Biological implications of global bifurcations

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1 Abstract

Bifurcation analysis is a powerful tool for the analysis of ODE-models with limited variables. Typically, a few parameters are selected for variation (so-called bifurcation parameters), after which it is evaluated how the qualitative behaviour of the system changes under these parameter variations (e.g. from stable steady state to limit cycle). Thusfar, these analyses have mostly been focussed on local phenomena, where the qualitative behavioural changes are coupled to bifurcations that can be found by evaluating the local state space (i.e. *Jacobian* matrix and its eigenvalues), so steady states and limit cycles.

In this research we have shifted the attention to bifurcations of orbits through state space, that cannot be found by means of the Jacobian matrix. Such orbits connect saddle equilibria and/or saddle cycles, known as *homo-* and *heteroclinic* orbits. The bifurcations of such orbits are named global bifurcations. Although their existence has been known for several decades already, the numerical techniques to actually detect and continue them as functions of parameters have only been developed quite recently, among them Beyn (1994), Champneys and Kuznetsov (1994), Dieci and Rebaza (2004). In collaboration with others we have developed new techniques, based on the adjoint Jacobian matrix, for the detection and continuation of point-to-cycle and cycle-to-cycle connecting orbits (Doedel et al., 2008, 2009). These techniques have been incorporated as extensions of the well-known bifurcation software package AUTO (Doedel et al. , Concordia University).

With the coming of the technique, there is also room for interpretation of the obtained results of the application. In the presentation I will focus mainly on some ecological examples of the application of the newly developed techniques. It is found that these techniques give robust results, and give rise to re-interpretation of old model analyses. One specific and simple example is the Allee-model, where it is found that a limit cycle, in contrast to earlier claims, does exist, but is almost immediately destroyed again by a homoclinic bifurcation (Van Voorn et al., 2007). Finally, despite the ecological context, there is room for a broad application of the discussed techniques, for instance in Systems Biology.

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