

Determination of the constants of the soil moisture retention function

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The soil moisture retention curve describes the relation between the soil-moisture stress  $\psi$  and the moisture content  $v$ . The moisture content may be expressed in percentage of the sample volume or in percentage of the pore space volume. In the first case the maximum moisture content is equal to the percentage of pore space and will be of the order of 40 to 50%. In the second case the maximum moisture content will be 100%.

The formula reads:

$$I \quad b(A - \log \psi) = p \log v - (1-p) \log (P - v)$$

$$II \quad b \left\{ \left[ A + \frac{(1-2p)}{b} \log P \right] - \log \psi \right\} = p \log \frac{v}{P} - (1-p) \log \left( 1 - \frac{v}{P} \right)$$

Expressed as in I the formula is given in practical units, II however may simplify comparison of different samples and has some advantage in fitting the curve to the observations.

Aim of curve fitting

The observations of the values of the desorption curve prove to be somewhat inaccurate due to the difficulties in assessing the degree of approach of the equilibrium in the laboratory. A higher accuracy may be required and may be obtained by determination of the average curve through a sequence of observations at increasing  $\psi$ .



For practical purposes it will be impossible to determine more than a minimum of  $\psi$  -levels and the relation at intermediate  $\psi$  -values of arbitrary value may be required.

For calculations of the indications, which may be deduced from the desorption curve for permeability and soil structure, often requiring a mathematical treatment of the desorption function, a formula is needed which requires the knowledge of the parameters of the formula.

The graphical representation

The desorption curve may graphically be represented in a nomogramme with three parallel axis, see Fig. 1. On the left axis  $\log v$  is plotted downward, on the right axis  $\log P-v$  is plotted upward and on the middle axis  $A-\log \psi$  is plotted with a scale,  $b$  times as large as the scale on the two other axis ( $b$  is of the order of 0.3). See an example in Fig. 7.

The distance between the right and left axis is arbitrary, the distance between the middle and right axis is  $p$  times this arbitrary distance, the distance between the left and middle axis is  $1-p$  times this arbitrary distance. A 10 cm distance as well as a 10 cm scale unit on the right and left axis prove to be handy.

Now a line, linking the point  $\log v$  on the left axis with the point  $\log P-v$  on the right axis intersects the  $A-\log \psi$  axis at the required value of  $\log \psi$ .

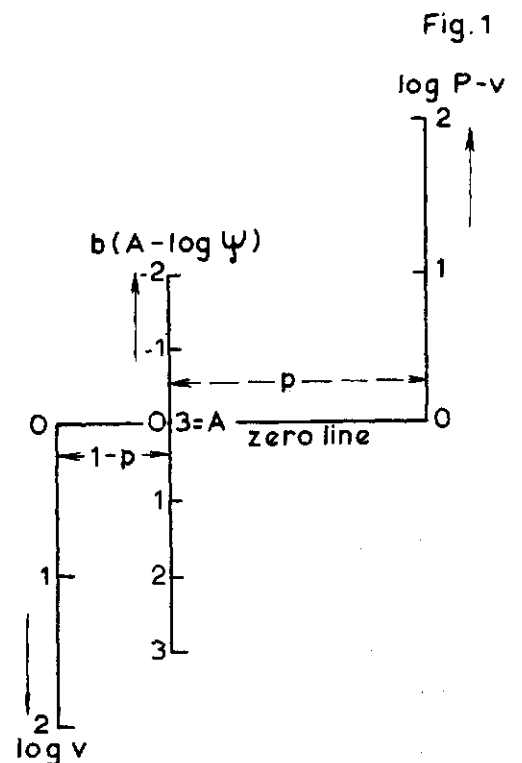


Fig. 1

Geometry of the nomogramme

In the triangle ABC the relation holds, see Fig. 2:

$$\frac{AB}{AC} = \frac{BE}{ED} \quad \text{or} \quad \frac{l}{\log v + \log(P-v)} = \frac{pl}{\log(P-v) + b(A - \log \psi)}$$

1 cancels out and is apparently unimportant. Therefore

Fig. 2

$$b(A - \log \psi) = p \log v + p \log(P-v) - \log(P-v)$$

$$b(A - \log \psi) = p \log v - (1-p) \log(P-v)$$

This proves that the nomogram depicts the formula. If the parameters are known, the nomogram may be constructed. Then at each  $\log \psi$  -value the  $\log v$  value may be read or the nomogram may be used with the opposite purpose.

Fitting of the formula with three observations

For any straight line linking  $\log v$  with  $\log P-v$ , the  $\log \psi$  -scale will be intersected at the required value of  $\log \psi$ . The middle scale is metric in  $\log \psi$ . If three pairs of values of  $\psi$  and  $v$  are given, a solution of the constants may be obtained by constructing the place of the middle axis where the intercepts occur at distances  $b(\log \psi_1 - \log \psi_2)$  and  $b(\log \psi_2 - \log \psi_3)$ . At this place the intercepts bear a linear relation to the values of  $\log \psi$ .

The left and right axis are drawn and the points for  $\log v$  and  $\log P-v$  are marked (see Fig. 3). The dashed lines 1 to 3 link the points for the same value of  $v$ .

Assume that this are curves for  $\log \psi = 1.0, 2.7$  and  $6.0$ . Now the distance between line 1 and 3 must be divided in such a way that the dotted line is traced at  $(6.0 - 2.7) / (6.0 - 1.0)$  times the distance between the two lines from line 3, and at  $(2.7 - 1.0) / (6.0 - 1.0)$  times the total distance from line 1.

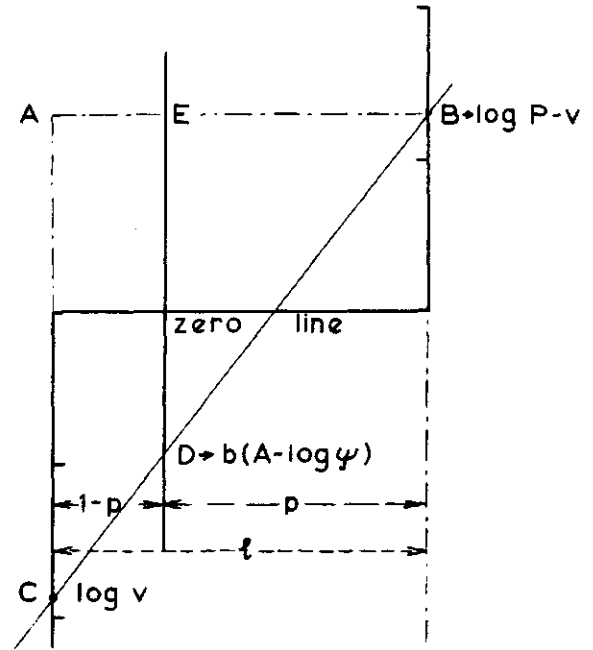
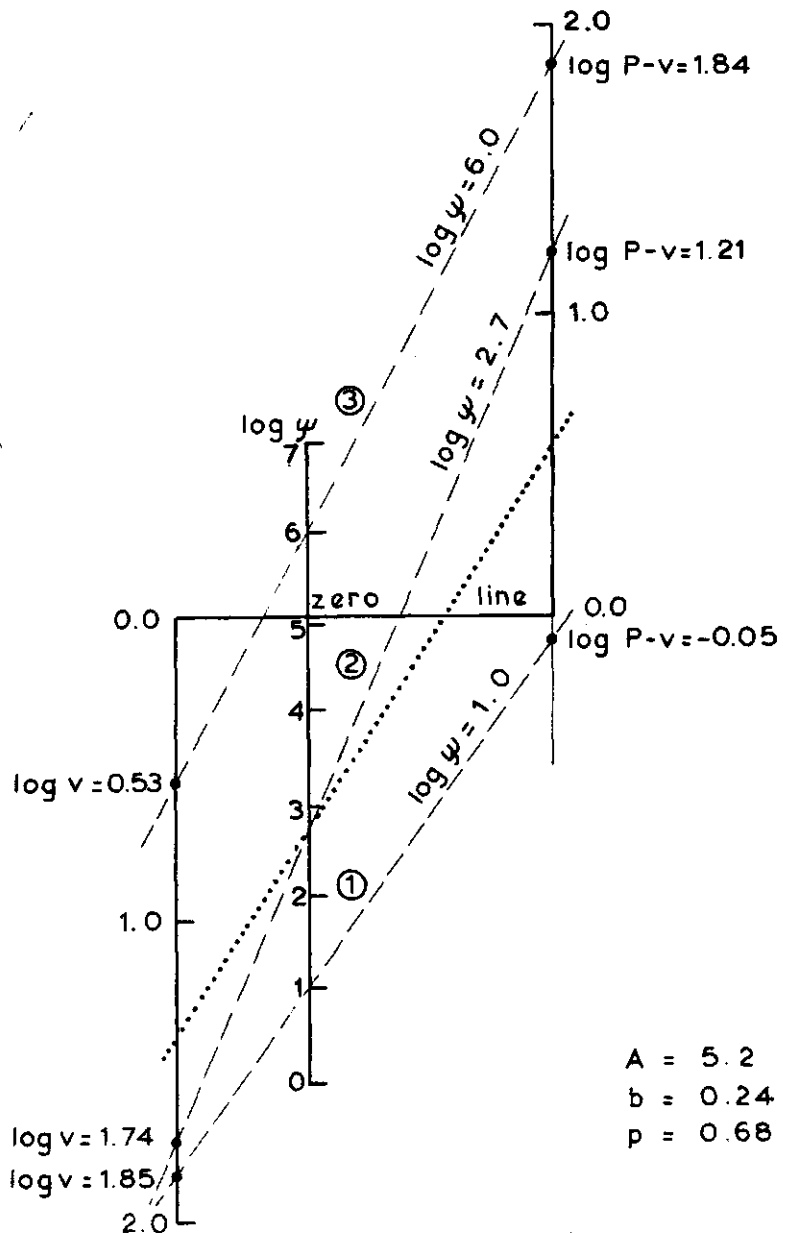


Fig. 3

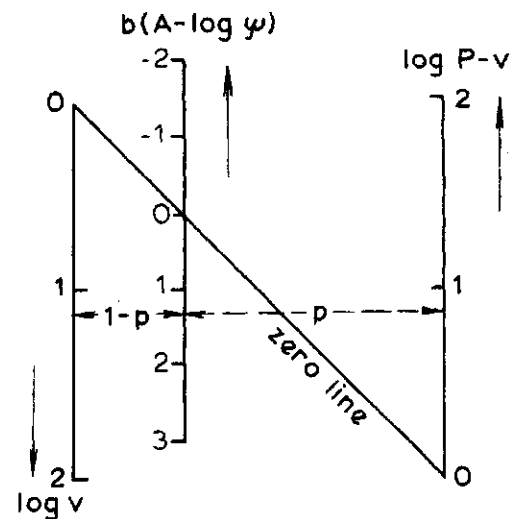


The distance between the intercepts on the  $\log v$  axis is therefore divided to a ratio 0.66 to 0.34 and the same is done for the  $\log P-v$  axis. At the place where the dotted line and line 2 intersect the vertical distance between the three dashed lines bears a linear relation to the  $\log \psi$  values. Here the  $\log \psi$  axis may be traced and a linear scale enables to read which  $\psi$  value belongs to any value of  $v$ . The determination of the value of  $v$  belonging to a certain  $\log \psi$  requires to trace a few  $v$ -lines in order to estimate the solution to the desired accuracy.

To make the best use of the paper, it is advisable not to draw the zero line at right angles to the axis, but to construct a N-nomogram as depicted in Fig. 4. The zero point of the  $\log P-v$  axis is taken at the same height as the point  $\log v = 2$ . The zero line has now an inclination, but the nomogram may be used in the same way. The value of  $A \cdot \log \psi$  is still zero on the now inclined zero-line.

In this way the formula accounts for three observations. When a good fit to more than three observations is desired, an other method should be followed.

Fig 4



Fitting of the formula with any given number of observations

The construction of the graph, from which the parameters of the desorption function are deduced, is given in Fig. 5 and 6. A square is drawn with a distance between the horizontal lines of 14 cm and of 20 cm between the vertical lines (see Fig. 5). On the vertical lines a  $\log \psi$  scale with 2 cm units is marked. On the lower horizontal lines  $\log P-v$  is marked increasing from right to left, on the higher horizontal line  $\log v$  is marked from left to right. On both horizontal lines the units are taken at 10 cm.

The values  $\log v$  and  $\log P-v$  are plotted against  $\log \psi$  in such a way that for  $v = P$ ,  $\log v$  coincides with the right hand vertical line, and for  $v = 0$ ,  $\log P-v$  coincides with the left hand side vertical line. This is the same as plotting  $v$  in percentage of the pore space. For the values of  $\psi$  and  $v$ , see the table, rendered in Fig. 6.

The plotting of the observations produces two curved lines with as asymptotes the right and left hand side vertical lines and two inclined lines, indicated as AC and BD. The inclined asymptotes have angles with the vertical lines of  $b/p$  for the  $\log v$  curve and  $b/1-p$  for the  $\log P-v$  curve.

By drawing smooth curves through the dots, the existence of the two asymptotes for each curve may be taken into consideration.

The line AB - a straight line - is drawn. This line divides for any value of  $\log \psi$  the horizontal distance between the two curved lines, equal to

$$EG = 2 + \log \frac{v}{P} + \log \left(1 - \frac{v}{P}\right)$$

into two parts with a relation:

$$\frac{EF}{p} = \frac{FG}{1-p}$$

It may be proved that the line AB depicts the desorption formula in its shape:

$$b \left\{ \left[ A + \left( \frac{1-2p}{b} \right) \log \frac{P}{100} \right] - \log \psi \right\} = p \log \frac{100v}{P} - (1-p) \log \left( 100 - \frac{100v}{P} \right)$$

The value of  $p$  is found by measuring for the various values of  $\log \psi$  the distance EF, which is plotted against the distance EG (see Fig. 6). A line through the origin of this graph and the dots has a tangent with the EG axis equal to  $p$ .

Now the asymptotes AC and BD may be corrected and the points of intersection A and B be determined more accurately. The values of  $b$  and  $p$  are now known in first approximation and the tangents  $b/p$  and  $b/1-p$  can be calculated. This will change the straight parts of the  $\log v$  and  $\log P-v$  line and change the points of intersection A and B. A revised line AB is drawn. In this way a better value of  $b$  is found. The value of  $p$  is not affected very much by experimental errors and the first approximation often will suffice.

The value of the parameter A in the formula is found in Fig. 5 by dividing the 20 cm between the two vertical lines in a distance  $20p$  from the right vertical line and  $20(1-p)$  from the left vertical line. A vertical line HK intersects the line AB at the point K at a value of  $\log \psi = \left\{ A + \left( \frac{1-2p}{b} \right) \log \frac{P}{100} \right\}$

The value of  $P$  may be estimated crudely by judging the place of the vertical asymptote on the  $\log v$  scale. A more precise estimation can be made by observing the shape of the  $\log P-v$  line in the part of that line, where it approaches the asymptote BD. A correction of the value of  $P$  which reforms the

right hand side of the log P-v curve so that the curve conforms best with the oblique asymptote BD gives the most probable value of P. The assessment of the value of P depends on the low values of P-v. The value of P may turn out to be significantly larger than the pore space volume. This enables to determine the air entry point.

The assessment of the air entry point, field capacity and wilting point

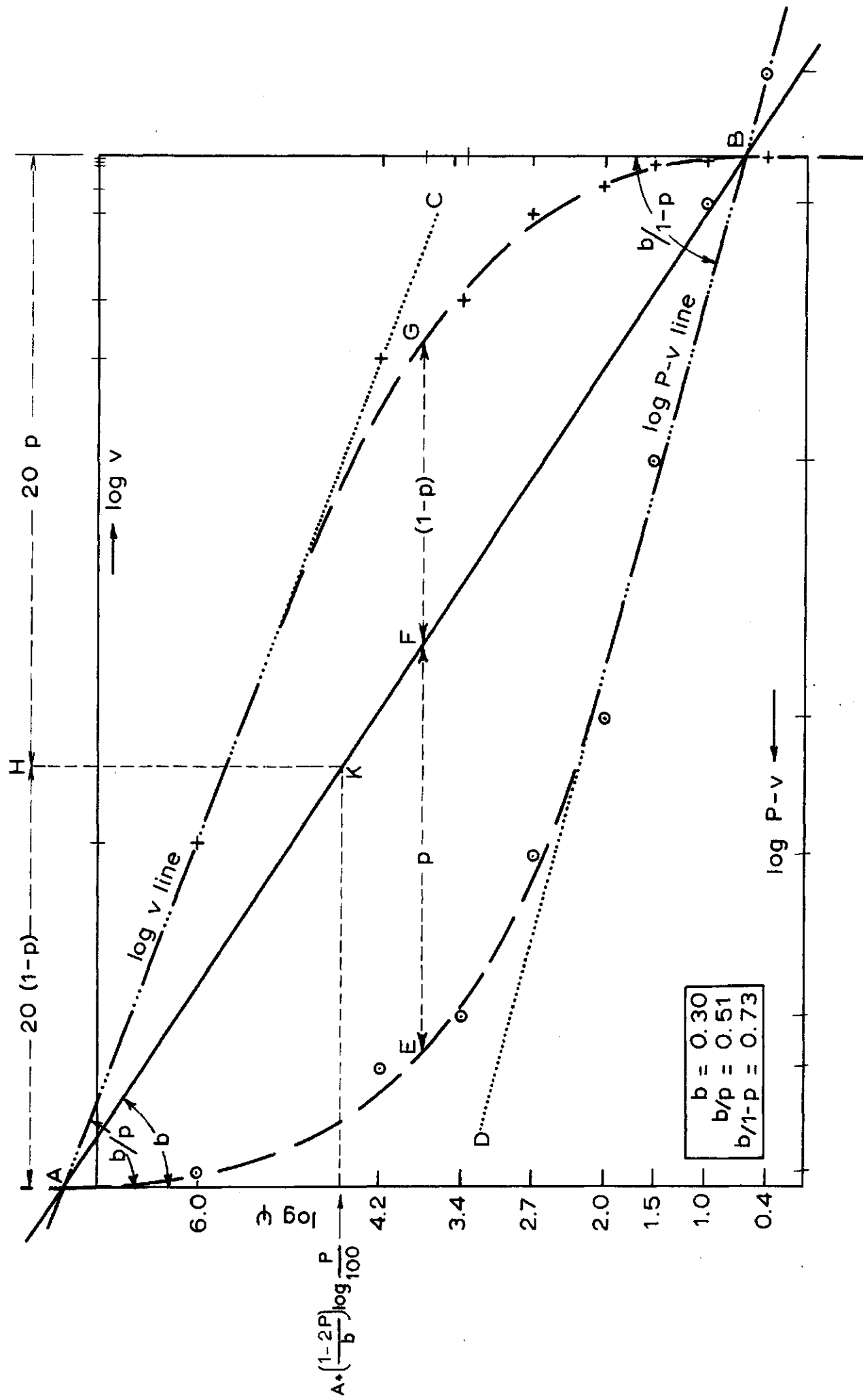
The air entry point is the log  $\psi$  value, at which the line AB intersects the coordinate line for  $\log v = \log Pv$ . Here Pv means the actual pore space volume as determined in the sample. The air entry point describes the radius of the largest pore present in the soil according:  $\log \psi = \log 0.15 - \log r$ .

In each sample a maximum radius must occur, which means that the desorption curve at some value of  $\psi$  must necessarily have a vertical part, where the value of  $\psi$  may decrease, but the value of v can no longer increase because all the pores are already saturated. In the modern physical description of the pore size distribution curve the stress on this saturated capillary zone has deminished strikingly and is capillarity equated with unsaturatedness.

The air entry point is an important parameter in assessing the unsaturated permeability of the soil and requires a fair amount of observations at values of log  $\psi$  between zero and 2.

The determination of available moisture is often met by the determinations at log  $\psi = 2.7$  and 4.2. The values of log  $\psi$  at field capacity and wilting point, however, are strongly dependent on the amount of flow in the soil and may vary considerably. This means that also above and below these values of log  $\psi$  the desorption curve should be known. The observations should be scattered over the whole range of log  $\psi$  values between zero and six. By fitting an average curve through these observations so that for every desired value of log  $\psi$  the moisture content may be calculated by a continuous relation, the observations of the water retention may be used so that any desired use of the information, contained in these data, may be made.

Fig. 5



log $\psi$	0,4	1,0	1,5	2,0	2,7	3,4	4,2	6,0
V	71,3	70,9	68,8	63,0	55,5	38,3	29,4	3,4
P-V	0,5	0,9	3,0	8,8	16,3	33,5	42,4	68,4
log V	1,853	1,851	1,838	1,799	1,744	1,583	1,468	0,532
log P-V	0,301	0,046	0,477	0,945	1,212	1,525	1,627	1,835

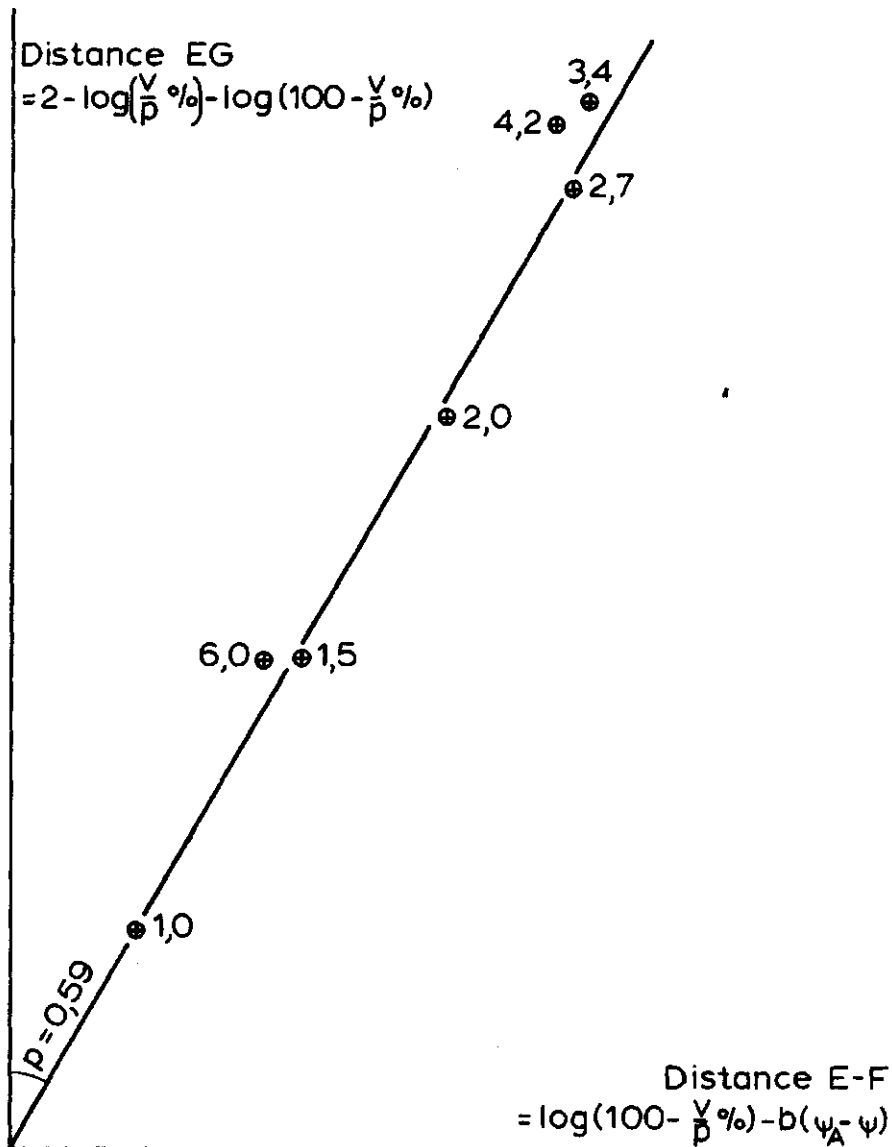


fig.6



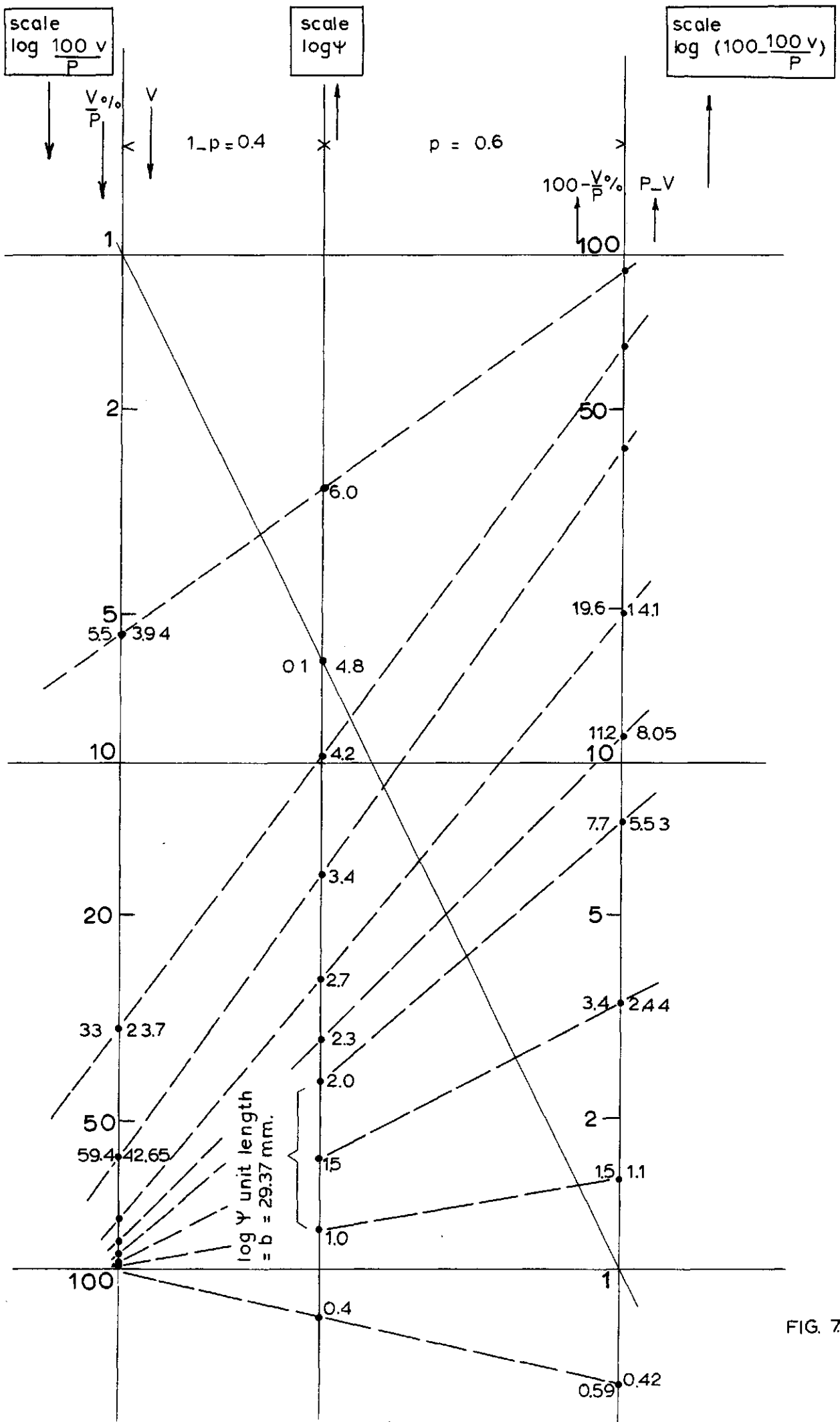


FIG. 7