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THE FRACTION OF DRY DAYS AS A PARAMETER IN ANALYTIC RAINFALL INTERSTATION CORRELATION FUNCTIONS

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C O N T E N T S

1. INTRODUCTION

Rainfall interstation correlation functions can be derived analytically from hypothetical storm models. In several publications the theory has been developped and it was shown that for three storm functions as suggested in the literature the analytic solution can be obtained. In this report an extension is given in that the fraction of dry days that might occur is used as a parameter as well. Finally results are given in dimensionless from using area size L as unit of length.

2. REVIEW

Publications devoted to the analytic approach of interstation correlation functions, written in the last few years, are the following:

- Empirical treatment of daily rainfall data observed in the eastern part of the province of Gueldre in the Netherlands to obtain correlation functions for each month (STOL, 1972).

- A first note on the possibility that rainfall interstation correlation functions can be obtained analytically (STOL, 1973).

- Basic theory, description of the model, statistical properties and the transformation of storm function variates to gage time series variates (STOL, 1977a).

- The solution of the required integrals to obtain the interstation correlation function for three different storm models (STOL, 1977b).

- Comparison of various storm models with respect to their statistical parameters in order to obtain correlation functions for different storms with the same diameter and mean value (STOL, 1977c).

- An article on the concept of an analytic approach to correlation functions on the basis of storm models in which a complete solution for the triangular storm model is given (STOL, 1977d).

The results thus far obtained can easily be extended to be suitable for rainfall measuring practice in which completely dry days occur. This completes the first stage of the model in which homogeneous probability fields are assumed.

3. MATHEMATICAL DESCRIPTION OF THE PROBLEM

It can be proved (STOL, 1977d) that homogeneity with respect to statiscal properties in a model of a rain gaging area is obtained by adding a distance equal to half the size of the storm diameter to both ends of the gaged area.

Let P denote probability, $\underline{g}_i$ the stochastic time series of rainfall amounts measured in gage G_i, and let B denote the storm diameter and L the length (cross section) of the gaged area perpendicular to the direction in which storms move, then

$$P(\underline{g}_i \neq 0) = \frac{B}{L + B}, \quad P(\underline{g}_i = 0) = \frac{L}{L + B}$$

which easily can be verified in Fig. 1. In this figure it is illustrated that storm passing points $\underline{\omega}$ for storms are uniformly distributed on the storm passing area (L + B) symbolized by:

$$\underline{\omega}_t = \underline{U}\left[z = -\frac{1}{2}B,\ z = L + \frac{1}{2}B\right]$$

where storm models are supposed to be symmetric about the mean.

The above given expression tells that the terminal points of the storm are distributed on the area-axis according to:

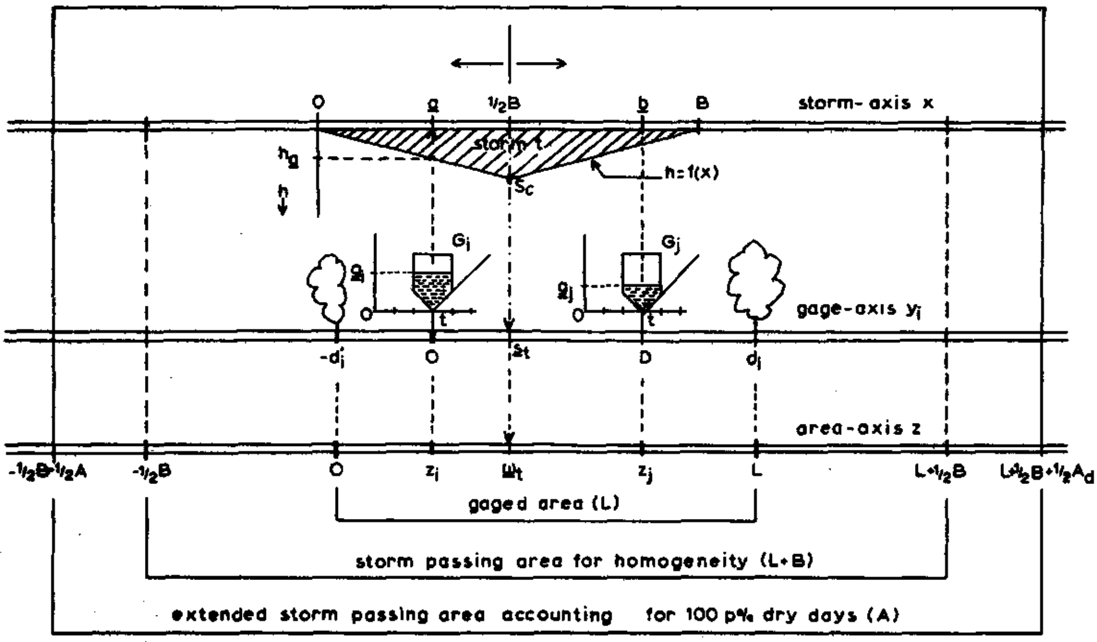


Fig. 1. Relationship between axes and areas used in the analytic approach to determine interstation-correlation functions

$$\underline{\omega}_t + \frac{1}{2} B = \underline{U}[z = 0,\ z = L + B]$$

$$-\frac{1}{2} B + \underline{\omega}_t = \underline{U}[z = -B,\ z = L]$$

Since

$$[z = 0,\ z = L + B] \cap [z = -B,\ z = L] = [z = 0,\ z = L]$$

We see that at least one point of the closed interval $[0, L]$ on the area-axis is hit by any storm. Or, on the gage-axis: at least one gage in the gaged area is hit by any storm.

If a storm function is given by:

$$h = f(x)$$

where h = rainfall amounts

x = location in the storm

and if we assume that f(0) = 0 and f(B) = 0 the realization of $\underline{h}$ in a gage at $z_i = 0$ (so at the left boundary) equals $\underline{h} = 0$, when $\underline{\omega}_t = -\frac{1}{2}B$ on the z-axis (Fig. 1).

Although, in this case, the gage at the left boundary is assumed to be hit by the very right most point of the storm, the rainfall amount measured in the gage equals zero. However, this event is not an impossible event but it occurs with probability zero. So

$$P(\underline{g}_i = 0 \text{ for all } G_i \text{ with } z_i \in [z = 0, z = L]) = 0$$

which will be abbreviated to

$$P(\underline{g}_i = 0 | \text{all } G_i) = 0 \qquad (1)$$

In our model we consider storms as a unit and each storm passing the storm area contributes to the statistical characteristics of the process of occurring and measuring rainfall. It is convenient to think in terms of days and so for our present purpose we will consider the situation that each day a storm passes the storm area. However, equation (1) expresses that at least one gage is hit by a storm each day, or that the probability that this is not the case equals zero.

Now we assume that this probability is greater than zero, which means that we introduce days at which no gages are hit by a storm: the (completely) dry days.

Suppose that in a certain area the fraction of dry days equals p, then we have

$$P(\underline{g}_i = 0 | \text{all } G_i) = p, \quad 0 \leq p < 1$$

Our model has to be extended so that it can cope with this complication.

4. THE EXTENDED AREA

Drawing storm passing points $\underline{\omega}_t$ on $\left[z = -\frac{1}{2}B,\ z = L + \frac{1}{2}B\right]$ from a uniform distribution means that the event

$$(\underline{g}_i = 0 \mid \text{all } G_i) \text{ viz.: a dry day}$$

has probability zero. The correct way of sampling (STOL, 1972 and SHARON, 1974) by taking into account only those 'days' (drawings) which are potentially rainy days is automatically simulated by this procedure. However, it is rather simple to generalize the procedure to a more often used way of sampling namely: to involve completely dry days in the sample as well.

Suppose it is found in a certain area that

$$P(\underline{g}_i = 0 \mid \text{all } G_i) = p$$

then we define the extended area by (see Fig. 1):

$$\left[z = -\frac{1}{2}B - \frac{1}{2}A_d,\ z = L + \frac{1}{2}B + \frac{1}{2}A_d\right]$$

where the total length of this interval namely $(L + B + A_d)$ will be denoted by A. If we take

$$\frac{A_d}{A} = \frac{A_d}{L + B + A_d} = p \qquad (2)$$

we have

$$A_d = \frac{p(L + B)}{1 - p}$$

with which the storm area has to be extended to account for a fraction of dry days equal to p. Since $A_d = A - (L + B)$ we have from (2):

$$A = \frac{L + B}{1 - p}$$

which defines the extended (storm passing) area A into

L = size of the gaged area

B = storm diameter

p = fraction of dry days

5. NOTATION

In the following sections we will employ the notation given below

$\underline{g}_i$ = gage time series for gage G_i

γ = expectation of $\underline{g}_i$ for all i

ϕ^2 = variance of $\underline{g}_i$ for all i

$h_{\underline{a}}$ = storm variate

μ = expectation of $h_{\underline{a}}$ for all a

σ^2 = variance of $h_{\underline{a}}$ for all a

η = expectation of random or exposure errors for all a

τ^2 = variace of random or exposure errors for all a

ρ = interstation correlation between gage time series in gages G_i and G_j with interstation distance D

ρ_{ab} = correlation between storm variates $h_{\underline{a}}$ and $h_{\underline{b}}$ at interstation distance D

ψ^2 = $\rho_{ab}\sigma^2$, covariance of $h_{\underline{a}}$ and $h_{\underline{b}}$ at interstation distance D

θ_{ab} = correlation between random or exposure errors at location a and b with interstation distance D

The correlations ρ, ρ_{ab} and θ_{ab} depend on the interstation distance D, so, in particular, $\rho = \rho(D)$ and we try to express ρ in storm characteristics and area parameters, viz.:

$$\rho = \rho(D;\ B,\ L,\ A,\ \mu,\ \sigma^2,\ \eta,\ \tau^2,\ \rho_{ab},\ \theta_{ab})$$

where

$$A = \frac{L + B}{1 - p}$$

and the fraction of dry days p is a parameter as well.

6. GENERAL SOLUTION

For the interstation correlation function STOL (1977a) derived general formulas that do not depend on specific storm functions. The solution given in the Appendix there, makes use of the area parameter

$$A = L + B$$

Using the new definition for A, viz.:

$$\boxed{A = \frac{L + B}{1 - p}} \quad (3)$$

which accounts for the fraction of dry days p, the general solutions still hold.

Since

$$A - B = \frac{L + pB}{1 - p} \quad (4)$$

we have for the statistical characteristics of gage time series the following expressions.

6.1. The expectation of $\underline{g}_i$

From previous results (Report No. 992, Appendix 1 and Section 12) we have immediately

$$\gamma = \frac{B}{A}(\mu + \eta)$$

or

$$\boxed{\gamma = \frac{(1 - p)\ B}{L + B}(\mu + \eta)}$$

6.2. The variance of $\underline{g}_i$

From the same Appendix we have

$$\phi^2 = \frac{B}{A}\left\{\sigma^2 + \tau^2 + \frac{A - B}{A}(\mu + \eta)^2\right\}$$

or

$$\phi^2 = \frac{(1-p)\ B}{L + B}\{\sigma^2 + \tau^2 + \frac{L + pB}{L + B}(\mu + \eta)^2\} \qquad (5)$$

6.3. The covariance between $\underline{g}_i$ and $\underline{g}_j$

The covariance finally is given by

$$\mathrm{Cov}(\underline{g}_i\underline{g}_j) = \frac{B}{A}\{\rho_{ab}\ \sigma^2 + \theta_{ab}\ \tau^2 + \frac{A - B}{A}(\mu + \eta)^2\}$$

which in the same way as the variance can be expressed in the fraction of dry days p, using equations (3) and (4).

In the above given formula it is not necessary to calculate ρ_{ab} explicitely since $\rho_{ab}\ \sigma^2 = \mathrm{Cov}(\underline{h}_a\underline{h}_b)$ which has to be determined first. Since nothing is known about the correlation between random exposure errors, and since nothing is assumed about it, we still write θ_{ab}. The notational convention of Section 5 now allows us to write

$$\mathrm{Cov}(\underline{g}_i\underline{g}_j) = \frac{(1-p)\ B}{L + B}\{\Psi^2 + \theta_{ab}\ \tau^2 + \frac{L + pB}{L + B}(\mu + \eta)^2\} \qquad (6)$$

6.4. The interstation correlation ρ

Since the interstation correlation is given by the ratio between (5) and (6) so by

$$\rho = \frac{\mathrm{Cov}(\underline{g}_i\underline{g}_j)}{\phi^2}$$

we have, multiplying numerator and denominator by $\frac{(L + B)^2}{(1 - p)\ B}$ for the correlation coefficient

$$\rho(D) = \frac{(L + B)\ (\Psi^2 + \theta_{ab}\ \tau^2) + (L + pB)\ (\mu + \eta)^2}{(L + B)\ (\sigma^2 + \tau^2) + (L + pB)\ (\mu + \eta)^2} \qquad (7)$$

which is the general solution for the interstation correlation expressed in the required parameters. We note that Ψ^2 and θ_{ab} are functions of the interstation distance D.

An alternative form for this solution reads

$$\rho(D) = 1 - (L + B) \frac{\sigma^2 - \Psi^2 + (1 - \theta_{ab}) \tau^2}{(L + B)(\sigma^2 + \tau^2) + (L + pB)(\mu + \eta)^2} \tag{8}$$

where correlations are expressed relative to unity.

7. THE PRACTICAL SOLUTION

The last formula can be used to determine interstation correlation relationships after the expectation μ, the variance σ^2 and the covariance Ψ^2 have been determined for a given storm model.

Calculations can be simplified by taking into account that

$$\sigma^2 = E(\underline{h}_a \underline{h}_a) - \mu^2$$

$$\Psi^2 = E(\underline{h}_a \underline{h}_b) - \mu^2$$

and so

$$\boxed{\sigma^2 - \Psi^2 = E(\underline{h}_a \underline{h}_a) - E(\underline{h}_a \underline{h}_b)}$$

where subtraction of μ^2 from the expectations and the cancelling of it by taking the difference is avoided.

To simplify the notation for intermediate results we define for the nominator and denominator, respectively

Case I: $(0 \leq D < \frac{1}{2} B < B)$, $\rho_I = 1 - (L + B)\frac{N_I}{D_I}$

Case II: $(0 < \frac{1}{2} B \leq D \leq B)$, $\rho_{II} = 1 - (L + B)\frac{N_{II}}{D_{II}}$

Case III: $(0 < \frac{1}{2} B < B < D)$, $\rho_{III} = 1 - (L + B)\frac{N_{III}}{D_{III}}$

The elaborations require to distinguish between D_I, D_{II} and D_{III} although they have the same expression since they are not a function of the interstation distance and so $D_I = D_{II} = D_{III}$.

8. THE RECTANGULAR STORM TYPE

Since the rectangular storm has a constant rainfall amount H, integrals are replaced by sums and so we use the general solution with (see Report No. 993, section 5A):

$$\mu = H$$

$$\sigma^2 = 0$$

Case I: $\Psi_I^2 = \frac{-DH^2}{B}$

Case II: $\Psi_{II}^2 = \frac{-DH^2}{B}$

Case III: $\Psi_{III}^2 = -H^2$

and so

$$\rho_{I,II} = \frac{(L+B)\left(\frac{-DH^2}{B} + \theta_{ab}\tau^2\right) + (L+pB)(H+\eta)^2}{(L+B)\tau^2 + (L+pB)(H+\eta)^2}$$

$$\rho_{III} = \frac{(L+B)(-H^2 + \theta_{ab}\tau^2) + (L+pB)(H+\eta)^2}{(L+B)\tau^2 + (L+pB)(H+\eta)^2}$$

which can be written

$$\rho_{I,II} = \frac{(L+B)(\theta_{ab} B\tau^2 - DH^2) + B(L+pB)(H+\eta)^2}{B(L+B)\tau^2 + B(L+pB)(H+\eta)^2}$$

$$\rho_{III} = \frac{(L+B)(\theta_{ab}\tau^2 - H^2) + (L+pB)(H+\eta)^2}{(L+B)\tau^2 + (L+pB)(H+\eta)^2}$$

We can divide denominator and numerator by L^2 and put

$$B' = \frac{B}{L} \quad \text{and} \quad D' = \frac{D}{L}$$

and drop the primes. Then the correlation is given in dimensionless form, the new B and D being expressed in units of L. So

$$\rho_{I,II} = \frac{(1 + B)\ (\theta_{ab}\ B\tau^2 - DH^2) + B(1 + pB)\ H + \eta)^2}{B(1 + B)\ \tau^2 + B(1 + pB)\ (H + \eta)^2}$$

$$\rho_{III} = \frac{(1 + B)\ (\theta_{ab}\ \tau^2 - H^2) + (1 + pB)\ (H + \eta)^2}{(1 + B)\ \tau^2 + (1 + pB)\ (H + \eta)^2}$$

It must be noted that all magnitudes related to rainfall amounts have to be expressed in the same units, but the unit can be chosen arbitrarily, viz. put

$$H' = \frac{H}{\alpha}\ ,\ \eta' = \frac{\eta}{\alpha}\ ,\ \tau' = \frac{\tau}{\alpha}$$

and drop the primes. The result then gives the same formulas as those given above.

Using the alternative form the correlation functions are

$$\rho_{I,II} = 1 - \frac{1 + B}{B} \cdot \frac{B(1 - \theta_{ab})\ \tau^2 + DH^2}{(1 + B)\ \tau^2 + (1 + pB)\ (H + \eta)^2}$$

$$\rho_{III} = 1 - (1 + B) \frac{(1 - \theta_{ab})\ \tau^2 + H^2}{(1 + B)\ \tau^2 + (1 + pB)\ (H + \eta)^2}$$

This solution reduces to the one given in Report No. 993, Annex 1a, by putting

$\eta = 0$ (mean exposure error zero)
$\tau^2 = 0$ (variance exposure errors zero)
$p = 0$ (no dry days)

then, dividing through by H^2,

$$\rho_{I,II} = 1 - \frac{1 + B}{B} D$$

$$\rho_{III} = - B$$

the most simple solution for rainfall interstation correlations.

9. THE TRIANGULAR STORM TYPE

The statistical parameters necessary to obtain the correlation function for this storm type are obtained from Report No. 993, Section 5B, viz.:

$$\mu = \frac{H}{2}$$

$$\sigma^2 = \frac{H^2}{12} = \frac{H^2}{3} - (\frac{H}{2})^2$$

Case I: $$\Psi_I^2 = \frac{H^2}{3B^3}(B^3 - 6BD^2 + 6D^3) - (\frac{H}{2})^2$$

Case II: $$\Psi_{II}^2 = \frac{2H^2}{3B^3}(B - D)^3 - (\frac{H}{2})^2$$

Case III: $$\Psi_{III}^2 = - (\frac{H}{2})^2$$

From which we have

$$\sigma^2 - \Psi_I^2 = \frac{2H^2D^2}{B^2} - \frac{2H^2D^3}{B^3} \tag{9}$$

$$\sigma^2 - \Psi_{II}^2 = \frac{H^2}{3} - \frac{2H^2}{3B^3}(B - D)^3 \tag{10}$$

$$\sigma^2 - \Psi_{III}^2 = \frac{H^2}{3} \tag{11}$$

This gives, starting with (9), for the first numerator N_I multiplied by B^3,

$$B^3N_I = H^2D^2(B - D) + B^3(1 - \theta_{ab})\ \tau^2$$

and for the denominator

$$D_I = (L + B)\ (\frac{H^2}{12} + \tau^2) + (L + pB)\ (\frac{H}{2} + \eta)^2$$

or,

$$12D_I = (L + B)\ (H^2 + 12\tau^2) + 3(L + pB)\ (H + 2\eta)^2$$

and finally, putting with auxiliary variables α and β,

$$B^3N_I = \alpha_I \quad \text{and} \quad 12D_I = \beta$$

we have now

$$\frac{N_I}{D_I} = \frac{\alpha_I}{B^3} \cdot \frac{12}{\beta} = \frac{12}{B^3}\frac{\alpha_I}{\beta}$$

so

$$(L + B)\frac{N_I}{D_I} = \frac{12(L + B)}{B^3}\ \frac{\alpha_I}{\beta}$$

giving

$$\boxed{\rho_I = 1 - \frac{12(L + B)}{B^3} \cdot \frac{2H^2(B - D)\ D^2 + B^3(1 - \theta_{ab})\ \tau^2}{(L + B)(H^2 + 12\tau^2) + 3(L + pB)\ (H + 2\eta)^2}}$$

which becomes in dimensionless form

$$\boxed{\rho_I = 1 - \frac{12(1 + B)}{B^3} \cdot \frac{2H^2(B - D)\ D^2 + B^3(1 - \theta_{ab})\ \tau^2}{(1 + B)\ (H^2 + 12\tau^2) + 3(1 + pB)\ (H + 2\eta)^2}}$$

The second case can be solved as follows, starting with (10),

$$3B^3 N_{II} = B^3 H^2 - 2H^2 (B - D)^3 + 3B^3 (1 - \theta_{ab}) \tau^2$$

Then, see D_I,

$$12 D_{II} = (L + B)\ (H^2 + 12\tau^2) + 3(L + pB)\ (H + 2\eta)^2$$

and finally, putting

$$3B^3 N_{II} = \alpha_{II} \quad \text{and} \quad 12 D_{II} = \beta$$

we have for

$$(L + B) \frac{N_{II}}{D_{II}} = \frac{4(L + B)}{B^3} \frac{\alpha_{II}}{\beta}$$

giving

$$\boxed{\rho_{II} = 1 - \frac{4(L + B)}{B^3} \cdot \frac{B^3 H^2 - 2H^2 (B - D)^3 + 3B^3 (1 - \theta_{ab}) \tau^2}{(L + B)\ (H^2 + 12\tau^2) + 3(L + pB)\ (H + 2\eta)^2}}$$

The third case can be solved as follows, starting with (11),

$$3N_{III} = H^2 + 3(1 - \theta_{ab}) \tau^2$$

and

$$12 D_{III} = (L + B)\ (H^2 + 12\ \tau^2) + 3(L + pB)\ (H + 2\eta)^2$$

resulting into

$$\boxed{\rho_{III} = 1 - 4(L + B) \frac{H^2 + 3(1 - \theta_{ab}) \tau^2}{(L + B)\ (H^2 + 12\tau^2) + 3(L + pB)\ (H + 2\eta)^2}}$$

which becomes in dimensionless form

$$\rho_{III} = 1 - 4(1 + B) \frac{H^2 + 3(1 - \theta_{ab})\,\tau^2}{(1 + B)\,(H^2 + 12\tau^2) + 3(1 + pB)\,(H + 2\eta)^2}$$

Also for the solution of the rectangular storm type the result can be simplified by putting

$\eta^2 = \tau^2 = p = 0$, dividing through by H^2,

Then we have for case I, II and III the correlation functions

$$\rho_I = 1 - \frac{24(1 + B)}{B^3} \cdot \frac{(B - D)\,D^2}{4 + B}$$

$$\rho_{II} = 1 - \frac{4(1 + B)}{B^3} \cdot \frac{B^3 - 2(B - D)^3}{4 + B}$$

$$\rho_{III} = 1 - \frac{4(1 + B)}{4 + B}$$

similar to the solution given in Report 993, Annex 1b

10. THE EXPONENTIAL STORM TYPE

The statistical parameters for the exponential storm type are given in Report No. 993, Section 5C, viz.:

$$\mu = \frac{H}{bB}(1 - e^{-bB})$$

or, putting

$$u = 1 - e^{-bB}$$

$$v = 1 + e^{-bB}$$

this becomes

$$\mu = \frac{H}{bB}u$$

$$\sigma^2 = \frac{H^2}{2bB} uv - (\frac{H}{bB} u)^2$$

Now, introducing

$$w = e^{-2bD}$$

we can write for the covariances:

Case I: $\Psi_I^2 = \frac{H^2}{2bwB}(w^2 - e^{-2bB} + 2bDw^2) - \mu^2$

Case II: $\Psi_{II}^2 = \frac{H^2 w}{B}(B - D) - \mu^2$

Case III: $\Psi_{III}^2 = - \mu^2$

The expression $\sigma^2 - \Psi^2$ thus becomes, using the equality $uv = 1 - \exp(- 2bB)$,

$$\sigma^2 - \Psi_I^2 = \frac{H^2}{2bwB} (uvw - w^2 + 1 - uv - 2bDw^2)$$

$$= \frac{H^2}{2bwB} \{(uv - w - 1)\ (w - 1) - 2bDw^2\}$$

$$\sigma^2 - \Psi_{II}^2 = \frac{H^2}{2bB}\{uv - 2bw(B - D)\}$$

$$\sigma^2 - \Psi_{III}^2 = \frac{H^2}{2bB} uv$$

Proceeding in the same way as for the other storms, we now have

$$2bwBN_I = H^2\{(uv - w - 1)\ (w - 1) - 2bDw^2\} + 2bwB(1 - \theta_{ab})\ \tau^2$$

$$D_I = (L + B)\ (\frac{H^2}{2bB} uv - \frac{H^2}{b^2B^2} u^2 + \tau^2) + (L + pB)\ (\frac{H}{bB} u + \eta)^2$$

so

$$2b^2B^2D_I = (L + B)\{H^2u(bBv - 2u) - 2b^2B^2\tau^2\} + (L + pB)\ 2(Hu + bB\eta)^2$$

Now we put in the same way as before

$$2bwBN_I = \alpha_I \quad \text{and} \quad 2b^2B^2D_I = \beta$$

so

$$\frac{N_I}{D_I} = \frac{\alpha_I}{2bwB} \cdot \frac{2b^2B^2}{\beta} = \frac{bB}{w} \cdot \frac{\alpha_I}{\beta}$$

which finally results in the equation for the interstation correlation function,

$$\boxed{\rho_I = 1 - (L + B)\frac{bB}{w} \cdot \frac{H^2\{...\} + 2bBw(1 - \theta_{ab})\,\tau^2}{(L + B)\,\{---\} + 2(L + pB)\,(Hu + bB\eta)^2}}$$

where

$$\{...\} = (uv - w - 1)\,(w - 1) - 2bDw^2$$

$$\{---\} = H^2u(bBv - 2u) + 2b^2B^2\tau^2$$

The second case gives the following elaborations

$$2bBN_{II} = H^2\{uv - 2bw(B - D)\} + 2bB(1 - \theta_{ab})\,\tau^2$$

and

$$\frac{N_{II}}{D_{II}} = \frac{N_{II}}{D_I} = \frac{\alpha_{II}}{2bB} \cdot \frac{2b^2B^2}{\beta} = bB\,\frac{\alpha_{II}}{\beta}$$

so, for the interstation correlation function

$$\boxed{\rho_{II} = 1 - (L + B)\,bB\,\frac{H^2\{uv - 2bw(B - D)\} + 2bB(1 - \theta_{ab})\,\tau^2}{(L + B)\,\{---\} + 2(L + pB)\,(Hu + bB\eta)^2}}$$

Finally we proceed with

$$2bBN_{III} = H^2uv + 2bB(1 - \theta_{ab})\ \tau^2$$

giving, with

$$\frac{N_{III}}{D_{III}} = \frac{N_{III}}{D_I} = bB\frac{\alpha_{III}}{\beta}$$

$$\boxed{\rho_{III} = 1 - (L + B)\ bB\ \frac{H^2uv + 2bB(1 - \theta_{ab})\ \tau^2}{(L + B)\ \{---\} + 2(L + pB)\ (Hu + bB\eta)^2}}$$

The simplification for $\eta = \tau^2 = p = 0$ needs more algebra than the other solutions. We start with the denominator including the factor $(L + B)\ bB$. Then we have

$$\frac{(L + B)\ bB}{(L + B)\ H^2u(bBv - 2u) + 2LH^2u^2}$$

$$= \frac{(L + B)\ bB}{H^2u\{bBvL - 2uL + bB^2v - 2Bu - 2Lu\}}$$

$$= \frac{(L + B)\ b}{H^2u\{(L + B)\ bv - 2u\}}$$

in accordance with the result obtained in Report No 993, Annex 1C.

Next we have for Case I

$$\frac{H^2}{w}\{(uv - w - 1)\ (w - 1) - 2bDw^2\}$$

$$= \frac{H^2}{w}\{uvw - uv - w^2 + 1 - 2bDw^2\}$$

$$= \frac{H^2}{w}\{uvw - 1 + e^{-2bB} - w^2 + 1 - 2bDw^2\}$$

$$= H^2\{uv - w(1 - \frac{1}{w^2}\, e^{-2bB}) - 2bDw\}$$

Which is in agreement with the results in Report No. 993, Annex 1C.

For Case II and Case III the comparison with the results in Report No. 993 is obvious.

11. DIMENSIONLESS FORM FOR THE EXPONENTIAL TYPE

It must be noted that after some precautionary measures the correlation coefficient for the exponential storm type also can be written in dimensionless form.

Putting $bL = b'$ we may write

$$bB = bL \cdot \frac{B}{L} = b'B'$$

and

$$bD = bL \cdot \frac{D}{L} = b'D'$$

giving

$$u = 1 - e^{-bB} = 1 - e^{-b'B'}$$

$$v = 1 + e^{-bB} = 1 + e^{-b'B'}$$

$$w = e^{-2bD} = e^{-2b'D'}$$

After having expressed L, B and D in units of L, and after having multiplied the storm constant b by L, the dimensionless form is obtained. Dropping the primes, the structure of the correlation function remains the same, however, in the original formula the symbol L should be replaced by the numer 1 to obtain dimensionless expressions.

12. FINAL REMARKS AND SUMMARY

In this report the correlation function for rainfall amounts measured in gages at various distances was extended with p, a parameter for dry days. This parameter finally appeared in the numerator

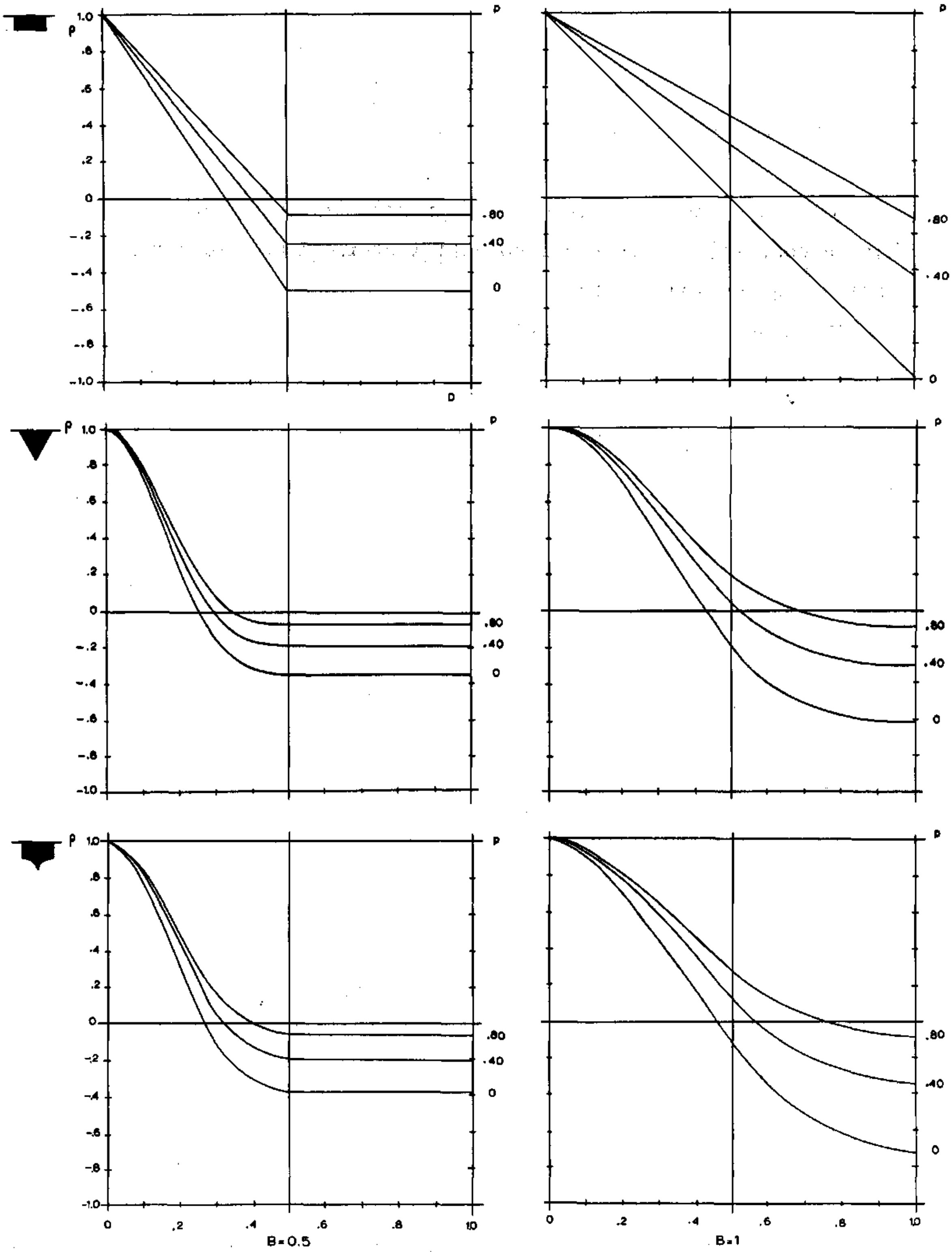
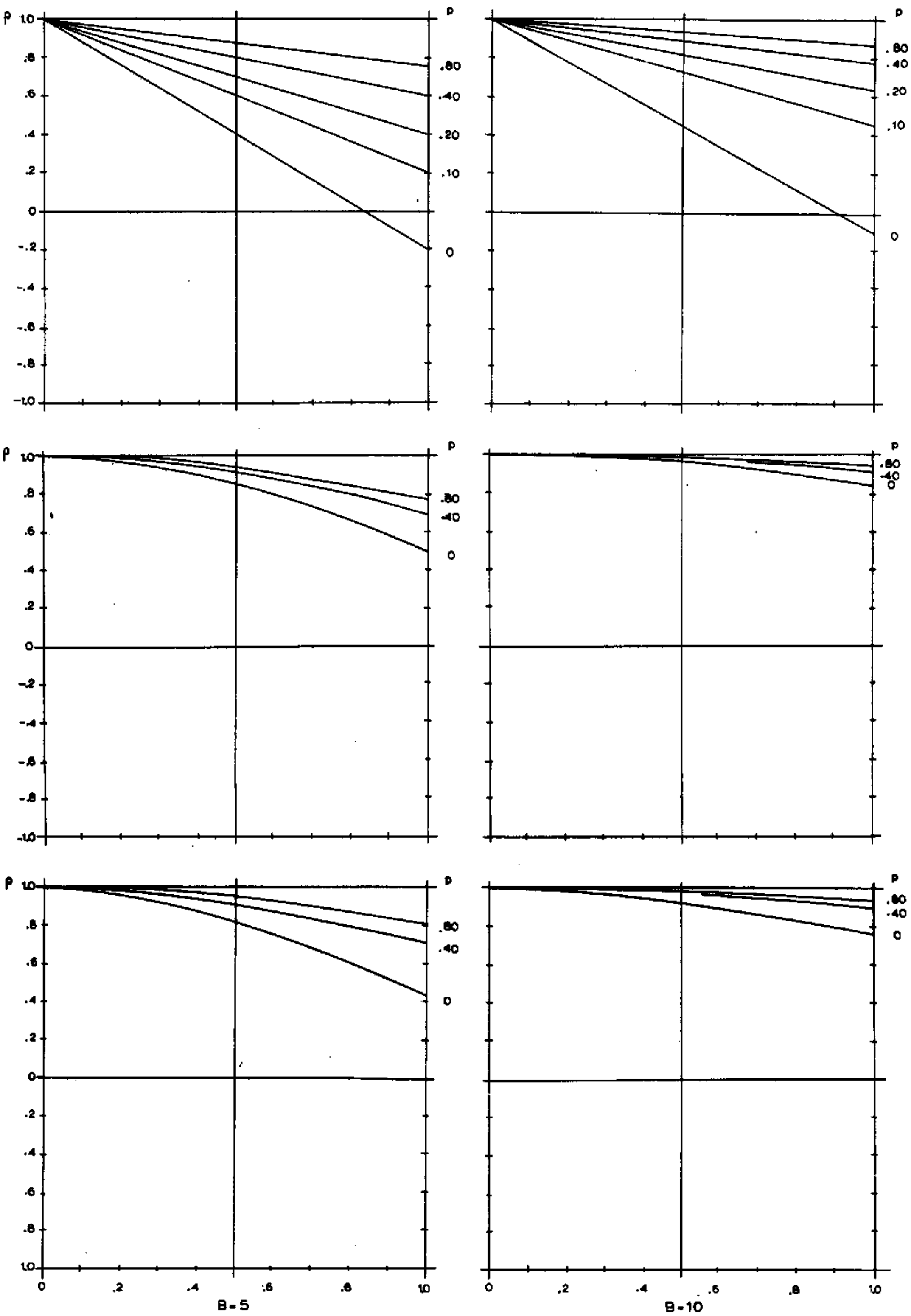


Fig. 2. Comparison of the influence of increasing values of the
tion function of three storm types (rectangular, triangu
various diameters B (B expressed in units of length of



parameter p (fraction of dry days) on the shape of the correla-
lar and exponential respectively) with the same mean value and
the gaged area)

of the correlation function. As a result, the correlation increases if p increases, for given values of the other parameters.

The effect of increasing values of p on the shape of the correlation function for three storm types is illustrated in Fig. 2. Specifications are:

$\eta = \tau = \theta = 0$

$B = 0.5 \quad 1 \quad 5 \quad 10$

$p = 0 \quad (0.10 \quad 0.20) \quad 0.40 \quad 0.80$

$\mu = 0.5$ for all storms considered

The upper row in Fig. 2 illustrates the rectangular storm type, the following rows the triangular- and exponential storm type respectively.

Form the figures it becomes clear that in practice it will be difficult to distinguish between the three storm types, especially in case of a wide scattering of estimated correlations.

We also see that it will be difficult, if not impossible, to distinguish between the effect of the storm diameter B and the fraction of dry days p. For instance for large values of B (e.g. $B \geq 5$) the correlation function for the triangular and the exponential storm type look alike.

For the exponential storm type with parameter values B = 5, p = 0.60 the correlation function is approximately equal to the one with B = 10 and p = 0. The interpretation of the figures has to be done carefully and needs further meteorological information than the one given by the estimated correlations only.

The generalization of the formulas of the interstation correlation with a parameter for the fraction of dry days completes the first stage of the model in which homogeneous probability fields are assumed.

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