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OUTLINE OF A PROCEDURE TO GENERATE THE TARGET STATE

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1. Introduction.

In the Scenario Generating System a target scenario for the development of a region is selected. In the selection a linear model is used. This model describes the hydrological system, the agricultural system and the interactions between these systems. The target state is considered to be a stationary state. It is selected by maximizing income in agriculture taking into account constraints with respect to environment and public water supply. When a target state has been selected it has to be shown that it can be reached from the current situation in the region.

In this paper an outline is given of a procedure to select the targetstate and to verify its reachability. The procedure is not finished yet
and can probably be improved. The linear model is described very briefly in section 2. In section 3 an approach to select the target state is outlined. The reachability of this target state is the subject of section 4.

2. The linear model.

The linear model describes the hydrological system, the agricultural system and the interactions between these systems. Both the hydrological system and the agricultural system depend on the weather conditions which are stochastic. So the linear model is stochastic, but it has a deterministic component. The restrictions in the model can be split up into stochastic (i.e. weather dependent) restrictions and deterministic restrictions. The objective function is stochastic. For year t the model can be stated as:

(1)

$$\max_{x(t),y(t)} \quad c1.x(t) + c2.y(t) \qquad \text{income in agriculture}$$

$$A1.\{x(t)-x(t-1)\} - A2.y(t-1) \le b1 \qquad \text{mutations in } x$$

$$B1.x(t) + B2.y(t) \le b2 \qquad \text{deterministic constraints}$$

$$D1.x(t) + D2.y(t) \le b3 \qquad \text{weather dependent constraints}$$

$$x(t),y(t) \ge 0$$

with c1,c2,D1,D2 and b3 — vectors or matrices with coefficients that depend on the weather. This means that when the weather is known then the coefficients are known too.

A1,A2,B1,B2,b1 and b2 — vectors or matrices with coefficients that do not depend on the weather

- x — the state vector with the allocation or the intensity of the technologies, the capacity of sprinkling etc.. The vector x is fixed during the year.
- y — the vector with variables that depend on the weather (i.e. that can be changed during the year). Examples of variables are subtechnologies, irrigation and the quantity applied for the different types of manure.

In (1) the general form of the model for one year is presented in the case that the weather is known. In reality the weather is not known. It is assumed that the climate can be described by a finite number of weather years each having its own probability of occurence. In this case the weather dependent constraints can be replaced by chance constraints. These chance constraints have to be met with a specified probability p. Or in other words the chance constraints have to be met in 100.p percent of the years. The objective function also depends on the weather. This can be handled by introducing the minimum earned income YDES.

If an income less then p is allowed with a probability (1-p), then the objective function of (1) can be replaced by (2) and (3)

$$
(2) \qquad \max_{x(t),y(t)} \quad YDES
$$

and the chance constraint (Pr(} indicates the probability that { })

$$
(3) \qquad Pr\{c1.x(t) + c2.y(t) - YDES \geq 0\} \geq p
$$

Inclusion of YDES in y and of c1,c2 and -1 in D1 and D2 and introduction of chance constraints leads to the following reformulation of (1)

$$
(4) \qquad \max_{x(t),y(t)} \quad YDES
$$

$$
\begin{array}{rll}
A1.\{x(t)-x(t-1)\} - A2.y(t-1) & \leq b1 & \\
B1.x(t) + B2.y(t) & \leq b2 & \\
Pr\{D1.x(t) + D2.y(t) \leq b3\} & \geq p & \\
x(t),y(t) & \geq 0 & \text{with YDES} \\
 & & \text{included in } y(t)
\end{array}
$$

It is also possible to introduce subsets of chance constraints with different probabilities. Introduction of these subsets into the model and into the proposed procedure is very simple. However the model stated in (4) is used for notational reasons.

3. The selection of the target state.

The target state selected in the Scenario Generating System has to be such that:

- restrictions stemming from the environment and the public water supply are met with a specified probability p.
- income in agriculture is as high as possible and equals or exceeds the income YDES with probability p
- the state can be reached starting from the current state within T years

The reachability of the target state is the subject of section 4. In the present section the selection of the target state is treated. The target state is considered to be a stationary state. In this concept the intensities of the technologies the capacities of sprinkling etc remain constant for an infinite number of years (i.e. the vector x) and there are no investments other then replacement investments. The vector y depends on the weather and changes from year to year.

Because the target state is a stationary state the constraints in (4) concerning the year to year mutations can be left out and the problem can be stated as

$$
(5)\quad \max_{x,y} \; YDES
$$

$$
\begin{aligned}
& B1.x + B2.y \le b2 \\
& Pr\{D1.x + D2.y \le b3\} \ge p \\
& x,y \ge 0 \quad \text{with YDES included in } y
\end{aligned}
$$

This problem can be solved using Deterministic Equivalents for the chance constraints. One of the disadvantages of the linear model is that it results in specialisation. It is the allocation of land to a minimum number of technologies and a specialisation with respect to the technologies that do not use land. This is caused by the fact that minor income differences between technologies have the same influence with respect to allocation as very large differences. This leads to a large difference between current state and target state.

In order to avoid this problem as much as possible the following procedure is proposed:

STEP 1 select a desired minimum income YDES* by solving (5) and reduce this income by a small percentage rho.

STEP 2 select the allocations and intensities of technologies which guarantee with probability p at least the the reduced optimum income ((1-rho).YDES*), which meets the other conditions stated in (5) and which minimizes the changes (especially investments) with respect to the current state. In doing this the reachability of the target state is enlarged and the specialisation is less strong then in the optimum solution generated in STEP 1.

The second step can be considered as a kind of a sensitivity analysis of the optimum solution in STEP 1. In STEP 2 the income variable YDES is replaced by by the constant (1-rho).YDES* , with YDES* the income generated in STEP 1. Let f[0,T] represent the changeswith respect to the current state. Find the limitations for the changes in a period of T years by extrapolating the mutation constraints

$$A1.\{x(t)-x(t-1)\} - A2.y(t-1) <= b1$$

The cumulative savings are not incorporated in the extrapolation because at this moment only technical constraints are considered. The cumulative savings are treated in section 4. The extrapolation results in the following set of constraints

$$(6) \qquad AT.\{x-x(0)\} <= bT$$

The problem that has to be solved in STEP 2 can be stated as

$$(7) \qquad \min_{x,y} \quad f[0,T]$$

$$\begin{array}{rl} AT.\{x-x(0)\} & <= bT \\ B1.x + B2.y & <= b2 \\ Pr\{D1.x + D2.y <= b3\} & >= p \\ x,y & >= 0 \end{array}$$

with the variable YDES replaced by the constant (1-rho).YDES*

When deterministic equivalents are used the problem becomes deterministic and can be solved.

4. The reachability of the target state.

When the target state is selected it has to be shown that it can be reached from the current state within T years. In this section an outline of an approach based on Markov chains is described. This approach can be divided into

1. the selection of changes in the state vector x. It is assumed that changes take place at the end of each period of for instance 5 years. The state vector is independent of the weather. The changes are limited by the income (via cumulative savings), by the constraints on the mutations and by the hydrological model.
2. the analysis of the behaviour of the system within the period of 5 years. This means the selection of the vector y that depends on the weather conditions. The vector y is choosen in such a way that, for a known state vector x, the income in agriculture is maximized conditional to the weather conditions. This is stated in (8)

$$
(8) \quad \max_{y(t)} \; c2.y(t)
$$
$$
\begin{aligned}
B2.y(t) &<= b2 - B1.x(t)\\
D2.y(t) &<= b3 - D1.x(t)\\
y(t) &>= 0
\end{aligned}
$$

with c2, D1, D2 and b3 weather dependent coefficients. It can be derived from (8) that the value of y does not depend on the years y(t-1) etc. it only depends on x and on the weather. So for a known state vector x each weather year corresponds with a unique solution y, income (c1.x + c2.y) and savings (income minus normative consumption). The sum of the savings during a period limits the possibility to invest at the end of the period.

For both problems a procedure has to be formulated. It has been shown that the analysis of the behaviour within the 5 years period can be reduced to seperate analysis for each of the five years. These yearshave the same mutually independent probability distribution for the the weather conditions. Because the savings in one year are directly linked to the weather year it is possible to generate the probability function for the cumulative savings in a period as the convolution over five years of the probability distribution of the weather years. The distribution of the cumulatove savings depends on the state vector x.

For the changes in the state vector x a decision rule has to be formulated. A possibility for this decision rule is minimisation of the differences between the target state and the actual state, taking into account the limitations posed by the hydrological system, the limitations with respect to the mutations and the cumulative savings.

Let f[t,T] represent the difference between the actual state and the target state (x(T)). Then the decision rule can be formulated as (the cumulative savings of period t-1 are included in y(t-1))

$$(9) \quad \min_{x(t)} \; f[t,T]$$

$$\begin{aligned} A1.(x(t)-x(t-1)) - A2.y(t-1) &\le b1 \\ B1.x(t) + B2.y(t) &\le b2 \\ Pr(D1.x(t) + D2.y(t) \le b3) &\ge p \\ x(t),y(t) &\ge 0 \end{aligned}$$

For the desired income a rather low value can be choosen, for instance the normative consumption. The problem stated in (9) can be solved when deterministic equivalents are used. Each value of the cumulative savings leads to a unique solution of the problem. This means that x(t+1) can be linked directly to the cumulative savings in period t (for x(t) known). Because the probability distribution of the cumulative savings (conditional to x(t)) is given it is easy to generate the probability distribution of x(t+1) (conditional to x(t)).

The procedure described corresponds to an analysis based on Markov chains. that is x(t+1) depends on x(t) and not on x(t-1),x(t-2),---. Moreover the probability that x(t+1)=x1 given that x(t)=x0 is equal to the probability that x(t+n)=x1 when x(t+n-1)=x0. This because the probability distribution of the mutations in a period depends only on the statevector at the begin of the period.

Whether this procedure can be applied depends on the number of potential future states. If this number is rather small then the procedure can be applied. The Markov chain contains two absorbing states i.e. the target state and the 'infeasible state'. A reduction in the number of states can be obtained by classifying the cumulative savings into a small number of classes. This is a subject for further investigations.