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Wageningen

RELATIVE HUMIDITY FROM WET AND DRY BULB THERMOMETER

(CENT. SCALE)

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I N D E X

1. INTRODUCTION

In humidity investigations often a thermodynamical hygrometer, called psychrometer, is used.

This instrument consists of two ordinary mercury-in-glass thermometers. One of them, the dry bulb thermometer, indicates the prevailing air temperature. The other, the wet bulb thermometer, is kept wet by means of a mouslin cloth which is wrapped around the bulb and is soaked with water. When the surrounding air is unsaturated water from this wet bulb will evaporate. The heat needed for this evaporation is obtained from the mercury of the wet thermometer, so that it registers a lower temperature. The difference between the dry and wet bulb temperature is an indication for the humidity of the air.

2. PSYCHROMETRIC CONSTANT

The humidity-temperature relationship is shown in the expression

$$\gamma = \frac{e_w - e}{TA - TW} \qquad (1)$$

where e = actual water vapour pressure at temperature TA (mm Hg)

e_w = saturated water vapour pressure at temperature TW (mm Hg)

TA = temperature dry bulb (^{o}C)

TW = temperature wet bulb (^{o}C)

γ = psychrometric constant (mm Hg . $^{o}C^{-1}$)

The psychrometric constant γ depends on temperature, atmospheric pressure and wind velocity.

Because of the physical incompleteness of the proportional constants, for the calculations of γ various assumptions may be made. One of the assumptions is that all the heat required to vaporize the mass of water is obtained from the surrounding air. Starting from the energy

balance equation

$$R_n = LE + H \qquad (cal.\ cm^{-2}\ .\ S^{-1}) \tag{2}$$

where R_n = the energy flux of net radiation

LE = the flux of latent heat into the air

H = the flux of sensible heat into the air

this assumption means that R_n is neglected.

From this it follows that

$$LE = H \tag{3}$$

Equation (3) can also be written as a transport equation

$$-\ (L\rho_a\ \varepsilon/p_a)\ K_v \frac{de}{dz} = -\ \rho_a\ c_p\ K_h \frac{dT}{dz} \tag{4}$$

where ρ_a = density of the air $(g.\ cm^{-3})$

L = latent heat of evaporization of water $(cal.g^{-1})$

ε = ratio of molecular weight of water vapour to dry air (= 0,622)

p_a = atmospheric pressure (mm Hg)

c_p = specific heat of dry air at constant pressure $(cal.g^{-1}.{}^oC^{-1})$

K_v = eddy transfer coefficient for vapour $(cm^2.S^{-1})$

K_h = eddy transfer coefficient for heat $(cm^2.S^{-1})$

The coefficients K_v and K_h are depending on wind speed, but in a different way, especially for low wind speeds.

For higher wind speeds it is usually assumed that $K_v = K_h$.
So, in a restricted range of wind velocities (between 4 and 10 $m.S^{-1}$) eq. (4) can be written as

$$-\ (L\rho_a\ \varepsilon/p_a) \frac{de}{dz} = -\ \rho_a\ c_p \frac{dT}{dz} \tag{5}$$

or, expressing it in finite form

$$-\ (L\rho_a\ \varepsilon/p_a) \frac{\Delta e}{\Delta z} = -\ \rho_a\ c_p \frac{\Delta T}{\Delta z} \tag{6}$$

Hence

$$\frac{\Delta e}{\Delta T} = \frac{c_p\ p_a}{L\ \varepsilon} \tag{7}$$

Substitution of $\Delta e = e - e_w$ and $\Delta T = TA - TW$ gives

$$\frac{e - e_w}{TA - TW} = \frac{c_p \, p_a}{L \, \varepsilon} \qquad (8)$$

Combination of eq. (1) and eq. (8) yields

$$\gamma = \frac{c_p \, p_a}{L \, \varepsilon} \qquad (\text{mm Hg} \; . \; {}^{o}C^{-1}) \qquad (9)$$

Putting $c_p = 0.24$ cal.g^{-1} . $^{o}C^{-1}$, $p_a = 760$ mm Hg and $L = 588$ cal.g^{-1} at $15^{o}C$ in equation (9) gives

$$\gamma = 0.499 \text{ mm Hg} \; . \; {}^{o}C^{-1} \qquad (10)$$

Note 1. L depends on temperature, so substituting L-values related to TW = 0, 10 and $20^{o}C$ one finds $\gamma = 0.492$, 0.497 and 0.501 mm Hg . $^{o}C^{-1}$ respectively. Several investigators (SMITHSONIAN METEOROLOGICAL TABLES (1951), PAGE 365) describe the relation between γ and TW as

$$\gamma = 0.000660 \; p_a (1 + 0.00115 \; TW) \qquad (11)$$

At an atmospheric pressure p_a of 760 mm Hg substitution in eq. (11) of TW = 0, 10 and $20^{o}C$ yields $\gamma = 0.502$, 0.507 and 0.513 mm Hg . $^{o}C^{-1}$ respectively.

Note 2. In cases with wet bulb temperatures below $0^{o}C$ and a frozen cloth, the psychrometric constant should be multiplied by 0.882 i.e. with the ratio of the latent heat of evaporation of water to that of ice.

3. RELATIVE HUMIDITY

Relative humidity is the actual water vapour pressure (e) at temperature TA as a fraction of the saturated water vapour pressure (e_s) at temperature TA

$$h = \frac{e}{e_s} \qquad (12)$$

Rearranging expression (1) one gets the psychrometric formula

$$e = e_w - \gamma(TA - TW) \qquad (\text{mm Hg}) \tag{13}$$

thus

$$h = \frac{e_w - \gamma(TA - TW)}{e_s} \tag{14}$$

To avoid decimal points, relative humidity can be expressed as a percentage, so

$$rh = 100\ h = 100 \frac{e_w - \gamma(TA - TW)}{e_s} \tag{15}$$

4. TABLE

With the aid of eq. (15) a table of relative humidity values is composed for a range of increasing values of TA and (TA - TW). The computations were carried out on an IBM 1130 computer with FORTRAN program.

The table gives the relative humidity rh directly from reading of dry bulb temperature TA and wet bulb temperature TW. The given values relate to a temperature of $15^{\circ}C$ and an atmospheric pressure of 760 mm Hg.

Errors resulting from the use of this table for air temperatures above $-10^{\circ}C$ and a barometric pressure between 775 and 710 mm Hg will usually be within the error of observation.

5. LITERATURE

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