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SECOND AND THIRD DEGREE EQUATIONS FOR THE DETERMINATION OF THE SPACING BETWEEN PARALLEL DRAINAGE CHANNELS

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C O N T E N T S

1. DRAINAGE OF A HOMOGENEOUS AQUIFER

In the Netherlands two formulae are rather widely used for the computation of discharge, drain spacing or phreatic level. Both formulae take into account the radial flow to the parallel, horizontal drains.

The older formula of these two is obtained by assuming a completely horizontal flow, but instead of the thickness H_o of the aquifer between drain level and impermeable base (fig. 1a), a reduced thickness d (fig. 1b) has to be introduced to take into account the influence of the radial flow (HOOGHOUDT, 1940).

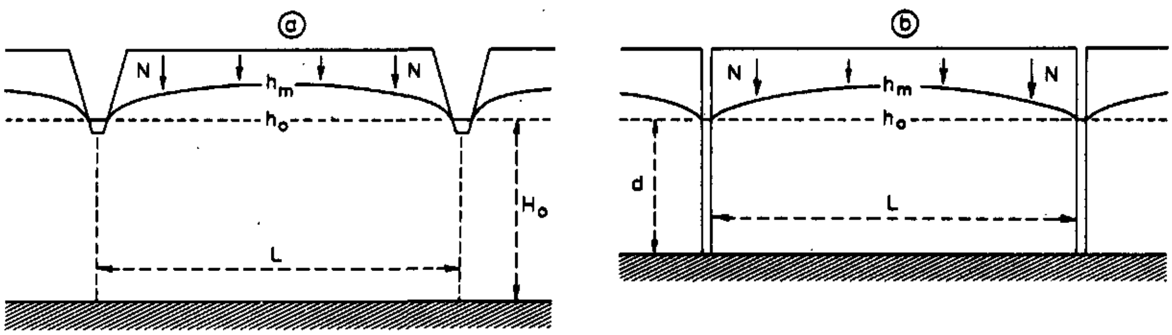

Fig. 1. Steady state groundwaterflow in a homogeneous aquifer.
a. Real situation with partially penetrating parallel drains
b. Imaginary situation with fully penetrating drains.
Inflow, outflow, drain spacing and potential difference are supposed equal in both cases. The equivalent layer must have a thickness $d < H_o$

$$N = U = \frac{4k(h_m - h_o)^2 + 8kd(h_m - h_o)}{L^2} \qquad (1)$$

N = downward flux through phreatic surface (in steady state equal to precipitation surplus P-E)

U = discharge by drains per unit of horizontal area

k = hydraulic conductivity

L = spacing between parallel drains

h_m = hydraulic head of the groundwater midway between the drains

h_o = hydraulic head of the open water in the drains

d = thickness of equivalent layer

H_o = thickness of the aquifer between drain level and impermeable base

B_{wp} = wet perimeter of the drainage channels

The parameter d depends solely on the thickness H_o of the aquifer below drainlevel, the drainspacing L and the wet perimeter B_{wp}. Hooghoudt used infinite series to compute the parameter d (see tables in the original paper, HOOGHOUDT, 1940). These infinite series can be replaced by a closed expression containing hyperbolic functions (LABYE, 1960). In spite of this simplification d is a rather complicated function of L, so that an explicite solution of L cannot be obtained in this way. However, equation (1) has the advantage, that it shows immediately that there is a second degree relation between $h_m - h_o$ and N or U.

Another formula valid for the same situation has been obtained by distinguishing a vertical, a horizontal and a radial component in the flow from land surface to drainage channel (see b, c and d in fig. 2 and ERNST, 1956, 1962, 1963). The potential difference between A and B has to be determined for each of the figures 2b, c and d. Addition will give the potential difference valid for fig. 2a:

$$h_m - h_o = \Delta h_{vert} + \Delta h_{hor} + \Delta h_{rad} \qquad (2)$$

with

$$\Delta h_{vert} = \frac{N}{k} \left(H_u - \frac{H_u^2}{2H_m}\right) \approx \frac{N}{k} (h_m - h_o) \qquad (3)$$

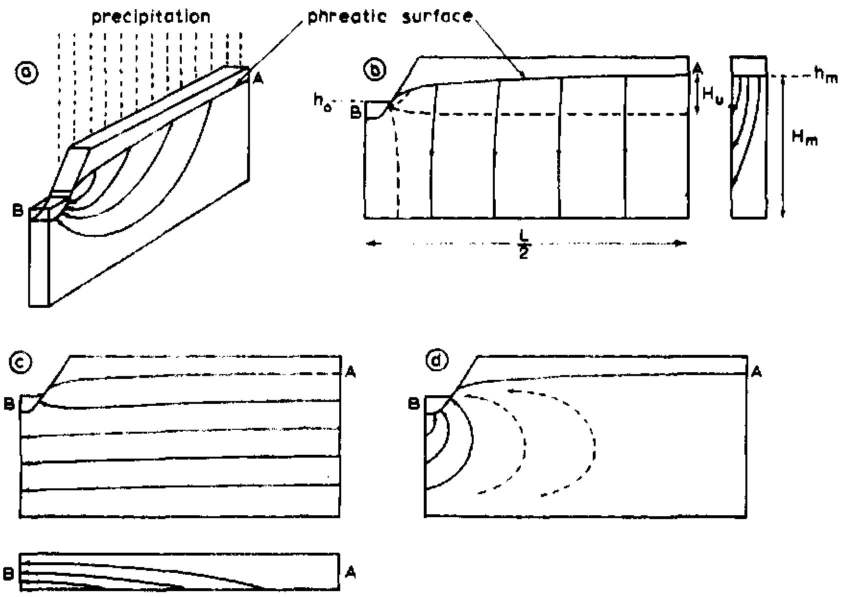

Fig. 2. Separation of the groundwaterflow into three components for vertical, horizontal and radial flow: a = b+c+d

$$\Delta h_{hor} = \frac{NL^2}{4k(H_o+H_m)} = \frac{NL^2}{8kH_{av}} \tag{4}$$

$$\Delta h_{rad} = NL\Omega \tag{5}$$

H_{av} = average thickness of aquifer

Ω = radial flow resistance

By substitution of eqs. (3), (4) and (5) into eq. (2):

$$h_m - h_o = \frac{N}{k}(h_m - h_o) + \frac{NL^2}{8kH_{av}} + NL\Omega$$

or:

$$\frac{k-N}{k}(h_m - h_o) = \frac{NL^2}{8kH_{av}} + NL\Omega \tag{6}$$

Eq. (6) seems to be quite close to a linear relation between h_m-h_o and N, because in nearly all practical cases the coefficient $(k-N)/k$ will ly between 0.9 and 1. However this is not a reason to consider eq. (6) very different from the second degree eq. (1). It should be born in mind, that not only H_{av} is depending on h_m-h_o, i.e. $H_{av} = H_o+\frac{1}{2}(h_m-h_o)$, but that there is also a slight decrease in the radial flow resistance Ω for increasing discharge.

For the moment all non-linear effects will be discarded by assuming that only situations with small N/k and consequently small h_m-h_o have to be dealt with. This implies that eq. (6) can be replaced by:

$$h_m-h_o = \frac{NL^2}{8kH_o} + NL\Omega_o \tag{7}$$

with

Ω_o = radial flow resistance for a nearly horizontal phreatic surface.

For a drainage channel with a half circular wet perimeter or with a width about equal to twice its depth (fig. 3a), the following expression can be used (ERNST, 1962):

$$k\Omega_o = \frac{1}{\pi}\ln\frac{H_o}{B_{wp}} \tag{8}$$

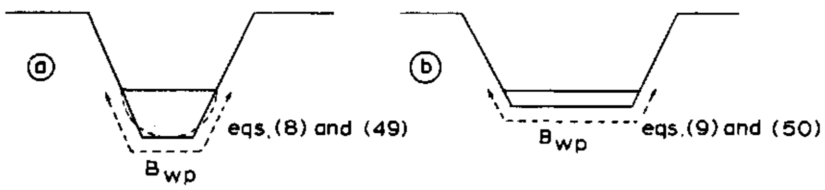

Fig. 3. Drainage channels with trapezoidal wet cross-sectional area. a. Width about equal to twice the depth, which case is fairly equivalent to a half circular shape. In the homogeneous aquifer eq. (8) can be used. In case of a two layer aquifer see fig.10 and eq. (49). b. Width much larger than depth, which case is about equal to a zero depth. For a homogeneous aquifer and a two layer aquifer the eqs. (9) and (50) can be used respectively

When the depth of the channel is small as compared with the width (fig. 3b), the radial flow resistance Ω_o can be determined by means of (ERNST, 1962):

$$k\Omega_o = \frac{1}{\pi} \ln \frac{4H_o}{\pi B_{wp}} \tag{9}$$

The decrease of Ω, with increasing discharge q_o = NL through each of the drainage channels, is a rather complicated problem, which has not been investigated thoroughly up to now. For a homogeneous aquifer both the depth H_o of the impermeable base, the shape of the drainage channel (e.g.: slope α, see fig. 4) and the discharge intensity q_o should be taken into account.

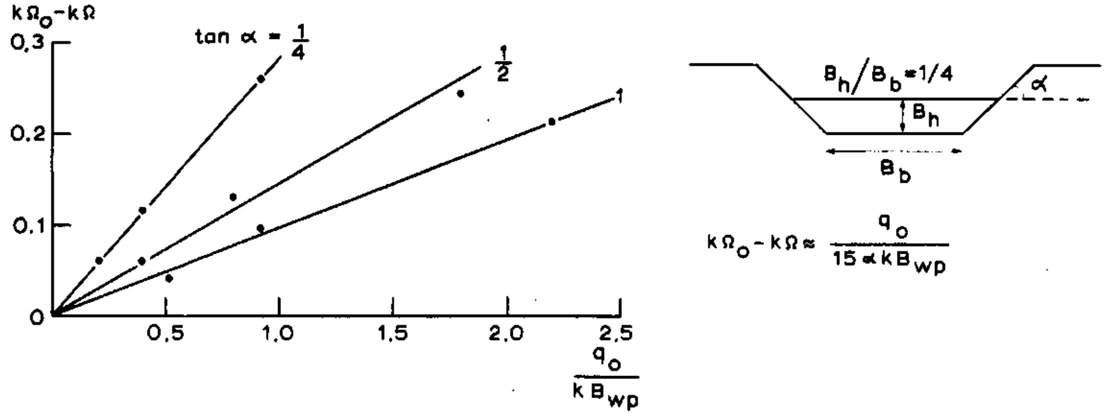

Fig. 4. Nomograph for the decrease in radial flow resistance Ω with increasing discharge q_o for a wet cross-sectional area like shown in right hand side of this figure (ERNST, 1962, fig. 28c). The water depth is assumed to remain constant

The magnitude of the decrease of Ω can be read from fig. 4. Because in most practical cases $q_o/kB_{wp} < 1$, it can be assumed that this decrease is not of much importance, except for flat slopes and very large discharge intensities.

Another question which has still to be discussed is about the applicability of the preceding formulae on very thick aquifers. Both expressions (8) and (9) are not to be used in case of very large H_o/L-values. It can be seen immediately, that use of these equations for $H_o = \infty$ would lead to $h_m - h_o = \infty$. It is obvious, that for increasing H_o there must be a gradually decreasing hydraulic head difference $h_m - h_o$, with the following minimum values for the cases corresponding to eqs. (8) and (9) (ERNST, 1956, 1962):

$$H_o \to \infty \begin{cases} h_m - h_o \to \dfrac{NL}{\pi k} \ln \dfrac{L}{B_{wp}} & (10) \\ h_m - h_o \to \dfrac{NL}{\pi k} \ln \dfrac{4L}{B_{wp}} & (11) \end{cases}$$

Substitution of eq. (8) or (9) in (7) and comparison with eq. (10) or (11) shows that the accuracy is satisfactory if $H_o < L/4$. Formulae containing radial flow resistances can therefore be accepted especially for those practical problems in which H_o/L and $(h_o - h_m)/L$ have no excessive values.

2. A THIRD DEGREE EQUATION FOR THE DRAIN SPACING

Still using the assumption that N/k and $h_m - h_o$ are relatively small, eq. (1) can be simplified to:

$$h_m - h_o = \frac{NL^2}{8kd} \qquad (12)$$

From (7) and (12) it follows immediately that

$$\frac{d}{H_o} = \frac{L}{L + 8kH_o\Omega_o} \qquad (13)$$

This expression for d can also be used in eq. (1) without neglecting the second degree term. Then only one condition has to be obeyed: $H_o < L/4$. For larger values of H_o eq. (13) can still be used by introducing a fictitious value H_* being about one fourth of the presumable value of L.

Substitution of (13) into (1) gives:

$$NL^2 = 8kH_o \frac{L}{L + 8kH_o\Omega_o} (h_m - h_o) + 4k(h_m - h_o)^2$$

or:

$$\frac{NL^2}{4k(h_m - h_o)^2} - 1 = \frac{2H_o L}{(L + 8kH_o\Omega_o)(h_m - h_o)} \tag{14}$$

A rather convenient method for the solution of this third degree equation in L is by means of a nomograph. For this purpose the following parameters are substituted:

$$\frac{L}{2(h_m - h_o)}\sqrt{\frac{N}{k}} = \lambda \tag{15}$$

$$\frac{4\ kH_o\Omega_o}{h_m - h_o}\sqrt{\frac{N}{k}} = \alpha \tag{16}$$

$$\frac{h_m - h_o}{2H_o} = \beta_1 \tag{17}$$

which results in:

$$\beta_1(\lambda^2 - 1) = \frac{\lambda}{\lambda + \alpha} \tag{18}$$

A graphical representation of eq. (18) is given in fig. 5. For any arbitrary combination of α and β_1 the corresponding λ-value can be read directly from this diagram. Finally eq. (15) has to be used in order to obtain the value of L satisfying the original eq. (14).

The increasing elevation of the phreatic surface with increasing precipitation must result in a non-linear relation between N or U and $h_m - h_o$. This has been taken into account by HOOGHOUDT in his equation (1) by assuming that the flow in the ground above drain level is the main reason for the non-linear behaviour. Neglect of the vertical component of the flow above drain level is not always allowed. The vertical flow is of importance if $0.2 < N/k < 1$ and

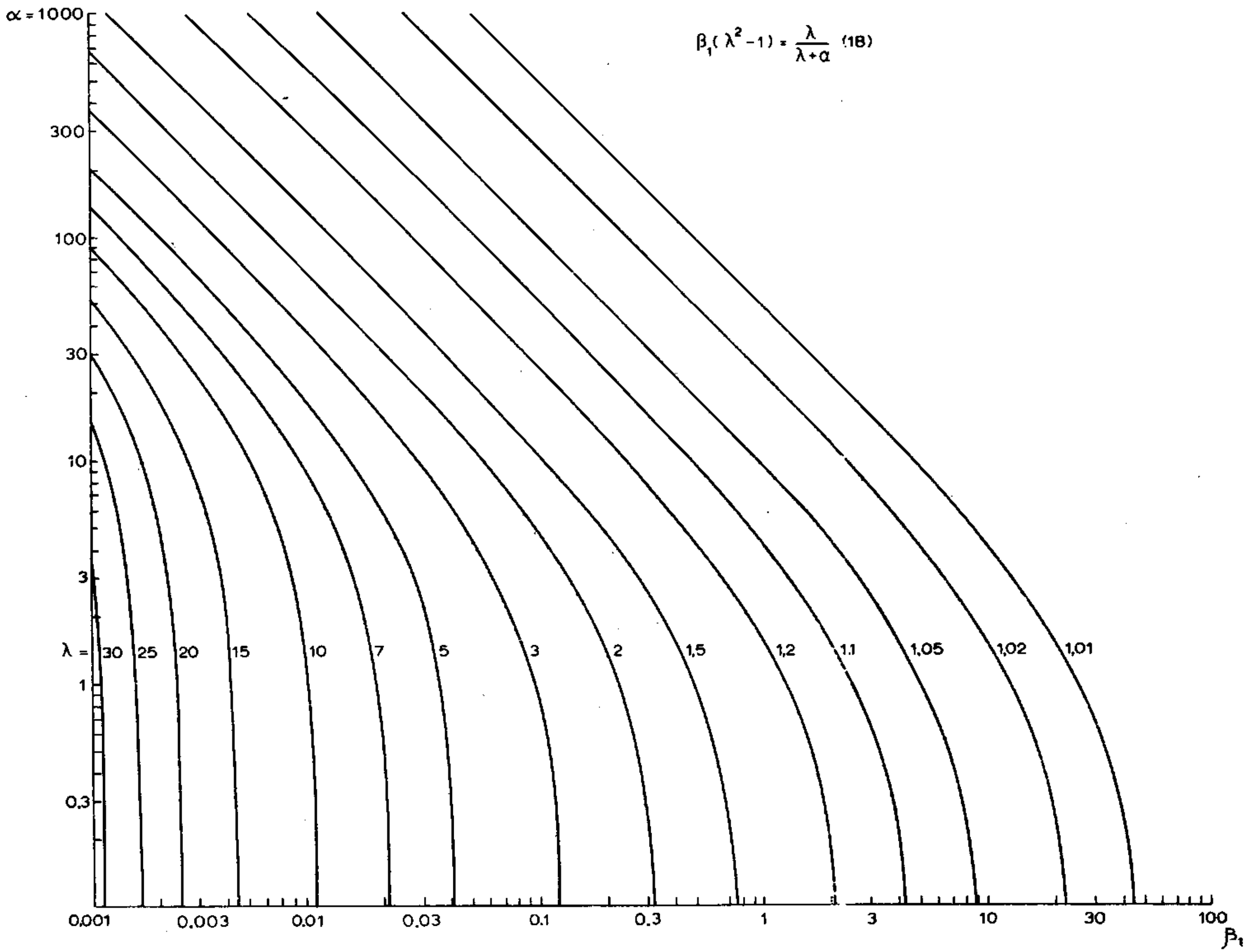

Fig. 5. Nomograph of equation (18)

also in two-layer aquifers with a rather small hydraulic conductivity in the upper region. Whatever the importance may be, the vertical flow component can be taken properly into account by addition of a coefficient k/(k-N), as has been shown by eqs. (3) and (6) (ERNST, 1956; KIRKHAM, 1961).

It is obvious, that this coefficient should also be added to eqs. (1) and (14) respectively resulting in:

$$N = \frac{4(k-N)(h_m-h_o)^2 + 8d(k-N)(h_m-h_o)}{L^2} \qquad (19)$$

and

$$\frac{N}{k-N} L^2 = \frac{L}{L + 8kH_o\Omega_o} 8H_o(h_m-h_o) + 4(h_m-h_o)^2 \qquad (20)$$

Equation (18) remains valid with the following expressions for λ, α and β_1, being slightly different from those used before:

$$\frac{L}{2(h_m-h_o)}\sqrt{\frac{N}{k-N}} = \lambda \qquad (21)$$

$$\frac{4kH_o\Omega_o}{h_m-h_o}\sqrt{\frac{N}{k-N}} = \alpha \qquad (22)$$

$$\frac{h_m-h_o}{2H_o} = \beta_1 \qquad (23)$$

In the analysis achieved by KIRKHAM and TOKSÖZ (KIRKHAM, 1958; TOKSÖZ and KIRKHAM, 1971a and 1971b) the horizontal flow above drain level has been neglected, which implies that always $d^2h/dN^2 > 0$. This involves that a comparison of these results with eq. (20) should be done in the first place for $N << K$. Under this condition Kirkham's formula (KIRKHAM, 1958) and formula (20) will give practically results except for $H_o > L/4$, where the combination of eq. (7) with (8) or (9) is failing.

3. THE INFLECTION POINT IN THE $h_m(N)$-RELATION

In many practical cases a linear relation between $N = U$ and $h_m - h_o$ can be assumed without the implication of large errors. Introduction of special assumptions for the flow direction above drainlevel - horizontal flow or vertical flow - will result in non-linear relations with exclusively negative or exclusively positive values for the second derivative d^2h/dN^2.

In the preceding chapter it has been shown, that from a fundamental point of view a more satisfactory relation can be obtained by means of the third degree eq. (20), with both positive and negative values of the second derivative d^2h/dN^2. The conditions under which for practical application a linear or non-linear relation might be assumed, can be most easily discussed by writing equation (19), in a slightly different way:

$$4\left[\left\{\frac{h_m - h_o + d}{L}\right\}^2 - \left\{\frac{d}{L}\right\}^2\right] = \frac{N}{k-N} \tag{24}$$

Equation (24) can also be written as:

$$4(y^2-a^2) = \frac{x}{1-x} \tag{25}$$

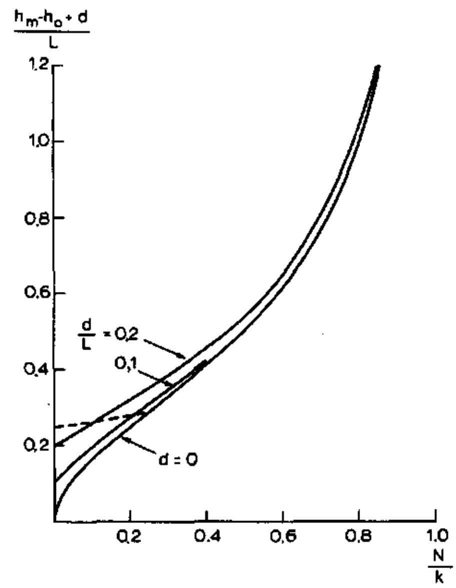

Fig. 6. Relation between hydraulic head $h_m - h_o$ and precipitation surplus N dependent on d, L and k according to eq. (24)
---------locus of inflection points

In each inflection point the second derivative must be equal to zero:

$$\frac{d^2y}{dx^2} = 0 \tag{26}$$

By elimination of a, from eqs. (25) and (26), the locus of the inflection points is found to be:

$$\frac{h_m - h_o + d}{L} = \frac{1}{4}\sqrt{\frac{k}{k-N}} \tag{27}$$

Eq. (24) and the locus of inflection points according to eq. (27) are shown in fig. 6. It can be seen that an obvious curvature is only possible for relatively large values of $(h_m-h_o)/L$ or (and) d/L. According to Hooghoudt's tables the maximum value of d/L = 0.34. Large values of $(h_m-h_o)/L$ can be considered as being exceptional under practical conditions. This implies that the influence of k/(k-N) will seldom be so large that the concave curvature will be predominating. The convex curvature, which follows from the Hooghoudt equation (1), is only of importance for relative small values of d/L. This cannot always be considered to be neglectable, especially for two-layer aquifers with $k_1 << k_2$, which case will be considered in the next chapter.

4. TWO-LAYER AQUIFERS

The heterogeneous aquifer, to which Hooghoudt's formula can be applied equally well as to the homogeneous aquifer, is made up of two layers with permeabilities k_1 and k_2 and divided by a horizontal boundary running through the level of the open water in the drainage channels (fig. 7). For this case formula (1) can be changed into:

$$N = U = \frac{8k_2d(h_m-h_o) + 4k_1(h_m-h_o)^2}{L^2} \tag{28}$$

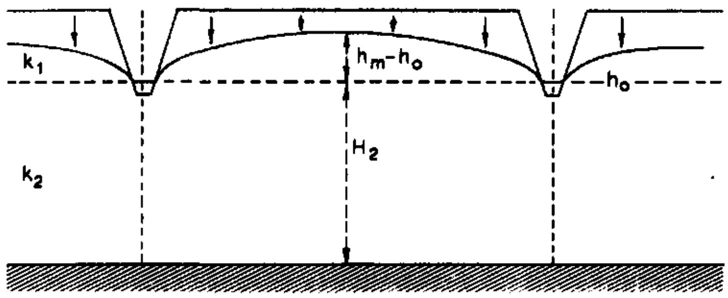

Fig. 7. Groundwater flow to partially penetrating drains. Situation comparable with fig. 1a, but in this case a two-layer aquifer with hydraulic conductivities k_1 and k_2, respectively above and below the level of the open water

In chapter 1 the vertical component of the flow above the drain level has been taken into account by eq. (3). The potential difference for the vertical flow in the upper layer can now be expressed by:

$$\Delta h_{vert} = \frac{N(h_m-h_o)}{k_1}\left[1 - \frac{\frac{1}{2}k_1(h_m-h_o)}{k_2H_2 + k_1(h_m-h_o)}\right] \qquad (29)$$

Analogous to the case of the homogeneous aquifer a simplification of the expression for Δh_{vert} can be accepted by neglecting the second term between the brackets, giving:

$$\Delta h_{vert} = \frac{N}{k_1}(h_m-h_o) \qquad (30)$$

The vertical flow can be incorporated in the Hooghoudt formula by adding Δh_{vert} to the potential difference used in eq. (28). The result is an expression similar to eq. (19):

$$\frac{N}{k_1-N} = \frac{U}{k_1-N} = \frac{8\frac{k_2}{k_1}d(h_m-h_o) + 4(h_m-h_o)^2}{L^2} \qquad (31)$$

The expression which has to be substituted for d can be found again in (13), but now with proper subscripts:

$$\frac{d}{H_2} = \frac{L}{L + 8k_2H_2\Omega_o} \qquad (32)$$

Substitution of eq. (32) in (31) and rearranging results in:

$$\frac{NL^2}{4(k_1-N)(h_m-h_o)^2} - 1 = \frac{2k_2H_2L}{k_1(h_m-h_o)(L+8k_2H_2\Omega_o)} \qquad (33)$$

Principally there is no difference between eqs. (14) and (33). Therefore eq. (18) and fig. 5 can be used again for a solution of L. The parameters λ, α and β_1 are now:

$$\frac{L}{2(h_m-h_o)}\sqrt{\frac{N}{k_1-N}} = \lambda \qquad (34)$$

$$\frac{4k_2H_2\Omega_o}{h_m-h_o}\sqrt{\frac{N}{k_1-N}} = \alpha \qquad (35)$$

$$\frac{k_1(h_m-h_o)}{2k_2H_2} = \beta_1 \qquad (36)$$

5. SIMPLIFIED EXPRESSIONS FOR THE DRAIN SPACING

Some attempts have been made to obtain simple expressions for the drain spacing. The first attempt was made by adding an empirical coefficient to the last term of equation (7):

$$h_m-h_o = \frac{NL^2}{8k_2H_2 + 4k_1(h_m-h_o)} + \beta_2NL\Omega_o \qquad (37)$$

with

$$\beta_2 = \frac{K_2H_2}{k_2H_2 + \frac{1}{2}k_1(h_m-h_o)} = \frac{1}{1+\beta_1} \qquad (38)$$

The introduction of the coefficient β_2 was done with the intention to avoid the use of more complicated expressions for Ω. This results in an equation of the second degree in L:

$$h_m - h_o = N \frac{L^2 + 8k_2H_2\Omega_oL}{8k_2H_2 + 4k_1h} \tag{39}$$

However, eq. (39) is only sufficiently accurate if $\alpha\beta_1 \approx 2k_1\Omega_o\sqrt{N/k} < 0.1$. This is enough reason to reject eq. (39).

By solving L^2 from eq. (7), adding a second degree term in h similar to eq. (1) and using the radial flow resistance $\beta_2\Omega_o$ similar to eq. (37), a simplified expression of satisfactory accuracy has been obtained, namely:

$$NL^2 = 4k_1(h_m-h_o)^2 + N\left[-4\beta_2k_2H_2\Omega_o + \sqrt{(4\beta_2k_2H_2\Omega_o)^2 + \frac{8k_2D_2(h_m-h_o)^2}{N}}\right]^2 \tag{40}$$

Introduction of the parameters λ, α and β_1 (with $\sqrt{N/k_1}$ instead of $\sqrt{N/(k_1-N)}$) permits a shorter writing of the last formula:

$$\lambda^2 = 1 + \left[-\frac{\alpha}{2(1+\beta_1)} + \sqrt{\frac{\alpha^2}{4(1+\beta_1)^2} + \frac{1}{\beta_1}}\right]^2 \tag{41}$$

$$\frac{L}{2(h_m-h_o)}\sqrt{\frac{N}{k_1}} = \lambda \tag{42}$$

$$\frac{4k_2H_2\Omega_o}{h_m-h_o}\sqrt{\frac{N}{k_1}} = \alpha \tag{43}$$

$$\frac{k_1(h_m-h_o)}{2k_2H_2} = \beta_1 \tag{44}$$

A nomograph of eq. (40) is shown in fig. 8. The ~~broken~~ interrupted lines in this figure make clear that the differences between eqs. (33) and (40) are so small that they will be hardly of importance in practical applications.

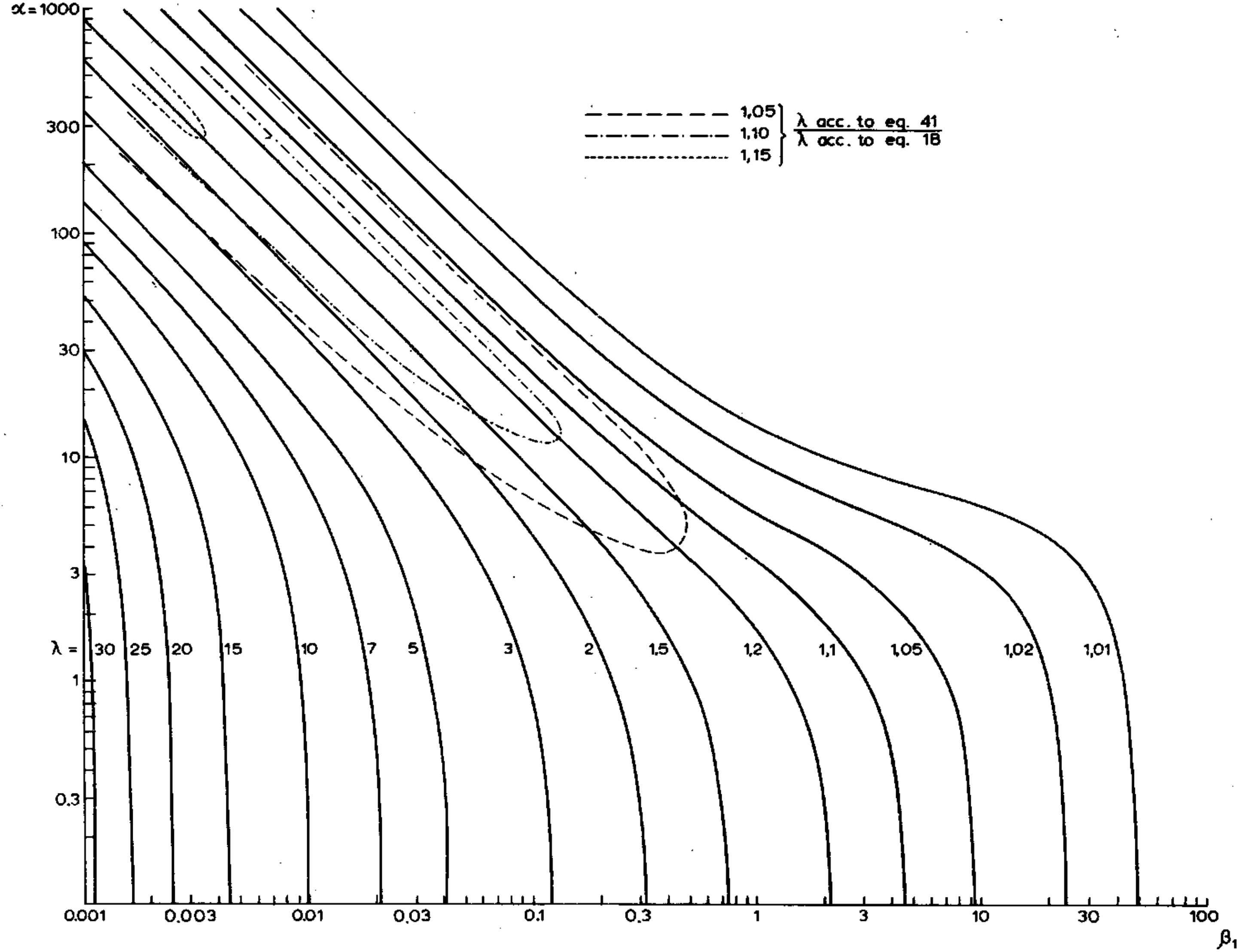

Fig. 8. Nomograph of equation (42)

6. THE DRAIN SPACING FOR A THREE-LAYER AQUIFER

Because eq. (33) is applicable to two-layer aquifers, with the restriction that the interface between the two layers has to be of the same depth as the level of the open water, it seems profitable for practical application to investigate also the case of an interface below the bottom of the drainage channels. This can be done by passing over immediately to the consideration of three-layer aquifers like shown in fig. 9.

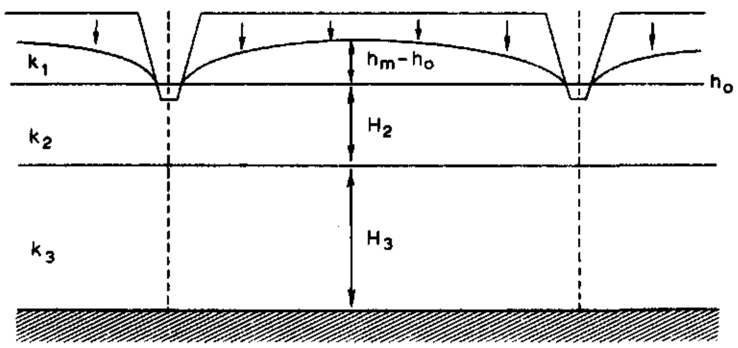

Fig. 9. Parallel drains in a three-layer aquifer

Eq. (33) can be adapted to such three-layer aquifers by writing $k_2H_2 + k_3H_3$ instead of k_2H_2. Moreover a resistance Ω_{20} will be introduced for the radial flow in the two layers below the open water level. This means that hardly anything new will be met in the following eqs. (45) .... (48).

$$\frac{NL^2}{4(k_1-N)(h_m-h_o)^2} - 1 = \frac{2(k_2H_2 + k_3H_3)L}{k_1(h_m-h_o)\{L + 8(k_2H_2 + k_3H_3)\Omega_{20}\}} \tag{45}$$

Eq. (18) is again valid when the following expressions for λ, α and β_1 are used:

$$\frac{L}{2(h_m-h_o)} \sqrt{\frac{N}{k_1-N}} = \lambda \tag{46}$$

$$\frac{4(k_2H_2 + k_3H_3)\Omega_{20}}{h_m-h_o}\sqrt{\frac{N}{k_1-N}} = \alpha \tag{47}$$

$$\frac{k_1(h_m-h_o)}{2(k_2H_2 + k_3H_3)} = \beta_1 \tag{48}$$

For the application of the preceding equations the determination of the transmissivity for horizontal flow in each of the three layers is required. This will give no special difficulties compared with the more simple cases. So there remains only the determination of the radial flow resistance Ω_{20} as a new problem, asking for a separate treatment.

Fig. 10 is showing a nomograph by means of which a determination of radial flow resistance in two-layer aquifers can be obtained. In this nomograph only k_3/k_2 and H_3/H_{20} are considered as variables, while variations in size or shape of the drainage channel and the phreatic surface are neglected (ERNST, 1962, 1963).

Fig. 10 depends mainly on the assumption that $B_{wp} = H_{20}$ and that moreover the discharge q_o is so small that variations, comparable to what has been shown in fig. 4, are of no importance. Only for these conditions fig. 10 is giving immediately the corresponding radial flow resistance Ω'_{20}. For arbitrary values of the wet perimeter B_{wp}, but anyhow not larger than H_{20}, the real radial flow resistance Ω_{20} can be computed by means of eqs. (49) or (50):

$$k_2\Omega_{20} = k_2\Omega'_{20} + \frac{1}{\pi}\ln\frac{H_{20}}{B_{wp}} \tag{49}$$

$$k_2\Omega_{20} = k_2\Omega'_{20} + \frac{1}{\pi}\ln\frac{4H_{20}}{\pi B_{wp}} \tag{50}$$

in which formulae:

Ω_{20} = radial flow resistance for a two layer aquifer with a nearly horizontal phreatic surface

H_{20} = thickness of layer with k_2 between drain level and lower boundary of this layer

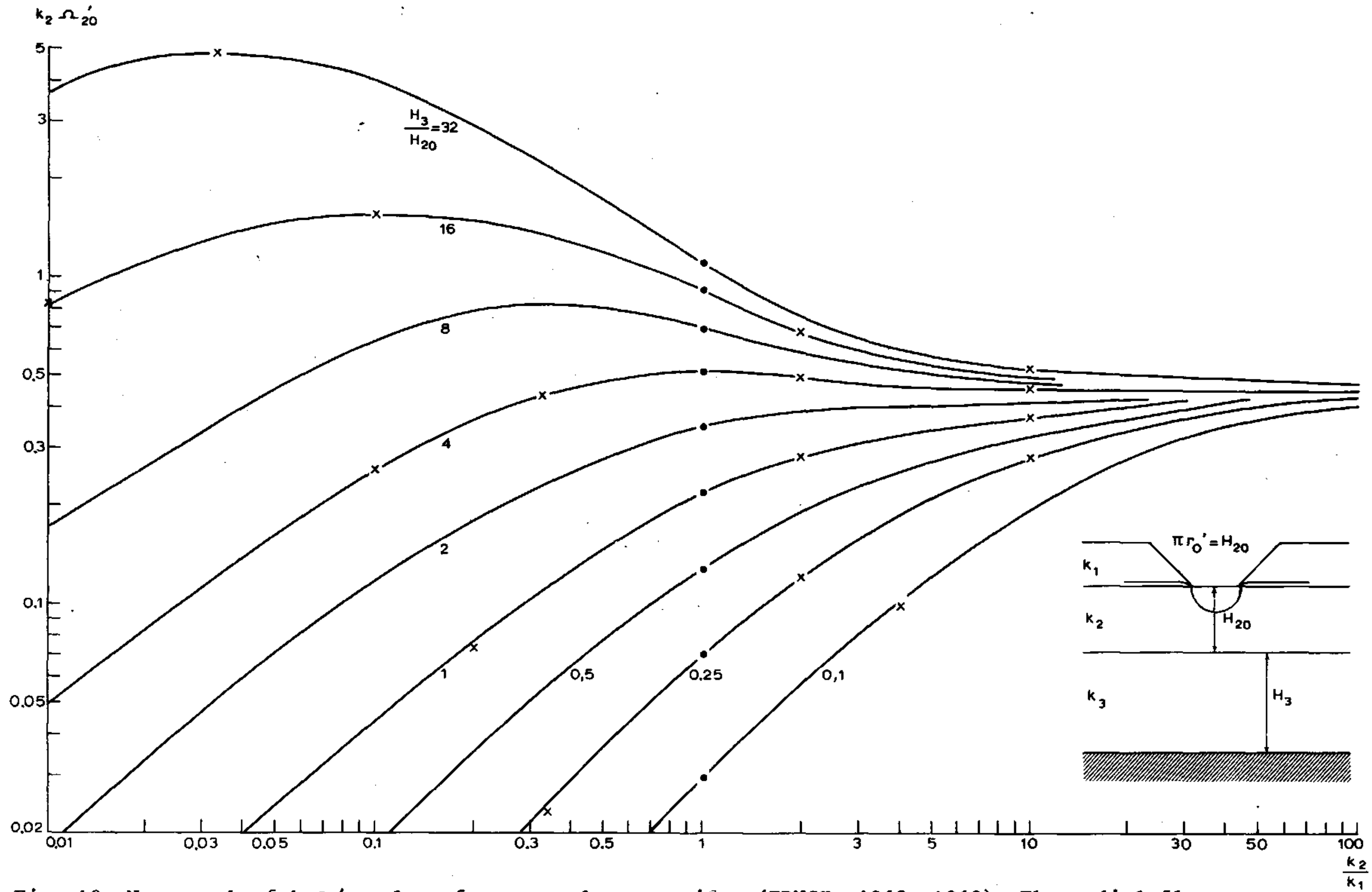

Fig. 10. Nomograph of $k_2\Omega'_{20}$-values for a two-layer aquifer (ERNST, 1962, 1963). The radial flow resistance Ω_{20} can be computed by means of eq. (49) or (50)

. . . . . values resulting from eq. (8)

x x x x x obtained by relaxation method

Which of the last two eqs. has to be applied depends on the general shape of the channel, respectively for relatively deep and relatively shallow channels as shown in fig. 3.

7. DISCUSSION

Use of equations with logarithms - like eq. (8) or (9) - for the determination of the radial flow resistance Ω_o may cause relatively large errors. This might be considered as a major imperfection in the presented system. In order to show to which extent this is an objection for practical use, eqs. (7) and (8) will be applied on the drainage situation in a homogeneous aquifer (fig. 11).

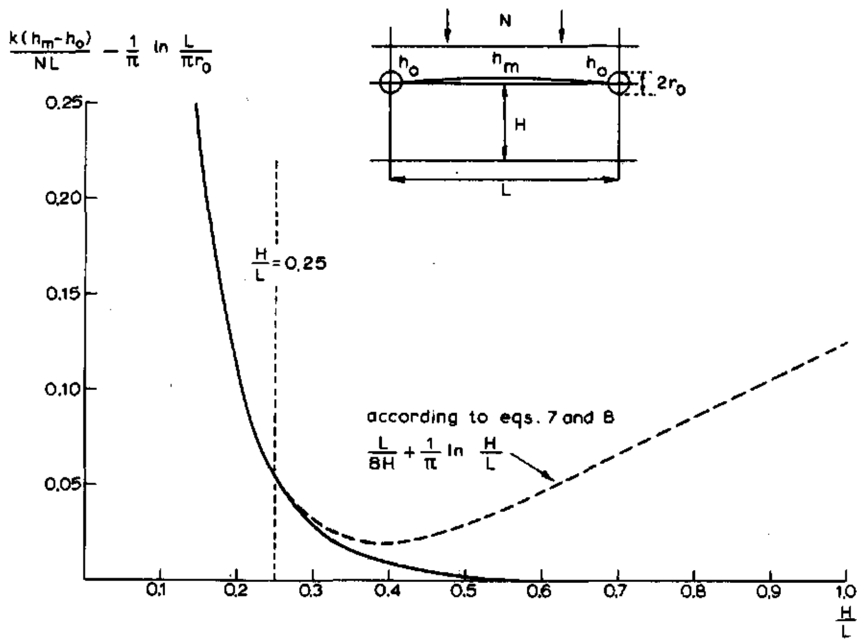

Fig. 11. Graphical representation of the hydraulic head difference $h_m - h_o$ for the symmetric drainage of a homogeneous aquifer of constant thickness H

By addition of the logarithmic term to the ordinates in fig. 11, the influence of the channel size ($\pi r_o = B_{wp}$) has been eliminated. The full drawn line is giving the exact relation, while the less accurate relation according to eqs. (7) and (8) is represented by the broken line. It is obvious that for increasing H/L above 0.25 the error is rapidly increasing, while for smaller H/L the error is completely unimportant.

For the same reason the new eqs. (14), (20), (33) and (40) should also be applied with caution when $H/L > 0.25$. This is even more valid for the heterogeneous aquifers dealt with in the preceding section when $(H_2+H_3)\ L^{-1} > 0.25$ and $k_3 >> k_2$.

Some authors (HOOGHOUDT, 1940; VAN BEERS, 1965) have stated that it makes hardly any difference if the aquifer at a depth below 0.25 L is permeable or impermeable. Neglecting the deeper part of the aquifer is fairly correct, in all those cases that the aquifer below 0.25 L will not have a very large conductivity.

In order to show the influence of the permeability of deeper layers, e.g. below a depth 0.25 L, fig. 12 has been constructed by means of available exact information (KIRKHAM, 1961; TOKSÖZ and KIRKHAM, 1971 b). This figure gives a comparison of required hydraulic head differences for two-layer aquifers, which are only different in k_2-values. The small H_1/r_o-values given in this figure, will be rather exceptional. When $H_1/r_o = 8$, it follows that $B_{wp}/L = 0.1$.

By fig. 12a it is shown, that the assumption of impermeability below a depth L/4 can be rather bad, especially for ~~large~~ small H_1/r_o-values. Even larger errors have to be expected when the second layer is neglected for values of H_1/L smaller than 0.25.

From fig. 12b it can be concluded, that in case of a complete ignorance about the deeper layers, the errors will stay between fair limits by assuming that the permeability k_1 holds also for the deeper layers below L/4.

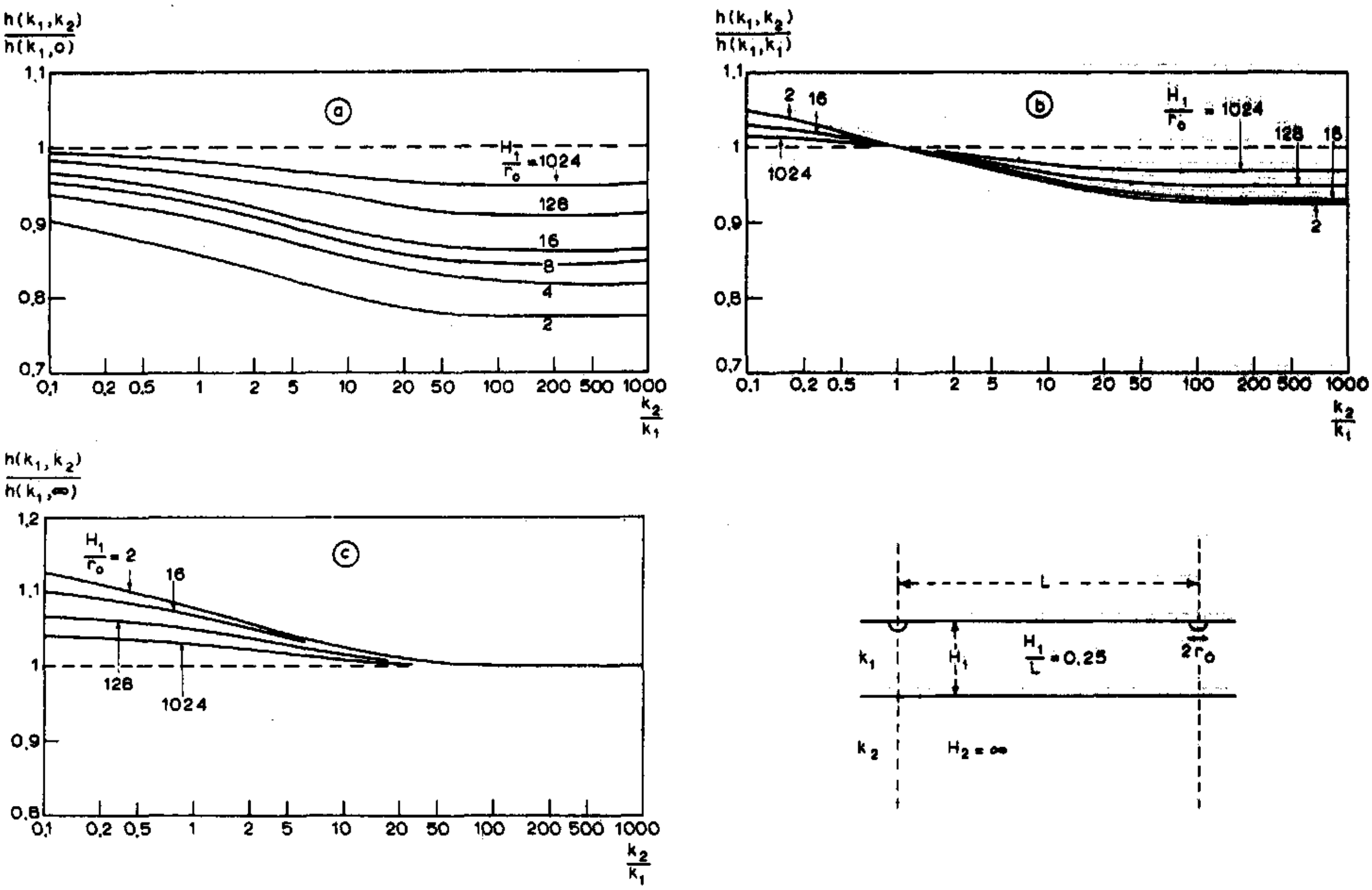

Fig. 12. The ratio between the hydraulic head differences for two symmetric drainage situations, which are only different in the k_2-value. a. Comparison with $k_2=0$. b. Comparison with $k_2=k_1$. c. Comparison with $k_2=\infty$

Fig. 12c shows that introduction of $k_2 = \infty$ can only be recommended in those cases that $k_2/k_1 > 3$.

A main result of fig. 12 is that it shows the relatively large influence of the H_1/r_o-values on the magnitude of the errors caused by introduction of wrong values for k_2. When relatively large drains are excluded, some ignorance about the deeper layers is much less harmful.

Finally it must be born in mind that the question about the errors caused by inaccurate values for the hydraulic conductivity of the deeper layers, has not to be confused with the applicability of the drainage formulae presented in this paper. The statement that these formulae should not be used for very thick aquifers ($H/L > 0.25$) has to be maintained. This restriction can hardly be weakened for relatively small drains, because even for $B_{wp}/L = 0.003$ with $H_1/L=1$, $H_2/L=1$ and $k_2/k_1=10$, it can be shown that a plain use of these formulae will result in a 25%-error.

SUMMARY

The drainage formula proposed by Hooghoudt can be combined with one proposed by the author. This results in a third degree polynomial equation for the drain spacing. The resulting formula can be applied to the steady state groundwater flow to parallel drains in homogeneous aquifers and in two-layer aquifers with the interface at the same level as the open water surface in the drainage channels. For some three-layer aquifers the same equation can be used in combination with a nomograph for the radial flow resistance in a two-layer aquifer. An attempt has been made to obtain a simpler formula by adding an empirical coefficient to the radial flow resistance. The result is a slightly less accurate second degree equation. A comparison of these formulae with the results of Kirkham and Toksöz did show only small differences. The formulae presented in this paper can therefore safely be used for practical applications, however with the condition that the drainspacing must be at least equal to four times the total thickness of the aquifer.

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