq. (42) is obtained:

$$\alpha_{exp}/\alpha_0 \sim Re^{0.5}$$  \hspace{1cm} (42)

If the assumed disturbances are indeed provoked, the experimentally measured ratio ($\alpha_{exp}/\alpha_0$) will approach the proposed dependence on the entrance Re-number beyond a certain value of this number. According to fig. (3–29) a transition occurred for 20 < Re < 30. The improvement in the heat transfer coefficient in this region is ascribed to unestablished flow conditions. Turbulence is created for Re > 30. It is evident that the heat transfer for Re < 20 cannot be improved by rigid measures such as the introduction of repeated narrowings and widenings of the duct cross section in combination with 90 degree changes in the flow direction. This illustrates the difficulties that will be encountered in heat transfer to highly viscous materials. The experiments also point out that in fluids that are less difficult to handle, the heat transfer coefficient showed a remarkable increase for still moderate fluid velocities when duct configurations similar to that of duct VII are used.

The next point of discussion will be the pressure drop losses measured in the
respective ducts. These losses can be predicted by means of eq. (6) introduced in Ch. 2.1.1.

For the three rectangular ducts I, II and III the pressure drops were determined isothermally, in order to compare them with the theoretical values of eq. (6). The results are given in fig. (3–30) and table (9). The table also includes the corresponding flow modulus and its predicted value according to eq. (42) derived by WEBER (1921).

\[ M = 1 - 0.630 \left( \frac{H}{B} \right) + 0.052 \left( \frac{H}{B} \right)^6 \]  

(43)

The small deviations from theoretical values are to be ascribed to experimental imperfections. The temperatures were measured accurately up to 0.3 °C, the viscosity values within 2.5%, while the pressure drops were accurate within 3%. Eq. (6) and (43) may therefore be recommended for practical applications.

The data of duct I made it possible to calculate the velocity gradient at

<table>
<thead>
<tr>
<th>( \frac{H}{B} )</th>
<th>0.10</th>
<th>0.079</th>
<th>0.035</th>
</tr>
</thead>
<tbody>
<tr>
<td>theor.</td>
<td>exp.</td>
<td>theor.</td>
<td>exp.</td>
</tr>
<tr>
<td>(f.Re)</td>
<td>21.0</td>
<td>21.9</td>
<td>22.7</td>
</tr>
<tr>
<td>(M)</td>
<td>0.937</td>
<td>0.951</td>
<td>0.978</td>
</tr>
</tbody>
</table>

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(x = 0; y = H/2), using eq. (2). It was found, for instance, that insertion of \( Q_v = 10^{-4} \text{ m}^3/\text{sec} \), \( \Delta p = 1200 \text{ N/m}^2 \) led to \( (s) = 59.6 \text{ sec}^{-1} \), which is almost exactly equal to the value obtained from \( (s) = 6 <v_z>/H \).

![Graph of non-isothermal pressure drops as a function of the distance to the entrance (duct I, oil I)](image)

As observed earlier in Ch. 2.3, there were no data available that could be compared with the pressure drops that had been obtained experimentally under non-isothermal conditions. To illustrate the pressure drop behaviour downstream from the entrance, fig. (3-31) has been composed from some experimental data. It shows that the deviation from the theoretical isothermal pressure drop (represented by the straight lines) becomes measurable at fifty centimeters from the entrance. Apparently the method of measurement was not sensitive enough to register the minor differences in pressure drop over the first fifty centimetres. However, a more sensitive method would not make this representation more attractive for an analysis of the quantitative results. Therefore a different approach was made, which will be introduced in the following paragraph. This led to the presentation of the results in figs. (3-32) and (3-33). The influence of non-isothermal conditions is marked by a decrease of the pressure drop as compared with the theoretical values for isothermal flow. In the following paragraph it will be supposed that buoyancy forces are at first negligible all over the duct. This means that the change in the pressure drop is caused by a variation of the viscosity, which mainly occurs in the layers close to the wall.

The rising wall temperature and the presence of the heat flux diminish the pressure drop by decreasing the viscosity both at the wall and in the axial direction. On the other hand they augment the pressure drop, since the velocity gradient at the wall is increased. The influence of this factor will be initially neglected in proportion to the effect of a decreasing viscosity.

The first step is the re-introduction of the expression that gives the best description of the temperature dependence of the viscosity, viz. eq. (22)

\[
(\mu/\mu_0) = \exp (- \varepsilon \Delta T_w)
\]  

(22)
Fig. 3-32. Non-isothermal pressure drops as a function of the axial and radial variations in viscosity

\[ \ln \left( \frac{\Delta p_i}{\Delta p} \right) \]

\[ \psi = \left( \frac{\varepsilon}{\rho_c p} \right) \left( \frac{L}{H} \right) \left( \frac{\Phi_w}{V_{z}} \right) \]

Fig. 3-33. Non-isothermal pressure drops as a function of the axial and radial variations in viscosity

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Eq. (22) is now used to relate the pressure drops for two different fluid temperatures with a zero radial temperature gradient. To this end eq. (44) is recalled.

\[
\frac{dp}{dz} = 12 \langle v_z \rangle \mu / H^2 \quad (44)
\]

\[
\frac{\Delta p_0}{\Delta p} = \frac{\mu_0 / \mu} = \exp (\varepsilon \Delta T_a) \quad (45)
\]

Eq. (45) states that the ratio of the pressure drop under isothermal conditions to that obtained non-isothermally, is expressed by the product of the temperature coefficient of the viscosity and a representative temperature difference. The viscosity in eq. (44) is generally evaluated at the arithmetic mean bulk temperature of the fluid, i.e. at \(T_e + T_o)/2 \) (°C) or \(T_e - T_o)/2 \) (°C) above the entrance temperature. This temperature difference will be rewritten into eq. (46) to show the dependence of the pressure drop on the experimental conditions.

\[
\frac{T_e - T_o)/2}{\Delta T_a} = \frac{(1/\rho c_p) (L/H)}{(\Phi_w/\langle v_z \rangle H)} \quad (46)
\]

A combination of eqs. (45) and (46) gives:

\[
(\Delta p_0/\Delta p) = \exp (\varepsilon \Phi_w L/\rho c_p \langle v_z \rangle H) = \exp \psi \quad (47)
\]

It will be clear that the cup mixed mean temperature difference leads to an overestimation of the actual pressure drops, since the pressure drop is determined predominantly by the temperature of the wall. Another unreality of the physical model is the assumption of a flat temperature distribution over the cross section. Chapter 2.3 has shown that a radial temperature gradient flattens the velocity profile. The resulting increase of the velocity gradient will raise the pressure drop or lower the ratio \(\Delta p_0/\Delta p\), at the same time counterbalancing the decrease in the pressure drop by a reduction of the viscosity. To correct the improper reference temperature as well as the change in the velocity gradient at the wall, eq. (47) is rewritten into:

\[
(\Delta p_0/\Delta p) = \exp \{C_5 (\varepsilon \Delta T_a)^n\} \quad (48)
\]

The introduction of eq. (46) gives:

\[
\ln (\Delta p_0/\Delta p) = C_5 (\varepsilon \Phi_w L/\rho c_p \langle v_z \rangle H)^n \quad (49)
\]

For \(\psi = \varepsilon \Delta T_a < 0.3\) the values of \(C_5\) and \(n\) have been determined to be respectively 2.92 and 0.75, see fig. (3-32), by which eq. (49) becomes:

\[
\ln (\Delta p_0/\Delta p) = 2.92 \psi^{0.75} \quad (50)
\]

It would be very interesting to use the experimental data to evaluate the mean velocity gradient belonging to a certain value of \(\Phi_w/\langle v_z \rangle\). This value could then be compared with the deviations from the isothermal solutions of the experimental local Nu-numbers.

By using RABINOWITSCH' equation, eq. (2), in combination with the experimentally found pressure drop according to eq. (50), the following expression can be derived for the velocity gradient at the wall as a function of \(z\) and
This equation shows that \( s \) increases gradually from the value for isothermal flow at the entrance \( (z = 0) \), as is expected. But it still remains difficult to predict the influence, which this changing gradient has on the local heat transfer coefficient.

Therefore eq. (51) is on the one hand of limited use, while on the other hand it may provide perhaps a more fundamental basis for interpreting experimental results on heat transfer with a temperature dependent viscosity than, for instance, the improved approach of Sieder and Tate, which was discussed before. This may be recognized from the consideration that the ratio of the experimentally observed Nu-value and the Nu-value from the 'isothermal' theory must be an unique function of \( \psi \), at least in the case of a uniform heat flux. This line of approach has not been pursued, because the correction proposed by Sieder and Tate in its improved form already proved to be of value. In addition the latter correction has the advantage of being very practical and simple.

The heat transfer results and flow patterns of ducts I, IV and V were found to be identical. Fig. (3-31) shows that the pressure drop measurements are in complete agreement with this.

This figure also gives a few data on non-isothermal pressure drops in duct VI. The ratios \( (\Delta \rho_0/\Delta \rho) \) appear to be larger than those for ducts I, IV and V. This is in agreement with the model of the unhampered core flow, as the shearstress at the boundaries that contact the fluid layers in the ribs will be less than the stress exerted in the normal situation, when the fluid flows past a solid wall with the same mean velocity. For isothermal conditions it was found that \( (\Delta \rho_0/\Delta \rho) = 1.22 \).

Only a few quantitative remarks can be made on the pressure drops in such a complicated passage as provided by duct VII. The presence of the narrowings and widenings and the ensuing changes in the flow direction will cause an additional pressure drop due to the rearrangement of the parabolic velocity profile for small entrance velocities. It is augmented by excess kinetic energy losses for more disturbed flows. This may be expressed as follows:

\[
\Delta \rho = f ( \mu Q_v, \varphi Q^2_v )
\]  

(52)

Since \( (\Delta \rho_v) \) is only a function of the viscous term \( (\mu Q_v) \) it may theoretically be expected that

\[
(\Delta \rho/\Delta \rho_v) = a + b \text{ Re}
\]  

(53)

The experimental data for isothermal flow showed \( (\Delta \rho_0/\Delta \rho) \) to be constant for \( \text{Re} < 20 \), see fig. (3-28). This supports the hypothesis, which was developed.

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to explain the heat transfer results, that the velocity profile had been influenced only slightly.

The non-isothermal pressure drops are collected in fig. (3–33). The outlined area comprises those values of \( \psi \) for which the 'critical' Re-number \( (Re = 20) \) may be exceeded. Dependent on the existing heat flux, the value \( \psi = 0.5 - 0.6 \) will be obtained for large or small entrance fluid velocities. In the first case the 'critical' Re-number is exceeded and the results cannot be compared since the flow patterns are different. The pressure drop ratio will be smaller than expected, because \( \left( \Delta p_o \right) \) increases. This explains the wide scatter of experimental data in this region. The pressure drop ratio for this duct shows once more dependence on the viscosity for \( 0.05 < \psi < 0.3 \), as observed before.

3.4 CONCLUSIONS

The theoretical heat transfer coefficients for the parallel plate case may also be regarded as the theoretical local values at \( (x = 0, y = H/2) \), see fig. (2–1), of rectangular ducts with aspect ratio \( H/B < 0.4 \). This followed from the fact that the velocity distributions were identical in both situations, as proved by isothermal measurement of pressure drops in these rectangular ducts. Transformation of the data into velocity gradients at the wall by means of the RABINOVITCH equation, eq. (2), resulted in values that were equal to those theoretically occurring in a parallel plate channel for the same mean fluid velocity. It was shown that the isothermal pressure drops were also in agreement with the theoretical predictions of CORNISH (1928) for rectangular ducts of arbitrary aspect ratio.

The non-isothermal pressure drops observed in the ducts with flat parallel walls obeyed eq. (50),

\[
\ln \left( \left( \Delta p_o / \Delta p \right) \right) = 2.92 \psi^{-0.75}
\]

The use of the arithmetic mean fluid temperature appeared to result in an incorrect prediction of non-isothermal pressure drops. This may be ascribed to an improper choice of the reference fluid temperature, and to changes in the velocity distribution due to radial viscosity and density differences.

For conditions near the entrance, the measured local heat transfer coefficients approach the LÉVÈQUE asymptote. The wall to bulk temperature difference determines the Graetz-value up to which this solution remains valid.

The limiting Nusselt number has never been reached owing to variable-physical properties. The deviations from theoretical predictions for small Graetz-values could be ascribed to radial viscosity variations. The obtained experimental correction factor \( (\mu_m/\mu_w)^{0.21} \) equals BEEK's numerically and is valid for viscosity ratios \( (\mu_m/\mu_w) < 2.0 \), subject to fulfillment of the condition \( \text{Ra}^* < 10^5 \). Beyond this limit there is a region in which the results are influenced by both viscosity and density differences. The effects cannot be disentangled up to \( \text{Ra}^* \approx 10^6 \).

Instead of its theoretical limiting value, the Nusselt number has always
reached a higher minimum value somewhere downstream from the entrance. For still larger Gz+-values it generally increased again, due to aiding free convective forces.

The results obtained with couples at \( z > 0.5 \) m agreed with the theoretical solutions of Han, Tao and Agrawal for combined free and forced convection, but this could not be completely explained. The assumptions of fully developed conditions for which the solutions have been derived, diverged too much from the experimental situation to justify this good agreement.

To describe the free convection effects it was found that the Rayleigh-number had to be multiplied by the factor \((z/H)^3\) to allow for the local situations. Proposals in this direction could not be traced in the limited amount of literature on combined free and forced convection.

It was observed that for \( Ra^* > 10^8 \), the Nusselt number ratio \((Nu/Nu_0)\) became a function of \((Ra^*)^{1/4}\). This indicated the existence of conditions similar to those of pure free convection. As a consequence, the assumed distorted velocity distribution no longer correctly described the actual situation, and it was supposed that instabilities would arise. However, a thorough analysis of the effects could not be made since the amount of data was insufficient. A further extension of the range of \( Ra^*\)-numbers could not be realised with the experimental equipment.

Neglection of viscosity variations and buoyancy forces was found to lead to serious under-estimations of the heat transfer coefficients. The investigations made it possible to correct the theoretical solutions for the whole entrance and developed region as follows:

\[
\begin{align*}
Ra^* &< 10^6; \quad (\mu_m/\mu_w) < 2.0 \quad & Nu = Nu_0 (\mu_m/\mu_w)^{0.21} \\
10^6 &\leq Ra^* < 5.10^7 \quad & Nu = 0.53 \ Nu_0 (Ra^*)^{0.06} \\
10^8 &< Ra^* < 6.10^8 \quad & Nu = 0.19 \ Nu_0 (Ra^*)^{0.25}
\end{align*}
\]

It was further observed that laminar flows of very low Re-numbers could only be disturbed by severe measures, as shown in the experiments with ducts of various wall profilations. The central fluid stream cannot be allowed to flow unhindered, since this has an unfavourable influence on the heat transfer. The best way to disturb the velocity distribution appeared to be a regular succession of narrowings and widenings in the duct cross section, although this considerably increased the pressure drops involved. The local Nu-numbers obtained may be used for the derivation of mean values by integration over the duct length. These values could serve as a basis for future investigations into the heat exchange performance of apparatus with a wider range of variations in wall configurations and duct cross sections.
The investigation has been actuated by the question whether, and if so to what extent, the heat transfer to laminar flows may be improved by the usual profilation of plate heat exchangers. Such a profilation could disturb the velocity distribution of laminar flows, thereby improving the heat transfer. This improvement could be demonstrated by comparing local heat transfer coefficients in model ducts with profiled walls to values for ducts with flat walls under similar conditions. However, an investigation into the existing literature showed a lack of information in this field, even in the ostensibly simple case of heat transfer in gaps; the problem has been solved in theory, but under circumstances that diverged too much from actual conditions, and hardly any trace could be found of experimental investigations. Therefore no data were available upon which a comparative investigation could be based. So first of all experiments with simple heat exchangers had to be carried out to obtain the necessary material for reference, and this meant a change in the accent of the investigation. In order to restrict the broad field, it was decided to perform the experiments for the case of a uniform heat flux to dynamically established laminar flows of Newtonian fluids only. In this way the interest was directed to the thermal entrance region. A further restriction was introduced by not extending the experiments beyond the maximal value of \( \text{Re} = 100 \).

The introduction (Ch. 1) is followed by a systematical discussion of the literature data available (Ch. 2). The subjects introduced here may be illustrated by the following conclusions.

Velocity distributions and pressure drops have apparently been studied both experimentally and theoretically for simple geometries and isothermal flows (2.1.1). In a few cases the hydrodynamic entrance length can be calculated. Little experimental work has been done to confirm the existing theoretical solutions (2.1.2).

The non-isothermal flow has been approached with a wide diversity of mathematical techniques (2.2). With a few exceptions, all theoretical solutions assumed constant physical properties. The growth of the thermal boundary layer in laminar flows between parallel plates has been solved for the whole thermal entrance region (2.2.2.1), but experimental confirmation is once again not available; besides, more complex situations have not or have hardly been analysed. The same goes for the simultaneous hydrodynamic and thermal development (2.2.2.2). There appears to be a considerable lack of knowledge concerning the influence of the temperature dependence of the fluid properties (2.3).

Finally, just for summarizing the conclusions (2.5), the influence of wall profilations on the flow behaviour is considered (2.4).

The experimental part is discussed in ch. 3. It starts with a description of the
main parts of the experimental equipment (3.1.1) in fig. (3-1). Then special attention is paid (3.1.2) to the construction of the eight different heat exchangers as illustrated in fig. (3-2). As the uniformity of the heat flux is very important, it was verified by two special tests based upon the principles described in (3.1.3).

Another point that deserves special attention is the measurement of the local wall temperatures (3.1.4). To this end 5–9 Philips Stick-on Thermocouples were attached to the duct wall in the flow direction and at regular intervals, see figs. (3-4) and (3-5). These measurements required corrections, which were experimentally obtained as a function of the heat flux intensity.

The experimental equipment also comprised a device to determine the pressure losses (3.1.5). These pressure drops were measured to provide additional information on the flow patterns in the various ducts. The main physical properties of both oils used in the investigations, are given in (3.1.6). The kinematic viscosity of these oils was 33 and 105 centi-Stokes respectively at the usual entrance temperature of 20 °C. The experiments also included a study of the flow in the heat exchangers. For this purpose transparent models were used. The lay-out of these experiments is described in (3.1.7).

The experimental programme is discussed in (3.2). It included a study of the following variables: fluid velocity, viscosity, heat flux intensity, distance to the entrance, aspect ratio and wall configuration. In this section an example will also be given of the way in which the local Nu-numbers were calculated.

The huge amount of data obtained in the experiments, made it necessary to process them before presenting the results. This method, which is explained in (3.3), allowed the presentation to be combined with a discussion.

The investigation produced a number of interesting conclusions (3.4). It was observed for example that the LéVêque solution indeed describes the thermal entrance region fairly well under certain conditions. On the other hand it seems unlikely that the theoretical limiting Nu-number for fully developed flows will be found in practice. The temperature dependence of the fluid properties results in large deviations from theoretical values. It appeared that the correction of Sieder and Tate on the radial viscosity variations led to an underestimation of the heat transfer coefficient. Free convection effects actually occur as early as the thermal entrance region, and they increase as the distance to the entrance grows. Furthermore it was observed that laminar flows can hardly be disturbed by wall profilations for Re < 100, if the cross sectional area is not varied simultaneously at regular intervals in the axial direction.

Finally it may be mentioned that the conclusions have been confirmed by the results of the pressure drop measurements and by studying the flow behaviour.
5 SAMENVATTING

De aanleiding tot het onderzoek was de vraag of, en zo ja in welke mate, de gebruikelijke profileren van de platen van een platenwarmtewisselaar een verbetering van de warmteoverdracht naar laminaire stromen tot gevolg heeft. Zo'n verbetering zou veroorzaakt kunnen worden doordat de profileren de snelheidsverdeling van dergelijke stromingen verstoort. De vraag zou beantwoord kunnen worden door vergelijking van plaatselijke warmteoverdrachtscoëfficiënten in model-buizen met geprofileerde wanden met waarden voor buizen met vlakke wanden onder overigens gelijke omstandigheden. Een literatuuronderzoek toonde evenwel aan dat er zelfs over het ogenschijnlijk eenvoudige geval van warmteoverdracht in vlakke spleten onvoldoende bekend is; de bestaande theoretische oplossingen van het probleem gelden voor omstandigheden die te sterk afwijken van de werkelijkheid, experimenteel onderzoek is tot nu toe weinig verricht. Betrouwbare gegevens voor een vergelijkend onderzoek waren derhalve niet voor handen. Er diende dus eerst referentiemateriaal vermeld te worden door uitvoering van experimenten met eenvoudige vlakke warmtewisselaars, hetgeen een sterke accentverschuiving van het onderzoek betekende. Teneinde het zeer uitgebreide terrein enigermate te beperken, werd besloten het onderzoek te verrichten voor het geval van een uniforme warmte-stroom naar dynamisch ingestelde laminaire stromingen van Newtonse vloeistoffen. Hierdoor werd het accent gelegd op het thermische instelgebied.

Een verdere beperking werd ingevoerd door slechts het gebied tot maximaal $Re = 100$ in de onderzoekingen te betrekken.

Na de inleiding (Hfdst. 1) volgt een systematische bespreking van de beschikbare literatuur gegevens (Hfdst. 2). Ter illustratie van de daarin behandelde onderwerpen mogen de volgende conclusies dienen.

Voor eenvoudige geometrieën en isotherme stromingen blijken snelheidsverdelingen en drukvallen zowel theoretisch als experimenteel bestudeerd te zijn (2.1.1). Een berekening van de benodigde lengte voor het instellen van de parabolische snelheidsverdeling is voor een beperkt aantal gevallen mogelijk. Er bestaan weinig experimentele metingen ter bevestiging van de bestaande theoretische oplossingen (2.1.2).

De niet-isothermische stroming is met gebruikmaking van een grote verscheidenheid aan wiskundige technieken behandeld (2.2). Met een enkele uitzondering echter, veronderstellen alle oplossingen constante stofeigenschappen. De indringing van het temperatuurfront in laminaire stromen tussen parallelle platen is voor eenvoudige gevallen voor het gehele thermische inloopgebied opgelost (2.2.2.1). Ook hier ontbreekt echter bevestiging door experimenten; bovendien zijn meer ingewikkelde situaties niet of nauwelijks in analyses betrokken. Hetzelfde geldt voor een gelijktijdige hydrodynamische en thermische instelling (2.2.2.2). Betreffende de invloed van de temperatuurafhankelijkheid der
Stofeigenschappen op de warmteoverdracht blijkt er een grote leemte in kennis te zijn (2.3).

Tenslotte wordt, voorafgaande aan de samenvatting der conclusies (2.5), een korte beschouwing gegeven over de invloed van wandprofileringen op het stromingsgedrag en dus op de warmteoverdracht (2.4).

Het experimentele gedeelte wordt beschreven in hoofdstuk 3. Allereerst worden in (3.1.1) de belangrijkste onderdelen van de proefopstelling besproken aan de hand van fig. (3-1). Vervolgens krijgt in (3.1.2) de constructie der acht verschillende warmtewisselaars speciale aandacht. Een beeld hiervan geeft fig. (3-2). Gezien het belang van de uniformiteit der warmtestroom, werden twee proefopstellingen ontwikkeld om deze te controleren. Het principe hiervan staat beschreven in (3.3.). Een ander punt, dat bijzondere aandacht verdient, is de meting der plaatselijke wandtemperaturen (3.1.4). Verdeeld over de buiswand waren op iedere buis in de stroomrichting 5–9 Phillips plak-thermokoppels bevestigd, zie fig. (3-4) en (3-5). De benodigde correcties om de wandtemperaturen te vinden, werden als functie van de warmtestroomdichtheid experimenteel bepaald.

De proefopstelling bevatte een meetinrichting voor de bepaling van drukverliezen (3.1.5). Deze drukvervallen werden gemeten om extra informatie te verschaffen omtrent de stromingstoestand in de verschillende pijpen.

In (3.1.6) worden daarop de belangrijkste fysische eigenschappen van de twee in het onderzoek gebruikte oliën vermeld. De kinematische viscositeit van deze oliën bedraagt bij de gebruikelijke ingangstemperatuur van 20°C respectievelijk 33 en 105 centi-Stokes. Tot onderdeel van de experimenten behoorde ook een bestudering van de stroming in de warmtewisselaars. Dit geschiedde met behulp van doorzichtige modellen. De hiervoor gebruikte opstelling wordt beknopt behandeld in (3.1.7).

Het onderzoek programma wordt hierop nader besproken in (3.2). Het omvatte de volgende variabelen: stroomsnelheid, viscositeit, warmtestroomdichtheid, afstand tot de instroming, dwarsdoorsnede verhouding en wandconfiguratie. Tevens wordt in deze paragraaf een voorbeeld gegeven van de berekening der plaatselijke Nu-getallen.

Het grote aantal waarnemingen maakte het wenselijk de resultaten te presenteren nadat deze gedeeltelijk bewerkt waren. De gevolgde methodes, verklaard in (3.3), maakte het mogelijk aan de presentatie tegelijkertijd een discussie te koppelen.

Het onderzoek leverde een aantal belangwekkende conclusies op (3.4). Zo werd onder meer geconcludeerd dat de oplossing van LÉVÈQUE onder bepaalde voorwaarden inderdaad het thermische instelgebied goed beschrijft. Daarentegen is het onwaarschijnlijk dat de theoretische limiet waarde voor volledig ingestelde stromingen in de praktijk zal gelden. Tengevolge van de temperatuurafhankelijkheid van de stofeigenschappen treden namelijk zeer grote afwijkingen van de theoretische waarden op. Het bleek dat de correctie op radiale viscositeitsverschillen van Sieder en Tate tot een onderschatting van de warmte-

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overdrachtscoefficient leidt. Ook in het instroomgebied treden reeds vrij spoedig vrije convectie effecten op, die toenemen met de afstand tot de instroming. Voorts werd waargenomen dat laminaire stromingen bij Re < 100 vrijwel niet te verstoren zijn d.m.v. wandprofileringen, tenzij gelijktijdig de buisdoorsnede in axiale richting regelmatig gevarieerd wordt.

Ten slotte kan vermeld worden dat de conclusies kwalitatief door de bestude-ring van de stroming en door drukvervalmetingen bevestigd zijn.
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