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THE MODEL OF THE WATER BALANCE AND NUTRIENT UPTAKE AS A BASIS FOR HYDROLOGICAL, AGRO-HYDROLOGICAL AND OTHER PROJECTS

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SUMMARY

Hydrological science can be summarized in hydrological and agro-hydrological models. The hydrological model best can be based on the water balance, which is a consequence of the law of conservation of matter. The agrohydrological model is based on the diffusion equation, which equation leads to the law of limiting factors. The importance of these models is, that the underlying principles are generally accepted. Further a wide range of processes can be described by these two physical relations.

The model can be used to check new scientific results for processes participating in the water balance or in plant growth. It can also result in indications for practical design. It may result in a description of capillarity or evaporation . But it also may result in an indication to carry out sprinkling irrigation.

The models both for the water balance and for plant growth are discussed and the state of affairs with respect to the least developed part of the adjustment technique is explained.

The advantage of using conceptual models

The mathematical description of a problem provides the possibility to investigate the qualities or costs and benefits of a design with mathematical instead of technical means. The problems usually are of a multi-dimensional, non-linear nature. It is expected that more powerful techniques for using these mathematical models will be found in future than are now available. This holds especially in the realm of curve fitting and adjusting of parameters to observations.

It is a valuable aspect of a mathematical model, dealing with a number of effects, that by checking on only one single effect it is possible to adjust the model in such a way that for many other effects accurate values can be derived.

The water balance model, if adjusted to the groundwater depth only, produces values for properties as the intensity of evaporation, the soil moisture content in the root zone or the required depth of drainage. The same results would also have been obtained if the adjustment would have been done on the data for river discharge or soil moisture stress.

The models for the water balance and the yield function enable one to assess a number of aspects of the presence of water in or on the soil. They can be used to determine moisture contents or moisture flow, physical parameters or practical design constants.

The great interest in this technique was proved by the discussions in Leningrad (1969), Tucson (1968) and Fort Collins (1967). These meetings centered mainly on hydrological problems. It is, however, of importance that the models are extended to other technical goals as agricultural problems. These are also complicated because of the great number of factors which govern plant response. In particular the adjustment technique is for these agricultural problems of considerable importance. All kinds of complicated technical or agricultural problems can be treated with the same adjustment technique, though with different models.

Type of problem

In the Netherlands the water consumption increases rapidly and it is difficult to let the water abstraction by pumped wells keep up with this increase. Surface water is considered to be less reliable in chemical and bacteriological respect and chemically treated water is expensive and less tasty. The use of groundwater for civil water supply has the disadvantage that agriculture may experience damage due to lowering of the water table and to the accompanying moisture deficiency. The increase in abstraction of water has put the rural population on the alert and financial compensation of the man-made drought damage is contemplated. This will increase the cost of water, a prime necessity, considerably. This increase in cost is considered undesirable. A compromise between the magnitude of the abstraction and the drought damage seems possible however, if the water is abstracted at sites with a deep water level, with soils with good moisture retention at sites where the extracted water is replenished sufficiently by seepage or by inflowing rivers or water courses. In the Netherlands the groundwater is found nearly everywhere less than 2 m below soil surface and an intensive system of drainage ditches is present. By weirs the ditch water table can be built up and runoff can be checked. It is the aim of the investigation to develop an optimum strategy for the extraction by deep wells, management of riverflow and agricultural cropping system, so that the returns from the viewpoint of the national income are optimum or the additional costs accountable to civil water supply, as well as the costs to the farmers are acceptably related to each other.

Problems of nature conservation, wildlife, fishing and recreation can also be taken into account, though in these fields of research the mathematical expression of the processes is still less well understood.

The intention with the mathematical model

The investigations dealing with the influence of water abstraction on agriculture and other interests will in the near future have to be repeated in several parts of the Netherlands. Therefore the research technique should be worked out to a routine method. The technique can also be used as a foundation for the design of river improvement, agricultural water management, tile drainage, cost - benefit calculations and as a criterion for agro-hydrological planning.

The model is therefore set up in such a way that expansion and increased accuracy for special calculations can be attained by attaching supplementary models. Use of the model can also be made to compare theoretical descriptions of several processes dealing with the water balance as to their accuracy by means of error calculation as part of the adjustment. In this respect the model can also provide an important series of theoretical applications.

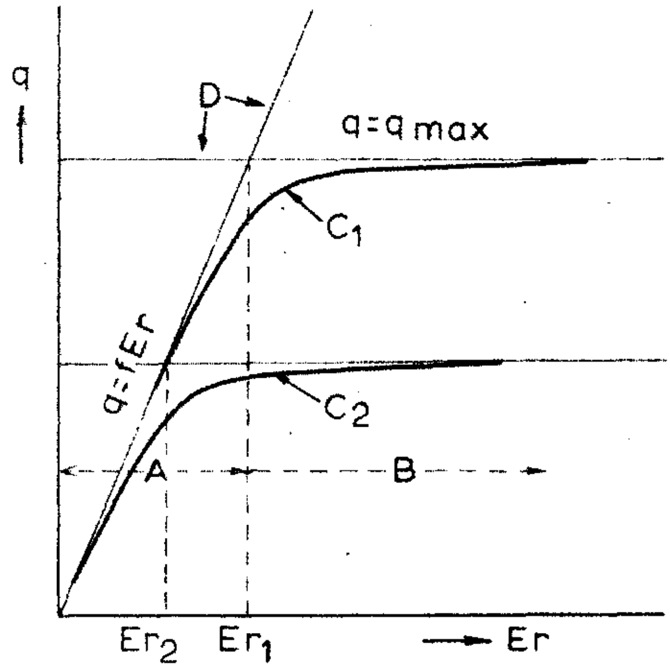
The model for plant response

Of the 750 mm of yearly rain in the Netherlands the largest user is the vegetation with an evaporation of 450 mm. The use by the plant therefore has to be determined most accurately. The vegetation uses water in dependence of the evaporative conditions of the climate or the potential evaporation E_o. The water use can also depend on the degree of dessication of the soil, expressed by the soil moisture stress ψ, which allows a lower actual evaporation E_r than the one dependent on the climate.

The amount of water, needed by the vegetation is governed by the law of limiting factors, formulated by Blackman. This law appeared to be the consequence of uptake of moisture and nutrients by diffusion or other linear relations of the same kind (VISSER, 1965, 1968 a, 1968 b, 1969 a, 1969 b).

The smaller the plant is in its first stages of growth, the less water is needed for maximum growth. If the plant consumes more water than is needed, a luxury consumption takes place. If the maximum yield q_{max} in Fig. 1 is low, then the amount of water which necessarily has to evaporate to enable maximum growth, is small. The higher the maximum yield, the higher the real evaporation E_r where the luxury consumption begins.

Curve C_1 has a lower limit of optimal growth at E_{r1}, curve C_2 at E_{r2}.



A: indispensable water
B: luxury consumption
C: yield curve
D: asymptotes

fig. 1

Fig. 1. The water needed for plant growth is restricted to the quantity A. The water quantity B, in excess of A, constitutes the luxury consumption, with no need for yield increase. The higher the yield, compare curve C_1 with C_2, the more evaporation Er is needed for dry matter production, compare Er_1 with Er_2

The model for crop response is given by the equation:

$$\left(1 - \frac{T-T_o}{a\,\Delta T}\,\frac{\Delta q}{Q}\left(\frac{1}{q} + \frac{1}{Q-q}\right)\right)\left(1 - \frac{\Delta q}{\overline{\Delta q}}\right)^{n}\Delta q\left(1 - \frac{\Delta q}{C_1 \psi^{c2}}\right)\left(1 - \frac{\Delta q}{f\,E_r}\right) = -F\left(1 - \frac{2\Delta q}{\overline{\Delta q}}\right) \qquad (1)$$

I   II   III   IV   V   VI   VII

Ia   Ib   Ic

< Growth factors >< Mathematical factors >

The model is devided in 7 terms, numbered I to VII, each describing a growth factor or a mathematical factor. These factors are:

I plant size
II optimum yield increase
III zero growth of ripened plant
IV aeration
V evaporation
VI integration constant
VII sign of the integration constant

The first term consists of three parts, all three dependent on the size and age of the plant. The three parts describe:

Ia occupation of area or space
Ib ageing of the plant
Ic ripening of the plant

These properties are often considered as separate influences. But they are all described as functions of the product of the time of the year t and the temperature τ, and by the yield q, together with the biological maximum yield Q. It seems advisable to call these three parts aspects of the growth factor of the time-temperature sum, than to indicate them by separate biological names as leaf area, root density, ageing or ripening, suggesting in this way that it are independent growth factors.

Explanation of the growth model

Term I for the plant size

This term can be expressed as:

$$\frac{\dfrac{dq}{T - T_o}}{\dfrac{q(Q - q)}{(T - T_o)^2}} = a\, dT \qquad (2) \qquad \text{or} \qquad \frac{dq}{q(Q - q)} = a \frac{dT}{T - T_o} \qquad (3)$$

Equation 2 expresses that the time - temperature sum $(T - T_o)$ and the crop response q are linearly related. A larger time - temperature sum means a higher growth rate. There is, however, a time t_o, at which the growth starts and a temperature τ_o below which the plant ceases to grow. This can, to ease calculation, be simplified from

$$(t - t_o)(\tau - \tau_o) \qquad \text{to} \qquad (T - T_o)$$

Part Ia indicates that with an increase of $T - T_o$, growth Δq slows down, presumably because the plant intercepts more and more radiation with its leafs and occupies more and more of the soil volume with its roots and starts to compete with itself.

Part Ib shows, that the increase in growth Δq of the plant is proportional to the size q, the plant already has reached. The more cells are present the larger is the number that by cell division partakes in the growth process.

Part Ic explaines that the growth in the latter stages of plant life is proportional to the number of cells, still able to partake in the cell division. The number of cells Δq, which no longer take part in the cell division is proportional to the amount of growth $Q - q$, which is still to be expected. It is presumed that the maximum growth Q, due to biological principles, is fixed in the biological blueprint of growth, which is predetermined in the plant.

Term II for the optimum additional growth

In term II is expressed that the yield increase reacts according

$$\Delta q = \overline{\Delta q} \qquad (5)$$

Here $\overline{\Delta q}$ is the limiting level above which the plant cannot increase its rate of growth. This limit works in a different way than the limiting value Q in equation 3. The effect in formula 5 can partly be due to other non-limiting factors with a somewhat higher value than Q or $\overline{\Delta q}$.

Still these non-limiting factors excert an influence which is the larger the closer the limiting level nears Q or $\overline{q}$. These non-limiting growth factors generally will be unknown. Assuming that the level is about equal to $\overline{\Delta q}$, and that n factors with about this value for its limiting level are present, the effect of these factors is approximately described by bringing term II to the n^{th} power.

Term III for the ripening of the plant

The ripening of the plant means the cessation of growth. This is described by inserting a term T_{III} as

$$\Delta q = 0 \quad \text{or} \quad T_{III} = 0 - \frac{\Delta q}{1} \quad \text{or} \quad T_{III} = - \Delta q \qquad (6)$$

This zero yield brings a negative sign in the formula.

Term IV for aeration of the root zone

This term expresses that the yield increase Δq depends on the soil moisture stress ψ, which in its turn determines the air content λ. According to CURRY (1961), the exponent c_2 is of the order of 4. BAKKER (1970) found a value of 3. The diffusivity is considered to be linearly related to the air content, so that the equation for plant growth in the model can be given by

$$\Delta q = C_1 \, \psi^{c_2} \qquad (7)$$

Term V for the availability of soil moisture

The availability of soil moisture is expressed by the real evaporation. The yield q or the yield increase Δq is directly proportional to the opening of the stomata and the uptake of carbondioxyde. This entering of carbondioxyde occurs at the same moment that water vapour flows out of the stomata. This leads to a direct relation between evaporation and growth, though the influence is indirect. (VISSER, 1963)

The increase in yield is expressed as

$$\Delta q = f \, E_r \qquad (8)$$

This formula holds up to the yield level due to the operating limiting factor.

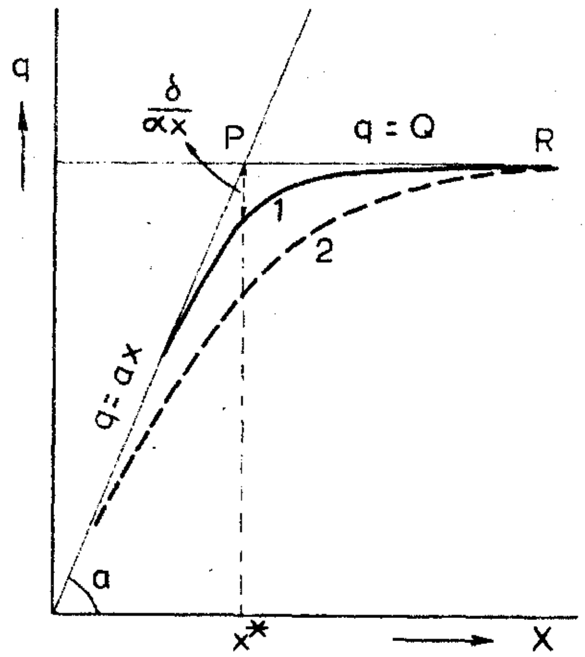


Fig. 2. The yield curve 1, a product of terms of the type of formula 9, shows that the integration constant F is the product of the quantities δ/ax between the yield curve and the intercept of the asymptotes. The neglected factors have a main influence on the magnitude of F and produce curves of the shape and position of curve 2

Term VI for the integration constant

The diffusion equation only can act as basis for the yield function because the plant possesses a certain flexibility. The uptake of growth factors needs not take place in an entirely fixed ratio. Therefore the yield observations do not follow precisely the two asymptotes in fig. 2, but follow a transition curve given by the broken line1. A difference between the yields with fixed and flexible ratio exists, which can be rendered by

$$\frac{\delta}{ax} = \left(1 - \frac{q}{ax}\right) \qquad (9)$$

If there are n growth factors inserted in the yield equation, then at the point of intersection P in fig. 2, with a value of each growth factor x_i^*, a distance δ_i is present, meaning that F has a value of

$$F = \left(\frac{\delta}{a_i x_i^*}\right)^n, \qquad (10)$$

Selecting the scales in such a way that $a_i x_i^* = 1$ leads to:

$$F = \delta^n$$

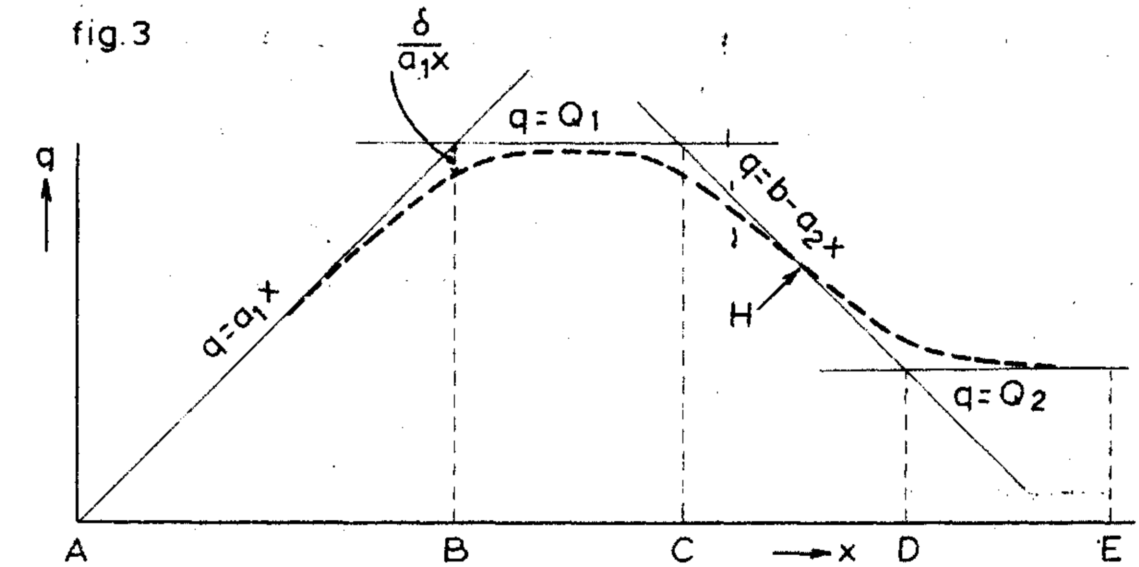
But F can also have a high value resulting from neglected factors. If such factors are not inserted in the yield equation, they are silently taken up in the value of F_1 according:

$$\left(1 - \frac{q}{a_1 x_1}\right)\left(1 - \frac{q}{a_2 x_2}\right) = \frac{F_1}{\left(1 - \frac{q}{a_3 x_3}\right)\left(1 - \frac{q}{a_4 x_4}\right)} = F_2 \qquad (11)$$

If $a_3 x_3$ is about equal to q, then such a term approximates the value 0. Is, however, $a_3 x_3$ much larger than q, then the value approaches 1.

If $a_3 x_3$ or $a_4 x_4$ are large, F_2 will be about the same as F_1. Are these values small, then F_1 is divided by a product of small values and F_2 will become much larger than F_1. This means that the distance $\delta / a_i x_i^*$ between the intersection of the asymptotes and the transition curve is large, as for curve 2 in fig. 2.

fig.3



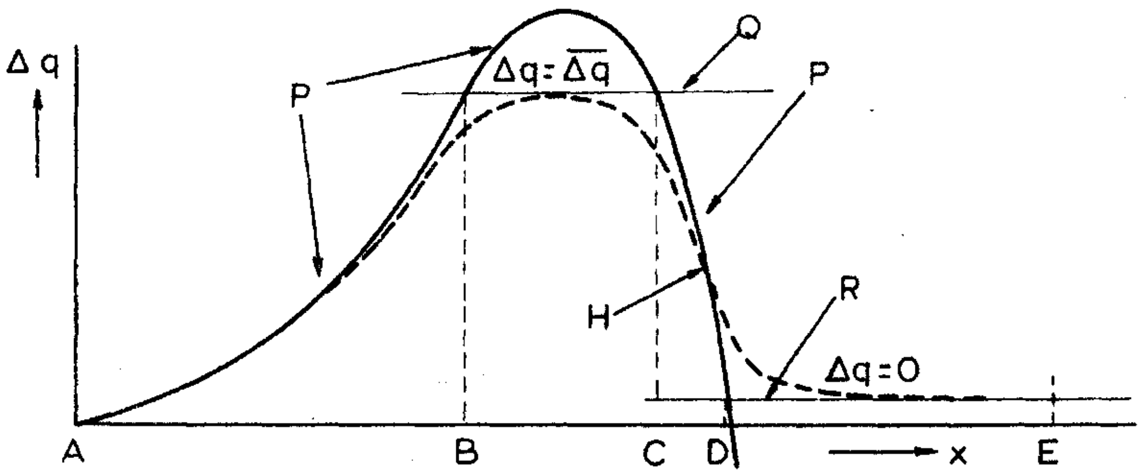


fig.4

Fig. 3. The value of the integration constant F is found as the product of the values δ/ax. At point B this value is positive, at D it is negative, and in the multiplication F has to become negative

Fig. 4. The change of sign of F is obtained by inserting a term of the type $(1 - \frac{q}{q^*})$ in which q^* has the value of q at point H. In the yield equation depicted by curve P, this value is assumed to be the mean $\overline{\Delta q}$ for asymptote Q and q = 0 for asymptote R

It is interesting to note that it is customary to consider the point R, where in fig. 2 line 2 and the asymptote coincide, as the end of the active influence of the growth factor x. As was explained, in the range PR the yield increase, shown by line 2, is dependent on the neglected factors and not on the growth factor x. The yield increase in this range should therefore not be used in a cost-benefit calculation for the growth factor x.

Term VII for the sign of the integration constant

The last term does not describe a growth factor but a mathematical relation, governing the sign of F.

As was explained, the value of F is found by multiplying the δ_1/a_1x_1 values. Now it is clear that in the range from A to H along the dotted yield curve in fig. 3 the value of δ/ax is positive. From H onward to higher values of x the value of $\delta/B-a_2x$ is negative, however.

If the yield curve shifts from a position below the asymptotes to a position above these curves, the value of F has to change sign.

This is done in fig. 3 by adding a term:

$$\left(1 - \frac{2q}{Q_1 + Q_2}\right) \qquad (12)$$

which term changes sign when

$$q = \frac{Q_1 + Q_2}{2} \qquad (13)$$

In a formula consisting of the terms I, II and III - see formula 1 - which is depicted in fig. 4, the value of Q_2 is zero and the value of Q_1 is given by $\Delta q = \overline{\Delta q}$.

Formula 12 therefore takes the shape of

$$-\left(1 - \frac{2\,\Delta q}{\overline{\overline{\Delta q}}}\right) \qquad (14)$$

The value of $\overline{\overline{\Delta q}}$ is for mathematical reasons often somewhat higher than $\overline{\Delta q}$ and has also to be distinguished from the third horizontal limit Q in term 1 of formula 1.

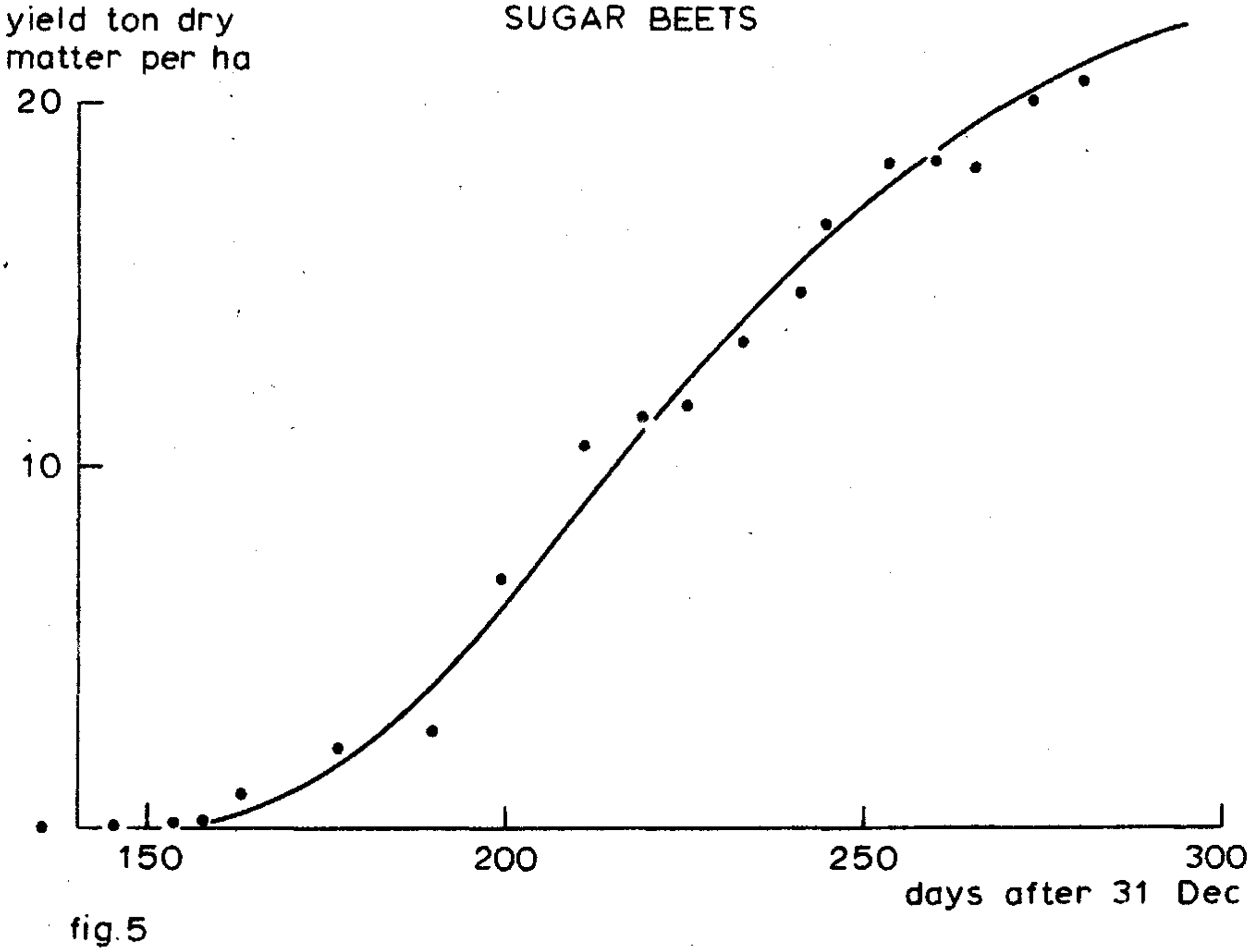


Fig. 5. Yield in tons of dry matter per ha as function of time according the formula

$$\frac{1}{Q} \log \frac{q}{Q-q} = a \log (T - T_0) + b$$

Data of SIBMA (1968) graphically adjusted. T = time t x temp ¶

The validity of the expression of the time factor

The yield model is expressed in formula 1 as the daily yield increase in order to be able to account for the daily changes in rainfall and evaporation. If the influence of time and temperature is a steady one, the formula can be integrated and the expression for the first term becomes:

$$\log \frac{q}{Q - q} = a\,Q \log (T - T_o) + b\,Q \qquad (15)$$

The flexibility of this formula, with 4 constants Q, T_o, a and b, to be assessed,is large, so that repeatedly determined yields can easily be fitted to the curve according formula 15. A good fit does not mean much, so that one has to rely on the theoretical base of formula 15. The adjustment of 4 unknowns, using data of Sibma (1968), could easily be carried out along graphical lines. The result is shown in fig. 5.

The values for the constants were found to be:

Q = 28 ton/ha $\qquad$ T_o = 775 degree days after 31 Dec.
$a\,Q$ = 2.75 $\qquad$ $b\,Q$ = 1.10

For the 8 crops for which Sibma gives observations the fit is very satisfying.

The model for the water balance

The model for the hydrological situation has to satisfy the requirements of good theoretical validity. It should be applicable in many different directions and it should allow extension into different comprehensive problems. The model for the water balance satisfies these requirements. It has in its simplest shape an absolute validity. It can deal with the more scientific problems as different kinds of discharge, capillary rise and with different processes of evaporation. The water balance can also express practical problems as the best depth and distance of tile drains, effects of water management with weirs, or time and quantity of water supply to the crop by sprinkling irrigation (VISSER, 1966 a).

Solution based on a two-layer problem

Single aspect solutions often are achieved by calculating the flow of water layer after layer through the profile as a multi-layer problem. This amounts to a numerical integration. If this is combined with a day to day calculation of the balance terms, it becomes a time consuming and expensive technique. A single layer problem is insufficiently able, however, to describe practically important problems, for instance to what depth of the capillary zone excess of water is present. Therefore a two-layer problem as depicted in fig. 6 is used to treat the water balance.

The upper layer

The moisture content of the upper layer increases by rainfall N and capillary rise V_c. It loses water by evaporation E_r and infiltration I. The balance is struck by increase or decrease of the temporary storage B_z. If this temporary storage rises above a certain value, then ponding will occur. But above a value $B_{z\,max}$ surface runoff A_{surf} will start.

The water balance of the first layer is given by:

$$B_{z1} - B_{zo} = N - I - E_r + V_c \qquad \text{if} \quad B_{zo} < B_{z\,max} \qquad (16)$$

$$B_{z1} - B_{z\,max} = A_{surf} \qquad \text{if} \quad B_{z1} > B_{z\,max} \qquad (17)$$

The second layer

The second layer obtains its moisture from infiltration and loses it by capillary rise and different drainage processes, as drainage to tiles and furrows A_{tile}, drainage to deeper ditches farther away A_{far} and drainage through the deep subsoil to the sea or to distant main rivers A_{seep}. This seepage through the deep subsoil can be positive or negative and can supply water to the second layer.

The water balance of this second layer is given by:

$$\Delta B_h = I - V_c - \sum A \qquad (18)$$

Here B_h stands for the storage of capillary tied water and varies only if the groundwater depth W or the capillary flow V_c is changed.

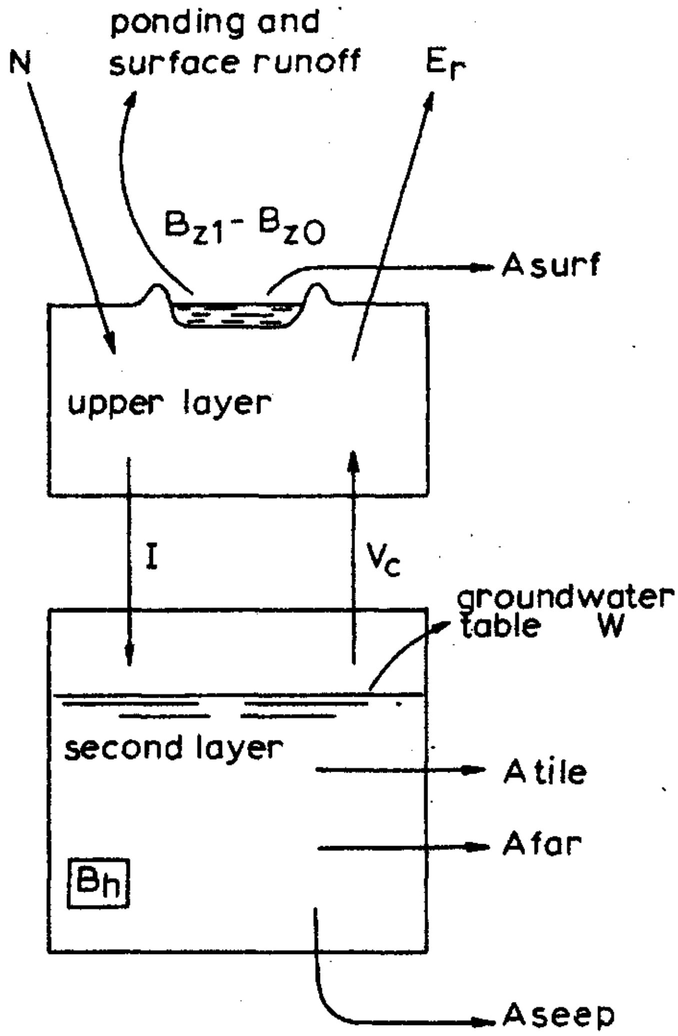


Fig. 6. The water balance is split up over an upper layer accounting for rainfall N, infiltration I, real evaporation E_r and capillary rise V_c. It also accounts for storage B_z, and the surface runoff and ponding, which are consequence of the storage.
The second layer accounts for the capillary storage B_h and several types of drain discharge A as well as infiltration I and capillary rise V_c

The quantitative values in the mathematical model for the upper layer

The terms in the model for the upper layer can be expressed as functions of the known quantities N and E_o and the starting values for the first day W_o and B_{zo}.

These equations are:

$$B_{z1} - B_{zo} = N - I - E_r \qquad (19)$$

$$- B_{zo} = N - I - E_r + V_c \qquad (20)$$

If B_{z1} is positive, the gradient will not allow a capillary rise of the soil moisture and in equation 19, V_c has to be zero. If, however, V_c as in formula 20 has a positive value, this means that B_{z1} has to be zero, as depicted in fig. 7. If $B_{z1} = 0$, the capillary rise will bring the water into the root zone and there it will be taken up by the roots and evaporated by the plant. E_r is in case of zero rain N and zero infiltration I equal to V_c. If, however, as in fig. 7, the upper layer is moist due to the temporary storage B_{z1}, the capillary rise will stop at the lower limit of the moist upper layer. The quantity A of moisture, situated in the lower triangle, will rise and fill up the upper triangle with volume B. If equilibrium is reached, A will be equal to B and the water level will, due to rain, sink from W_o to W_1.

The infiltration rate I is described by

$$I = (N + B_{zo})\, e^{-\beta W} \qquad (21)$$

For E_r two situations are possible. In the first situation the evaporation is limited by the climate and depends directly on the potential evaporation E_o. In the second situation the evaporation depends on the moisture content of the soil and is calculated with the use of the soil moisture stress ψ.

The alternatives are

$$E_r = g\, E_o \qquad (22)$$

$$E_r = d_1\, \psi^{d_2} \qquad (23)$$

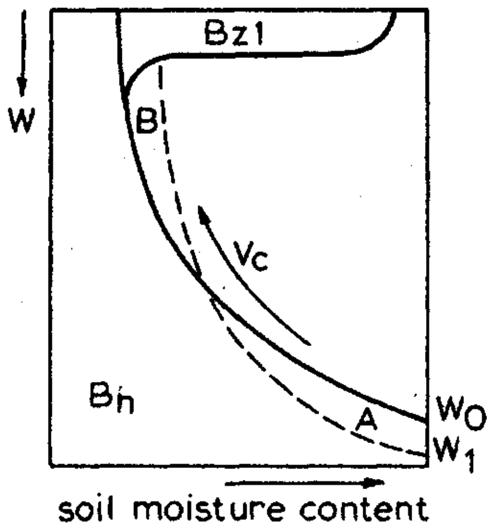


Fig. 7. If rain causes an increase of the temporary storage B_z, then this water decreases the gradient in the upper layer and the quantity of water A will be deposited at B below the moist zone. Due to rainfall the water table will go down

The criterion, which of the two formulae is valid, is found in the two equations

$$V_c = I - N - B_{zo} - E_r \tag{24}$$

and

$$\psi = \frac{-1}{\alpha} \ln\left\{1 - \left(1 - e^{-\alpha W_o}\right)\left(1 + \frac{V_c}{k_o}\right)\right\} \tag{25}$$

together with formula 22 or 23. If formula 22 is used and the logarithm becomes imaginary, then formula 23 should be substituted in formula 25 and the value of ψ can be solved. The value of B_{z1} can after solving be calculated from equation 19, because B_{zo} is known from the previous day. N is given, I can be calculated and E_r follows from the explanation, given by formulae 22 and 23.

The quantitative values in the model for the lower layer

The general equation 18 for B_h is given in quantitative value by inserting the formulae 21 and 22 or 23, whilst V_c is calculated according formula 24 in which negative values for V_c are taken to be zero.

The different values of the discharge A are calculated with the drainage formula according Hooghoudt

$$A_{tile} = B_1 (S_1 - W) + B_2 (S_1 - W)|S_1 - W| \tag{26}$$

$$A_{far} = B_3 (S_2 - W) \tag{27}$$

$$A_{seep} = B_4 (D_p - W) \tag{28}$$

The absolute value $|S_1 - W|$ in formula 26 has to be used to ensure that the sign of the two parts of the equation change at the same value of $(S_1 - W)$.

The larger distance of drainage in formula 27 enables one to neglect the second power term. The discharge as seepage through the deep subsoil in formula 28 does not influence the slope of the water table very much, so that the gradient can be taken constant. The area of flow is equal to the depth of the pervious layer D_p minus the water depth W.

The value for the water storage in the lower layer ΔB_h in equation 18 depends on the depth of the water table W_o and W_1 on two consecutive days and the rate of capillary rise V_{co} and V_{c1} on the same days.

The storage capacity, the amount of water stored per unit rise of the water table, is described by

$$\frac{dB_W}{dW} = C_1 \, \psi^{C_2} \tag{29}$$

This is a simplification of a more accurate formula for the desorption curve (VISSER, 1969 b) as this better formula resists integration. An other simplification is, that the relation between $\Delta\psi$ and ΔW is assumed to be

$$\Delta \psi = \gamma \Delta W \tag{30}$$

and

$$\gamma = \frac{d\psi}{dW} = \frac{(1 + \frac{V_c}{k_o}) \, e^{-\alpha W}}{1 - (1 + \frac{V_c}{k_o})(1 - e^{-\alpha W})} = \frac{e^{-\alpha W}}{e^{-\alpha W} - \frac{V_c}{V_c + k_o}} \tag{31}$$

Formula 31 was derived by differentiating formula 25.
The formulae 29, 30 and 31 lead to an equation for ΔB_h of the following type:

$$\frac{dB_h}{dW} = C_1 (\gamma W)^{C_2} \tag{32}$$

$$\Delta B_h = \frac{C_1}{1 + C_2} \left(\gamma_o^{C_2} W_o^{C_2+1} - \gamma_1^{C_2} W_1^{C_2+1} \right) \tag{33}$$

$$\Delta B_h = \frac{C_1}{1 + C_2} \left\{ \gamma_o^{C_2} \left(W_o^{C_2+1} - W_1^{C_2+1} \right) + W_1^{C_2+1} \left(\gamma_o^{C_2} - \gamma_1^{C_2} \right) \right\} \tag{34}$$

The value of W_1 is calculated from:

$$\gamma_1^{C_2} W_1^{C_2+1} = \gamma_o^{C_2} W_o^{C_2+1} - (I - V_c - \Sigma A) \tag{35}$$

The term between brackets is taken from formula 18.

The non-steady case

In fig. 8b a non-steady case is represented in which the capillary rise below the moist root zone supplies water from zone A to the dryer zone B and redistributes the stored moisture B_h.

This is calculated by assuming that infiltration does not reach the lower layer and that capillary rise does not cause water to leave this layer. Only discharge ΣA can affect ΔB_h and can change W_1. This change in the value of W is calculated by the first part of formula 34. Then the redistribution takes place. In that case ΔB_h is zero. The flow of water of zone A to zone B in fig. 8a causes the water table to fall. This is described by formula 33 in the shape of:

$$\frac{1 + C_2}{C_1} \Sigma A + \gamma_1^{C_2} W_1^{C_2+1} = \gamma_{1a}^{C_2} W_{1a}^{C_2+1} \qquad (36)$$

The first part of formula 36 is known. From the second part W_{1a} can be solved and the difference $W_{1a} - W_1$ is added to the lowering W $W_o - W_{1a}$ of the water table, calculated from the first part of formula 33, while accounting for the drainage in the same time interval, see fig. 8a and 8b.

Accounting for further details

In order to keep time and costs of computation on a reasonable level each aspect of the water balance is kept rather simple. If a certain aspect meets with more attention, it may be advisable to elaborate that aspect in more detail. A few instances are given.

The rainfall

Rainfall data are often obtained from rain gauges, other than the rain gauges at soil surface, which are considered the most accurate. A correction formula can be inserted in the model, describing the areal rainfall N_a as related to the point rainfall N_p of a rain gauge as

$$N_a = a_1 N_p + a_2 \qquad (37)$$

As the rainfall is the only absolute quantity in equations 16, 17 and 18 and is not multiplied by a factor, to be assessed afterwards, the

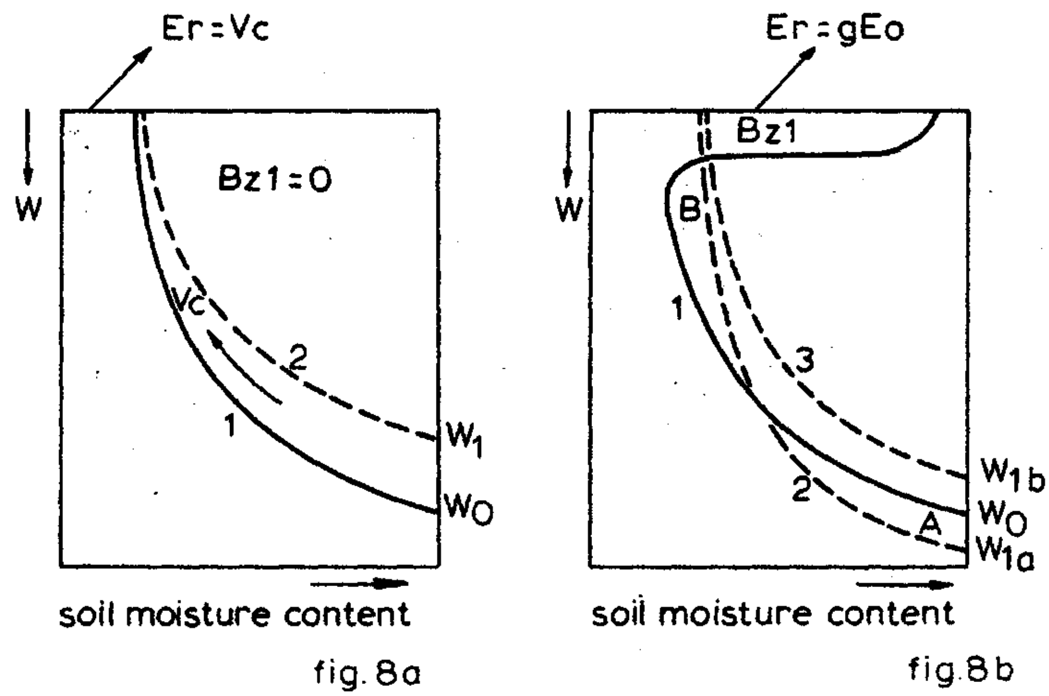


Fig. 8a, 8b. If rainfall and infiltration are of equal intensity, no storage B_{z1} will occur and the groundwater level will rise as in fig. 8a. Evaporation will be equal to capillary rise.
If rainfall has a higher intensity, the moist upper layer will stop the capillary rise and the volume B is filled with water from A.
Water table W_0 will change to W_{1a}. Then the stored water disappears due to infiltration and evaporation, the capillary rise is restored and the water table changes to W_{1b}

values for a_1 and a_2 can only be determined if the river discharge as a known absolute value, measured by a measuring weir, takes the place of the rainfall. in providing an absolute scale.

The river discharge, measured directly, is equated with A_{tile}, A_{far}, A_{surf} and a percentage m of A_{deep}, where m is an areal reduction factor, accounting for the deep flow, as far as it drains upon the river above the measuring weir.

In this way the catchment area is used as a large size rain gauge, the quantity of rain being equal to the discharge with corrections for evaporation, storage and part of the deep discharge. Out of such a large catchment area no water is lost by splashing. Neither is the reading influenced by turbulence of the air around the rain gauge. In this way the measurements of rainfall can be checked and the real rainfall can be determined with the easily readable gauge in its position at the up to now customary height.

The variation of the water level in the rivers

The discharge on the water courses is considered to be governed by S-W with S constant. Often this will be sufficiently accurate, but not if some type of river training is aimed at. In that case S is to be expressed as a variable quantity. This may be achieved by expressing S as $S_r - S = h$, in which formula S_r is the depth of the river bottom below soil surface and h the depth of the water in the river.

The water depth h is given by the formula of Manning:

$$Q = K_m \, h^{2.67} \, I^{0.5} \, \frac{(b/h + 1)^{1.67}}{(b/h + 2\sqrt{2})^{0.67}} \qquad (38)$$

The b/h function is simplified by assuming the slope of the river talus equal to 1 and can be further simplified by writing

$$\frac{(b/h + 1)^{1.67}}{(b/h + 2\sqrt{2})^{0.67}} = (0.49 + 0.08\, b/h) \qquad (39)$$

With the value for h, solved from formula 38, the discharge can more accurately be predicted. The effect of river improvement due to a change in b or h, or to the building up of the water table by weirs and changing the gradient I of the water surface of the river, can with the expansion of the model in this way be calculated in more detail.

The influence of water abstraction by wells

The primary target of the elaboration was to assess the damage by desiccation caused by water abstraction by pumped wells. Dependent on the hydrological situation different equations will have to be used for the drawdown curve.

The theoretical formulae, all derived on the assumption that the extraction per unit area is constant, irrespective of the distance to the well, are incorrect if used as part of the water balance model, because in this model the amount of extractable water varies with the drain discharge and the real evaporation, both dependent on the depth of the groundwater.

To calculate this effect the area around the well is devided in a number of rings for which the amount of extractable water is not taken constant but as an approximation is given by:

$$Ex = A - Br \tag{40}$$

This relation, inserted in the formula for the water abstraction, gives as result

$$h_2 - h_1 = \frac{A}{2kD}\left(R^2 \ln\frac{r_2}{r_1} - \frac{r_2^2 - r_1^2}{2}\right) - \frac{B}{3kD}\left(R^3 \ln\frac{r_2}{r_1} - \frac{r_2^3 - r_1^3}{3}\right) \tag{41}$$

Here r_1 and r_2 are the radius of the inner and outer circle of a ring and R is the radius of the area that is affected by the abstraction. It may represent the density of a network of wells. In h the height of the water table above some zero plain is expressed.

The values of A and B are calculated for each day and have such values that the extraction Ex fits exactly in the following version of formula 18 extended with the abstraction term:

$$\Delta B_h = I - V_c - \sum A - Ex \tag{42}$$

Investigations are carried out, whether instead of formula 40 other functions as inverse or logarithmic formulae for r in E_x give a better value for the abstraction or allow the use of wider rings in smaller numbers.

Formulae as 41 will seldom be used for adjustment because too few pumping stations with sufficiently frequent read observation wells in the neighbourhood are available. The formulae will more frequently be used for simulation.

The process of adjustment

Much about the adjustment technique can be found in the thesis of IBBITT (1970). However, also with the vast technique developed in this thesis still the experience is, that the last difficulties have not yet been overcome.

The aim of the adjustment technique is, to make the elaboration entirely automatic and the calculation as independent of the starting values as possible. Further it is expected that with some 200 repeats of the calculation of the model a sufficiently accurate result will be obtained.

For the calculation of the 15 parameters:

β, B_1, B_2, B_3, B_4, S_1, S_2, D_p, C_1, C_2, D_1, D_2, g, α and k_o

this aim has not yet been reached.

From the day after day calculated value of W, for every day at which W is also observed, the difference is taken and the error is calculated.

By changing the parameters it can be established, whether the error increases or decreases. The latter result shows that the change of the parameter was in the correct direction.

The step size, the change in the parameter which is applied to lower the error, should preferably be based on three principles. These are:

1. The step size for the separate parameters should be chosen, so that the lowering of the error is about the same for all parameters.
2. The step sizes which are small, due to a large influence on the error, are enlarged to arrive at small errors more quickly.
3. The distance from the starting value to the parameter value P_{min} at which the minimum error is obtained, all other parameters kept constant,

is calculated and if large, the step size is enlarged to get to the parameter combination with the minimum error more quickly.

For each set of constants the step sizes are calculated and applied to move from the starting values to the absolute minimum point with a minimum of calculations and a maximum of successful elaborations. This approach to the minimum point follows a poligon. The direction to the minimum is calculated anew after a small number of steps have been taken in the same direction.

The number of successful elaborations is mentioned to allow for the uncertainty in the calculation technique, due to which a number of calculations do not produce an acceptable result, often without an indication as to the cause of the irrational values. The technique still needs improvement to lower the percentage of these less useful results.

The step size Δp_i^* which is ultimately applied in the adjustment technique, is calculated according:

$$\Delta p_i^* = \underbrace{\frac{\Delta p_i}{\Delta Sp_i}}_{1} \; \underbrace{\frac{n(Sp_i - Sp_{i+\Delta p_i})}{\sum_{i=1}^{n}(Sp_i - Sp_{i+\Delta p_i})}}_{2} \; \underbrace{\frac{n(Sp_i - Sp_{i\,min})}{\sum_{i=1}^{n}(Sp_i - Sp_{i\,min})}}_{3} \; \underbrace{\overline{\Delta p}}_{4} \qquad (43)$$

Here p_i stands for parameter i, Sp_i for the error, calculated at this value of p_i, the other parameters remaining constant. $Sp_{i\,min}$ stands for the error if p_i is changed till the error becomes minimum.

The first term calculates the step size for a unit value of the decrease in error. The fourth term is a constant value applicable to every parameter, used to be able to use step sizes of larger or smaller magnitude if desired.

The second term accounts for the second principle of increasing the step size of a separate parameter so that the error diminishes more rapidly.

The third term accounts for the second principle of increasing the step size for the parameter value which differs most from the value at the minimum error. In this way the best estimate of the parameter combination for the solution is used to obtain the absolute minimum of the ultimate solution more rapidly and with more certainty.

Principle 1 is used to arrive at the solution with a same number of steps for each parameter.

Principle 2 is inserted to decrease the error more quickly and deminish the number of cases in which with formula 23 and 25 the value of ψ has to be solved. With the tentative approximations of the parameters this number of time consuming calculations may be heigh and a quick decrease of the error may reduce the number of these calculations.

Principle 3 is built in to strengthen the influence of the available indications of the solution in the calculation and weaken the influence of the starting values for the parameters.

The accuracy of the model for the water balance

The accuracy with which the model can describe the observations depends on the correctness with which the model accounts for the physical relations well as for the efficiency of the adjustment technique. As was already mentioned, the latter is the more uncertain part.

In fig. 9 an instance is given for an adjustment technique which is selected because it more easily allows graphical representation. The technique consists of two parts both based on principle 3.

First the distance $\Delta p_{i\ min}$ is determined. Each of the parameters is corrected with half the distance between the starting values and the parameter at the minimum, all other factors remaining constant. The next parameter is calculated after the first is corrected. The starting values gave a sum of squares of deviations $\sum r^2$ of 27390. After successively applying the $\Delta p_{i\ min}/2$ correction to all parameters, the error diminished to 10922.

In the second part of the calculation the value of $\Delta p_{i\ min}$ again was determined, but now all parameters obtained a n times $\Delta p_{i\ min}$ correction at the same time with n varying from 0 to 1.2.

The $\sum r^2$-values are indicated in fig. 7 along the vector 0 and vary from 10922 to 7405 with a minimum at n = 1.1 of 7327.

It appeared during the calculation of $\sum r^2$ for every separate parameter that the resulting sum of squares mainly was of the order of 10 000, and was lowest for parameter G with $\sum r^2$ = 7882.

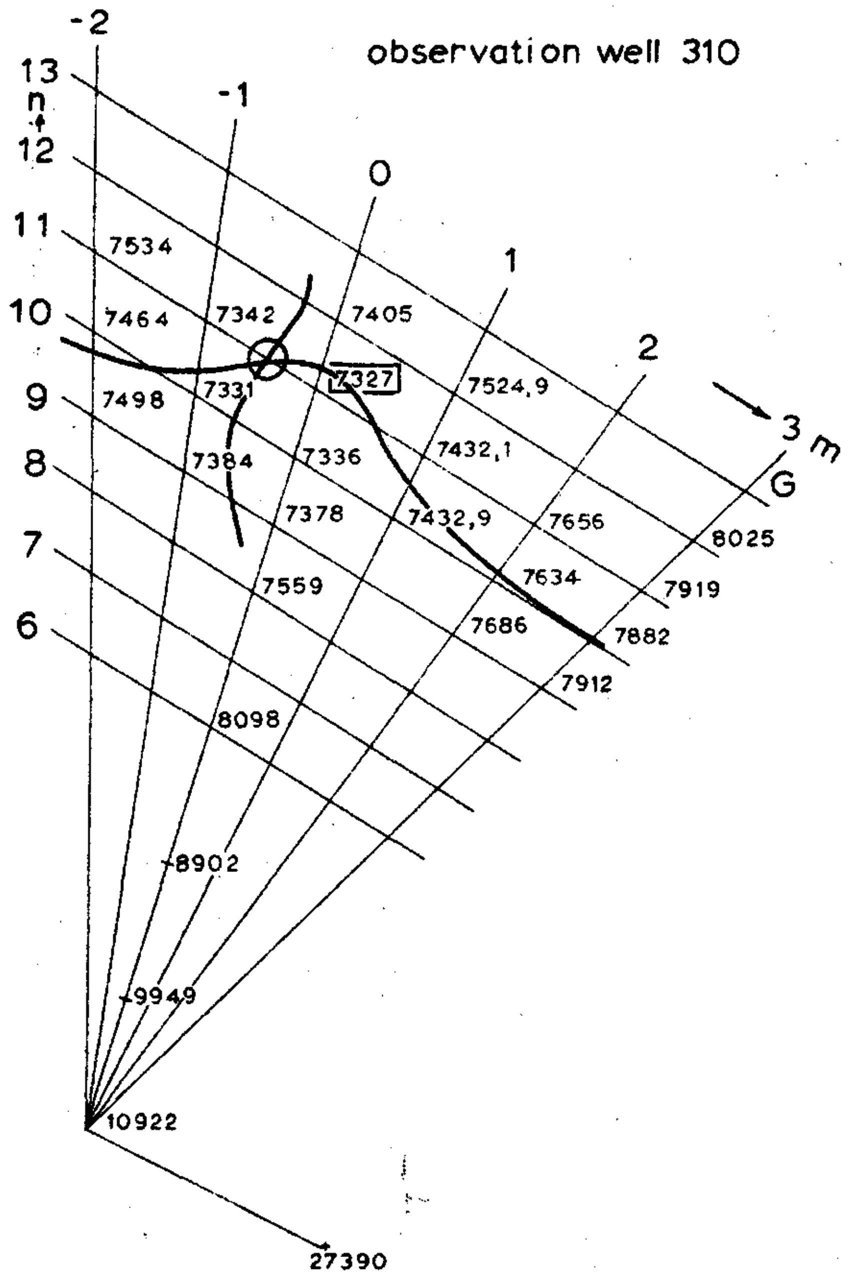


Fig. 9. An automatic calculation of the combination of parameters, which produces the minimum error, gives besides acceptable results also deviating parameters, of which deviations the cause is not yet understood nor the remedy known

It is, however, the authors opinion that the models and the solution of the adjustment technique on which this article is based still leaves room for quite some further development.

As G apparently can give extra information as to the minimum sum