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APPLICATION OF THE m^{th} EXTREME VALUE DISTRIBUTION IN LAND USE PLANNING

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SUMMARY

In this paper the statistical theory of m^{th} extreme values is elaborated by the calculation of the theoretical values of the expected mean and standard deviation of the m^{th} extremes as functions of the number of observations n, in case, n = 1(1) 20,50 and 100 respectively (table 2). This calculation allows to estimate the probability function of the m^{th} extreme value of a stochastic variate, based on a limited number of observations.

Calculation of the probability functions of the use intensity on the first to tenth most crowded day per annum is discussed in chapter 3 for six different types of recreation and traffic facilities, based on observations of the daily use intensity of these facilities during a couple of years. With the aid of these functions (fig. 5) a justified choice for the normative day of the designing capacity of similar projects can be made.

1. INTRODUCTION

In the fields of hydraulic engineering and hydrology application of the theory of extreme values is common practice. As a rule, assessment of the normative height of dykes for instance, is based on a frequency analysis of the heights of floods, whereas determination of the capacity of sewerage systems in urban areas is based on a frequency analysis of extreme values of precipitation. In all cases an exceedance of the capacity of the project will result in disaster (loss of life or substantial economic losses). Consequently hydraulic engineers/hydrologists will be interested primarily in the return period of the first extreme value of the observed phenomena (flood, precipitation).

Unlike this, in the field of land use planning, exceedance of the carrying capacity of a man made facility like a highway or a recreation site rarely leads to disaster. Exceedance of the capacity of a highway for instance will result in traffic congestion and consequently in loss of time for road-users; exceedance of the capacity of a recreation site means less enjoyment for recreational visitors. According to economic principles the designing capacity of this kind of land use projects will be based on predicting the use intensity.

We suggest to equate the designing capacity of a facility with the expected number of visitors on the m^{th} crowded day per annum with a probability of P percent. M and P should be chosen by policymakers depending on the type of facility and based on

- the extent of exceedance as obtained by frequency analysis of use intensity
- the physical and social impact of exceedance of the capacity and

- the marginal costs of construction and maintenance of facilities designed.

In order to know the probability of m exceedances of a certain use intensity per period, the probability function of the m^{th} extreme value of the number of users per day is needed. Data about the use intensity of this kind of objects are almost exclusively available for only a couple of years. Calibration of the probability function of the m^{th} extreme value of the number of users, requires the calculated values of $\bar{Y}_{n,m}$ and $\sigma_{n,m}$ (n = registration period in years).

The procedure with respect to the calculation of these model-parameters is discussed in chapter 2, the procedure as proposed for the assessment of a criterion for the designing capacity of a man made facility in land use planning is illustrated in chapter 3.

2. DISTRIBUTION OF m^{th} EXTREME VALUE

In this paper the distribution of the m^{th} extreme among n observations taken from an initial distribution of the exponential type will be considered. We speak of m^{th} values in case m is counted from above in the series given in increasing order x_1, x_2,.... x_{n-1}, x_n, and of m^{th} extremes if m is small compared to n. GUMBEL (1960) has given the asymptotical distribution of the m^{th} extreme, viz.

$$\phi_m(x_m) = \frac{m^m \cdot \alpha_m}{(m-1)!} \exp(- m\, y_m - m\, e^{-y_m}) \qquad (1)$$

$$y_m = \alpha_m(x_m - u_m) \qquad (2)$$

where:

x_m = the m^{th} extreme among the observations of x

$\phi_m(x_m)$ = distribution of the m^{th} extreme

y_m = reduced m^{th} extreme variate

α_m = scale parameter; reciprocal of the measure of dispersion of the distribution

u_m = mode of the distribution

The asymptotic probability $\Phi_m(x_m)$ of the m^{th} extreme value of the observations is given by:

$$\phi_m(x_m) = e^{-me^{-y_m}} \sum_{v=o}^{m-1} \frac{m^v \cdot e^{-vy_m}}{v!} \qquad (3)$$

Which becomes for m = 1 (1) 5:

$$m = 1 \qquad \Phi_1(x_1) = e^{-e^{-y_1}} \qquad (3^1)$$

$$m = 2 \qquad \Phi_2(x_2) = e^{-2e^{-y_2}}(1 + 2e^{-y_2}) \qquad (3^2)$$

$$m = 3 \qquad \Phi_3(x_3) = e^{-3e^{-y_3}}(1 + 3e^{-y_3} + \frac{3^2}{2!} e^{-2y_3}) \qquad (3^3)$$

$$m = 4 \qquad \Phi_4(x_4) = e^{-4e^{-y_4}}(1 + 4e^{-y_4} + \frac{4^2}{2!} e^{-2y_4} + \frac{4^3}{3!} e^{-3y_4} \qquad (3^4)$$

$$m = 5 \qquad \Phi_5(x_5) = e^{-5e^{-y_5}}(1 + 5e^{-y_5} + \frac{5^2}{2!} e^{-2y_5} + \frac{5^3}{3!} e^{-3y_5} + \frac{5^4}{4!} e^{-4y_5}) \qquad (3^5)$$

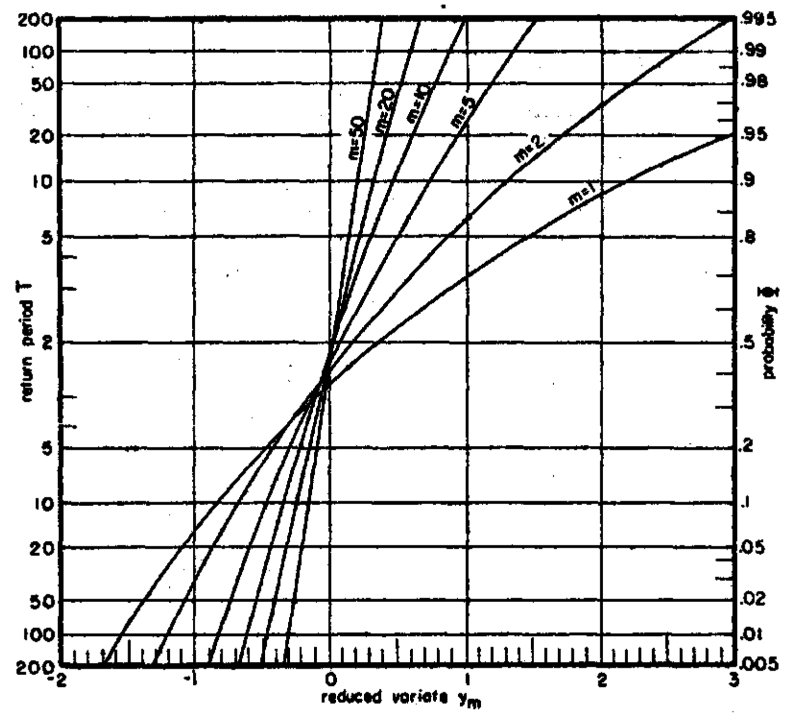

Fig. 1. Asymptotic probabilities of m^{th} extremes traced on normal paper (GUMBEL, 1960)

In Fig. 1 the asymptotic distribution of the reduced first to fifth extreme value (y_m) are plotted on normal probability paper, showing that the distributions of the reduced extremes contract with increasing values of m and converge to normality.

In this graph the probability (Φ) as well as the return period (T) is given. The return period T is defined by

$$T(x_m) = \frac{1}{1 - \Phi_m(x_m)} \quad (4)$$

The fitting of the observations to the distribution Φ_m requires an independent estimate of the distribution parameters α_m and u_m. Following GUMBEL (1954):

$$1/\alpha_m = s_m/\sigma_{n,m} \quad (5)$$

$$u_m = \bar{x}_m - \bar{y}_{n,m}/\alpha_m \quad (6)$$

where:

$\bar{y}_{n,m}$ = expected mean deviation of the population of m^{th} extremes

$\sigma_{n,m}$ = expected standard deviation of the population of m^{th} extremes

$\bar{x}_m$ = mean of the observations of m^{th} extremes

s_m = standard deviation of the observations of m^{th} extremes

The theoretical values $\bar{y}_{n,m}$ and $\sigma_{n,m}$ are the mean and the standard deviation of n values of the reduced variate taken at equidistant probability intervals, according to the formula for the plotting positions:

$$\Phi(y_i) = p(y \leq y_i) = \frac{i}{n+1}, \quad i = 1(1)\ n \tag{7}$$

For the extremes value (m = 1) tables of $\bar{y}_n$ and σ_n are calculated by GUMBEL (1960) and in more detail by STOL (1978) from the definitions

$$\bar{y}_n = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \overline{y_n^2} = \frac{1}{n}\sum_{i=1}^{n} y_i^2 \tag{8}$$

$$\sigma_n = \sqrt{\overline{y_n^2} - (\bar{y}_n)^2} \tag{9}$$

Values for y_i were obtained by equalling the 'plotting positions' according to (7) to the probabilities according to (3^1) written $\Phi_1(y_i)$

$$y_i = -\lg(-\lg \frac{i}{n+1}) \tag{10}$$

which is the inverse of the probability function.

For n large ($n \to \infty$) the theoretical values tend towards

$$\lim_{n \to \infty} \bar{y}_n = \gamma = 0.57722 \qquad \text{(Eulers constant)} \tag{11}$$

$$\lim_{n \to \infty} \sigma_n = \frac{\pi}{\sqrt{6}} = 1.28255 \tag{12}$$

The theoretical values of the mean and standard deviation of the m^{th} extreme population, in the case n tends towards infinity are

$$\bar{y}_{\infty,m} = \lg m - \sum_{v=1}^{m-1} 1/v + \gamma \tag{13}$$

$$\sigma_{\infty,m} = \sqrt{\sum_{k=m}^{\infty} 1/k^2} \tag{14}$$

For m = 1 (1) 10 the mean and standard deviation calculated from these equations are given in table 1.

Table 1. Expected mean $\bar{y}_{\infty,m}$ and expected standard deviation $\sigma_{\infty,m}$ of the distribution of reduced m^{th} extremes (m = 1 (1) 10)

m	$\bar{y}_{\infty,m}$	$\sigma_{\infty,m}$	m	$\bar{y}_{\infty,m}$	$\sigma_{\infty,m}$
1	0.57722	1.28255	6	0.08564	0.42582
2	0.27036	0.80308	7	0.07313	0.39185
3	0.17583	0.62844	8	0.06380	0.36488
4	0.13018	0.53275	9	0.05658	0.34280
5	0.10332	0.47045	10	0.05083	0.32429

In the case the number of observations of the m^{th} extreme value is finite the theoretical values of $\bar{y}_{m,n}$ and $\sigma_{m,n}$ are not published, as far as we know. This lack on theoretical expected values obstructs the calculation of the parameters α_m and u_m following equations (5) and (6) for the m^{th} extreme values from a limited number of observations x_m. Therefore the table which contains the theoretical values $\bar{y}_{m,n}$ and $\sigma_{m,n}$ have been calculated for m = 2 (1) 10 and n = 1 (1) 20, 50 and 100. The solutions of $y_{i,m}$ are obtained by an iterative process because for m $\geq$ 2 the inverse function of equations (3) are implicit.

Values of y_i are obtained by using the tangent lines to the curves of the probability function for a given value of the reduced variate. The principle of this iterative process is laid down in Fig. 2.

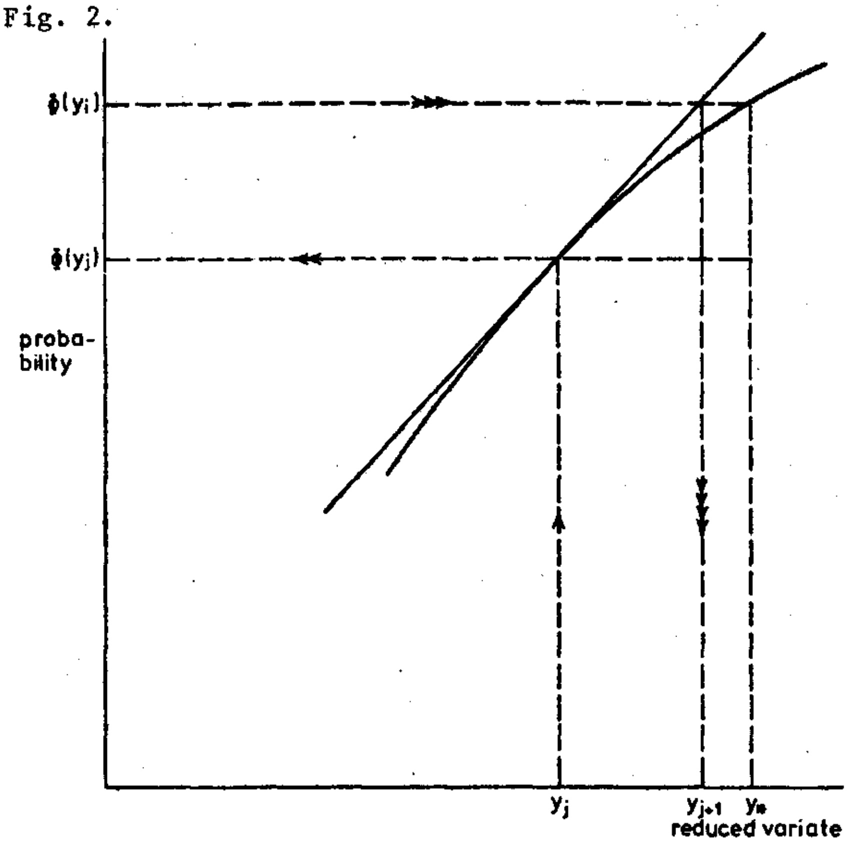

Fig. 2. Basic principle of the iterative process used to calculate y_i

The iteration process terminates when y_{j+1} and y_j are equal in the fifth decimal, see equation (15)

$$\Phi(y_i) - \Phi(y_j) = \Phi'(y_j)\{y_{j+1} - y_j\} \tag{15}$$

with

$$y_{j+1} = \frac{\Phi_m(y_i) - \Phi_m(y_j)}{\Phi'_m(y_j)} + y_j \qquad i = 1, \ldots, n$$

where

$\Phi_m(y_i)$ is known by eq. (7)

$\Phi'(y_j)$ = first derivative of Φ with respect to y, evaluated at y_j

Tabel 2

Expected Mean Y(N) and expected standard deviation S(N) of the M-th extremes as functions of the number of observations N

	M = 1		M = 2		M = 3		M = 4		M = 5		M = 6		M = 7		M = 8		M = 9		M = 10	
	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)	Y(N)	S(N)
N = 1	.3665	.0000	.1753	.0000	.1150	.0000	.0855	.0000	.0681	.0000	.0565	.0000	.0483	.0000	.0422	.0000	.0375	.0000	.0337	.0000
N = 2	.4043	.4984	.1925	.3276	.1261	.2610	.0937	.2233	.0745	.1983	.0619	.1801	.0529	.1662	.0462	.1551	.0410	.1459	.0369	.1382
N = 3	.4286	.6435	.2035	.4210	.1332	.3349	.0989	.2862	.0787	.2540	.0653	.2307	.0558	.2128	.0487	.1985	.0432	.1867	.0389	.1769
N = 4	.4458	.7315	.2113	.4768	.1382	.3788	.1026	.3236	.0816	.2871	.0677	.2607	.0578	.2404	.0505	.2242	.0448	.2109	.0403	.1997
N = 5	.4588	.7928	.2172	.5153	.1419	.4090	.1053	.3493	.0837	.3098	.0695	.2812	.0594	.2593	.0518	.2418	.0460	.2274	.0413	.2154
N = 6	.4690	.8388	.2218	.5440	.1449	.4314	.1075	.3682	.0855	.3265	.0709	.2963	.0606	.2732	.0529	.2548	.0469	.2396	.0422	.2269
N = 7	.4774	.8749	.2255	.5664	.1473	.4489	.1093	.3830	.0869	.3395	.0721	.3081	.0616	.2841	.0538	.2649	.0477	.2491	.0429	.2359
N = 8	.4843	.9043	.2287	.5845	.1493	.4629	.1107	.3949	.0880	.3500	.0730	.3176	.0624	.2928	.0545	.2730	.0483	.2567	.0434	.2431
N = 9	.4902	.9288	.2313	.5995	.1510	.4746	.1120	.4047	.0890	.3586	.0738	.3254	.0631	.2999	.0551	.2797	.0488	.2630	.0439	.2490
N = 10	.4952	.9496	.2336	.6121	.1524	.4844	.1131	.4130	.0898	.3659	.0745	.3320	.0637	.3060	.0556	.2853	.0493	.2683	.0443	.2540
N = 11	.4996	.9676	.2356	.6230	.1537	.4928	.1140	.4201	.0906	.3722	.0751	.3376	.0642	.3112	.0560	.2901	.0497	.2728	.0447	.2583
N = 12	.5035	.9833	.2373	.6325	.1548	.5001	.1148	.4262	.0912	.3776	.0757	.3425	.0646	.3157	.0564	.2943	.0501	.2767	.0450	.2620
N = 13	.5070	.9971	.2389	.6408	.1558	.5066	.1155	.4317	.0918	.3824	.0761	.3468	.0650	.3196	.0568	.2979	.0504	.2801	.0453	.2652
N = 14	.5100	1.0095	.2402	.6482	.1567	.5123	.1162	.4364	.0923	.3866	.0766	.3506	.0654	.3231	.0571	.3012	.0506	.2832	.0455	.2681
N = 15	.5128	1.0206	.2415	.6549	.1575	.5174	.1168	.4407	.0928	.3903	.0769	.3540	.0657	.3262	.0574	.3041	.0509	.2859	.0457	.2706
N = 16	.5154	1.0306	.2426	.6609	.1582	.5220	.1173	.4446	.0932	.3937	.0773	.3571	.0660	.3290	.0576	.3067	.0511	.2884	.0459	.2730
N = 17	.5177	1.0397	.2437	.6663	.1589	.5261	.1178	.4481	.0936	.3968	.0776	.3598	.0663	.3316	.0579	.3090	.0513	.2906	.0461	.2750
N = 18	.5198	1.0481	.2446	.6713	.1595	.5299	.1182	.4513	.0939	.3996	.0779	.3623	.0665	.3339	.0581	.3112	.0515	.2926	.0463	.2769
N = 19	.5217	1.0557	.2455	.6758	.1600	.5334	.1186	.4542	.0942	.4021	.0782	.3646	.0668	.3360	.0583	.3131	.0517	.2944	.0464	.2787
N = 20	.5236	1.0628	.2463	.6800	.1605	.5366	.1190	.4569	.0945	.4045	.0784	.3668	.0670	.3379	.0585	.3150	.0519	.2961	.0466	.2803
N = 50	.5485	1.1607	.2575	.7368	.1677	.5797	.1242	.4928	.0986	.4359	.0818	.3950	.0699	.3638	.0610	.3390	.0541	.3186	.0486	.3015
N = 100	.5600	1.2065	.2626	.7625	.1709	.5990	.1266	.5088	.1005	.4498	.0833	.4075	.0712	.3752	.0621	.3495	.0551	.3285	.0495	.3108

y_j = starting value of the jth step in the iteration process

y_{j+1} = final value of the jth step

The result of the calculation is given in Table 2 in which for the number of observations $n = 1\ (1)\ 20, 50, 100$ the expected mean $\bar{y}_{m,n}$, y(N), and standard deviation $\Phi_{m,n}$, s(N), is given for $m = 1\ (1)\ 10$.

The employed computer program in Fortran IV is reproduced in Appendix a.

3. APPLICATIONS IN LAND USE PLANNING

For the assessment of a criterion for the designing capacity of, for instance sportfishery facilities, the following procedure can be employed.

For a representative facility the number of visitors on the ten most crowded days per annum are selected for the period 1972 up to 1979. To eliminate the autonomous growth in the participation in sportfishery in this period the number of visitors per day is divided by the total number of visitors per annum. The magnitudes of the relative number of visitors to the object are considered as statistical variates of the exponential type. To calculate the parameters α_m and u_m according to equations (5) and (6) and the probabilities of $\Phi(\underline{x}_m < x)$ for series of values of x a computer-program has been written (Appendix b).

The observed and calculated cumulative distributions of the relative number of visitors to the focussed sportfishery facility is plotted in Fig. 3. The fit of the curves seems to be reasonable.

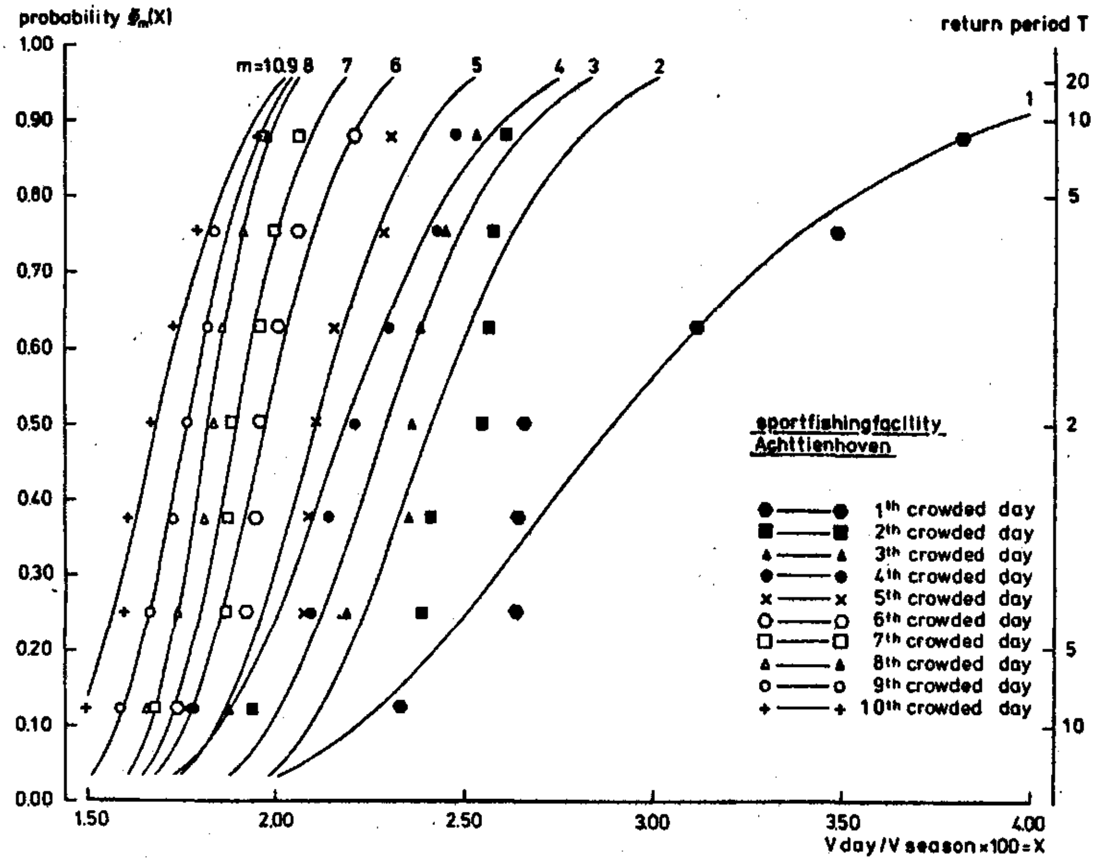

Fig. 3. Estimated and observed probabilities for the first to tenth crowded day a year related to the seasonal number of visitors to sportfishing water Achttienhoven (data collected by the Directorate of Fisheries, The Hague)

A similar procedure is followed for the number of passages along a point of a highway near Lemmer. The statistical variate is obtained by dividing the number of passages a day by the annual average as registered from 1973 up to 1979. Results are shown in Fig. 4.

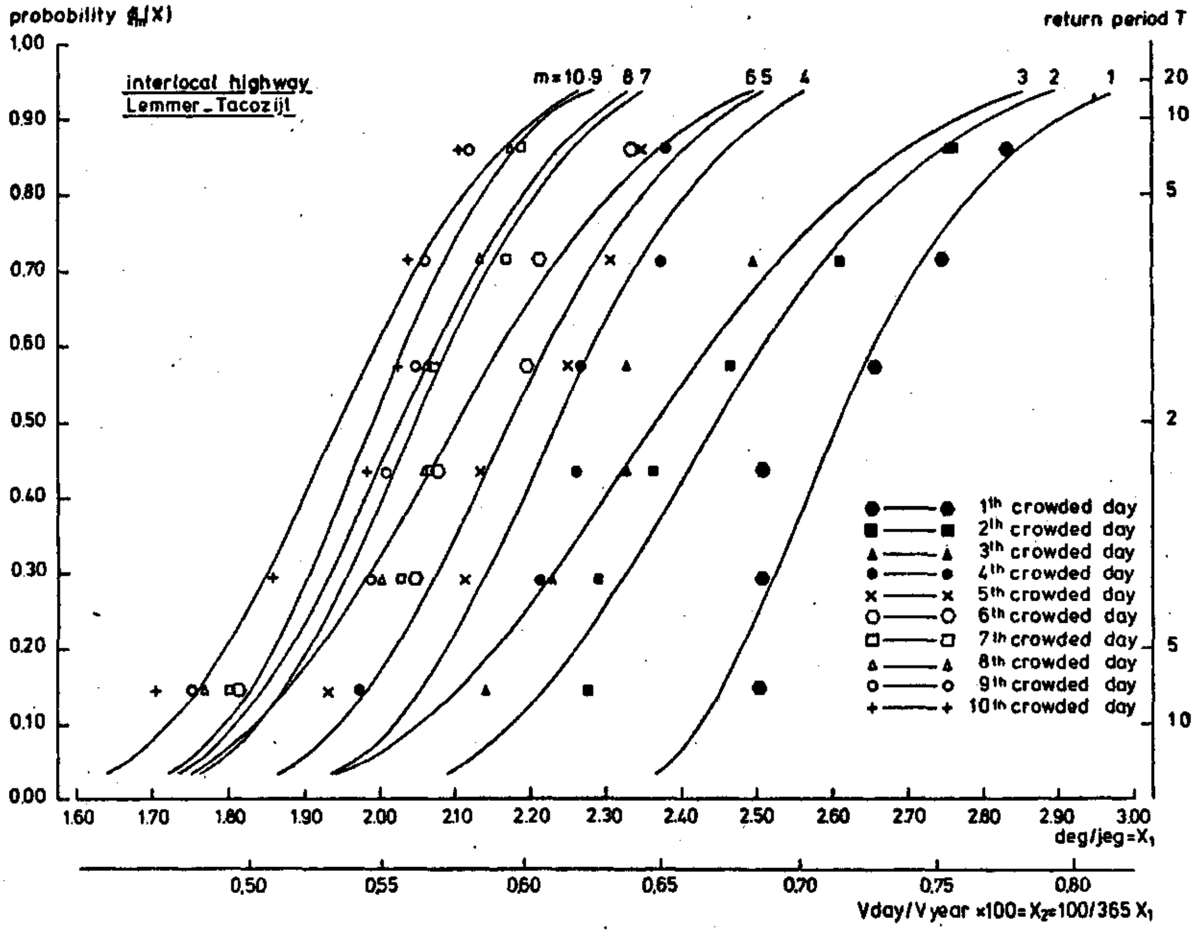

Fig. 4. Estimated and observed probabilities for the first to tenth crowded day a year related to the annual average of numbers of passages a day and the annual number of passages along a highway near Lemmer (data disposed by Mr. Jaarsma, Agricultural University, Wageningen)

If as a starting point in the planning it is taken that there has to be sufficient space as to allow anybody who wishes to participate in sportfishing on for instance the fifth crowded day in a modal (Φ_5 = 0,50) season, we may apply Fig. 3. From this figure can be derived; that we may, in this case,

accept the capacity of the supply to be insufficient during

1 day a year with a probability of 94%

3 days a year with a probability of 78%

5 days a year with a probability of 50%

7 days a year with a probability of 11%
10 days a year with a probability of 2%

With regard to the maintenance of man made facilities and the conservation of natural resources, project managers are particularly interested however in the extent of exceedance of the capacity. From Fig. 3 it can be derived that the choice of the fifth day with a probability of 50% leads to the acceptance of an intensity of use of twice the capacity every 15 years and of one and a half the capacity every 3 years.

For the type of highways like the one near Lemmer (Fig. 4) the consequence of two possible capacity norms are given in Table 3.

Table 3. Probability of combination of extent and number of exceedance of capacity as calculated for two capacity norms regarding a highway near Lemmer (Fig. 4)

Capacity norm	5th day, $\Phi_5 = 0.50$					10th day, $\Phi_{10} = 0.50$				
Number of exceedance/year	1x	3x	5x	7x	10x	1x	3x	5x	7x	10x
Extent of exceedance										
≥ 1.50xcap	1%	0%	0%	0%	0%	9%	5%	0%	0%	0%
≥ 1.25xcap	30%	13%	1%	0%	0%	88%	40%	11%	3%	1%
≥ 1.00xcap	100%	77%	50%	26%	14%	100%	96%	90%	72%	50%

The results of the statistical analysis of extreme values of the number of visitors/users of different kinds of facilities are shown in Fig. 5 as curves, that indicate the expected values of the number

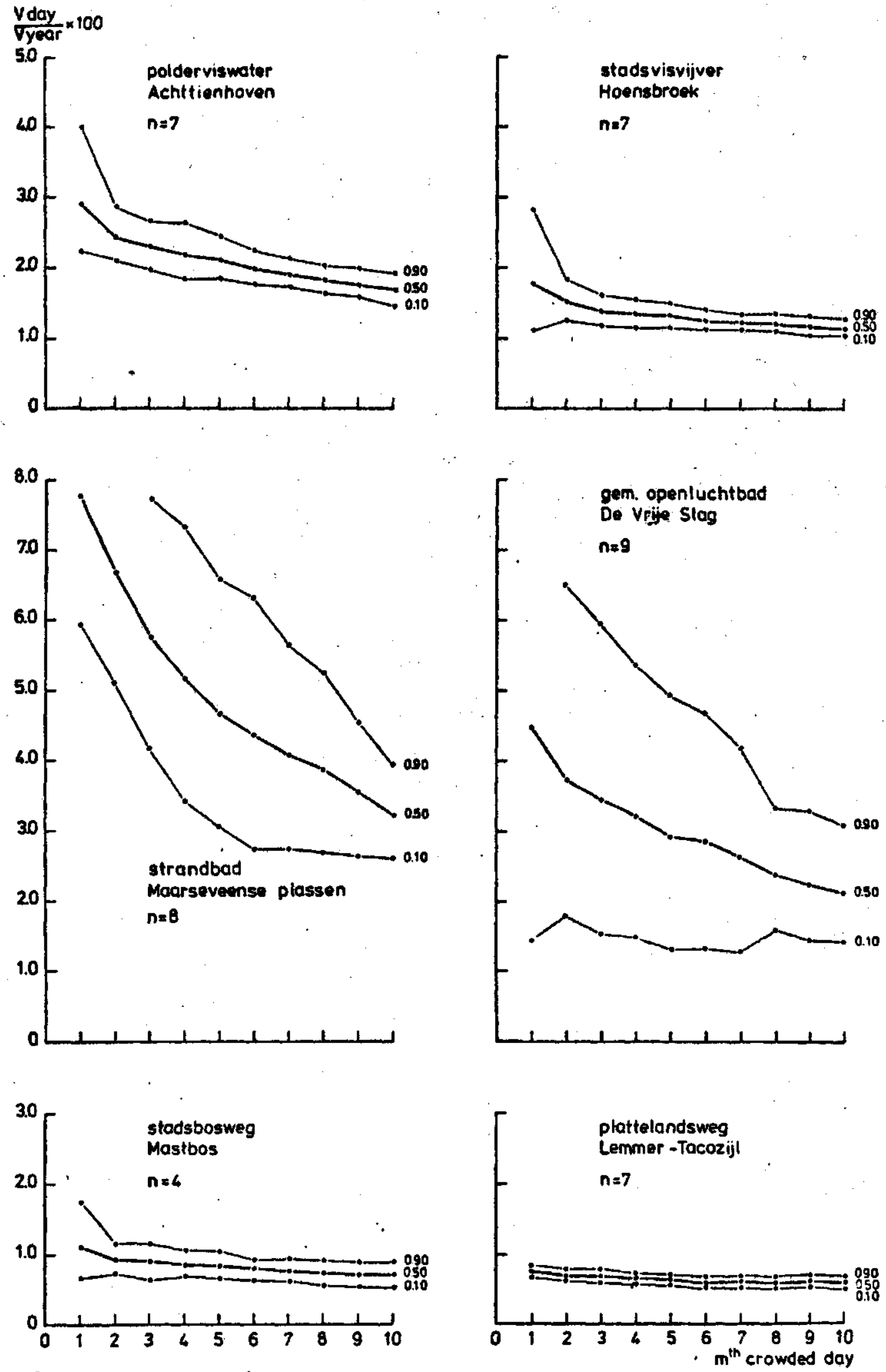

Fig. 5. Curves of the number of visitors expected on the 10 most crowded days per annum (p = 0.10, 0.50 and 0.90) for six types of facilities

of visitors on the 10 most crowded days per annum. In this case a sportfishing water within a rural area (Achttienhoven), a sportfishingpond within an urbanised district (Hoensbroek), an inland beach (Maarseveense Plassen), a local swimming pool (De Vrije Slag), a highway in the countryside (Lemmer) and a road along a townpark (Mastbos) are concerned. In all graphs the daily number of visitors is expressed as a part of the annual number of visitors.

Fig. 5 gives a clear indication of the frequency of occurrence of high use intensity of land use facilities. For inland beaches it appears that 49% of the annual number of visitors in a modal year occur on the 10 most crowded days; in contrast to the intensity of use of roads which is only 6 - 8%. Differences in level and course of the six curves clearly demonstrate the urgency of the availability of such curves for the observed type of facilities, for making a choice as to which extreme value of the number of visitors will be considered normative for the designing capacity.

In this way the elaborated theory for the m^{th} extreme value might be applied for assessment of capacities as an instrument in land use planning.

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——— 1960. Statistics of extremes. Columbia University Press, New York, USA.

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```
      PROGRAM GUMBEL (INPUT,OUTPUT,TAPE5=OUTPUT)
      DIMENSION FAC(10),YS(10),SIG(10)
      INTEGER UNITO,UNITI
      MMAX=10

      UNITI=0
      UNITO=5
      FAC(1)=1.
      DO 70 IV=2,MMAX
   70 FAC(IV)=FAC(IV-1)*IV
      DO 10 M=1,MMAX
      N=100
      SOMY=0.
      SOMYZ=0.
      Y=0.00
      DO 30 K=1,N
      FM=M
      FN=N
      FK=K
      F=FK/(FN+1.)
      NITER=0
    9 DO 40 IM=1,M
      FIM=IM
      SOM=1.
      VSOM=0.
      IF(IM.EQ.1) GOTO 42
      IIV=M-1
      DO 50 IV=1,IIV
      FIV=IV
      A=(FIM**FIV*EXP(-FIV*Y))/FAC(IV)
      SOM=A+SOM
   50 VSOM=A*FIV+VSOM
   42 HERM=-FIM*EXP(-Y)
      TERM=EXP(HERM)
      P=TERM*SOM
   40 PAC=TERM*(-HERM*SOM-VSOM)
      YN=(F-P)/PAC+Y
      YH=YN-Y
      Y=YN
      NITER=NITER+1
      YHA=ABS(YH)
      IF(YHA.GT.0.00001)GOTO 9
      IF(NITER.GT.100) GOTO 99
      SOMY=YN+SOMY
   30 SOMYZ=YN*YN+SOMYZ
      YS(M)=SOMY/FN
      SIG(M)=SQRT(SOMYZ/FN-(YS(M)*YS(M)))
      T=TIME(A)
   10 WRITE(UNITO,106) T,M
  106 FORMAT(1A10,I2)
      WRITE(UNITO,100)
  100 FORMAT(1H1///3X"EXPECTED MEAN Y(N) AND EXPECTED STANDARD DEVIATION
     * S(N) OF THE M-TH EXTREMES AS FUNCTIONS OF THE NUMBER OF OBSERVATI
     *ON N.")
      WRITE(UNITO,105) (M,M=1,5)
      WRITE(UNITO,101)

      WRITE(UNITO,110) (N,(YS(M),SIG(M),M=1,5))
      WRITE(UNITO,100)
      WRITE(UNITO,105) (M,M=6,10)
      WRITE(UNITO,101)

      WRITE(UNITO,110) (N,(YS(M),SIG(M),M=6,10))
  105 FORMAT(//1H ,30X,5(8X,"M="I2,8X))
  101 FORMAT(//1H ,30X,5(3X,"Y(N)        S(N)    "))
  110 FORMAT(/1H ,10X,"N="I2,18X,5(F6.4,3X,F6.4,5X))
   99 STOP 0
      END
```

Appendix b.

```
      PROGRAM GUMBFL(INPUT,OUTPUT,TAPE8=INPUT,TAPE5=OUTPUT)
C PROGRAMMA TOEPASSING GUMBEL
C
C ADMINISTATIE
C
      DIMENSION JAAR(10),ALFA(10),FAC(10),FHI(A0,10),OBJECT(70)
      DIMENSION X(10,10),G(10),S(10),U(10),Y(10),SIG(10)
      INTEGER UNIT0,UNIT1
      UNIT0=5
      UNIT1=8
C
C INVOERGEGEVENS LEZEN:NMAX  =AANTAL GETELDE JAREN
C                      MMAX  =AANTAL EXTREME WAARDEN PER JAAR
C                      X(N,M)=AANTAL BEZOEKERS OP M-DE DAG IN JAAR N
C                      Y(M)  =GEMIDDELDE VAN GEREDUCEERDE VARIABELE
C                      S(M)  =STANDAARDAFWYKING  VAN Y
      READ(UNIT1,301) OBJECT
      READ(UNIT1,300) NMAX,MMAX
      DO 11 N=1,NMAX
   11 READ(UNIT1,305) JAAR(N),(X(N,M),M=1,MMAX)
      READ(UNIT1,310) (Y(M),M=1,MMAX)
      READ(UNIT1,310) (S(M),M=1,MMAX)
  300 FORMAT(2I6)
  305 FORMAT(I6,10F6.3)
  310 FORMAT(6X,10F6.4)
  301 FORMAT(70A1)
C
C BEREKENING VAN GEMIDDELDE X(M) EN STD.AFW.S(M) VAN WAARNEMINGEN
C
      DO 12 M=1,MMAX
      SOMX=0.
      SOMXX=0.
      DO 13 N=1,NMAX
      SOMX=SOMX+X(N,M)
   13 SOMXX=SOMXX+X(N,M)**2
      FN=NMAX
      G(M)=SOMX/FN
   12 SIG(M)=SQRT(SOMXX/FN-G(M)**2)
C
C PRINT INVOERGEGEVENS EN G(M) EN S(M)
C
      WRITE(UNIT0,400) OBJECT
      WRITE(UNIT0,405) (I,I=1,10)
      DO 14 N=1,NMAX
   14 WRITE(UNIT0,410) JAAR(N),(X(N,M),M=1,MMAX)
      WRITE(UNIT0,415)(Y(M),M=1,MMAX)
      WRITE(UNIT0,420)(S(M),M=1,MMAX)
      WRITE(UNIT0,430)(G(M),M=1,MMAX)
      WRITE(UNIT0,435)(SIG(M),M=1,MMAX)
  400 FORMAT(1H1,70A1)
  405 FORMAT(//1H ,"     JAAR     ",3X,10(4X,"M=",I2,4X),//)
  410 FORMAT(4X,I4,5X,10F12.4)
  415 FORMAT(//1H ,"     Y(M)     ",10 F12.4)
  420 FORMAT(//1H ,"     S(M)     ",10 F12.4)
  430 FORMAT(///1H ,"     G(M)     ",10 F12.4)
  435 FORMAT(//1H ,"   SIG(M)     ",10 F12.4)
C

C BEREKENING VAN PARAMETERS ALFA(M) EN MU(M) VAN GUMBELVERDELING
C
      DO 15 M=1,MMAX
      ALFA(M)=S(M)/SIG(M)
   15 U(M)=G(M)-Y(M)/ALFA(M)
      WRITE(UNIT0,440) (ALFA(M),M=1,MMAX)
      WRITE(UNIT0,425) (U(M),M=1,MMAX)
  440 FORMAT(///1H ,"   ALFA(M)  ",10F12.4)
  425 FORMAT(//1H ,"     U(M)     ",10F12.4)
C
C BEREKENING VAN VERDELINGSFUNCTIE VAN X(M)
C
      FAC(1)=1.
      DO 16 IV=2,MMAX
   16 FAC(IV)=FAC(IV-1)*IV
      WRITE(UNIT0,201)
  201 FORMAT(1H1,///)
      READ(UNIT1,498)A,B
  498 FORMAT(2F10.3)
      DO 3 IX=1,60
      FX=IX*A+B
      DO 17 M=1,MMAX
      FIM=M
      FY=ALFA(M)*(FX-U(M))


      SOM=1.
      IF(M.EQ.1) GOTO 22
      IIV=M-1
      DO 50 IV=1,IIV
      FIV=IV
   50 SOM=(FIM**FIV*EXP(-FIV*FY))/FAC(IV)+SOM
   22 TERM=EXP(-FIM*EXP(-FY))
   17 FHI(IX,M)=TERM*SOM



    3 WRITE(UNIT0,202) FX,(FHI(IX,M),M=1,MMAX)
  202 FORMAT(1H ,"X="F6.2,1X,10F12.4)
      STOP
      END
```