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Instituut voor Cultuurtechniek en Waterhuishouding
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I. HYDROLOGICAL MODELS FOR THE SOUTHERN PEEL AREA: COLLECTION OF DISCUSSION PAPERS CONCERNING WATER QUANTITY AND WATER QUALITY MODELS, PREPARED BETWEEN MARCH AND SEPTEMBER 1983

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II. EXPLANATION TO THE MAP OF THE SUBREGIONS IN THE SOUTHERN PEEL AREA

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Projectgroep Zuidelijk Peelgebied 22

C O N T E N T S

PART I. HYDROLOGICAL MODELS FOR THE SOUTHERN PEEL AREA: COLLECTION OF DISCUSSION PAPERS CONCERNING WATER QUANTITY AND WATER QUALITY MODELS, PREPARED BETWEEN MARCH AND SEPTEMBER 1983

1. INTRODUCTION

1.1. Background of this report

The general background of this report is given by DRENT (1983): 'The Institute for Land and Water Management Research (ICW) started in 1981 with a project: Development of a model approach to analyze and evaluate alternatives for regional water management. In the ICW various models are already present or will be developed during the study which can be used as submodels in the approach. An important part of the study is the integration of the different submodels. The International Institute for Applied Systems Analyses (IIASA) is very interested in this project especially in connection with the system analytical aspects and the policy oriented modelling. This interest resulted in May 1982 in a collaborative agreement between the two Institutes on scientific Cooperation in the Field of Water and related Land Resources Management.

According to this agreement dr. S.A. Orlovski of IIASA visited the ICW on the 6th and 7th of December 1982. During this brief stay dr. Orlovski and the Working Group on Models of the project came to useful ideas for further cooperation. The ICW arranged a workshop to elaborate these ideas on 19, 20 and 21 January 1983 in a Conference Centre at Hapert.'

Part of the agreement of Hapert concerned the groundwater quantity and quality modelling. They resulted in discussions between ICW and IIASA members between March 1983 and September 1983. Finally simplified models were constructed during and after the visit of P.J.T. van Bakel, P.E. Rijtema, E. Smidt, J. Vreke and P.E.V. van Walsum (ICW) to IIASA from 12-16 September 1983 (see VAN WALSUM, 1983).

Two reasons led to the decision to make one report of all written notes between March 1983 and September 1983. First they clearify the way in which simplifications were introduced into the hydrological models. The time step in the models became larger and equations simulating hydrological processes were replaced by influence matrices. Second some ideas explained in the notes e.g. the coupling of the unsaturated and saturated zone, the coupling between a groundwater quantity and a groundwater quality model, seem to have some future value. However, the reader should take into account the status of the notes. They are only contributions to a discussion and thus subject to uncompleteness and errors. In some places they differ from the original notes because slight improvements or additions have been made.

1.2. Contents of the report

After the workshop at Hapert a first simulation model (ZUPE) has been designed at the ICW (Chapter 2). To test the assumptions on the spatial discretization some calculations have been made using the finite difference method for different grids (appendix A). Comment on this model and ideas concerning a simplified model based on the mathematics of linear systems by dr. S. Kaden of IIASA are given in Chapter 3. After the discussions at the ICW from 17-20 May 1983 with dr. S. Orlovsky from IIASA a reply to the comment of Dr. S. Kaden and a new proposal for simplified models has been written in June 1983 (Chapters 4 and 5). In this proposal the coupling between the groundwater quantity and groundwater quality model has been indicated vaguely. A detailed proposal has been made in September 1983 (Chapter 6).

1.3. Acknowledgement

I am very much indebted to dr. S.A. Orlovski (IIASA) with whom very stimulating discussions have been held in May 1983.

1.4. Literature

DRENT, J., 1983. Working plan for developing a system of models for the analysis of alternatives for regional water management. Nota ICW 1409. 15 p.

WALSUM, P.E.V. VAN, 1983. Report on Southern Peel research session at IIASA, 12-16 September 1983. Nota ICW 1463. 12 p.

Wageningen, October 1983 E.H. Smidt

2. CALCULATION OF GROUNDWATER FLOW IN THE SOUTHERN PEEL AREA (ZUPE-MODEL)

2.1. Introduction

During the workshop of the ZUPE modelling group in January (19th-21th) the quantitative description of the groundwater flow has been discussed. To guarantee the practicability of the optimization process two restrictions have been put on this description:

1. the spatial discretization should be restricted to some tenths subareas characterized by underground and soil type characteristics;
2. equations describing the groundwater flow should be linear.

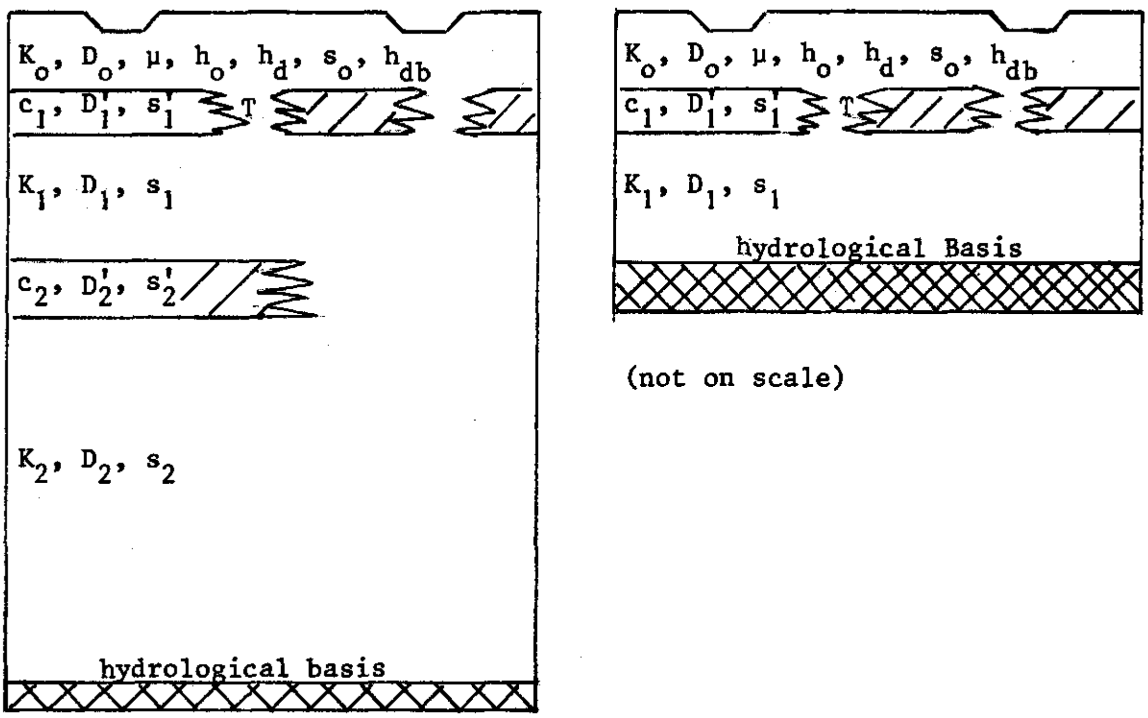
This note gives a preliminary description of the groundwater flow.

2.2. Hydrogeological schematization

Wit proposes the following hydrogeological schematization based on earlier studies and a number of recent pumping tests and drillings:

Roermond Valley Slenk Area — Peel Horst Area

Northern part — Southern part



(not on scale)

Fig. 2.1. Hydrogeological schematization and hydrogeological characteristics

In Fig. 1 the hydrogeological characteristics are:

$c_\ell = \frac{D'_\ell}{K_\ell}$	hydraulic resistance of the ℓth aquitard	(d)
D'_ℓ =	thickness of the ℓth aquitard	(d)
D_ℓ =	thickness of the ℓth aquifer	(m)
h_d =	water level in the ditches	(m)
h_{db} =	drainage basis	(m)
h_o =	phreatic water level	(m)
$h_\ell ; \ell$ =	1, 2 piezometric head in the ℓth aquifer	(m)
$K_\ell ; \ell$ =	0, 1, 2 hydraulic conductivity of the ℓth aquifer	(m/d)
s_ℓ =	specific storativity of the ℓth aquifer	(m^{-1})
s'_ℓ =	specific storativity of the ℓth aquitard	(m^{-1})
T =	drainage resistance	(d)
μ =	storage coefficient in the phreatic zone	(-)

As can be seen in Fig. 2.2 three different systems can be distinguished:

1. The northern Slenk area: three aquifers and two aquitards (R_1).
2. The southern Slenk area: two aquifers and one aquitard, aquitard 2 is absent (R_2).
3. The horst area : two aquifers and one aquitard, aquifer 2 is absent (R_3).

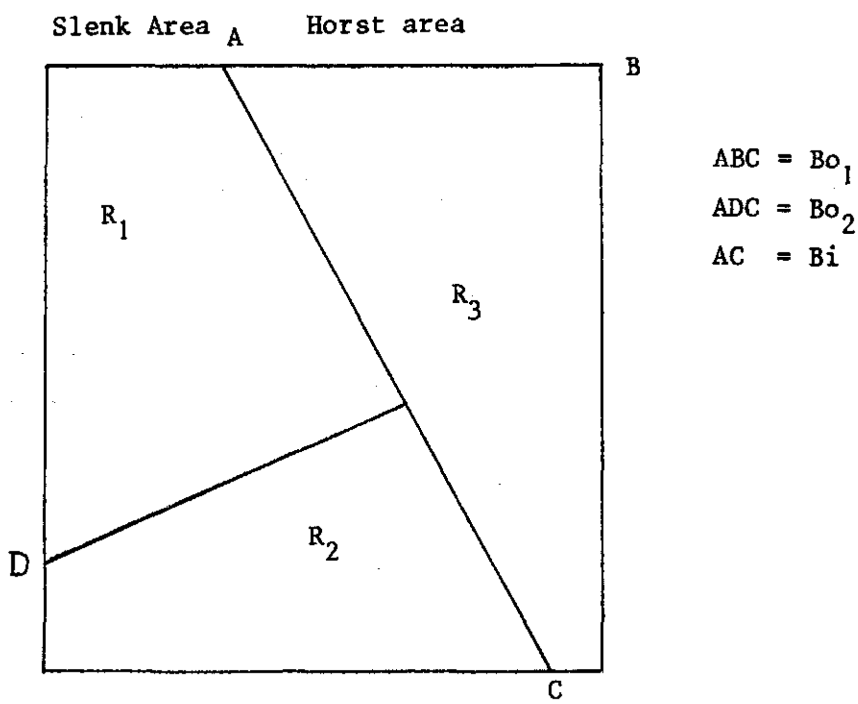


Fig. 2.2. Schematic regional distribution of different hydrogeological subsystems

Bloemen distinguishes 9 different soil-physical units. The regional extension of these units is given in nota 1374.

The combination of the soil-physical units and the hydrogeological subsystems will lead to about 20 subareas (in preparation).

2.3. Modelling of the groundwater flow

2.3.1. Assumptions

The following assumptions will be made:

1. Because of the small storativity and relatively thin aquifers and aquitards storage due to elasticity of water and grains is neglected ($s_1 = 0$, $s'_1 = 0$).
2. In the aquifers only horizontal flow takes place.
3. In the aquitards only vertical flow takes place.
4. Because of the low transmissivity in the phreatic zone the contribution to the regional horizontal flow in this zone can be neglected.

   The hydrological schematization for the model is simplified into a maximum of four layers (see Fig. 2.3).

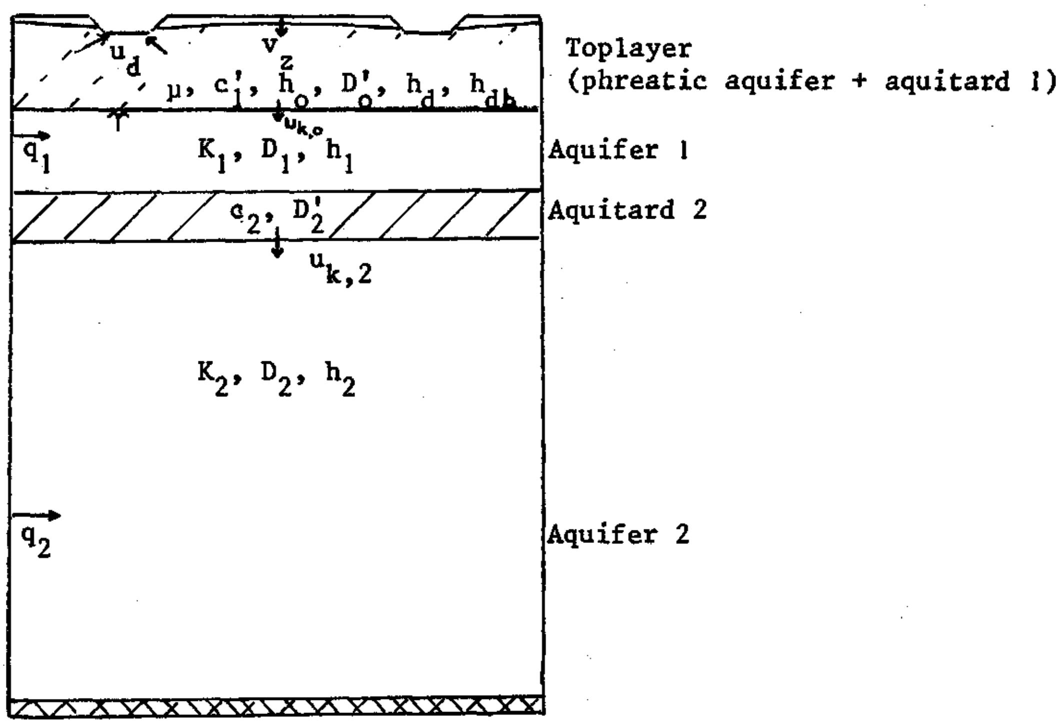


Fig. 2.3. Model schematization

with c'_o = hydraulic resistance of the toplayer (a combination of c_1, K_o from Fig. 2.1) (d)

D'_o = thickness of the toplayer ($D'_o = D_o + D'_1$)

The symbols for the flow in Fig. 3 are:

v_z	= flux though the phreatic water table	(m/d)
u_d	= flux to the ditches	(m/d)
$u_{k,o}$	= vertical flux between aquifer 1 and the toplayer	(m/d)
$u_{k,2}$	= vertical flux into/out aquifer 2	(m/d)
q_i	= horizontal flux in aquifer i	(m/d)

In the present notation a flux into a layer has a positive sign and a flux going out a layer has a negative sign.

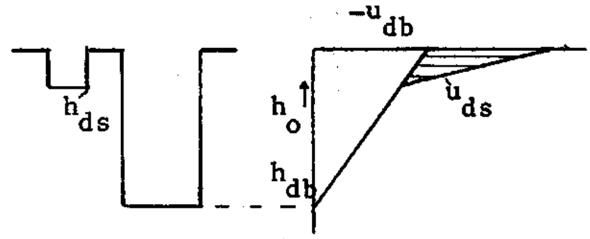
5. The flux to or from the ditches can be calculated as one dimensional flow linearly dependent on the difference between mean phreatic groundwater level and the water level in the ditches. Within one subarea three different surface water regimes can be distinguished (Fig. 2.4):

- free draining ditches

$$u^t_{db} = \alpha_{db}(h_{db} - h^t_0) \qquad (2.1a)$$

$$\alpha_{db} = \frac{1}{T_{db}}, \quad h^t_0 > h_{db} \qquad (2.1b)$$

$$\alpha_{db} = 0, \quad h^t_0 \leq h_{db} \qquad (2.1c)$$



free draining ditches

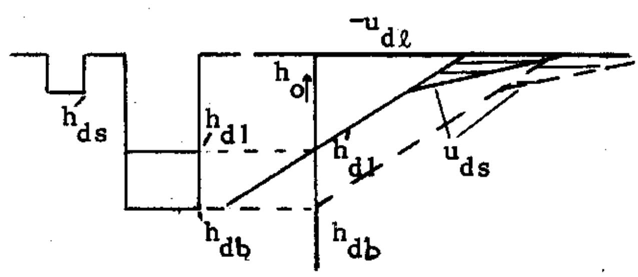
- controlled water level

$$u^t_{dc} = \alpha_{dc}(h^t_d - h^t_0) \qquad (2.2a)$$

$$\alpha_{dc} = \frac{1}{T_{dc}}, \quad h^t_d > h_{db} \qquad (2.2b)$$

$$\alpha_{dc} = 0, \quad h^t_d = h_{db}, \quad h^t_0 \leq h_{db} \qquad (2.2c)$$

$$\alpha_{dc} = \frac{1}{T_{dc}}, \quad h^t_d = h_{db}, \quad h^t_0 > h_{db} \qquad (2.2d)$$



controlled water level

Fig. 2.4. Relations between surface and groundwater

- flow through shallow ditches and over land flow

$$u^t_{ds} = \alpha_{ds}(h_{ds} - h^t_0) \qquad (2.3a)$$

$$\alpha_{ds} = \frac{1}{T_{ds}}, \quad h^t_0 > h_{ds} \qquad (2.3b)$$

$$\alpha_{ds} = 0, \quad h^t_0 \leq h_{ds} \qquad (2.3c)$$

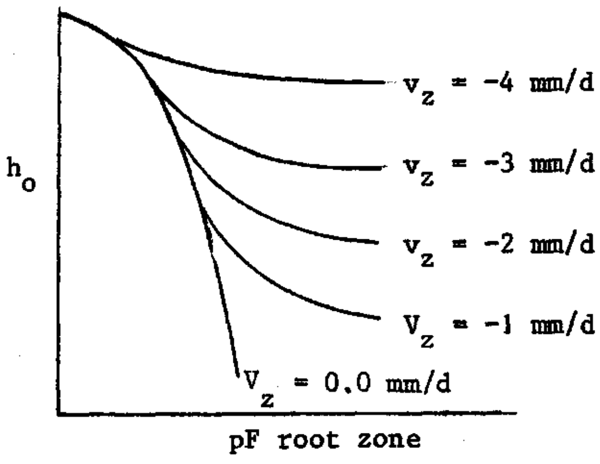
The total flux in one subarea to/from the surface water is:

$$u_d^t = u_{db}^t + u_{dc}^t + u_{ds}^t \tag{2.4a}$$

$$u_d^t = \alpha_{db}(h_{db} - h_0^t) + \alpha_{dc}(h_{dc}^t - h_0^t) + \alpha_{ds}(h_{ds} - h_0^t) \tag{2.4b}$$

with α_{db}, α_{dc} and α_{ds} as defined in 2.1b, 2.1c, 2.2b-2.2d, 2.3b, 2.3c.

6. The capillary rise can be expressed as a function of the phreatic water level and the pF in the root zone (Fig. 2.5) or as a function of the phreatic water level only (Fig.2.6). The last assumption is based on the fact that at a given phreatic water level an increase of pF in the root zone eventually does not lead to an increase in the stationary capillary rise (see Fig. 2.5).



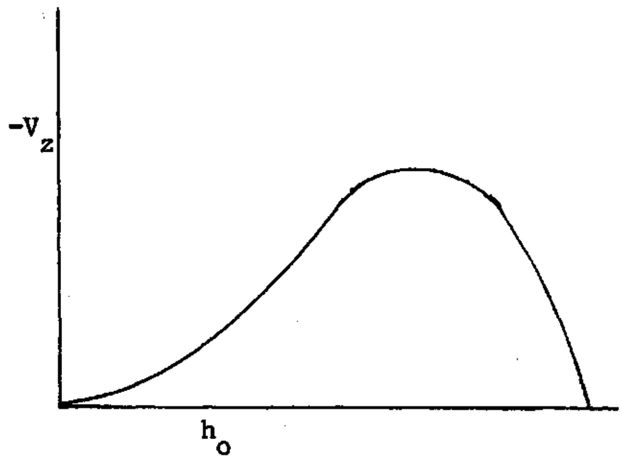


Fig. 2.5. Relation between pF in the root zone, phreatic water level and stationary capillary rise

Fig. 2.6. Simplified relation between phreatic water level and capillary rise

In this simplification the percolation is incorporated by introducing the waterbalance equation for the unsaturated zone

$$\frac{V^{t+\Delta t} - V^t}{\Delta t} = P^t + E^t + v_z^t \tag{2.5}$$

with V^t = storage in the unsaturated zone per area unit (m)

P^t = precipitation (m/d)

E^t = actual evapotranspiration (m/d)

(depending on meteorological, soil-physical and plant characteristics and the amount of water in the unsaturated zone; calculated by some Feddes formula)

$$\text{if } V^t > V^t_{eq}\,,\ v^t_z = \frac{V^t - V^t_{eq}}{\Delta t} \tag{2.6}$$

with $V^t_{eq}(h^t)$ being the equilibrium storage.

7. The storage coefficient can be related uniquely to the freatic water level (see Fig. 2.7):

$$\mu^t = f(h^t) \tag{2.7}$$

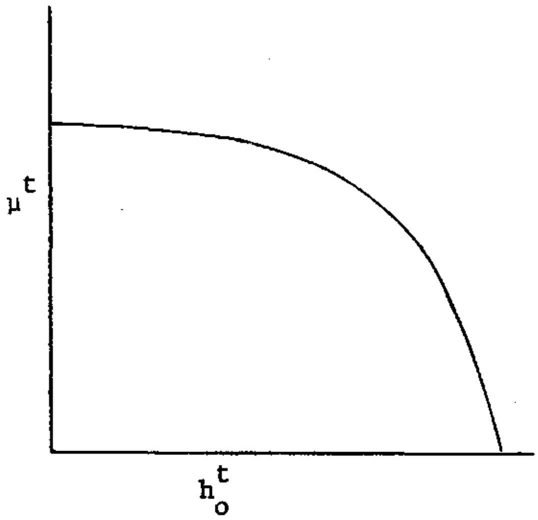


Fig. 2.7. Relation between phreatic water level and storage coefficient

Another procedure results from using Fig. 2.5 to find the storage coefficient as a function of the groundwater level and the capillary rise.

8. The horizontal flux from one subarea to another subarea takes place parallel to the connection line of the two centres of gravity of the subareas.

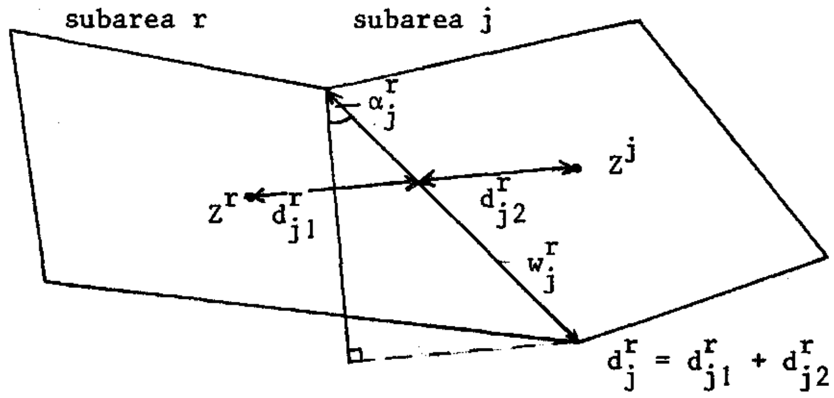


Fig. 2.8. Flow between subareas

The flow between the two subareas is:

$$Q_{j,\ell}^{r,t} = w_j^r \cos \alpha_j^r \, \bar{T}_\ell^{r,j} \, \frac{h_\ell^{j,t} - h_\ell^{r,t}}{d_j^r} \tag{2.8}$$

in which r = the sub area r

j = the neighbouring sub area j

ℓ = aquifer ℓ

d_j^r = distance between centre of gravity of subarea r and j (Z^r and Z^j)

$w_j^r \cos \alpha_j^r$ = the projection of the common boundary of r and j perpendicular on the flow line

$\bar{T}_\ell^{r,j} = \frac{(d_{j1}^r + d_{j2}^r) T_\ell^r T_\ell^j}{d_{j1}^r T_\ell^j + d_{j2}^r T_\ell^r}$ (the harmonic weighted average of the transmissivity of subarea r and subarea j)

T_ℓ = $K_\ell D_\ell$ (m^2/d), transmissivity

Then the total horizontal flow in sub area r in aquifer ℓ is:

$$Qs_\ell^{r,t} = \sum_{j=1}^{m} Q_{j,\ell}^{r,t} = \sum_{j=1}^{m} w_j^r \cos \alpha_j^r \, \bar{T}^{r,j} \, \frac{h_\ell^{j,t} - h_\ell^{r,t}}{d_j^r} \tag{2.9a}$$

with m the number of neigbouring subareas.

Some restriction are caused by the boundaries and the differences in hydrologeological schematization. If w_j^r is on the outer boundary (Bo) the flow is given*. If w_j^r is on the inner boundary (Bi)

*Neuman type boundary condition.Another type of boundary condition is the Dirichlet boundary condition. Then the head in the boundary areas must be given: $h_j^{r,t} = \underline{h}_j^{r,t}$, $r \in$ boundary elements. The flux will be calculated.

(see Fig.2.2)the flow in aquifer 2 is zero and the flow in aquifer 1 is diminished by the Peelrand fault system. in formula:

$$Q_{j,\ell}^{r,t} = \underline{Q}_{j,\ell}^{r,t} \;,\; w_j^r \in Bo_1 \cup Bo_2 \tag{2.9b}$$

$$Q_{j,2}^{r,t} = 0 \qquad ,\; w_j^r \in Bo_1 \cup Bi \tag{2.9c}$$

$$Q_{j,1}^{r,t} = \beta_j^r w_j^r \cos \alpha_j^r \, \bar{\bar{T}}_\ell^{r,j} \, \frac{h_\ell^{j,t} - h_\ell^{r,t}}{d_j^r} \;, \qquad w_j^r \in B_i \tag{2.9d}$$

with β the fault coefficient $(0 \leq \beta_j \leq 1)$

9. The vertical flow into/out the aquifer is dependent on the hydrogeological subdivision.

$$\text{Toplayer: } u_{k,o}^{r,t} = \frac{h_1^{r,t} - h_o^{r,t}}{c_o'^{r,t}} \tag{2.10a}$$

$c_o'^{r,t}$ is the hydraulic resistance of the toplayer which depends on the phreatic groundwater level and thus on time.

Aquifer 1: for $r \in R_1$

$$u_{k,1}^{r,t} = \frac{h_0^{r,t} - h_1^{r,t}}{c_1'^{r,t}} + \frac{h_2^{r,t} - h_1^{r,t}}{c_2^r} \tag{2.10b}$$

for $r \in R_2$ the assumption will be made that:

$$h_1^{r,t} = h_2^{r,t} = h_{12}^{r,t} \tag{2.10c}$$

The vertical flux into/out aquifer 2 will result as a rest in the waterbalance calculation in this area:

$$u_{k,1}^{r,t} = \frac{h_0^{r,t} - h_{12}^{r,t}}{c_1'^{r,t}} + u_{k,12}^{r,t} \tag{2.10d}$$

$$\text{for } r \in R_3 \;,\; u_{k,1}^{r,t} = \frac{h_o^{r,t} - h_1^{r,t}}{c_1'^{r,t}} \tag{2.10e}$$

$$\text{Aquifer 2: For } r \in R_1 \text{ , } u_{k,2}^{r,t} = \frac{h_1^{r,t} - h_2^{r,t}}{c_2^{r,t}} \tag{2.10f}$$

$$r \in R_2 \text{ , } u_{k,2}^{r,t} = - u_{k,f2}^{r,t} \tag{2.10g}$$

$$r \in R_3 \text{ , } u_{k,2}^{r,t} = 0 \tag{2.10h}$$

10. In the expression for the capillary rise (Fig. 2.5 and 2.6), the storage coefficient (eq.2.7) and the vertical flux from the toplayer (eq.2.10b) nonlinearily has been introduced. This is eliminated by the following procedure:
    At the beginning of a time step $h_o^{r,t}$ is known thus $v_z^{r,t}$, $\mu^{r,t}$ and $c_1'^{r,t}$ can be calculated (or read from tables). These values are thought to be valid during t till t + Δt. Then $h_o^{r,t+\Delta t}$ is calculated. Again $v_z^{r,t+\Delta t}$, $\mu^{r,t+\Delta t}$ and $c_1'^{r,t+\Delta t}$ are determined etc. Thus the time dependent soil characteristics are calculated explicitly while the groundwater levels are calculated implicitly.

2.3.2. Balance equation

For the toplayer and the aquifers the waterbalance equation will be given implicitly. Boundary conditions are given by the formulas given in the preceding sections. The flux in the toplayer is written as a time-centered flux, which means that the average of h^t and $h^{t+\Delta t}$ is used to calculate the flux during t till t + Δt.

$$\text{toplayer} : v_z^{r,t} + u_d^{r,t+\frac{1}{2}\Delta t} + u_{k,o}^{r,t+\frac{1}{2}\Delta t} + Q_{ir}^{r,t+\frac{1}{2}\Delta t} = \mu^{r,t} \frac{h_o^{r,t+\Delta t} - h_o^{r,t}}{\Delta t} \tag{2.11}$$

in which $Q_{ir}^{r,t+\frac{1}{2}\Delta t}$ is the extraction rate of water for irrigation (m/d). With (2.4b) and (2.10a):

$$v_z^{r,t} + \alpha_{db}^r \left[h_{db}^r - 0,5\left(h_o^{r,t} + h_o^{r,t+\Delta t}\right)\right] + \alpha_{dc}^r \left[h_{dc}^{r,t+\frac{1}{2}\Delta t} - 0,5\left(h_o^{r,t} + h_o^{r,t+\Delta t}\right)\right]$$

$$+ \alpha_{ds}^r \left[h_{ds}^r - 0,5\left(h_o^{r,t} + h_o^{r,t+\Delta t}\right)\right] + \frac{h_1^{r,t} + h_1^{r,t+\Delta t} - h_o^{r,t} - h_o^{r,t+\Delta t}}{2c_D'^{r,t}} +$$

$$+ Q_{ir}^{r,t+\frac{1}{2}\Delta t} = \mu^{r,t} \frac{h_o^{r,t+\Delta t} - h_o^{r,t}}{\Delta t} \tag{2.12a}$$

$$\text{with } \alpha^r_{db} = \frac{1}{T^r_{db}} \;,\; h^{r,t}_o > h^r_{db} \tag{2.12b}$$

$$\alpha^r_{db} = 0 \;,\; h^{r,t}_o \le h^r_{db} \tag{2.12c}$$

$$\alpha^r_{dc} = \frac{1}{T^r_{dc}} \;,\; h^{r,t}_d > h^r_{db} \tag{2.12d}$$

$$\alpha^r_{dc} = \frac{1}{T^r_{dc}} \;,\; h^{r,t}_d = h^r_{db} \;,\; h^{r,t}_o > h^r_{db} \tag{2.12e}$$

$$\alpha^r_{dc} = 0 \;,\; h^{r,t}_d = h^r_{db} \;,\; h^{r,t}_o \le h^r_{db} \tag{2.12f}$$

$$\alpha^r_{ds} = \frac{1}{T^r_{ds}} \;,\; h^{r,t}_o > h^r_{ds} \tag{2.12g}$$

$$\alpha^r_{ds} = 0 \;,\; h^{r,t}_o \le h^r_{ds} \tag{2.12h}$$

(2.12b-2.12h are written explicitly. This results in small errors in the phreatic water level if a change in α^r_{db} or α^r_{ds} takes place).

Aquifer 1 and 2: In general:

$$\frac{1}{A^r} \sum_{j=1}^{m} Q^{r,t+\Delta t}_{j,\ell} + u^{r,t+\Delta t}_{k,\ell} + \frac{Q^{r,t+\Delta t}_{w,\ell}}{A^r} = 0 \tag{2.13a}$$

With m the number of neighbouring subareas

$Q_{w,\ell}$	extraction rate	(m^3/d)
A^r	area of the r-th subarea	(m^2)

$$Q^{r,t+\Delta t}_{j,\ell} = \beta^r_j w^r_j \cos \alpha^r_j \, \bar{T}^r_\ell \, \frac{h^{j,t+\Delta t}_\ell - h^{r,t+\Delta t}_\ell}{d^r_j} \tag{2.13b}$$

With $\beta_j^r = 1$ $\quad$, $w_j^r \notin Bi$ (2.13c)

$$0 \le \beta_j^r \le 1 \qquad , w_j^r \in Bi \tag{2.13d}$$

Neuman condition:

$$Q_{j,\ell}^{r,t+\Delta t} = \underline{Q}_{j,\ell}^{r,t+\Delta t} \quad , w_j^r \in Bo_1 \cup Bo_2 \tag{2.13e}$$

or Dirichlet condition:

$$h_\ell^{r,t+\Delta t} = \underline{h}_\ell^{r,t} \quad , r \in \text{Boundary elements} \tag{2.13f}$$

$$Q_{j,4}^{r,t+\Delta t} = 0 \quad , w_j \in Bi \text{ or } r \in R_3 \tag{2.13g}$$

$$u_{k,1}^{r,t+\Delta t} = \frac{h_o^{r,t+\Delta t} - h_1^{r,t+\Delta t}}{c_1^{\prime r,t}} + \frac{h_2^{r,t+\Delta t} - h_1^{r,t+\Delta t}}{c_2^r} \quad r \in R_1 \tag{2.13h}$$

$$u_{k,1}^{r,t+\Delta t} = \frac{h_o^{r,t+\Delta t} - h_{12}^{r,t+\Delta t}}{c_1^{\prime r,t}} + u_{k,12}^{r,t+\Delta t} \quad r \in R_2 \tag{2.13i}$$

$$u_{k,1}^{r,t+\Delta t} = \frac{h_o^{r,t+\Delta t} - h_1^{r,t+\Delta t}}{c_1^{\prime r,t}} \quad r \in R_3 \tag{2.13j}$$

$$u_{k,2}^{r,t+\Delta t} = \frac{h_1^{r,t+\Delta t} - h_2^{r,t+\Delta t}}{c_2^r} \quad r \in R_1 \tag{2.13k}$$

$$u_{k,2}^{r,t+\Delta t} = - u_{k,12}^{r,t+\Delta t} \quad r \in R_2 \tag{2.13l}$$

$$u_{k,2}^{r,t+\Delta t} = 0 \quad r \in R_3 \tag{2.13m}$$

Let n_i be the number of subareas in R_i. The equations and conditions 2.12a-h and 2.13a-m define a set of N linear equations with N unknows, $h_\ell^{r,t+\Delta t}$ and $u_{k,12}^{r,t+\Delta t}$, that can be solved by several algorithms. N is given by:

$$N = 3 * (N_1 + N_2) + 2N_3 \qquad (2.14)$$

Wageningen, March 1983

P.J.T. van Bakel
E.H. Smidt
K.E. Wit

3. CONTRIBUTION TOWARDS THE CALCULATION OF GROUNDWATER FLOW IN THE SOUTHERN PEEL AREA

Dr. Stefan Kaden, IIASA, May 1983

3.1. Preliminary remarks

The basis for my considerations are the working plan Southern Peel Area (J. Drent)[1], the note on the calculation of groundwater flow (P.J.T. van Bakel et al.)[2] and some discussions held with S. Orlovsky as well as with your colleagues L.J. Vreke and L. Locht. Due to the vagueness of my imagination on the geographical, hydrological and hydrogeological situation in the Southern Peel Area and the absence of the surface water flow and groundwater quality models, I have some difficulty in classifying the position of the groundwater model in the whole model system. I think it would be nice (if not necessary) to have a detailed analysis of the system with the inputs and outputs of subsystems as well as the interdependencies of subsystems, with the position of subsystems and the constraints and objectives. Reading the submitted submodels, I have doubts that the aim of 'application of simple equations in a regional model' (see working plan Southern Peel Project, January 1983) is realizable.

First of all, I would like to discuss the simplified groundwater model proposed by Van Bakel et al. and second, I would like to propose a simplified groundwater model based on linear system mathematics. These are only preliminary ideas. Due to my new assignment at IIASA, I have not had enough time for a more detailed description.

3.2. Discussion of the simplified groundwater flow model, proposed by Van Bakel et al. in their note on the calculation of groundwater flow in the Southern Peel Area, March 1983[3]

This model is based on two fundamental suppositions (paragraph 2.1). First, the spatial discretization should be restricted

1) J. Drent. 1983. Working plan for developing a system of models for the analysis of alternatives for regional water management, Nota ICW 1409. 15 p.
2) Chapter 2 of this note
3) The discussed paragraphs, points, etc. of this paper are marked by brackets.

to some tenth subareas characterized by underground and soil type characteristics. This is a sensible and necessary simplification considering the objectives of the Southern Peel Project. But I think that the word "discretization" should be used only in the sense of schematization, not in the mathematical sense (Finite-Difference Method). I will return to this point in the next paragraph.

In the context of the first supposition, I missed the suppositions on time discretization. Restrictions of the number of time steps are just as important as restrictions of the spatial discretization considering the expense of the data-preparation and computation. From the mathematical point of view, big time steps are possible, due to the rough spatial discretization. Time discretization is mainly determined by the variety of outer and inner boundary conditions (irrigation rates, etc.), and the time-scale of the simulation. If you are interested in long-term planning, time steps of a year, or at most of a quarter, with average boundary conditions will be useful. In the short-term control, of course, time steps of some days are necessary.

Second, equations describing the groundwater flow should be linear. In my opinion, not only some equations should be linear, but the mathematical model as a whole. Only in this case can the model be integrated into a complex model using linear methods of systems analysis. The assumed step-by-step linearization results in linear finite-difference equations for one time step, though not in a linear model. For example,

$$\nabla(T(\bar{h})\nabla h)^{t} = \mu(h^{t-\Delta t}) \cdot \frac{h(t)-h(t-\Delta t)}{\Delta t} - w^{t} \qquad (3.1)$$

is a linear equation, but the whole system for all time steps is nonlinear.

Now to hydrogeological schematization (paragraph 2.2). In Fig. 2.1 , hydraulic connections are shown between the aquifers--in other words, windows in the aquitards (if I have not misunderstood the Figure). Taking into account all the other extensive simplifications, it could be sufficient to work with only one or two aquifers, assuming the windows are big enough and the groundwater levels do not differ between the aquifers significantly.

The method of model development is a usual physical-based one. The resulting discret model is of the same type as the simple finite difference or finite elements models with rough spatial discretization. Similar models, for instance Haimes et al., are used for the first stage in hierarchical groundwater model systems with the aim of determining the boundary conditions between subareas. The use of only such models is problematic due to the description of the groundwater flow in the subareas with one average groundwater level. For diffuse inner-boundary conditions, like irrigation rates, this is a possible simplification. If we have to consider point-source or line-source inner-boundary conditions (wells, streams, etc.), the model results have restricted practical value. Primarily the results cannot be employed for the coupling with surface water flow models (if the subarea is not identical with a river segment). In this context, I did not understand the integration of irrigation ditches and over-land flow in the model. Has an outlet been provided for to describe the interdependency between three different ditch-types in a subarea with the groundwater flow only through one average phreatic water table (paragraph 2.3.1, point 5)? I would have some doubts. Further, the results are not useable for the prediction of the water level in or near wells. As you know, these values are necessary, for instance, to optimize energy consumption. Because of my limited knowledge about the Southern Peel Region, I have no feeling of the importance of these problems. Probably they can be neglected.

For the transmissivity between subareas, the arithmetic mean has been used (paragraph 2.3.1, point 8)[1]. This mean is not really adequate to the assumption of constant transmissivities in the subareas. The flow between two subareas can be described by the flow equation

$$Q = \Delta h / R_{hydr.} \quad , \quad R_{hydr.} \text{ hydraulic resistance } [s/m^2] \tag{3.2}$$

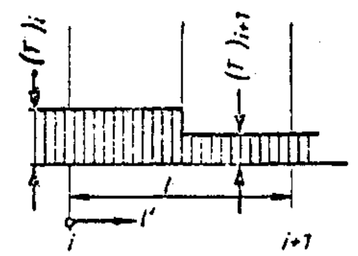
$$R_{hydr.} = \frac{1}{b} \int_{l} \frac{dl}{T} \tag{3.3}$$

with l = flood length, and b = characteristic flood width.

1) In the present version of Chapter 2 the harmonic mean has been used. See 4.2.

Subject to the function of T on the length l, you can obtain a certain means of transmissivity. Two examples:

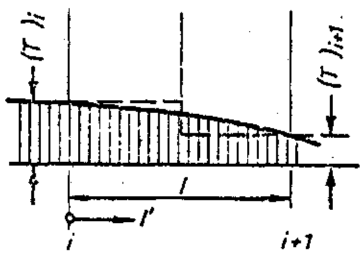
-- step function



$$R_{hydr.} = 1/(b \cdot \overline{T})$$

$$\overline{T} = \frac{2 \cdot T_i \cdot T_{i+1}}{T_i + T_{i+1}}$$

-- root function of 2. degree



$$R_{hydr.} = 1/(b \cdot \overline{T})$$

$$\overline{T} = \frac{T_i + T_{i+1}}{2}$$

If the transmissivities differ largely, the arithmetic mean is wrong. For instance, in the case of $T_{i+1} \sim 0$ ($Q \sim 0$) using the arithmetic mean you would obtain $\overline{T} = T_i/2$ ($Q >> 0$!).

I propose to use the following expression based on the harmonic mean (step function)

$$\overline{T}_l^{r,j} = \frac{T_l^r \cdot T_l^j \cdot d_r^j}{d_{j1}^r \cdot T_l^r + d_{j2}^r \cdot T_l^j} \qquad (3.4)$$

The boundaries are not described exactly (equ. 2.8, 2.9a, etc.). In the case of outer-boundaries, no neighboring subarea j exists, which is used to describe the boundary w_j^r.

Above, I emphasized the importance of linear models. It is necessary to look for possibilities of linearization beyond your proposed method (paragraph 2.3.1, point 10). According to Figure 2.6 $v_z(h_o)$ could be approximated by a linear function for each different subarea, assuming a bounded variation of h_o in a subarea.

The hydraulic resistance of the top layer is a combination of the hydraulic resistance of the first aquitard and the hydraulic resistance of the phreatic aquifer (which is dependent on h_o). Certainly the hydraulic resistance of the aquitard is manifold greater than the resistance of the aquifer. Resulting changes of the resistance of the aquifer can be neglected.

The main problems, in my opinion, are connected with the nonlinear storage coefficient μ. Because I am unfamiliar with the real hydrogeologic data for the region, it is difficult to propose a simplification. The best way would be to use constant μ^r. A time dependent $\mu^r(t)$ would also be possible. Based on a first approximation of $\mu^r(t)$, the problem can be solved and, if necessary, by using the computed h_o-values new $\mu^r(t)$-values can be obtained and the problem again solved. Probably one iteration will satisfy. Moreover, the same procedure could be used for the linearization of nonlinear v_z and c_1'.

Finally, some remarks about the balance equation (paragraph 2.3.2). Equation (2.11) is a combination of the Cranc-Nicholson method (time-centered u_d, $u_{k,o}$, Q_{ir}) and the explicit method (v_z, μ). From a mathematical point of view, this is an unusual and doubtful method, due to the nonlinearity with reference to mathematical stability. Supposing a linear model, you should use the time-centered method for all terms. In the nonlinear case, the simple implicit method seems to be more reliable. If you use time-centered values for the description of flux through aquitards, these values should be used in all equations. In your model the output/input of the top layer $u_{k,o}$ is time-centered but the input/output of the first aquifer is not. Consequently, this results in a wrong balance.

I propose for the aquifers 1 and 2 (equ. 2.13a) to use the same time-discretization as for the top layer. In this case, the equations of the top layer and the first aquifer can be combined and the unknown h_1^r-values eliminated (from equation 2.11), h_1^r can be estimated explicitly!). Consequently, the number of unknowns would be reduced (eq 2.14) by $N_1+N_2+N_3$

$$N = 2\cdot(N_1+N_2)+N_3 \quad . \tag{3.5}$$

This number has to be multiplied by the number of time steps if we want to introduce this model into complex linear models; the number of equations would be more than 1000.

3.3. Proposal for a Simplified Model Based on the Mathematics of Linear Systems

In many cases, groundwater systems are approximately linear, time-invariant systems. It is a well-known fact that such systems can be characterized by one function, the unit impulse kernel (or Green's Function). The response of this system to any excitation pattern can be predicted by the convolution equation

$$q(t) = \int_0^b k(t-\tau)r(\tau)d\tau \tag{3.6}$$

with q = response of the system

k = impulse kernel function

r = excitation pattern

t = time

Using a step kernel function K, the above equation can be rewritten in the form

$$q(t) = K(t)\cdot r(0^+) + \int_0^t K(t-\tau)\frac{\partial r(\tau)}{\partial \tau}d\tau \tag{3.7}$$

For practical solutions, instead of the continuous description of the convolution equation, a discrete description will be used considering the unit pulse

$$q(n) = \sum_{\nu=1}^{n} \sigma(n-\nu+1)r(\nu) \tag{3.8}$$

with n, ν = discrete times

$\delta(n) = \int_0^1 k(n-\tau)d\tau$ = discrete pulse kernels

$r(\nu)$ = constant excitation pattern for the interval $(\nu-1,\nu)$

This equation is well-known in hydrology--the unit hydrograph concept.

In the case of m different excitation patterns you obtain

$$q(n) = \sum_{i=1}^{m} \sum_{\nu=1}^{n} \delta_i(n-\nu+1) \cdot r_i(\nu) \tag{3.9}$$

In many cases, the unit step will be used:

$$q(n) = K(n) \cdot r(0) + \sum_{\nu=1}^{n} K(n-\nu) \cdot (r(\nu) - r(\nu-1)) \tag{3.10}$$

with $K(\nu)$ = discrete step kernel

In the last decade, modeling for groundwater management, with special regard to the conjunctive use of surface and groundwater, has been increasingly based on the above-mentioned approach. Two similar directions are most important--the discrete kernel approach (Morel-Seytoux et al.) and the algebraic technological functions (Haimes et al.). In the GDR, we used this method for the development of simple models for the short-term control of groundwater extraction for municipal water supply. Generally, the discrete kernels have to be obtained numerically. For the numerical generation of the discrete kernels, more or less complicated flow models (finite difference or finite elements models) are used.

The advantage of the discrete kernel approach over other approaches results from the following facts: First, a finite difference (or finite elements) model is used only to generate basic response functions to specialized excitations. Once these basic response functions have been calculated and saved, simulation of the system behavior to any excitation is obtained without ever making any more use of the (most costly) numerical model. Second, because the numerical model is used only to generate the response functions or influence coefficients, smaller grid sizes and time increments can be used to accurately calculate the influence coefficients than is usually feasible when performing a large number of complex simulation runs. Third, the systems behavior is described by the equations of the sought-after system responses on the system impacts or influences only. Such a model is best suited for its integration in complex linear models of systems analysis.

The use of the discrete kernel approach for the Southern Peel Project would necessitate the following main working steps: First, the system responses have to be defined. Such system responses could be the phreatic water table or piezometric head in characteristic control points (irrigation areas, wells, etc.) and the flux between subareas (certainly these values are necessary for the modeling of groundwater quality) or from surface water sources. Second, the influence values (or excitation patterns) have to be fixed, for instance, irrigation rates in subareas, groundwater withdrawals, etc. Third, the time discretization has to be determined. Fourth, the discrete kernels have to be computed using finite difference models or finite elements models like FEMSAT or a model similar to the proposed model from van Bakel et al., but with a narrower discretization. Fifth, the discret kernel model can be described as a set of linear equations

$$q_j(n) = \sum_{i=1}^{m} \sum_{\nu=1}^{n} \delta_{ij}(n-\nu+1) \cdot r_i(\nu) \tag{3.11}$$

for $j = 1,\ldots,p$ (number of responses)

Probably a great part of the discrete kernels is equal to zero, considering that not all influence values affect all defined responses.

The discrete kernel approach is useable not only for short-term control problems but also for long-term planning. If you are interested in using this approach, I could develop the model concept in a more detailed manner, taking into account the conditions of the Southern Peel area. For this, the elucidation of the problems, described in paragraph 2.2 and a better understanding of the groundwater and surface water flow and quality problems is necessary.

By the way, the linearization of the model is desireable but not indispensable. The method of algebraic technological functions also is useable for nonlinear problems. I hope to establish contacts with Y.Y. Haimes and H.J. Morel-Seytoux (USA) who have high practical experience using algebraic technological functions and discret kernels, respectively.

4. Reply to Contribution Towards the Calculation of Groundwater Flow in the Southern Peel Area by Dr. Stefan Kaden

drs. Ebel Smidt
july, 1983

4.1. INTRODUCTION

First of all I want to express my gratitude for the detailed analysis of the proposed model and the new ideas put forward. I hope that your experience will be of great help in tackling the Southern Peel problems.

A number of remarks made in your contribution have been discussed with Sergei in detail and influenced our new discussion paper.[1] Not all of them will be discussed here. In the next pages I will give a reply to the comment on our Groundwater Flow Model and your proposal for a simplified model.

4.2. COMMENT ON THE SIMPLIFIED GROUNDWATER MODEL DISCUSSION BY DR. KADEN, MAY, 1983[2]

As for the timestep considered we agreed upon a ten days period during the growing season. This relative small timestep is necessary for the simulation of the actual evapotranspiration and the demand for subirrigation and sprinkler irrigation during the growing season. For the autumn/wintermodel a smaller time step (one day) is necessary to be able to calculate the surface runoff. If this autumn/winter model has to be included in the scenario generating system (SGS) is subject to discussion (see the discussion paper).

Your remark about the linearity of the model probably is the heart of the matter. Concerning the saturated groundwater flow system with constant boundary conditions and linear relations between the groundwater and surface water system a linear distributed model will give a good comparison with reality. If nonlinear surface water-groundwater relations, nonlinear functions describing, capillary rise and storage coefficients a.s.o. are involved, linearization of the model might lead to unrealistic results. Therefore your final

1) Chapter 5 in this note; 2) Chapter 3 in this note

remark about the application of algebraic technological functions (ATF) in modelling of nonlinear problems is of great importance.

As explained in the first note[1)] important hydraulic connections between the aquifers are restricted to one part of the region. The phreatic aquifer and the first aquitard (with some windows in it) will be regarded as the toplayer in which horizontal regional flow can be neglected (see fig. 2.3).

During the visit of Sergei additional information on the surface water system has been given. Due to the dense network of ditches the surface water system cannot be included as a fully distributed system. The relation between the average phreatic groundwater level, the surface water level and the flow to or from the ditches is described by ERNST (1978). The drainage resistance can be calculated using field data on the geometry of the surface water system or by comparing the discharge from a drainage basin with the average phreatic groundwater level. Both approaches are followed in the Southern Peel Study.

The drawdown in wells indeed cannot be predicted by the proposed model. However in the municipal water supply model costs of pumping are fixed per m^3 and only additional costs for purification due to nitrate load will be taken into account.

The influence of the spatial discretization can be very important. I made some calculations using the finite difference method for the steady state calculation in a confined aquifer in an arbitrary region of 120 ha. With a rectangular grid 120 elements have been formed. The region has been divided also in 18 polygons. The transmissivity in the region varies from 50 to 200 m^2/d. An extraction of 400 m^3/d has been introduced. If the mean value of the rectangular grid model over a polygon is compared with the polygonal model result the differences in drawdown mount to 26.4 cm or 40% and the absolute difference in flux between two polygons mounts to 40 m^3/d at an absolute value of 125 m^3/d. Relative differences in flux can be larger than 100%. If these differences are tolerable depends on the objectives of the particular study. If the polygon model is used to calculate the boundary conditions for a detailed analysis by the rectangular grid model the drawdown in the pumping cell differs only 6 cm from the rectangular grid model with the original boundary conditions.

1) Chapter 2 in this note

Coming to your remark about the transmissivity between subareas I fully agree with your argumentation. Before our presentation in the internal modelling group in April 1983 we had changed this point already.

Concerning the capillary rise (v_z) figure 1 in FEDDES and RIJTEMA (1983) has to be used. We discussed this with Sergei. Your comment on the hydraulic resistance of the toplayer is correct. The dependency of μ on the groundwater level is very important: for a sandy loam it varies from 0,04 at a groundwater depth of 20 cm to 0,29 at a groundwater depth of 300 cm. Indeed it will be better to calculate μ and v_z implicitly. For μ this can be done without much problems if an iteration procedure is used for the solution of the finite difference equation. If v_z has to be calculated implicitly iterations between the unsaturated zone and saturated zone model are needed. This is computing time consuming. Secondly by taking v_z explicitly a retardation effect is taken into account.

Because the flow in the deep aquifers is assumed to be steady during each timestep in which only $Q^r_{j.\ell}$ might change in equation (2.12a) the term

$$h_1^{r,t} + h_1^{r,t+st}$$

has to be changed into

$$2h_1^{r,t+st}$$

From this equation h_1^r values can be eliminated to reduce the number of unknowns.

4.3. COMMENT ON THE PROPOSAL FOR A SIMPLIFIED MODEL BASED ON THE MATHEMATICS OF LINEAR SYSTEMS

To incorporate the dynamics of the ground and surface water system into an optimization system three ways can be distinguished:

1. Application of a simple linear programming model to calculate a scenario for the distribution of water demand.
   The water demand being an input for a simulation model the feasibility of the scenario is tested (for example see DE RIDDER and EREZ, 1977).

2. Using the balance equation of the hydro(geo)logical system as constraints in the optimization model (see for example BEAR (1979), p 505).
3. Using technological functions or kernels (the work of HAIMES and MOREL SEYTOUX that you mentioned, the work done in Israël see for example GABLINGER and SCHWARZ, 1979), the work by GORELICK and REMSON (1982) and DE MARSILY et al. (1978)[1)]

Our first proposal was based on the second method, and your proposal is based on the third method. During our discussion with Sergei in May, we designed a balance model for the rootzone and discussed the possibility of incorporating the groundwater balance equations as well in the optimization model. Our new proposal includes technological functions for the flow into/out the toplayer for each subregion. As explained in the discussion paper these will be nonlinear. Therefore we are very much interested in the way of using algebraic technological functions for non-linear problems.

1) In fact they apply a combination of the first and third approach

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5. HYDROLOGICAL MODELS FOR THE ZUIDELIJKE PEEL STUDY: A DISCUSSION PAPER

5.1. DESCRIPTION OF THE HYDROLOGICAL SYSTEM

5.1.1. Hydrogeology

Two hydrogeological systems can be distinguished (see fig. 1). In the Central Slenk region the system consists of:

- the Nuenen Group (to 15-20 m -GL[1]) consisting of fine sand, sandy loam and loam. The transmissivity varies between 20 and 625 m^2/d. Because this value is relatively low compared to the deeper aquifers, the horizontal regional flow in these areas can be neglected and it can be regarded as the top layer with a hydraulic resistance varying from a few days to over 1000 days. The flow to or from the ditches within one subregion is thought to be dependent on the drainage resistance (see paragraph 2.3.1);
- the combined Veghel-Sterksel Formation consisting of coarse sometimes gravel bearing sand. It reaches from 15-20 m to 50-70 m -GL, and the transmissivity varies from 1150-5700 m^2/d. At the base of this formation clayey deposits at the top of the Kedichem Formation form an aquitard. The Kedichem and Tegelen Formation consist of locally gravel-bearing sand, fine sand and clayey deposits. The combined Kedichem-Tegelen Formation reaches from 50-70 m to 140-200 m -GL. The base of the Kedichem Formation and/or the top of the Tegelen Formation consists of clayey deposits with a height between 8 and 30 m. At the base of the Tegelen Formation also thick clayey layers occur (7-31 m thick). Transmissivity of the formations is relatively low (Kedichem Formation 250-2200 m^2/d, Tegelen Formation 350-3000 m^2/d). Because of the low transmissivity and the presency of the clayey layers the Kedichem-Tegelen Formation will be regarded as an aquitard in the model;
- the Kieseloolite Formation consisting of fine sand, coarse gravel bearing sand and humic clay beds. It extends from 140-200 m to 300-350 m -GL. It will be regarded as the second aquifer in the model. The transmissivity varies from 5000-12 000 m^2/d;
- the hydrological base (300-350 m -GL) is the top of the Breda Formation consisting of fine marine deposits.

The second system is the Peel Horst area, divided from the Central Slenk by the Peelrand fault. In this region only the top layer (Nuenen

1) GL = ground level

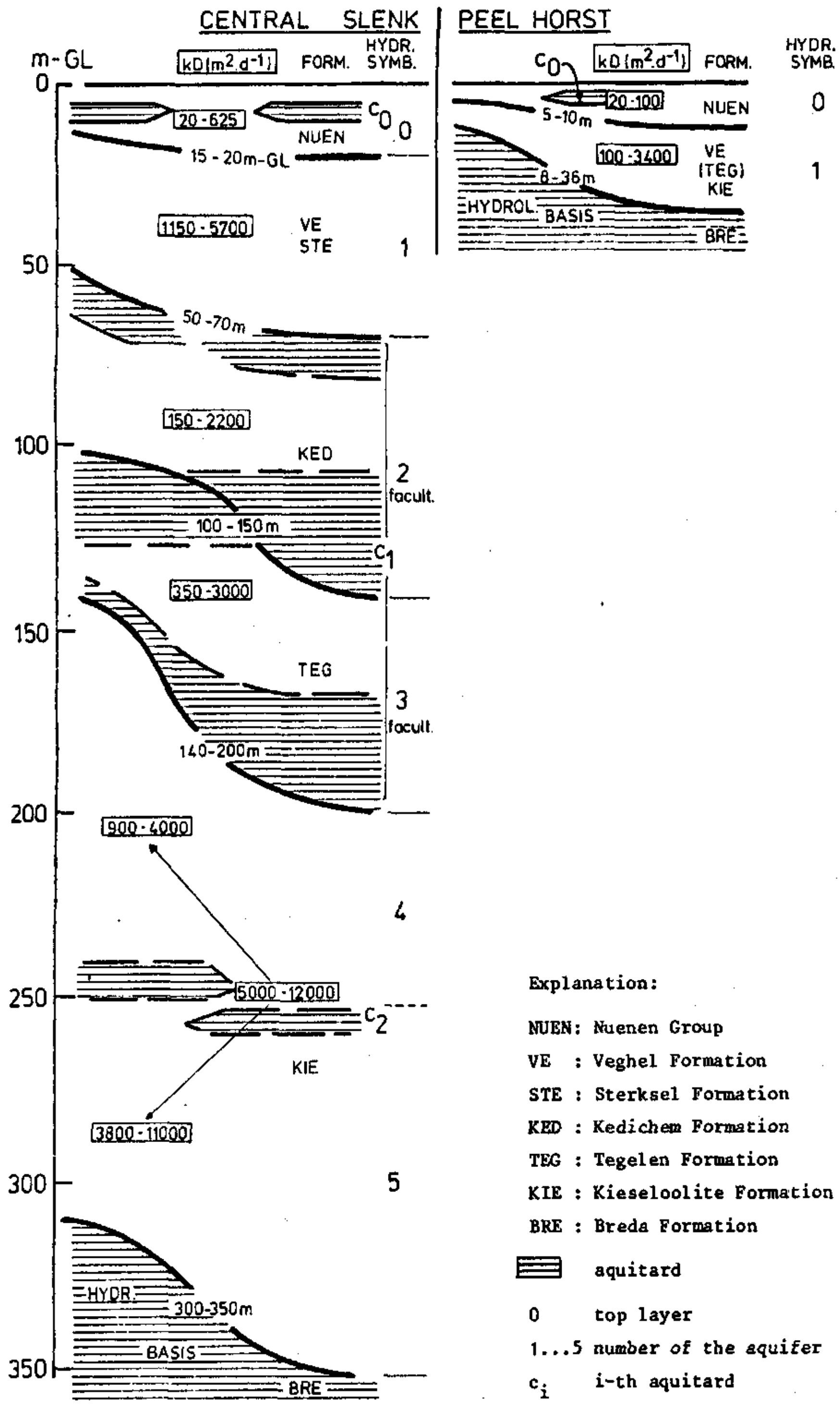

Fig. 5.1. Hydrogeological schematization in the Southern Peel area (based on the work of Van Rees Vellinga, not yet published)

Group, to 5-10 m -GL) and one aquifer (Veghel and Kieseloolite Formation, 5-10 m -GL to 8-36 m -GL with a transmissivity of 100-3400 m^2/d) can be distinguished. The hydrological base is the 100-3400 boundary with the Breda Formation.

5.1.2. Surface water system

The surface water system consists of some larger canals (Zuid-Willemsvaart, Noordervaart, Helenavaart, Kanaal van Deurne and the Peelkanaal)and a dense network of ditches and brooks. The larger canals distribute the water into and out the system of ditches and brooks. During the summer water from the river Meuse can be imported into the area while during the winter the precipitation surplus is drained.However, some (artificial) drainage basins are draining permanently due to the large storage capability (some nature areas) or seepage from other basins. To some areas external water cannot be allocated. How to describe the process mathematically has already been explained in the preceeding short note.

5.1.3. Pumping

Withdrawal of water for different purposes takes place from the following aquifers:

irrigation	Veghel and Sterksel Formation
industrial supply	Veghel and Sterksel Formation
municipal water supply	Veghel to Kieseloolite Formation

Total industrial and municipal water extraction is about 10.10^6 m^3/year. Data on irrigation extraction are not yet available. Presently part of the irrigation water ($\pm$ 25 %) is still pumped directly from the surface water. Due to new legislation this will be stopped. For the scenario generating system we will asume that all the irrigation water is taken from the toplayer.

5.2. SIMPLE HYDROLOGICAL MODEL

5.2.1. Introduction

Two models have been designed one for the spring/summer and one for the autumn/winter situation. The assumptions made in the first short note on the elastic storativity, the direction of flow, the capillary rise and the storage coefficient have not been changed. The main differences with the short note are:

- the surface water balance is included;
- during the spring/summer the surface water system can be controlled completely. Storage in the surface water system will be neglected during the spring/summer period;
- the influence of industrial municipal and irrigation pumping on the flow into/out the top layer will be taken into account in the following way:
  . Select representative summer and winter boundary conditions, called the zero summer and winter situation. Calculate with the steady state version of FEMSAT the deep percolation out or seepage into the toplayer for each subregion:

$$v^{o}_{z,1}\,(i,s) \qquad i = 1, N \qquad s = 1,2 \quad s = 1 \text{ summer},\; s = 2 \text{ winter}$$

  . Select besides the existing industrial and municipal pumping sites some new sites. Calculate with FEMSAT for each well with different pumping intensities the extra vertical fluxes leaving the top layer for each subregion: (steady state)

$$v^{ws}_{z,1}\,(i,\ Q_{ws}(\ell),\ s) \qquad i = 1, \ldots .\ N \qquad \ell = 1, \ldots .\ L \qquad s = 1,2$$

  Note 1. Because the influence of any withdrawal on the flux into/out the top layer depends on the interaction with the surface water, restrictions have to be incorporated on the maximum amount of subirrigation water available for one subregion. This will result in a non linearily in $v^{ws}_{z,1}\,(i,\ Q_{ws}\ (\ell),\ s)$

  . The same procedure is repeated for the irrigation. For each subregion a diffuse irrigation intensity is applied which gives the extra vertical fluxes flowing out the top layer in the neighbouring regions and flowing into the top layer in the irrigation region:

$$v^{irr}_{z,1}\,(i,\ Q_{irr}(k)) \qquad i = 1, \ldots .\ N \qquad k = 1, \ldots .\ N$$

Note 2. Probably some research has to be done to test the assumption of zero specific storage. If the assumption is not correct, the time effect has to be taken into account

$$v_{z,1}^{ws} \quad (i,\ Q_{ws}\ (\ell, n), \nu\ ,\ s) \qquad \nu = n,\ n+1,\ \ldots.$$

and

$$v_{z,1}^{irr} \quad (i,\ Q_{irr}\ (k,n), \nu\ ) \qquad \nu = n,\ n+1,\ \ldots.$$

In the model equations the effect of an activity during the preceding time steps has to be incorporated

$$\sum_{\ell \varepsilon L} \sum_{\nu=1}^{n} v_{z,1}^{ws} \quad (i,\ Q_{ws}\ (\ell, n - \nu + 1),\ n, s)$$

- the root zone system is modelled for each technology in the subregion during the spring/summer. The subsoil-phreatic water zone is modelled for each subregion. This means one value for the capillary rise for one subregion whereas the value of v_z^+ (percolation through the rootzone) and Q_{irr} (irrigation pumping) depends on the technology. However, due to the diffusity of the technologies over the subregion the effect of v_z^+ and Q_{irr} on the phreatic water table can be averaged over the subregion;
- the actual evapotranspiration depends on the available water in the rootzone;
- the surface runoff is modelled only during the autumn/winter in the following way:

  surface runoff = net precipation - maximum possible drainage - vertical flux to/from the deep aquifer - saturation deficit. For the calculation of the surface runoff a timestep of one day will be necessary because of the averaging effect on the precipitation at timesteps of 10 days or more.

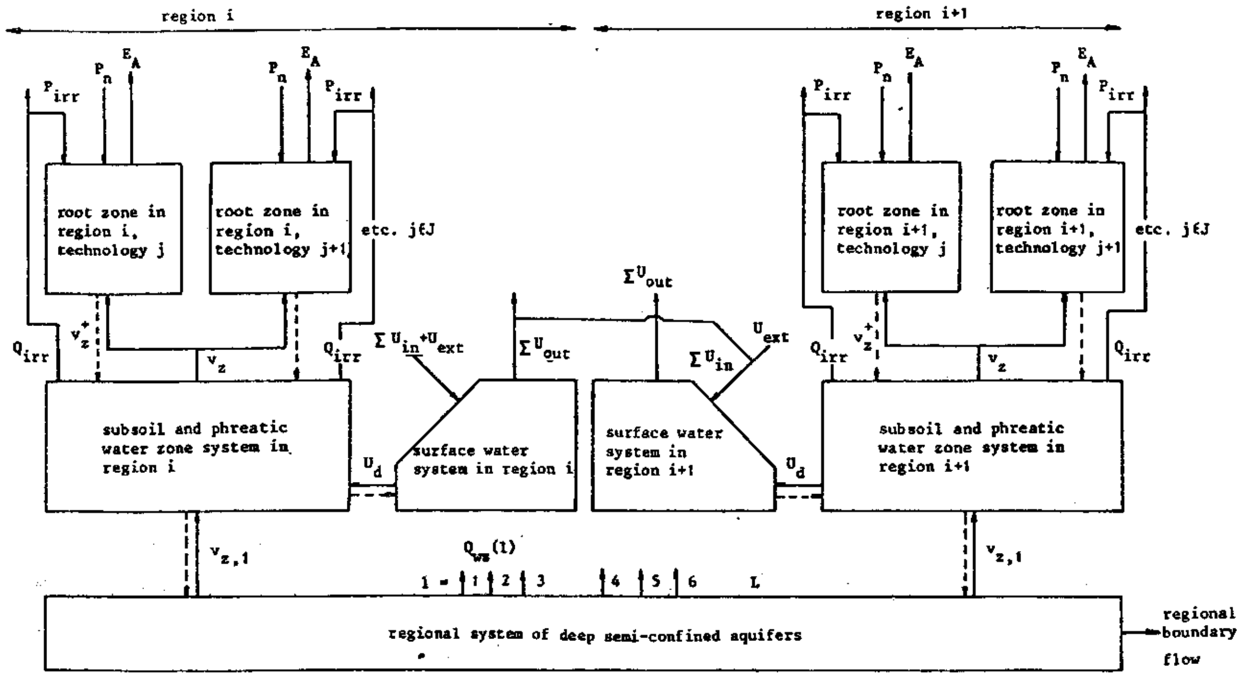

Fig. 5.2. Relations between subsystems in the spring/summer hydrological model for two neighbouring subregions

<u>Balance for the root zone per technology</u>

$$V(i,j,t + \Delta t) = V(i,j,t) + P_n(j,n) - \alpha(i,j,n)\, E_p(j,n) + v_z\left[h(i,t)\right]\Delta t + P_{irr}(i,j,n) \qquad (5.1)$$

$$\text{with: } \alpha(i,j,n) = \frac{V(i,j,t) + V(i,j,t + \Delta t)}{2\, V_{max}(i)}$$

$$V(i,j,t + \Delta t) = \frac{V(i,j,t)\left(1 - \dfrac{E_p(j,n)}{2\, V_{max}(i)}\right) + v_z\left[h(i,t)\right]\Delta t + P_n(j,n)}{1 + \dfrac{E_p(j,n)}{2\, V_{max}(i)}} \qquad (5.2)$$

<u>Actual evapotranspiration</u>

$$E_A(i,j,n) = \frac{V(i,j,t + \Delta t) + V(i,j,t)}{2\, V_{max}(i)}\, E_p(j,n) \qquad (5.3)$$

Percolation

$$V^{+}(i,j,t+\Delta t) = V(i,j,t+\Delta t) - V_{eq}(i,t) \tag{5.4}$$

if $V^{+}(i,j,t+\Delta t) > 0$

$$v_z^{+}(i,j,n) = -\frac{V^{+}(i,j,t+\Delta t)}{\Delta t} \tag{5.5a}$$

$$V(i,j,t+\Delta t) = V_{eq}\left[h(i,t)\right] \tag{5.5b}$$

Balance for the phreatic water and subsoil zone

$$\mu(i,\bar{h})\,\frac{h(i,t+\Delta t)-h(i,t)}{\Delta t} = -\sum_{j\in J} X_r(i,j,t)\,v_z^{+}(i,j,n) - v_z\left[h(i,t)\right] + v_{z,1}^{o}(i,1) + \sum_{\ell\in L} v_{z,1}^{ws}(i,Q_{ws}(\ell,n),1) + \sum_{k\in N} v_{z,1}^{irr}(i,Q_{irr}(k,n)) + \frac{U_d(i,n)}{\Delta t} - \frac{1}{X(i)\,\Delta t}\sum_{j\in J} Q_{irr}(i,j,n) \tag{5.6}$$

Relation between Q_{irr} and P_{irr}

$$P_{irr}(i,j,n) = \frac{0.95\,Q_{irr}(i,j,n)}{X_r(i,j,n)\,X(i)} \tag{5.7}$$

Surface water balance

$$-U_d(i,n)\,X(i) + \sum_{k\in N_i} U_{in}(k,i,n) - \sum_{k\in N_i} U_{out}(k,i,n) + U_{ext}(i,n) = 0 \tag{5.8}$$

with $U_{in}(k,i,n) = U_{out}(i,k,n)$

Constraint on inlet of water on external sources

$$U_{ext}^{min}(i,n) < U_{ext}(i,n) < U_{ext}^{max}(i,n) \tag{5.9}$$

Constraint on inlet of water for each subregion

$$U_{in}^{min}(k,i,n) < U_{in}(k,i,n) < U_{in}^{max}(k,i,n) \tag{5.10}$$

Constraint on possibility to infiltrate/drain

$$U_d^{min}(i) < U_d\ (i,n) < U_d^{max}\ (i) \tag{5.11}$$

Constraint on the maximum inlet of water for the whole region

$$\sum_{i \in N} U_{ext}\ (i,n) \leq U_{ext}^{tot}\ (n) \tag{5.12}$$

Constraint on maximum and minimum groundwater levels

$$h^{min}\ (i,t + \Delta t) < h\ (i,t + \Delta t) < h^{max}\ (i,t + \Delta t) \tag{5.13}$$

Constraint on industrial and municipal water supply

$$Q_{ws}^{min}\ (\ell,n) < Q_{ws}\ (\ell,n) < Q_{ws}^{max}\ (\ell,n) \tag{5.14}$$

Constraint on the total demand for industrial and municipal water supply

$$\sum_{\ell \in L} Q_{ws}\ (\ell,n) \leq Q_{ws}^{tot}\ (n) \tag{5.15}$$

Constraint on irrigation water supply

$$0 < Q_{irr}\ (i,j,n) < Q_{irr}^{max}\ (i,j,n) \tag{5.16}$$

Constraint on demand to be met

$$E_A\ (i,j,n) - V\ N(i,j,n) \geq 0 \qquad \forall i \in N,\ \forall j \in J,\ \forall n \in T \tag{5.17}$$

Objective of the model

Minimize $E_A\ (i,j,n) - VN\ (i,j,n)$

5.2.3. Autumn/ winter model (see fig. 5.3)

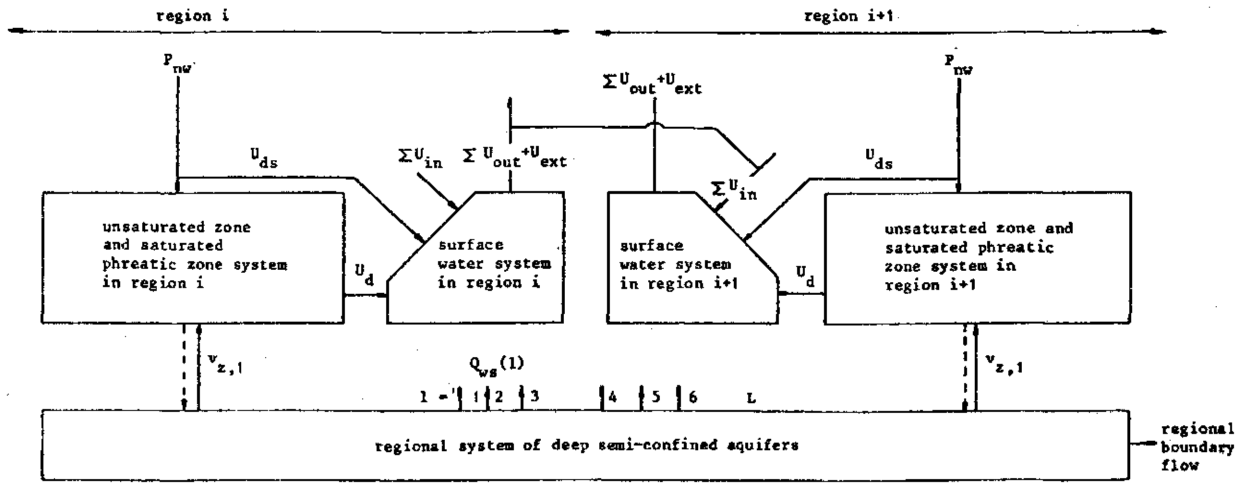

Fig. 5.3. Relations between subsystems in the autumn/winter hydrological model for two neighbouring subregions

Surface runoff

$$U_{ds}(i,n) = (1 - a(i))\ P_{nw}\ (n) - \frac{z(i) + h(i,t) - h_o(i,t+\Delta t) - h_o(i,t)}{2\ \gamma(i,\bar{h})}\ \Delta t$$

$$- (1-a(i))\ V_s(t) + v^{o}_{z,1}\ (i,2)\ \Delta t + \Delta t \sum_{\ell\in L} v^{ws}_{z,i}\left[i,Q_{ws}(\ell,n),2\right] \qquad (5.18)$$

with $U_{ds}(i,n) > 0$

Drainage to the ditches

$$U_d(i,n) = \frac{h(i,t+\Delta t) + h(i,t) - h_o(i,t+\Delta t) - h_o(i,t)}{2\ \gamma(i,\bar{h})}\ \Delta t \qquad (5.19)$$

Surface water outflow from the subregion

$$U_{out}(i,k,n) = a(i)\ X(i)\{\ \eta(k,i)\ \frac{h_o(i,t) + h_o(i,t+\Delta t)}{2} - h_{db}(k,i)\}^{\zeta(k,i)}\ \Delta t \qquad (5.20)$$

$$U_{ext}(i,n) = a(i)\ X(i)\{\eta_{ext}(i)\ \frac{h_o(i,t) + h_o(i,t+\Delta t)}{2} - h_{db}\ (i)\}\ ^{\zeta_{ext}(i)}\ \Delta t \qquad (5.20)$$

Storage in the surface water system

$$S_{sw}(i,t+\Delta t) - S_{sw}(i,t) = a(i)\left[h_o(i,t+\Delta t) - h_o(i,t)\right] \tag{5.21}$$

Surface waterbalance

$$a(i)\ X(i)\ P_{nw}(n) + X(i)\ U_d(i,n) + X(i)\ U_{ds}(i,n) + \sum_{k\in N_i} U_{in}(k,i,n) -$$

$$- \sum_{k\in N_i} U_{out}(k,i,n) - U_{ext}(i,n) = S_{sw}(i,t+\Delta t) - S_{sw}(i,t) \tag{5.22}$$

Phreatic/unsaturated zone balance

$$V_s(t+\Delta t) - V_s(t) = (1 - a(i))\ P_{nw}(n) - U_d(i,n) + v^{o}_{z,1}(i,2)\Delta t +$$
$$+ \Delta t \sum_{\ell\in L} v^{ws}_{z,1}(i,Q_{ws}(\ell,n),2) \tag{5.23}$$

Constraint on minimum water level

$$h(i,t+t) > h^{min}(i) \tag{5.24}$$

Constraint on industrial and municipal water supply

$$Q^{min}_{ws}(\ell,n) < Q_{ws}(\ell,n) < Q^{max}_{ws}(\ell,n) \tag{5.25}$$

The equations for the autumn/winter model are nonlinear. They can be solved by iteration. This means that problems arise if this model has to be integrated in the scenario generating system (SGS) of models. Probably the model has to be used for calculating the following functions:

$$U_{ds}\left[i,n,Q_{ws}(1,n),\ \ldots\ldots\ldots,\ Q_{ws}(\ell,s)\right]$$
$$U_d\left[i,n,Q_{ws}(1,n),\ldots\ldots\ldots,\ Q_{ws}(\ell,s)\right]$$
$$U_{out}\left[i,k,n,Q_{ws}(1,n),\ldots\ldots\ldots,\ Q_{ws}(\ell,n)\right]$$
$$U_{ext}\left[i,n,Q_{ws}(1,n)\ \ldots\ldots\ldots,\ Q_{ws}(\ell,n)\right]$$
$$h\left[i,t+\Delta t,\ h(i,t)\ Q_{ws}(1,n),\ \ldots\ldots\ Q_{ws}(\ell,n)\right]$$

For the SGS it would be nice if these functions can be written as a linear combination:

$$U = U^{o} + \sum_{\ell \varepsilon L} U\left[i, n, Q_{ws}(\ell, n)\right] \tag{5.26}$$

However, the model to generate these functions being highly non-linear, the functions will be non-linear.

An other argument for running the autumn/winter model seperately from the SGS is the absence of control variables in the surface water system during the autumn/winter. All weirs are set in their lowest possibility to drain the precipitation surplus. Only if the drainage system itself is part of the optimization ($\gamma(i)$ being a control variable) the autumn/winter model has to be incorporated in the SGS.

5.3. RELATIONS WITH OTHER MODELS

Production model and agriculture model

FEDDES and RIJTEMA (1983) give the production for different crops as a function of actual evapotranspiration during periods of 10 days.

VREKE (1983) selects three levels of production, which determined the water demand during each timestep for a certain production level, for each technology.

Given a set of technologies with a given production level and a relative area in a subregion the hydrological model calculated the actual evapotranspiration and compares it with the water demand.

Nature model

From the nature model boundary conditions for the water level in nature areas result.

N-P load on surface water

STEENVOORDEN (1983) distinguishes three types of drainage surface water:

- surface runoff
- shallow drainage
- deep drainage

The surface runoff component (U_{ds}) will result from the autumn/ winter model.
The shallow drainage occurs in subregions where $v_{z,1}$ is downward: U_d is shallow drainage.
The deep drainage occurs in subregions where $v_{z,1}$ is upward: $v_{z,1}$ Δt is the deep drainage component

$U_d - v_{z,1}\Delta t$ is the shallow drainage component

Given the total load of N and P on the surface water and $Q_{out}(i,n)$ the concentration can be calculated.
During the spring/summer the amount of infiltrated water will be an input for the N-P load model.

Groundwater quality model

The following idea exists:

For the groundwater quality model the long term influence on the public water supply is essential.A steady state calculation for different public water supply strategies will give long term flow components (see 5.2.1). Assuming complete mixing in the vertical and horizontal direction within one region the long term N and P load in public water supply extractions can be estimated.

The calculation of the N and P load of deep seepage water is not clear yet.

June, 1983

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P.J.T. van Bakel

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List of Symbols

i, $i = 1, \ldots\ldots, N$	subregion
k, $k = 1, \ldots\ldots, N_i$	subregion
j, $j = 1, \ldots\ldots, J$	technology
ℓ, $\ell = 1, \ldots\ldots, L$	pumping station
s, $s = 1, 2$	season
n	timestep

$a(i)$	relative area of surface water in region i	(i)
$E_a(i,j,n)$	actual evapotranspiration in region i of technology j during timestep n	(m)
$E_p(j,n)$	potential evapotranspiration for technology j during timestep n	(m)
$h(i,t)$	phreatic groundwater level in region i at time t	(m)
$\bar{h}\ (i,n)$	phreatic groundwater level in region i during timestep n	(m)
$h_o(i,t)$	surface water level in tegion i time t	(m)
$h_{db}(k,i)$	height of the bottom of the channel at the discharge measurement structure between subregion k and i	(m)
J	number of technologies	
L	number of pumping stations	
N	number of subregions	
N_i	number of neigbouring regions of region i with exchange possibility of surface water	
$P_n(i,n)$	net precipitation (precipitation minus interception) for technology j during timestep n in the spring/summer	(m)
$P_{nw}(n)$	net precipitation (precipitation minus surface interception) during timestep n in the autumn/winter	(m)
$P_{irr}(i,j,n)$	irrigation on technology j in region i and during timestep n	(m)
$Q_{ws}(\ell,n)$	extraction for municipal and industrial water at pumping station ℓ during timestep n	(m^3)
Q_{ws}^{tot}	total demand for industrial and municipal water supply during timestep n	(m^3)
$Q_{irr}(k,n)$	total extraction for irrigation use in region k during timestep n	(m^3)
$Q_{irr}(i,j,n)$	extraction for irrigation use in region i for technology j during timestep n	(m^3)
$S_{sw}(i,t)$	storage in the surface water system	(m)
$U_d(i,n)$	flow into/out the drainage system in region i during timestep n per unit area	(m)
$U_{in}(k,i,n)$	surface water inflow from region k into region i during timestep n	(m^3)

$U_{out}(k,i,n)$	surface water outflow from region i to region k during timestep n	(m^3)
$U_{ext}(i,n)$	surface water flow into/out the total region during timestep n in subregion i	(m^3)
U_{ext}^{tot}	inlet capacity for the whole region during timestep n	(m^3)
$U_{ds}(i,n)$	surface runoff in region i during timestep n per unit area	(m)
$v_z^{\mp}(i,t)$	capillary rise in region i at time t	(m/d)
$v_z^{+}(i,j,t)$	percolation in region i for technology j at time t	(m/d)
$v_{z,1}^{o}(i,s)$	deep vertical flux into/out the toplayer in region i for steady state conditions during spring/summer or autumn/winter	(m/d)
$v_{z,1}^{ws}(i,Q_{ws}(\ell,n),s)$	deep vertical flux into/out the toplayer in region i due to industrial or municipal pumping centre ℓ during timestep n and season s	(m/d)
$v_{z,1}^{irr}(i,Q_{ws}(k,n),s)$	deep vertical flux into/out the toplayer in region i due to irrigation pumping from region k during timestep n at season s	(m/d)
$V(i,j,t)$	soil moisture storage in the root zone in region i for technology j at time t	(m)
$V_s(i,t)$	saturation deficit above the groundwater table in region i at time t	(m)
$V_{max}(i)$	maximum amount of soil moisture in the root zone in region i (= soil moisture in the root zone at field capacity)	(m)
$V^+(i,j,t+\Delta t)$	surplus of soil moisture in the root zone for technology j at time $t+\Delta t$	(m)
$V_{eq}(i,t)$	equilibrium soil moisture in the root zone in region i at time t	)m)
$VN(i,j,n)$	water demand of technology j in region i during time n	(m)
$X(i)$	total area of region i	(m^2)
$X_r(i,j,t)$	relative area of technology j in region i at time t	(-)
$z(i)$	surface ground level of region i	(m)
$\alpha(i,j,n)$	actual evapotranspiration coefficient in region i for technology j during timestep n	(-)
$\mu(i,h)$	storage coefficient in region i	(-)
$\gamma_{db}(i,h)$	drainage resistance of region i	(d)
$\eta(k,i)$, $\zeta(k,i)$, $\eta_{ext}(i)$, $\zeta_{ext}(i)$	coefficients for the discharge formula of the discharge measurement structure between region k and i	
ν	timestep	

In general: Y^{max} and Y^{min} maximum and minimum value of the variable Y

6. Note on the coupling between the groundwater quantity and -quality model, based on the discussion between van Bakel, Drent, Rijtema, Smidt and van Walsum at the ICW on August 29th, 1983

6.1. IMPORTANCE OF THE GROUNDWATER QUALITY

In the SGS groundwater quality relations are needed because the total amount of manure and fertilizer used in each subregion influences the chemical composition of the water extracted for drinking water supply and the composition of the water in nature areas. For the drinking water company the nitrate content is the most important parameter. For the evaluation of the chemical composition of water in nature areas other ions (Ca^{+2}, Na^{+} , etc.) are important as well. However, as a first simplification only nitrate load will be evaluated.

6.2. ASSUMPTIONS FOR THE WATER QUALITY MODEL

a. Steady state calculations

It is obviously not feasible to base the groundwater quality modelling on simulation runs of non-steady groundwater flow. So for a start the calculations will be based on steady state flow, that is computed for boundary conditions that are derived from long term averages. The RPMA will be interested in two aspects of NO_3^--pollution:

1) given a 'steady state' NO_3^--load on groundwater, what will in the long run the NO_3^--concentration of the extracted water be?
2) how long does it take for pollution in subregion r to reach an extraction 1?

For 2) times of residence would have to be computed, e.g. by a rough method of adding up the residence times of the volumetric elements along a streamline.

The assumption of a steady contamination pattern (= concentrations are not time-dependent) implies that effects of longitudinal dispersion are not included in the analysis.

b. Within one volumetric element in an aquifer there is instant mixing of all incoming fluxes. This means that no exact flow lines can be evaluated and only one concentration per element is calculated. This procedure leads to errors in both the longitudinal and lateral movement of NO_3^--loads.

c. Adsorption can be neglected.

d. Decomposition of NO_3^- can be approximated by a first order reaction

$$\frac{\delta C}{\delta t} = -k_r C \tag{6.1}$$

in which C = the concentration of NO_3^- (ML^{-3})

k_r = the decomposition coefficient (T^{-1})

The solution of (1) is:

$$C = C_0 e^{-k_r t} \tag{6.2}$$

in which C_0 = initial concentration of NO_3^- at $t = 0$ (ML^{-3})

In the steady state calculation the decomposition in a volumetric element is governed by the mean residence time in the element ($\bar{t}_{res}$):

$$\bar{t}_{res} = \frac{\varepsilon V}{\sum_i Q_i^+} \tag{6.3}$$

in which V = volume of the element (L^3)

Q_i^+ = incoming flux from the boundary element i (L^3T^{-1})

ε = porosity

Then (6.2) becomes:

$$C = C_0 e^{-k_r \bar{t}_{res}} \tag{6.4}$$

e. The N-concentration does not influence convection by gravity flow or differences in viscosity. This means that the groundwater flow problem can be solved independently from the quality model.

f. The NO_3^--concentration of the boundary fluxes is known. All influences of activities outside the total region are known or neglected. Inside the region the NO_3^--load on the phreatic water is known depending on the technology and the depth of the mean groundwater table during the winter (h_w^*)

$$NO_{3\ ph}^- = NO_{3\ la}^- \ \alpha(h_w^*) \tag{6.5}$$

with $NO_{3\ ph}^-$ = NO_3^--load on the phreatic water table per unit area per timestep ($ML^{-2}T^{-1}$)

$NO_{3\ la}^-$ = total NO_3^--load on the land surface per unit area per timestep ($ML^{-2}T^{-1}$)

$\alpha(h^*)$ = function describing the denitrification process in the unsaturated zone depending on the mean groundwater table depth during the winter h_w^* (see Fig. 6.1)

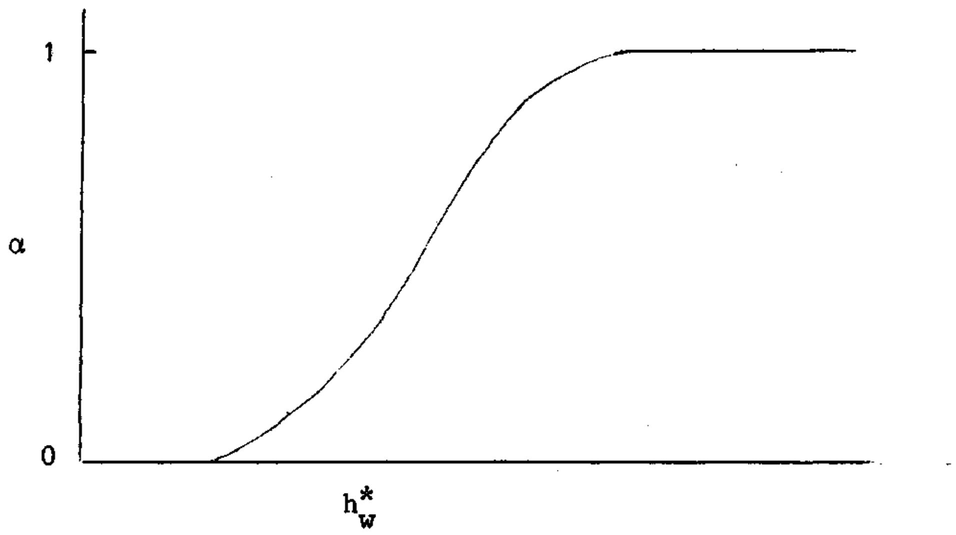

Fig. 6.1. α as function of h_w^*

g. The infiltrated imported surface water does not influence the chemical composition of water at the pumping sites or the nature areas.

6.3. MODEL FORMULATION

For each element i in each aquifer and the top layer a mass balance can be written:

$$e^{-k_r(i,l)\bar{t}_{res}(i,l)}\left[\sum_{j\varepsilon J^+(i,l)} Q_h(i,j,l)C(j,l) + \sum_{k\varepsilon K^+(i,l)} Q_v(i,k,l)C(i,k)\right] +$$

$$+ C(i,l)\left[\sum_{j\varepsilon J^-(i,l)} Q_h(i,j,l) + \sum_{k\varepsilon K^-(i,l)} Q_v(i,k,l)\right] = 0 \qquad (6.6)$$

in which i = element index
- j = index for neighbouring element
- k = index for neighbouring layers
- l = index for the layer
- C = concentration of NO_3^- (ML^{-3})
- J = set of neighbouring elements in the horizontal direction
- K = set of neighbouring layers. The superscript + denotes the subset of J or K from which water flows into element i in layer l. The supercript - denotes the subset of J or K to which water flows from element i in layer l
- k_r = decomposition coefficient (T^{-1})
- Q_h = horizontal flow (L^3T^{-1})
- Q_v = vertical flow (L^3T^{-1})
- $\bar{t}_{res}$ = mean residence time (T)

The horizontal and vertical discretization is equal to the one used in the steady state calculations with FEMSAT from which the horizontal and vertical flow per element results. The phreatic water level and the boundary flow also results from the FEMSAT calculations with a given extraction pattern. Then the mean residence time per element can be calculated by using eq. (6.3).

Given the NO_3^--concentration of the boundary fluxes and the $NO^-_{3\ la}$ values the set of equations given by (6.6) can be solved, e.g. by the Gauss-Seidel iteration procedure.

6.4. DISCUSSION

a) The model of which the draft has been presented is non-linear (decomposition in the unsaturated and saturated zone is described by a non-linear function). To generate useful relations for the SGS the model has to be run many times.
Suppose we select L possible pumping stations with E extraction levels, we have N_a agricultural subregions with C_a $NO_3^-{}_{1a}$-levels and N_n nature areas. To find the $NO_3^-{}_{1a}$-concentrations in the nature areas (C_{nat}) and in the pumped water (C_{ws}) the model has to be run $L \times E \times N_a \times C_a \times N_n$ times to find the influence matrices. For example let $L = 5$, $E = 3$, $N_a = 30$, $C_a = 3$ and $N_n = 3$, 1350 runs are needed. To make these matrices useful in the SGS-system some correlation program has to be applied to find the following expressions:

$$C_{nat}(m) = C^0_{nat}(m) + \sum_{i=1}^{N_a} \beta(m,i)NO^-_{3\ 1a}(i) + \sum_{l=1}^{L} \gamma(m,l)Q(l) \qquad (6.7)$$

$$C_{ws}(l) = C^0_{ws}(l) + \sum_{i=1}^{N_a} \zeta(l,i)NO^-_{3\ 1a}(i) + \sum_{k=1}^{L} \kappa(l,k)Q(k) \qquad (6.8)$$

in which l,k = index for the subregion with a pumping station included
m = index for the nature subregion
C_{nat} = NO_3^--concentration in a nature area without any activity in the total region (ML^{-3})
C^0_{ws} = NO_3^--concentration in the subregion with a pumping station without any activity in the total region (ML^{-3})
β,γ,ζ and κ are coefficients relating a unit activity in one subregion to its effects in the nature subregion or the regions with a pumping station

b) In the model the implicit assumption is made that the extractions for irrigation during the summer do not influence the chemical composition of the water at the pumping sites and the nature areas. Because sprinkling reduces the load on the deeper groundwater, the NO_3^--concentrations will be exaggerated. This problem can be solved for example by introducing a sprinkling coefficient f_{spr} in the calculation of the NO_3^--load on the phreatic water:

$$NO_{3\ ph}^{-\ eff} = f_{spr}\ NO_{3\ ph}^{-} \tag{6.9}$$

with $NO_{3\ ph}^{-\ eff}$ = effective NO_3^--load on the phreatic water per unit area (ML^{-2})

September 1983 E.H. Smidt P.E. van Walsum

COMPARISON BETWEEN FINITE DIFFERENCES METHODS FOR A DENSE RECTANGULAR AND A WIDE POLYGONAL GRID

1. Methods

To study the reliability of the ZUPE-groundwater model a theoretical region of 99 ha has been chosen. For reasons of simplicity only one layer is assumed in which only horizontal steady flow takes place. The transmissivity value varies strongly over the region (see fig. A.1). To calculate hydraulic heads and water balances two methods have been applied.

1) Finite differences method with a rectangular grid (REC-model)

   The regular grid distance is 200 m, resulting in 120 nodal points (see fig. A.2). The area of influence of each internal nodal point is 1 ha. Finite differences equations has been solved with fixed hydraulic heads at the boundaries (Dirichlet conditions). The results are used for the calibration of the polygon method.

2) Finite differences method with a polygonal grid (POL-model), equivalent to the ZUPE-saturated flow model

   The region has been divided into 8 internal subregions and 9 boundary regions (see fig. A.2).
   The transmissivity of a subregion has been calculated as a weighted average over the subregion. The results are shown in fig. A.2. For the calculation of Z^r, A^r and $w_j^r \cos \alpha_j^r \, \overline{T}^{r,j}$ simple TI-59 programs have been used. The results for the areas of the subregions are given in table A.1.

Table A.1. Area of the polygonal subregions (ha)

r	A^r	r	A^r	r	A^r
1	15,125	7	12,250	13	2,250
2	14,125	8	14,125	14	1,000
3	10,500	9	3,250	15	2,250
4	12,625	10	2,250	16	1,750
5	2,000	11	2,250	17	1,500
6	18,250	12	2,250	18	1,250

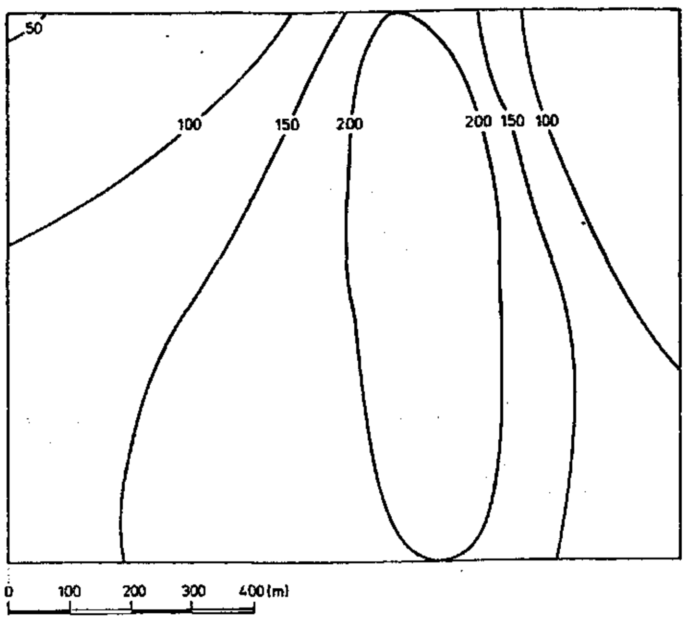

Fig. A.1. Distribution of the transmissivity (T) in m^2/d

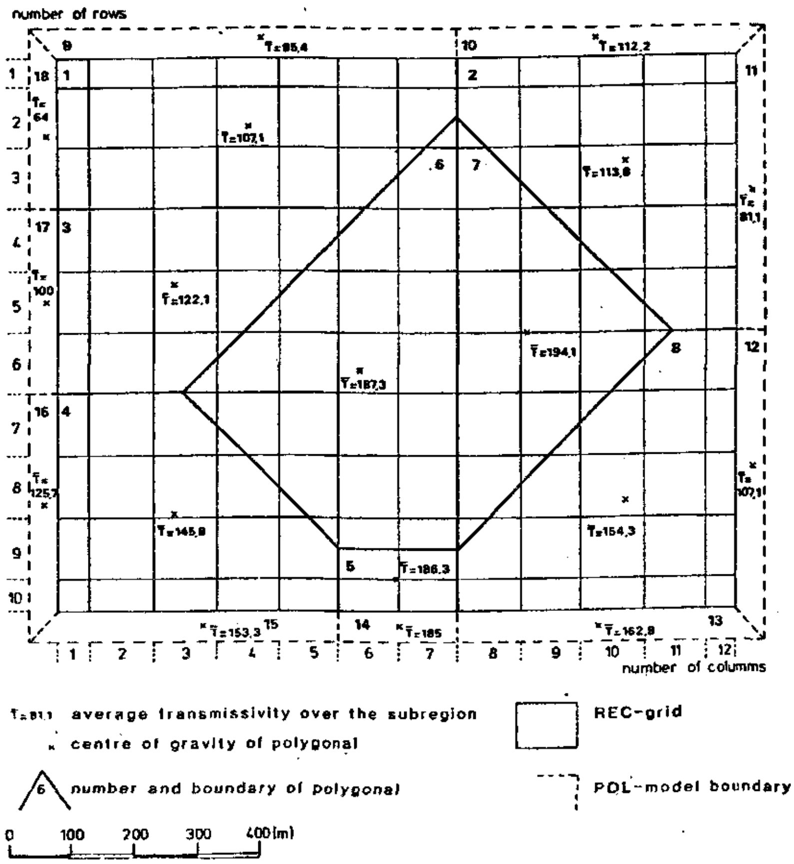

Fig. A.2. REC-grid and POL-polygons

To compare the POL-model with the REC-model hydraulic heads have been averaged over het POL-subregions and fluxes into/out these regions have been calculated. Secondly the REC interpolated head in the centre of gravity of the POL-subregion has been used in the comparison.

Three different boundary conditions for the POL-model have been analyzed:

- Neuman conditions (fixed flow into/out subregions 1, 2, 3, 4, 5 and 8);
- mixed boundary conditions (fixed flow into/out subregions 1, 2, 3, 4 and 8, fixed head in element 5);
- Dirichlet conditions (fixed heads in element 9-18).

The data for these boundary conditions are the results of the REC-model.

To analyze the effect of an extraction the models have been used to calculate the effect of an extraction of 400 m^3/d from cell (5,6) in the REC-model and in element 6 in POL-model.

2. Results

2.1. REC-model

The results of the REC-model with and without extraction are shown in tables A.2 and A.3. The values of h in row 1 and 10 and column 1 and 11 are fixed.

Table A.2. Results of the REC-model without extraction

	1	2	3	4	5	6	7	8	9	10	11	12
1	19.000	19.750	20.500	21.250	22.000	22.750	23.000	23.000	23.500	24.000	24.500	25.000
2	20.000	20.762	21.408	22.014	22.586	23.049	23.298	23.466	23.780	24.188	24.668	25.200
3	21.000	21.610	22.146	22.633	23.059	23.385	23.614	23.807	24.061	24.403	24.829	25.300
4	22.000	22.359	22.763	23.140	23.464	23.723	23.924	24.115	24.336	24.622	24.986	25.400
5	22.500	22.899	23.258	23.572	23.832	24.050	24.232	24.410	24.602	24.842	25.147	25.500
6	23.000	23.378	23.699	23.974	24.193	24.379	24.539	24.697	24.863	25.062	25.307	25.600
7	23.500	23.853	24.141	24.373	24.551	24.707	24.846	24.982	25.119	25.275	25.467	25.700
8	24.000	24.346	24.599	24.773	24.915	25.041	25.158	25.269	25.372	25.487	25.630	25.800
9	24.500	24.880	25.077	25.198	25.299	25.391	25.476	25.555	25.629	25.702	25.794	25.900
10	25.500	25.550	25.600	25.650	25.700	25.750	25.800	25.850	25.900	25.930	25.960	26.000

NORTHERN FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-27.273	-62.324	-65.137	-61.899	-55.804	-39.811	-59.224	-90.671	-36.164	-17.136	-13.803	-7.241
3	-33.333	-63.344	-62.524	-58.965	-53.794	-52.120	-67.577	-72.842	-43.802	-20.363	-13.833	-3.871
4	-39.844	-65.034	-61.040	-56.561	-56.366	-56.392	-68.075	-65.537	-50.011	-23.672	-13.931	-4.121
5	-23.487	-56.195	-58.948	-56.696	-57.864	-58.945	-66.182	-62.681	-51.916	-28.436	-15.140	-4.371
6	-26.860	-56.257	-59.506	-59.663	-60.105	-61.318	-63.931	-60.879	-52.233	-32.824	-17.207	-4.622
7	-28.111	-58.131	-60.782	-64.870	-62.147	-62.759	-62.522	-59.539	-51.059	-34.601	-19.643	-4.919
8	-29.362	-62.897	-62.879	-68.550	-64.565	-64.352	-62.491	-59.278	-50.724	-35.504	-20.800	-5.292
9	-31.765	-72.009	-70.334	-70.820	-65.868	-64.641	-62.779	-60.730	-50.603	-35.483	-21.692	-5.622
10	-68.727	-95.403	-79.790	-73.404	-66.753	-64.635	-62.371	-61.809	-52.909	-35.001	-21.969	-5.872

WESTERN FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	-19.643	-22.344	-26.116	-30.682	-38.571	-18.387	0.000	-36.000	-25.714	-21.176	-18.667
2	0.000	-49.221	-48.266	-50.804	-53.792	-55.807	-43.512	-35.170	-52.974	-45.356	-42.161	-42.125
3	0.000	-47.233	-45.419	-46.952	-49.299	-46.721	-42.435	-41.898	-49.109	-42.902	-39.547	-39.444
4	0.000	-32.196	-41.190	-43.343	-43.160	-41.737	-39.210	-41.189	-44.002	-42.125	-37.400	-36.203
5	0.000	-43.791	-43.764	-43.139	-40.162	-38.003	-35.684	-37.968	-39.780	-39.529	-35.157	-33.097
6	0.000	-43.379	-41.481	-40.122	-34.988	-32.943	-31.453	-32.827	-34.133	-35.313	-33.547	-31.098
7	0.000	-42.246	-37.443	-35.332	-31.603	-29.185	-27.603	-27.689	-27.940	-28.266	-27.318	-26.155
8	0.000	-43.201	-34.028	-26.609	-24.347	-23.035	-22.755	-22.441	-20.967	-21.083	-21.103	-20.215
9	0.000	-52.274	-28.886	-19.426	-16.794	-15.966	-15.916	-16.346	-15.231	-12.955	-13.438	-13.148
10	0.000	-3.561	-3.686	-3.871	-4.062	-4.304	-4.622	-4.872	-4.937	-2.524	-2.076	-2.496

BOUNDARY FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	-46.916	-65.024	-68.910	-66.464	-63.694	-19.627	-40.837	******* -126.67	-25.879	-12.598	-11.293	11.425
2	-55.281											45.495
3	-53.744											39.194
4	-15.839											35.953
5	-47.165											32.847
6	-44.630											30.801
7	-43.497											25.782
8	-45.604											19.885
9	-89.237											12.898
10	65.166	95.278	79.606	73.214	66.510	64.317	62.121	61.744	55.322	35.449	21.549	8.368

SUM OF BOUNDARY FLOW IS: .01624

*EOR

Table A.3. Results of the REC-model with extraction of 400 m^3/d in cell (5,6)

	1	2	3	4	5	6	7	8	9	10	11	12
1	19.000	19.750	20.500	21.250	22.000	22.750	23.000	23.000	23.500	24.000	24.500	25.000
2	20.000	20.711	21.309	21.871	22.408	22.863	23.136	23.330	23.670	24.114	24.631	25.200
3	21.000	21.519	21.967	22.363	22.711	23.006	23.296	23.553	23.862	24.265	24.761	25.300
4	22.000	22.241	22.524	22.768	22.942	23.073	23.439	23.761	24.074	24.442	24.895	25.400
5	22.500	22.771	22.997	23.149	23.157	22.857	23.596	24.000	24.315	24.647	25.047	25.500
6	23.000	23.261	23.466	23.615	23.685	23.734	24.044	24.337	24.603	24.884	25.212	25.600
7	23.500	23.761	23.960	24.106	24.205	24.315	24.503	24.709	24.913	25.131	25.390	25.700
8	24.000	24.285	24.479	24.599	24.699	24.806	24.942	25.090	25.232	25.387	25.577	25.800
9	24.500	24.850	25.018	25.113	25.195	25.279	25.372	25.466	25.558	25.651	25.767	25.900
10	25.500	25.550	25.600	25.650	25.700	25.750	25.800	25.850	25.900	25.930	25.960	26.000

NORTHERN FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-27.273	-59.174	-58.049	-50.318	-38.826	-15.004	-27.039	-64.251	-21.946	-10.401	-10.765	-7.241
3	-33.333	-60.357	-55.679	-46.907	-34.452	-22.279	-34.099	-47.654	-29.868	-14.305	-11.134	-3.871
4	-39.844	-62.678	-55.205	-45.112	-32.132	-11.172	-31.574	-44.148	-38.696	-19.078	-11.897	-4.121
5	-23.487	-55.198	-56.336	-49.944	-33.916	38.886	-33.695	-50.845	-46.949	-26.567	-14.298	-4.371
6	-26.860	-57.544	-63.218	-69.271	-87.828	*******	-93.185	-71.470	-57.491	-35.281	-17.839	-4.622
7	-28.111	-61.234	-67.903	-79.826	-90.427	*******	-93.314	-77.883	-62.004	-40.201	-21.784	-4.919
8	-29.362	-66.787	-71.401	-84.504	-87.550	-94.512	-88.157	-78.645	-63.969	-42.917	-23.804	-5.292
9	-31.765	-76.209	-79.243	-85.625	-85.080	-87.464	-84.766	-79.790	-64.232	-43.454	-25.150	-5.622
10	-68.727	-99.657	-88.725	-87.212	-84.078	-84.786	-82.462	-80.424	-66.739	-42.894	-25.581	-5.872

WESTERN FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	-19.643	-22.344	-26.116	-30.682	-38.571	-18.387	0.000	-36.000	-25.714	-21.176	-18.667
2	0.000	-45.916	-44.705	-47.102	-50.465	-54.881	-47.693	-40.706	-57.304	-49.354	-45.407	-45.062
3	0.000	-40.178	-37.922	-38.282	-40.166	-42.382	-53.578	-56.022	-59.545	-50.661	-45.981	-45.182
4	0.000	-21.585	-28.924	-28.001	-23.124	-21.258	-71.324	-69.164	-62.487	-54.081	-46.565	-44.187
5	0.000	-29.758	-27.574	-20.771	-1.380	52.405	*******	-85.916	-65.243	-54.734	-46.062	-42.508
6	0.000	-29.964	-26.481	-21.764	-11.132	-8.642	-60.992	-60.876	-54.480	-49.976	-45.065	-41.112
7	0.000	-31.280	-25.810	-22.251	-17.641	-20.488	-37.178	-42.266	-41.453	-39.517	-36.849	-34.799
8	0.000	-35.586	-26.189	-18.247	-17.109	-19.524	-26.590	-29.949	-28.803	-28.482	-27.927	-26.575
9	0.000	-48.169	-24.658	-15.208	-13.619	-14.595	-17.333	-19.560	-18.936	-16.399	-16.962	-16.535
10	0.000	-3.561	-3.686	-3.871	-4.062	-4.304	-4.622	-4.872	-4.937	-2.524	-2.076	-2.496

BOUNDARY FLUXES

	1	2	3	4	5	6	7	8	9	10	11	12
1	-46.916	-61.874	-61.821	-54.884	-46.715	5.180	-8.652	*******	-11.660	-5.863	-8.256	11.425
2	-51.977							-100.251				48.432
3	-46.688											44.932
4	-5.228											43.937
5	-33.132											42.257
6	-31.214											40.815
7	-32.531											34.425
8	-37.989											26.245
9	-85.131											16.285
10	65.166	99.532	88.540	87.021	83.836	84.469	82.212	80.359	69.152	43.342	25.161	8.368

SUM OF BOUNDARY FLOW IS: 400.31023
*EOR

2.2. POL-model

Neuman conditions

By calculating the boundary fluxes from the REC-model the 8 linear equations could be solved (see table A.4).

Table A.4. Boundary fluxes and heads in the POL-model. No extraction, Neuman boundary conditions

Element no. i	Q_i (m^3/d)	h_i (m)	i	Q_i (m^3/d)	h_i (m)
1	-480,50	-3,066	5	126,44	0,746
2	- 11,55	-0,066	6	0	-0,324
3	-107,63	-1,622	7	0	0,108
4	201,44	0,236	8	271,80	1,000

The results show the principal uncertainty in the steady state calculation with Neuman conditions. The absolute value of the hydraulic head has no meaning. Only the differences between the heads can be used to analyze the flow between subregions. Therefore the mixed boundary conditions have been applied.

Mixed boundary conditions

It is assumed that the value of h_5 is known. In our case this value is known from the REC-model calculations (h_5 = 25,604 m)*. The results are given in tables A.5 and A.6 for the case with and without extraction. The main conclusions are:

- Without extraction the differences in head vary from 0,204 to -0,280 m, whereas in the extraction case the differences vary from -0,088 to -0,591 m. The sum of the square of the difference between h_{rec} and h_{pol} ($\sum_i \Delta_i^2$) equals 0,328 and 1,012 respectively. The reasons for these deviations are the linearization of the gradients over relatively large distances and the influence of the averaging of boundary flows, transmissivity and hydraulic heads.

*25,505 m for the extraction case

Table A.5. Heads and fluxes in the POL- and REC-model: mixed boundary conditions, no extractions

El.	h_{rec} (m)	h_{pol} (m)	Δh (m)	1			2			3			4			5			6			7			8		
				Q_{rec}	Q_{pol}	ΔQ % [1)]	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
1	22,006	21,792	0,204				35,17	52,94	33,6	278,845	262,49	6,2							166,40	165,07	0,1						
2	24,465	24,792	-0,227	- 35,17	- 52,94	33,6																24,90	38,80	35,8	21,83	25,68	15,0
3	23,084	23,236	-0,152	-278,85	-262,49	6,2							147,02	134,66	9,2				239,54	235,47	1,7						
4	24,756	25,094	-0,238							-147,02	-134,66	9,2				+12,29	20,32	39,5	- 66,76	- 87,10	23,4						
5	25,604	25,604	0										- 12,29	20,32	39,5				-129,44	-116,01	11,6				12,49	9,88	26,4
6	24,405	24,534	-0,129	-166,40	165,07	0,1				-239,55	-235,47	1,7	+ 66,76	87,10	23,4	129,44	116,01	11,6				211,65	197,43	7,2			
7	24,686	24,966	-0,280				-24,90	-38,80	35,8										-211,65	-197,43	7,2				228,84	236,24	3,1
8	25,613	25,858	-0,245				-21,83	-25,68	15,0							-12,49	9,88	26,4				-228,84	-236,24	3,1			

$\Sigma \Delta h_i^2 = 0,328$

1) ΔQ as a percentage of Q_{pol}

$\sum_{ij} \Delta Q_{ij}^2 = 1884,8$

$\Delta Q_{max} = +20,34 = \Delta Q_{4.6}$

Table A.6. Heads and fluxes in the POL- and REC-model: mixed boundary conditions; extraction: $Q_6 = -400\ m^3/d$

El.	h_{rec} (m)	h_{pol} (m)	Δh (m)	1			2			3			4			5			6			7			8		
				Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
1	21,877	22,112	-0,235				47,69	50,51	5,6	234,97	207,02	13,5							98,85	116,81	15,4						
2	24,383	24,974	-0,591	- 47,69	- 50,51	5,6																-46,77	- 38,03	23,0	22,46	23,59	4,8
3	22,848	23,250	-0,402	-234,97	-207,02	13,5							157,25	131,19	19,9				147,59	145,41	1,5						
4	24,694	25,061	-0,367							-157,25	-131,19	19,9				11,60	19,51	40,5	-122,93	-156,86	21,6						
5	25,550	25,550	0										-11,60	- 19,51	40,5				-172,23	-162,45	6,0				14,46	15,69	7,8
6	23,964	24,052	-0,088	- 98,35	-116,81	15,4				-147,59	-145,41	1,5	122,93	+156,93	21,6	172,23	162,45	6,0				353,98	342,91	3,2			
7	24,425	24,803	-0,378				46,77	+38,03	23,0										-353,98	-342,91	3,2				279,35	304,88	8,4
8	25,552	25,953	-0,401				-22,46	-23,59	4,8							-14,65	-15,69	7,8				-279,35	-304,88	8,4			

$\Sigma \Delta h_i^2 = 1,012$

$\sum_{ij} \Delta Q_{ij}^2 = 3958,7$

$\Delta Q_{max} = +33,93 = \Delta Q_{4.6}$

- The maximum flux difference is 20.34 m^3/d and 33.03 m^3/d for the no extraction respectively extraction case. The relative difference in the fluxes mount to 40%, respectively 41%.
- Probably the most striking feature is the discrepancy between the values of h_{pol} in the no-extraction case and those in the extraction case, the latter being higher than the first in some elements. This means that in this case the POL-model leads to erroneous results. The main reason for this error is the linearization of the gradients over large distances.

Dirichlet boundary conditions

To apply the Dirichlet boundary conditions the hydraulic heads in the boundary subregions 9-18 have been set equal to the weighted average of the boundary cells of the REC-grid in a subregion. The results of this procedure are shown in table A.7 (no extraction) and table A.8 (with extraction). In table A.9 the drawdown due to pumping in the REC-model and in the POL-model is presented. Tables A.10 and A.11 give the boundary flows. These tables show that

- the differences in heads vary from -0,213 to 0,124 m in case of no extraction and from -0,165 to 0,294 m in case of extraction. $\sum_i \Delta h_i$ is 0,092 and 0,192 respectively;
- the maximum flux difference for the no extraction and extraction case is 56,67 m^3/d respectively 39,88 m^3/d. The relative difference mounts to 1047% and 946% respectively for the small flux between element 8 and 5. Without these data the relative differences are 54,7% and 59%;
- the differences in the lowering of the head vary from -0,012 to 0,264 m;
- the differences in boundary flow mount to 56,02 m^3/d for the no extraction case and -48,37 m^3/d for the extraction case. The maximum relative difference is 32,4% and 27,8% respectively.

Interpolated h_{rec} data

For the above mentioned cases the POL-model results have been compared also to the interpolated REC-model data (table A.12). If these results are compared to the data in table A.5-8, it can be seen that

Table A.7. Heads and fluxes in the POL- and REC-model, Dirichlet boundary conditions, no extraction

El.	h_{rec}	h_{pol}	Δh	1			2			3			4			5			6			7			8		
	(m)	(m)	(m)	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
1	22,006	21,953	0,033				35,17	48,09	26,9	278,85	222,18	25,5							166,40	145,80	14,1						
2	24,465	24,678	-0,213	- 35,17	- 48,09	26,9																24,90	19,10	30,3	21,83	19,99	9,2
3	23,084	23,175	-0,091	-278,85	-222,18	25,5							147,02	117,67	24,9				239,548	217,63	10,1						
4	24,756	24,799	-0,043							-147,02	-117,67	24,9				12,29	27,16	54,7	- 66,76	- 65,93	1,3						
5	25,604	25,480	0,124										-12,29	-27,16	54,7				-129,46	-119,83	8,0				12,49	1,09	1047,2
6	24,405	24,375	0,035	-166,40	-145,80	14,1				-239,55	-217,63	10,1	66,76	65,93	1,3	129,44	119,83	8,0				211,65	177,74	19,1			
7	24,686	24,764	0,078				-24,90	-19,10	30,3										-211,65	-177,74	19,1				228,84	197,09	16,1
8	25,613	25,508	0,105				-21,83	-19,99	9,2							-12,49	- 1,00	1047,2				-228,84	-197,09	16,1			

$\sum_i \Delta h_i^2 = 0,092$

$\sum_{ij} \Delta Q_{ij}^2 = 7783,7$

$\Delta Q_{max} = 56,67 = \Delta Q$[illegible]

Table A.8. Heads and fluxes in the POL- and REC-model, Dirichlet boundary conditions. Extraction: $Q_6 = -400\ m^3/d$

El	h_{rec}	h_{pol}	Δh	1			2			3			4			5			6			7			8		
	(m)	(m)	(m)	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
1	21,877	21,836	0,041				47,69	47,86	0,4	234,97	195,09	20,4							98,85	110,41	10,5						
2	24,383	24,548	-0,165	- 47,69	- 47,86	0,4																- 46,77	- 42,21	10,8	22,46	20,23	11,0
3	22,848	22,909	-0,061	-234,97	-195,09	20,4							157,25	125,57	25,2				147,59	138,01	6,9						
4	24,694	24,642	0,052							-157,25	-125,57	25,2				11,60	28,31	59,0	-122,93	-151,15	18,7						
5	25,550	25,352	0,198										-11,60	-28,31	59,0				-172,23	-182,41	5,6				14,65	1,4	946
6	23,964	23,670	0,294	- 98,85	-110,41	10,5				-147,59	-138,01	6,9	122,93	151,15	-18,7	172,23	182,41	5,6				353,98	314,36	12,6			
7	24,425	24,358	0,067				46,77	+42,21	10,8										-353,98	-314,36	12,6				279,35	272,86	2,4
8	25,552	25,388	0,164				-22,46	-20,23	11,0							-14,65	- 1,4	946				-279,35	-272,86				

$\sum_i h_i^2 = 0,192$

$\sum_{ij} \Delta Q_{ij}^2 = 5611,4$

$\Delta Q_{max} = 39,88 = \Delta Q_{31}$

Table A.9. Drawdowns due to the extraction of Q_6 = -400 m^3/s

El.	Δh_{rec}	Δh_{pol}	Δh
1	-0,129	-0,117	-0,012
2	-0,082	-0,130	0,048
3	-0,236	-0,266	0,030
4	-0,062	-0,157	0,095
5	-0,054	-0,128	0,074
6	-0,441	-0,705	0,264
7	-0,261	-0,406	0,145
8	-0,061	-0,120	0,059

$\sum_i \Delta h_i^2 = 0,112$

Table A.10. Boundary flow in the REC- and POL-model, Dirichlet boundary conditions, no extraction

El.	1			2			3			4			5			8		
	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
9	348,01	280,88	23,9															
10				170,75	135,50	26,0												
11				-159,20	-144,38	10,3												
12																- 93,55	- 83,14	12,5
13																-178,25	-134,62	32,4
14													-126,44	-146,21	13,5			
15										-347,19	-291,17	19,2						
16										145,76	134,85	8,1						
17							107,63	113,17	4,9									
18	132,48	135,47	2,2															

$\sum_{ij} \Delta Q_{ij}^2 = 11668$, $\Delta Q_{max} = -56,02 = \Delta Q_{4.15}$

Table A.11. Boundary flow in the REC- and POL-model, Dirichlet boundary conditions, extraction: Q_6 = -400 m^3/d

El.	1			2			3			4			5			8		
	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %	Q_{rec}	Q_{pol}	ΔQ %
9	252,22	226,74	11,2															
10				120,32	104,43	15,2												
11				-185,27	-174,04	6,5												
12																-121,95	-120,09	1,6
13																-222,20	-173,83	27,8
14													-250,52	-209,66	19,5			
15										-307,78	-347,46	11,4						
16										123,07	98,66	24,7						
17							69,58	68,57	1,5									
18	122,12	126,43																

$\sum_{ij} \Delta Q_{ij}^2 = 7231$ $\Delta Q_{max} = -48,37 = \Delta Q_{8.13}$

Table A.12. h_{pol} and h_{rec}; h_{rec} as a interpolated value between cell values

Compared values		Mixed boundary conditions $Q_6 = 0$			Dirichlet boundary conditions $Q_6 = 0$			Mixed boundary conditions $Q_6 = -400\ m^3/d$			Dirichlet boundary conditions $Q_6 = -400\ m^3/d$		
REC-model	POL-model	h_{rec}	h_{pol}	Δh	h_{rec}	h_{pol}	Δh	h_{rec}	h_{pol}	Δh	h_{rec}	h_{pol}	Δh
$h_{2.4}, h_{25}, h_{3.4}, h_{3.5}$	h_1	22,285	21,792	+0,493	22,285	21,953	0,332	22,098	22,112	0,014	22,098	21,836	0,262
$h_{3.10}, h_{3.11}, h_{2.10}, h_{2.11}$	h_2	24,479	24,792	-0,313	24,479	24,678	-0,199	24,380	24,974	-0,594	24,380	24,548	-0,168
$h_{5.3}, h_{5.2}, h_{4.3}, h_{4.2}$	h_3	23,006	23,236	-0,230	23,006	23,175	-0,169	22,787	23,250	-0,463	22,787	22,908	-0,121
$h_{8.3}, h_{8.2}, h_{9.3}, h_{9.2}$	h_4	24,755	25,094	-0,339	24,755	24,799	-0,044	24,680	25,061	-0,381	24,680	24,642	+0,038
$h_{9.6}, h_{9.7}, h_{10.6}, h_{10.7}$	h_5	25,604	25,604	0	25,604	25,480	0,124	25,550	25,550	0	25,550	25,352	+0,198
$h_{6.6}, h_{6.5}, h_{7.6}, h_{7.5}$	h_6	24,416	24,534	-0,118	24,416	24,375	0,041	23,850	24,052	-0,202	23,850	23,670	0,180
$h_{5.9}, h_{5.8}, h_{6.9}, h_{6.8}$	h_7	24,658	24,966	-0,308	24,658	24,764	-0,106	24,337	24,803	-0,466	24,337	24,358	-0,021
$h_{8.10}, h_{8.11}, h_{9.10}, h_{9.11}$	h_8	25,596	25,858	-0,262	25,596	25,508	0,088	25,520	25,953	-0,433	25,520	25,388	0,132
		$\sum_i \Delta h_i^2 = 0,686$			$\sum_i \Delta h_i^2 = 0,205$			$\sum_i \Delta_i h_i^2 = 1,158$			$\sum_i \Delta_i h_i^2 = 0,202$		

the averaged value show a slightly better correspondence with the POL-model results than the interpolated values.

2.3. POL-model results as input for the REC-model

For several hydrological problems detailed data are needed only locally. A regional model with low spatial discretization can be used to calculate boundary conditions for a detailed local model. In this special case this is done by using the POL-model hydraulic heads in subregions 1, 2, 3, 4 and 5 calculated with Dirichlet boundary conditions as new boundary conditions for the REC-model II. The results can be compared with the results of the REC-model I applied to the whole region. As can be seen from table A.13 the differences between the two model calculations mount to 0.13 m close to the boundary, whereas the differences close to the extraction point are less than 0.09 m.

Table A.13. Differences between groundwater levels (in m) in REC-model I and II with different boundaries. In REC-model I the boundaries coincide with region boundaries. In REC-model II the boundaries are based on the centres of gravity of subregions 1, 2, 3, 4, 5 and 8

columns / rows	2	3	4	5	6	7	8	9	10	11
	-	0,094	-0,016	-0,268	-0,159	-0,217	0,001	0,037	-0,136	0,081
	0,047	-0,064	-0,072	-0,133	-0,020	-0,114	-0,044	-0,015	-0,016	0,103
	-0,106	-0,063	-0,067	-0,086	-0,084	-0,074	-0,046	-0,027	-0,013	0,037
	0,029	-0,003	-0,050	-0,061	-0,061	-0,052	-0,037	-0,029	-0,033	-0,083
	0,088	-0,010	-0,043	-0,052	-0,047	-0,035	-0,024	-0,019	-0,017	0,008
	-0,063	-0,053	-0,058	-0,056	-0,041	-0,020	-0,002	-0,004	-0,020	-0,064
	-0,113	-0,078	-0,080	-0,077	-0,041	0,001	0,039	0,024	-0,004	0,037
	-0,040	-0,070	-0,004	-0,133	-0,042	0,019	0,132	0,060	-0,055	-0,111
	-	-	-	-0,010	-0,020	-0,020	-0,010	-	-	-

3. Conclusions

The application of the POL-model can lead to physically erroneous results. This danger exists especially if Neuman boundary conditions or mixed boundary conditions are used. Using Dirichlet boundary conditions the errors in the calculated heads in a special case are as much as 0,30 m and the maximum error in the drawdown is 0,264 m at a drawdown of 0,705 m. If the POL-model is used to calculate boundary conditions for a more detailed REC-model errors in this REC-model are less than 0,13 m. If these errors are tolerable depends on the objective of the specific study.

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PART II. EXPLANATION TO THE MAP OF THE SUBREGIONS IN THE SOUTHERN PEEL AREA

II. Explanation to the map of the subregions in the Southern Peel area

For the subdivision of the Southern Peel area into subregions the following factors should be taken into account.

a. hydrogeological schematization: The 'Peelrand' fault divides the area into the horst area east of the fault and the slenk area west of the fault. In the horst area only the Veghel-Sterksel semi-confined aquifer is present, whereas in the slenk area the Kedichem to Kiezeloolite formation form one or two semi-confined aquifers
b. flow characteristics in the unsaturated and saturated zone like capillary rise, deep percolation and drainage to the ditches. In the linear programming model these fluxes depend on a mean groundwater table over the whole subregion. Thus a relative homogeneous relation over a subregion is required. To assume this requirement the classification into groundwater table depth classes can be used in combination with the classification into soil physical units. (see table 1 and fig. 1).

Table 1. Classification of the depth of the groundwater table

Gc	I	II	III	IV	V	VI	VII
MHW	-	-	<40	>40	<40	40-80	>80
MLW	<50	50-80	80-120	80-120	>120	>120	>120

Gc = groundwater class

MHW = depth of the mean highest groundwater table (cm)

MLW = depth of the mean lowest groundwater table (cm)

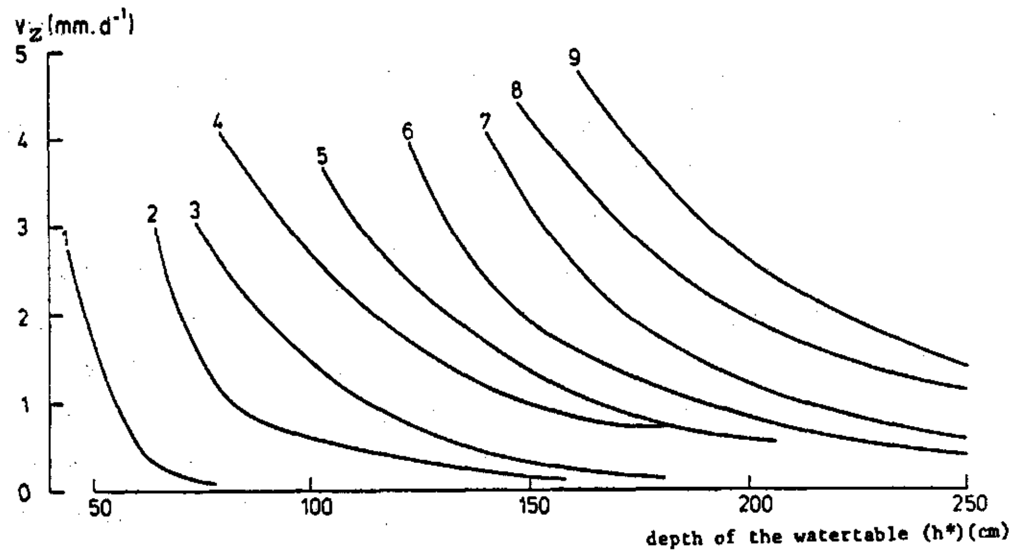

Fig. 1. Mean v_z -h* relations of the soil physical units 1 - 9 (BLOEMEN, 1982)

c. nature areas: nature areas should be regarded as seperate areas.
d. possibility of water import for subirrigation. This factor depends on the man-made drainage basins and the possibility to allocate water to a certain basin.
e. maximum number of subregions. As a result of the large number of equations in the linear programming model the number of subregions has to be not more than about thirty.

In general boundaries based on a, b and c will not coincide with boundaries based on d. For the present subdivision priority is given to the factors a-c. Factors depending on the channel network e. g. the supply capacity of region i during time t, Smax(i, t) will be calculated by using a weighted average of the values of all SMAX(i, k, t) with k indicating the drainage basins within one subregion.

In the preparation of the map the following procedure has been applied. Groundwater table classes I, II, III and V (IV does not occur) and classes VI and VII have been combined. Soil maps of the Netherlands (1: 50 000) sheets 51E, 52W, 57E and 58W have been used. Introducing the 'Peelrand' fault as a boundary and the three most important nature areas ('de Berken', 'Grote Peel' and 'Mariapeel/Deurnschepeel') as separate areas 31 subregions have been constructed.

In some cases a relative large area with a different groundwater table depth class had to be tolerated to fulfil the requirement of the maximum number of subregions (For example subregion 11 and 12, see table 2). On the map it is shown which groundwater table depth class has the most frequent occurrence within one subregion.

Table 2 also shows soil types occurring in each subregion and the most frequent occurring soil type. Generally speaking the relatively low areas (Gc I - V) coincide with peat and peaty soils, low 'enk' earth soils, high black 'enk' earth soils, 'beek' earth soils, 'goor' earth soils and 'veld' and 'laar' podzol soils. The relatively high areas (Gc VI and VII) have podzol soils, black 'enk' earth soils and vague soils. BLOEMEN(1982) combined some soil types and distinguished 9 different soil physical units based on the maximum capillary rise that can reach the surface level (v_z) dependent on the groundwater table depth (h*) (see fig. 1).[1] Because the maximum number of

[1] The v_z seems to be overestimated for a deep ground water table in soils with good capillary characteristics (6-9). SWATRE-calculations using $K(\psi)$ relations based on the Bloemen calculations also overestimate the capillary rise (Wit, personal communication).

Table 2. Characteristics of the subregions in the Southern Peel

Subregion number 1)	Gc combination	Most frequent Gc	Occurring soil types 2)	Most frequent soil type 3)	Most frequent soil physical unit
1	> V	VII	zEZ21, cHn21	zEZ21	7
2	≤ V 4)	V	pZg21, EZg21, vWz, pZN21, pZN23, Hn21, zEZ21, Hn/pZn23	Hn/pZn23	8
3	≤ V	II	zVz, pZg23, zEZ21, vWz	zVz	2
4	> V	VII	Hn21, Hd21, Zd21, Zn21, zEZ21, pZg23	Hn21	5
5	> V	VII	Hn21, Zd21, zEZ21	Hn21	5
6	≤ V	III	Hn21, pZn21, EZg21	Hn21(EZg21)	5
7	> V 5)	VII	Hn21, zEZ21, Zd21, Hd21, EZg21	zEZ21	5
8	≤ V	III	zVz, vZ, pZg23, EZg21, EZg23	pZg23	2
9	> V 5)	VII	Hn21, Hn23, Hd21, zEZ21, Zd21	zEZ21	8
10 N	III-VI	III	pZg23, Hn21, pZn21	Hn21 (pZg23)	5-7
11	≤ V 4)	III	EZg21, zEZ21, pZn21, pZg23	pZg23	5
12	> V 5)	VI	Hn21, Zd21	Hn21	5
13	≤ V	III	EZg23w, zVz/pZg23, pZg21, pZg23, EZg21, cHn21	pZg23	8
14	> V 5)	VII	zEZ21, cHn21, Hn21, zVz/pZn21	Hn21	5
15	≤ V 4)	V	vWp, zWp, Hn21, Vz/vWz	Hn21	5
16 N	I, I/II, III, V, VI	I/II	Vs/Vp, Vp, zVp, Vs, Hn21, vWp, Hn23, vWp/Hn23, Hn/Hd21	Vs/Vp	3
17	> V	VII	zEZ21, Hn21, Zd21	Hn21 (zEZ21)	7
18	< V 4)	V	Hn23, zWp, zWz, Hn/pZn23, zVs/zVz, pZn23, pZg23, vWp	Hn23	8
19	> V	VII	zEZ23	zEZ23	9
20	> V	VII	zEZ21, Zd21, Hn21, vWp	zEZ21	9
21	≤ V	V	pZn21, pZn23, zEZ21, EZg21w, Hn/pZn23, vWp, zWp, zVz, zWz, Vs, Hn21, Vp/vWp, zWp/Hn21	Hn21 (Hn/pZn23)	8
22	> V	VI	Zd21, Hn21	Hn21	5
23	≤ V 4)	V	Hn21, zVz/zWz, vWp, Vp, zVp, zWp, vWz/pZn21, EZg21, zEZ21	Hn21 (vWz/pZn21)	5
24	> V 5)	VI	Hn21, zWp/Hn21, zWp, Hn23, vWp/Hn23	Hn21	5
25	> V 5)	VI	Hn21, vWp, gHn30, zEZ21	Hn21	5
26	≤ V	V	Hn21, gpZN30, zVp, vWp, zWp, pZg23	Hn21	5
27 N	≤ V 4)	III	Vs, vWp, Vp, Vp/vWp, Hn21, aVp, zVs	Vs	3
28	≤ V	V	aVp, Vs, Vp, vWp, zWp, zWp/Hn21, zVp/zWp, Hn/Zd21, vWz, Hn23, pZn21/23, Hn21	Hn21 (zVp)	5
29	> V	VII	Hn21, cHn21, gHN30, zEZ21/23, Zd21	Hn21 (zEZ21)	7
30	> V	VI	Hn21, Hn23, Zd21, pZn23	Hn21	5
31	≤ V 4)	III	Vp, zVp, zWz, vWp, Hn21, Hn23, pZn23, cHn23, Hn/pZn23	Hn21 (zVp)	5

1) N denotes nature area
2) see for the explanation the soil maps
3) between brackets the second important soil type is given
4) including relative large parts with Gc > V
5) including relative large parts with Gc ≤ V

subregions had been reached already no extra subdivisions based on the soil physical units could be made. The map shows only the soil physical unit with the most frequent occurrence.

Literature: BLOEMEN, G.W., 1982. Bodemfysische interpretatie van de bodemkundige gegevens van het Zuidelijk Peel-gebied. ICW-nota 1374, 26 p.

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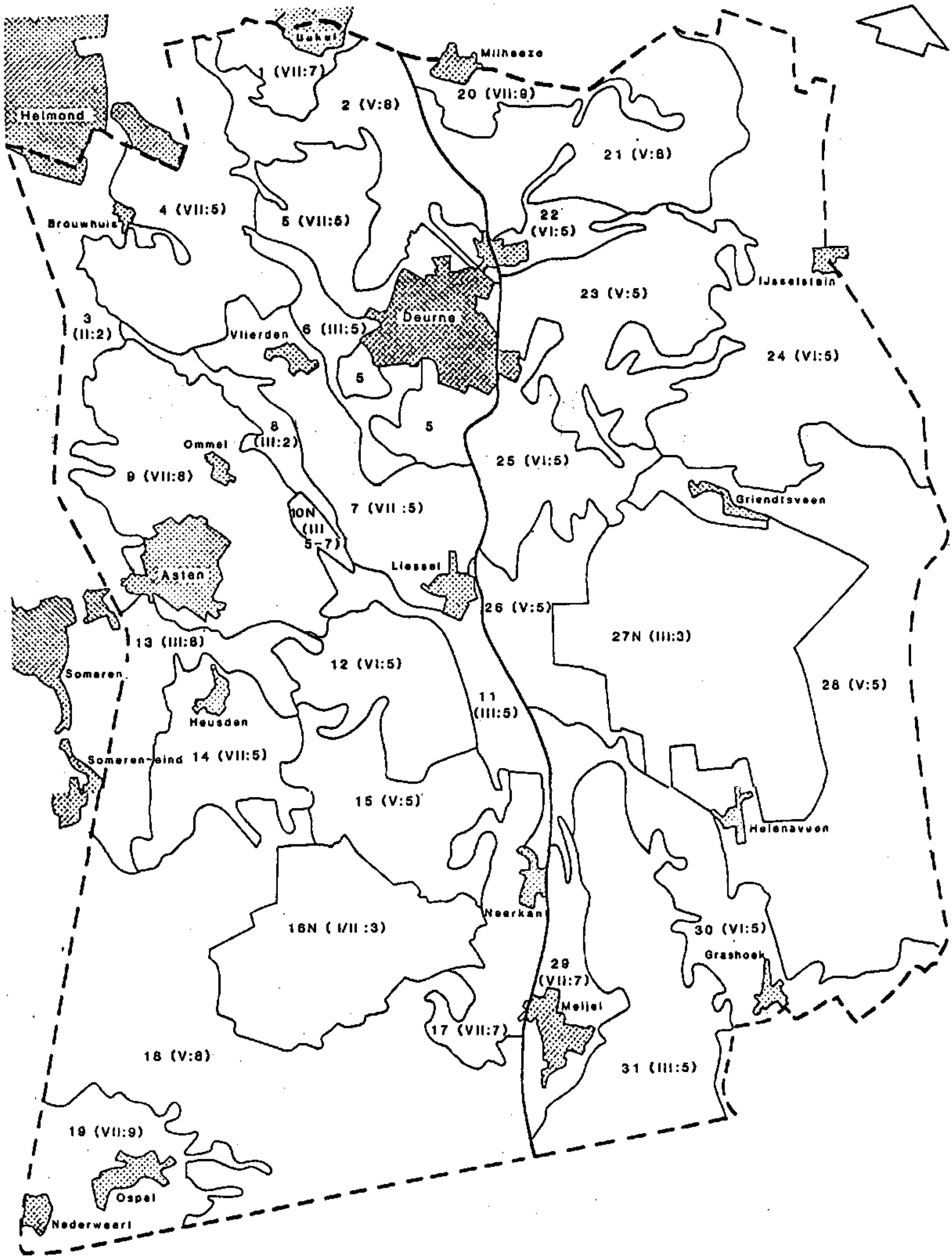
ANNEX A

SUBREGIONS IN THE SOUTHERN PEEL AREA

based on hydrological and soil physical characteristics



boundary of the area

"peelrand" fault

built-up area

27N (III :3)

27	number of the subregion
N	nature area
III	most frequent groundwater depth class
3	most frequent soil physical unit