: CULTUURTECHNIEK EN WATERHUISHOUDING

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The graphical solution of the constants for the relation between real evapotranspiration and open water evaporation or soil moisture content

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Principle of elaboration

According to the formulae for evapotranspiration developed by VISSER (1963) the curves relating real evapotranspiration (E_r) to open water evaporation (E_o) give identically shaped curves for different moisture conditions of the soil (V). These curves can be schematically represented by a system of one oblique and a number of horizontal asymptotes. The same holds when plotting E_r against V^m - the moisture content raised to the power m - for different values of E_o (see fig. 1).

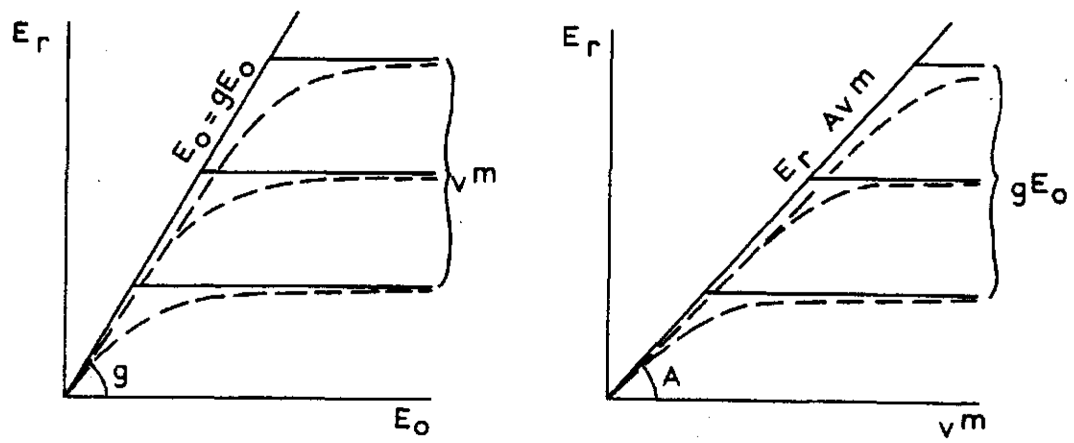


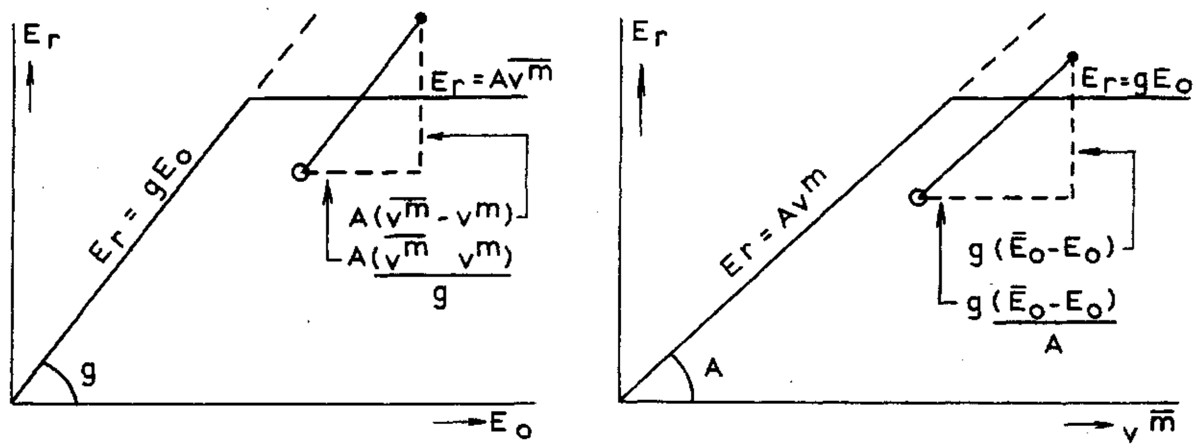
Fig. 1

The oblique asymptotes are coinciding with the same sloping line, the horizontal asymptotes are on a level of $gE_o = E_r$ or $AV^m = E_r$. The similarity of the shape of the curves is an important property because it greatly enlarges the possibilities to determine the constants describing the curves.

Because the oblique asymptotes are part of one and the same line all curves can be brought to coincide by shifting the oblique asymptotes along this sloping line till also the horizontal asymptotes cover each other.

Often only the data for a small number of periods with ample variation of the value for the third variable are available. To determine the relations between E_r and V or E_r and E_o, the corrections towards an arbitrarily chosen mean value $\bar{V}$ or $\bar{E}_o$ for the third variable can be applied arithmetically. See fig. 2.

Fig. 2



The corrections in question are composed of a vertical and a horizontal shift of the horizontal asymptote from a level E_o to a level $\bar{E}_o$ or from AV^m to $A\overline{V^m}$. This results in a displacement of the point representing the observation in a direction parallel to the oblique asymptote. The vertical correction is a change in the value of E_r. The horizontal correction is a change in the value of E_o or V^m or in the logarithms of these two magnitudes. The corrections transpose a three dimensional relation between E_r, E_o and V^m into a two dimensional one, between E_r and E_o or E_r and V^m by eliminating the third variable. The constants A, g or m, linked with the third variable, still unknown at this stage of the analyses have to be estimated for this first elaboration. The resulting graphs (see fig. 3) show what the value of g, A and m should have been.

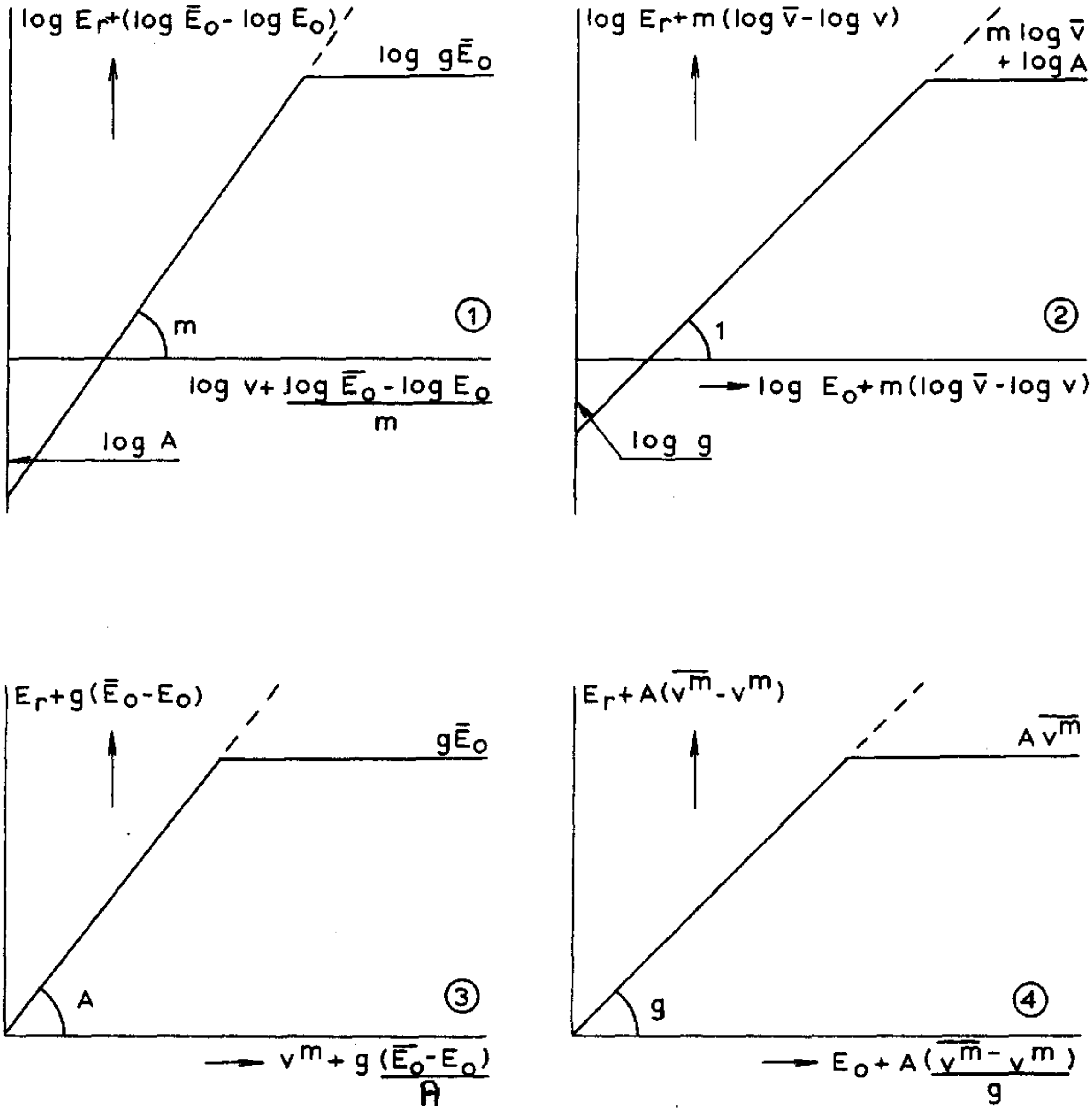


Fig. 3

It follows from the formula $E_r = AV^m$ and $E_r = g\bar{E}_o$ that in the diagrams in fig. 3, in which the constants g and A show as tangents, when plotting E_r against E_o, the correction is calculated as:

$$\text{correction } E_r = A(\bar{V}^m - V^m)$$

$$\text{correction } E_o = \frac{A(\bar{V}^m - V^m)}{g} \qquad \text{(see fig. 3, diagram 4)}$$

and when plotting E_r against V^m, the following correction is valid:

$$\text{correction } E_r = g(\overline{E}_o - E_o)$$

$$\text{correction } V^m = \frac{g(\overline{E}_o - E_o)}{A} \qquad \text{(see fig. 3, diagram 3)}$$

When plotting the logarithm of E_r against V, the constant m shows as a tangent because $\log E_r = m \log V + \log A$. So in fig. 3, diagram 1, it is obvious that the formulae hold:

$$\text{correction } \log E_r = \log \overline{E}_o - \log E_o$$

$$\text{correction } \log V = \frac{\log \overline{E}_o - \log E_o}{m} \qquad \text{(see fig. 3, diagram 1)}$$

When $\log E_r$ is plotted against $\log E_o$, like in fig. 3, diagram 2, because of the relation $\log E_r = \log E_o + \log g$, the absense of an exponent shows up as a tangent equal unit. To make the data for different values of V^m comparable, it is clear that the correction may be calculated as follows:

$$\text{correction } \log E_r = m(\log \overline{V} - \log V)$$

$$\text{correction } \log E_o = m(\log \overline{V} - \log V) \qquad \text{(see fig. 3, diagram 2)}$$

The angle of inclination of 45^o is reflected by identical corrections.

Checking the graphical analyses

In case that all four graphs are made, they mutually give a good check. See fig. 3. The logarithmic graphs should show tangents m and 1 and intercepts of log A and log g. The metric graphs have tangents A and g and zero intercepts. The tangent 1 and the zero intercepts are solid directives for fitting lines through the scatter diagram. Further A and log A have to conform as well as g and log g. In case the oblique lines fit with these requirements as well as with the scatter of the corrected data, then the value of m for which there is no check may be expected to be estimated correctly.

Example

An example of the computations is elaborated in table 1 and accessory diagrams 1, 2, 3 and 4.

The values for E_r, E_o and V which are the initial data, are given in columns 1, 2 and 3. E_r and E_o are expressed in mm per day, V in percentages of volume.

The best start is plotting the logarithms of E_r and V because the graph is showing the factors m, A and g. The value of these constants is of importance for the construction of the other diagrams. Especially the factor m, used as a power of V, will have a major influence on the value for A as elaborated in diagram 3.

In columns 4, 5 and 6 the logarithms of the initial data are given. In columns 7 and 8 the corrections for log E_r and log V are calculated with provisional estimates for the values of the constants g and m. The corrected values for log E_r and log V are plotted in diagram 1. It appears that m = 3.8. It is also possible to determine a value for factor A. When log $E_r = 0$ it holds that $1 = AV^m$, so $A = \frac{1}{V^m}$. The intersection of the oblique asymptote with a horizontal through log $E_r = 0$ gives $A = \frac{1}{21.4^{3.8}} = 93 \times \frac{1}{10^7}$. The horizontal asymptote which answers to $E_r = gE_o$ shows that $2.57 = 0.965 \times 2.66$. So $g = 0.965$.

In columns 9, 10 and 11 the corrections are prepared for diagram 2 in which log E_r is plotted against log E_o. For g and A the values according to diagram 1 are used. Because of the logarithmic scales the oblique asymptote in this diagram is at 45^o to the horizontal. The point of intersection with the horizontal through log E_r = 0 gives a value for g = 0.9. The horizontal asymptote answers to $E_r = AV^m$. When m = 3.8, than $A = 85 \times \frac{1}{10^7}$. However, should $A = 93 \times \frac{1}{10^7}$ which is a possibility, according to diagram 1, than m = 3.74.

In columns 13, 14 and 15 the corrections for the diagram 3, in which E_r and V are plotted on metric scales, are calculated with g = 0.9 and $A = 100 \times \frac{1}{10^7}$. The value for V^m in column 16 is calculated with m = 3.8. In diagram 3 the oblique asymptote has to go through the origin. Factor A turns out to be $75 \times \frac{1}{10^7}$. In this diagram for g is found $\frac{2.6}{2.66} = 0.977$.

In columns 16, 17 and 18 the corrections for the diagram in which E_r and E_o are plotted on metric scales, are calculated with g = 0.95, being the average of the values determined in the other diagrams and with $A = 80 \times \frac{1}{10^7}$, being the mean value according to diagrams 2 and 3. The oblique asymptote has to go through the origin. With concern to this not too much attention is payed to the data with a negative sign for [redacted] the corrected E_r, because these data come from periods in early spring and the mechanism of evapotranspiration may differ from that for later periods as far as the value for g is concerned due to growth of leaves. The result is that g = 0.91. The level of the horizontal asymptote answers to $E_r = AV^m$ and is giving the value $A = 83 \times \frac{1}{10^7}$.

In this elaboration V was expressed in percentages of volume. It can also be expressed in parts per unit, so 30% = 0.30 parts per unit. In that case the value V^m changes as well as the value A. Either calculation will do because the results from one can be transposed into those of the other by the multiplication factors

$$\frac{1}{100^m} \quad \text{for } V^m$$

and $$100^m \quad \text{for } A$$

When the first elaboration results in a first approach of the values of the constants, a second elaboration, using mean values or most probable estimation s, would be necessary to check these. The diagrams have to correspond closely so that the values for the constants read from each diagram should be the same.

A point of careful consideration should be the thickness of the layer for which the average moisture contents are established. Here this layer was 50 cm. A method of calculation which accounts accurately for the influence of the deeper layers will be too complicated for general use so that the depth of the extraction zone is better dealt with by inspection.

The accuracy of the results to be attainable will markedly improve by spacing the moments of moisture determination closer together. The interval of 14 days of the example is rather long.

In table 2 the initial data of a second case for elaboration are given.

Literature

VISSER, W.C. 1964. Moisture requirements of crops and rate of moisture depletion of the soil. Techn. Bull. 32, Institute for Land and Water Management Research, Wageningen.

TABLE 1. Example of elaboration part 1

1	2	3	4	5	6	7	8	diagram 1	
E_w	E_o	V	$\log E_w$	$\log E_o$	$\log V$	$\overline{\log gE_o}-\log gE_o$	$\frac{\overline{\log gE_o}-\log gE_o}{m}$	4+7	6+8
2.18	2.4	34.3	0.3385	0.3802	1.5353	+0.0068	+0.0023	0.3453	1.5376
3.32	2.98	32.9	0.5211	0.5999	1.5172	-0.2129	-0.0710	0.3082	1.4462
4.0	3.3	28.8	0.6021	0.5185	1.4594	-0.1315	-0.0438	0.4706	1.4156
3.8	3.5	24.9	0.5798	0.5441	1.3962	-0.1571	-0.0524	0.4227	1.3438
1.42	3.74	24.7	0.1523	0.5729	1.3927	-0.1859	-0.0620	-0.9664	1.3307
2.5	2.54	27.9	0.3979	0.4048	1.4456	-0.0178	-0.0059	0.3801	1.4397
1.14	1.52	37.4	0.0569	0.1818	1.5729	+0.2052	+0.0684	0.2621	1.6413
2.1	2.3	36.8	0.3222	0.3617	1.5658	+0.0253	+0.0084	0.3475	1.5742
2.5	2.1	91.6	0.3979	0.3222	1.4997	+0.0648	+0.0216	0.4627	1.5213
1.46	3.28	27.6	0.1644	0.5159	1.4409	-0.1289	-0.0430	0.0355	1.3979
2.83	3.05	25.8	0.4518	0.4843	1.4116	-0.0973	-0.0328	0.3545	1.3788
2.46	3.28	26.1	0.3909	0.5159	1.4166	-0.1289	-0.0430	0.2620	1.3736
1.50	2.33	28.2	0.1761	0.3674	1.4502	+0.0196	+0.0065	0.1957	1.4567
1.70	1.44	31.3	0.2304	0.1584	1.4955	+0.2286	+0.0762	0.4590	1.5717
1.52	1.14	31.5	0.1818	0.0569	1.5502	+0.3301	+0.1100	0.5119	1.66
0.7	1.33	32.6	-0.1549	0.1239	1.5132	+0.2631	+0.0877	0.1082	1.6009
2.0	1.95	31.2	0.3010	0.2900	1.4942	-0.0970	+0.0323	0.3980	1.5265
2.14	1.73	31.6	0.3304	0.2380	1.4997	+0.1490	+0.0497	0.4794	1.5494
2.30	3.12	29.8	0.3617	0.4942	1.4742	-0.1072	-0.0357	0.2545	1.4385
1.58	2.75	26.4	0.1987	0.4393	1.4216	-0.0523	-0.0174	0.1464	1.4042
2.40	3.16	25.6	0.3802	0.4997	1.4082	-0.1127	-0.0376	0.2675	1.3706
1.03	2.66	25.1	0.0128	0.4249	1.3997	-0.0379	-0.0126	-0.0251	1.3871
mean value	2.66			0.3870	1.4709				
factors (estimated values)						g=1.0	m=3		

9	10	11	diagram 2	
$\overline{\log V}-\log V$	$m(\overline{\log V}-\log V)$	$\frac{m(\overline{\log V}-\log V)}{g}$	4+10	5+11
-0.0644	-0.25	-0.242	0.0885	0.1382
-0.0463	-0.18	-0.174	0.3411	0.4259
+0.0115	+0.045	+0.044	0.6471	0.5625
+0.0747	+0.29	+0.28	0.8698	0.8241
+0.0784	+0.306	+0.296	0.4583	0.8689
-0.0253	+0.100	+0.096	0.4979	0.5008
-0.1020	-0.29	-0.28	-0.2331	-0.1018
-0.0949	-0.37	-0.358	-0.0478	0.0037
-0.0288	-0.11	-0.106	0.2879	0.2162
+0.0300	+0.117	+0.113	0.2814	0.6289
+0.0593	+0.23	+0.222	0.6818	0.7063
+0.0543	+0.21	+0.201	0.6009	0.7169
+0.0207	+0.08	+0.078	0.2561	0.4454
-0.0246	-0.096	-0.093	0.1344	0.0654
-0.0793	-0.31	-0.299	0.1282	-0.2421
-0.0423	-0.165	-0.159	-0.3199	-0.0351
-0.0233	-0.08	-0.077	0.2210	0.2130
-0.0288	-0.112	-0.108	0.2184	0.1300
-0.0033	-0.013	-0.0125	0.3487	0.4817
+0.0493	+0.19	+0.183	0.3887	0.6223
+0.0627	+0.245	+0.236	0.6252	0.7357
+0.0712	+0.274	+0.264	0.2868	0.6889

factors (determined values) — column 10: m=3.8; column 11: m=3.8, g=0.965

TABLE 1. Example of elaboration part 2

	12	13	14	15	diagram 3	
	$\bar{E}_o - E_o$	$g(\bar{E}_o - E_o)$	$\frac{g(\ddot{E}_o - E_o)}{A}$	V^m	1+13	15+14
	0.26	0.23	2[illegible]000	683000	2.41	706000
	-1.32	-1.19	-11[illegible]000	582000	2.13	463000
	-0.64	-0.58	-58000	355000	3.42	297000
	-0.84	-0.76	-76000	209000	3.04	133000
	-1.08	-0.97	-97000	200000	0.45	113000
	0.12	0.11	11000	311000	2.61	322000
	1.14	1.03	103000	948000	2.17	1051000
	0.36	0.32	32000	891000	2.42	923000
	0.56	0.50	50000	502000	3.00	552000
	-0.62	-0.56	-56000	299000	0.9	243000
	-0.39	-0.35	-35000	231000	2.48	196000
	-0.62	-0.56	-56000	263000	1,90	207000
	0.33	0.30	-30000	323000	1.80	293000
	1.22	1.10	110000	479000	2.80	369000
	1.52	1.37	137000	777000	2.89	640000
	1.33	1.20	120000	563000	1.90	683000
	0.71	0.64	64000	457000	2.64	521000
	0.93	0.84	84000	501000	2.98	585000
	-0.46	-0.41	-41000	398000	1.89	357000
	-0.09	-0.08	08000	251000	1.50	331000
	-0.50	-0.45	45000	224000	1.95	269000
	0	0	0	204000	1.03	204000
mean value				439000		
factors (estimated values)	g=0.9		g=0.9, $A=100x\frac{1}{10^7}$			
factors (determined values)				m=3.8		

	16	17	18	diagram 4	
	$\bar{V^m} - V^m$	$A(\bar{V^m} - V^m)$	$\frac{A(\bar{V^m} - V^m)}{g}$	1+17	2+18
	-244000	-1.92	-2.06	0.23	0.34
	-143000	-1.14	-1.07	2.18	1.91
	+ 84000	+0.67	+0.70	4.67	4.0
	+ 23000	+1.85	+1.94	5.64	5.44
	+239000	+1.93	+2.02	3.35	5.76
	+128000	+0.73	+0.77	3.23	3.31
	-509000	-4.07	-4.28	-2.93	-2.79
	-452000	-3.62	-3.81	-1.52	-1.51
	- 63000	-0.51	-0.54	2.05	1.56
	+140000	+1.11	+1.17	2.57	4.45
	+208000	+1.66	+1.75	4.49	4.80
	+176000	+1.41	+1.48	3.87	4.76
	+116000	+0.93	+0.98	2.43	3.31
	- 40000	-0.03	-0.032	1.67	1.41
	-338000	-2.70	-2.84	-1.18	-1.70
	-124000	-0.99	-1.04	-0.29	0.29
	- 18000	-0.14	-0.11	1.86	1.84
	- 62000	-0.49	-0.51	1.65	1.22
	+ 41000	+0.32	+0.34	2.62	3.46
	+188000	+1.51	+1.59	3.09	4.34
	+215000	+1.72	+1.80	4.12	4.96
	+235000	+1.88	+1.98	2.91	4.64
factors (determined values)		$A=80x\frac{1}{10^7}$	g=0.95		

TABLE 2. Lysimeter Binnenveld

Container nr. 4

Initial data

nr	E_r	E_o	V	log E_r	log E_o	log V
1	1.0	1.7	54	0	0.2304	1.7324
2	2.2	2.0	53	0.3424	0.3010	1.7243
3	2.8	2.5	56	0.4472	0.3979	1.7482
4	1.8	1.9	53	0.2553	0.2788	1.7243
5	3.5	2.6	51	0.5441	0.4150	1.7076
6	2.7	2.0	50	0.4314	0.3010	1.6990
7	3.9	3.6	49	0.5911	0.5563	1.6902
8	4.5	4.6	48	0.6532	0.6628	1.6812
9	2.9	2.9	46	0.4624	0.4624	1.6628
10	2.4	3.4	46	0.3802	0.5315	1.6628
11	7.0	7.1	43	0.8451	0.8513	1.6335
12	2.8	3.1	42	0.4472	0.4914	1.6243
13	4.0	4.2	41	0.6021	0.6232	1.6128
14	3.8	3.5	39	0.5798	0.5441	1.5811
15	2.8	3.3	40	0.4472	0.5185	1.6021
16	2.4	3.9	39	0.3802	0.5911	1.5911
17	2.8	4.5	38	0.4472	0.6532	1.5798
18	2.8	5.3	37	0.4472	0.7243	1.5682
19	4.4	6.9	35	0.6435	0.8389	1.5441
20	4.7	7.8	33	0.7621	0.8921	1.5185
21	2.4	3.2	34	0.3802	0.5051	1.5315
22	1.8	1.8	38	0.2553	0.5253	1.5798
23	3.8	4.7	36	0.5798	0.6721	1.5563
24	5.5	5.0	37	0.7404	0.6990	1.5682
25	3.9	4.6	32	0.5911	0.6628	1.5051
26	2.4	5.4	30	0.3802	0.7324	1.4771
27	2.2	6.0	30	0.3424	0.7782	1.4771
28	3.7	4.9	29	0.5682	0.6902	1.4624
mean values		$\overline{E}_o = 4.0$	$\overline{V} = 42$		0.5552	1.6160

DIAGRAM 1

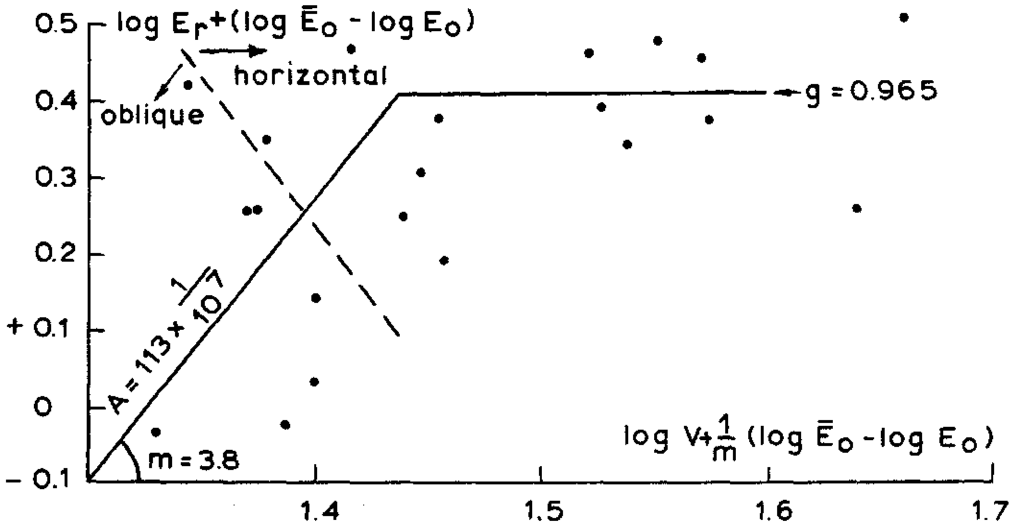
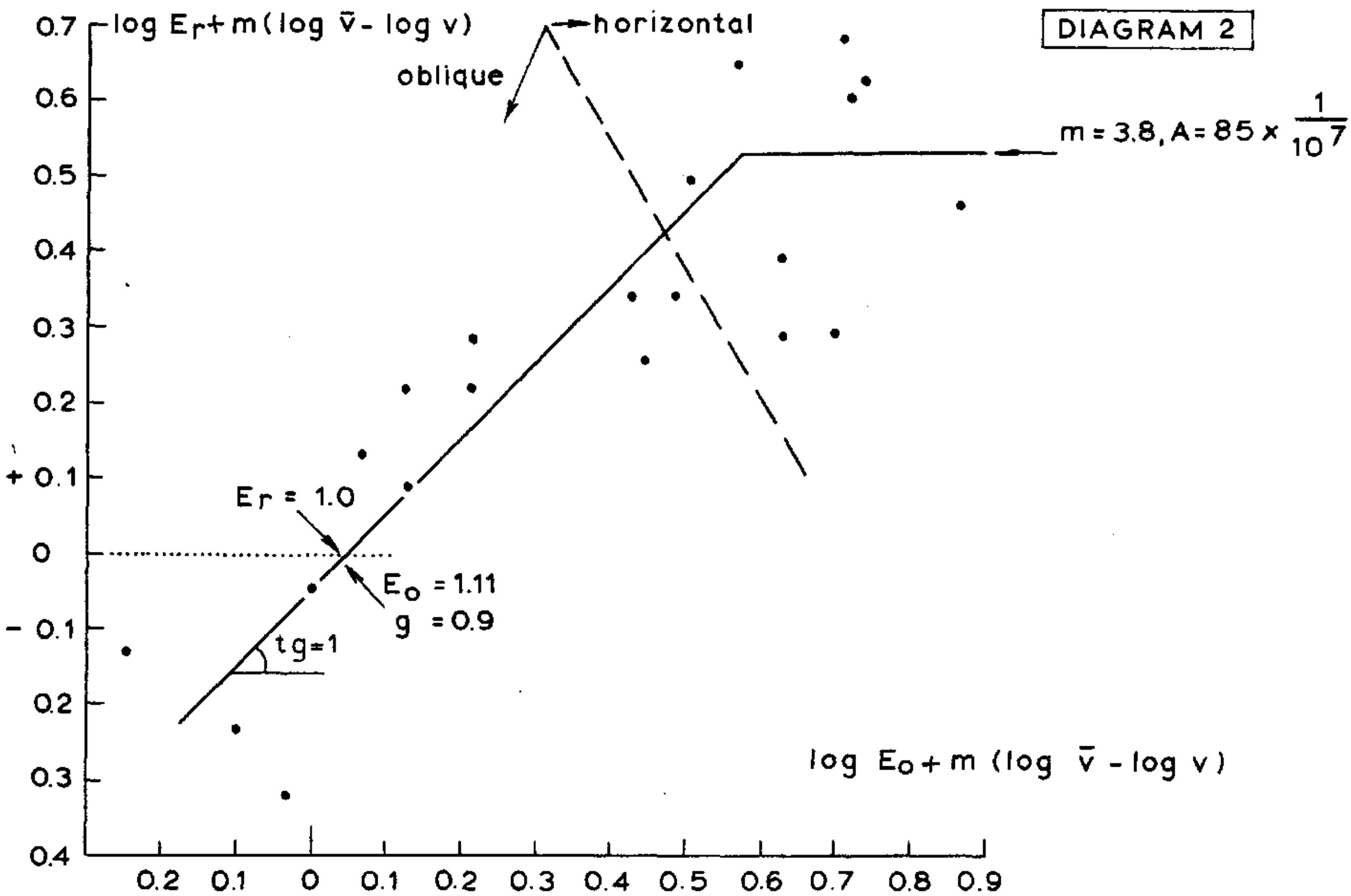


DIAGRAM 2



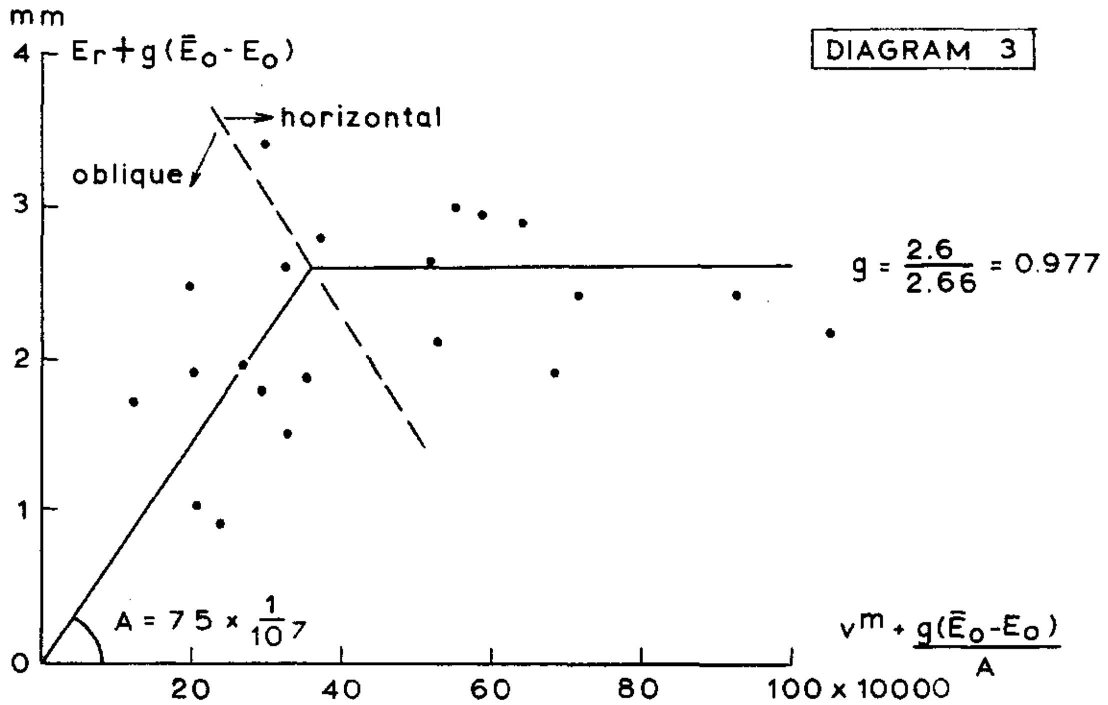


DIAGRAM 3
mm
$E_r + g(\bar{E}_o - E_o)$
horizontal
oblique
$g = \frac{2.6}{2.66} = 0.977$
$A = 75 \times \frac{1}{10^7}$
$v^m + \frac{g(\bar{E}_o - E_o)}{A}$
0
1
2
3
4
20
40
60
80
100 x 10000

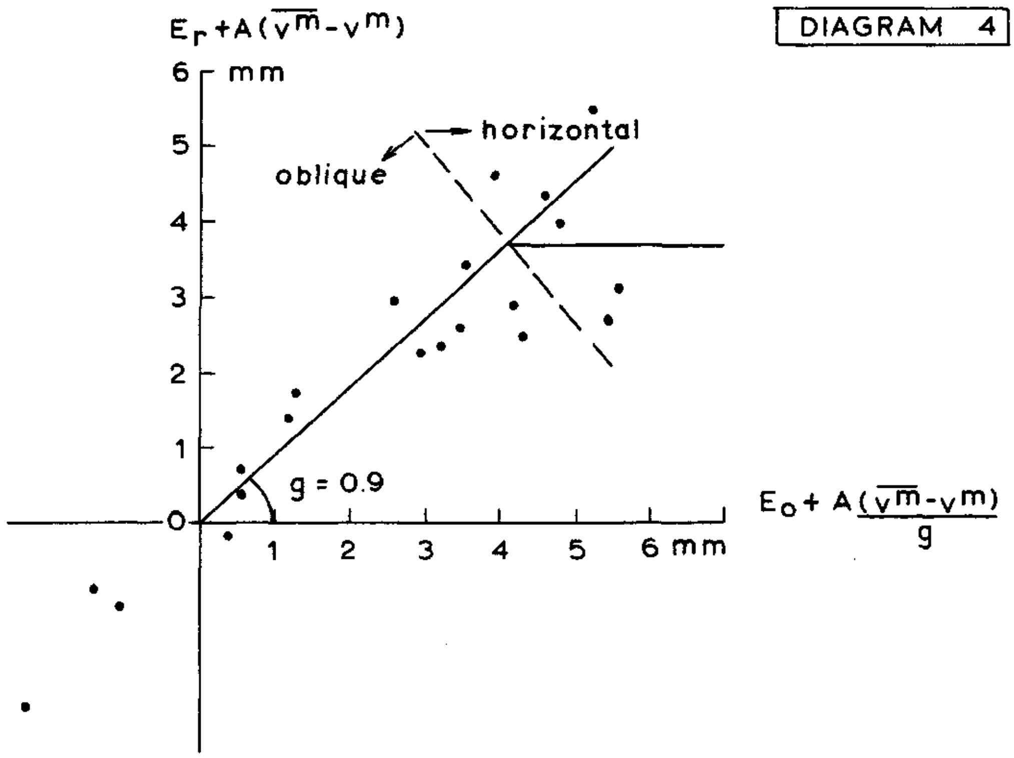


DIAGRAM 4
$E_r + A(\overline{v^m} - v^m)$
mm
horizontal
oblique
g = 0.9
$E_o + \frac{A(\overline{v^m} - v^m)}{g}$
0
1
2
3
4
5
6
1
2
3
4
5
6 mm