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**BIBLIOTHEEK  
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THE DESORPTION CURVE EQUATION

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## INTRODUCTION

It is very important to get a clear concept with respect to the desorption curve, representing the relation between the soil moisture stress and the soil moisture content, for studying the behaviour of water movement in the unsaturated zone.

If we could express the equation for the desorption curve mathematically, this equation will be available for solving many soil-water problems. Therefore, a desorption curve equation was evolved to make a check available on the accuracy of separated soil moisture stress - soil moisture content observation. Especially, the important properties as the capillary conductivity of the soil or the soil moisture stress can be calculated in a continuous way by using the desorption curve equation.

But, at the present time, no extensively authorized desorption curve is available. For example, the desorption curve equation

$$\psi = \frac{G(P^* - v)^n}{v^m}$$

(method 'ICW'<sup>1</sup>) is probably too laborious to calculate the parameters by numerical methods. Therefore, applying the mathematical expression of sigmoidal shape which relates the probability distribution of particles to their energy status for the desorption curve as well as the adsorption isotherm curve, the desorption curve equation

$$pF = A + B \ln \left[ (v/v_0)^{-c} - 1 \right]$$

is obtained.

This paper presents some practical applications of this desorption curve equation and considers the results.

## 1. DEVELOPMENT OF THE DESORPTION CURVE EQUATION

FINK and JACKSON (1973)<sup>2)</sup> have applied a mathematical expression of sigmoidal shape which relates the probability distribution of particles to their energy status for the adsorption isotherm curve and have obtained a good agreement between the experimental values and the calculated values by using this equation.

This equation is obtained as follows:

$$Pr = 1 / \left[ 1 + \exp (\alpha + \beta E) \right] \quad (1)$$

The parameters Pr and E are empirically related to the relative vapour pressure  $p/p_0$  and the water content W, respectively, subject to the conditions

$$W \rightarrow \infty \quad \text{for } p/p_0 \rightarrow 1 \quad (2a)$$

$$W \rightarrow 0 \quad \text{for } p/p_0 \rightarrow 0 \quad (2b)$$

and 
$$d^2 W / d(p/p_0)^2 = 0 \quad (2c)$$

these conditions hold for a single value of  $p/p_0$  in the range of  $0 < p/p_0 < 1$ . Conditions (2a, 2b and 2c) are met by defining  $Pr = (p/p_0)^c$  and  $E = \ln W$ , where c is a constant to be determined. Substituting into equation (1) and rearranging yields:

$$Pr = 1 / \left[ 1 + \exp (\alpha + \beta E) \right]$$

$$\exp (\alpha + \beta E) = (1 - Pr) / Pr$$

$$\alpha + \beta E = \ln (1 - Pr) / Pr$$

$$E = \frac{1}{\beta} \ln (1/Pr - 1) - \frac{\alpha}{\beta}$$

$$A = -\frac{\alpha}{\beta} \quad B = \frac{1}{\beta}$$

$$\ln W = A + B \cdot \ln \left[ (p/p_0)^c - 1 \right] \quad (3)$$

where A, B and c are adjustable constants by fitting the equation to the experimental data. Also  $p_0$  can be found by adjustment, though  $p_0$  can also be determined in the laboratory.

Next, the application of the equation (3) to the desorption curve is as follows.

Plotting the pF-value along the vertical axis and  $(v/v_0)$ -value along the horizontal axis, this  $pF \sim (v/v_0)$ -curve will be sigmoidal

in shape as shown in fig. 1 (where  $v_0$  represents the total pore space, so  $v/v_0$  represents relative soil moisture content).



(VI-1)

pF	6.0	4.2	3.4	2.7	2.3	2.0	1.5	1.0	0.4	0
v	2.5	8.0	13.2	25.2	28.5	34.3	42.7	45.2	48.5	50.0
$v/v_0$	0.05	0.16	0.26	0.50	0.57	0.69	0.85	0.90	0.97	

Fig. 1 Example of desorption curve obtained by plotting the pF-value along the vertical axis and  $v/v_0$ -value along the horizontal axis.

Substituting the pF for  $\ln W$  and  $(v/v_0)$  for  $p/p_0$  in equation (3), equation (4) is obtained:

$$pF = A + B \ln \left[ (v/v_0)^{-c} - 1 \right] \quad (4)$$

Equation (4) is an alternative for the desorption curve equation. The determination of the constants A, B and c is carried out by the following procedure:

At first,  $x = \ln \left[ (v/v_0)^{-c} - 1 \right]$  is calculated using the v-values which were determined in the laboratory ~~and~~ choosing an initial value for c. The value of c is chosen <sup>by</sup> within a range of 1.0~10.0 with 0.1 steps, sequentially. From  $pF = A + Bx$ , A and B are determined by the least squares method. Using the obtained values for A and B,  $pF' = A + Bx$  is calculated. The correlation coefficient ( $r_f$ ) between the experimental pF-value and the calculated pF'-value is calculated by linear regression. This procedure is repeated for different values of c until the maximum value of  $r_f$  is obtained for the particular set of pF-v data. This procedure can be calculated by using a simple computer program.

One example of the desorption curve equation obtained by the above mentioned technique is as follows. The experimental data are given in fig. 1. The A, B and correlation coefficient ( $r_f$ ) and standard deviation (S), calculated for the range of  $c = 2.0 \sim 4.0$  with 0.1 steps, are shown in table 1.

Table 1. Review of the constants A, B and the correlation coefficient ( $r_f$ ), standard deviation (S) between the experimental pF-values and calculated pF'-values by equation (4). (In the range of  $c = 2.1 \sim 4.0$ )

c	A	B	$r_f$	S
2.1	1.927	0.6134	0.9966	0.1517
2.2	1.878	0.5967	0.9971	0.1407
2.3	1.832	0.5807	0.9975	0.1313
2.4	1.790	0.5653	<del>0.9977</del> 0.9977	0.1238
2.5	1.752	0.5505	0.9980	0.1180
2.6	1.716	0.5363	0.9981	0.1140
2.7	1.682	0.5227	0.9982	0.1117
2.8	1.651	0.5097	0.9982	0.1110
2.9	1.622	0.4972	0.9982	0.1116
3.0	1.595	0.4851	0.9981	0.1135
3.1	1.570	0.4736	0.9980	0.1163
3.2	1.547	0.4625	0.9979	0.1198
3.3	1.525	0.4518	0.9977	0.1240
3.4	1.505	0.4416	0.9976	0.1286
3.5	1.485	0.4317	0.9974	0.1334
3.6	1.467	0.4223	0.9972	0.1385
3.7	1.450	0.4132	0.9970	0.1437
3.8	1.434	0.4044	0.9967	0.1489
3.9	1.419	0.3959	0.9965	0.1542
4.0	1.405	0.3878	0.9963	0.1594

From the relation  $c$  and  $r_f$  shown in fig. 2,  $r_f$  becomes the maximum value (0.9982) when  $c$  is equal to 2.8. In this way, the equation  $pF = 1.6510 + 0.5097 \ln \left[ \left( \frac{v}{v_0} \right)^{-2.8} - 1 \right]$  is obtained.

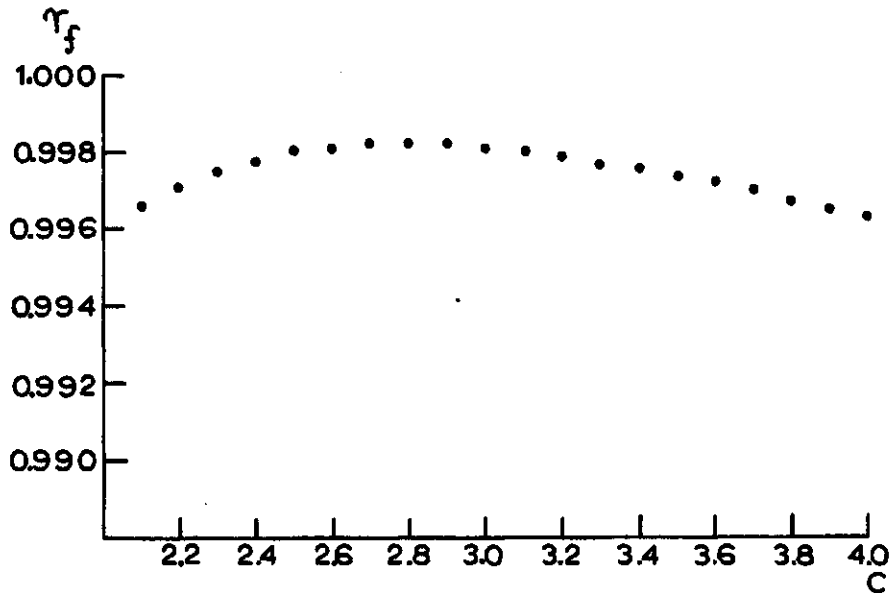


Fig. 2. Relation between  $c$ -value and correlation coefficient ( $r_f$ )

To demonstrate how well this equation fits the desorption data ( $pF$ - $v$ ),  $pF$ -values are calculated using this equation and these values are compared with the experimental data. This result indicates a good agreement between the calculated curve and the experimental data as shown in fig. 4(1).

And, plotting the  $pF$ -value along the vertical axis and  $x = \ln \left[ \left( \frac{v}{v_0} \right)^{-2.8} - 1 \right]$  along the horizontal axis, a straight line  $pF = 1.6510 + 0.5097 \cdot x$  is shown in fig. 4 (2). Also, the  $pF \sim v$ -curve calculated by given  $pF$  in equation  $v = v_0 \left[ e^{\frac{pF-A}{B}} + 1 \right]^{-\frac{1}{c}}$  is shown in fig. 3.

Then, the correlation coefficient between the calculated  $v$ -value is 0.9975. In fig. 5, the change of the shape of the desorption curve and the observed  $v$ -value

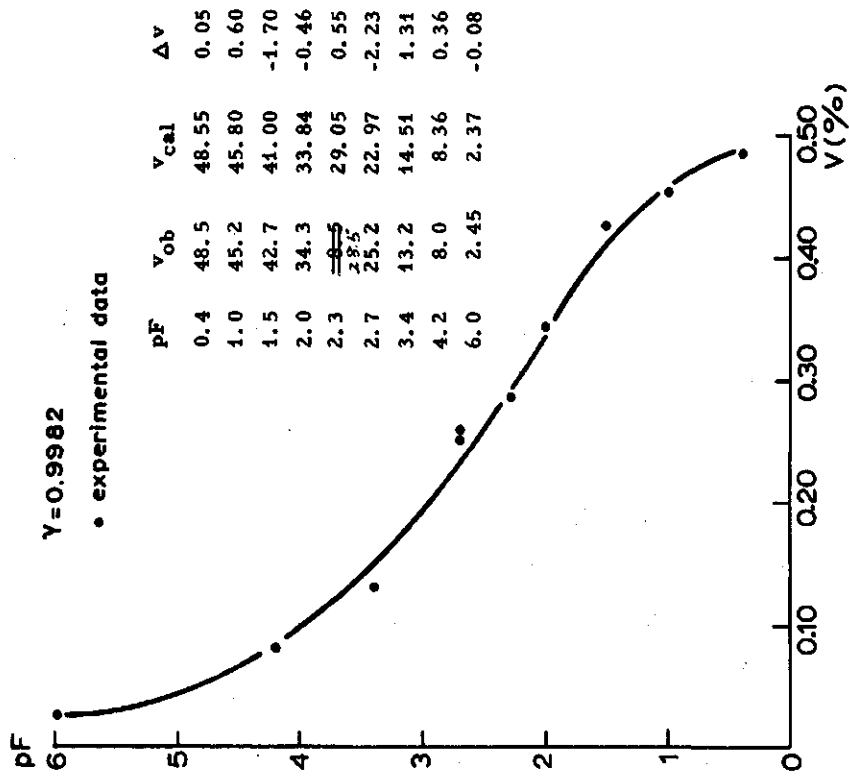


Fig. 3. pF-v curve calculated when c-value is equal to 2.8

$$v = v_0 \left( e^{\frac{pF-1.651}{0.5097}} + 1 \right)$$

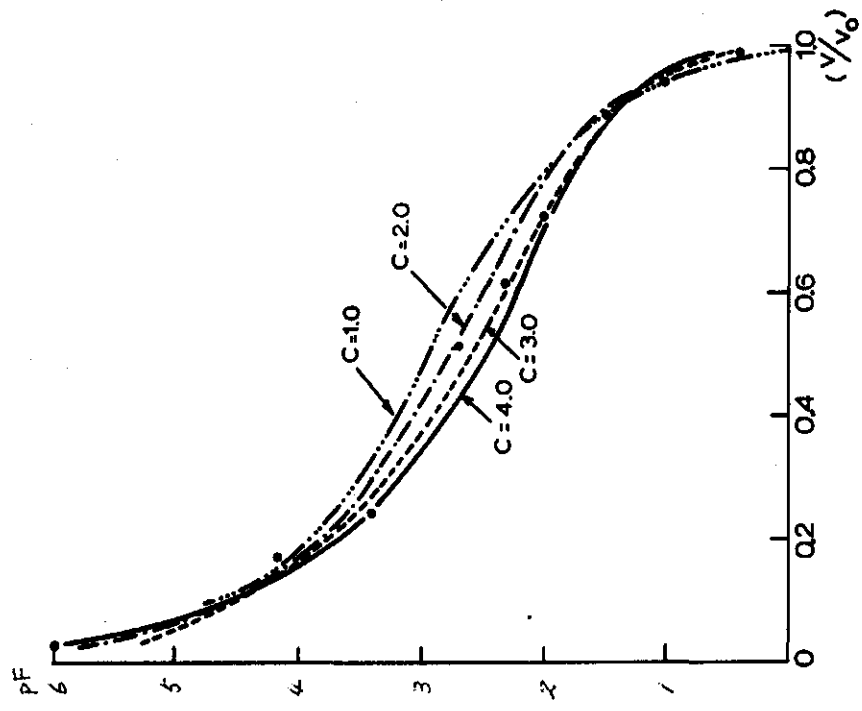


Fig. 5. Desorption curves calculated by equation (4) using various values of c (c= 1.0, 2.0, 3.0, 4.0)



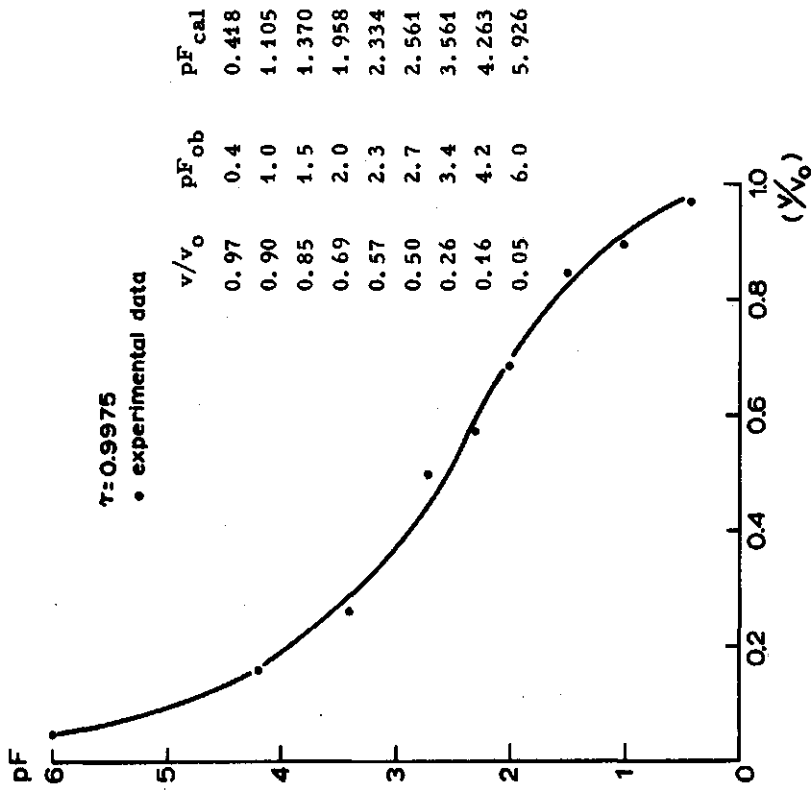


Fig. 4(1). Desorption curve calculated when c-value is equal to 2.8

$$pF = 1.651 + 0.5097 \ln \left[ \left( \frac{v}{v_0} \right)^{-2.8} - 1 \right]$$

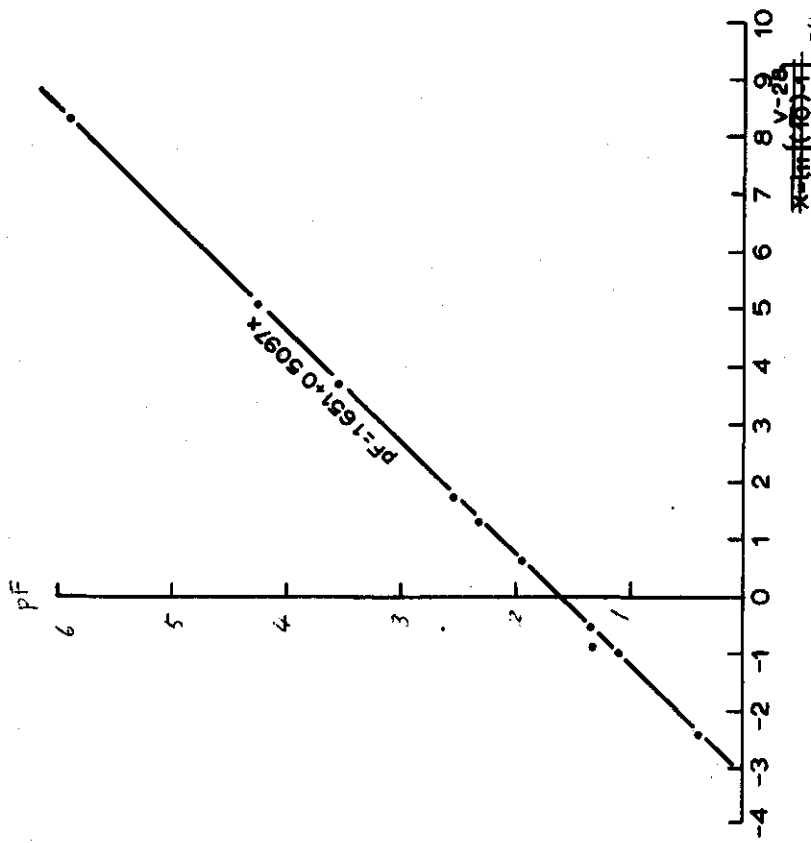


Fig. 4(2). Straight line of  $pF = A + Bx$ , when  $A = 1.651$ ,  $B = 0.5097$  and

$$c = 2.8$$

$$\left( x = \ln \left[ \left( \frac{v}{v_0} \right)^{-c} - 1 \right] \right)$$

Table 2. Data of  $(v/v_0)$  for each sample

Sample	pF								
	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2	6.0
I -2	0.98	0.94	0.84	0.50	0.37	0.28	0.15	0.11	0.03
I -3	0.98	0.91	0.85	0.49	0.32	0.21	0.09	0.06	0.02
II -1	0.98	0.94	<del>0.80</del>	0.72	0.60	0.50	0.25	0.16	0.05
II -2	0.98	0.94	0.87	0.61	0.46	0.32	0.17	0.13	0.04
II -3	0.99	0.93	0.90	0.53	0.39	0.27	0.10	0.08	0.03
III -1	0.97	0.95	0.93	0.79	0.71	0.61	0.40	0.26	0.06
III -2	0.97	0.93	0.88	0.70	0.52	0.40	0.23	0.16	0.05
III -3	0.98	0.95	0.92	0.61	0.47	0.36	0.17	0.13	0.02
IV -1	0.98	0.89	0.80	0.47	0.32	0.24	0.14	0.10	0.03
IV -2	0.98	0.91	0.80	0.47	0.34	0.25	0.13	0.09	0.03
IV -3	0.99	0.92	0.80	0.31	0.20	0.13	0.05	0.04	0.01
V -1	0.97	0.85	0.78	0.57	0.45	0.34	0.18	0.11	0.04
V -2	0.97	0.86	0.77	0.53	0.39	0.30	0.16	0.10	0.03
V -3	0.98	0.90	0.83	0.53	0.32	0.24	0.09	0.06	0.03
VI -1	0.97	0.90	0.85	0.69	0.57	0.50	0.26	0.16	0.05
VIII-1	0.98	0.90	0.83	0.58	0.44	0.33	0.18	0.12	0.04
VIII-2	0.98	0.93	0.85	0.55	0.41	0.29	0.18	0.12	0.04
VIII-3	0.98	0.91	0.81	0.54	0.39	0.28	0.16	0.10	0.04
VIII-4	0.97	0.88	0.79	0.46	0.32	0.23	0.13	0.08	0.03
VIII-5	0.98	0.90	0.83	0.53	0.32	0.24	0.09	0.06	0.03
IX -1	0.98	0.89	0.84	0.63	0.49	0.40	0.24	0.15	0.04
IX -2	0.98	0.91	0.83	0.62	0.45	0.37	0.24	0.14	0.04
IX -3	0.98	0.90	0.82	0.54	0.38	0.28	0.19	0.12	0.04
IX -4	0.98	0.92	0.83	0.58	0.37	0.26	0.16	0.09	0.04
IX -5	0.98	0.90	0.84	0.67	0.43	0.33	0.21	0.13	0.02
X -1	0.98	0.94	0.90	0.77	0.65	0.62	0.27	0.19	0.05
X -2	0.97	0.91	0.88	0.68	0.52	0.42	0.19	0.11	0.03
X -3	0.98	0.93	0.86	0.61	0.43	0.37	0.10	0.06	0.02
XI -1	0.98	0.93	0.90	0.81	0.69	0.64	0.35	0.25	0.05
XI -2	0.96	0.90	0.83	0.73	0.56	0.50	0.26	0.14	0.05
XII -2	0.99	0.93	0.90	0.83	0.75	0.62	0.44	0.29	0.09

Table 3. Constants A, B, c and correlation coefficient ( $r_f$ ) between the observed pF-values and the calculated pF-values, correlation coefficient ( $r_e$ ) between the observed v-values and the calculated v-values for each sample

Sample	A	B	c	$r_f$	$r_e$
I -2	1.05295	0.22965	5.99	0.9944	0.9914
I -3	1.04167	0.21741	5.36	0.9873	0.9848
II -1	1.68613	0.44041	3.19	0.9976	0.9979
II -2	1.21661	0.28625	5.00	0.9935	0.9942
II -3	1.11983	0.20360	6.18	0.9830	0.9874
III -1	2.39197	0.64538	2.00	0.9965	0.9984
III -2	1.43410	0.39888	3.74	0.9957	0.9962
III -3	1.59300	0.36020	3.15	0.9935	<del>0.9754</del> 0.9954
IV -1	0.88822	0.20231	6.94	0.9934	0.9877
IV -2	0.92824	0.20752	6.61	0.9930	0.9901
IV -3	0.84550	0.12662	8.08	0.9831	0.9716
V -1	1.11392	0.34829	4.20	0.9968	0.9966
V -2	1.07042	0.31867	4.31	0.9979	0.9964
V -3	1.02802	0.22793	5.40	0.9773	0.9852
VI -1	1.65110	0.50968	2.80	0.9982	0.9978
VIII-1	1.13959	0.29591	4.94	0.9966	0.9956
VIII-2	1.05841	0.24461	6.06	0.9948	0.9947
VIII-3	0.99134	0.23814	6.14	0.9932	0.9930
VIII-4	0.86472	0.21483	6.39	0.9915	0.9874
VIII-5	1.02802	0.22793	5.40	0.9773	0.9852
IX -1	1.38339	0.38958	3.70	0.9987	0.9979
IX -2	1.29771	0.35344	4.13	0.9989	0.9972
IX -3	0.98318	0.24171	6.27	0.9957	0.9916
IX -4	1.01925	0.23384	6.09	0.9892	0.9890
IX -5	1.58321	0.41823	2.75	0.9960	0.9936
X -1	2.02247	0.52721	2.47	0.9960	0.9945
X -2	1.62676	0.44627	2.71	0.9960	0.9955
X -3	1.46912	0.34302	3.08	0.9878	0.9917
XI -1	2.38955	0.63574	1.91	0.9972	0.9983
XI -2	1.73478	0.56765	2.47	0.9978	0.9969
XII -2	<del>2.02835</del> 2.02885	0.52316	3.20	0.9966	0.9982

Table 4. Depth of sampling, silt content, humus content, volume weight for each sample  
(silt) : 0.002 ~ 0.050 mm

Sample	Horizontal	Depth (cm)	Silt %	Humus %	Volume weight gr/cc	
I -2	B <sub>2</sub>	25 ~ 40	5.8	2.3	1.503	} grassland
I -3	C	> 40	5.2	0.6	1.678	
II -1	A <sub>p</sub>	0 ~ 25	14.0	5.4	1.385	} grassland
II -2	B <sub>2</sub>	25 ~ 40	14.9	3.0	1.450	
II -3	C	> 40	14.8	0.5	1.696	
III -1	A <sub>p</sub>	0 ~ 30	25.2	6.6	1.301	} grassland
III -2	B <sub>2</sub>	25 ~ 45	26.8	2.9	1.443	
III -3	C	> 45	22.3	0.6	1.720	
IV -1	A <sub>p</sub>	0 ~ 30	7.5	4.5	1.274	} farmland
IV -2	B <sub>2</sub>	30 ~ 45	6.8	3.0	1.397	
IV -3	C	> 45	4.5	0.6	1.610	
V -1	A <sub>p</sub>	0 ~ 30	13.1	5.1	1.262	} farmland
V -2	B <sub>2</sub>	30 ~ 45	13.4	2.1	1.432	
V -3	C	> 45	12.6	1.0	1.601	
VI -1	A <sub>p</sub>	0 ~ 30	26.5	6.4	1.277	} farmland
VIII-1	A <sub>1p</sub>	0 ~ 30	13.3	4.7	1.281	} farmland
VIII-2	A <sub>1.2</sub>	30 ~ 50	13.3	3.8	1.357	
VIII-3	A <sub>1.3</sub>	50 ~ 80	14.0	3.2	1.331	
VIII-4	B <sub>2</sub>	80 ~ 100	13.2	1.3	1.449	
VIII-5	C	> 100	12.6	1.0	1.601	
IX -1	A <sub>1p</sub>	0 ~ 30	22.4	5.0	1.281	} farmland
IX -2	A <sub>1.2</sub>	30 ~ 50	21.8	4.2	1.338	
IX -3	A <sub>1.3</sub>	50 ~ 80	22.1	2.6	1.344	
IX -4	B <sub>2</sub>	80 ~ 100	21.2	2.4	1.459	
IX -5	C	> 100	25.6	0.7	1.678	
X -1	A <sub>pg</sub>	0 ~ 20	14.1	4.3	1.362	} grassland
X -2	C <sub>1.1g</sub>	20 ~ 40	14.1	0.3	1.693	
X -3	C <sub>1.2g</sub>	> 40	7.5	0.3	1.699	
XI -1	A <sub>pg</sub>	0 ~ 20	22.6	6.4	1.242	} grassland
XI -2	C <sub>1.1g</sub>	20 ~ 40	21.7	0.7	1.708	
XII -2	C <sub>1.1g</sub>	20 ~ 40	43.3	0.7	1.647	} grassland

obtained under the influence of several different values of  $c$  (1.0, 2.0, 3.0 and 4.0) is shown.

## 2. RESULTS

Using the  $pF \sim v$ -data of 31 samples measured by STIBOKA<sup>3)</sup> (table 2), the constants  $A$ ,  $B$  and  $c$  in equation (4) are calculated with  $c$ -steps (0.01) with the computer. The list of these results is shown in table 3.

The sampling depth, soil texture (silt content), humus content and the volume weight of each sample are shown in table 4. All samples belong to the loamy-sand soil.

~~(tables 2, 3 and 4)~~

### 2.1. Discussion of the results for the calculated value by the equation

#### a. The correlation coefficient between the observed and the calculated values

Comparing the observed value with the value calculated by equation (4), the correlation coefficients ( $r_f$ ) between the observed  $pF$ -values and the calculated  $pF$ -values in each sample are shown in table 4.<sup>3</sup>

Of the total 31 samples, 24 samples have a correlation coefficient ( $r_f$ ) higher than 0.99, 5 samples are between 0.98 and 0.99 and the lowest is 0.97. The relations between the observed  $pF$ -values and the desorption curve ( $pF \sim v/v_0$ ) obtained by equation (4) for each sample are illustrated in fig. 6 (see appendix). The soil moisture content ( $v$ ) for each  $pF$  are computed using the formula  $v = v_0 \left[ e^{\frac{pF-A}{B}} + 1 \right]^{-\frac{1}{c}}$  derived from equation (4). The desorption curves ( $pF \sim v$ ) are illustrated in fig. 7 (see appendix). And the correlation coefficients ( $r_e$ ) between the observed  $v$ -value and the calculated  $v$ -value are shown in table 3.

These figs 6 and 7 (appendix) indicate that the equation (4) is fairly well apt at fitting the desorption curve for loamy-sand soil.

But, considering the application of this equation, it is necessary to notice that the samples with less than  $r_e = 0.99$  are all sampled from the C-layer or from greater depths.

b. Difference between the observed v-value and the calculated v-value

Examining the difference between the observed v-value and the calculated v-value ( $\Delta v = v_{\text{obs}} - v_{\text{calc}}$ ) for each pF, the standard deviation (S) and the coefficient of variation (standard deviation / mean value) ( $C_v$ ) of these differences ( $\Delta v$ ) for all samples are computed. These results are shown in table 5.

Table 5. Standard deviation (S), coefficient of variation ( $C_v$ ) of difference between the observed v-values and the calculated v-values by using the equation (4) for each pF

pF	All samples (n = 31)			Samples ( $> r_e = 0.990$ ) (n = 23)			Samples ( $> r_e = 0.995$ ) (n=14) <sup>e</sup>		
	$\bar{v}$	S	$C_v$	$\bar{v}$	S	$C_v$	$\bar{v}$	S	$C_v$
0.4	<del>42.59</del> <sup>42.39</sup>	0.3109	0.73	43.43	0.2663	0.61	43.60	0.2240	0.51
1.0	<del>37.64</del> <sup>37.77</sup>	0.9791	2.50	41.73	0.8049	1.92	41.05	0.7452	1.81
1.5	32.37 <sup>7</sup>	5.3936	16.65	34.53	3.5207	10.19	36.14	2.4758	6.85
2.0	<del>34.81</del> <sup>34.81</sup>	1.5639	6.30	<del>37.35</del> <sup>37.35</sup>	1.2852	0.69	<del>39.63</del> <sup>39.63</sup>	1.2843	4.33
2.3	20.67	1.2589	6.08	22.92	1.2244	5.34	<del>35.03</del> <sup>35.03</sup>	1.0315	4.12
2.7	15.95	1.4324	8.98	17.90	1.5008	8.43	19.77	1.0125	5.12
3.4	9.81	1.5736	16.04	11.17	1.5393	13.78	12.48	1.4217	11.39
4.2 <sup>3</sup>	5.52	0.5942	10.76	6.36	0.6424	10.10	7.15	0.6666	9.32
6.0	1.46	0.2507	16.82	1.75	0.1870	10.68	1.97	0.1589	8.06

The standard deviations of the difference at lower pF-values (pF 0.4, 1.0) and higher pF-values (pF 4.2, 6.0) are less than 1.0. At the middle pF-values (pF 2.0, 2.3, 2.7 and 3.4), the deviations are about 1.5 but at pF 1.5, the error is much larger than at any other pF. And the correlation coefficient describing the relation between  $\Delta v$  at pF 1.5 and the values of constant c used in equation (4) is high ( $r = 0.9370$ ). The greater constant c is, the greater  $\Delta v$  is calculated as shown in fig. 8. Moreover, the coefficients of deviation at pF 6.0, 1.5 and 3.4 are greater than at other pF-values and their magnitudes are 16.82, 16.66, and 16.04%, respectively.

Next, the samples (the number of the samples is 23) with the correlation coefficient ( $r_e$ ) higher than 0.990 and the samples (the

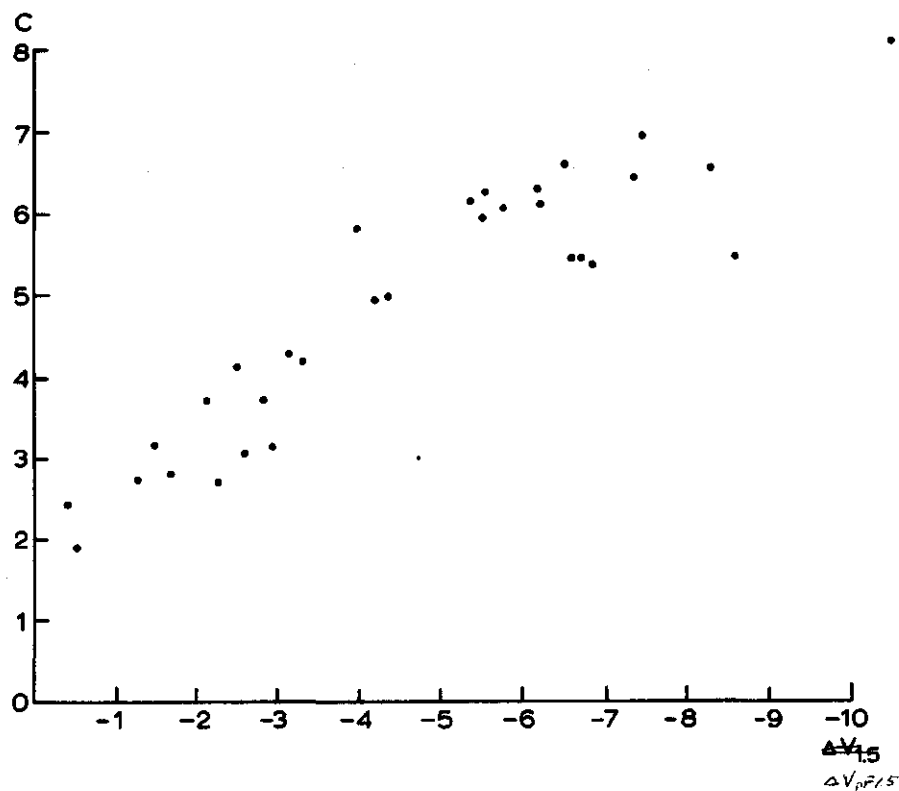


Fig. 8. Relation between difference ( $\Delta v_{pF1.5}$ ) and constant  $c$

number of the samples is 14) with the correlation coefficient ( $r_e$ ) higher than 0.995 are selected from all samples, the standard deviation ( $S$ ) and the coefficient of variation ( $C_v$ ) of  $\Delta v$  for these two groups are computed, respectively (table 5). In the case of the samples with the correlation coefficient ( $r_e$ ) higher than 0.995,  $S$  and  $C_v$ -values at pF 1.5 decrease remarkably and  $C_v$ -values become less than 10% except at pF 3.4.

These results suggest that the agreement between the observed  $v$ -value and the calculated  $v$ -value will be excellent and the equation (4) will be applicable to the desorption curve equation, when the  $r_e$ -value is obtained higher than 0.995. So, it is necessary to adjust the parameters in equation (4) by method mentioned in paragraph 3, if the  $r_e$ -value is obtained less than 0.995.

But, at the present time, it is not clear whether these large deviations at pF 1.5 and higher pF are due to the limitation of equation (4) or to the experimental errors in measuring.

c. Comparison with the results by the graphical method

The v-values in the desorption equation  $pF = n \log (P^* - v) - m \log v + \log G$  (method 'ICW') are computed using the parameters (m, n, log G and P\*) obtained by the graphical method (FONK's results). The correlation coefficients ( $r_g$ ) between the observed v-value and v-value obtained by the graphical method are shown in table 6.

Comparing  $r_g$  and  $r_e$ , both values in two thirds of all samples are approximately identical. On the III-3 and X-1 samples which present a little difference with  $r_g$  and  $r_e$ ,  $r_g$ -values are less than 0.99. This will be due to the errors in processing the data by the graphical method, because these curves show also a little difference regarding the desorption curve obtained by other numerical method as shown in fig. 9.

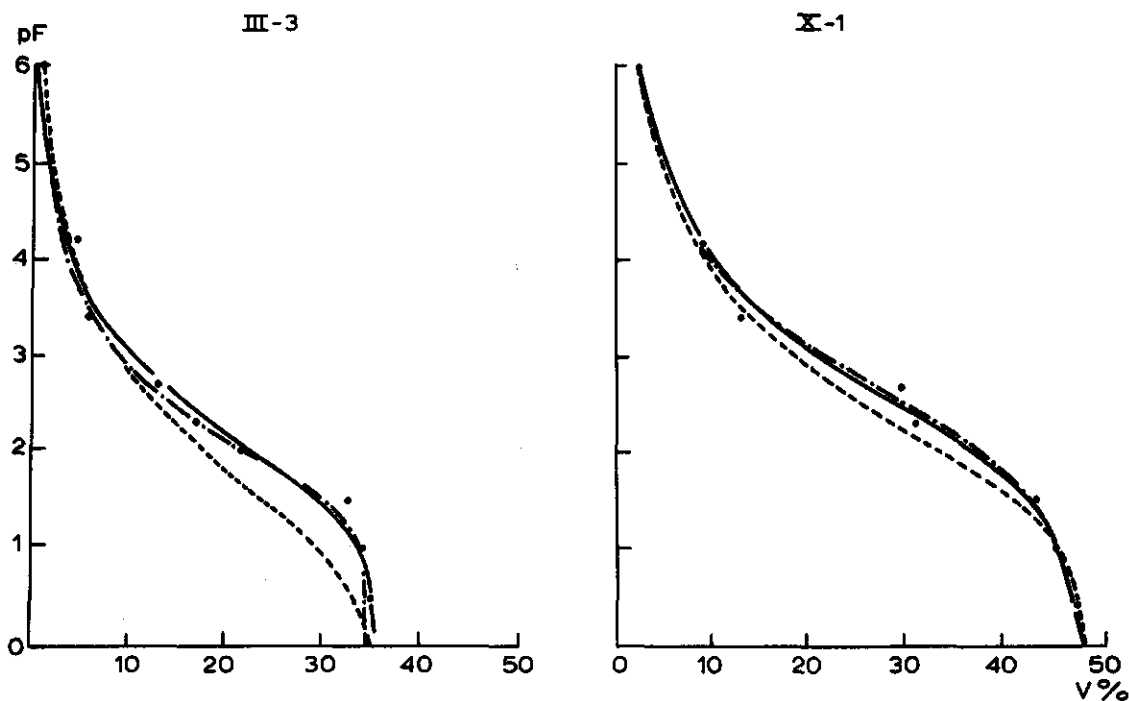


Fig. 9. Comparison of desorption curves obtained by various methods

- graphical method
- equation (4)
- · - · - numerical method
- · · · · experimental data



Table 6. The v-values observed by graphical method and correlation coefficient ( $r_g$ ) between the observed v-values and the v-values obtained by the graphical method

pF	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2	6.0	$r_g$	$r_e$
Sample											
I - 2	40.8	38.6	30.4	21.6	17.2	12.7	7.28	3.82	0.882	0.9934	0.9914
I - 3	34.8	31.9	23.2	15.4	11.8	8.24	4.34	2.06	0.382	0.9849*	0.9848*
II - 1	45.4	43.5	39.0	31.8	27.2	21.4	13.5	7.64	2.00	0.9973	0.9979
II - 2	41.9	39.8	32.8	24.7	20.4	15.7	9.74	5.58	0.56	0.9935	0.9942
II - 3	35.3	32.4	28.8	20.7	16.2	11.2	5.67	2.51	0.384	0.9931	0.9874*
III - 1	46.7	45.5	43.3	39.0	35.3	29.7	20.0	11.4	2.75	0.9985	0.9984
III - 2	43.1	41.4	37.0	29.8	25.2	19.5	11.9	6.48	1.56	0.9967	0.9962
III - 3	33.7	29.6	23.8	17.8	14.7	11.1	6.71	3.67	0.910	0.9834*	0.9954
IV - 1	47.6	41.6	32.0	23.3	19.1	14.5	8.88	5.02	1.37	0.9874*	0.9872*
IV - 2	43.5	40.1	30.8	21.8	17.4	12.8	7.36	3.88	0.904	0.9927	0.9901
IV - 3	37.7	36.4	25.3	15.7	11.3	7.10	3.06	1.15	0.123	0.9854*	0.9716*
V - 1	48.8	44.6	35.1	25.9	21.3	16.2	9.98	5.67	1.56	0.9945	0.9966
V - 2	42.2	40.0	33.5	24.3	19.2	13.6	7.11	3.29	0.556	0.9973	0.9964
V - 3	38.4	36.7	30.4	20.9	15.8	10.6	4.98	2.04	0.266	0.9939	0.9850*
VI - 1	48.4	46.0	40.6	32.9	28.1	22.1	14.0	8.00	2.14	0.9970	0.9978
VIII - 1	47.3	43.7	35.0	26.1	21.5	16.5	10.3	5.88	1.65	0.9944	0.9956
VIII - 2	43.9	41.3	33.1	24.4	19.9	15.1	9.18	5.13	1.36	0.9956	0.9947
VIII - 3	44.1	41.3	32.7	23.7	19.2	14.4	8.51	4.63	1.16	0.9952	0.9930
VIII - 4	41.5	37.6	28.5	20.0	16.0	11.7	6.76	3.56	0.829	0.9913	0.9874*
VIII - 5	38.3	36.2	29.4	20.1	15.3	10.3	4.93	2.06	0.282	0.9937	0.9852*
IX - 1	47.4	45.0	39.5	31.5	26.6	20.7	12.8	7.11	1.80	0.9971	0.9979
IX - 2	45.6	42.9	35.2	26.7	22.2	17.3	10.9	6.40	1.89	0.9968	0.9972
IX - 3	45.6	41.1	32.4	24.0	19.8	15.2	9.39	5.38	1.51	0.9912	0.9916
IX - 4	42.4	39.8	31.3	22.4	18.0	13.3	7.76	4.14	0.990	0.9916	0.9890*
IX - 5	37.4	36.1	31.8	24.4	19.9	14.6	8.07	3.95	0.749	0.9931	0.9936
X - 1	47.6	45.7	41.0	33.6	28.7	22.7	14.3	8.08	2.12	0.9887*	0.9945
X - 2	35.3	34.0	30.1	23.4	19.2	14.3	8.12	4.08	0.824	0.9972	0.9955
X - 3	34.8	33.7	28.8	20.6	15.8	10.8	5.24	2.21	0.306	0.9959	0.9917
XI - 1	51.2	49.6	47.0	42.6	38.1	33.1	22.2	12.0	2.30	0.9973	0.9983
XI - 2	34.0	32.5	29.1	23.8	20.5	16.3	10.4	6.02	1.65	0.9963	0.9969
XII - 2	38.1	36.8	34.6	31.1	28.4	24.5	19.3	11.1	3.40	0.9972	0.9982

On the II-3, V-3 and VIII-5 samples, on the contrary,  $r_e$ -values are less than  $r_g$ . At the present time, the reason for this difference in outcome is not clear but all these samples are sampled from the C-layer.

The standard deviation (S) and the coefficient of variation ( $C_v$ ) of the difference between the observed v-value and v-value obtained by the graphical method are shown in table 7.

Table 7. Standard deviation (S), coefficient variation of difference between observed v-values and calculated v-values by graphical method for each pF

pF	Standard deviation (S)	Mean (v)	Coefficient of variation ( $C_v$ )
0.4	0.2683	42.02	0.64 %
1.0	1.2731	39.46	3.23
1.5	4.0533	33.08	12.25
2.0	1.8220	25.29	7.20
2.3	2.1319	20.97	10.17
2.7	2.1045	16.03	13.13
3.4	1.3458	9.77	13.77
4.2	0.4788	5.29	9.05
6.0	0.4201	1.30	32.32

Comparing with the results in table 5, there are little differences between both methods but in the graphical method, at pF 6.0,  $C_v$  is the largest value (larger than 30%) and at pF 1.5, 2.3, 2.7 and 3.4,  $C_v$ -values are larger than 10%, and at pF 1.5,  $C_v$ -value is less than the  $C_v$ -value obtained by equation (4).

Therefore, considering from only these results, it seems that there is little difference to choose between these two methods.

## 2.2. Consideration of the constant c

### a. Relation with the soil texture (silt content)

The values of the constant c in equation (4) obtained from 31 samples are within 1.91 ~ 8.08 range. Approximately speaking, this range is 2.0 ~ 8.0. Except 8.08 for sample IV-3, these are not very extreme values. This will be due to the fact that the soil

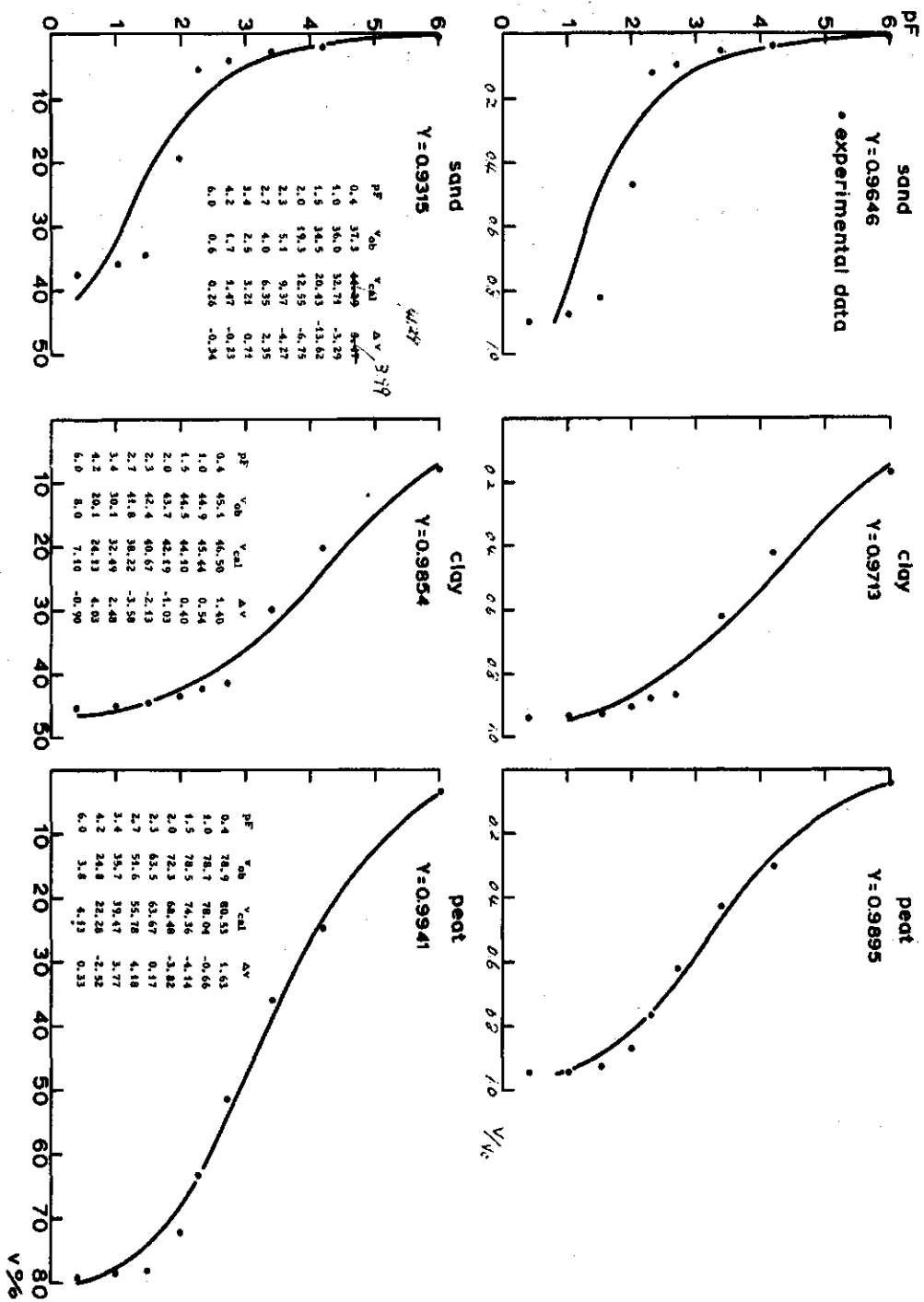


Fig. 10.  $PF \sim (v/v_0)$  curves and  $PF \sim v$  curves obtained by equation (4) on typical soils

texture of all samples is similar (the loamy-sand soil). For few examples, the values of constant  $c$  obtained from samples of clay, sand and peat as typical soil texture types are situated beyond this range as shown in table 8 and the shapes of curve on these soils are different from the loamy-sand soil, respectively (fig. 10).

Table 8. Constants A, B,  $c$  and correlation coefficient between observed pF-values and calculated pF-values by using equation (4) on typical soils (peat, sand and clay)

Soil	Constant			
	A	B	$c$	$r$
peat	3.12189	0.82047	1.18	0.9895
sand	0.77237	0.11995	8.56	0.9646
clay	4.73972	1.18576	0.71	0.9713

These results suggest that the value of the constant  $c$  is a main factor in determining the shape of curve and it has some relation to the soil texture.

Then, plotting the  $c$ -value along the vertical axis and the silt content along the horizontal axis (fig. 11), there are no distinct relations between the  $c$ -value and the silt content but when the silt content increases, the  $c$ -value decreases, generally. (fig. 11, see p. 19). Classifying all samples into three classes by their silt content, namely, loamy-sand (silt content 0 ~ 10%), coarser loamy-sand (10 ~ 17.5%) and finer loamy-sand (17.5 ~ 32.5%), the ranges of  $c$ -value in each class are approximately 5.0 ~ 7.0, 3.0 ~ 5.0 and 2.0 ~ 4.0 with some exceptions. But the samples which indicate exceptional values in each class are sampled from the C-layer. This fact suggests that the  $c$ -value has relations not only with the soil texture but also with the sampling layer.

Using the data of samples in all classes together, the  $c$ -values in each class are computed. These results are shown in table 9.

The  $c$ -value further has no relations with the humus content and the volume weight of the soil.

Table 9. Constant A, B and c in each soil class obtained by using data of samples in each class altogether

Constant	Loamy sand		coarser loamy sand		finer loamy sand	
	A-layer	C-layer	A-layer	C-layer	A-layer	C-layer
A	0.9562	1.1730	1.2811	1.1308	1.7257	1.5338
B	0.2121	0.2392	0.3274	0.2555	0.9863	0.9630
c	6.52	4.36	4.30	4.90	2.91	3.35

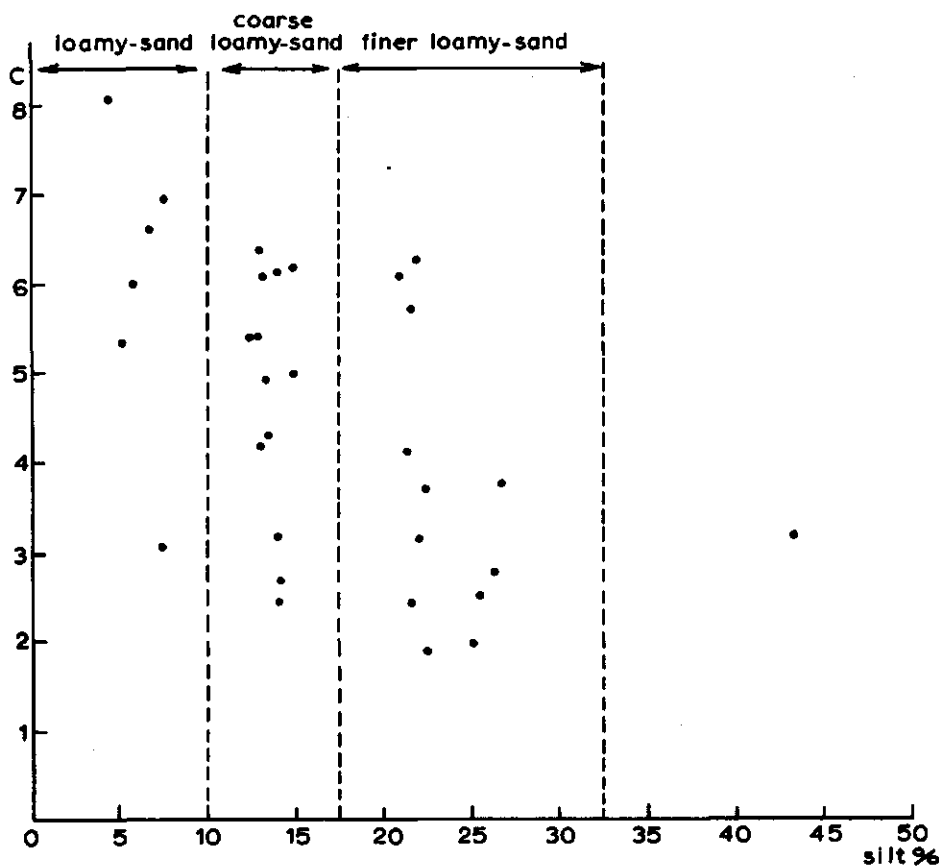


Fig. 11. Relation between constant c and soil texture (silt content %)

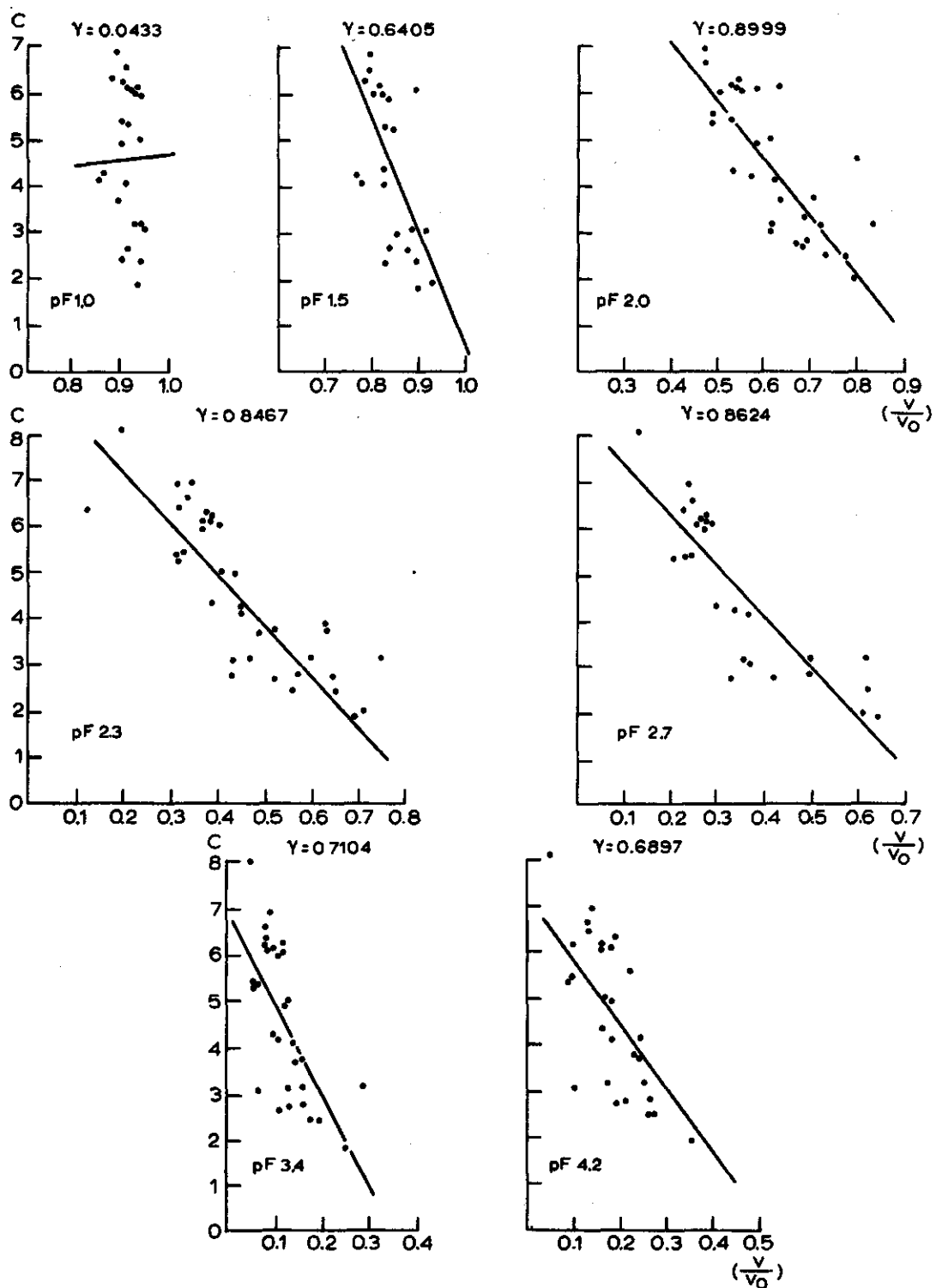


Fig. 12. Relation between the relative soil moisture content  $(\frac{v}{v_0})$  and constant  $c$

- (1)  $(\frac{v}{v_0})$  at pF 1.0    (2)  $(\frac{v}{v_0})$  at pF 1.5    (3)  $(\frac{v}{v_0})$  at pF 2.0  
 (4)  $(\frac{v}{v_0})$  at pF 2.3    (5)  $(\frac{v}{v_0})$  at pF 2.7    (6)  $(\frac{v}{v_0})$  at pF 3.4

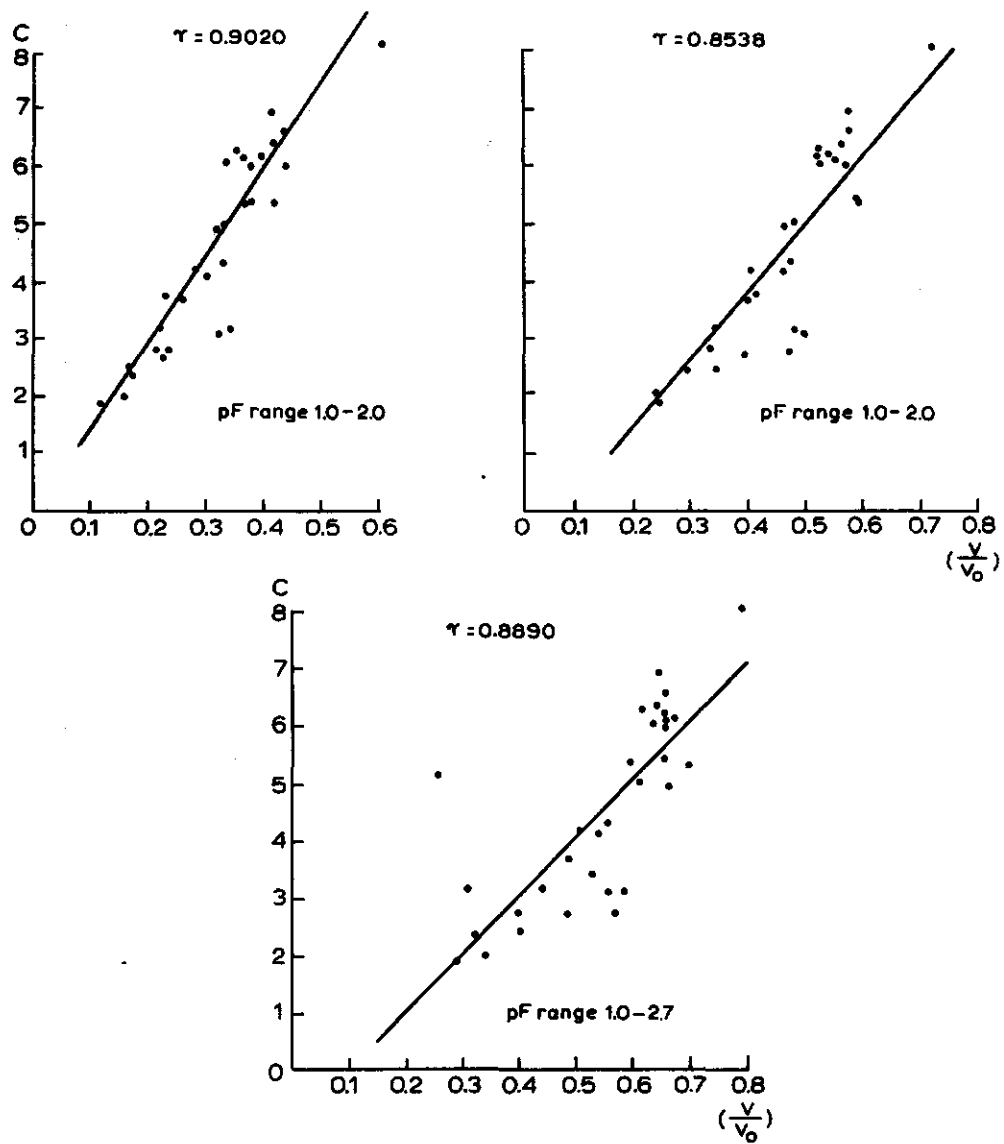


Fig. 13. Relation between soil moisture class and constant  $c$

- (1)  $(\frac{v}{v_0})_{1.0} - (\frac{v}{v_0})_{2.0}$
- (2)  $(\frac{v}{v_0})_{1.0} - (\frac{v}{v_0})_{2.3}$
- (3)  $(\frac{v}{v_0})_{1.0} - (\frac{v}{v_0})_{2.7}$

b. Relation with the soil moisture content

The c-value was plotted along the vertical axis and the relative soil moisture content ( $v/v_0$ ) at each pF along the horizontal axis (fig. 12) in order to find the relations between the c-value and the soil moisture content. The correlation coefficients between the c-value and the relative soil moisture content in each case have been computed (table 10).

Table 10. Correlation coefficient between relative soil moisture content ( $v/v_0$ )<sub>pF</sub> and constant c

( $v/v_0$ ) pF	r
1.0	0.0433
1.5	0.6405
2.0	0.8999
2.3	0.8467
2.7	0.8624
3.4	0.7104
4.2	0.6897

For lower and higher pF-values, the correlation coefficients are less than 0.70, but in the middle of the pF (pF 2.0, 2.3 and 2.7), the values of  $v/v_0$  and c are more closely related ( $r = 0.85 \sim 0.90$ ). Therefore, the soil moisture content for these pF-values have an important effect on the shape of the desorption curve but, naturally, it is impossible to evaluate the c-value from the correlation coefficient with the soil moisture content and the pF.

Secondly, representing the soil moisture content with some classes, the relations between this class and the c-value are examined. The relative soil moisture content on (pF 1.0 - 2.0), (pF 1.0 - 2.3) and (pF 1.0 - 2.7) are adopted as the soil moisture classes. As shown in fig. 13 (p. ~~22~~<sup>24</sup>), the correlation coefficients between the c-value and the soil moisture class are  $0.85 \sim 0.90$  and these relations are almost identical with the in fig. 12 mentioned correlations for the soil moisture contents for pF 2.0, 2.3 and 2.7.

Fig. 5 indicates how the shape of the desorption curve is for the same soil shaped by changing the values of c. The change of the curve at higher and lower pF is very small but in the range between pF 1.0 and pF 4.2 the shape of the curve is very changeable. Therefore, the



relation between the c-value and the inclination of the curve in this range is found as follows.

Taking the experimental points for pF 1.0 and 4.2 as fixed standard points, these two points are linked with other pF-points (pF 2.0, 2.3, 2.7 and 3.4) according a straight line (fig. 14).

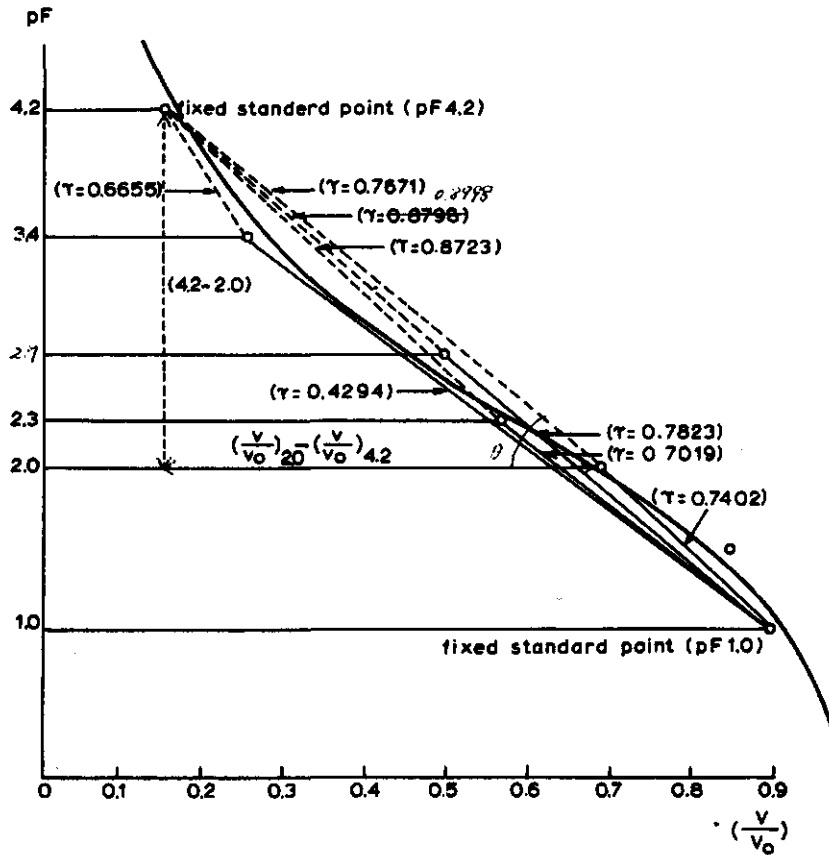


Fig. 14. The straight lines connected fixed standard points (pF 1.0, 4.2) with the other points (pF 2.0, 2.3, 2.7 and 3.4)

r: the correlation coefficient between the gradient of the straight line and c-value

Then the gradients of these eight straight lines for each sample are computed. For example, the gradient of a straight line (pF 4.2  $\rightarrow$  pF 2.0) is  $\tan \epsilon = 4.2 - 2.0 / (v/v_0)_{2.0} - (v/v_0)_{4.2}$  as shown in fig. 14. The correlation coefficients with the gradient of the straight line and the c-value for each straight line are given in table 11.

Table 11. Correlation coefficient between gradient of line connected a standard point with direction point and constant c

Direction point (pF) Standard point (pF)	2.0	2.3	2.7	3.4
1.0	0.7402	0.7019	0.7805	0.4295
4.2	0.7671	0.8723	0.8998	0.6665

The gradient of the straight line (pF 4.2  $\rightarrow$  pF 2.7) is the most closely related with the c-value ( $r = 0.8998$ ).

#### c. Relation of c with the constants A and B

The relations between the logarithm of the c-value and other constants A and B in equation (4) are shown in fig. 15. According to these results, these relations are very intimate, especially the correlation coefficient between the log c and the constant B is with 0.9769 rather high.

Thus, it is possible to estimate the c-value from the silt content or the soil moisture content, but, at the present time, this estimation will be only available for a first approach to its value, to lessen the complexity of the calculation.

### 2.3. Adjustment of the parameters A, B, c and $v_0$

On the samples with a correlation coefficient ( $r_e$ ) less than 0.995, the parameters A, B, c and  $v_0$  in equation (4) are adjusted by the following curve fitting adjustment method <sup>4)</sup>.

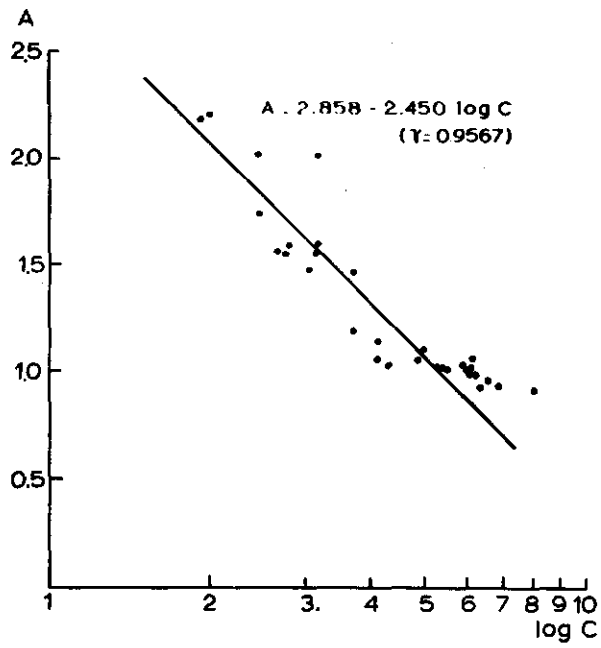


Fig. 15(1). Relation between constant A and log c

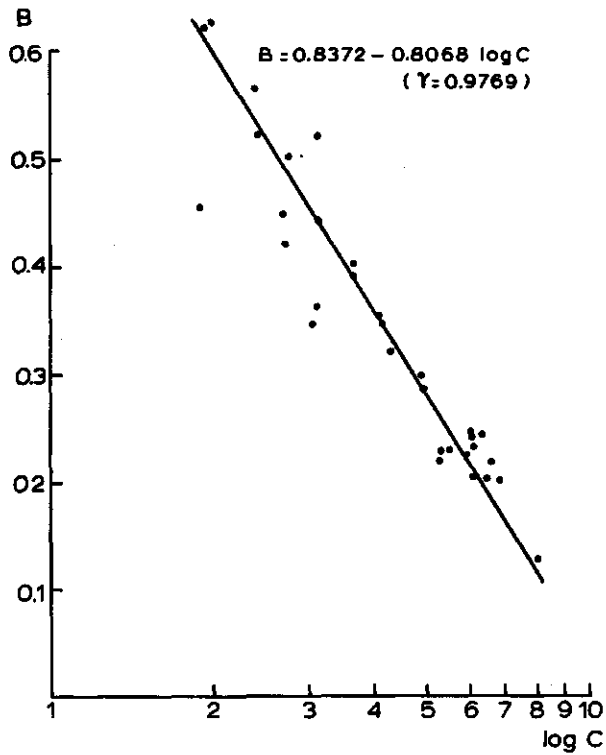


Fig. 15(2). Relation between constant B and log c



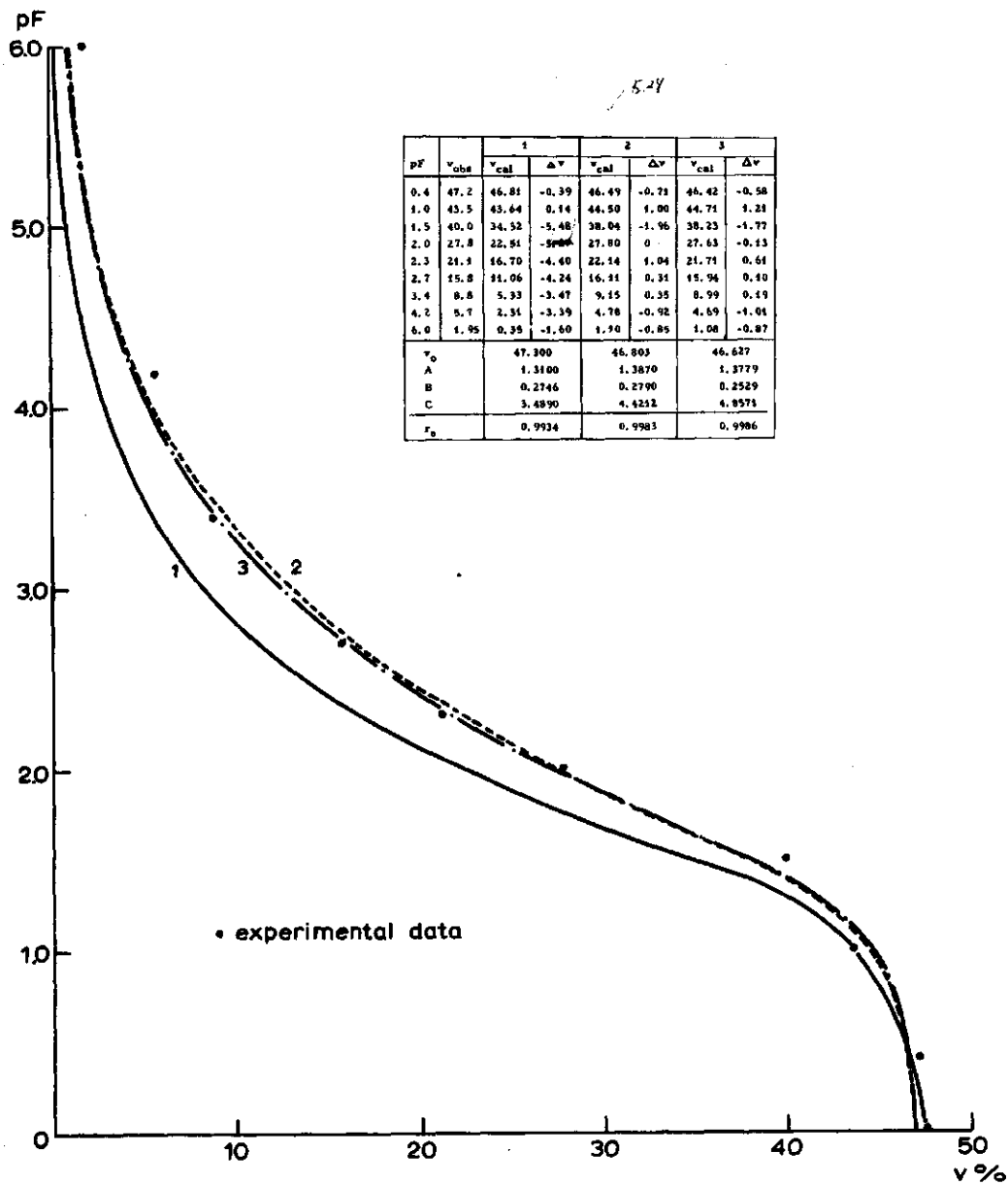


Fig. 16(2). Desorption curves obtained using adjusted parameters by formula (5)  
 1: Initial curve      3: Final curve

$$\begin{pmatrix} f_a f_a & f_a f_b & f_a f_c & f_a f_v \\ f_b f_a & f_b f_b & f_b f_c & f_b f_v \\ f_c f_a & f_c f_b & f_c f_c & f_c f_v \\ f_v f_a & f_v f_b & f_v f_c & f_v f_v \end{pmatrix} \begin{pmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta v_o \end{pmatrix} = \begin{pmatrix} f_a f_o \\ f_b f_o \\ f_c f_o \\ f_v f_o \end{pmatrix} \quad (5)$$

$$\begin{aligned} a_i &= a_o + \Delta a_i \\ b_i &= b_o + \Delta b_i \\ c_i &= c_o + \Delta c_i \\ v_{oi} &= v_{oo} + \Delta v_{oi} \end{aligned} \quad (6)$$

where  $v = v_o \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{c}}$

$$f_a = \frac{\partial v}{\partial A} = \frac{v_o}{B \cdot c} e^{\frac{pF-A}{B}} \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{c}-1}$$

$$f_b = \frac{\partial v}{\partial B} = \frac{v_o}{B \cdot c} \frac{pF-A}{B} \cdot e^{\frac{pF-A}{B}} \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{c}-1}$$

$$f_c = \frac{\partial v}{\partial c} = \frac{v_o}{c^2} \ln \left( e^{\frac{pF-A}{B}} + 1 \right) \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{c}}$$

$$f_v = \frac{\partial v}{\partial v_o} = \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{c}}$$

$$f_o = v_{obs} - v_{calc}$$

and  $\Delta a$ ,  $\Delta b$ ,  $\Delta c$  and  $\Delta v_o$  are the parameter residuals.

The adjusted parameters will be found by substituting parameter residuals obtained from formula (5) into formula (6).

The practical examples of this adjustment method are shown in fig. 16. In the upper row, the calculated  $v$ -value, the difference between the observed  $v$ -value and the calculated  $v$ -value and in the lower 4 lines the adjusted parameters  $A$ ,  $B$ ,  $c$ ,  $v_o$  after each iterative calculation are given. The  $r_e$ -value obtained with the first approximation for the parameters is 0.9612 and is less than 0.995. But this value becomes 0.9986 by repeating the adjustments three times.

This curve fitting calculation is at this moment not yet finished. The adjustment is, however, continuing on the samples used in this paper.

According to above mentioned considerations, it is clear that the equation (4) proposed in this paper will be applicable for the desorption curve on the loamy-sand soil with an accuracy of considerable height.

Moreover, it is possible to calculate the constants A, B and c in equation (4) by a simple computer program and to estimate roughly the c-value from the relations with the soil moisture characters and the soil texture.

But this conclusion only is based on a few kinds of soil, so it is necessary to investigate the applicability to more kinds of soil.

#### ACKNOWLEDGEMENT

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Appendix

Fig. 6.  $pF \sim (\frac{v}{v_0})$  curves calculated by using  $pF = A + B \ln \left[ \left( \frac{v}{v_0} \right)^{-c} - 1 \right]$

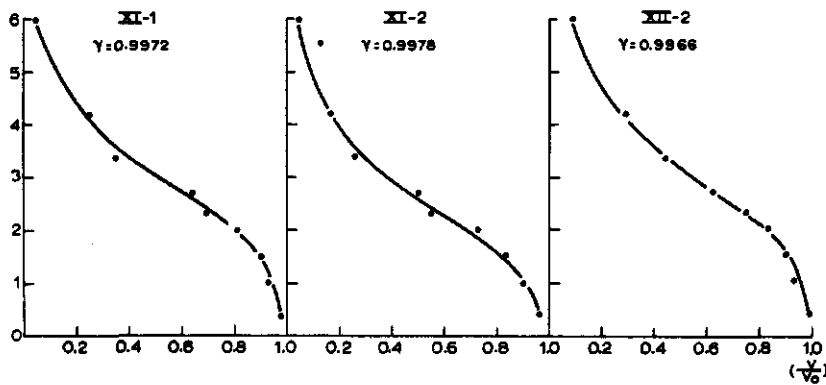
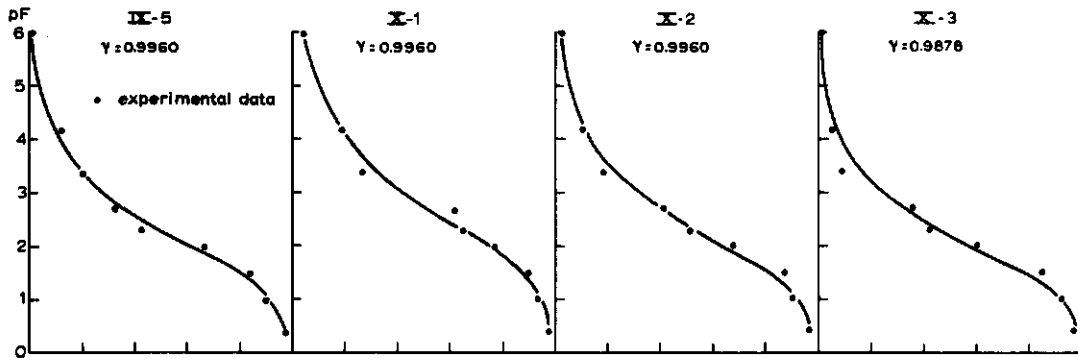
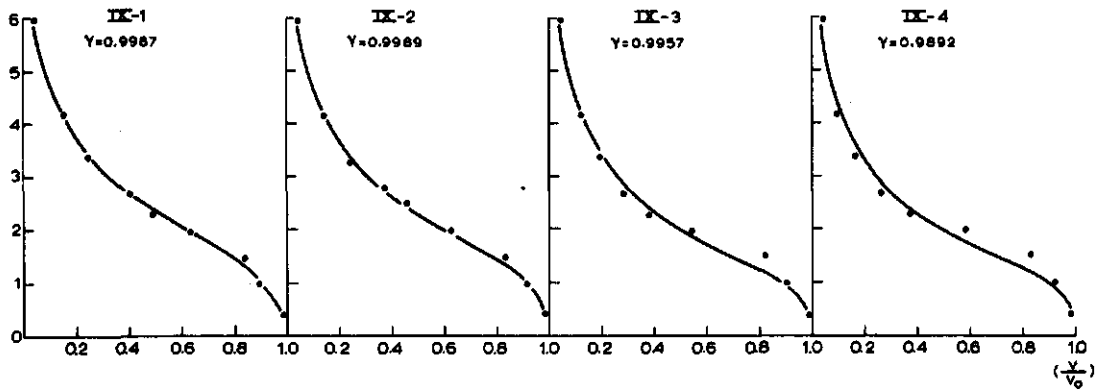
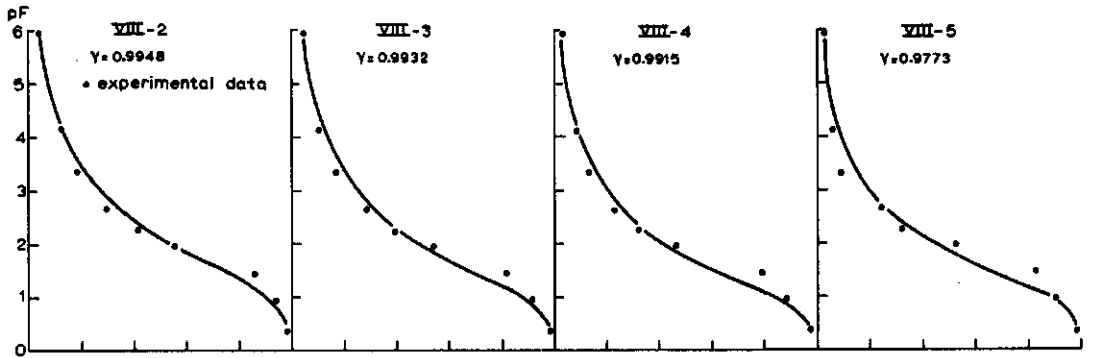
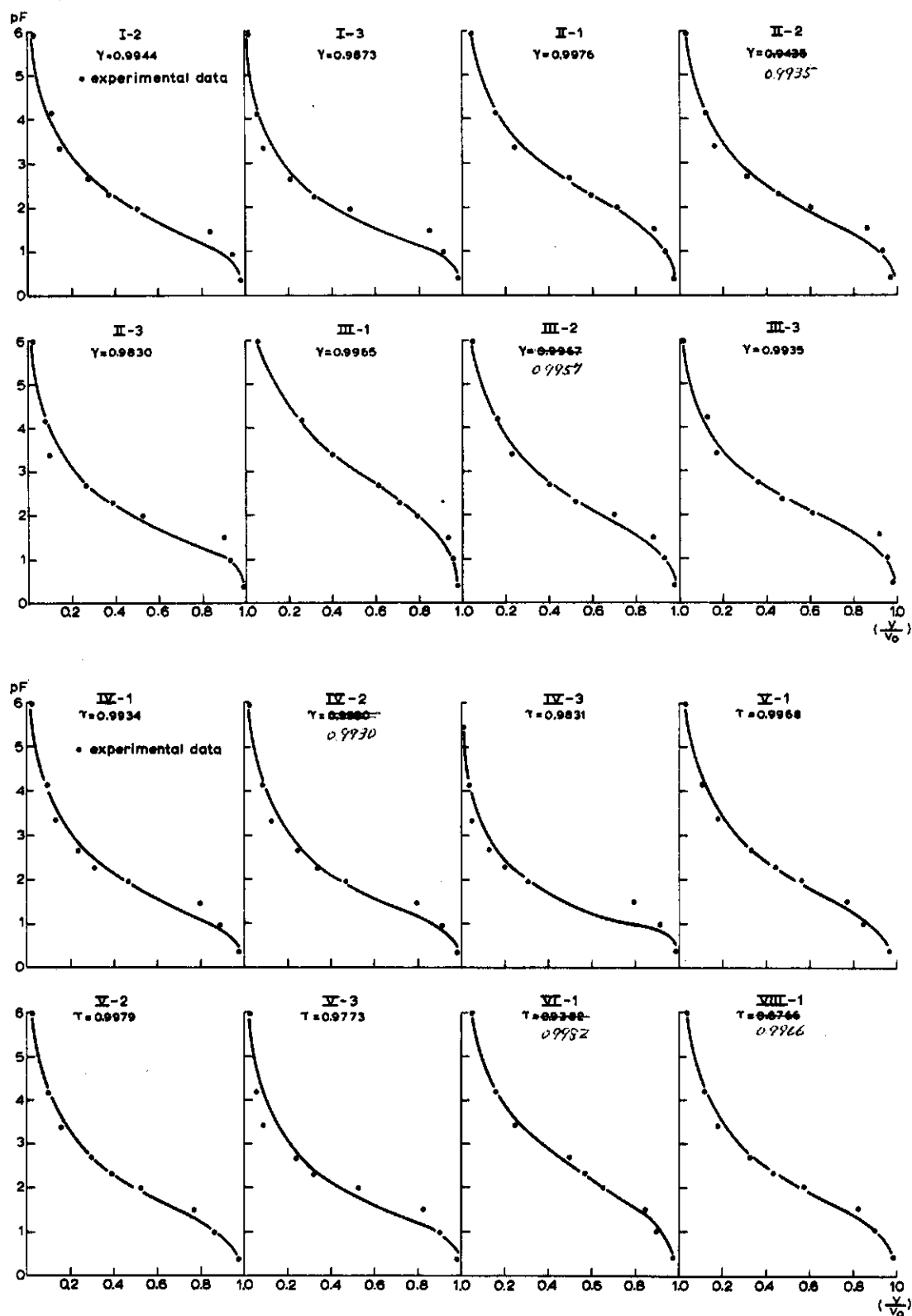
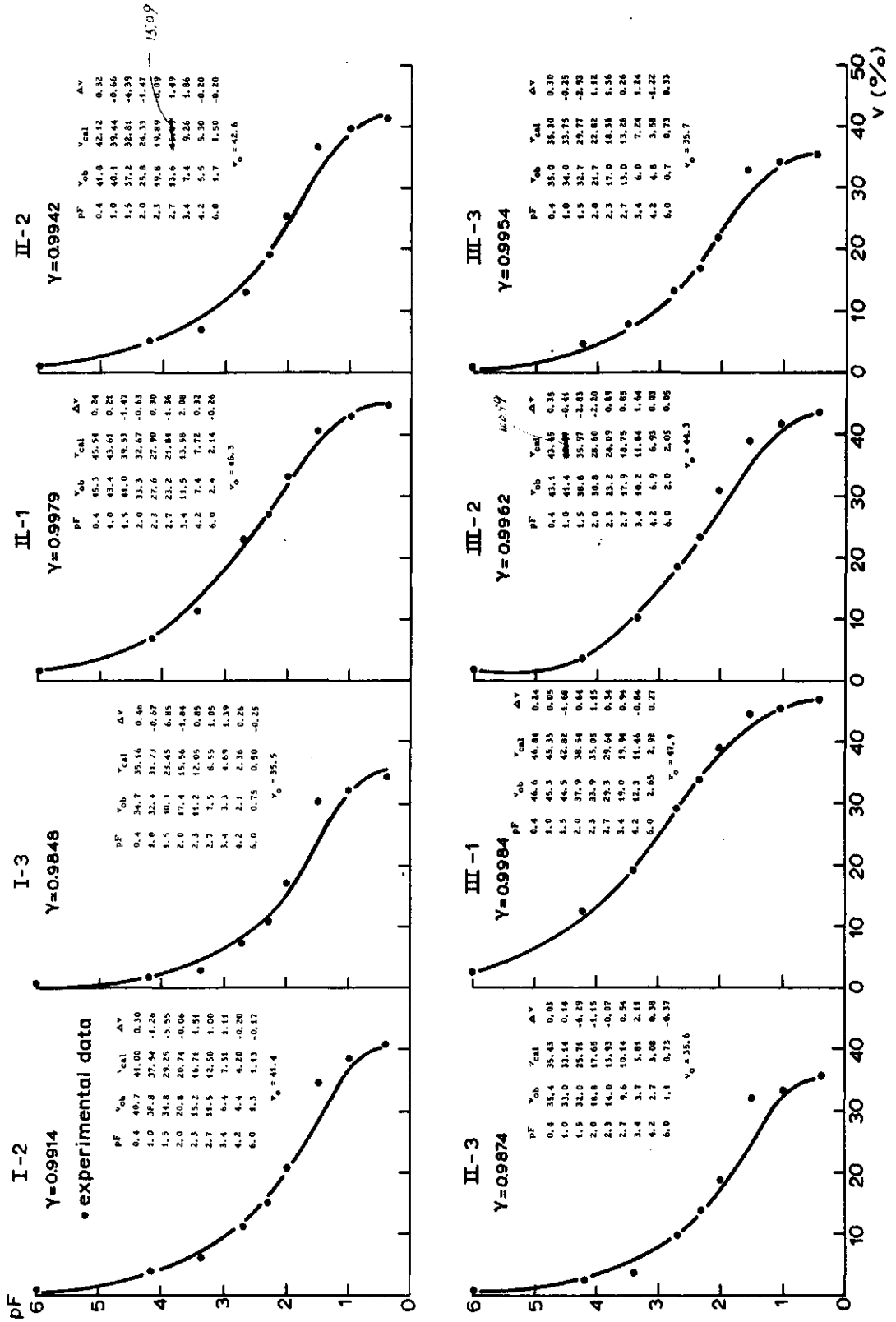


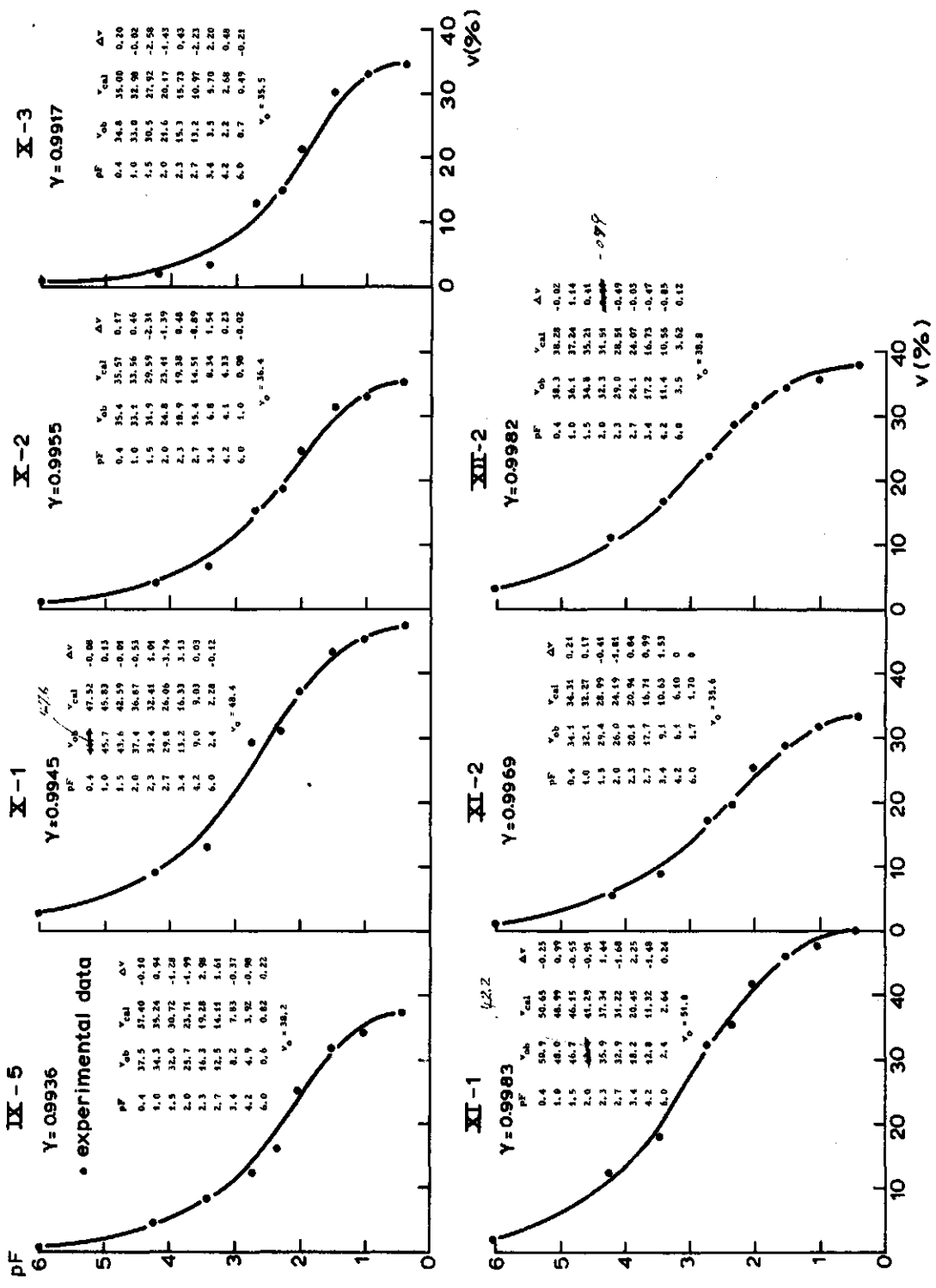
Fig. 6  $PF \sim (v/v_0)$  curves calculated by using  $PF = A + B \ln[(v/v_0)^c - 1]$

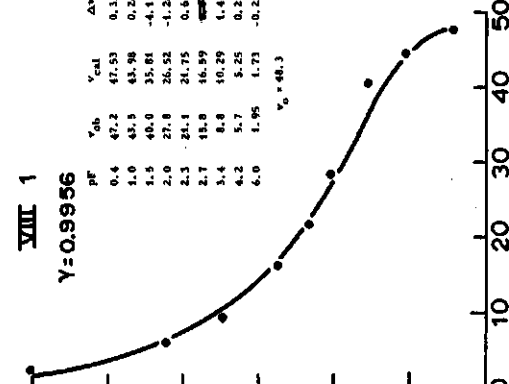
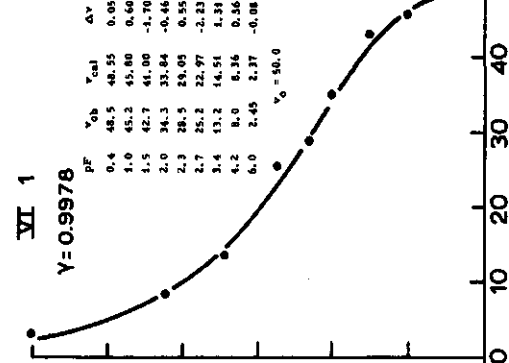
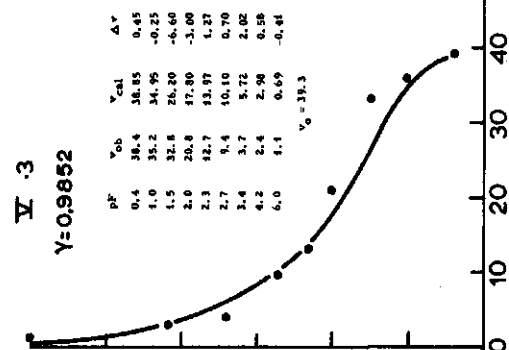
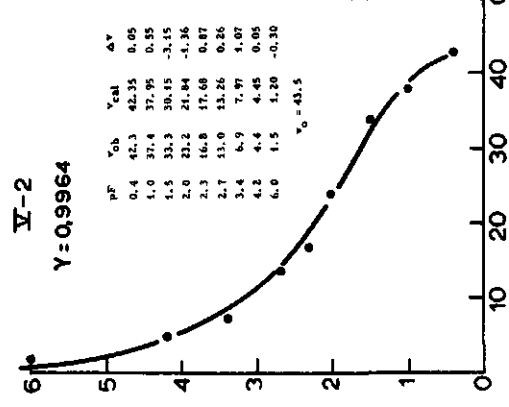
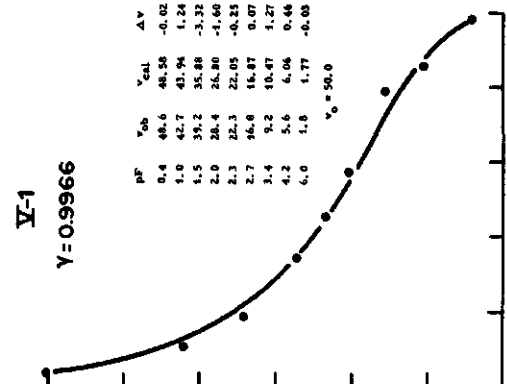
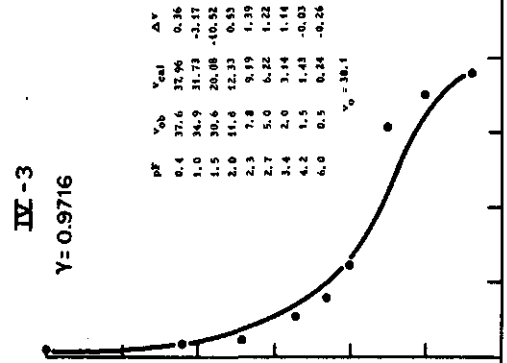
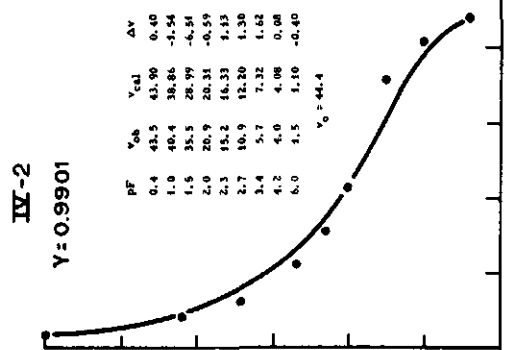
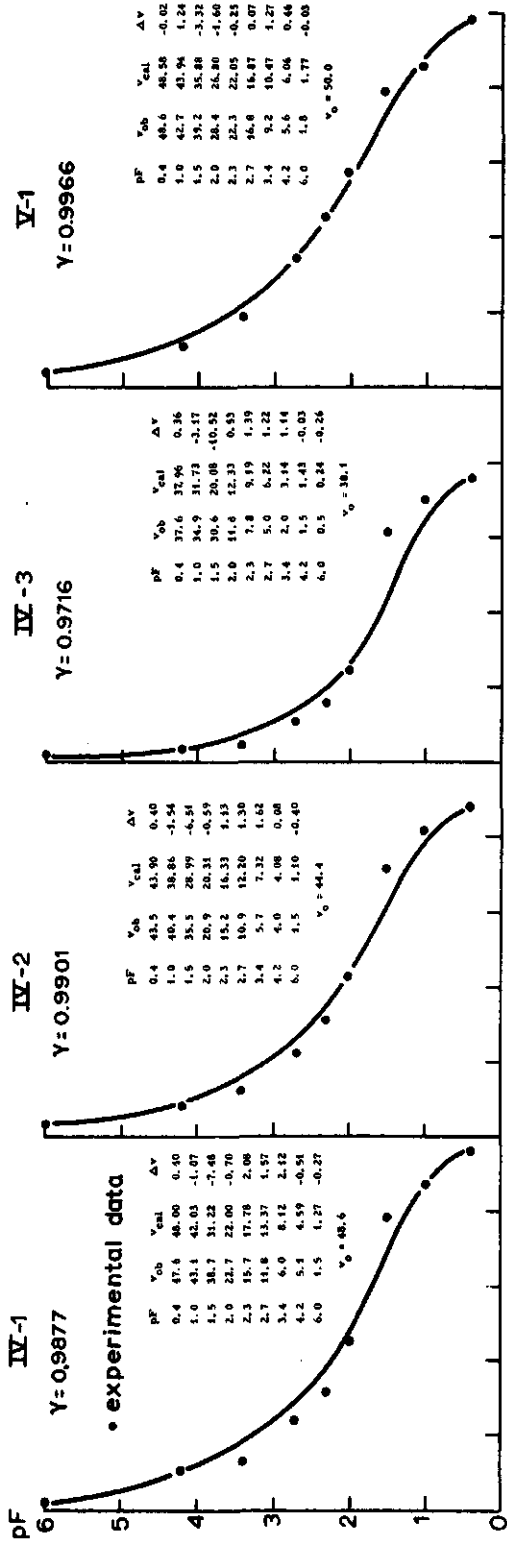


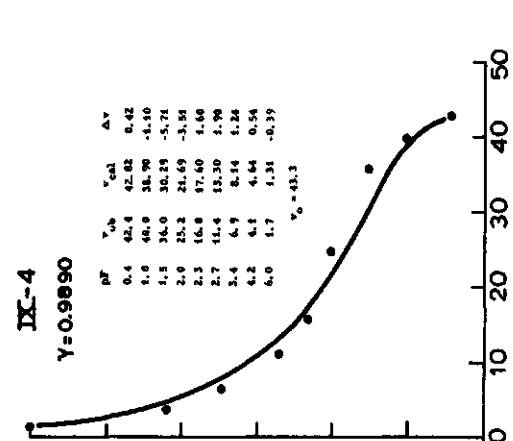
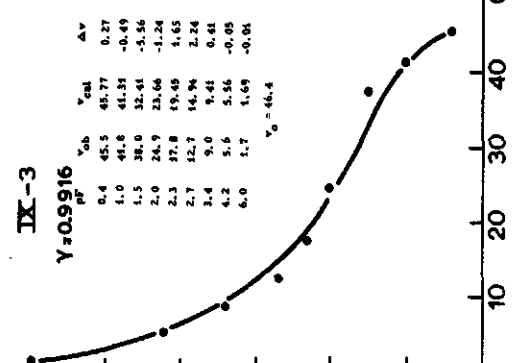
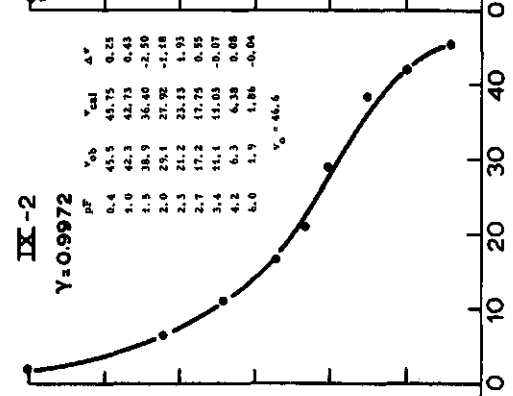
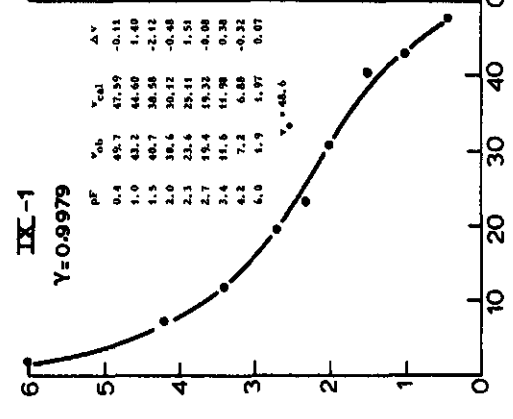
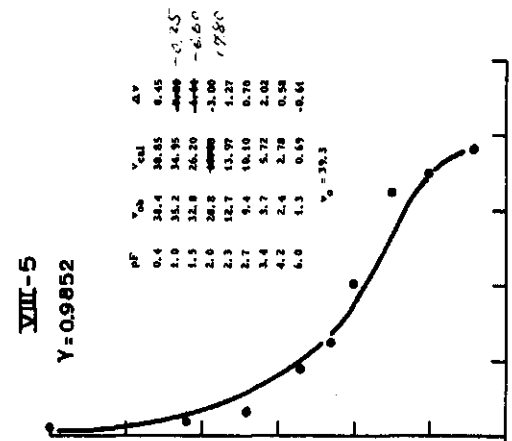
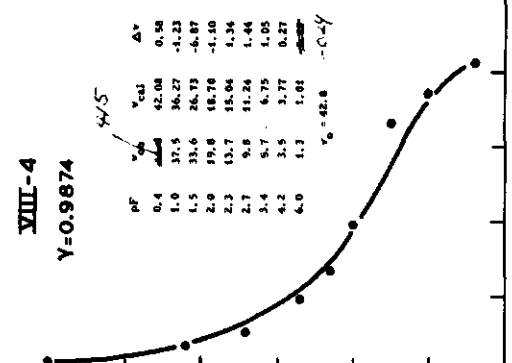
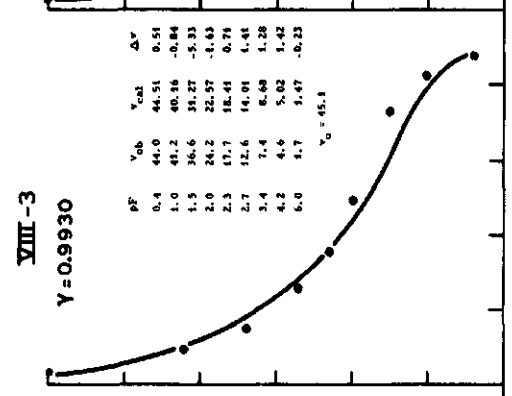
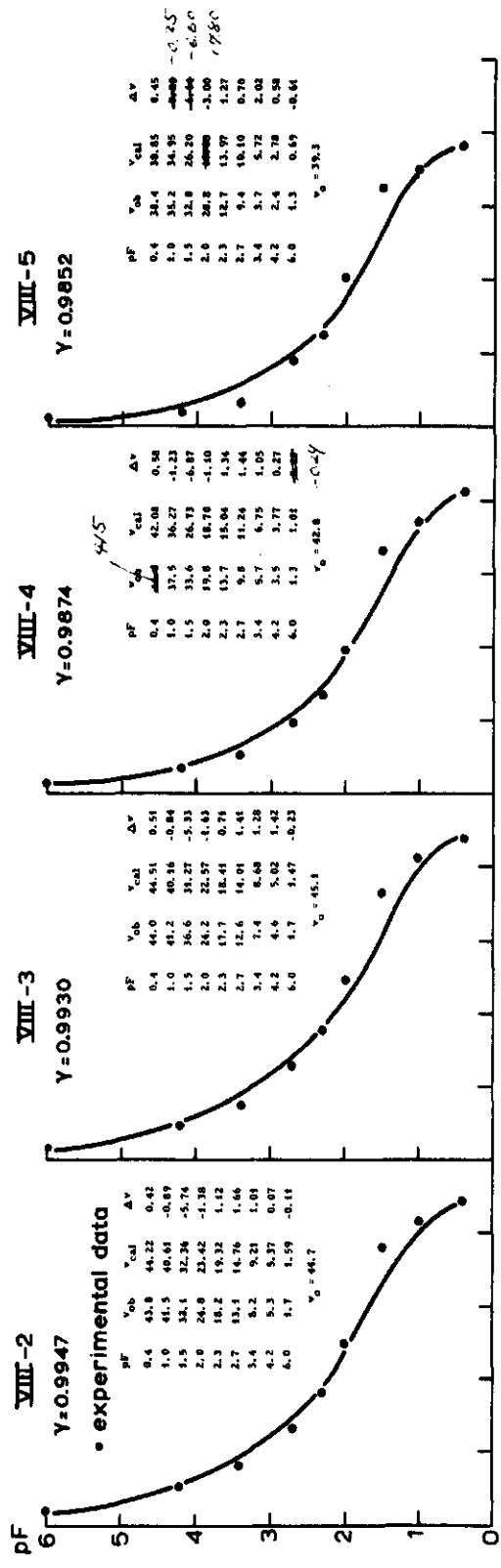
Appendix

Fig. 7. pF -- v curves calculated by using  $v = v_0 \left( e^{\frac{pF-A}{B}} + 1 \right)^{-\frac{1}{C}}$   
 (. experimental data)









Afgekend.

Schrijft vliegt om 25 copie!  
Nou beide Note's 705- en 825  
Heeft hy de afdrucker nou zyn  
3de note oitvengen? W!

ERRATA ( NOTA 825 )

- page 2 line 1; JACKSON(1973) is JACKSON(1973)<sup>23</sup>  
line 9; P/P<sub>0</sub> is P/P<sub>0</sub>  
linell;  $W \rightarrow \infty$  is  $W \rightarrow \infty$   
line 23; eq.(3) read:  $\ln W = A + B \ln \left[ \left( \frac{P}{P_0} \right)^c - 1 \right]$
- page 3 in subscript Fig.1; write: Fig.1 Example of  
description curve obtained by plotting the  
pr-value along the vertical axis and  $v/v_0$ -  
value along the horizontal axis.
- page 4 line 5; and is by  
in Table1, line 5; 0.9997 is 0.9977
- page 5 line 14; insert between v-value and is:  
and the observed v-value
- page 6 in subscript Fig.5; equation is equation  
in Fig.5; write on vertical axis:  
 $pr, 1, 2, 3, 4, 5, 6$   
in Fig.3; 8.5 is 28.5
- page 7 in Fig.4(2);  $x = \ln \left[ \left( \frac{v}{v_0} \right)^{-2.8} - 1 \right]$  is  $x = \ln \left[ \left( \frac{v}{v_0} \right)^{-2.8} - 1 \right]$   
in Fig.4(2); write on vertical axis;  
 $pr, 1, 2, 3, 4, 5, 6$
- page 8 in Table 2, line 4; 0.80 is 0.89
- page 9 in subscript Table 3; (r) is (r<sub>f</sub>)  
in Table 3, line<sup>9</sup>8; 0.9754 is 0.9954  
last line; 2.02835 is 2.02885
- page 11 line 11; ( table 2, 3 and 4 ) is needless  
line 19; table 4 is table 3
- page 12 line 8; value (C<sub>v</sub>) is value) (C<sub>v</sub>)  
in Table 5, line 4; 42.59 is 42.29  
line 5; 37.04 is 39.04  
line 6; 32.37 is 32.39  
0.85 is 6.85

Page 12 in Table 5, line 7; 34.81 is 24.81  
 0.69 is 4.69  
 37.35 is 27.35  
 39.63 is 29.63  
 line 8; 35.03 is 25.03  
 line 11; 4.3 is 4.2

page 13 in Fig.8;  $\Delta v_{1.5}$  is  $\Delta v_{PF1.5}$   
 in subscript Fig.8; ( $\Delta v$  pF 1.5) is ( $\Delta v_{PF1.5}$ )

page 14 in subscript Fig.9:  
 ----- graphical method  
 \_\_\_\_\_ equation (4)  
 ----- numerical method  
 . experimental data

page 15 in table 6, line 5; 0.56 is 1.56

page 17 in Fig.10; Write on horizontal axis of upper  
 graph; 0.2, 0.4, 0.6, 0.8, 1.0,  $v/v_0$   
 in lower graph; 44.29 is 41.29, 3.47 is 3.99

page 22 in subscript Table 10;  $(v/v_0)_{PF}$  is  $(v/v_0)_{PF}$

page 23 line 19; (p.22) is (p.21)  
 in Fig. 14;  $r=0.8798$  is  $r=0.8998$

page 26 in upper row on Fig.16(1);  $v$  is  $v$

page 27 in upper row on Fig.16(2); line 6; -5.27 is -5.29

page 29 line 1; eq.(5)

$$\begin{matrix} f_1 & f_2 & f_3 & f_4 & \Delta a & f_5 \\ f_1 & f_2 & f_3 & f_4 & \Delta b & f_5 \\ f_1 & f_2 & f_3 & f_4 & \Delta c & f_5 \\ f_1 & f_2 & f_3 & f_4 & \Delta v & f_5 \end{matrix} \text{ is } \begin{pmatrix} f_1 & f_2 & f_3 & f_4 \\ f_1 & f_2 & f_3 & f_4 \\ f_1 & f_2 & f_3 & f_4 \\ f_1 & f_2 & f_3 & f_4 \end{pmatrix} \begin{pmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta v \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

line 10;  $(\dots)^{\frac{1}{c}}$  is  $(\dots)^{\frac{1}{c}-1}$

$$\text{line 11; } f_b = \frac{\partial v}{\partial B} = \frac{v_0}{B \cdot c} \frac{PF-A}{B} \cdot e^{\frac{PF-A}{B}} \left( e^{\frac{PF-A}{B}} + 1 \right)^{-\frac{1}{c}-1}$$

$$f_b = \frac{\partial v}{\partial B} = \frac{v_0}{B \cdot c} \frac{PF-A}{B} \cdot e^{\frac{PF-A}{B}} \left( e^{\frac{PF-A}{B}} + 1 \right)^{-\frac{1}{c}-1}$$

line 12  $\frac{v_0}{c^2}$  is  $\frac{v_0}{c^2}$



Appendix

- page (1) in subscript Fig. 6; write Fig.6  $pF_{\sim}(v/v_0)$   
curves calculated by using  $pF = \frac{1}{N} \ln \left[ \left( \frac{v}{v_0} \right)^{-1} \right]$   
in II-2;  $r=0.9435$  is  $r=0.9935$   
in III-2;  $r=0.9967$  is  $r=0.9957$   
in IV-2;  $r=0.9980$  is  $r=0.9930$   
in VI-1;  $r=0.9382$  is  $r=0.9982$   
in VIII-1;  $r=0.8766$  is  $r=0.9966$
- page (2) subscript of Fig.6 is needless
- page (3) in Fig.7  
in II-2; 15.89 is 15.09  
in III-2; 40.97 is 40.99
- page (4) in VIII-1; 0.09 is 0.79
- page (5) in VIII-4; 44.5 is 41.5, -0.27 is -0.29  
in VIII-5; 7.25 is -0.25, -6.64 is -6.60  
17.83 is 17.80
- page (6) in X-1; 46.7 is 47.6  
in XI-1; 42.4 is 42.2  
in XII-2; -0.99 is -0.79

\*\* Order of appendix sheets is mistake, so arrange in regular order

