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Mathematical models in soil productivity studies exemplified by the response to nitrogen

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Hun inhoud varieert sterk en kan zowel betrekking hebben op een eenvoudige weergave van cijferreeksen, als op een concluderende discussie van onderzoeksresultaten. In de meeste gevallen zullen de conclusies echter van voorlopige aard zijn omdat het onderzoek nog niet is afgesloten.
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The purpose of models

In many branches of science the use of mathematical models has proved to be valuable. It necessitates the construction of an integrated working hypothesis, describing the cooperation of all factors influencing crop growth. The models show that not only the relations governing the action and interaction of factors is of importance, but that also the value and often the variation in the value of the parameters in a growth function should claim a considerable part of the researcher's attention.

The value of such models is twofold. They enable one to treat the results of field experiments with accurate mathematical methods as curve fitting or least square calculations. They also can lead directly to an advice to the farmer, based on calculations carried out with computers. This computer-based advice is so important because these machines can easily cope with problems of the high degree of complexity inherent to agricultural advisory work, in which intricate relations govern the economic and technical results of farm management and investment.

The least square and other treatments of experimental results, using well-conceived models, are important because they make it possible to analyze the plant growth relations into far more detail than the usual, mainly graphical analyses. In this way from the same observations a considerably deeper insight into the reaction of a plant to its environment can be obtained.

Earlier models

Of the earlier models that of Mitscherlich is the best known. The Mitscherlich equation has, notwithstanding many years of concentrated attention given to it, not proved to be very successful. The concept to describe the growth relations along quantitative lines, was sound and valuable. The reason that after years of experimentation no sustained success was attained is, so it seems, partly to be attributed to the simplifications of the mathematical concept. The expectation that the growth parameters should be constant and invariant with respect to environmental conditions, is of restricted importance. The neglect of the full solution of the differential equations was of more importance.

However, VAN UVEN gave the full solution in 1932. The shortcoming materially restricting the success, was the lack of a biological foundation to the mathematical concept. The basic assumption was, that the increase in yield, per unit increase of the growth factor, is linearly related to the yield deficit. The resulting equation does not give the same results as the field or pot experiments and especially does not possess the required flexibility in experiments in which the number of growth factors , which are varied experimentally, increases.

The main objection however is, that a not existing physical quantity as the yield deficit is considered as a governing factor. No biological principle is known, that makes this preponderant position of a quantity which has no reality, credible. And the credibility of the equation depends entirely on the credibility of the hypotheses on which the equation is founded.

The unfavourable factors

The research usually is concentrated on the study of factors with a favourable influence on plant growth. There are, however, in an integrated research always factors involved which have an unfavourable influence. Salinity and excess of water in the soil are perhaps the factors which drew the greatest attention. These factors also have to have a place in an integrated model.

Three different types of unfavourable reactions have been recognized. The factor may work unfavourable because it counteracts the uptake of nutrients in the plant or in photosynthesis. Another type of reaction is the one in which the factor destroys part of the mechanism of plant life. The third type is the one, where the factor reacts proportional to the size of the plant or to some other dimensional property, usually indicated as 'the heaviness' of the crop.

This study deals with the last type of reaction, and investigates the reduction of the crop yield due to excess of nitrogen, where lodging is proportional to the amount of nitrogen applied and to the size of the crop. The yield data are taken from an article of SINGH et al (1967) in which data are given for an orthogonal experiment with three different gifts of potassium and phosphate and four gifts of nitrogen to coxfoot grass.

Basis of the growth equation

The process of combining in a formula the growth factors in the required form deals with subjects as uptake of nutrients, transfer of energy, influence of genetic properties of the crop. The uptake of nutrients can depend on mass flow, diffusion or root activity. All these effects, however, can be described by the linear relation of the type:

$$y = \frac{x_b - x_a}{r_b} \qquad g_x = \frac{y}{q} \qquad q = \frac{x_b - x_a}{g_x r_b} \tag{1}$$

Here y is the amount of the growth factor that is transported, x_a is the flow potential at the interface where assimilation takes place, x_b is the potential at a distance b from the interface, r_b is the flow resistance in the interval b of the flow path. The quantity of dry matter, synthesized due to the uptake of y, is equal to q. The ratio between y and q is the content g_x of y in q.

This formula holds for every interval of the flow path, but will at first be taken to hold for the boundary plane where assimilation takes place.

In case the assimilation is able to use the growth factors with the same velocity as they are supplied by the process of transfer, the potential x_a at the boundary plane will be small or even zero, depending on the nature of the assimilation process. In writing x_b instead of $x_b - x_a$ and writing $a = 1/g_x r_b$, formula 1 changes as follows:

$$q = a\,(x_b - x_a) \qquad\qquad a - \frac{q}{x_b} = \Delta \tag{2}$$

The value of Δ tends to be small.

Formula 2 holds for all growth factors, but before uniting them into some general function, they have to be expressed in a scale of equal activity. An unhampered healthy growth requires a certain ratio of the different contents a_i. This ratio may be considered as an innate equilibrium which is preserved by the checks and balances which protect plant life. The value of will be small and a change $d\Delta$ of Δ will upset the equilibrium, the more so the larger the change $d\Delta$ is with respect to its original value Δ. The equilibrium of the Δ_i-values for different growth factors x_i will depend on the ratio $d\Delta/\Delta$ and is expressed by the sum of these ratios equal to zero. This can be given by:

$$\sum_{i=1}^{n} \frac{d\,(a_i - q/x_i)}{a_i - q/x_i} = 0 \tag{3}$$

This equation not only accounts for neglecting the value of x_a but also for the fact that the plant possesses a certain flexibility, so the value of a_i needs not be constant but may deviate between certain margins.

Extension of the applicability

The formula has been constructed for the situation existing at the assimilation interface. If the diffusion path is extended to include other parts of the plant and the transfer through the soil, potentials x_c, x_d.... x_g along the flow path can be inserted in the formula as follows:

$$\begin{array}{rcl} g_x r_b q &=& x_b - x_a \\ g_x r_c q &=& x_c - x_b \\ & \text{etc.} & \\ g_x r_n q &=& x_n - x_{n-1} \\ \hline g_x (r_a + r_b + \dots r_n) q &=& x_n - x_a \end{array} \qquad (4)$$

The length of the flow path can be increased at will by inserting the potentials x_n and x_a at the beginning and the end, and by replacing the resistance r_i of a separate flow interval by the sum of resistances over the full length. The equation may be used to describe internal situations as well as the influence of conditions outside the plant.

Further, equation 3 was derived from the assumption that the assimilation capacity was exceeding the transport capacity. This is, however, not a necessary condition. If the transport capacity is larger, then the actual transfer Q still will be governed by an equation of type 2, but Q, a and x_b being given, the assumption that x_a is small or zero no longer holds. The concentration x_a will have to increase with the same amount as x_b to keep the transfer y or the dry matter production q equal to the assimilation capacity Q. By inserting Δx instead of $x_b - x_a$, in order to show that only the difference between the two potentials is a necessary variable, and not x_b or x_b and x_a, the following formulae are obtained:

$$Q = a \Delta x \qquad q = Q \qquad \frac{d(a - q/x)}{a - q/x} \rightarrow \frac{d(1 - q/Q)}{1 - q/Q} \qquad (5)$$

The formal equality of the formulae for a limiting rate of transfer and a limiting assimilation rate is maintained and the same type of formula holds, though the biological significance is different.

The growth equation

Integration of formula 3 gives the growth equation. In equation 3 one of the terms may take the shape of formula 5, inserting in the equation that an assimilation rate dependent on the genetic properties of the plant will limit the rate of growth, even if the rate of transfer might make a quicker uptake of the growth factor in the process of synthesis possible.

Writing equation 3 more in detail, the following relations hold:

$$\frac{d(1 - q/Q)}{1 - q/Q} + \frac{d(a_1 - q/x_1)}{a_1 - q/x_1} + \frac{d(a_2 - q/x_2)}{a_2 - q/x_2} + \dots + \frac{d(a_n - q/x_n)}{a_n - q/x_n} = 0$$

$$\ln(1 - q/Q) + \ln(a_1 - q/x_1) + \ln(a_2 - q/x_2) + \dots + \ln(a_n - q/x_n) = \ln F$$

$$(1 - q/Q)(a_1 - q/x_1)(a_2 - q/x_2) \dots (a_n - q/x_n) = F \qquad (6)$$

Formula 6 provides the yield increase q per unit time as a function of the growth factors x_i and the maximum biological growth capacity Q.
The ultimate yield is the sum of the yield increases q over the full growth period. If the formula is used over a time interval equal to the growth period, it will be clear that this simplification means that any variation in the growth constants in the course of the growth period is neglected and that the value of these constants is taken equal to some average of the values for the separate time intervals. It should be noted that the integration constant F appears as a constant of mathematical, not biological origin, and in practice will allow for the unknown growth factors, which are not taken up in the description of the plant environment (VISSER W.C., 1964).

Construction of the model

The field experiment of Singh et al (1967) contained 3 levels of potassium and phosphate application and 4 levels of nitrogen. The experiment started in 1949 and in the six years of which data are given, an adverse influence of high applications of nitrogeneous fertilizer was observed. In the following discussion the model will first be constructed and afterwards it will be shown that the observations comply with the model.

In the growth equation a model for two influences has to be constructed. As the first one the change in level of fertility of the soil after repeated applications of fertilizer will have to be inserted. The second point is the construction of the part of the formula accounting for the adverse influence of the high nitrogen gifts.

<u>Change in level of the soil nutrient stock</u>

The repeated fertilizer applications will increase the availability of fertilizer stored in the soil. The storage will be equal to the original storage z and is increased by the quantity x that is supplied and decreased by the quantity x_e that is extracted. A factor a_2 accounts for the ratio between the amount of fertilizer stored and the effect of the stored nutrient on the increase in fertility level, expressed in units of the direct activity of the fertilizer.

The following equation is proposed as supplement on the terms given in formula 6:

$$f(x) = (a_1 - {}^{q}/x+z_g) \qquad z_g = z_o + a_2\,(1 - e^{-a_3 t})(x - x_e) \qquad (7)$$

The equation for z_g states that the original available quantity of nutrient z_o increases proportional to the difference between the applied and the extracted quantity $(x - x_e)$ and depends on the number of years of application t. The availability increases till a maximum value $z_o + a_2(x-x_e)$ for t infinite. For low values of a_3t the increase of the e-function is nearly proportional to t as is shown by the first term of the series expansion. A strong fixation is rendered by a small value of a_2, but also by a high value of a_3. An example of this increase to a maximum is given by DE VRIES, O. and VISSER, W.C. (1934).

In the experiment of Singh et al the effect of a gradual increase in fertility level by yearly application of fertilizer cannot be assessed by simple means as fig. 1 shows.

The data of each year, after condensing in a way as explained later, lead to a scatter diagramme in which for phosphate and the year 1958 the yields $\Delta Q + q$ are plotted against the gifts P_1, P_2 and P_3. Here ΔQ stands for the difference in maximum yield Q for the yield curves in which all the other growth factors are optimally applied but for the factor against which $\Delta Q + q$ is plotted. Along the horizontal axis $x_i + \Delta Q/a_i$ is plotted. This system of plotting brings the curves with different applications of factors $x_{jk} \neq i$ to coincidence.

The intercept on the horizontal axis, marked with z_g, represents the data of Singh et al about the nutrient available in the plots with zero phosphate application. If for the higher gifts z_g increases, this will be shown by a shift of the yield curve to the left increasing with the magnitude of the gift, which shift will follow from fitting the curve through the crosses and dashes. In this accurate experiment after 9 years of fertilizer application the shift is still too small to be assessed. For experiments of long duration, however, the model of formula 7 can be used to give an integrated description of the building up or depletion of the stock of fertilizer in the soil as dependent on the number of years and the magnitude of the gifts. It should be noted, that the extraction x_e may depend on the amount of dry matter produced, the amount of water that has leached the profile or whatever factor is governing the depletion of the stock of fertilizer in the soil.

The influence of adverse factors

A factor with an adverse influence on plant growth, considered in the most general way, will possess a range where the influence is favourable, but above a certain value the effect becomes unfavourable. The favourable range may be small and approach zero, but for a number of growth factors the existence of the two effects is well-known. The biological origin of the two effects will be different. The favourable effect may be due to the necessity of the factor as building stone of the dry matter, the unfavourable effect to the exclusion of another factor as is the case in antagonistic action.

For the unfavourable action of an excess in nitrogen the size of the plant is commonly mentioned. The yield reduction is due to lodging because the crop was "too heavy". This means that above a certain value of q the harmful factor x_n influences the yield according to a negative factor b_n.

In fig. 2 this is shown in the section ABC of the three-dimensional diagramme by the line BC. In the favourable range the yield curve is given by line AB. The term of formula which depicts such an adverse activity of the growth factor is given by:

$$f(x) = \left(1 - \frac{q}{Q - b_n x_n + c}\right) \quad \text{or} \quad fx = \left(1 - \frac{q}{a_i x_i - b_n x_n}\right) \qquad (8)$$

The formula states that some excess of nitrogen $(x_n - x_Q)$ will decrease the yield as soon as the yield is in excess of Q. This yield potential is, however,

governed by the growth factor x_i which is limiting the yield, so that Q can be replaced by $a_i x_i$. Because it can be shown that in case in formula 6 the numerator q is zero, the denominator $a_i x_i$ has also to be zero. And this is only possible if in formula 8 the constant c is zero. In fig. 2 this zero-value of c shows itself in the fact that line CD goes through the origin of the $x_i - x_n$ system. Between the x_n-axis and the line CD an area of zero yield is present, which means that no use can be made of a certain part of the available nutritive value of x_i.

The equation 8 accounts for this difference between the total stock z_g and the available stock z_o in case of adverse effects of every factor x_i, as illustrated in fig. 3.

Determination of the constants

If the section DBE is taken from fig. 2, as depicted in fig. 3, it is obvious that five unknowns which have to be determined are z_g, z_n, α, a_i and Q, of which one follows from the others, and as fifth F. A first approximation is obtained by a graphical solution. In this solution a simplification is used. The curvature of the yield curve at point B is assumed to be constant and not influenced by the level of the other growth factors. This simplification is acceptable because, due to the place in the formula, F is subjected to a rather large error.

If for different values of x_i the oblique and horizontal asymptotes are brought to coincidence and the curvature is assumed invariant, the curves will fully coincide. This coincidence can be attained by adding to any value of q a shift $Q - Q_i = \Delta Q$ and to every x a value $\Delta Q/a_i$, in which Q has the highest value with which the other curves have to coincide and Q_i is the maximum yield which has to be brought to coincide. This can be done by shifting the two scatter diagrammes over each other. The magnitude of the shifts is indicated by marking the zero points of the x and q-axes at the position of coinciding. If the availability z_g of the soil nutrient is constant, then the markings of the zero points are situated on a line parallel to the oblique asymptote. Is an adverse factor present, then the oblique asymptote and the line OE of the zero points diverge as indicated in fig. 3.

Now from fig. 2 can be derived that the following relations hold:

$$\alpha_i = \frac{a_n a_i}{a_n + b_n} \qquad z_n = \frac{b_n}{a_n a_i} Q \qquad (9)$$

Often the determination of α_i is more accurate than that of a_i. In case a_n and b_n are known, a_i can be determined via α, a_n and b_n.
If on the other hand α_i and a_i are known, then with a_n an estimate can be made about b_n. This may be useful in case the harmful excess of fertilizer is for practical reasons kept so small that b_n becomes inaccurate.

For elaboration of experimental data over a number of years, taking into account that the value of b_n may vary, causing variation in the apparant availability of the soil nutrients, for z_o a formula has to be used reading:

$$z_o = z_g - z_n - z_t$$

$$z_o = z_g - \frac{b_n Q}{a_n a_i} - a_3(1 - e^{-a_3 t})(x - x_e) \qquad (10)$$

The value of z_g is obtained in fig. 3 as the distance BE where Q is zero, in case z_t is zero. Otherwise this distance BE renders a value $z_g - z_t$.

The experimental results for nitrogen

The data of Sing et al (1967) for yields q of coxfoot grass, to which different gifts of nitrogen x_n, potassium x_k and phosphate x_p were given, were plotted as $q + \Delta Q$ against $x_n + \Delta Q/a_n$. For $Q = Q - Q_i$ the value of Q was arbitrary chosen as the maximum yield for the few combinations of P and K which produced the highest average maximum yield. The nitrogen gifts were 0, 57, 114 and 171 kg/ha.

In fig. 4 the results of the graphical shifting technique as well as the points along the AB lines marking the size of the shifts are given. The years 1949 and 1951 do not show the unfavourable effect, so that b_n is small or zero. In the successive years the depressions due to excess of nitrogen increase regularly. The results are given in table 1.

Table 1

Year	a_n	b_n	z_o	$10^{-2}Q$
49	58	-0.5	23	70
51	60	-0.5	20	52
54	54	5	40	54
58	53	8	43	59
60	55	13	40	59
62	58	10	34	60

The value of Q shows that in the years '51 and '54 the grass was probably cut somewhat early, and in '49 the well-known abundant grass growth of the first year after sowing is obvious. The values of z_o are low in '49 and '51, possibly due to a low organic matter content and a still somewhat tardy nitrification. The value of a_n is rather constant.

The only systematic variation in the parameters is the gradual increase of b_n with time. In fig. 5, b_n was plotted against the successive years and it appears that b_n can be expressed as:

$$b_n = 0.95\ t$$

Further can be used

$$a_n = 55$$
$$z_o = 39$$

For the favourable effect of nitrogen the formula can be given as

$$f(n) = \left(1 - \frac{q}{a_n(x_n + z_{no})}\right) \qquad f(n) = \left(1 - \frac{q}{55\,(N + 39)}\right) \qquad (11)$$

For the unfavourable effect the parameters of P and K have first to be assessed.

The results for potassium

The results of the elaboration of the yield data and the potassium application are given in fig. 6. For the year 1949 the effect of potassium on the yield was practically absent and no potassium curve could be constructed. For 1951 a curve has been made but in this year the yield depressions are also very small and the reliability of curve and shifts is slight. In the curves the value of α_k can be more accurately assessed. This was therefore done and with the help of a_n and b_n in formula 9 the value of a_k was determined.

In table 2 the values, which were obtained for the parameters from fig. 6 are given.

Table 2

Year	α_k	a_n/a_n+b_n	a_k	z_{kg}	$10^{-4}z_n/q$	$10^{-2}Q$	z_{kn} for Q
51	100	1.00	100	36	-1	51	-0.5
54	93	0.91	102	25	9	50	4.6
58	96	0.87	110	29	14	51	7.3
60	87	0.81	103	32	22	54	11.7
62	98	0.85	115	32	15	58	8.4

The value of a_k is reasonable constant and an average value of 109 can be used for all years. The value of z_g, measured as the difference between the intercepts of the asymptote and the line of the shifts with the line q = Q also is sufficiently constant, though the differences in z_g certainly will be significant. That z_n/q increases with the successive years was to be expected, because b_n is a function of t. Instead of the data obtained for the separate years, the average values can be used. The formula for the activity of potassium can be written as follows:

$$f(k) = 1 - \frac{q}{a_k(x_k + z_{kg} - \frac{b_n q}{a_n a_k}) - b_n(x_n + z_{no})}$$

$$= 1 - \frac{q}{a_k(x_k + z_{kg}) - b_n(\frac{q}{a_n} + x_n + z_{no})}$$

$$= 1 - \frac{q}{109(K + 31) - 0.95\, t(\frac{q}{55} + N + 39)} \qquad (12)$$

In this formula 12 the equations 7, 8, 9 and 10 are combined. The formula accounts for the effect of the asymptotic plane BCD of fig. 2 and represents the unfavourable effect of excess nitrogen as well as the effect of the potassium.

The results for phosphate

The diagrammes for the yield as influenced by phosphate are given in fig. 7. The elaboration of the diagrammes is the same as for potassium. The data for 1949 did not allow to construct a curve, the data for 1951 are less accurate than those for potassium and the curve was made by relying

heavily on the curves for the other years.

In table 3 the values for the parameters for the effect of phosphate are represented.

Table 3

Year	α_p	a_n/a_n+b_n	a_p	z_{pg}	$10^{-5}z_n/q$	$10^{-2}Q$	z_{kn} for Q
51	315	1.00	315	31	-3	50	-0.1
54	290	0.91	315	21	27	51	1.3
58	210	0.87	240	34	60	52	3.1
60	200	0.81	245	29	100	53	5.3
62	175	0.85	205	28	88	56	4.9

From the data of table 3 it is obvious that the reaction of the yield on phosphate is not as exhaustively explained by the formulae as was the case for potassium. The value of z_g is tolerably constant, but a_p quite clearly decreases with time.

The average value appears to be: $a_p = 360 - 12\,t$

It is not clear whether this effect is an artefact due to small inaccuracies in the elaboration of the data, or the result of some unknown effect. It will be assumed here that the first explanation holds and that a_p on the average is 250.

The formula for the activity of phosphate can now be written as follows:

$$f(p) = 1 - \frac{q}{a_p(x_p + z_{pg}) - b_n(\frac{q}{a_n} + x_n + z_{no})}$$

$$= 1 - \frac{q}{250(P + 29) - 0.95\,t(\frac{q}{55} + N + 39)} \qquad (13)$$

As was the case in formula 12 this equation accounts for two effects on the crop yields for P as well as for the adverse influence of N. If results for separate years are desired, the parameters of that year instead of the average value for 6 years should be inserted.

The effect of unknown factors

In formula 6 one of the terms deals with the influence of the maximum yield Q_{max} which the plant is able to produce, and which depends on its genetic constitution. In this experiment the genetic limitation for the years 1954 through 1962 cannot be shown easily, because here the maximum yield depends on the intersection of the asymptotes for the two effects of nitrogen. For the years 1949 and '51 the maximum yield might depend on the genetic capacity.

If only a small number of growth factors is known, there is always a possibility that there are limiting levels not much higher than the point of intersection of the two branches of the nitrogen asymptotes, and this will influence the shape of the curves. In this experiment no simple graphical treatment can show what the value of this limit Q_i is, and this has to be found by calculation.

The mathematical treatment can do this in some detail by assuming that the successive limiting levels can be replaced by one average level $\bar{Q}$ for each of the influences. This means that in the mathematical elaboration the unknown factors can be accounted for by taking up in the formula a term:

$$f(Q_i) = \left(1 - \frac{q}{\bar{Q}}\right)^m \qquad f(Q_i) = \left(1 - \frac{q}{7500}\right) \tag{14}$$

Both $\bar{Q}$ and m are unknowns, but a non-linear multivariate curve fitting technique is able to solve the values for these unknowns which fit best in the solution of the problem.

A very simple calculation, in which m was assumed unity, rendered for $\bar{Q}$ the value of 75, which is strikingly of the same order as the value of Q in table 1 for 1949, the year in which no adverse effect of excess nitrogen was apparent. This can be considered as an indication that even the unknown factors can be accounted for by introducing a constant limiting level for them in the calculations.

The final model

The final model may seem of a confusing size. Computers will be needed to use it. The set-up of the model was discussed step by step to show that at any separate point the relevant biological knowledge can be formulated and given its proper place.

The model that has been evolved is:

$$\left(1 - \frac{q}{a_n(x_n + z_{no})}\right)\left(1 - \frac{q}{a_k(x_k + z_{kg}) - b_n(x_n + z_{no} + \frac{q}{a_n})}\right) \times$$

$$\left(1 - \frac{q}{a_p(x_p + z_{pg}) - b_n(x_n + z_{no} + \frac{q}{a_n})}\right)\left(1 - \frac{q}{Q}\right)^m = F \qquad (15)$$

This formula, containing 11 unknowns, can be used in a curve fitting technique in which the unknowns are solved. The elaboration can at will be carried out for separate years or for a range of years together.

The parameters, solved in the graphical elaboration, can be inserted in the formula in order to give a means of predicting the effect of an application of one of the fertilizers on the yield.

The value of F has not yet been solved in the preceding paragraphs. This value is obtained by calculating the difference $Q - q$ for the point of intersection of the oblique and the horizontal asymptote for all terms expressing the differences in parts of Q and multiplying these values.
In this example F is found to be of the order:

$$F = 2.10^{-4}$$

It should be remembered that no technique of calculation is able to give an accurate value for F, but the consequence of this is, that a deviation of F from the exact value has generally not much influence on the result for q.

The formula in which the provisional results of the graphical analyses are inserted, reads:

$$\left(1 - \frac{q}{55(N + 39)}\right)\left(1 - \frac{q}{109(K + 31) - 0.95t(N+39+q/55)}\right) \times$$

$$\left(1 - \frac{q}{250(P+29) - 0.95t(N+39+q/55)}\right)\left(1 - \frac{q}{7500}\right) = 0.0002 \qquad (16)$$

By inserting the values of the fertilizer application as well as the length of the time which the regime of fertilization has lasted, the yield is obtained as an equation of the fourth power.

Summary

For project design and advisory work it is important that a prognosis can be made of the yield of crops as a result of a complex cooperation of favourable and adverse growth factors. To give such a system of estimation of yields the largest possible general significance, the calculation has to be based on a mathematical model of which every part has to reflect as correctly as possible the relevant biological and agricultural knowledge. A model has been constructed which - taking into account that the complexity of the representation of the biological relations has to be able to explain a sufficient part of the variance of the data - shows how, based on acceptable biological principles, a model can be constructed.

One of the problems on which this investigation was directed, is the correct understanding of the action of adverse factors. The special type studied here is the factor which affects the level of the yield known commonly as lodging due to "the heaviness" of a crop.

It appears that the solution, based on the assumption of the level of the yield as the determinative factor in the definition, is not contradicted by the experimental data. It is, however, found that not only the level of the yield and the excess of the growth factor influences the yield depression, but that also a time dependent variable is active. The more years a certain fertilizer regime lasts, the larger the yield decrease becomes. This may be considered as an indication that some favourable factor is gradually depleted or some unfavourable factor is gradually built up. Here again, the same mathematical formula may define entirely different biological processes.

The model was constructed in order to have available a means of analyzing effects of growth factors, which are less manageable than fertilizers, as is the case with the factors dependent on the water management. A formal expression of the interaction of the growth factors is thought to be an advantage in dealing with problems of which the number and quality of the data often is on a lower level than is the case with the more easily manageable fertilizer experiments.

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