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M. Elena Sáiz, Eligius M.T. Hendrix, Niels J. Olieman

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Mansholt Graduate School

Hollandseweg 1, 6706 KN Wageningen,
The Netherlands
Phone: +31 317 48 41 26
Fax: +31 317 48 47 63
Internet: http://www.mansholt.wur.nl/
e-mail: office.mansholt@wur.nl

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M. Elena Sáiz, Eligius M.T. Hendrix, Niels J. Olieman[a]

[a] Operations Research and Logistics Group, Wageningen University, Hollandseweg 1, 6706 KN Wageningen, The Netherlands.

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Determination of stable coalitions in a CO_2 emission game

M. Elena Sáiz [1] , Eligius M.T. Hendrix, Niels J. Olieman [a]

[a] *Operations Research and Logistics Group, Wageningen University, 6706 KN Wageningen, The Netherlands*

Abstract

This paper presents an implementable description of a game on Kyoto protocol coalition formation. Concepts from literature about the so-called CO_2 emission game where stability of coalition structures in an Open Membership Game and Exclusive Membership Game are applied, are translated into a new notation for specifying the stability and facilitating the implementation into computer coding. We consider multiple coalitions that groups of world regions can join. Implementation aspects are outlined and results are shown.

Key words: Environment, Game theory, Coalition formation, Stability, Implementation

1 Introduction

A consensus seems to appear in the world on the existence of the greenhouse effect. Using financial means to abate greenhouse gas emissions by individual countries looks inefficient from a cost perspective, as some countries can abate more cheaply than others. This gives an economic drive to co-operate in the reduction of greenhouse gas emissions. The real co-operation of countries and regions, such as observed in the Kyoto protocol, is merely a political question. Concepts from economy and game theory provide means of analyses to see whether at least economic incentives exist to form coalitions. Extensive literature on coalition formation of economic coalitions, such as Bloch, [1], [2], Ray and Vohra [5], [11] and Yi [13], [14], describe formation of multiple coalitions and examine the equilibrium number and size of coalitions. However, each of

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the authors adopts a different notion of the stability of a coalition structure. A difference between the rules of coalition formation lies in what can happen to the membership of a coalition once it has been formed: Can an existing coalition break apart, admit new members, or merge with other coalitions? *Different rules of coalition formation lead to different predictions about stable coalition structures.*

Despite the amount of papers on this topic, few of them showed implemented results. To derive analytical results, often players are assumed to be identical; this is called a symmetric game. Application in empirical studies where players are probably heterogeneous, is rare. Recently, the idea has been applied to get a feeling for incentives of coalition formation around the reduction of the emission of greenhouse gasses (the so-called Kyoto discussions). Finus and Eykmans [4] show a study where the world has been divided into 6 regions and the tendency to co-operate is analysed using estimated payoff models. As will be described, determination of stability of all (multiple) coalition structures implies comparing some 200 possible structures when there are 6 players. One could check 200 structures by hand. Their study is elaborated and extended by Finus e.a. [8] using new estimates of the payoff function for 12 world regions and algorithms developed in co-operation with the authors of the current paper. In [8] only cartels (one coalition is possible) are checked, leading to some 4000 possible coalitions. The real challenge is to determine all (multiple) coalition structures, which for 12 regions counts up to more than 4 million. This requires exact implementable formulations of the problem and efficient data handling.

The current paper describes this development and facilitates studying multiple coalition formation among players in the Kyoto protocol, implement it and obtain so-called stable coalition structures. Particular attention is paid to the analysis of the stability under different membership rules. The case of Finus and Eykmans [4], is used for validation purposes to check the final implementation. Results of applying the suggested approach to the case described in [8], gave new insight to economic researchers about economic incentives for coalition formation with respect to CO_2 emission reduction.

This paper is organized as follows: in section 2 we apply a new notation to introduce the concept of neighbourhood and definitions of stability in the open membership game and the exclusive membership game with unanimity and majority vote. In section 3 we discuss the implementation used. In section 4 the result of different cases is shown.

2 Multiple Coalition Game

2.1 Game

[2] The Kyoto protocol game, as introduced in [8], is a *two-level game*. The players are world regions each with a particular benefit and cost structure on the abatement of CO_2 emissions. On the *first level* they decide simultaneously on their membership in a coalition; on the *second level* coalitions choose their abatement strategies. This choice depends on the coalition structure appearing on the first level.

Let J be the set of regions and $j \in J$ represent the region index. Let q_j be the individual abatement of region j, $B_j(\cdot)$ the benefits function from global abatement ($\sum_{r=1}^{n} q_r$) and $AC_j(\cdot)$ the abatement costs function from individual abatement q_j (see [8] for details). The abatement decision on the second level is based on the following payoff function:

$$\varphi_j(\underline{q}) = B_j(\sum_{r=1}^{n} q_r) - AC_j(q_j) \tag{1}$$

The *Cartel Game* (see e.g. [8]) assumes the existence of at most one coalition. Regions of the coalition are called signatories and all other regions non participating are called singletons or non-signatories. On the first level of this game, two membership strategies are available to regions:

- strategy $\sigma_j = 0$: means region j is a non-signatory,
- strategy $\sigma_j = 1$: means region j is a signatory

In the *Multiple Coalition Game* regions on the the first level can either decide to be member of a non-trivial coalition, signatories, or can form a singleton coalition, non-signatories. In this game several coalitions may exist and regions can choose which coalition to join. The Multiple Coalition Game has a larger membership strategy set than the Cartel Game.

We introduce formally the concept of a ***coalition structure*** $c = \{\kappa_1, \ldots, \kappa_m\}$ as a collection of coalitions κ_i of one or more regions, with m the total number of coalitions in coalition structure c. In other words, a coalition structure $c = \{\kappa_1, \ldots, \kappa_m\}$ is a partition of the set of regions, J, where a particular coalition is denoted by κ_i, $i \in \{1, \ldots, m\}$, $\kappa_i \cap \kappa_j = \emptyset$ for all $i \neq j$, $\cup \kappa_i = J$.

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[2] Some general remarks about syntax conventions in this paper: An underscore under a variable depicts a vector, such as $\underline{x}$. The i^{th} element of the vector $\underline{x}$ is denoted as x_i.

A coalition with only one region j, $|\kappa_j| = 1$, is called a **singleton** and the coalition where all regions participate, $\kappa_1 = J$, is called the **Grand Coalition**. The two level game is defined as follows:

- Region $j \in J = \{1, \dots, n\}$ decides in which coalition κ_i to participate ($i \in \{1, \dots, m\}$)
- In the Emission Abatement game, now the coalitions are considered as a set of regions who decide on their common abatement, such that the aggregate payoff (we will call welfare) of a coalition (outcome) is maximized.

The input for level 2 is defined by coalition structure $c = \{\kappa_1, \dots, \kappa_m\}$. The welfare functions of level 2 for each coalition κ_i in the coalition structure c are defined by

$$wel_i^c(\underline{q}) \quad = \quad \sum_{j \in \kappa_i} \varphi_j(\underline{q})$$

where $\varphi_j(\underline{q})$ is the individual payoff for player (world region)j with abatement vector $\underline{q}$

The optimal abatement level $\underline{q}^*(c)$ depending on structure c leads to an output of level 2 being a welfare vector that can be translated into values of the payoffs of the individual world regions $j \in J$ possibly after applying transfers between the members of a coalition. Actually, the payoff vector only depends on the output c of level 1 with elements $\Pi_j(c) = \varphi_j(\underline{q}^*(c))$:

$\underline{\Pi}(c) =$ vector of payoff values following from level 2 under coalition structure c

$\Pi_j(c) = \varphi_j(\underline{q}^*(c))$ payoff of region j based on optimal abatement vector $\underline{q}^*(c)$

Because the strategy in the second stage is fixed, given a coalition structure c the entire coalition formation game reduces to one single stage.

We now introduce the so-called **"Eyckmans notation"**, see [4], for a coalition structure. The coalition structure in this notation is expressed as a string of numbers indicating the number of the coalition, a region belongs to. A "1" indicates that a region is part of coalition "1"; a "2" indicates that a region is part of the coalition "2"; and so on until the maximum number of possible coalitions "m = n". For example for six regions:

$$\begin{aligned}\underline{c} = [1,2,2,3,4,2] \Rightarrow \quad & c = \{\{1\},\{2,3,6\},\{4\},\{5\}\} \text{ with} \\ & |\kappa_1| = 1, \kappa_1 = \{1\}, \\ & |\kappa_2| = 3, \kappa_2 = \{2,3,6\} \\ & |\kappa_3| = 1, \kappa_3 = \{4\} \\ & |\kappa_4| = 1, \kappa_4 = \{5\}\end{aligned}$$

represents: region $j = 1$ called "1" is a singleton, κ_1 and $|\kappa_1| = 1$, regions $j = 2, 3, 6$ form a coalition called "2", κ_2 and $|\kappa_2| = 3$, and regions $j = 4$ and $j = 5$, called "3" and "4" respectively, κ_3, κ_4; $|\kappa_3| = 1$, $|\kappa_4| = 1$, are singletons. In this notation, the same coalition can be represented by:

$$\underline{c} = [2,1,1,3,4,1]$$

The idea is to **translate** alternative representations by ordering via an algorithmic rule: for $j = 1, \ldots, n$ if a region is not in an existing coalition, give it coalition number $i := i + 1$.

The Grand Coalition, $c = \{\kappa_1\}$, $|\kappa_1| = n$, in vector notation is $\underline{c} = [1, 1, \ldots, 1]$ and the coalition structure with only singleton members, $c = \{\kappa_1, \kappa_2, \ldots, \kappa_n\}$, $|\kappa_j| = 1$, for all j, is $\underline{c} = [1, 2, \ldots, n]$.

In this paper we use both notations; that of subsets and of strings. The string notation allows defining the so-called "neighbourhoods" that can be used in the implementation.

The total number L of all possible coalition structures is not known analytically, but can be derived numerically. A unique index function $1, \ldots, L$ to number the coalition structures is not straightforward either. However, the generated coalition structures in the (ordered) Eyckmans notation defines an ordered list of numbers consisting of n digits. This gives the possibility to find a particular structure from a list by bisection in (average) $\log L$ steps. The number of multiple coalition structures generated by an algorithm is:

n	:	6	7	8	9	10	12
L	:	203	877	4140	21147	115975	4213597

One can determine all optimal payoff values $\Pi_j(\underline{c}_l)$ for all possible coalition structures c_l, $l = 1, \ldots, L$. As a result, we obtain a large $L \times n$ **payoff matrix Π**, of optimal payoff values for every individual region j when coalition structure l applies. The matrix can be used to study decisions on level 1.

We introduce some new concepts that are useful in the study of stability of coalition structures. In the Cartel Game (single coalition game) a region j has only one alternative action, either leave or join the coalition. In the multiple coalition game, inside a coalition structure $c_l = \{\kappa_1, \dots, \kappa_{m_l}\}$, each region j usually has more than one alternative:

Let region j be a member of a *non-singleton coalition* κ_i, $i \in \{1, \dots, m_l\}$ in the coalition structure c_l. Region j has two possible alternatives: one alternative is to leave coalition κ_i to become a singleton forming another coalition, κ_{m_l+1}, $|\kappa_{m_l+1}| = 1$; the other alternative is to leave coalition κ_i to join another coalition κ_t, $t \in \{1, \dots, m_l\} \backslash \{i\}$.

Now, suppose j forms a singleton coalition by itself κ_i, $|\kappa_i| = 1$. Region j has "only" one alternative: to join another coalition κ_t, $t \in \{1, \dots, m_l\} \backslash \{i\}$.

The number of possible alternatives depends on the number of different coalitions in the coalition structure. If there is more than one coalition κ_t, $t \in \{1, \dots, m_l\}$ in the coalition structure c_l, then a region can join any other coalition except its current coalition, i.e. $m_l - 1$ coalitions (m_l possibilities if it is a singleton).

The players of the game on level 1 are the world regions $j = 1, \dots, n$. Given a coalition structure c_l they can decide (strategy) on leaving a coalition and entering another. Here the stability definitions of the following section play a role. It is important to note that the deviation of one region while the others do not change strategy, leads to a so-called **neighbour** coalition structure.

For the check of stability we need all the neighbours of c_l. The exact number of neighbours is not easy to identify in the multiple coalition game, in contrast to the cartel formation game. Let c_l be a coalition structure, $k(j)$ be the number of the coalition which j belongs to and let $\kappa_{k(j)}$ be that coalition. The neighbour coalition structures are represented by $c_l^{neig(j,t)}, j \in J$, $t \in \nu_{lj}$, with ν_{lj} the possible deviation index set of region j in structure c_l defined as follows:

If coalition $\kappa_{k(j)}$ is formed by a singleton region j, $|\kappa_{k(j)}| = 1$, the neighbours generated by region j are defined by the possible alternatives in the strategy set:

$$\nu_{lj} = \{1, 2, \ldots, m_l\} \backslash \{k(j)\} \qquad \text{and} \qquad |\nu_{lj}| = m_l - 1 \tag{2a}$$

If j is non-singleton, $|\kappa_{k(j)}| > 1$, the possible alternatives in the strategy set are given by:

$$\nu_{lj} = \{1, 2, \ldots, m_l + 1\} \backslash \{k(j)\} \quad \text{and} \quad |\nu_{lj}| = m_l \tag{2b}$$

as j can also choose to proceed alone.

Note that each neighbour of the Grand Coalition corresponds to a Cartel Coalition Structure: a coalition formed by $n - 1$ regions and only one non-signatory region, a singleton.

Example:

Let $\underline{c}_l = [1, 1, 2, 2, 3, 4, 3, 3, 2, 1, 1, 2]$ a coalition structure in the Eyckmans notation.

For non-singleton region 1 the possible deviation index set is:

$$k(1) = 1,\ \nu_{l1} = \{2, 3, 4, 5\}$$

and corresponding neighbour coalition structures are:

$$\begin{aligned}
\underline{c}^{neig(1,2)} &= [\mathbf{2}, 1, 2, 2, 3, 4, 3, 3, 2, 1, 1, 2] &=\quad& [\mathbf{1}, 2, 1, 1, 3, 4, 3, 3, 1, 2, 2, 1] \\
\underline{c}^{neig(1,3)} &= [\mathbf{3}, 1, 2, 2, 3, 4, 3, 3, 2, 1, 1, 2] &=\quad& [\mathbf{1}, 2, 3, 3, 1, 4, 1, 1, 3, 2, 2, 3] \\
\underline{c}^{neig(1,4)} &= [\mathbf{4}, 1, 2, 2, 3, 4, 3, 3, 2, 1, 1, 2] &=\quad& [\mathbf{1}, 2, 3, 3, 4, 1, 4, 4, 3, 2, 2, 3] \\
\underline{c}^{neig(1,5)} &= [\mathbf{5}, 1, 2, 2, 3, 4, 3, 3, 2, 1, 1, 2] &\underset{\uparrow}{=}\quad& [\mathbf{1}, 2, 3, 3, 4, 5, 4, 4, 3, 2, 2, 3]
\end{aligned}$$

which after

translating

becomes

For singleton region 6 the possible deviation index set is:

$$k(6) = 4,\ \nu_{l6} = \{1, 2, 3\}$$

and corresponding neighbour coalition structures are:

$$\underline{c}^{neig(6,1)} = [1, 1, 2, 2, 3, \mathbf{1}, 3, 3, 2, 1, 1, 2]$$
$$\underline{c}^{neig(6,2)} = [1, 1, 2, 2, 3, \mathbf{2}, 3, 3, 2, 1, 1, 2]$$
$$\underline{c}^{neig(6,3)} = [1, 1, 2, 2, 3, \mathbf{3}, 3, 3, 2, 1, 1, 2]$$

which do not require translation.

The total neighbourhood of a structure is defined by all the alternatives of all world regions. Due to overlap of (translated) neighbours, we only have an upper bound of the total number of neighbours of $\underline{c}_l$; it is not bigger than

$$\sum_j |\nu_{lj}|$$

From now on we leave out the sub-index l and we only write c to denote a coalition structure, m for the total number of coalitions in c and ν_j for the deviation index set of a region j.

2.3 Stability definition

In a single coalition game (Cartel Game), a distinction can be made between *internally* and *externally* stable (see [8], [10]). *Internally stable* means that there is no region *in* the coalition with the incentive to leave to become a singleton. *Externally stable* means, that there is no singleton region with the incentive to enter the coalition. In the Multiple Coalition Game we have more than one coalition and we have to be more careful defining stability.

We use the concept of *Inter-Coalition stability* (see [3]). This concept means for structure c^* that there is no region belonging to a coalition $\kappa_k \in c^*$ which would be better off by leaving the coalition to join another non-singleton coalition $\kappa_t \in c^*$.

Formally, the definition of stability is:

Definition 1 Stability in a Multiple Coalition Game:

- ***Internal Stability:*** *no cooperating region j would be better off by leaving coalition $\kappa_k \in c^*$ to form a singleton κ_{m+1};*
- ***External Stability:*** *no singleton region j would be better off by joining any coalition $\kappa_t \in c^*$, $t \in \nu_j$;*
- ***Inter-coalition Stability:*** *no region belonging to $\kappa_k \in c^*$, $|\kappa_k| > 1$, would be better off by leaving κ_k to join another coalition $\kappa_t \in c^*$.*

Note the following:

- A coalition structure without singletons is *externally stable*.
- In the definition of external stability we include the possibility that a singleton can join another singleton, that is, coalition $\kappa_t \in c^*$, $t \in \nu_j$ in the definition above can be a coalition with only one region.
- In the definition of inter-coalition stability we assume that coalition $\kappa_k \in c^*$ has more than one region, that is, it is not a singleton.

Before proceeding to the stability check we summarise the notation used:

- c: **coalition structure** in set notation; $\underline{c}$ in vector (Eyckmans) notation;
- $J = \{1, \ldots, n\}$: **set of world regions**;
- $k(j)$: **current coalition number of world region** j;
- m: **number of coalitions in coalition structure** c.
- κ_i: **coalition** inside coalition structure c with one or more members, $i \in \{1, \ldots, m\}$, $m \leq n$;
- ν_j: **set of alternative strategies of world region** j in structure c;

Consider a situation with the following existing coalition.

$$\underline{c}^* = [1, 1, \underset{\substack{\uparrow \\ \text{region } 3}}{1}, 1, 2, 3, 2, 4, 1, 2, 3, 5]$$

A region, for instance region three, has six possible strategies and deviation set: $v_3 = \{2, 3, 4, 5, 6\}$. The first possible strategy is to stay in its current coalition 1; the strategies two to five are to leave the current coalition and join coalition number i, $i \in \{2, 3, 4, 5\}$ which is a subset of v_3; the last possible strategy, number six, is to leave the current coalition and form a coalition number 6 to become a singleton; depending on what is the most favorable option. In the situation that none of the 12 regions in $\underline{c}^*$ has the incentive to change the current situation, the coalition structure is considered to be *in* Nash equilibrium. This implies the following criterion for determining the stability of a coalition:

if the payoff for each region in the current situation is higher than the payoff in all of the neighbour alternatives, then coalition structure $\underline{c}^*$ *is stable.*

This means, for instance, that the coalition structure

$$\underline{c}^* = [1, 1, 1, 2, 3, 2, 4, 1, 2, 3, 5]$$

has to be compared with the following *neighbours* of $\underline{c}^*$ with respect to region $j = 3$

- $\underline{c}^{neig(3,2)} = [1, 1, 2, 2, 3, 2, 4, 1, 2, 3, 5]$
  region 3 enters a coalition called 2

- $\underline{c}^{neig(3,3)} = [1, 1, 3, 2, 3, 2, 4, 1, 2, 3, 5]$
  region 3 enters a coalition called 3

- $\underline{c}^{neig(3,4)} = [1, 1, 4, 2, 3, 2, 4, 1, 2, 3, 5]$
  region 3 enters a coalition called 4

- $\underline{c}^{neig(3,5)} = [1, 1, 5, 2, 3, 2, 4, 1, 2, 3, 5]$
  region 3 enters a coalition called 5(=m)

- $\underline{c}^{neig(3,6)} = [1, 1, 6, 2, 3, 2, 4, 1, 2, 3, 5]$
  region 3 forms a singleton coalition called 6(=m+1)

The following can happen:

- If option $\underline{c}^{neig(3,t)}$, for all $t \in \upsilon_3$, does not result into a higher payoff for region 3, then region 3 will not enter coalition t

There is no incentive to change anything as far as region 3 is concerned. The same applies to other regions, where the region has possibly a different number of alternatives. If for each other region the departure from the coalition will not result in a higher payoff, then it will not leave the coalition. Therefore: if in all of these cases the alternative of the region does not give an expected improvement, no action will be taken and thus coalition structure $\underline{c}^*$ is *stable*.

2.3.1 Open membership stability:

The term *Open Membership* is used to indicate that for the current regions of coalition κ_t, any other region is allowed to enter it.

To check the stability of $c^* = \{\kappa_1, \ldots, \kappa_m\}$ let $t \in \nu_j$ be an index indicating the possible coalitions that j can join and $c^{neig(j,t)}$, the possible **neighbour coalition structure** arising from coalition structure c^* as a result of region j changing its status and forming coalition t (singleton) or joining another coalition t.

Consider region j changing its strategy from $k(j)$, $(\kappa_{k(j)})$, to $t \in \nu_j$, (κ_t). This results into neighbour coalition structure:

(1) If j is a singleton

$$c^{neig(j,t)} = \{\kappa_1, \kappa_2, \ldots, \kappa_t \cup \{j\}, \ldots, \kappa_{m-1}\}$$

(2) If j is a non-singleton

- either join other coalition κ_t

$$c^{neig(j,t)} = \{\kappa_1, \kappa_2, \ldots, \kappa_t \cup \{j\}, \ldots, \kappa_{k(j)} \backslash \{j\}, \ldots, \kappa_m\}$$

- or form a singleton κ_{m+1}

$$c^{neig(j,t)} = \{\kappa_1, \kappa_2, \ldots, \kappa_t, \ldots, \kappa_{k(j)} \backslash \{j\}, \ldots, \kappa_{m+1}\}$$

with

$$\kappa_{m+1} = \{j\}$$

Therefore, a coalition structure (c^*) is defined:

internally stable in the open membership game if:

$$\Pi_j(\underline{c}^*) \geq \Pi_j(\underline{c}^{neig(j,m+1)}) \quad \text{for all } j \text{ with } |\kappa_{k(j)}| > 1 \tag{3}$$

externally stable in the open membership game if:

$$\Pi_j(\underline{c}^*) \geq \Pi_j(\underline{c}^{neig(j,t)}) \quad \text{for } t \in \nu_j \text{ and } j \text{ with } |\kappa_{k(j)}| = 1 \tag{4}$$

inter-coalitionally stable in the open membership game if:

$$\Pi_j(\underline{c}^*) \geq \Pi_j(\underline{c}^{neig(j,t)}) \quad \text{for } j \text{ with } |\kappa_{k(j)}| > 1, \; t \in \nu_j \backslash \{m+1\} \text{ with } |\kappa_t| \geq 1 \tag{5}$$

2.3.2 *Exclusive membership stability:*

The exclusive membership in the single coalition game (Cartel Game) implies there is an additional condition in external stability: a non-coalition region is only allowed to join the existing coalition if the payoff for the existing coalition members will not decrease. Specifically one can consider two types of voting rules:

(1) *Majority Voting*: if a region has an incentive to join coalition κ_t we have to check if the current members are better or worse off. If more than 50% are in favor of accession, then the candidate is accepted and the original coalition structure is not stable. We assume that if 1 is in favor and 1 against then accession is not accepted.
(2) *Unanimity Voting*: only if all coalition members are in favor of accession, then the candidate is allowed to enter. That is, if there is one region against (veto), then accession is not possible.

In the multiple coalition game this rule differs depending on if we are testing the external stability or the inter-coalition stability. Consider region j wants to change its strategy from $k(j)$ to $t \in \nu_j$

- *Internal Stability*: in the exclusive membership game is still defined by 3.
- *External Stability*: Region j forms a singleton coalition by itself $\kappa_{k(j)} = 1$. If there is any region p in κ_t (unanimity) such that its payoff decreases, then region j is not allowed to join coalition κ_t.
- *Inter-coalition Stability*: Region j is a region of a non-trivial coalition, $|\kappa_{k(j)}| > 1$. If there is any region p in κ_t (unanimity) such that its payoff decreases, then region j is not allowed to join coalition κ_t.

The concept of majority voting requires introduction of a new binary variable yes/no symbol, δ_p, that tell us whether the current region in a neighbour coalition structure is against or in favor of another region entering:

$$\text{For } p \in \kappa_t : \begin{cases} \delta_p = 1 & \text{, if} \quad \Pi_p(\underline{c}^*) > \Pi_p(\underline{c}^{neig(j,t)}) \quad \text{(against)} \\ \delta_p = 0 & \quad\quad \text{otherwise.} \end{cases} \tag{6}$$

and the majority is against when:

$$\textstyle\sum_{p\in\kappa_t} \delta_p \geq |\kappa_t|/2$$

More specifically, a coalition Structure c^* is defined externally or inter-coalitionally stable in the exclusive membership game if

	Externally Stable if for all j with $\lvert\kappa_{k(j)}\rvert = 1$, $t \in \nu_j$	Inter-coalitionally Stable if for all j with $\lvert\kappa_{k(j)}\rvert > 1$, $t \in \nu_j$
	$\Pi_j(\underline{c}^*) \geq \Pi_j(\underline{c}^{neig(j,t)})$	
Majority Voting Rule	OR $\Pi_j(\underline{c}^*) < \Pi_j(\underline{c}^{neig(j,t)})$ and $\sum_{p \in \kappa_t} \delta_p \geq \lvert\kappa_t\rvert/2$	
Unanimity Voting Rule	OR $\Pi_j(\underline{c}^*) < \Pi_j(\underline{c}^{neig(j,t)})$ and there exists p in κ_t such that $\Pi_p(\underline{c}^*) > \Pi_p(\underline{c}^{neig(j,t)})$	

The challenge is now to create a procedure to find for the first level game, where coalition formation takes place, so-called Nash equilibria i.e. stable coalition structures. Consequently, on the level of the abatement decisions, for each structure the equilibrium of that level has to be determined. Such a procedure is outlined in the next section.

3 Implementation of the algorithm and non stability indicators

We have implemented an algorithm for finding equilibria of the multiple coalition game in MATLAB. As a consequence of the large number of coalition structures we must consider a big matrix with all necessary data to test stable coalition structures. For each coalition structure we determine:

- index for coalition structure.
- coalition structure.
- payoff vector for each region of coalition structure.
- three indicators 0-1 for internal, inter-coalition and external stability of the coalition structure (0 = non-stable; 1 = stable).
- sum of differences between payoffs for all regions in coalition structure in case of non-stability (internal, inter-coalition and external).

To test stability, all possible neighbours of current coalition structure, $\underline{c}^*$, are considered and the payoff for each region for each neighbourhood $\underline{c}^{neig(j,t)}$ (one at a time) is looked up. Subsequently, we compare the differences between the payoff for region j, $j \in J$, in the current coalition structure against its payoff in its neighboring coalition structure, i.e.:

$$\Delta_{j,t} = \Pi_j(\underline{c}^*) - \Pi_j(\underline{c}^{neig(j,t)}) \tag{7}$$

We sum this quantity in cases of non-stability (negative) for all regions in each coalition structure for each indicator of stability: internal, inter-coalition and external, showing how far this coalition structure is from becoming internally, inter-coalitionally or externally stable. These are called ***Non-Stability Indicators*** and are defined as follows:

$$\textit{Non-stability indicators}\colon\ \Delta^p_{c^*} = -\sum_j \sum_{t\in\nu_j} \left[\Pi_j(\underline{c}^*) - \Pi_j(\underline{c}^{neig(j,t)})\right]^-$$

where $x^- = \max(0, -x)$ only counts negative deviations. Index $p = 1, 2, 3$ denotes internal (1), inter-coalition (2) and external (3) stability deviation for each coalition structure.

Furthermore, by adding the three indicators one gets a picture of the total deviation from stability of the coalition structures. This "global indicator" is:

$$\Delta_l = \sum_{p=1}^{3} \Delta^p_{c_l} \tag{8}$$

It gives the total amount of means necessary to make a coalition structure stable by compensating the regions for not changing their strategy. Finding stable structures means to maximise equation (8) over the coalition structures since the optimal value will be 0 and corresponds to a Nash equilibrium, i.e.

$$\max_l \quad \Delta_l$$

The implementation of the algorithm contains the following ingredients:

STEP 1 *Settings*: Region specific parameter values, used in calculation of the payoffs. In the case of Finus et al., [8], such parameters concern data about benefits, costs, abatement, payoff, emissions and concentration:

- Share of global benefits vector
- Abatement cost parameters
- Reference year CO_2 emission data in Gtons carbon in 2010 (see [8])

STEP 2 *Generate*: generate all possible coalition structures in vector notation and save in matrix structure. In addition we save a special index indicating the Eyckmans code.

STEP 3 *Abatement*: calculate the optimal payoff for all possible coalition structures. The strategies are continuous decisions, $\underline{q}$ (see [8] for details), on abatement, reduction of emissions from a maximum level e_j^{max}, $0 \leq q_j \leq e_j^{max}$. The payoff for an individual region j in a coalition structure c is:

$$\varphi_j(\underline{q}) = B_j(\sum_{r=1}^{n} q_r) - AC_j(q_j) \tag{9}$$

and depends on:

- $B_j(\sum_r^n q_r)$: benefits from global abatement. Benefits from global abatement are derived from reduced environmental damages caused by greenhouse gas emissions. $B_j(\cdot)$ is increasing ($dB_j/dq > 0$) and either strictly concave ($d^2B_j/(dq)^2 < 0$) or linear ($d^2B_j/(dq)^2 = 0$) function in the sum of all abatement.
- $AC_j(q_j)$: abatement cost from abatement in region j, which is increasing ($dAC_j/dq_j > 0$) in the abatement and (strictly) convex ($d^2AC_j/(dq_j)^2 > 0$).

The payoff φ_j of region j and thus the aggregate welfare wel_i^c of coalition i in coalition structure c, not only depends on its own strategy but also on those in other world regions. The game may now be seen as a *positive externality game*: payoff in region j increases with abatement in region p. After the players of the game have been identified one can try to find the optimal abatement levels from the game-theoretic concept that every coalition is maximising its payoff,

$$\max_{q_j} wel_i(\underline{q}), \qquad j \in \kappa_i, \qquad i = 1, \ldots, m$$

where $wel_i(\underline{q}) = \sum_{j \in \kappa_i} \varphi_j(\underline{q})$. The optimum is found by solving the set of equalities determined by the first order conditions:

$$\frac{\partial wel_i(\underline{q})}{\partial q_j} = 0 \qquad j \in \kappa_i \quad ,$$

if the optimum is interior with respect to $0 \leq q_j \leq e_j^{max}$ for $i = 1, \ldots m$. Regions belonging to the same coalition maximize the aggregate welfare of their coalition. The equilibrium abatement strategy vector $\underline{q}^*$ for coalition structure $\underline{c}$ is derived as a Nash equilibrium between coalitions. Here we want to specify that in our cases we take as the region payoff simply the payoff that an individual region gets when carrying out the common optimal abatement strategy $\underline{q}^*$. We understand that it is more realistic to assume transfers within coalitions, but our focus is mainly on the implementation and we keep the model as simple as possible. The outcome of this step consists of an abatement vector $\underline{q}$ and a payoff matrix Π which contains all individual payoffs Π_j for region$j$ in all possible payoff vectors for the coalition structures, $\underline{c}_l$:

$$\Pi = \begin{bmatrix} \underline{\Pi}_1 \\ \underline{\Pi}_2 \\ \dots \\ \underline{\Pi}_L \end{bmatrix} = \begin{bmatrix} \Pi_1(c_1) & \dots & \Pi_j(c_1) & \dots & \Pi_n(c_1) \\ \Pi_1(c_2) & \dots & \Pi_j(c_2) & \dots & \Pi_n(c_2) \\ \dots & & & & \\ \Pi_1(c_L) & \dots & \Pi_j(c_L) & \dots & \Pi_n(c_L) \end{bmatrix}$$

STEP 4 *Stability*: main procedure. Now we proceed to look for those coalitions which are stable, that is, internal, inter-coalition and externally stable. The new notation introduced in the last sections helps us in the implementation of this procedure. We use the same formulas that have been introduced to determine the stability of each coalition structure.

4 Cases

In this section results for the world regions of the Kyoto protocol are shown. We apply majority and unanimity games. We elaborate a 6 region case reported by Finus and Eyckmans in [4]. For the illustration we show a 12 region case based on Olieman [10] which requires the determination of more than 4 millions coalition structures. We get stable coalition structures in both games. In addition, if there is no stable coalition structure we evaluate the non-stability indicators and we also report the least non-stable coalition structure, i.e., those that are most near to stability (internal, external and inter-coalition). If there are stable coalition structures, all have a non-stability indicator value of zero.

The source of the data is due to Finus and Altamirano-Cabrera, [7], and Dellink et.a. [6]. They set up an empirical model, the so-called *STA*bility of *CO*alition model, STACO. For emissions in 2010 they choose the value of the DICE model (see [9]), which amounts to 11.96 gigatons CO_2. Olieman in [10] shows a 12 region case and looks for a stable cartel. The data of that study are used to derive a 6, 8 and 10 region case and the results are reported in [12]. The regions distinguished in these studies are: USA (USA), Japan (JPN), European Union (EEC), other OECD countries (OOE), Eastern European countries (EET), former Soviet Union (FSU), energy exporting countries (EEX), China (CHN), India (IND), dynamic Asian economies (DAE), Brazil (BRA) and "rest of the world" (ROW) (see [8]).

4.1 *Parameters of the payoff function*

With respect to the parameter values of the payoff function, the analysis of Finus and Altamirano-Cabrera, [7] is used. It starts in 2010 and covers a time period of 100 years in order to capture the long-run effects of the global warming problem. Benefits are expressed in the form of discounted reduced damages due to accumulated abatement over the entire period, $q = \sum_{t=2011}^{2110} q_t$. They come to $B(q) = 37.40q$ where allocation of global benefits from reduced environmental damages to the various world regions is based on the assumption:

$$B_j(q) = s_j B(q) \tag{10}$$

where s_j is the share of region j.

For the *Abatement Cost Function*, they assume an annual abatement cost function of the shape:

$$AC_{jt}(q_{jt}) = \frac{1}{3}\alpha_j(q_{jt})^3 + \frac{1}{2}\beta_j(q_{jt})^2$$

where simply q_{jt} is taken as $q_j/100$ assuming stationary strategies ($q_{j,2011} = \ldots = q_{j,2110}$). In the model, abatement means emission reduction with respect to (business-as-usual-scenario)-emissions. A total initial emission of 11.96 gigatons is allocated to the 12 regions. The total abatement cost for region j is the discounted sum over $t = 2011, \ldots, 2110$ leading to

$$AC_j(q_j) = 43.1 AC_{jt}(q_{jt}) \tag{11}$$

Table 1 shows parameter values about:

- Share of total of Emissions in 2010 in Gigatons, $E_{j,2010}$
- Share of Global Benefits, s_j. The sum is equal to 1.
- Abatement cost parameter α_j
- Abatement cost parameter β_j

that are used in the 12 region case.

Table 1
Parameters of STACO model (12 region) AC_j and B_j (equations 10, 11)

Region (j)	$E_{j,2010}$ *(Gigatons)*	s_j	α_j	β_j
1 USA	2.416	0.2263	0.0005	0.00398
2 JPN	0.557	0.1725	0.0155	0.18160
3 EEC	1.399	0.236	0.0024	0.01503
4 OOE	0.621	0.0345	0.0083	0
5 EET	0.519	0.013	0.0079	0.00486
6 FSU	1.003	0.0675	0.0023	0.00042
7 EEX	1.219	0.030	0.0032	0.03029
8 CHN	2.356	0.062	0.00007	0.00239
9 IND	0.639	0.050	0.0015	0.00787
10 DAE	0.405	0.0249	0.0047	0.03774
11 BRA	0.128	0.0153	0.5612	0.84974
12 ROW	0.698	0.068	0.0021	0.00805
WORLD	$\sum$=11.96	$\sum s_j$=1		

4.2 12 region case

A Fortran implementation of the algorithm generates the 4.213.597 possible coalition structures and performs the stability checks as outlined on section 2.3. Table 2 depicts the number of stable structures classified towards the different definitions of stability. Only in the exclusive membership game where members can apply a veto for other regions not to enter, stable structures appear.

Table 2
Results 12 region case. STACO Model

	OPENM	EXMMAJ	EXMUNAN
Stable	0	0	8
Internally Stable	98	98	98
Intercoalitionally Stable	7	1.834.950	3.922.082
Externally Stable	988.476	1.619.763	2.681.807

The stable coalition structures and their corresponding monetary values are listed in table 3 and are interpreted further by economists. The results on payoff, called welfare here, can be used to analyse the economic incentive for coalitions to appear. For instance one can observe that USA and Japan are not a member of any stable coalition and for Brazil the incentive to co-operate is bigger. In reality there may be many other reasons for coalitions to be formed. The model only generates economic sound possibilities for coalition formation.

Table 3
Stable Coalition Structures 12 region case. STACO Model. Unanimity Game

Coalitions	***Coalitions***				
Non-singleton Coalition	{OOE; IND; BRA}	{FSU; ROW}	{EEX; CHN}		
Welfare	86 + 122 + 40	164 + 165	75 + 130		
Singleton Coalitions	USA	JPN	EEC	EET	DAE
Welfare	536	447	590	34	65
World	2.452				
Non-singleton Coalition	{FSU; BRA; ROW}	{OOE; IND}	{EEX; CHN}		
Welfare	162 + 39 + 163	87 + 124	75 + 130		
Singleton Coalitions	USA	JPN	EEC	EET	DAE
Welfare	536	447	590	34	65
World	2.451				
Non-singleton Coalition	{OOE; IND; BRA}	{FSU; ROW}	{CHN; DAE}		
Welfare	84 + 120 + 39	161 + 163	131 + 61		
Singleton Coalitions	USA	JPN	EEC	EET	EEX
Welfare	527	440	581	33	76
World	2.415				
Non-singleton Coalition	{FSU; BRA; ROW}	{OOE; IND}	{CHN; DAE}		
Welfare	159 + 39 + 160	85 + 122	131 + 61		
Singleton Coalitions	USA	JPN	EEC	EET	EEX
Welfare	527	440	581	33	76
World	2.414				
Non-singleton Coalition	{OOE; IND; BRA}	{FSU; ROW}	{EEX; DAE}		
Welfare	79 + 112 + 37	150 + 151	71 + 59		
Singleton Coalitions	USA	JPN	EEC	EET	CHN
Welfare	489	411	542	31	133
World	2.263				
Non-singleton Coalition	{FSU; BRA; ROW}	{EEX; DAE}	{OOE; IND}		
Welfare	148 + 36 + 149	71 + 59	80 + 114		
Singleton Coalitions	USA	JPN	EEC	EET	CHN
Welfare	489	411	542	31	133
World	2.262				
Non-singleton Coalition	{FSU; BRA; ROW}	{OOE; DAE}	{EEX; IND}		
Welfare	148 + 36 + 149	81 + 58	69 + 114		
Singleton Coalitions	USA	JPN	EEC	EET	CHN
Welfare	489	411	541	31	133
World	2.261				
Non-singleton Coalition	{FSU; BRA; ROW}	{OOE; EEX}	{IND; DAE}		
Welfare	147 + 36 +149	80 + 70	114 + 58		
Singleton Coalitions	USA	JPN	EEC	EET	CHN
Welfare	488	410	540	31	132
World	2.255				

4.3 Finus-Eyckmans data

Michael Finus and Johan Eyckmans present in [4] results for a six region case and obtain stable coalition structures in the exclusive membership game. They obtain results for a case where coalition members share the additional benefits, due to so called transfers and a case where the transfers are absent as in the 12 region case. With their payoff data set and region set (USA, JPN, EU (EEC), CHN, FSU, ROW) and our implementation we get the same *stable coalition structures* in the open membership game and the exclusive membership game with and without transfers. The tables A.1, A.2 and A.3, presented in appendix A, show the stable coalitions structures with their data for unanimity and majority game.

4.3.1 Case where payoff is determined without transfers

The number of stable, internally, inter-coalitionally and externally stable coalition structures in the **open, exclusive majority and exclusive unanimity** membership game are given in table 4

Table 4
Results 6 region case Finus-Eyckmans

	OPENM	EXMMAJ	EXMUNAN
Stable	0	4	7
Internally Stable	11	11	11
Intercoalitionally Stable	2	120	152
Externally Stable	54	102	150

In the open membership game only the Grand Coalition and the coalition structure with all members singletons are Inter-Coalitionally stable.

Table 5. Least Non-Stable Coalition Structure, No Transfers, Open Membership

Number	***Coalition Structure***	$\mathbf{\Delta_l = \sum_{i=1}^{3} \Delta^i_{c_l}}$
29	{USA; JPN; ROW} {EU; CHN; FSU}	-98.26

In the following figures non-stability indicators are illustrated. The y axis of the figure represents the value of the indicator and the x axis the number of the coalition structure.

Figure 1 shows a case where Finus and Eykmans [4] do not apply transfers in an open membership game. The least non-stable coalition structure, number 29 in table 5, is the coalition structure that requires least total compensation for individual regions (in this case) not to leave one of the coalitions. Figure 2 shows results for the same case where a majority voting rule is applied. Stable coalition structures appear. In the figure they are marked as structures 49, 51, 198 and 199 (see appendix A). The stability indicator of these coalition structures fall exactly in the zero x-axis.

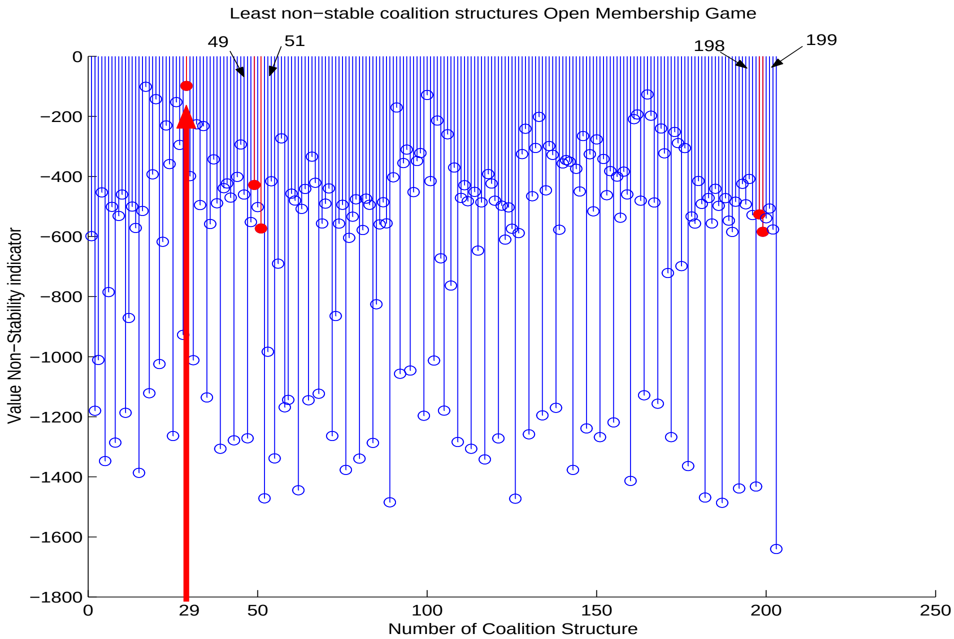

Figure 1. Non-stability indicators. Case No Transfers: Open Membership Game

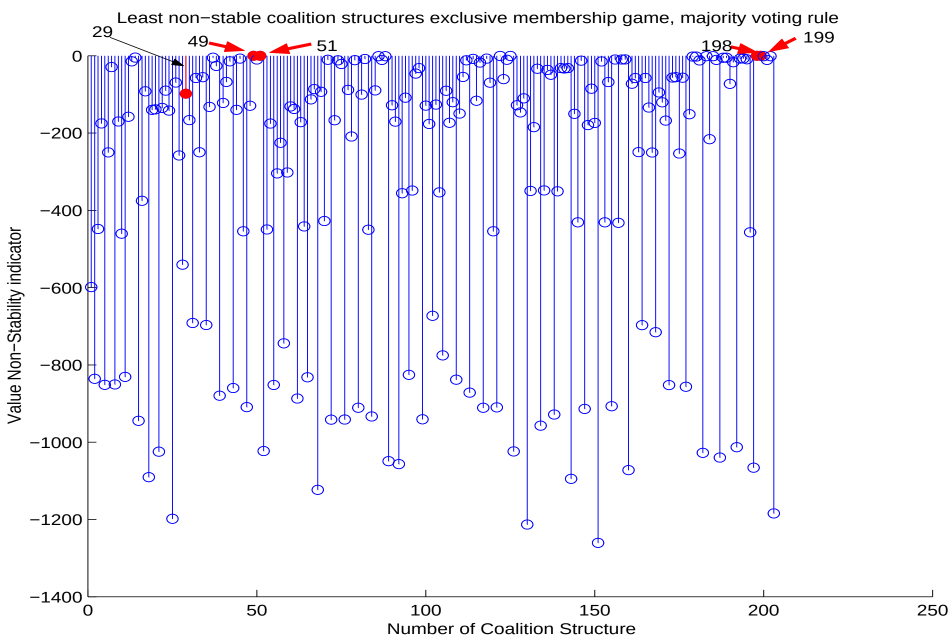

Figure 2. Non-stability indicators. Case No Transfers: Majority Game

4.3.2 Case where payoff is determined with transfers

Finus and Eyckmans elaborate another case where the payoff of the individual regions within a coalition is determined by sharing the benefits of the coalition in a certain way, according to a so-called transfer rule. We use the data of the resulting payoff matrix for determining stable coalition structures. The number of stable, internally, inter-coalitionally and externally stable coalition structures in the **open, exclusive majority and exclusive unanimity** membership game are given in table 6. One can observe that use of transfers may lead to more stable coalition structures.

Table 6
Results 6 region case Finus-Eyckmans with Transfers

	OPENM	EXMMAJ	EXMUNAN
Stable	0	5	5
Internally Stable	35	35	35
Intercoalitionally Stable	4	90	90
Externally Stable	68	91	91

The least non-stable coalition structure in the Open Membership Game is:

Table 7. Least Non-Stable Coalition Structure, Transfers, Open Membership

Number	***Coalition Structure***	$\mathbf{\Delta_l = \sum_{i=1}^{3} \Delta_{c_l}^{i}}$
133	{USA; CHN} {JPN; FSU} {EU; ROW}	-29.07

A table with the stable structures can be found in the appendix. For the description and interpretation of the stable structures see [4].

4.4 Implementation Aspects

All results are obtained using a simple processor Pentium IV. The implementation procedures are written in Matlab. The main program, *run.m*, call sequentially to the other subprograms, representing each of one part of the algorithm. The subprograms are:

- step 1 of the algorithm: *settings*
- step 2 of the algorithm: *generate*
- step 3 of the algorithm: *abatement*
- step 4 of the algorithm: *stability*

The original Matlab code was translated to Fortran code with the aim to speed-up calculation and improve memory use. The total CPU times obtained with the Fortran code are displayed in table 8:

Table 8
CPU Times of data processing

Cases StaCo Model [7]		**6 regions**	**8 regions**	**10 regions**	**12 regions**
Total Number of Coalition Structures		203	4.140	115.975	4.213.597
CPU Times Per Case (Seconds)					
Exclusive Memb. Game	Majority Voting	0,016	0,500	29,312	1.987,078
	Unanimity Voting	0,016	0,484	28,516	1.953,750
Open Membership Game		0,001	0,453	27,594	1.894,641

The table shows the development of the total number of coalition structures and the time to process them when the number of regions increases from 6 to 12.

5 Conclusions

The concept of the multiple coalition game as outlined in among others [3], [4], [5] and [11], has been defined in an exact way. The new vector notation and neighbourhood notation allows implementation in an algorithmic context.

The results of our implementation coincide what has been found in an earlier study (Finus & Eyckmans [[4]]) with 6 regions. Our implementation provides the feasibility to study a multiple coalition game with 12 regions within 1.950 seconds (approx. 1/2 hour).

The research in this paper shows how by mathematically redefining concepts, computer coding has been facilitated that made it possible to generate relevant results for such a huge case. To our knowledge, empirical results for such large cases were not reported before in the theory of coalition formation.

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A Appendix 1: Finus - Eyckmans stable coalition structures

Table A.1
Majority Game: No Transfers

Non-singleton Coalition	{USA,JPN}	{EU,ROW}		
Welfare	78639.741 + 43048.671	103131.988 + 81182.549		
Singleton Coalitions	CHN	FSU		
Welfare	9175.922	23910.032		
World	339088,903			
Non-singleton Coalition	{USA, ROW}			
Welfare	78600.713 + 81219.556			
Singleton Coalitions	JPN	EU	CHN	FSU
Welfare	43034.755	103094.343	9172.367	23896.640
World	339018,374			
Non-singleton Coalition	{USA,JPN}	{FSU,ROW}		
Welfare	78563.437 + 43012.936	23869.687 + 81261.914		
Singleton Coalitions	EU	CHN		
Welfare	103032.233	9167.649		
World	338907,856			
Non-singleton Coalition	{JPN,ROW}			
Welfare	43009.414 + 81245.427			
Singleton Coalitions	USA	EU	CHN	FSU
Welfare	78560.760	103024.030	9166.958	23875.586
World	338882,175			

Table A.2
Unanimity Game: No Transfers

Non-singleton Coalition	{USA,FSU,ROW}			
Welfare	78643,359 + 23898,355 + 81200,181			
Singleton Coalitions	JPN	EU	CHN	
Welfare	43056,722	103158,638	9177,722	
World	339134,977			
Non-singleton Coalition	{USA,JPN,ROW}			
Welfare	78635,362 + 43049,961 + 81180,319			
Singleton Coalitions	EU	CHN	FSU	
Welfare	103149,841	9177,001	23912,676	
World	339105,16			
Non-singleton Coalition	{USA,JPN}	{EU,ROW}		
Welfare	78639,741 + 43048,671	103131,988 + 81182,549		
Singleton Coalitions	CHN	FSU		
Welfare	9175,922	23910,032		
World	339088,903			
Non-singleton Coalition	{JPN,FSU,ROW}			
Welfare	43031,200 + 23882,409 + 81235,884			
Singleton Coalitions	USA	EU	CHN	
Welfare	78607,735	103090,042	9172,303	
World	339019,573			
Non-singleton Coalition	{USA,ROW}			
Welfare	78600,713 + 81219,556			
Singleton Coalitions	JPN	EU	CHN	FSU
Welfare	43034,755	103094,343	9172,367	23896,640
World	339018,374			
Non-singleton Coalition	{USA,JPN}	{FSU,ROW}		
Welfare	78563,437 + 43012,936	23869,687 + 81261,914		
Singleton Coalitions	EU	CHN		
Welfare	103032,233	9167,649		
World	338907,856			
Non-singleton Coalition	{JPN,ROW}			
Welfare	43009,414 + 81245,427			
Singleton Coalitions	USA	EU	CHN	FSU
Welfare	78560,760	103024,030	9166,958	23875,586
World	338882,175			

Table A.3
Majority and Unanimity Game: Transfers

Non-singleton Coalition	{USA,ROW}	{EU,CHN}
Welfare	78552,913 + 81453,237	103163,670 + 9187,516
Singleton Coalitions	JPN	FSU
Welfare	43082,561	23938,224
World	339378,121	
Non-singleton Coalition	{JPN,CHN}	{EU,ROW}
Welfare	43054,689 + 9181,829	102988,511 + 81424,765
Singleton Coalitions	USA	FSU
Welfare	78687,395	23928,538
World	339265,727	
Non-singleton Coalition	{EU,CHN}	{FSU,ROW}
Welfare	103096,241 + 9180,089	23847,647 + 81405,256
Singleton Coalitions	USA	JPN
Welfare	78661,292	43058,973
World	339249,498	
Non-singleton Coalition	{JPN,ROW}	{EU,CHN}
Welfare	42996,960 + 81390,804	103092,776 + 9179,701
Singleton Coalitions	USA	FSU
Welfare	78658,543	23915,769
World	339234,553	
Non-singleton Coalition	{USA,EU}	{CHN,ROW}
Welfare	78592,325 + 103068,831	9157,986 + 81315,745
Singleton Coalitions	JPN	FSU
Welfare	43031,480	23888,088
World	339054,555	